Status and Prospects of Nonleptonic B Decays

K. Keri Vos

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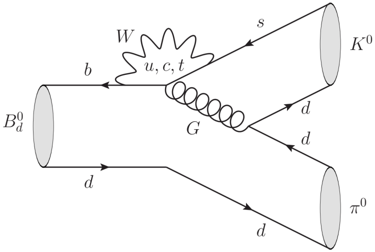
The challenge of nonleptonic B decays

- Nonleptonic decays are important probes of CP violation
  - Direct CP violation due to different strong and weak phases
  - Mixing-induced CP violation in neutral decays probe mixing phase $\phi_{d,s}$
  - Sensitivity to NP in loops (penguins)
- CP violation in the SM is too small and peculiar!
  - CKM CP violating effects only from flavour changing currents
  - Flavour diagonal CP violation tiny in SM (EDMs)
  - Large CP asymmetries with processes with tiny BRs and vice versa



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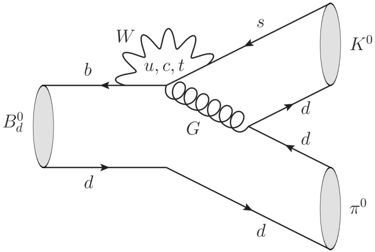
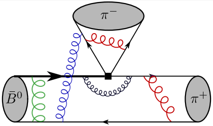


Challenge: Calculation of Hadronic matrix elements

How to handle nonleptonic B decays?



QCD Factorization Beneke, Buchalla, Neubert, Sachrajda

- Disentangle perturbative (calculable) and non-perturbative dynamics using HQE
- Systematic expansion in α_s and $1/m_b$ (studied up to α_s^2) Bell, Beneke, Huber, Li

$$\langle \pi^+\pi^- | \mathcal{Q}_i | B \rangle = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^-} + T_i^{II} \otimes \Phi_{\pi^-} \otimes \Phi_{\pi^+} \otimes \Phi_B$$

- Non-perturbative form factors and LCDAs
  - from data, lattice or Light-Cone Sum Rules
- No systematic framework to compute power corrections (yet?)
- Strong phases suffer from large uncertainties
- Theoretical challenge: reliable computations of observables
- QEDxQCD factorization also explored! Beneke, Boer, Toelstede, KKV [2020]

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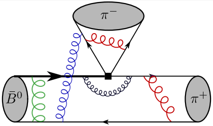

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Flavour symmetries (Isospin or $SU(3)$)

- Many studies e.g. Fleischer, Jaarsma, KKV, Malami [2017,2018]
- Global $SU(3)$ fit to $B \to PP$ decays Huber, Tetlalmatzi-Xolocotzi [2111.06418]

Light-cone sumrules

- Work in progress by Jung, Melic, Khodjamirian

SM predictions for non-leptonic B decays

$$A_{M_1 M_2} \equiv i \frac{G_F}{\sqrt{2}} m_B^2 F_0^{BM_1} f_{M_2}$$

Amplitude parametrization a la QCDF [Beneke, Neubert [2003]]

$$\begin{aligned}
\mathcal{A}_{B^- \to \pi^- \bar{K}^0} &= A_{\pi K} \hat{\alpha}_4^p \, , \\
\sqrt{2}\mathcal{A}_{B^- \to \pi^0 K^-} &= A_{\pi K} \left[\delta_{pu} \alpha_1 + \hat{\alpha}_4^p\right] + A_{K\pi} \left[\delta_{pu} \alpha_2 + \delta_{pc} \frac{3}{2} \alpha_{3,\mathrm{EW}}^c\right] \, , \\
\mathcal{A}_{\bar{B}^0 \to \pi^+ K^-} &= A_{\pi K} \left[\delta_{pu} \alpha_1 + \hat{\alpha}_4^p\right] \, , \\
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\end{aligned}$$

- α_1 and α_2 color-allowed and color-suppressed tree coefficients
- α_4 and $\alpha_{3,\mathrm{EW}}$ penguin and electromagnetic penguin coefficients
- contain all perturbative effects up to NNLO (α_s^2) e.g. [Bell, Beneke, Huber, Li]

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- contain all perturbative effects up to NNLO (α_s^2) e.g. [Bell, Beneke, Huber, Li]
- QED can be included! Beneke, Boer, Toelstede, KKV, JHEP 11 (2020) 081 [2008.10615]

Different QED effects

$$\mathcal{A}(M_1 M_2) \equiv i \frac{G_F}{\sqrt{2}} m_B^2 \mathcal{F}_{Q_2}^{BM_1}(0) \mathcal{F}_{M_2}$$

$$\langle M_1 M_2 | Q_i | B \rangle = \mathcal{A}(M_1 M_2) \alpha_i (M_1 M_2) = A_{M_1 M_2} \left(\alpha_i^{\rm QCD}(M_1 M_2) + \delta\alpha_i(M_1 M_2) \right)$$

- Electroweak scale to m_B: QED corrections to the Wilson coefficients
- m_B to μ_c: QED corrections to the hard-scattering kernels, form factors and decay constants
- below $\Lambda_{\rm QCD}$: Ultrasoft QED effects (for the rate!)

$$\delta\alpha_i(M_1 M_2) \equiv \delta\alpha_i^{\rm WC}(M_1 M_2) + \delta\alpha_i^{\rm K}(M_1 M_2) + \delta\alpha_i^{\rm F,V}(M_1 M_2) + \delta\alpha_i^{\rm F,sp}(M_1 M_2)\,.$$

$\rightarrow$ $\delta\alpha_i^{\rm WC} = \mathcal{O}(10^{-3})$ [Huber, Lunghi, Misiak, Wyler [2006]]

$\rightarrow$ $\delta\alpha_i^{\rm K} = \mathcal{O}(10^{-3})$

$\rightarrow$ $\delta\alpha_i^{\rm F,V} =??$ [Beneke, Boer, Toelstede, KKV [2021]]

$\rightarrow$ $\delta\alpha_i^{\rm F,sp} =??$ but $\mathcal{O}(\alpha_{\rm em}\alpha_s)$

Ultrasoft Contribution

- Ultrasoft effects dress braching ratio
- Key point: scale dependence cancels!!

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2 + Q_{M_1}^2\left[1+\ln\frac{m_{M_1}^2}{m_{B_q}^2}\right] + Q_{M_2}^2\left[1+\ln\frac{m_{M_2}^2}{m_B^2}\right]\right)}$$

- Recover the standard QED factor
- ΔE is the window of the πK invariant mass around m_B
- Theory requires $\Delta E \ll \Lambda_{\rm QCD} = 60\,\text{MeV}$

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- Experimentally usoft effects included using PHOTOS
- Challenging to compare theory with experiment! Work in progress!

