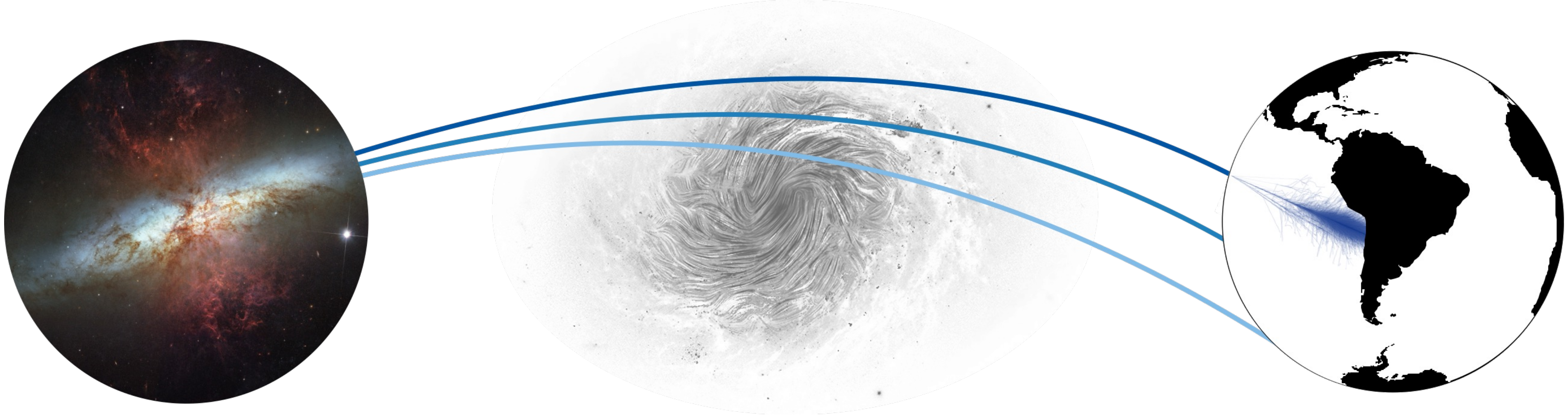
Application of Information Field Theory to UHECR Deflections in the Galactic Magnetic Field

Auger Youngsters 2024



Frederik Krieger, Martin Erdmann

Bayesian statistics

- probability = *degree of belief in event*
- incorporates prior knowledge about event

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Comparison to frequentist statistics

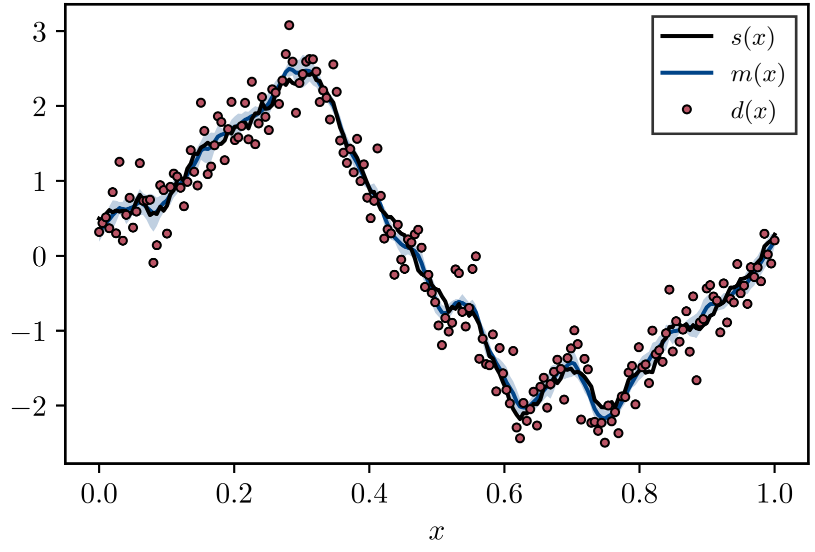
	Frequentist	**Bayesian**
probability definition	relative frequency of event after (infinitely) many trials	degree of belief in event
hypothesis test	check data against null-hypothesis → p-value	probability of hypothesis → Bayes factor
parameter inference	point estimate $\hat{\theta}$ using MLE	posterior distribution $p(\theta\|d)$

Posterior approximation

- Markov chain Monte Carlo (MCMC)
- Bayesian neural networks, normalizing flows, variational autoencoders
- **Information Field Theory** (IFT)
- …

