Non-stabilizerness (Magic) in the Assymetric Quantum Rabi Model

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WORKSHOP: GEOMETRY OF QUANTUM DYNAMICS

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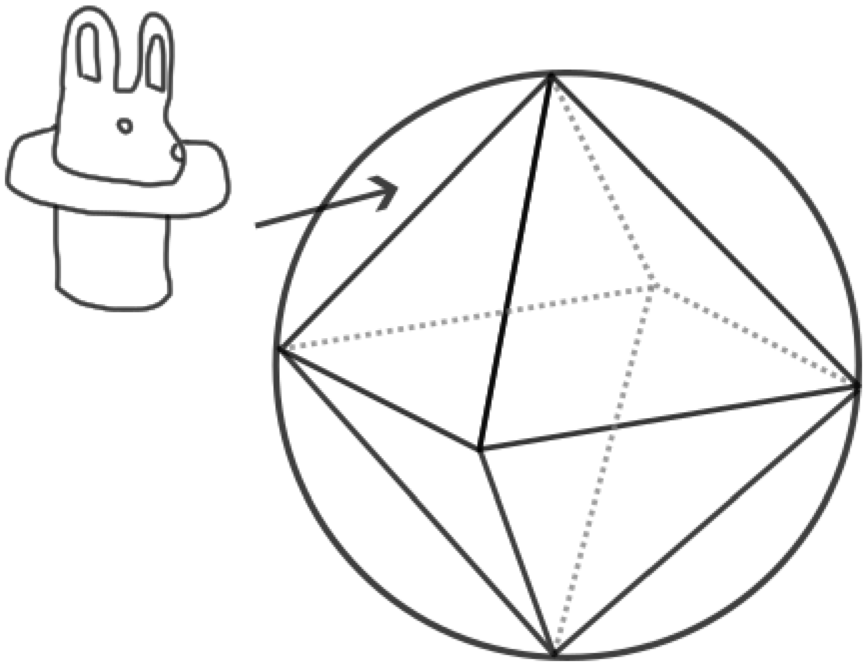
Non-Stabilizerness or Magic

What is magic?

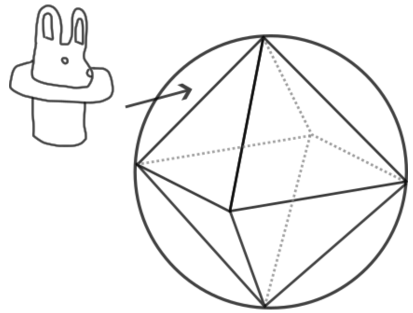
- Magic or Non-Stabilizerness is a resource for quantum computation, which differentiates quantum from classical computation.
- A resource theory for non-stabilizerness is necessary in order to work within a theoretical frame.
- Resource theories of magic in discrete and continuous regimes have been developed.



New J. Phys. 16, 013009 (2014), Phys. Rev. A 98, 052350 (2018)

Quantifying magic

- Quantifying magic has been an issue to tackle since its proposal in the fault tolerant quantum computation context.
- Distillation of magic states was the first process that used the resource theory of non-stabilizerness.
- Efforts to understand the meaning of magic in other branchs of Physics, like many body Physics, are currently under deep scrutiny.



Appl. Phys. B 86 *367* (2007), PRL 118 090501 (2017), PRL 128 *050402* (2022)

Quantifying magic

- Relative entropy of magic

$$r_M(\rho) = min\left[S\left(\rho||\sigma\right)\right]_{\sigma\in STAB(H_d)}$$

ρ - state of the system

$STAB(H_d)$ - set of stabilizer states of the space

d - dimension of the Hilbert space

New J. Phys. 16, 013009 (2014)

Quantifying magic

- Sum negativity

$$sn(\rho) = \sum_{W_\rho(u)<0} \frac{1}{2} |W_\rho(u)| = \frac{1}{2} \left[\sum_u |W_\rho(u)| - 1 \right],$$

- Mana

$$mana(\rho) = \mathcal{M}(\rho) = \log [2sn(\rho) + 1]$$

$W_\rho(u)$ - element of the discreteWigner function of ρ

New J. Phys. 16, 013009 (2014)

Quantifying magic

There is not an ultimate measure of magic!!

- Dai's M Witness

$$M(\rho) = \sum_{\mu,\nu} \sqrt{tr\left(\rho D_{\mu,\nu}\right) tr\left(\rho D_{\mu,\nu}^{\dagger}\right)}$$

$D_{\mu,\nu}$ - displacement operator of Heisenberg-Weyl algebra

Monotones and witnesses $\rightarrow$ not increasing under free operations

New J. Phys. 16, 013009 (2014), Int. J. Theor. Phys. 61 35 (2022)

Quantifying magic

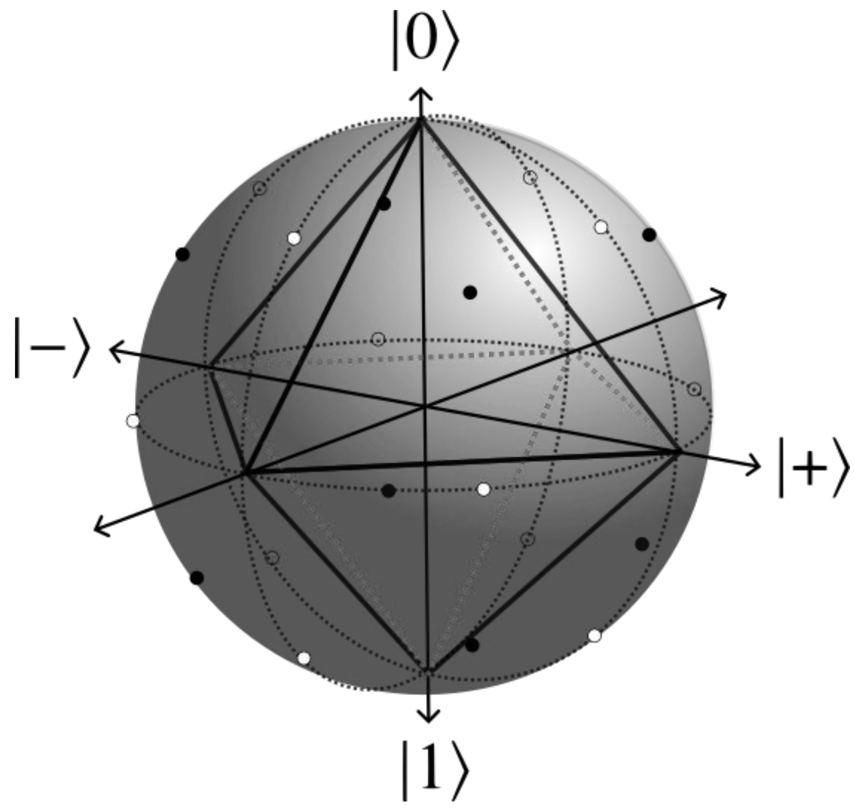
- Bloch sphere and octahedron

The white dots are the H-type states

$$|H\rangle\langle H| = \frac{1}{2}\left[I + \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)\right]$$

The black dots are the T-type states

$$|T\rangle\langle T| = \frac{1}{2}\left[I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z)\right]$$



Quantifying magic

- What about the continuous regime?

