Power Corrections in Mass and Scattering Angle at High Energy

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May 2023

Topics discussed

- QCD at high energy
  - *Sudakov and Regge limits*
  - *power corrections and renormalization group*
- Sudakov limit
  - *Higgs production at NLL* $\mathcal{O}(m_q^2/s)$
  - *Higgs production at LL* $\mathcal{O}(m_q^4/s^2)$
- Regge limit
  - *factorization and resummation at LL* $\mathcal{O}(t/s)$
  - *unitarity problem and its possible solution*
- *some magic relations*

Based on

*C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)*

*A.A. Penin, JHEP **04**, 156 (2020)*

*T. Liu, S. Modi, A.A. Penin, JHEP **02**, 170 (2022)*

High energy behaviour of gauge theories

- Sudakov limit
  - *high-energy fixed-angle* $m^2/s \to 0,\ t/s \sim 1$
  - *Sudakov suppression* $e^{-\frac{\alpha}{4\pi}\frac{C_R}{2}\ln^2(s/m^2)}$

Sudakov (1956); Frenkel, Taylor (1976)

- Regge limit
  - *high energy small angle* $t/s \to 0,\ m^2/t \sim 1$
  - *Regge-like enhancement* $s^{\gamma(\alpha)}$ with $\gamma(\alpha) > 0$

Gribov, Lipatov, ... (1960s-1970s)

? *Theory beyond the leading power in* $m^2/s,\ t/s$

High energy limit beyond the leading power

- Power *vs* perturbative corrections
  - Λ_X^2/Q^2 *vs* α_X^n
- Logarithmically enhanced power corrections
  - *phenomenologically crucial (b-loop in Higgs production, etc.)*
  - *intriguing from QFT point of view*
  - ↪ *eikonal charge nonconservation, exponential enhancement, universality, etc.*
- Recent progress for NLP in
  - *mass, angle, momentum, threshold, jetiness, ...*
- *We focus on corrections in mass and scattering angle*

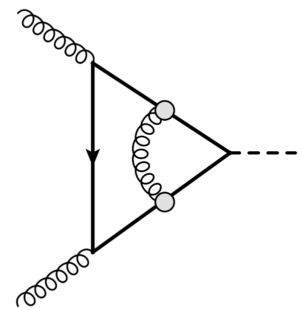
$gg \to H$ amplitude at NLP LL

T. Liu, A.A. Penin, Phys.Rev.Lett. **119** (2017) 262001

- soft quark emission ➪ eikonal color nonconservation

➥ *non-Sudakov double logs*

$$\mathcal{M}^{q\,LL}_{gg\to H} = Z_g^{2\,LL}\, g(z)\, \mathcal{M}^{q\,(0)}_{gg\to H}$$



$$Z_g^{2\,LL} = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{-s^{\varepsilon}}\right],\; z = (C_A - C_F)x,\;\; x = \frac{\alpha_s}{4\pi}\ln^2(m_H^2/m_q^2)$$

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta\;\; e^{2x\eta\xi} \;\; = {}_2F_2\,(1,1;3/2,2;x/2)$$

soft gluon loop exponent

$gg \to H$ amplitude at NLP NLL

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0} Z_g^{2\,NLL} \left[-\frac{3}{2}\frac{m_b^2}{m_H^2}L^2\,\mathcal{M}^{t\,(0)}_{gg\to H}\right]$$

Yukawa RG factor *gluon Sudakov form factor* *LO amplitude*

$$C_b = \left[g(z) + \frac{\alpha_s L}{4\pi}\left(2\gamma_q^{(1)} g_\gamma(z) - \beta_0 g_\beta(z)\right)\right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$c_1 = \frac{z}{6} + C_F\frac{\alpha_s L}{4\pi}, \quad c_2 = \frac{z^2}{45} + \frac{z}{5}\frac{\alpha_s L}{4\pi}\left[\frac{3}{2}C_F - \beta_0\left(\frac{5}{6}\frac{L_\mu}{L} - \frac{1}{3}\right)\right],$$

$$c_3 = \frac{z^3}{420} + \frac{z^2}{5}\frac{\alpha_s L}{4\pi}\left[\frac{5}{21}C_F - \beta_0\left(\frac{2}{9}\frac{L_\mu}{L} - \frac{2}{21}\right)\right], \quad \ldots$$

$$L = \ln(s/m_q^2),\; L_\mu = \ln(s/\mu^2)$$

3-loop coefficient c_2 *agrees with* Czakon, Niggetiedt, JHEP **05**, 149 (2020)