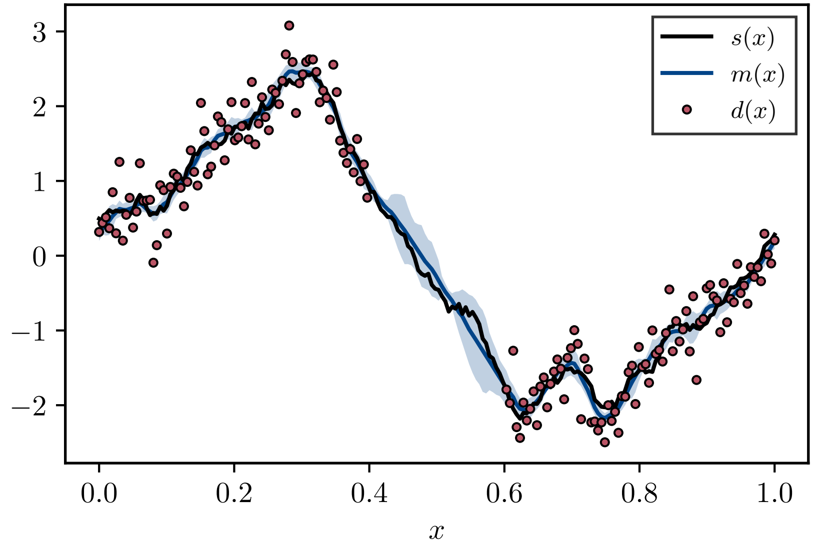
Information Field Theory (IFT)

- information theory for fields
- inference of **physical fields** with many degrees of freedom from noisy & incomplete data
- leverage physical assumptions via priors



Theory basics

- Bayesian probability theory: $\mathcal{P}(s|d) = \frac{\mathcal{P}(d,s)}{\mathcal{P}(d)} \equiv \frac{e^{-\mathcal{H}(d,s)}}{\mathcal{Z}(d)}$
  - s: signal of interest
  - d: data
  - $\mathcal{P}(s|d)$: posterior distribution
  - $\mathcal{H}(d,s)$: information Hamiltonian
- measurement equation: $d = R[s] + n$
  - R: operator
  - n: noise



Approximating posteriors with IFT

NIFTy

- **N**umerical **I**nformation **F**ield **T**heory
- software package written in python
- multiple inference algorithms, e.g.
  - Metric Gaussian Variational Inference (MGVI)
  - Geometric Variational Inference (geoVI)

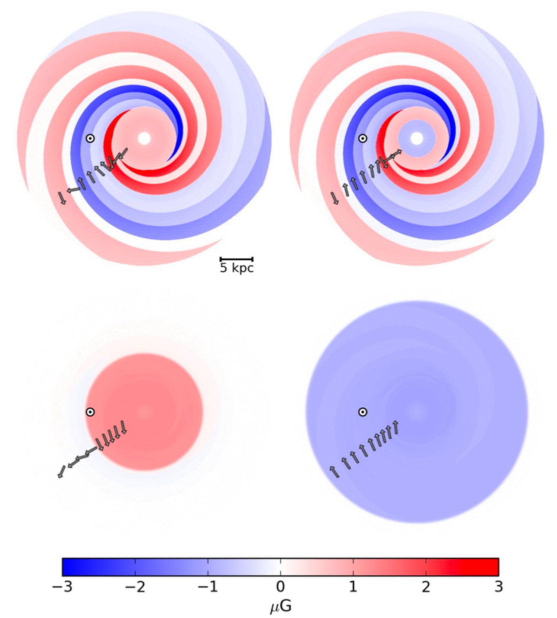
JAX

- open-source machine learning framework from Google
- numpy-like syntax
- key features:
  - automatic differentiation
  - just-in-time compilation (JIT)
  - execution on GPUs and TPUs

UHECR deflections in the Galactic magnetic field

Galactic magnetic field (GMF)

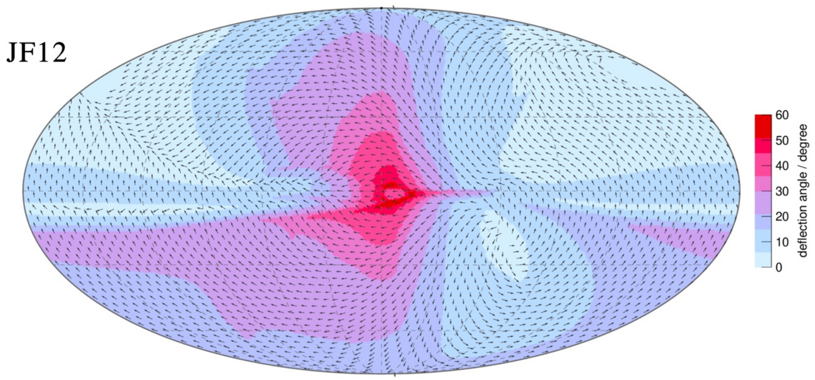
- order of magnitude: μG
- **coherent** and **turbulent** components
- measured by Faraday rotation and polarized synchrotron radiation