Ratios and isospin sumrules

Beneke, Boer, Toelstede, KKV, JHEP 11 (2020) 081 [2008.10615]

- QED gives sub-percent corrections to Branching ratios

Ratios and isospin sumrules

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- Beneficial to consider ratios in which QCD is suppressed

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\, \delta_{\mathrm{E}} + \delta_U$$

- new structure dependent QED corrections enter linearly, QCD only quadratically

$$\delta_E = (-1.12 + 0.16i) \cdot 10^{-3}$$

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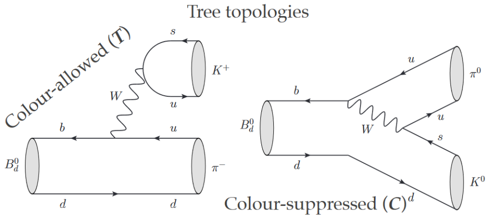
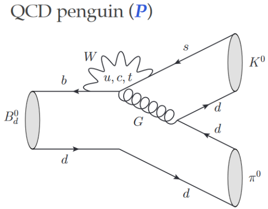
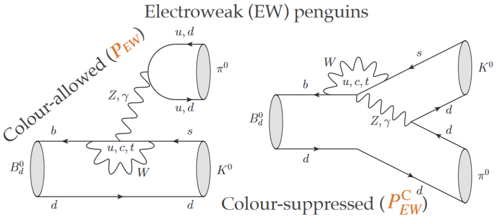
- Combined QED effect larger than QCD uncertainty!

$B \to \pi K$ puzzle

Why $B \to \pi K$ decays?







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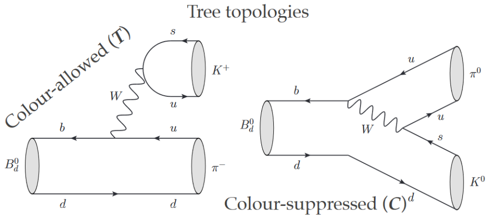

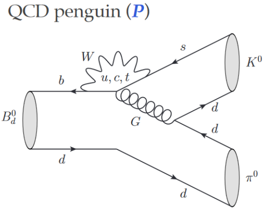

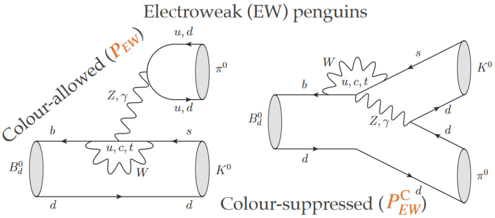

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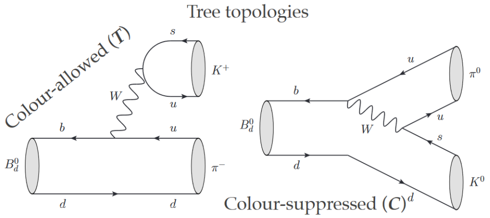

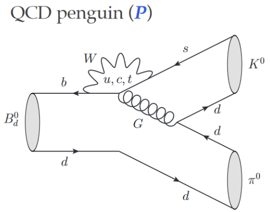

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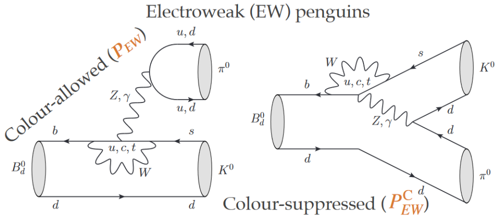
  - Search for tiny deviations of SM predictions



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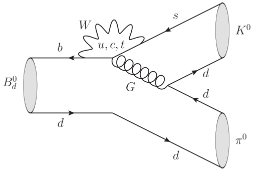
(Longstanding) Puzzling patterns in $B \to \pi K$ data

- First Example:

$$\delta(\pi K) \equiv A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

- Recent LHCb measurement for $A_{\rm CP}(K^- \pi^0)$
  LHCb Collaboration, PRL 126, 091802 [2021]
- Confirms and enhances the observed difference
  - $\delta(\pi K)^{\rm exp} = (11.5 \pm 1.4)\%$
  - 8σ from 0



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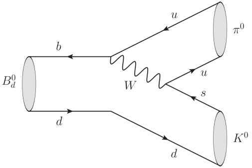

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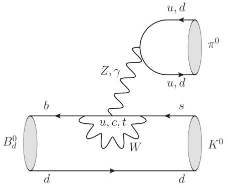
- Hint for NP in the EWP sector?



Light-cone sumrules predictions

Work in progress Jung, Melic, Khodjamirian (see MITP workshop 2019)

Preliminary!

Decay mode	BR-exp (in 10^{-6})	$A_{CP} = -C_{CP}$	BR-th	A_{CP}-th
		$\Delta S = -1$		
$B^- \to \pi^0 K^-$	12.7 ± 0.6	0.040 ± 0.021	13.74	0.050
$B^- \to \pi^- \bar{K}^0$	23.3 ± 0.8	-0.017 ± 0.016	24.56	-0.012
$\bar{B}^0 \to \pi^+ K^-$	20.0 ± 0.6	-0.082 ± 0.006	20.10	0.057
$\bar{B}^0 \to \pi^0 \bar{K}^0$	10.1 ± 0.5	-0.01 ± 0.10	8.87	-0.021

- LCSR calculations (+some QCDF input)
- More reliable than for $B \to \pi\pi$
- Different sign for $B \to K^+\pi^-$ (as in QCDF)

Isospin sumrule

e.g. Gronau [2005]; Gronau, Rosner [2006]

$$\Delta(\pi K) \equiv A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\rm CP}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\rm CP}(\pi^0 \bar{K}^0) \equiv \Delta(\pi K)^{\rm QCD} + \delta\Delta(\pi K)$$

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- Isospin sumrule also robust against QED effects!
- Updates of modes with neutral pions necessary $\rightarrow$ Belle II
- Or can be used to predict the direct CP in $B \rightarrow \pi^0 K^0$
- Mixing-induced CP asymmetry in $B \rightarrow \pi^0 K^0$ provides additional test Fleischer, Jaarsma, Malami, KKV [2016,2018]

QCDF: Quo Vadis?

Next Islay workshop?

- Power corrections?
- New study with updated weak annihilation parametrization
- SU(3) + QCDF analysis
- Light-cone sum rules for suppressed effects
- charm-mass effects [Beneke, Finauri, KKV - in progress]

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- Improving PHOTOS ?!

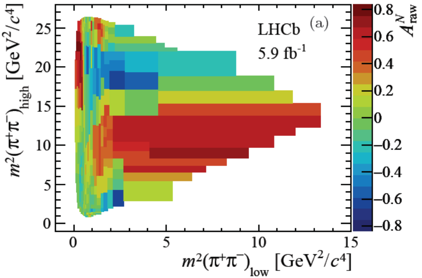
QCDF for three-body decays

Focus on $B \to \pi\pi\pi$ but can be adapted for $B \to hhh$ decays

Motivation

Multibody decays form a large part of the non-leptonic decays

- Rich structure of CP violation
- May contain non-perturbative strong not suppressed by Λ/m_b



Historic isobar model

- Sum of Breit-Wigner shapes and non-resonant background

$$\frac{1}{q^2 - m_R^2 + i\Gamma_R m_R}$$

Requires a QCD-based factorization approach [Kraenkl, Mannel, Virto '15]

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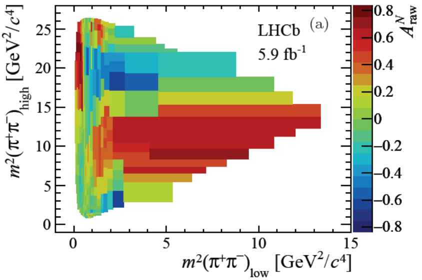