Top-bottom interference in threshold cross section

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

	LO	NLO	NNLO	N^{3}LO
$\delta\sigma^{\rm LL}_{pp\to H+X}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma^{\rm NLL}_{pp\to H+X}$	-1.420	-2.048	-2.170	-2.189
$\delta\sigma_{pp\to H+X}$	-1.023	-2.000		

- NLL K-factors with full threshold $\delta\sigma^{NLO}_{pp\to H+X}$

$$\delta\sigma^{\rm NNLO}_{gg\to H+X} \approx -0.12\ pb \qquad \delta\sigma^{\rm N^3LO}_{gg\to H+X} \approx -0.02\ pb$$

- *New uncertainty interval* -0.32 *to* 0.08 *pb* *(factor 2 reduction)*

NNLP LL: why?

- Theory
  - *proof of principle*
  - *the first step into terra incognita*
- Phenomenology
  - *validate the quark mass expansion*
  - *NNLO Higgs pair production at high* $p_\perp$

NNLP LL: how?

- Brut-force approach

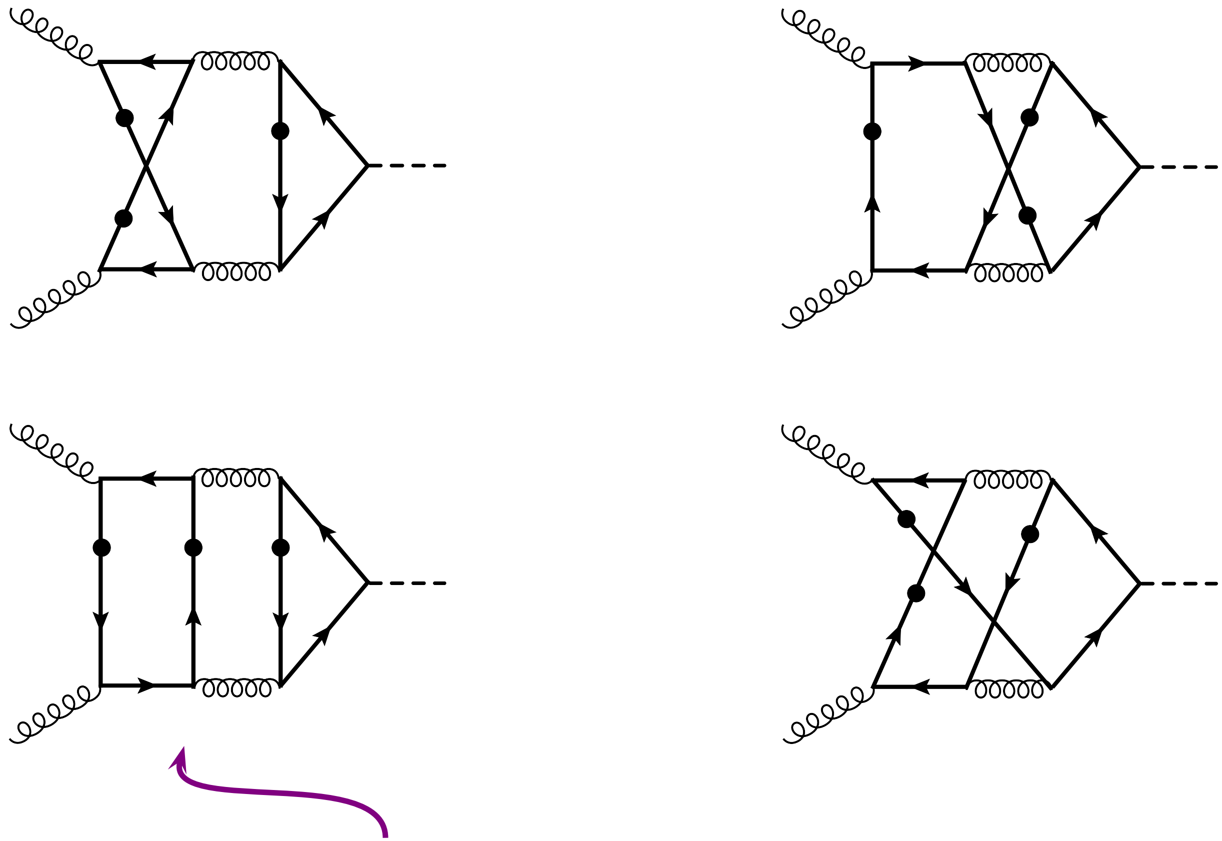
  A.A. Penin, Phys.Lett. B **119** (2015) 262001