coherent GMF component of JF12 model

Deflections of UHECRs

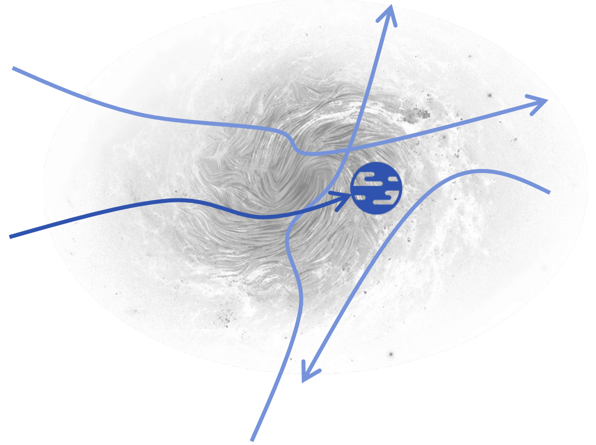
- Lorentz force: $\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = q\,\vec{v} \times \vec{B}(\vec{x})$
- dependent on initial direction, GMF model (JF12, UF23) and rigidity $R = \frac{E}{Ze}$ of particle



deflection direction & strength of 20 EV particle

Galactic magnetic field lenses for UHECR deflections

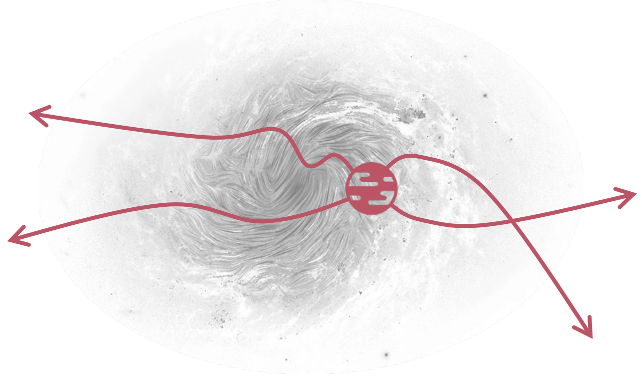
- only tiny fraction of UHECRs that enter the Galaxy arrive at Earth
  → forward simulation is expensive
- **idea**: backpropagation of UHECRs with opposite charge



GMF lenses

- backtracking of N particles originating at Earth in direction of pixel i and count how many CRs are deflected in direction j
- lens is matrix L with entries $L_{ij} = \frac{n(i \to j)}{N}$
- relates detected sky map at Earth $\vec{a}$ to extragalactic illumination map $\vec{b}$ via

$$\boxed{\vec{a} = L\,\vec{b}}$$



Modeling UHECR deflections for IFT

Goal: find sources of UHECRs

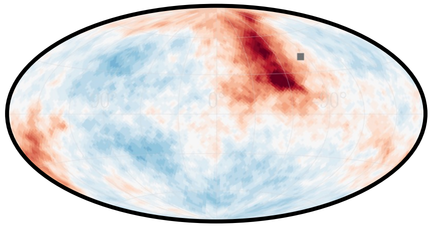
Challenges:

- deflections in GMF & EGMF
- uncertainty of GMF models
- no exact charge information, only $X_{\max}$

Idea:

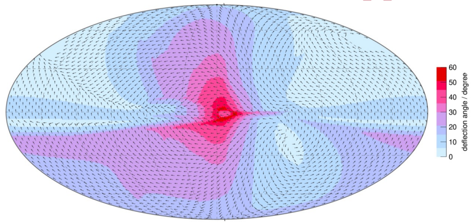
- reconstruct extragalactic illumination map to find overdensities in UHECR flux
  $\rightarrow$ possible origin direction of UHECRs
- IFT measurement equation: $d = R[s] + n$
  - s: extragalactic illumination map with charge and energy distribution
  - R: forward model $\rightarrow$ lensing, draw $X_{\max}$ from charge
  - n: measurement noise (Poisson)
  - d: measured flux, energy and $X_{\max}$ distribution at Auger