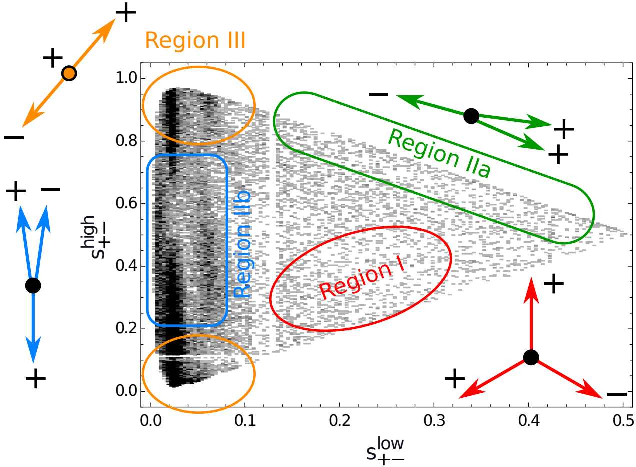
Impact beyond nonleptonics:

Vector mesons (ρ, K^*) are not stable particles

- Form factor calculations are done in the narrow-width limit
- Naively finite-width effect scale as: $\mathcal{W} \sim 1 + \text{coeff}.\ \Gamma/M$ where $\Gamma/M \sim 20\%(\rho), 6\%(K^*)$

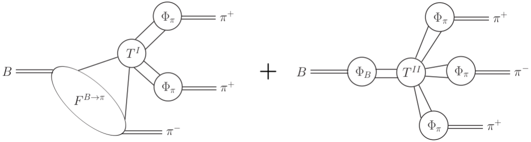
Dalitz distribution - Kinematics

- $B^+ \to \pi^+(k_1)\pi^-(k_2)\pi^+(k_3)$ Symmetric Dalitz plot
- Kinematic variables $s_{+-}^{\rm low} = \frac{(k_1+k_2)^2}{m_B^2}$ and $s_{+-}^{\rm high} = \frac{(k_2+k_3)^2}{m_B^2}$



Factorization in three-body decays - Central Region

Kraenkl, Mannel, Virto [2015]

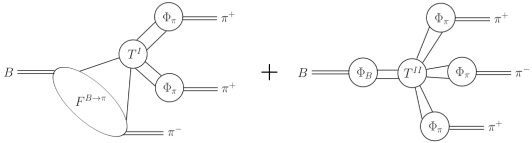


$$\langle\pi^+\pi^+\pi^-|\mathcal{O}_i|\bar{B}\rangle = F^{B\to\pi}\int du\, dv T_i^I(u,v)\Phi_\pi(u)\Phi_\pi(v)$$
$$+\int du\, dv\, dz\, d\omega\, T_i^{II}(u,v,z,\omega)\Phi_B(\omega)\Phi_\pi(u)\Phi_\pi(v)\Phi_\pi(z)$$

- Hard kernels depend on momentum fractions
- At leading order all convolutions are finite
- $1/m_b^2$ and α_s suppressed compared to the edge
- $A_{\rm CP} = \mathcal{O}(\alpha_s/\pi) + \mathcal{O}(\Lambda/m_b)$

Factorization in three-body decays - Central Region

Kraenkl, Mannel, Virto [2015]



$$\langle \pi^+\pi^+\pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B\to\pi} \int du\, dv T_i^I(u,v)\Phi_\pi(u)\Phi_\pi(v)$$
$$+ \int du\, dv\, dz\, d\omega\, T_i^{II}(u,v,z,\omega)\Phi_B(\omega)\Phi_\pi(u)\Phi_\pi(v)\Phi_\pi(z)$$

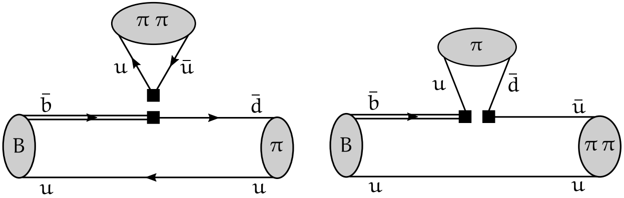
Perturbatively calculable region might not exist for $m_B = 5$ GeV

- Interesting to study QCD factorization properties
- Study power-corrections/weak annihilation? Bediaga, Frederico, Magalhaes [2017]

Kraenkl, Mannel, Virto [2015]; Klein, Mannel, Virto, KKV [2018]

Breakdown of factorization at edges requires new input

- Resonances only close to the edges
- Three-body decays resemble two-body

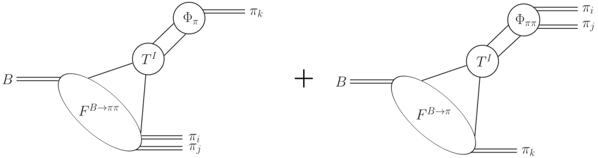


Leading contributions to hard kernels

Same operators as in two-body case, different final states

- Always an improvement over quasi-two-body decays
- Reduces to $B \to \rho\pi$ for ρ dominance and zero-width approximation

Kraenkl, Mannel, Virto [2015]



$$\langle \pi^+\pi^+\pi^- | \mathcal{Q}_i | B\rangle_{s_{+-}\ll 1} = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^+\pi^-} + T_i^I \otimes F^{B\to\pi^+\pi^-} \otimes \Phi_{\pi^+}$$

New non-perturbative input $\rightarrow$ new strong phases

- Two-pion light-cone distribution amplitude Polyakov, Diehl, Gousset, Pire, Gozin, ...
  - at this order: time-like pion form factor from $e^+e^- \to$ hadrons data
- Generalized Form Factor Virto, Descotes-Genon, Feldmann, Khodjamirian, Faller, Mannel, van Dyk, ...
  - P-wave $B \to \pi\pi$ form factors studied using LCSRs [Cheng, Khodjamirian, Virto JHEP 05 (2017) 157 [1701.01633]] [Cheng, Khodjamirian, Virto Phys.Rev.D 96 (2017) 5, 051901 [1709.00173]]

Study of CP violation in $B^+ \to \pi^+\pi^-\pi^+$

R. Klein, Th. Mannel, J. Virto, KKV; K. Olschewsky, Th. Mannel, KKV

JHEP 1710(2017) 117 [arXiv:1708.020407]; JHEP 06 (2020) 073 [arXiv:2003.12053]

$B \to \pi\pi\pi$ decay amplitude

At leading order, leading twist

$$\mathcal{A}_{s_\pm^{\mathrm{low}}<<1} = \frac{G_F}{\sqrt{2}} m_B^2 \left[f_\pi \frac{m_\pi}{m_B^2} (\lambda_u (a_1 + a_4^u) + \lambda_c a_4^c) F_t(s_\pm^{\mathrm{low}}, \zeta) \right.$$
$$\left. + (\lambda_u (a_2 - a_4^u) - \lambda_c a_4^c)(2\zeta - 1) F_\pi(s_\pm^{\mathrm{low}}) f_0(s_\pm^{\mathrm{low}}) \right],$$

- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
- $\lambda_u = |\lambda_u| e^{i\gamma}$ weak phase (constant!)