  - As an extrapolation from the discret theory, the negativity of the continuous variable Wigner function can be considered as a resource.
  - Wigner Logarithmic Negativity (WLN) can be written as

$$WLN(\rho) = log\left(\mathcal{N}(\rho) + 1\right), \quad \mathcal{N}(\rho) = \int d\mathbf{r} |W_\rho(\mathbf{r})| - 1$$

Phys. Rev. A 98, 052350 (2018)

Quantifying magic

Quantifying discrete magic

- The discrete Wigner function is not well defined in all dimensions, only in prime odd dimensions.
- One of the ways to surpass this issue is to take additional constrictions to the systems.
- We take the discrete Wigner function for SU(2) proposed in M. A. Marchiolli and D. Galleti, J. Phys. A: Math. and Theo. 52 405305 (2019):

$$W(\mu,\nu) = \frac{1}{2} tr \left\{ \rho \left[\mathbf{1} + (-1)^{\nu}\sigma_x + (-1)^{\mu+\nu+1}\sigma_y + (-1)^{\mu}\sigma_z \right] \right\}$$

$$0 \leq \mu, \nu \leq 1$$

Quantifying magic

- Quantifying continuous magic

  - To propose measures of continuous magic besides Wigner logarithmic negativity.
  - In our case we want to quantify magic in Fock basis for bosons.
  - To extrapolate measures from the discrete regime to the continuous (as first approach we take just a methodology of discretization, not a theory).
  - To tackle the hybrid scenario to see the behavior of magic in subspaces of a bipartite system.

The Assymetric Quantum Rabi Model

The Quantum Rabi Model

- ❑The model describes the coherent interaction between one two-level atom and a quantized mode of light.
- ❑It is one of the more simplest hybrid systems.
- ❑Searching for hidden symmetries.
- ❑It has promising applications, for instance, atom chip connected to a waveguide and superconducting qubits.

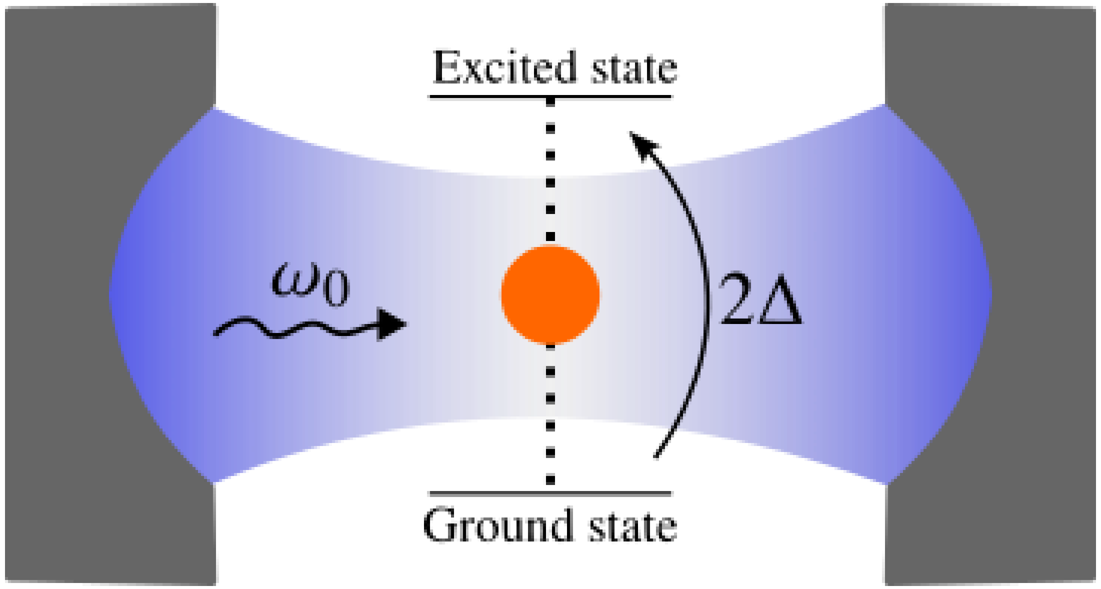


Figure: Scheme of the Quantum Rabi model

PRL 107 100401 (2011), Symmetry 11, 1259 (2019)

The Quantum Rabi Model

- The **Quantum Rabi Model** describes a two-level system coupled to a quantum harmonic oscillator (mode of the elctromagnetic field),

$$H_R = \Delta\sigma_z + \omega_0 a^\dagger a + g\sigma_x \left(a^\dagger + a\right)$$

2Δ- the level splitting betwen the two levels

ω_0- the frequency of the quantized light field

g- the coupling strength between the light field and the two-level system ($\hbar$ = 1).

PRL 107 100401 (2011), Symmetry 11, 1259 (2019)

The Assymetric Quantum Rabi Model

- The **Assymetric Quantum Rabi Model** Hamiltonian,

$$H_A = \Delta\sigma_z + \epsilon\sigma_x + \omega_0 a^\dagger a + g\sigma_x\left(a^\dagger + a\right)$$

which has a bias extra term, proportional to ϵ, that is related with spontaneous tunnelling between the two levels.

  - $\epsilon\sigma_x$ - Breaks symmetry of the QRM — Restauration of parity symmetry when ϵ/ω_0 is half integer.
  - To study the hidden symmetries of the AQRM could give us information about light matter interaction and its possible applications, for instance, superconducting qubits.

