Geometry of quantum dynamics 2024

28th August 2024, Siegen

Newton's laws of motion can generate gravity-mediated entanglement

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arXiv:2401.07832

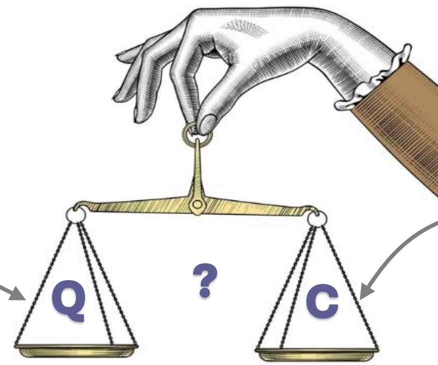
1. Motivations

   - Is Gravity fundamentally Quantum or Classical?
   - Can we use Gravity-mediated entanglement
     to investigate the fundamental nature of Gravity?

2. Classical phase-space model for Gravity-mediated entanglement

Results & Outlook

Do we need to quantize Gravity?



- **No consensus** on quantum gravity
- **Lack of direct experimental evidence**
- **Weakest interaction** Planck mass $E \sim 10^{19} GeV$

- **Semi-classical models**

✖ ~~Kafri–Taylor–Milburn (KTM)~~ → Atom interferometry experiments

Kafri, D., et al, New Journal of Physics **16.6**, 065020 (2014).

Altamirano, N., Classical and Quantum Gravity **35**, 145005 (2018).

Measurement-feedback

Carney, D., et al, arXiv:2301.08378 (2023).

Khosla, K. E., arxiv:1812.03118 (2018).

- **Reduction models**

Diósi, L., Physical Review A, **40**(3), 1165 , (1989).

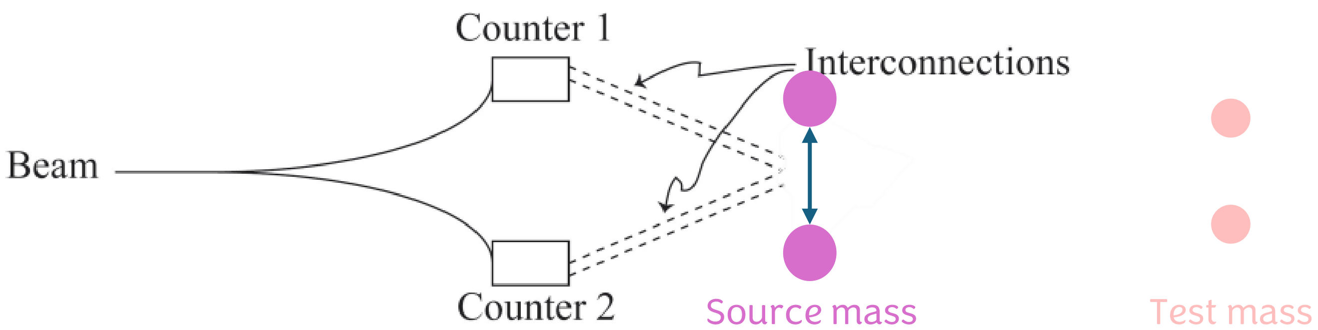
Tilloy A., Physical Review D **93**, 024026 (2016).

- **... and many others**

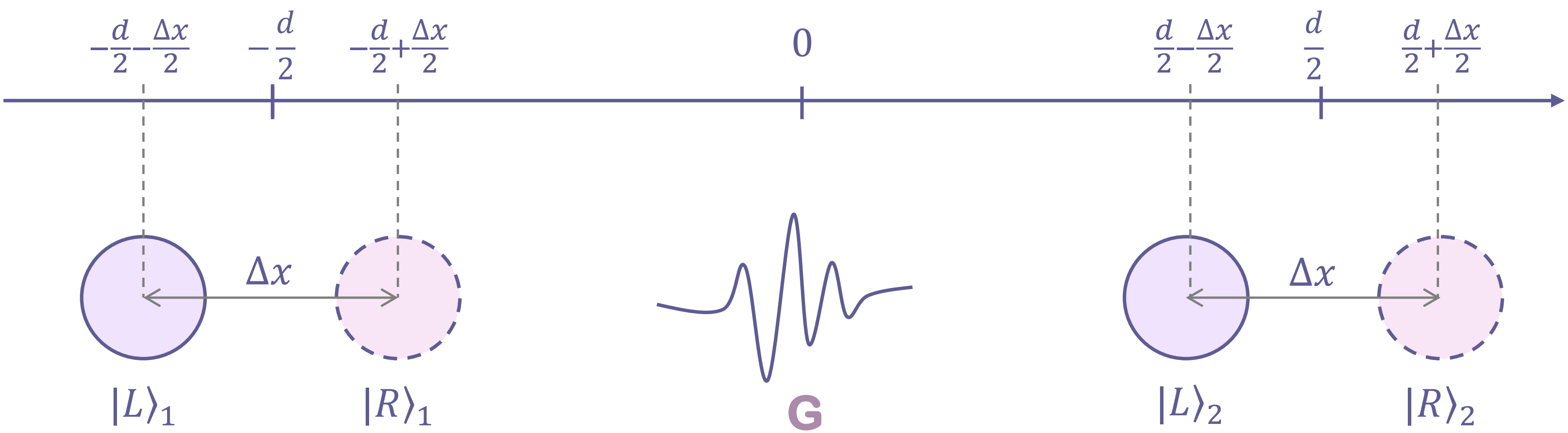
Tilloy, A., Journal of Physics: Conference Series, Vol. **1275** (2019) .