Only 4 inputs that can be obtained from data

- $B \to \pi$ form factor f_0
- Single pion DA gives the pion decay constant f_π
- $B \to \pi\pi$ form factor F_t
- 2π LCDA gives F_π

$B \to \pi\pi\pi$ decay amplitude

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- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
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Amplitude can always be expressed as resonances $\times (a^u e^{\pm i\gamma} + a^c)$

- a^u and a^c contain strong phases
- Preferred over experimental parametrization where strong and weak phase are mixed: $x \pm \delta x + i(y \pm \delta y)$

$B \to \pi\pi\pi$ decay amplitude

At leading order, leading twist

$$\mathcal{A}_{s_\pm^{\rm low} << 1} = \frac{G_F}{\sqrt{2}} m_B^2 \left[f_\pi \frac{m_\pi}{m_B^2} (\lambda_u (a_1 + a_4^u) + \lambda_c a_4^c) F_t(s_\pm^{\rm low}, \zeta) \right.$$
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- a_i as in two-body decay, contain perturbative strong phases $\mathcal{O}(\alpha_s)$
- $\lambda_u = |\lambda_u| e^{i\gamma}$ weak phase (constant!)

CP violation requires two strong phases $F_t \neq F_\pi$

- Both isoscalar (S-wave) and isovector (P-wave) contribute

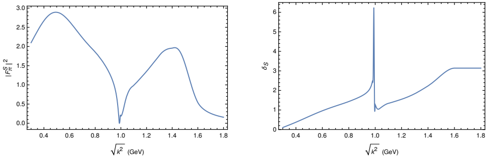
$$F_t = F_t^{I=0} + F_t^{I=1}$$

Daub, Hanhart, Kubis, Passemar, Cirigliano

F_π^S scalar pion form factor (analogous to F_π)

$$\langle \pi^-(k_1)\pi^+(k_2)| m_u \bar{u}u + m_d \bar{d}d |0\rangle = m_\pi^2 F_\pi^S(k^2) \; .$$

- Dispersion theory, coupled Omnes-equations (only non-strange)
- Only reliable* up to about 1.3 GeV



LCSR inspired model similar to $F_t^{I=1}$: not necessary in future!

β constant fit parameter

$$F_t^{I=0}(q^2) = \frac{m_B^2}{m_\pi f_\pi}\beta F_\pi^S(q^2)$$

Daub, Hanhart, Kubis, Passemar, Cirigliano

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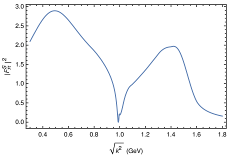

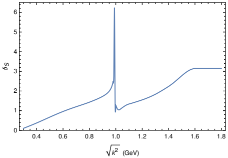

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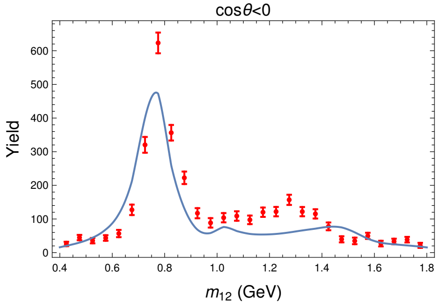
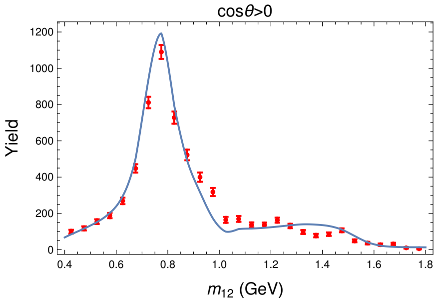
Dalitz Distribution

KKV, Virto, Mannel, Klein

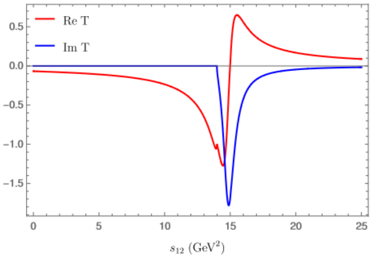


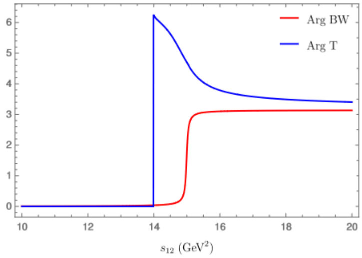


- Cannot be reproduced with our current = 2017 inputs
- Full Dalitz distribution preferred over projections

A model ansatz

Olschewsky, Mannel, KKV [2020]





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- Challenging to calculate: simple parametrization

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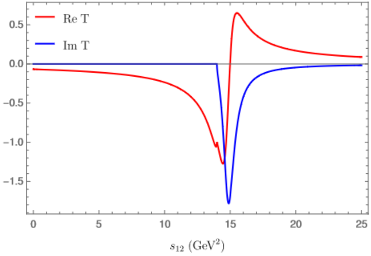
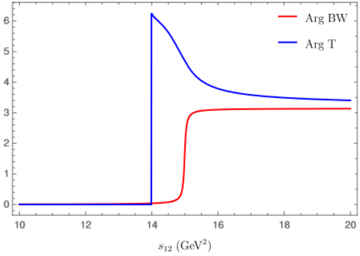




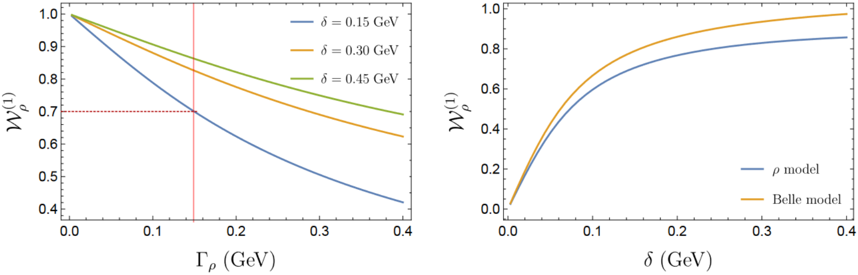
- A_c contains (Breit-Wigner-like) resonances, but also charm threshold effects
- Challenging to calculate: simple parametrization
- Modified propator $T_R = \frac{1}{s_{12} - m_R^2 + \left[\Sigma_R(s_{12}) - \mathrm{Re}\Sigma_R(m_R^2)\right]}$ with

$$\Sigma_R(s_{12}) = g_R m_R \sqrt{s_{\rm thres} - s_{12}} \arctan\left(\frac{1}{\sqrt{\frac{s_{\rm thres}}{s_{12}} - 1 + i\epsilon}}\right)$$

- Could explain the large CP asymmetry at high invariant mass (to be implemented)

A quick word on heavy-to-heavy three body decays

KKV, Virto, Huber [2007.08881]



- Only $B \to D$ form factor enters (as in two-body) for $B \to DM\pi^0$
- Give access to di-pion (or πK) LCDAs: modeled here by single-resonance Breit-Wigners
- Perturbative corrections known up to α_s^2 Huber, Kraenkl

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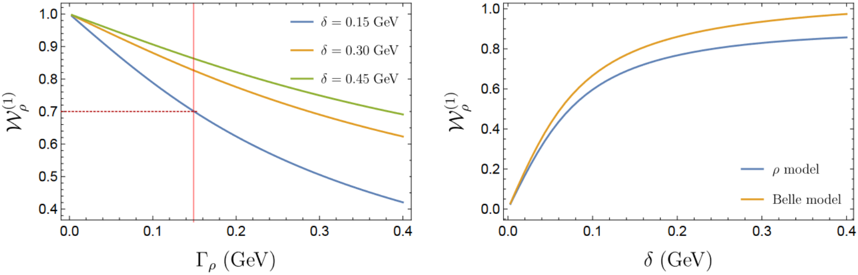