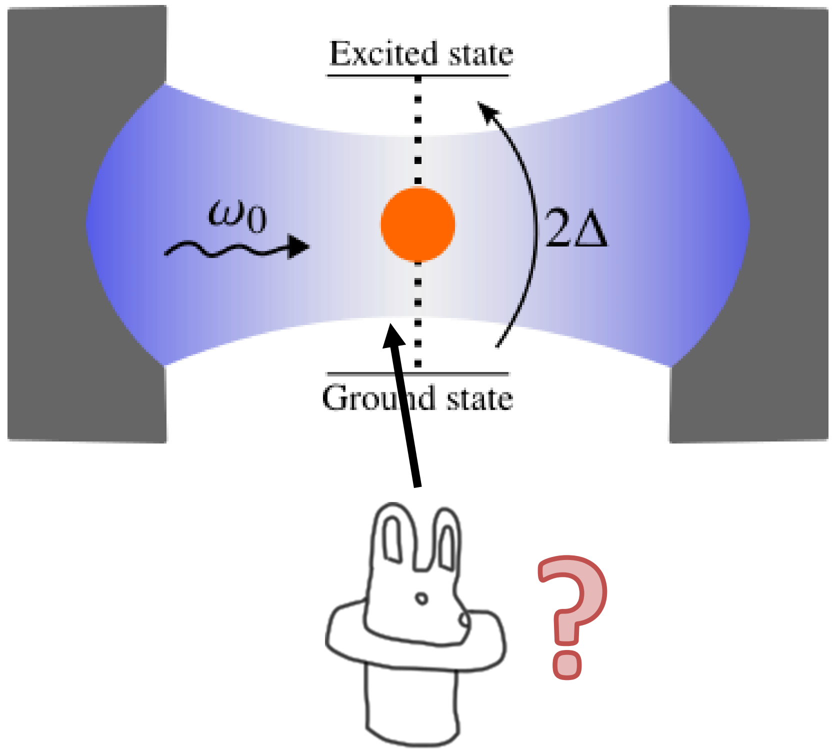
Phys. Rev. A 103 023719 (2021)

Magic + AQRM = Magical AQRM

Quantifying Magic in the AQRM

1. We numerically obtain the energy spectrum of the complete system for various values of the constants of the model, Δ, ϵ, ω_0 and g.
2. As a first approach we partially trace the systems.

- ❑Tracing over harmonic oscillator.
- ❑Tracing over two-level system.
- ❑We quantify discrete magic for both subsystems.



Results

Considerations

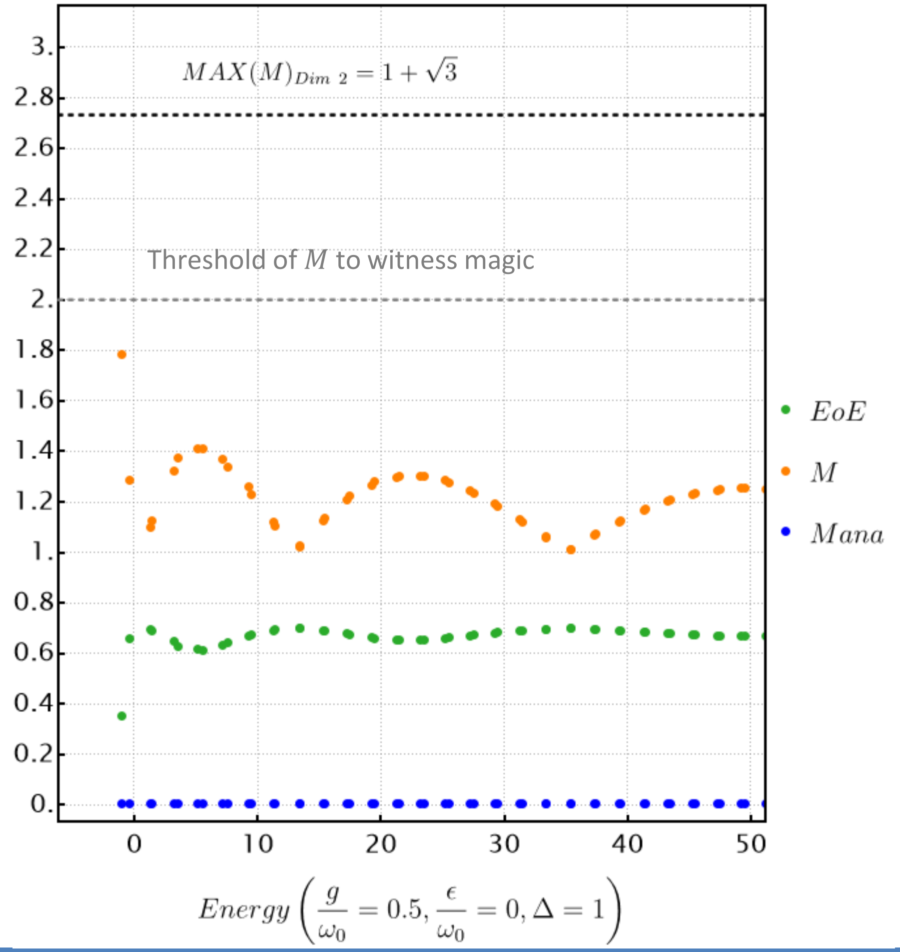
- To make our numerical computations we normalize the Hamiltonian with respect to ω_0.

$$\frac{H_A}{\omega_0} = a^\dagger a + \frac{g}{\omega_0}\left(a^\dagger + a\right)\sigma_x + \frac{\Delta\sigma_z + \epsilon\sigma_x}{\omega_0}$$

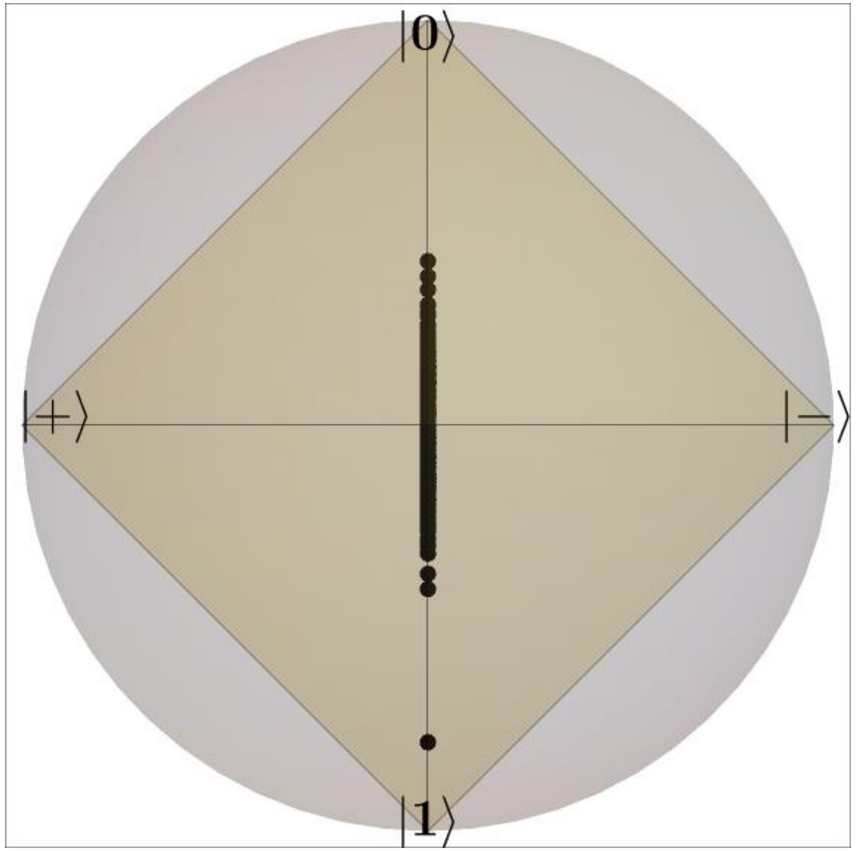
Then we diagonalize the Hamiltonian to obtain the eigen-energies and eigen vectors to construct the state of the system, $\rho_A(E_k)$. We trace such system to obtain the partially traced subsystems.