Signal s



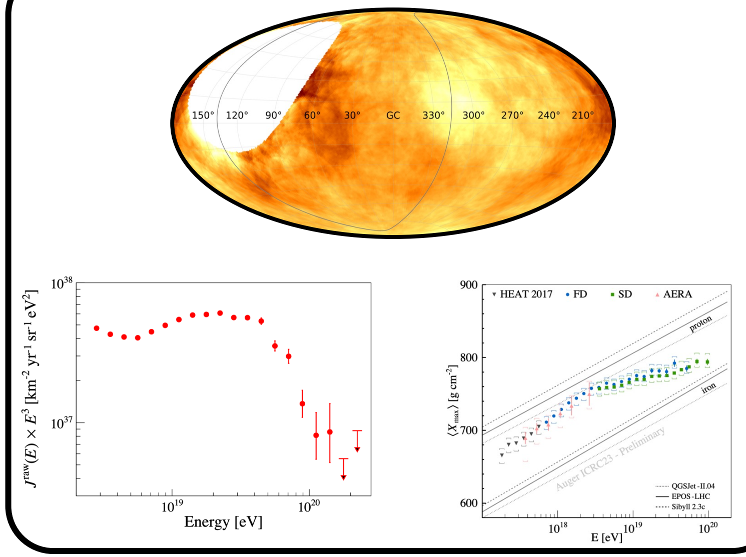
$\Downarrow$

Forward model $R[\cdot]$



$=$

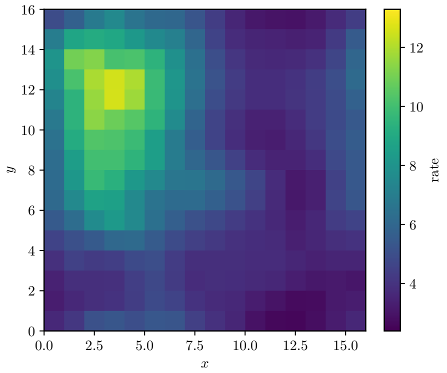
Data d



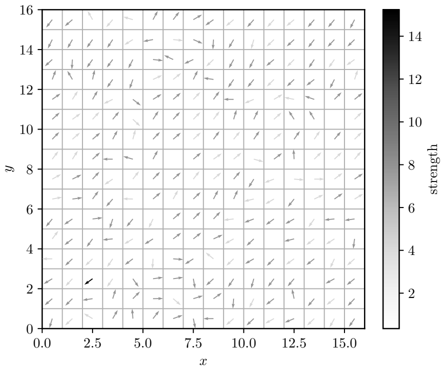
Toy model: 2D Euclidian deflections

- $N \times N$ Euclidian grid with periodic boundaries
- signal: extragalactic illumination map with correlation structure and R^{-1} rigidity spectrum
- forward model: GMF lenses constructed from deflection maps
- simulated measurement: flux at Earth per rigidity bin with Poisson noise