Oppenheim, J. , Physical Review X **13**, 041040 (2023) .

Feynman thought experiment



DeWitt, C. M., & Rickles, D., The role of gravitation in physics: report from the 1957 Chapel Hill Conference, (2011).



$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1) \times \frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$

$$H(x_1, x_{2,}, p_1, p_2) = \frac{p_1^2 + p_2^2}{2m} - \frac{G\, m^2}{|x_1 - x_2|}$$

Schrödinger equation

$$|\psi_Q(t)\rangle = \frac{e^{i\phi}}{\sqrt{2}}\left[|LL\rangle + e^{i\Delta\varphi_{LR}\, t}|LR\rangle + e^{i\Delta\varphi_{RL}\, t}|RL\rangle + |RR\rangle\right]$$

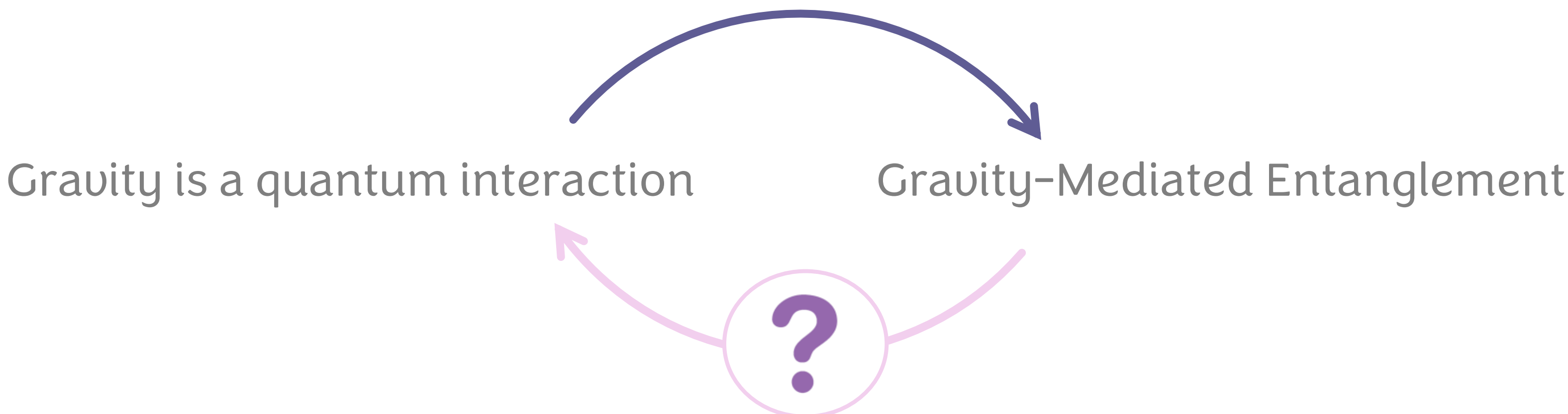
Bose, S., et al, Phys. Rev. Lett. **119**, 240401 (2017)

Marletto, C., et al, Phys. Rev. Lett. **119**, 240402 (2017)

GME as testable quantum feature!

Entanglement between two systems cannot be created

by Local Operations and Classical Communication (LOCC)



Is the detection of GME a proof of quantumness?

GME detection can be explained by classical descriptions

Concerns

- GME by **virtual gravitons**

  → LOCC assumption: interaction mediated by physical systems ⚠

- GME detection implies that the **state of the field was entangled with the particle**

  → Entanglement monogamy: a pure state cannot be entangled with any other state ⚠

- **Operator valued interaction**

  → Classical systems described in the Hilbert space with Koopman-von Neumann theory ⚠

Fragkos, V., et al, AVS Quantum Science **4**, (2022).
Bose, S.,et al., Physical Review D **105**.10 : 106028 (2022).
Piasecki, D., Arxiv: 2112:05619 (2021).