- $\rho_{light} = Tr_{atom}\left(\rho_A(E_k)\right)$
- $\rho_{atom} = Tr_{light}\left(\rho_A(E_k)\right)$

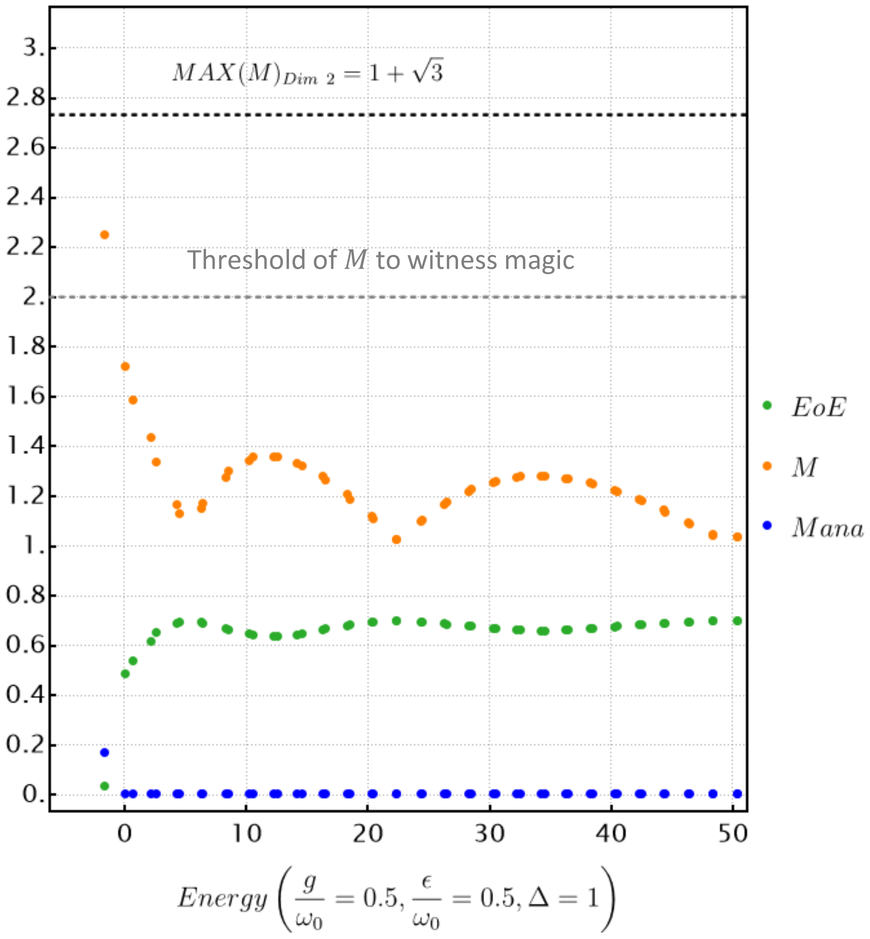
Magic in the atomic subspace



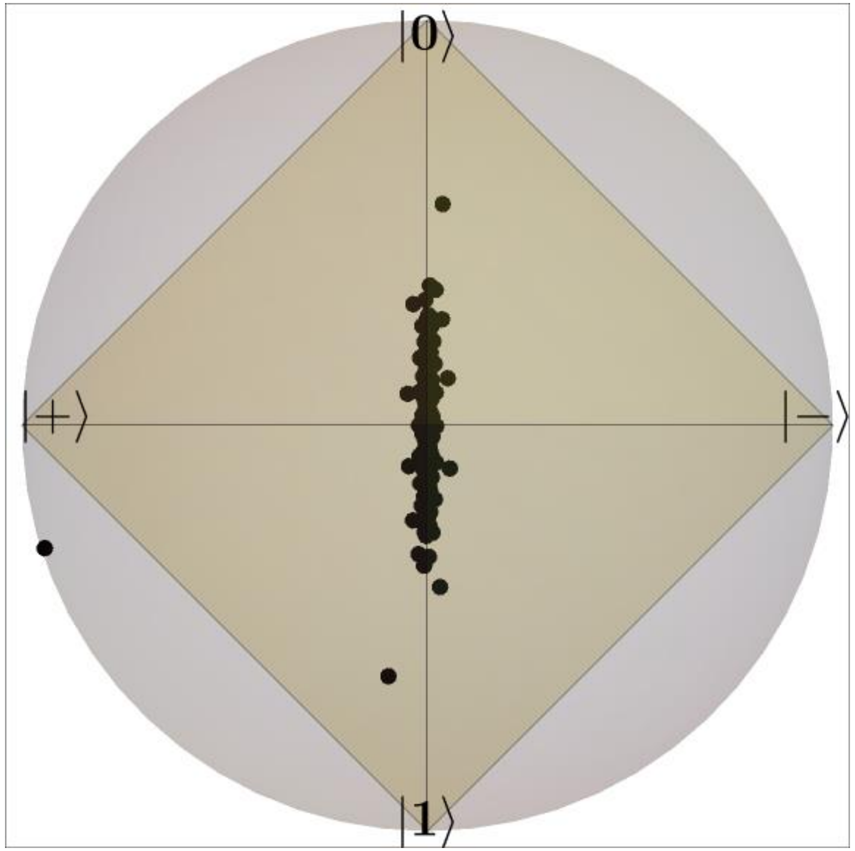