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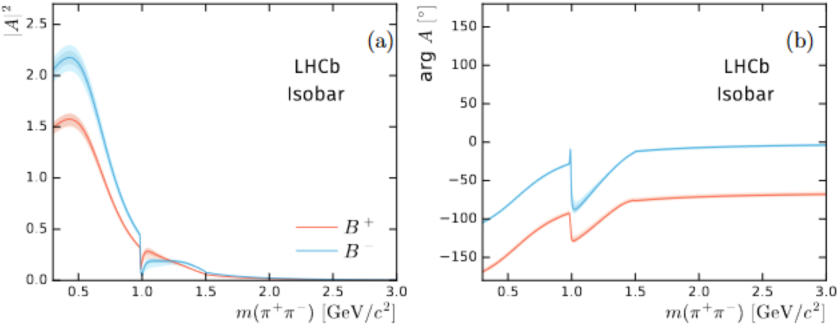
- Give access to di-pion (or πK) LCDAs: modeled here by single-resonance Breit-Wigners
- Perturbative corrections known up to α_s^2 Huber, Kraenkl
- Identify ratios that test QCDF for three-body decays: $z \equiv \cos\theta$

$$\mathcal{R}_{MM} = \frac{\int_{z_1}^{z_2} dz \frac{d\Gamma(B\to DM\pi^0)}{dzdk^2}}{\int_{z_3}^{z_4} dz \frac{d\Gamma(B\to DM\pi^0)}{dzdk^2}} = \frac{\int_{z_1}^{z_2} dz|a_1|^2}{\int_{z_3}^{z_4} dz|a_1|^2}$$

Quo Vadis?

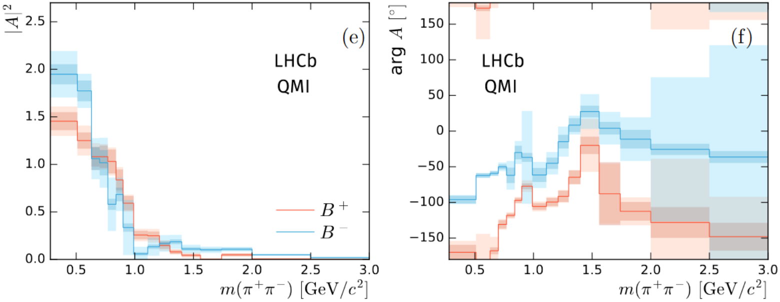
Improved form factors

LHCb-PAPER-2019-017



Improved form factors

LHCb-PAPER-2019-017



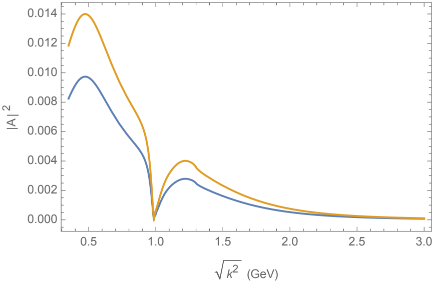
Insights from data on form factors in Quasi-model independent approach?

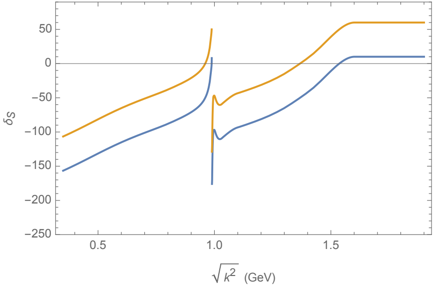
Scalar CP violation (example)

Example to show importance of perturbative phases

$$A_S^+ = (a_T e^{i\gamma} + a_P e^{i\delta}) F_\pi^S$$
$$A_S^- = (a_T e^{-i\gamma} + a_P e^{i\delta}) F_\pi^S$$

- Include $\mathcal{O}(\alpha)$ strong phases from QCD penguins
- Can give large CP violation in S-wave that agrees with data





Insights from light-cone sum rules

- Goal: Constrain $B \to K\pi$ form factors by imposing what we know from QCD
- Light-cone sum rule analysis

P-wave $B \to \pi K$ form factors

[J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

- Improvement over assuming K^* is a stable state
- Finite width effects in P wave at 20% level for BR
- Higher resonances large impact $\to$ can be constrained by moment analysis

S-wave $B \to \pi K$ form factors [S. Descotes-Genon, A. Khodjamirian, J. Virto, KKV] [in progress..]

- S wave even more challenging; generally broad resonances
- Requires coupled-channel analysis?

What do LCSR tell us about $B \to \pi K$ form factors?

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = \mathcal{S}_{0,t}^{(\text{OPE})}(q^2, s_0, M^2)$$

Key points:

- No closed expression for the $F_{0,t}^{(\ell=0)}(s,q^2)$!
- Only information on a weighted integral over the $K\pi$ invariant mass
- Use sum rule to constrain parameters of your favourite $K\pi$ P/S-wave model

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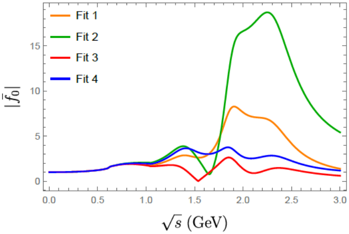
Inputs:

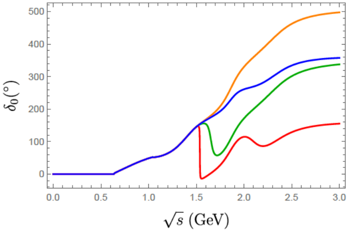
- $F_S(s)$ from data
- s_0 from two-point sum rule using $K\pi$ form factor from data

S wave πK form factor

von Detten, Noël, Hanhart, Hoferichter, Kubis, Eur. Phys. J. C 81 (2021) 420 [ArXiV:2103.01966]

- Based on rescattering πK phase shifts using Omnes parametrization at low energies
- Includes inelastic effects through higher resonances
- Applied to $\tau^- \to K_S \pi^- \nu_\tau$ data to fit resonance parameters (both for P and S wave)
- Four different fit assumptions for source term give four scalar form factors

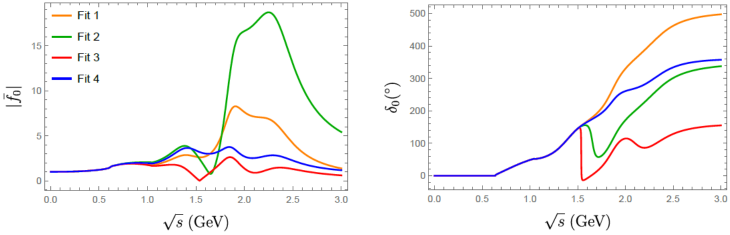




LCSR for $B \to \pi K$ form factors

Kubis, Hanhart, von Detten Descotes-Genon, Khodjamirian, Virto, JHEP 1912 (2019) 083

Descotes-Genon, Khodjamirian, Virto, KKV, JHEP [2304.02973]



- Different high-energy behavior depending on which resonances are include
- For P-wave; Simple ansatz $K\pi$ states decays via set of Breit-Wigner-type resonances
- For S-wave; link to πK form factor w_i kinematical factor

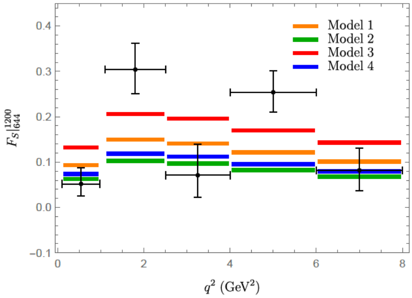
$$F_i^{(\ell=0)}(s, q^2) = w_i(s, q^2)\rho(q^2)F_S(s)$$

- ρ determined from sum rule

What can data do for us?