  - *expansion by regions* ⇨ *homogeneous integrals* ⇨ *log corrections*
  - *Ward identities + momentum shifts + eikonal factorization* ⇨ *resummation*

- Classification of NNLP terms for $gg \to H$

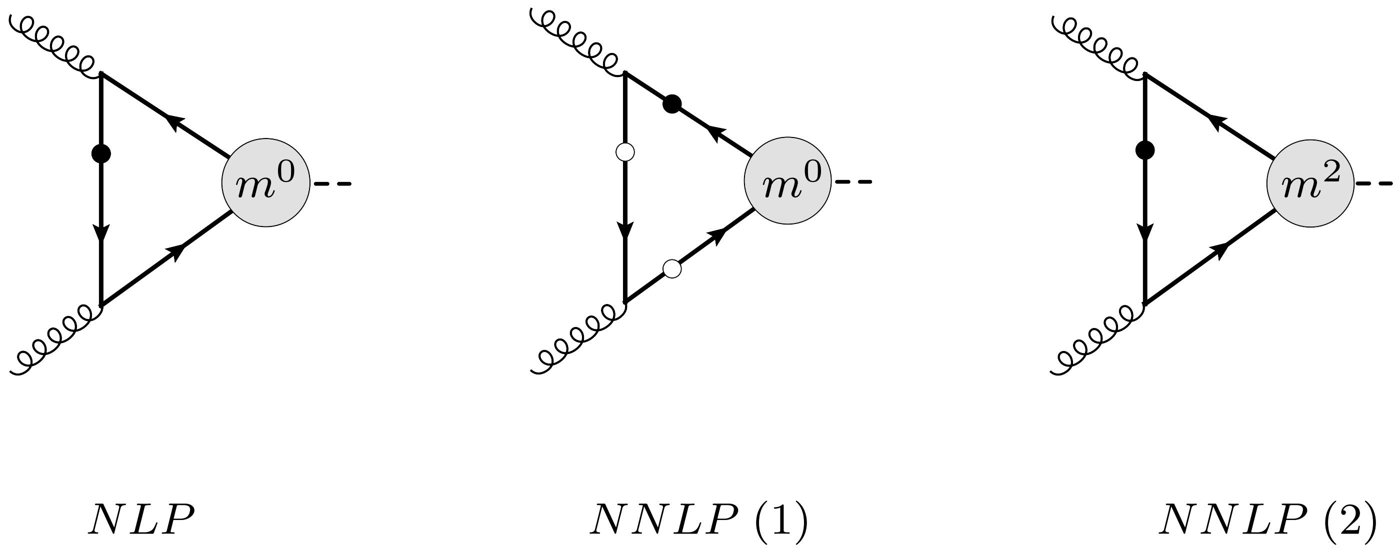
  - *soft quarks exchange: single or triple*
  - *topology: factorizable/nonfactorizable*

Triple soft quark exchange



only planar graph contributes ⇨ factorizable topology

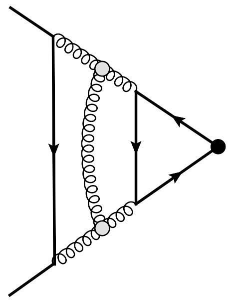
Factorizable contribution

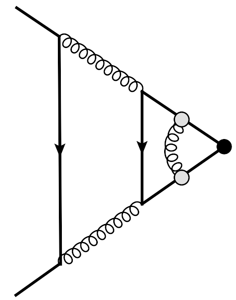


- *NNLP (1) power corrections to $gg \to q\bar{q}$ amplitude*
  - *non helicity-flip contribution for the first time*
- *NNLP (2) power corrections to the off-shell FF*
  - *can be inferred from on-shell result*

Scalar form factor NLP LL

- On-shell FF — T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)





$$F_S = Z_q^2 \sum_{n=0}^{\infty} \frac{m_q^2}{Q^2}^{n} F_S^{(n)}, \qquad F_S^{(1)} = -\frac{C_F T_F}{3} x^2 f(-z)$$

- *Magic #1: universal function for all NLP form factors*

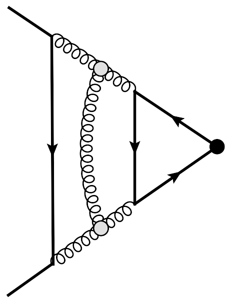
$$f(z) = 1 + \frac{z}{5} + \frac{11}{420} z^2 + \frac{z^3}{378} + \ldots$$