Fixed g/ω_0, $\epsilon/\omega_0 = 0$



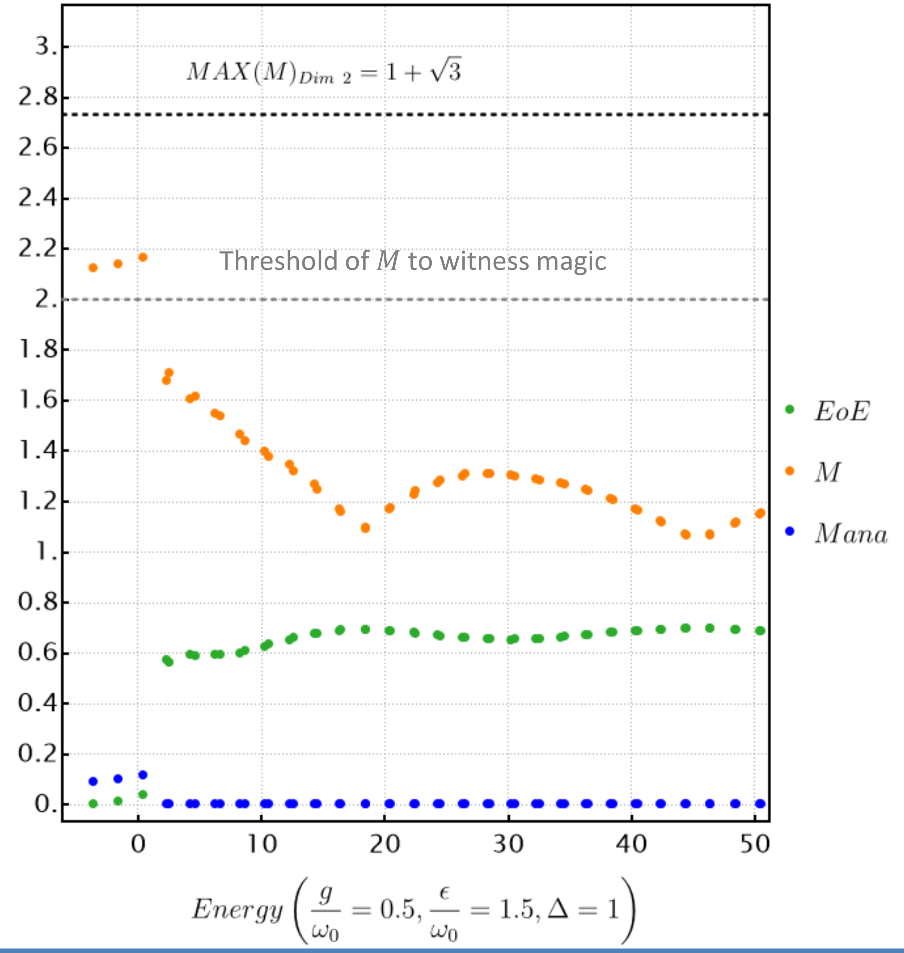
Magic in the atomic subspace



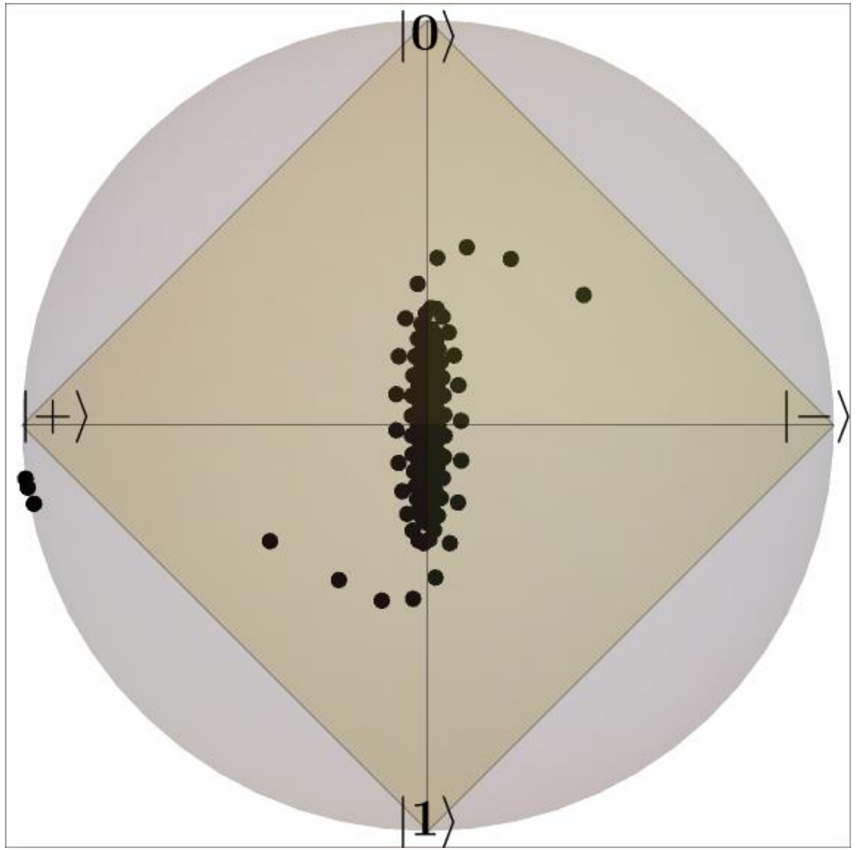
Fixed g/ω_0, ϵ/ω_0 = 0.5



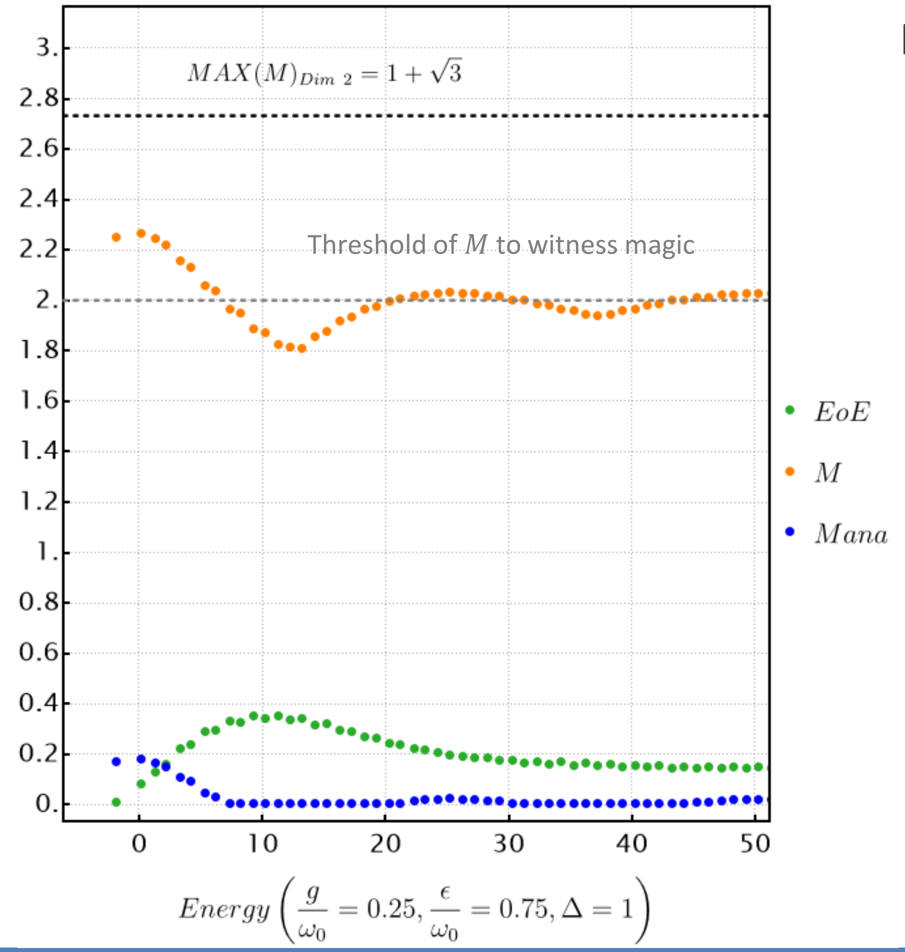