LHCb [JHEP12(2016)065] [arXiV:1609.04736]; Virto, Khodjamirian, Descotes-Genon, KKV [2304.02973]

- LHCb measured 41 moments depending on S, P, D waves around $m_{K\pi} \in [1.3, 1.5]$ GeV with $q^2 \in [1.1, 6]$ GeV^2
- 2 combinations only depend on S-wave [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]
- Current uncertainties too large to draw strong conclusions
- S-wave component in $B \to K^* \ell\ell$ exhibits some strange behavior Stay tuned!



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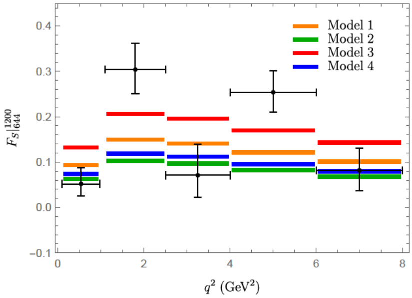

- Constrains S-wave contribution in $B \to K^*\ell\ell$ from QCD

Quo Vadis?

Measurements of angular moments of $B \to V\ell\ell$ in bins across q^2 and k^2 spectra very useful!

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To do list:

- Same approach for $B \to \pi\pi$ form factors?
- test QCDF in $B \to DM\pi^0$
- QCDF predictions for $B \to \pi KK/\pi$?
- Combined S and P-wave analysis of the differential rate and angular observables in $K^*(1410)$ region
- Charm three body decays?
- ...

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Thank you for your attention!

Backup

$B \to D\pi$ puzzle

$B_s^0 \to D_s^+ \pi$ and $B_d^0 \to D^+ K^-$ puzzle

see also Cai, Deng, Li, Yang [2103.04138], Endo, Iguro, Mishima [2109.10811], Gershon, Lenz, Rusov, Skidmore [2111.04478]

Discrepancies between data and theory for $B_s \to D_s^{+(*)} \pi^-$ and $B \to D^{+(*)} K^-$

- pure tree decays (no color-suppressed nor penguin contributions)
- NNLO predictions in QCDF Huber, Kraenkl [1606.02888]
- Same form factors as for exclusive V_{cb}
- Updated and extended calculations give $\sim 4\sigma$ deviation Bordone, Gubernari, Huber, Jung and van Dyk, [2007.10338]

$B_s^0 \to D_s^+\pi$ and $B_d^0 \to D^+K^-$ puzzle

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- Updated and extended calculations give $\sim 4\sigma$ deviation Bordone, Gubernari, Huber, Jung and van Dyk, [2007.10338]
- QED corrections cannot explain the tension* Beneke, Boer, Finauri, KKV [2107.03819]
- Possible NP explanations have been studied Iguro, Kitahara [2008.01086], Bordone, Greljo, Marzocca [2103.10332]
- Also puzzling patterns in $B_s \to D_s K$ are revealed Fleischer, Malami [2110.04240]

Interesting puzzle that requires both experimental and theoretical attention!

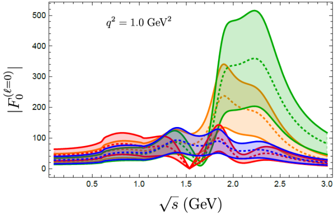
$B \to \pi K$ S wave form factors

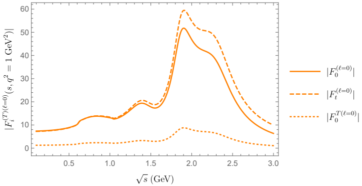
- Ansatz:

$$F_i^{(\ell=0)}(s,q^2) = \sqrt{\lambda}\rho_i(q^2)F_S(s)$$

- ρ parameters only depend on q^2 and are fixed by sum rule

$$\int_{s_{\rm th}}^{s_0} ds\ e^{-s/M^2}\omega_i(s,q^2)|F_S(s)|^2\rho_i(q^2) = \mathcal{S}_i^{\rm OPE}(q^2,s_0,M^2)$$





- Next different q^2 points and combined S and P wave

P-wave example

- Sum rule allows to determine model parameters $\mathcal{G}_{R,i}(q^2)$ $c_{R,i}$ kinematical factors

$$\sum_R \mathcal{G}_{R,i}(q^2)\, c_{R,i}(q^2)\, H_R(s_0, M^2) = \mathcal{S}_i^{\rm OPE}(q^2, s_0, M^2)$$

$$H_R(s_0, M^2) = \frac{1}{16\,\pi^2} \int_{s_{\rm th}}^{s_0} ds\; e^{-s/M^2}\; \frac{g_{RK\pi}\; \lambda_{K\pi}^{1/2}(s)\, |F_S(s)|}{s\sqrt{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}}$$

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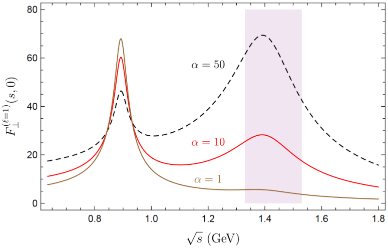
Application: [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]

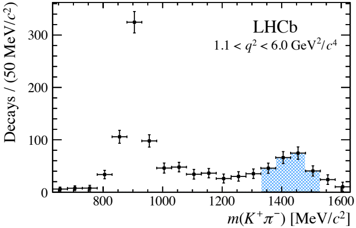
- Study finite-width effects for single K^* resonance:
  Width ratio $\mathcal{W} \equiv \frac{\mathcal{G}}{\mathcal{G}|_{\Gamma\to 0}} = 1 + 1.9\frac{\Gamma}{M} \sim 1.09 \to 20\%$ effect on BRs!
- Study effect of higher resonances beyond the $K^*(892)$: $\mathcal{G}_{K^*(1410)} = \alpha \mathcal{G}_{K^*(892)}$



What can data do for us?

LHCb [JHEP12(2016)065] [arXiV:1609.04736]

- LHCb measured 41 moments depending on S, P, D waves around $m_{K\pi} \in [1.3, 1.5]$ GeV with $q^2 \in [1.1, 6]$ GeV^2
- 4 combinations only depend on P-wave [Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]



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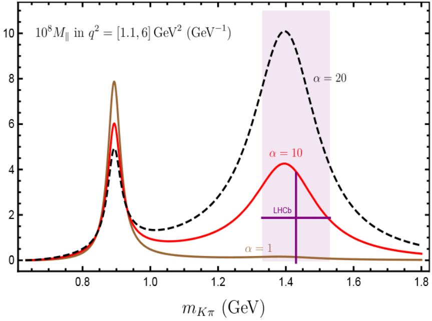


- Example of use of the data to constrain higher-partial waves
- Simultaneous analysis of S and P wave gives more information (in progress!)

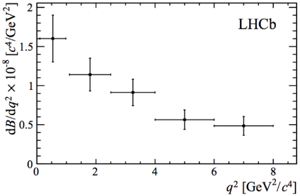
What can data do for us?