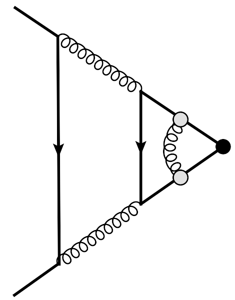
confirmed to 3 loops in M. Fael, et al. Phys.Rev.D **106**, 034029 (2022)

Scalar form factor NLP LL

- On-shell FF

T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)





$$F_S = Z_q^2 \sum_{n=0}^{\infty} \frac{m_q^2}{Q^2}^n F_S^{(n)}, \qquad F_1^{(1)} = -\frac{C_F T_F}{3} x^2 f(-z)$$

- *Magic #1: universal function for all NLP form factors*

$$f(z) \sim 6\left[\ln\left(\frac{z}{2}\right) + \gamma_E\right]\left(\frac{2\pi e^z}{z^5}\right)^{1/2}$$

$$f(-z) \sim \left[(\ln(2z) + \gamma_E)^2 - \frac{\pi^2}{2}\right]\frac{3}{z^2},$$

asymptotic behaviour at $z \to \infty$

Scalar form factor NLP LL

- Off-shell FF

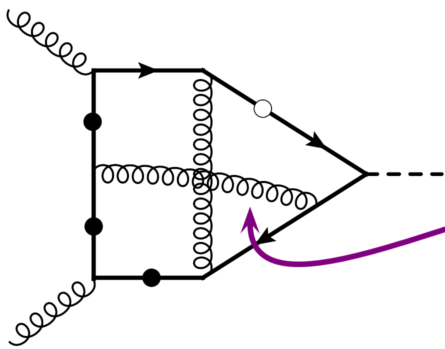
- *change of IR cutoff* $m^2 \to p_i^2$

$$f(z) \to 12 \int_0^{\eta} \mathrm{d}\eta_2 \int_0^{\xi} \mathrm{d}\xi_2 \int_0^{\eta_2} \mathrm{d}\eta_1 \int_0^{\xi_2} \mathrm{d}\xi_1 \, e^{-2z\eta_2\xi_2} e^{2z\eta_1\xi_1}$$

$\eta,\ \xi \propto \ln(p_i^2/Q^2)$

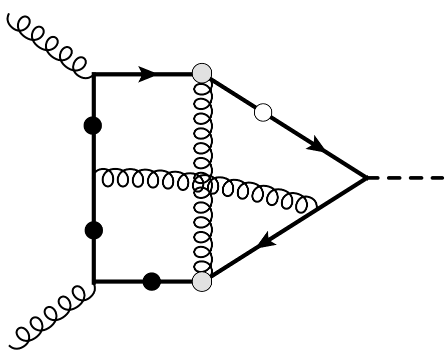
Nonfactorizable contribution

- *Abelian diagram*
  - *eikonal gluon/photon coupled to soft quark*



eikonal gluon (Wilson line)

- *Non-Abelian case* $C_F \to C_F - C_A$



Results

T. Liu, S. Modi, A.A. Penin, JHEP **02**, 170 (2022)

- Scalar amplitude

$$M^{q}_{ggH} = Z_g^2 \ln^2\left(\frac{m_q^2}{m_H^2}\right)\sum_{n=0}^{\infty}\left(\frac{m_q^2}{m_H^2}\right)^n M^{(n)}_{ggH}, \qquad M^{(0)}_{ggH} = g(z)$$

- NNLP

$$M^{(1)}_{ggH} = \left[-4g(z) + \left(\frac{T_F C_F}{45}h(z) - \frac{(C_A - C_F)(C_A - 2C_F)}{9}j(z)\right)x^2\right]$$

large-N_c

  - three loops

$$M^{(1)}_{ggH} = \left[-4 - \frac{2}{3}(C_A - C_F)x + \left(\frac{T_F C_F}{45} - \frac{14}{45}C_F^2 + \frac{23}{45}C_F C_A - \frac{9}{45}C_A^2\right)x^2\right]$$

  - *agrees with num. calc.* M.Czakon, M.Niggetiedt, JHEP **2005**, 149 (2020)

Results

- All-order
  - *factorizable single soft quark:* $g(z) = {}_2F_2\,(1,1;3/2,2;x/2)$
  - *factorizable triple soft quark*

$$h(z) = 6!\int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{\eta} \mathrm{d}\eta_2 \int_0^{\xi} \mathrm{d}\xi_2 \int_0^{\eta_2} \mathrm{d}\eta_1 \int_0^{\xi_2} \mathrm{d}\xi_1\, e^{2z(\eta\xi-\eta_2\xi_2+\eta_1\xi_1)}$$