Magic in the atomic subspace



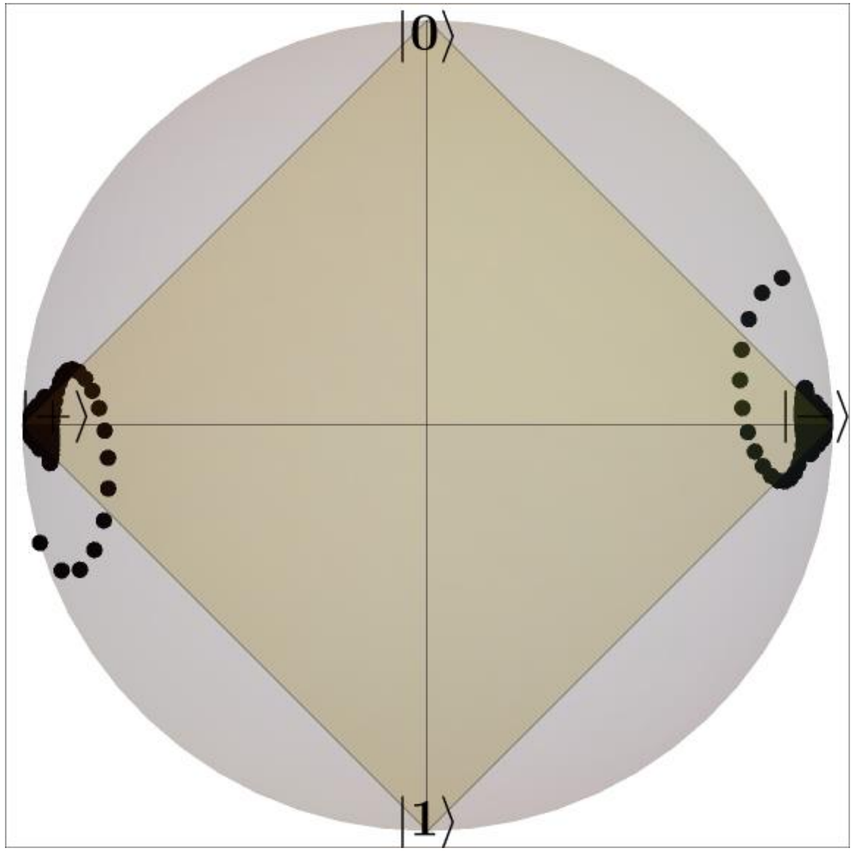
Fixed g/ω_0, ϵ/ω_0 = 1.5



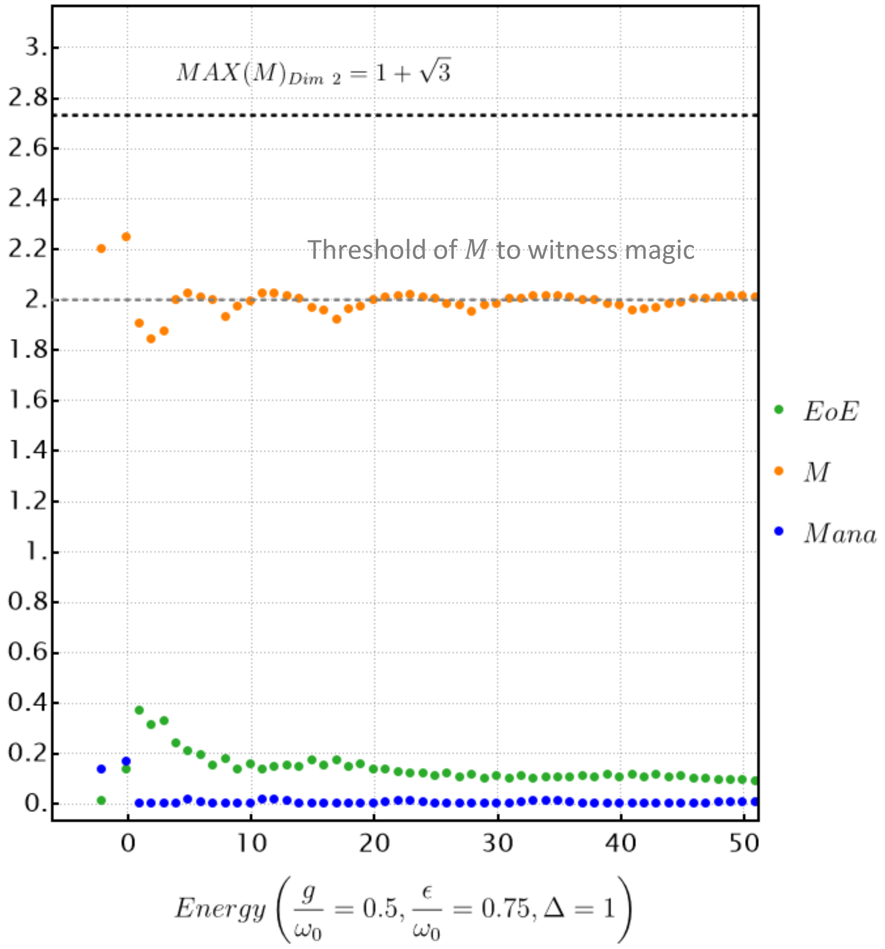
Magic in the atomic subspace



Fixed ϵ/ω_0, g/ω_0 = 0.25