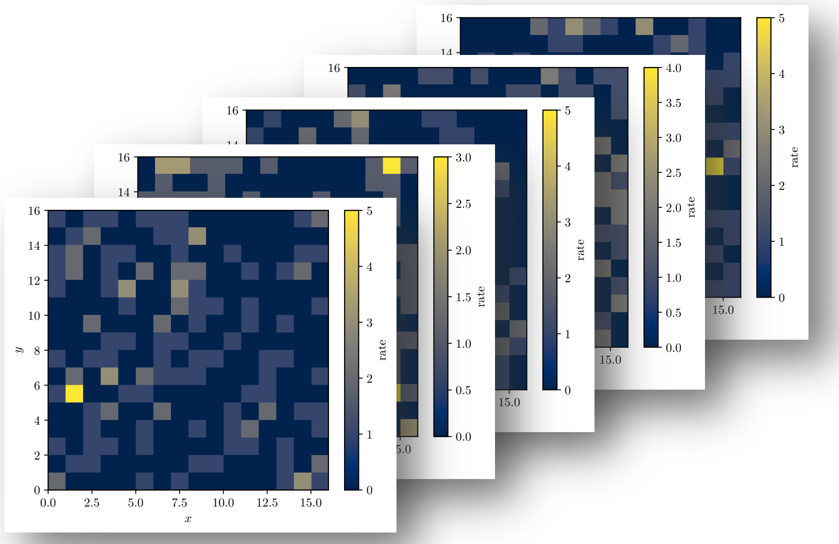
sampled with NIFTy



extragalactic illumination map

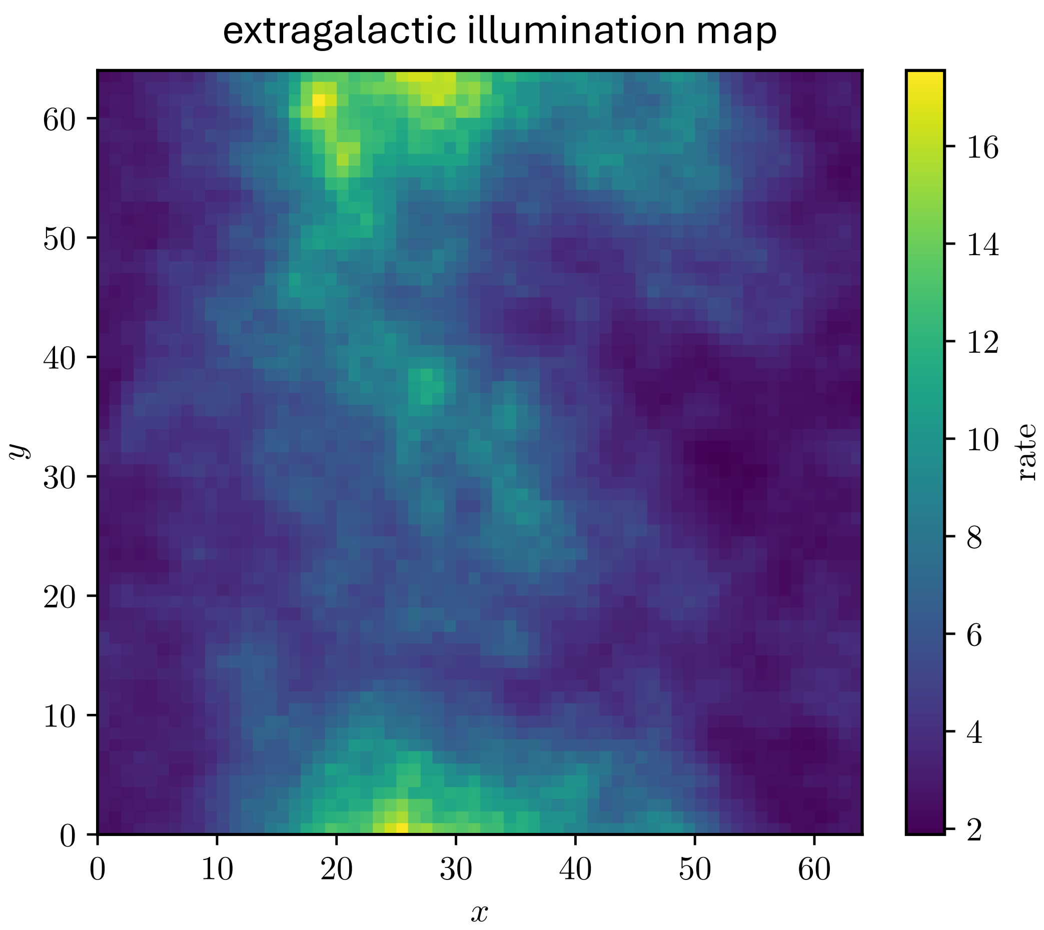