Quantum theory

- Initial state (superposition)
- Time evolution (Schrödinger equation)

Observe dynamical effects that cannot be explained by any classical approximation

Phase-space

Quantum Moyal equation (= Schrödinger equation)

(Quantum contribution)

$$\dot{W} = \{H, W\} + \mathcal{O}(\hbar^2)$$

(Poisson brackets)

Quantum Moyal equation

(= Schrödinger equation)

(Quantum contributions)

$$\dot{W} = \{H, W\} + \mathcal{O}(\hbar^2)$$

(Poisson brackets)

$W \to P$

Classical Liouville equation

$$\dot{P} = \{H, P\}$$

(= Newton's Law of motion)

Compare with evolution given by Schrödinger equation

W. P. Schleich, *Quantum optics in phase space*, JohnWiley & Sons, 2011

Reproduce GME with:

- Classical time evolution in **phase-space**
- Suitable **approximation of gravitational potential**

If $U(x) \approx U(\bar{x}) + (x - \bar{x})U'(\bar{x}) + (x - \bar{x})^2\, U''(\bar{x})$

Quantum Moyal equation coincide with Classical Liouville equation

Wigner function will evolve with classical trajectory in phase space

(Quantum and Classical evolution indistinguishable)

Marchese, M. M., et al, ArXiv:2401.07832 (2024)

Quantify entanglement

A quantum state $\rho(t)$ is pure and separable, i.e., $Tr[\rho(t)^2] = 1$ $\rho(t) = \rho_A(t) \otimes \rho_B(t)$

Iff all the marginals $Tr_B[\rho(t)] = \rho_A(t)$ are pure i.e., $Tr[\rho_A(t)^2] = Tr[\rho_B(t)^2] = 1$

$$Tr[\rho_A(t)^2] < 1 \longrightarrow \rho(t) \neq \rho_A(t) \otimes \rho_B(t)$$

Purity for particle 2 with the Quantum evolution

$$\gamma_Q = \frac{3 + \cos[(\Delta\varphi_{LR} + \Delta\varphi_{RL})t]}{4} < 1$$

The total final state is entangled

Newtonian Gravitational potential

$$V_G(\boldsymbol{x_{rel}}) = -\frac{Gm^2}{d + \sqrt{2}\boldsymbol{x_{rel}}}$$

$$x_{rel} = \frac{1}{\sqrt{2}}(x_2 - x_1 - d)$$

Taylor expansion

$$V_{Taylor}(\boldsymbol{x_{rel}}) \sim -\frac{Gm^2}{d} + \frac{\sqrt{2}Gm^2}{d^2}\boldsymbol{x_{rel}} - \frac{2Gm^2}{d^3}\boldsymbol{x_{rel}^2}$$

Second order polynomial

$$V_{fit}(\boldsymbol{x_{rel}}) \sim -\frac{Gm^2}{d} + \frac{\sqrt{2}Gm^2}{d^2 - \Delta x^2}\boldsymbol{x_{rel}} - \frac{2Gm^2}{d(d^2 - \Delta x^2)}\boldsymbol{x_{rel}^2}$$