Magic in the atomic subspace



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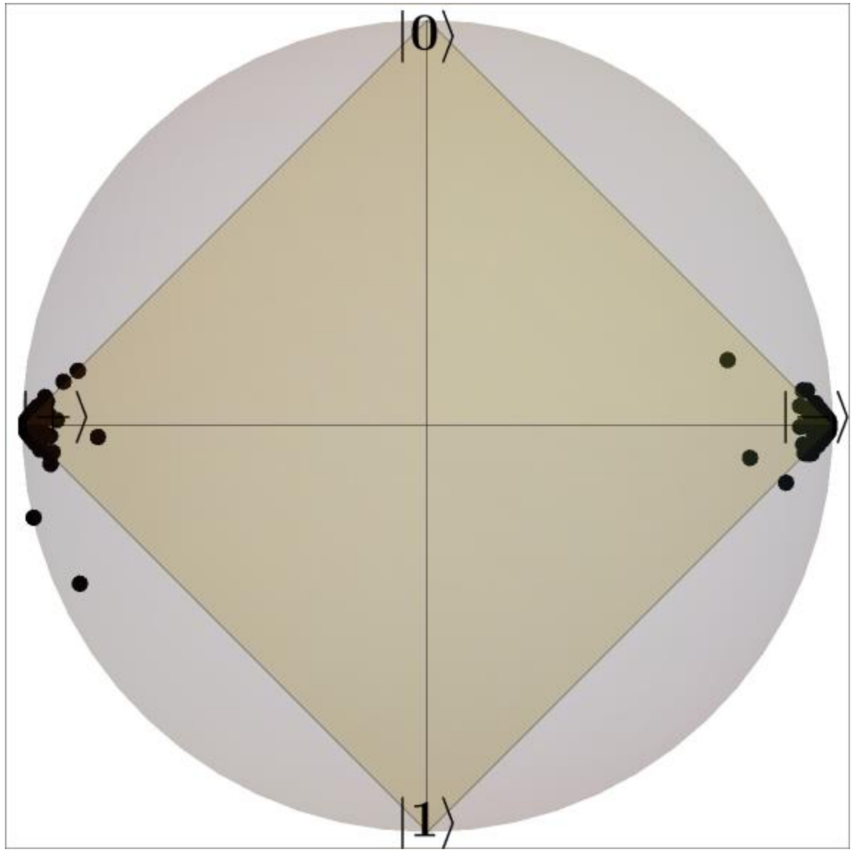
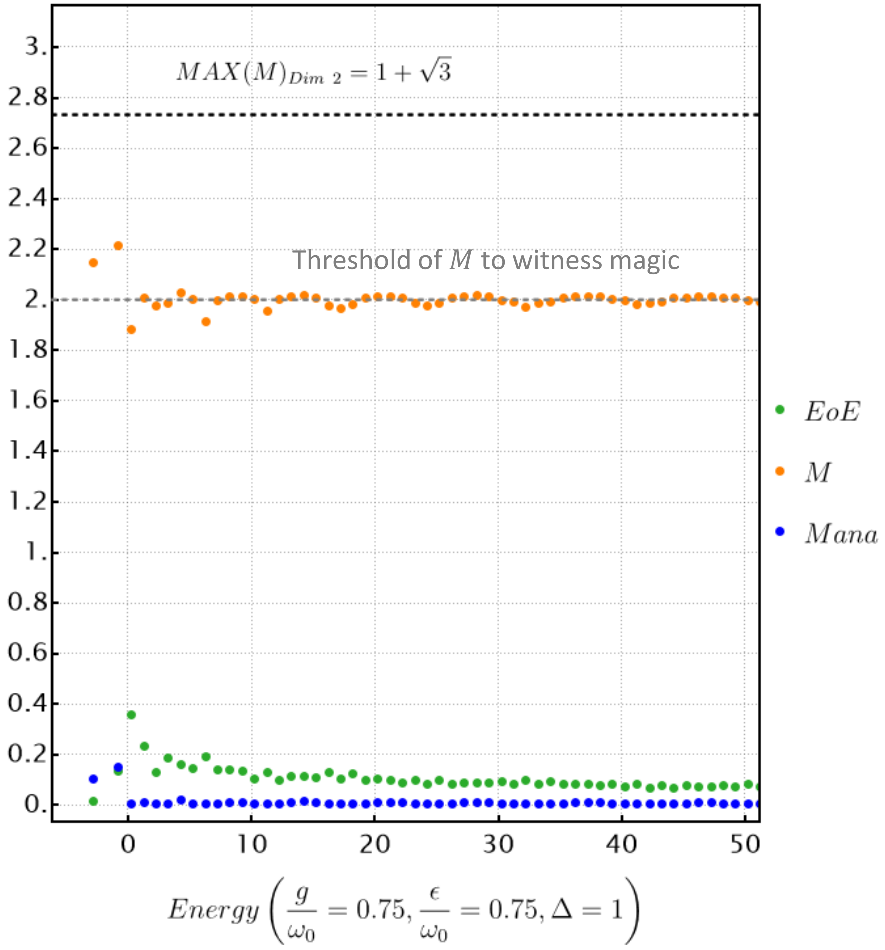
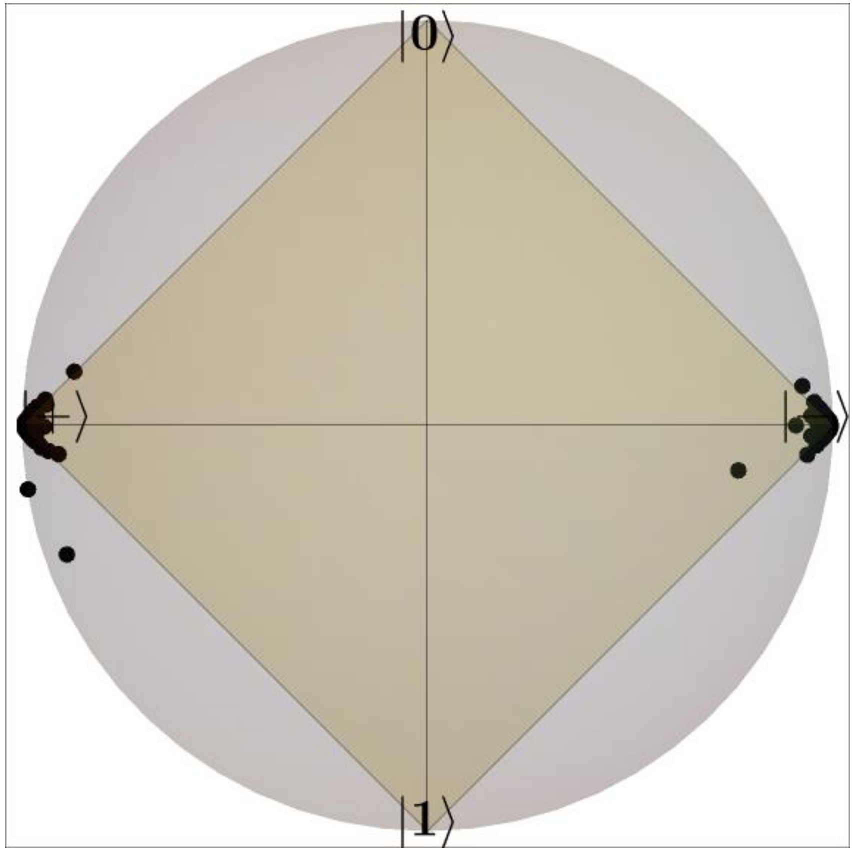