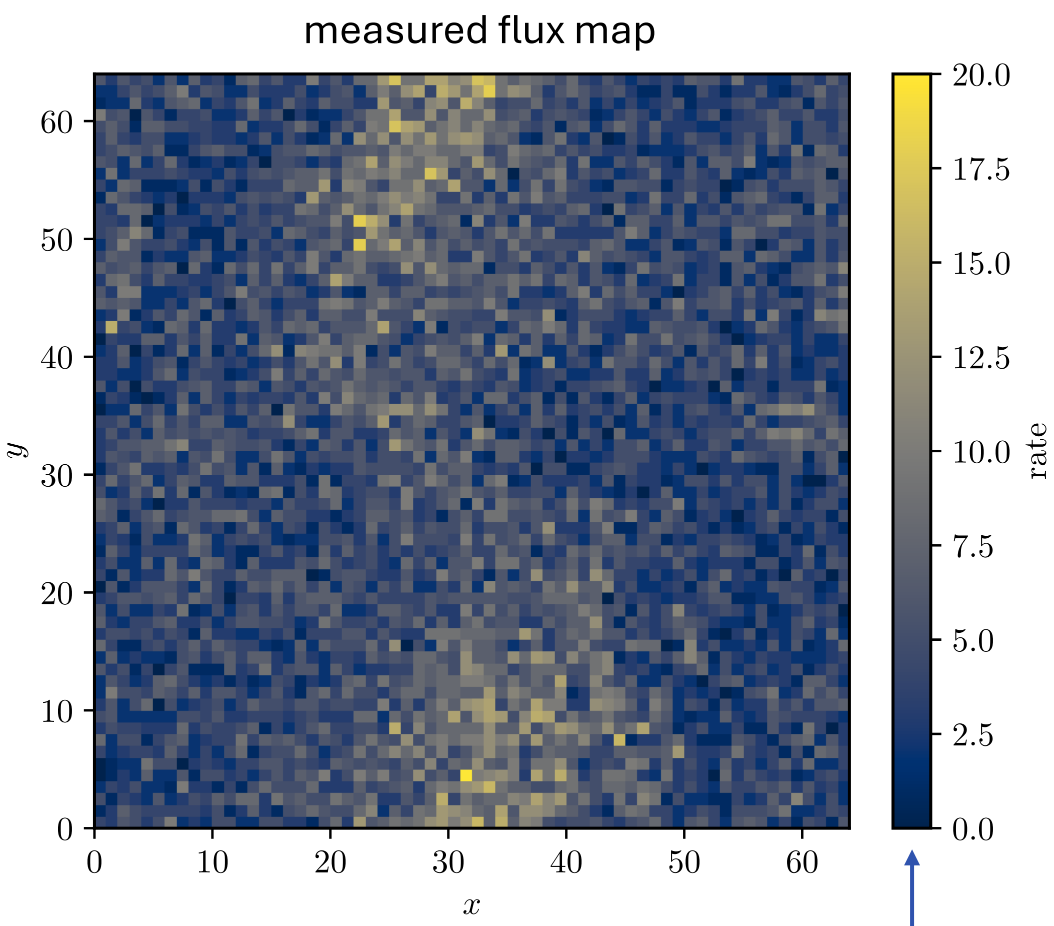
deflection map



measured flux per rigidity

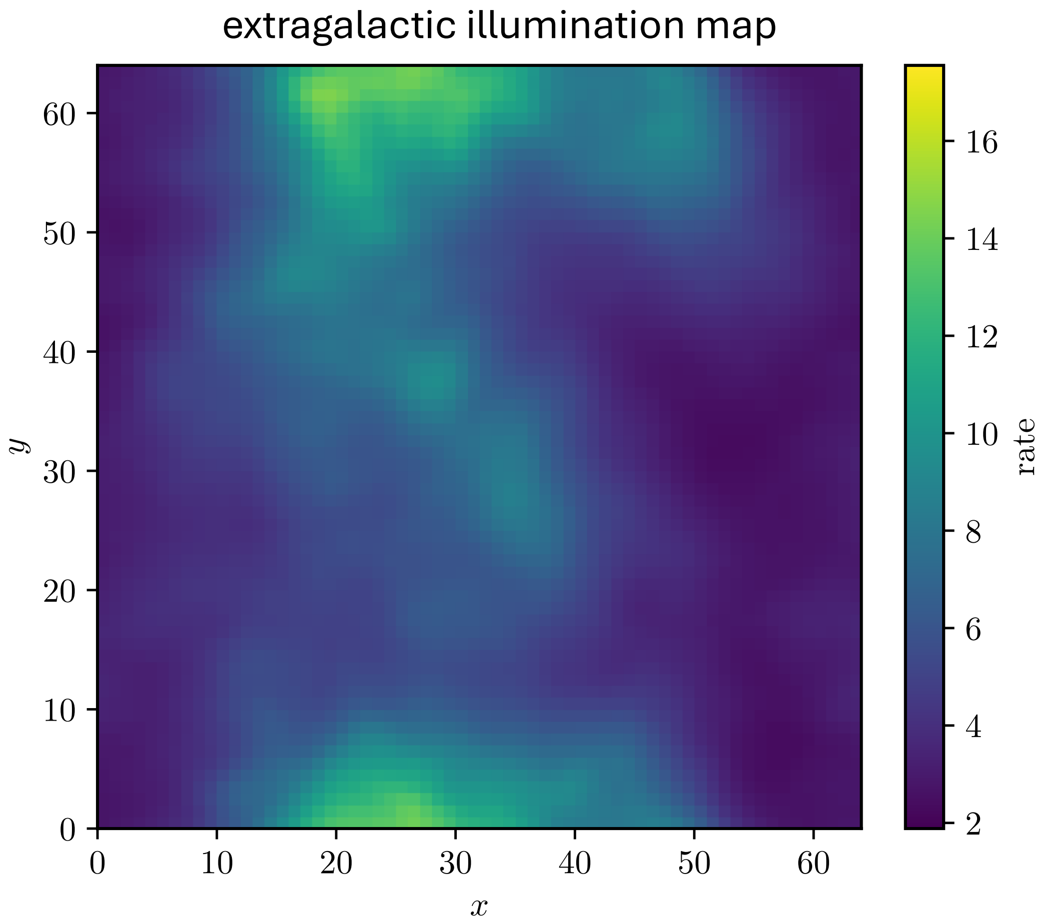
Toy model: ground truth