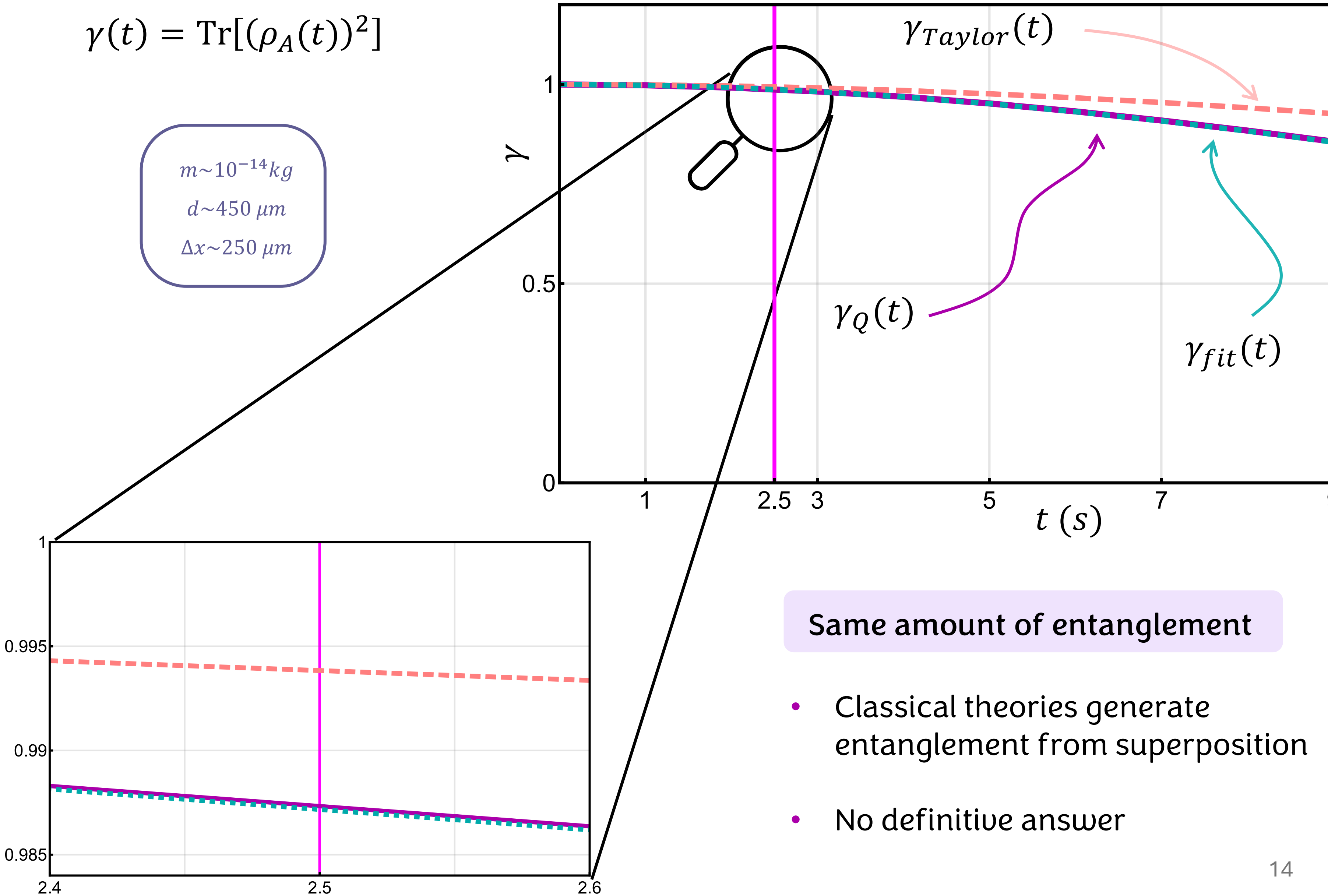
Fitting points

$x_{rel} = 0 \qquad x_{rel} = \pm {}^{\Delta x}/_{\sqrt{2}}$

Results

$$\gamma(t) = \mathrm{Tr}[(\rho_A(t))^2]$$

$m \sim 10^{-14} kg$
$d \sim 450\ \mu m$
$\Delta x \sim 250\ \mu m$



Same amount of entanglement

- Classical theories generate entanglement from superposition
- No definitive answer

High order terms?

Stepwise potential

$$V_{Step}(\boldsymbol{x_{rel}}) = -\frac{m^2 G}{\bar{x}_j} \qquad F_j = \frac{m^2 G}{\bar{x}_j{}^2} \qquad \bar{x}_j = |x_{0,j} - y_{0,j}|$$

Negativity in the marginals arises at $\boldsymbol{t \geq 1.6\ s}$

$$\frac{1}{2}\left[\int |W(p_1, p_2, t)| dp_1 dp_2 - 1\right] \approx 0.1\%$$

Classical dynamics leads to unphysical states!

Noise model

$$\dot{W}(x_1, x_2, p_1, p_2) = D\left[\int dq_1 dq_2\, g\,(q_1, q_2)\, W(x_1, x_2, p_1 - q_1, p_2 - q_2) - W(x_1, x_2, p_1, p_2)\right]$$

- Even the minimal diffusion wash out the entanglement

Conclusions

- **Classical evolution** in phase-space can generate **Gravity-mediated Entanglement**

  ☑ Second-order approximation indistinguihable from quantum dynamics

- The proposed experiments for **GME do not prove that gravity is quantum**

Design of **new experiments** to rule out **ALL** classical models

- Longer experimental times
- Different arms separations

Test different quantum feature as signature of quantum Gravity

$$\dot{W} = \{H, W\} + \mathcal{O}(\hbar^2)$$

Thank you for your attention!

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|TQO⟩ Theoretical Quantum Optics