[Virto, Khodjamirian, Descotes-Genon JHEP 1912, 083 (2019)]

- Differential branching ratio also limits P- (and S-)wave

$$\frac{d\Gamma}{dq^2 dk^2} = \tilde{\Gamma}_1 = |\widehat{S}^L|^2 + |\widehat{S}^R|^2 + |\widehat{A}_{\|}^L|^2 + |\widehat{A}_{\|}^R|^2 + |\widehat{A}_{\perp}^L|^2 + |\widehat{A}_{\perp}^R|^2 + |\widehat{A}_0^L|^2 + |\widehat{A}_0^R|^2 + \ldots$$

- Considering only P wave gives:



$$10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} = 1.41 \pm 0.27 \to \alpha \lesssim 5$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} = 1.60 \pm 0.29 \to \alpha \lesssim 6$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} = 1.37 \pm 0.26 \to \alpha \lesssim 5$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} = 1.12 \pm 0.26 \to \alpha \lesssim 4$$
$$10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} = 0.98 \pm 0.23 \to \alpha \lesssim 3$$

- Simultaneous analysis with S-wave in progress

QCD to constrain S wave models

- Use light-cone sum rules to constrain $B \to (K\pi)_S$ parametrizations/models
- Simple sum of Breit-Wigners (used for P-wave case) does not suffice

Model requirements:

- appropriate analytical properties
- poles corresponding to known resonances
- cuts for the relevant open channels
- simple (linear) dependence on the parameters to be constrained by the sum rules

Electromagnetic Effects

Beneke, Boer, Toelstede, KKV, JHEP 11 (2020) 081 [2008.10615]

$$\Gamma[\bar{B} \to M_1 M_2](\Delta E) \equiv \Gamma[\bar{B} \to M_1 M_2 + X_s]\Big|_{E_{X_s} \leq \Delta E},$$

- IR finite observable (width) must include ultra-soft photon radiation
- X_s are soft photons with total energy less than ultrasoft scale ΔE
- Factorizes in non-radiative amplitude and ultrasoft function

$$\Gamma[\bar{B} \to M_1 M_2](\Delta E) = |\mathcal{A}(\bar{B} \to M_1 M_2)|^2 \sum_{X_s} |\langle X_s|(\bar{S}_v^{(Q_B)} S_{v_1}^{\dagger(Q_{M_1})} S_{v_2}^{\dagger(Q_{M_2})})|0\rangle|^2 \, \theta(\Delta E - E_{X_s})$$

Simple classification:

- Ultra-soft photons: eikonal approximation, well understood

$$\Delta E \ll \Lambda_{\mathrm{QCD}}$$

- NEW: Non-universal, structure dependent corrections Beneke, Boer, Toelstede, KKV [2020]
- Both effects important: virtual photons can resolve the structure of the meson!

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Simple classification:

- Ultra-soft photons: eikonal approximation, well understood

$$\Delta E \ll \Lambda_{\rm QCD}$$

- Often done: Assume pointlike approximation up to the scale m_B [Baracchini, Isidori]
  - $\to$ fails to account for all large logarithms (and scales)!
  - $\to$ photons with energy $\gtrsim \Lambda_{\rm QCD}$ probe the partonic structure of the mesons

A brief dive into Light-Cone Sum Rules (LCSR)

- Example: Strange scalar current to interpolate the πK state: $j_S = (m_s - m_d)\bar{s}d$
- Start with correlation function:

$$\mathcal{S}_b(k,q) = i \int d^4x\, e^{ik\cdot x} \langle 0| \mathrm{T} \left\{ j_S^\dagger(x), j_b(0) \right\} |\bar{B}^0(q+k)\rangle \,,$$

- $\mathcal{S}$ calculated using light-cone OPE in terms of B-meson LCDAs for $k^2 < 0$ and $q^2 \ll m_b^2$
- Use dispersion relation in the variable k^2:

$$\mathcal{S}^{(\mathrm{OPE})}(k^2, q^2) = \frac{1}{\pi} \int\limits_{s_{\mathrm{th}}=(m_K+m_\pi)^2}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{S}(s,q^2)}{s-k^2} \,.$$

- Obtain spectral density by inserting a full set of states

$$2\,\mathrm{Im}\mathcal{S}_b^{(K\pi)}(k,q) = \sum_{K\pi} \int d\tau_{K\pi} \langle 0| j_S^\dagger |K(k_1)\pi(k_2)\rangle^* \langle K(k_1)\pi(k_2)| j_b |\bar{B}^0(q+k)\rangle \,,$$

$$\mathrm{Im}\mathcal{S}(s,q^2) = \mathrm{Im}\mathcal{S}^{(K\pi)}(s,q^2) + \mathrm{Im}\mathcal{S}^{(h)}(s,q^2)\theta(s-s_h)\,.$$

- $\mathcal{S}^{(h)}$ all contributions above s_{th}

- Assume quark-hadron duality for the states above threshold

$$\int_{s_h}^{\infty} ds \, \frac{\mathrm{Im}\mathcal{S}^{(h)}(s, q^2)}{s - k^2} = \int_{s_0}^{\infty} ds \, \frac{\mathrm{Im}\mathcal{S}^{(OPE)}(s, q^2)}{s - k^2} \, ,$$

- Perform Borel transformation in the variable k^2

$$\frac{1}{\pi} \int\limits_{(m_K+m_\pi)^2}^{s_0} ds \, e^{-s/M^2} \, \mathrm{Im}\mathcal{S}^{(K\pi)}(s, q^2) \quad = \frac{1}{\pi} \int\limits_{m_s^2}^{s_0} ds \, e^{-s/M^2} \, \mathrm{Im}\mathcal{S}^{(OPE)}(s, q^2)$$

$$\equiv \mathcal{S}^{(\mathrm{OPE})}(q^2, s_0, M^2)$$

- Borel trafo suppressed the effect of higher-order resonances
- $\mathcal{S}^{\mathrm{OPE}}(q^2, s_0, M^2)$ OPE expression after subtracting the above-threshold contribution from the dispersive integral
- s_0 and M^2 can be determined from two-point sum rule

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = \mathcal{S}_{0,t}^{(\mathrm{OPE})}(q^2, s_0, M^2)$$

- s_0 effective threshold
- $\omega_{0,t}(s,q^2)$ kinematic factors
- $F_S(s)$ scalar form factor:$(m_s - m_d)\langle K^-(k_1)\pi^+(k_2)|\bar{s}d|0\rangle \equiv F_S((k_1+k_2)^2)$
- $\mathcal{S}_{0,t}^{(\mathrm{OPE})}$ pert. calculable in terms of B-LCDA parameters
- Analogous expressions for P wave [J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = \mathcal{S}_{0,t}^{(\mathrm{OPE})}(q^2, s_0, M^2)$$

Key points:

- No closed expression for the $F_{0,t}^{(\ell=0)}(s,q^2)$!
- Only information on a weighted integral over the $K\pi$ invariant mass
- Use sum rule to constrain parameters of your favourite $K\pi$ P/S-wave model

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Inputs:

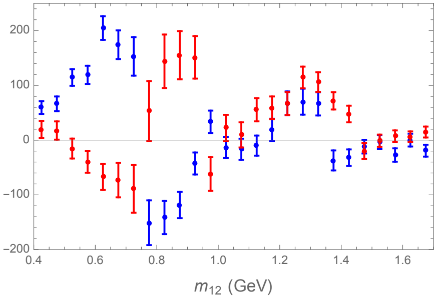
- $F_S(s)$ from data
- s_0 from two-point sum rule using $K\pi$ form factor from data