Magic in the atomic subspace



Fixed ϵ/ω_0, g/ω_0 = 0.75



Magic in the bosonic subspace

- Remember the Fock Wigner function,

$$W(\hat{\rho},\alpha,\bar{\alpha}) = \mathrm{tr}\Big(\hat{\rho}\ \hat{W}(\alpha,\bar{\alpha})\Big), \qquad \hat{W}(\alpha,\bar{\alpha}) = \hat{D}(\alpha)\hat{\Pi}\hat{D}^{\dagger}(\alpha)$$

where $\hat{\Pi} = \sum_{n=0}^{\infty}(-1)^n\,|n\rangle\langle n|$ is the parity operator and

$\hat{D}(\alpha) = \exp\big(\alpha\hat{a}^{\dagger} - \bar{\alpha}\hat{a}\big)$ the Fock displacement operator. And

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\Big(\hat{q} + \frac{i\hat{p}}{m\omega}\Big), \qquad \alpha(q,p) = \sqrt{\frac{m\omega}{2\hbar}}\Big(q + \frac{ip}{m\omega}\Big)$$

Fortschr. Phys. 65 1600092 (2017)

Magic in the bosonic subspace

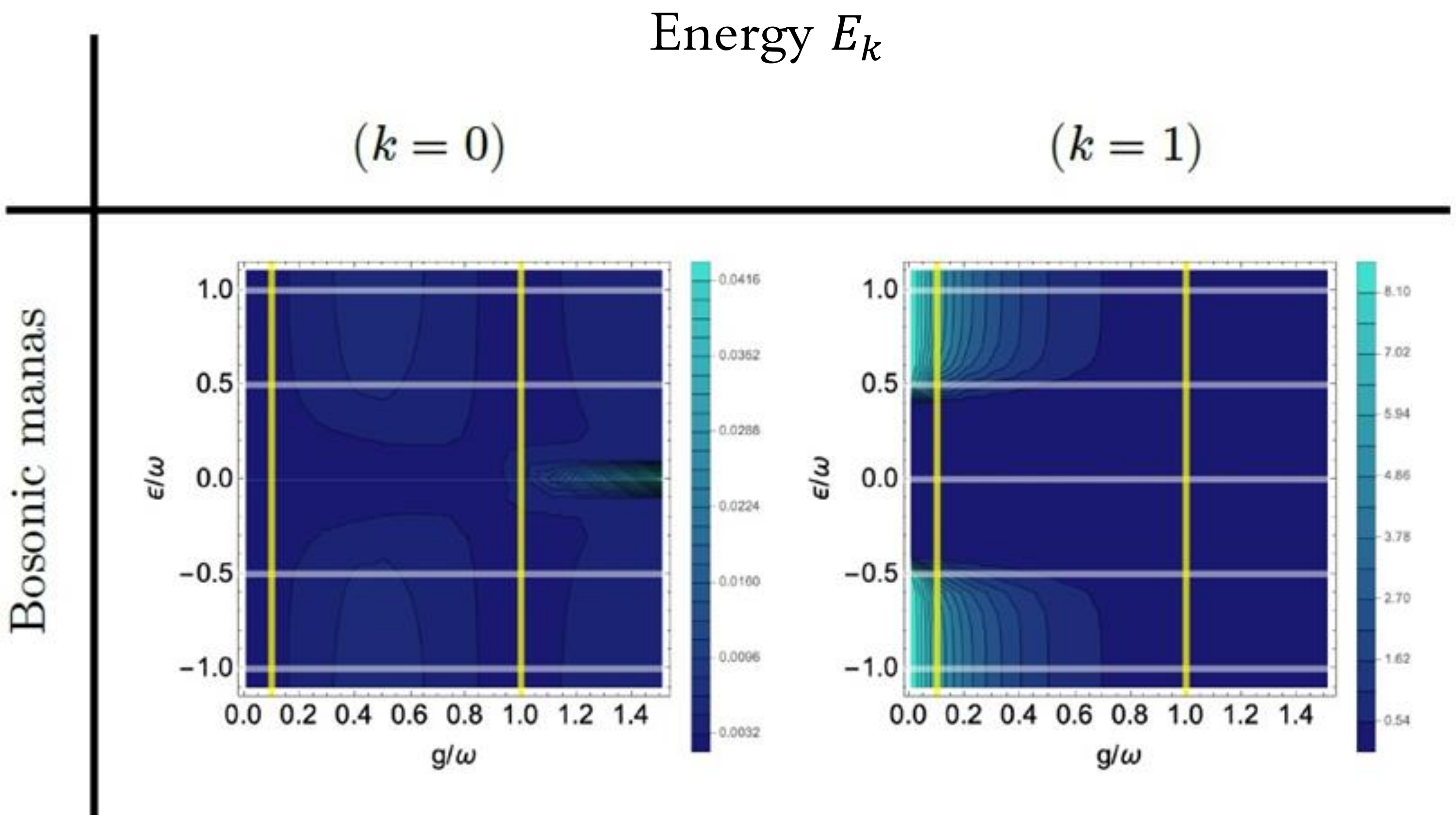
To numerically quantify magic in bosonic space we made a grid of the phase space of the traced subsystem,

$$\text{Grid} = \Big\{(\alpha_R, \alpha_I) \;:\; \alpha_R = [-\ell, \ell],\; \alpha_I = [-\ell, \ell]\Big\}$$

And, we obtain plots for a given energy E_k,

$$\Big\{(\alpha_R, \alpha_I, W[E_k, \alpha, \bar{\alpha}]) \;:\; \alpha \in \text{Grid}\Big\}$$

Magic in the bosonic subspace

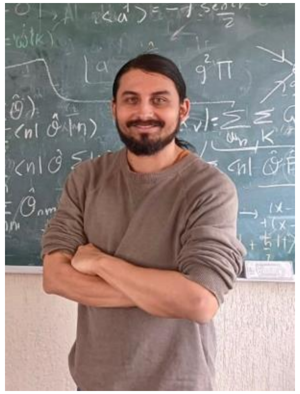


Conclusions

- We characterize magic in the AQRM subsystems, namely, atomic and bosonic.
- We compare the behaviour of a variety of measures of magic in each subsystem.
- We also check the behaviour of magic, comparing mana for fixed parameters in both subsystems.
- Our endevours pave the way towards a hybrid measure of magic in AQRM.

Acknowledgments

Collaborators



Titular Proffesor - Miguel Angel Bastarrachea Magnani **(UAMI)**

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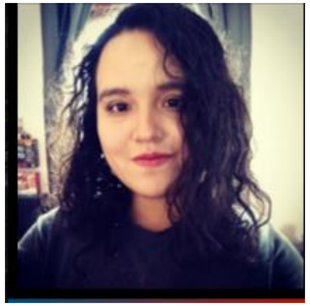

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Thank you!