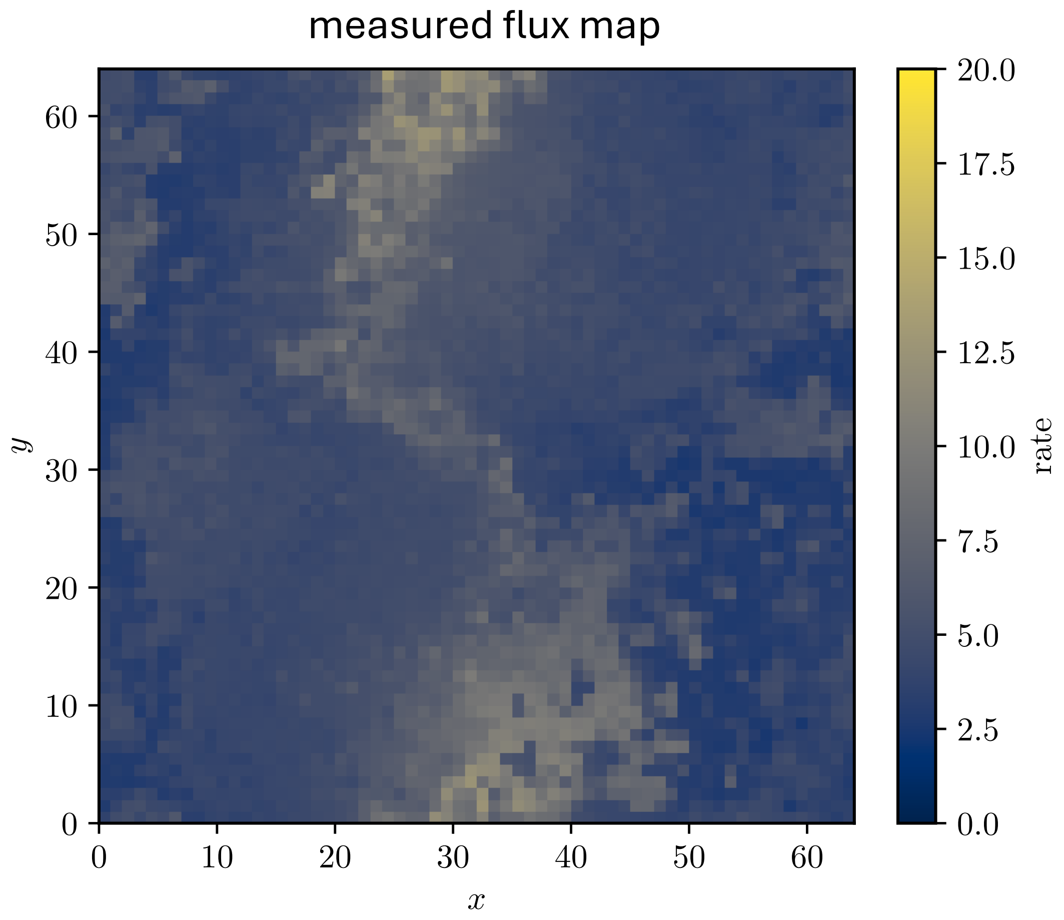




summed over all rigidities

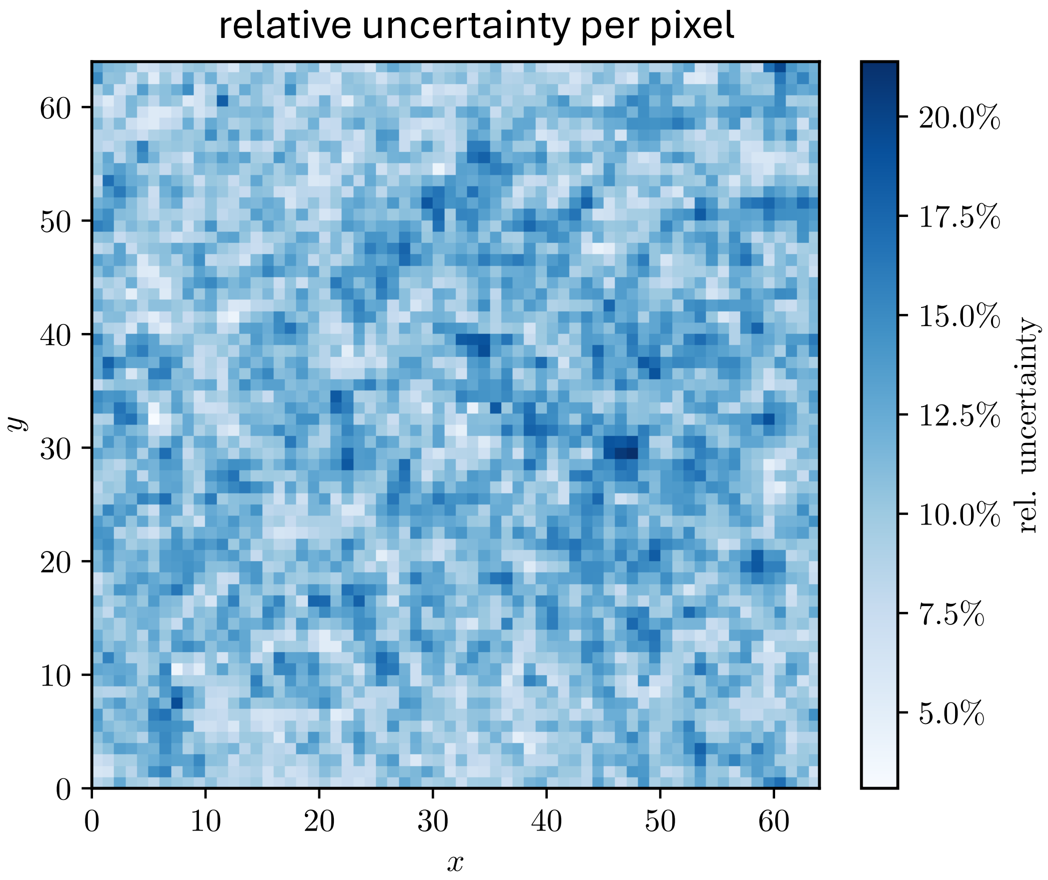
Toy model: fit results (mean prediction)