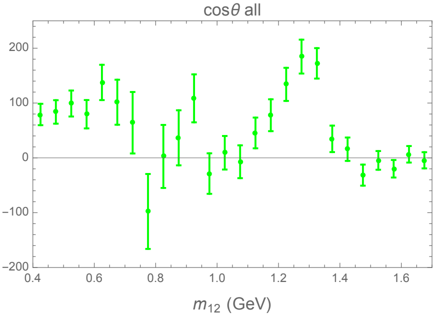
CP Distributions

Data from LHCb

$$A_{\rm CP} \propto \beta \sin\gamma \sin\phi \cos\theta + \beta' \sin\phi' \cos^2\theta + \beta'' \sin\phi'' \cos^4\theta$$

- Distinguish between region above and below $m_{12} = 1.0$ GeV
- Include higher-twist and $\mathcal{O}(\alpha)$ corrections



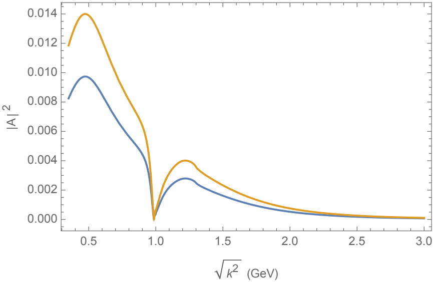


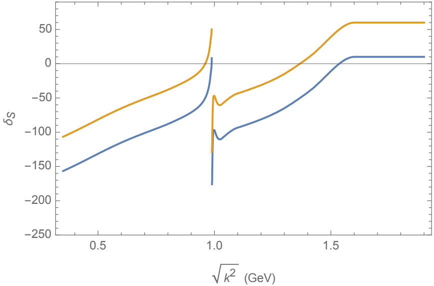
Scalar CP violation (example)

Example to show importance of perturbative phases

$$A_S^+ = (a_T e^{i\gamma} + a_P e^{i\delta}) F_\pi^S$$
$$A_S^- = (a_T e^{-i\gamma} + a_P e^{i\delta}) F_\pi^S$$

- Include $\mathcal{O}(\alpha)$ strong phases from QCD penguins
- Can give large CP violation in S-wave that agrees with data





Dipion and $K\pi$ form factors

2π LCDA

Polyakov, Diehl, Gousset, Pire, Teryaev

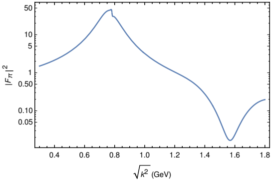
Reduces at leading order to the normalization

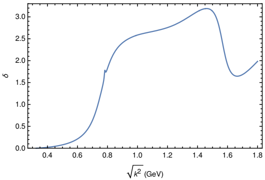
- Both isoscalar ($I = 0$) and isovector ($I = 1$) contribute

$$\int du\, \phi_{\pi\pi}^{I=1}(u,\zeta,s) = (2\zeta - 1) F_\pi(s) \qquad \int du\, \phi_{\pi\pi}^{I=0}(u,\zeta,s) = 0$$

Time-like pion formfactor $F_\pi(s)$: Babar data on $e^+e^- \to \pi\pi(\gamma)$

Hanhart, Kubis, Shekhovtsova, Roig, Was, Predzinski





$B \to \pi\pi$ form factor

Only vector form factor relevant [Faller, Feldmann, Khodjamirian, Mannel, van Dyk '14]

- Partial wave expansion: P wave always $I = 1$ and S wave has $I = 0$

$$k_{3\mu} \langle \pi^+(k_1)\pi^-(k_2)|\bar{b}\gamma^\mu\gamma^5 u|B^+(p)\rangle = -\sqrt{k_3^2} F_t(s,\zeta)$$

Theory efforts: [Boër, Feldmann, van Dyk '17, Feldmann, van Dyk, KKV '18]

- $B \to \pi\pi$ form factors factorize at large k^2
- Relevant kinematics in regime of Light-Cone Sum Rules
- P-wave studied with B-meson and dipion LCSRs [Khodjamirian, Virto, Cheng '17]
- S-wave in progress! [Descotes-Genon, Khodjamirian, Virto, KKV [in progress]]]

$B \to \pi\pi$ form factor from B-meson LCSRs

Correlation function with pseudoscalar heavy-light current

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|T\bar{d}(x)\gamma_\mu u(x), im_b\bar{u}(0)\gamma_5 b(0)|\bar{B}^0(q+k)\rangle$$

Light-cone OPE in terms of B-meson LCDA and dispersive relation:

$$F_\mu^{OPE}(k^2,q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds\frac{2\mathrm{Im}F_\mu}{s-q^2}$$

Unitarity Relation [Kang, Kubis, Hanhart, Meissner '14, Khodjamirian, Virto, Cheng [2017]]

$$\begin{aligned}2\mathrm{Im}F_\mu &= m_b\int d\tau\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle\langle\pi\pi|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle + \dots\\ &\propto F_\pi^*(s)F_t^{l=1}(s,q^2) + \dots\end{aligned}$$

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$$\propto F_\pi^*(s)F_t^{l=1}(s,q^2) + \dots$$

Phase F_π = Phase $F_t^{l=1}$

$B \to K\pi$ form factors

- Generated by the (axial-)vector and (pseudo)tensor $b \to s$ transition currents

$$j_A^\mu = \bar{s}\gamma^\mu(\gamma_5)b\,, \quad j_T^\mu = \bar{s}\sigma^{\mu\nu}q_\nu(\gamma_5)b\,.$$

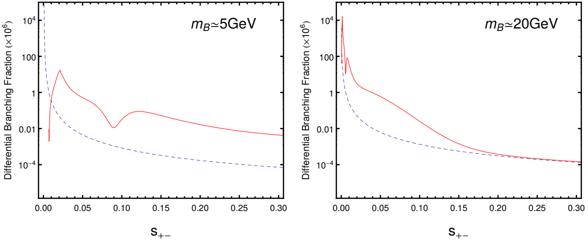
- Form factors $F_i(k^2, q^2, q \cdot \bar{k})$ defined as

$$\begin{aligned}
i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu b|\bar{B}^0(p)\rangle &= F_\perp\, k_\perp^\mu\,,\\
-i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}^0(p)\rangle &= F_t\, k_t^\mu + F_0\, k_0^\mu + \mathrm{F}_\parallel\, k_\parallel^\mu\,,\\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu b|\bar{B}^0(p)\rangle &= F_\perp^T k_\perp^\mu \|\,,\\
\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b|\bar{B}^0(p)\rangle &= F_0^T k_0^\mu + F_\parallel^T\, k_\parallel^\mu\,,
\end{aligned}$$

- Isolate P or S-wave part via partial wave expansion:

$$\begin{aligned}
F_{0,t}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\, F_{0,t}^{(\ell)}(k^2,q^2)\; P_\ell^{(0)}(\cos\theta_K)\,,\\
F_{\perp,\parallel}(k^2, q^2, q\cdot\bar{k}) &= \sum_{\ell=1}^{\infty}\sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2,q^2)\; \frac{P_\ell^{(0)}(\cos\theta_K)}{\sin\theta_k}\,,
\end{aligned}$$

Kraenkl, Mannel, Virto [2015]



Full 2πLCDA (red) and perturbative contribution (dashed)

Two approaches do not merge for realistic B meson mass

- Power-corrections not suppressed enough
- No part of the Dalitz plot is really center-like