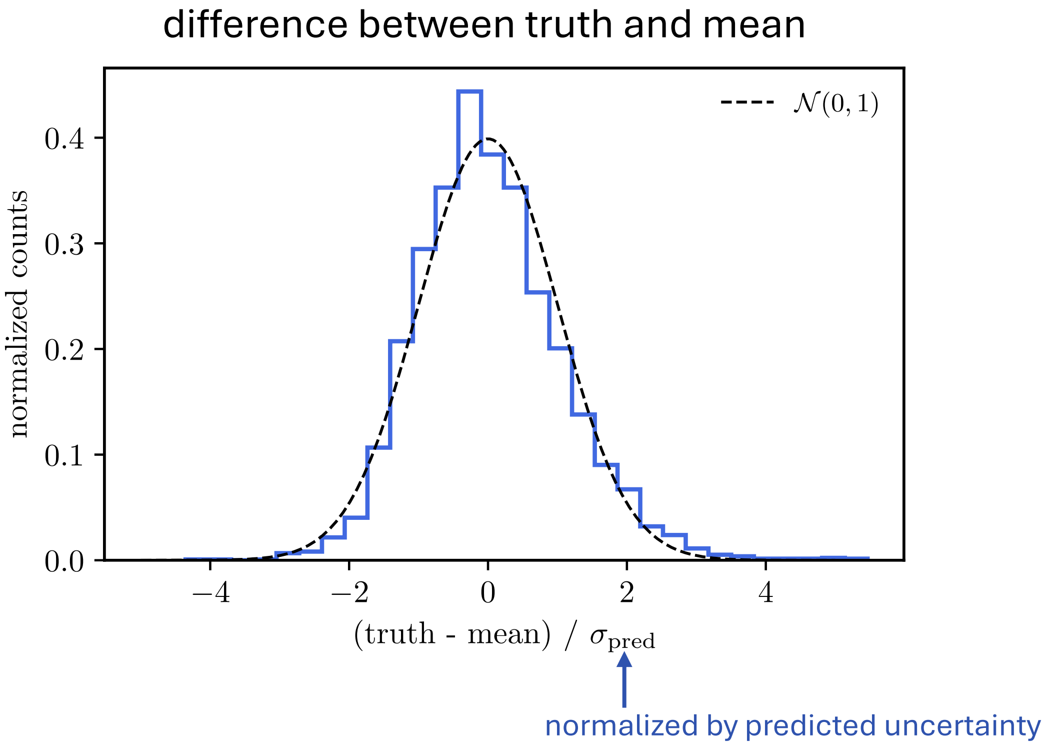




→ mean reconstruction works well

Toy model: fit results (predicted uncertainties)



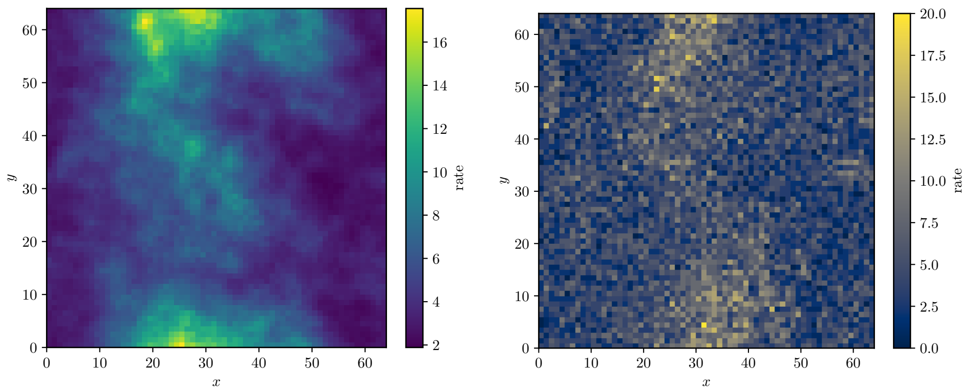


→ uncertainty reconstruction works as expected

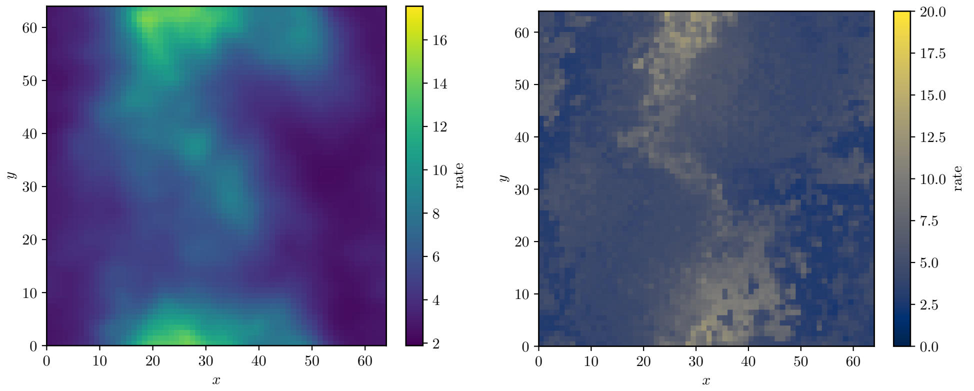
Summary and outlook

- Information Field Theory as Bayesian framework to infer properties of physical fields
- leverage physical assumptions via priors
- posterior distributions provide uncertainty measure
- NIFTy with JAX: fast computation on GPUs

Truth



Reconstruction



Next steps

- spherical coordinates (HEALPix)
- rigidity $\rightarrow X_{\mathrm{max}}$ and energy
- physical motivated simulations