  - *nonfactorizable (Abelian)*

$$j^{\mathrm{A}}(z) = 72\int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta_1 \int_0^{1-\eta_1-\xi} \mathrm{d}\xi_1\, \eta\xi_1 e^{2z\eta(\xi+\xi_1)}$$

$$\times\left[1+\frac{e^{-2z\eta\xi}-1}{2}+\frac{e^{-2z\eta\xi}-1+2z\eta\xi}{4z\eta\xi_1}\right],$$

  - $C_F \to C_F - C_A$ *rule is still to be proven beyond three loops*

Summary I

- Bottom effect in Higgs boson production in gluon fusion *(NLP-LL-threshold)*

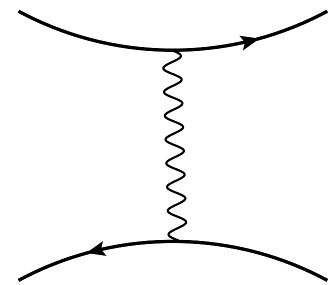
  ↪ *uncertainty interval reduced to* $0.40\ pb$ *(factor 2)*

- Power corrections to $gg \to H$ amplitude
  - *first ever NNLP LL result, new effects emerge*
  - *full analytic 3-loop, all-order (Abelian, large* N_c*)*
  - *small quark mass expansion parameter* $4m_q^2/s$

Regge Limit of QED

- *Kinematics:*

$$e(p_1^+) + \bar{e}(p_2^-) \to e(p_3) + \bar{e}(p_4)$$



$p_i^2 = 0, \qquad q = p_3 - p_1, \qquad q^2 = t, \qquad 2p_1p_2 = s$

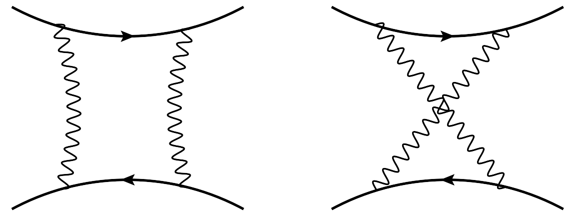
$\boxed{q_\perp \sim t^{1/2}}$, $\quad \boxed{q_\pm \sim t/s^{1/2}}$ $\quad$ similar to Coulomb scaling $\boldsymbol{q}^2 \sim q_0$

- *Small angle scattering:* $\quad \boxed{|t/s| \sim \theta^2 \to 0}$

- *Leading power approximation:*
  - *Glauber photons* $\quad q^2 \approx q_\perp^2$
  - *light-cone gauge currents* $\quad j^\mu \approx j^\pm$

Leading power

- One loop



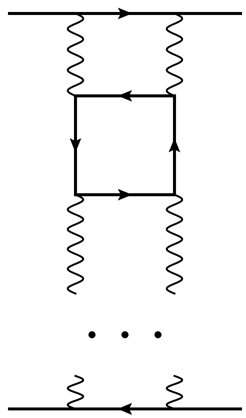
  - *eikonal fermion propagators* $\frac{1}{\not{p}_{1,2}+\not{l}+i\epsilon} \to \frac{\gamma^\pm}{2l^\pm+i\epsilon}$
  - *sum of planar and nonplanar* $\frac{1}{2l^\pm+i\epsilon} - c.c. = -i\pi\delta(l^\pm)$

- *Decoupling of light-cone and transversal dynamics*
  - *on-shell fermions on the light-cone*
  - *Glauber photons in the transversal space* $\int\frac{\mathrm{d}^2 l_\perp}{l_\perp^2(l_\perp - q_\perp)^2}$

- Exponentiation and Glauber phase
  - *all-order amplitude* $e^{i\alpha\ln|t/\lambda^2|}\mathcal{M}_B$

Reggezation

- Light-by-light scattering ladder
  - *induced interaction between Glauber photons*
  - *Schrödinger equation in Laplace space*
  - *spectral representation ⇨ Regge theory*
  - *right-most singularity in the spectrum*
  - ➥ *high-energy asymptotic behaviour*



- Regge cut contribution

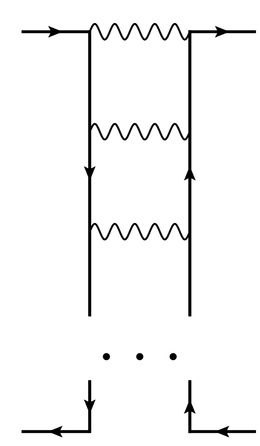
$$e^{\gamma(\alpha)\ln|s/t|}\mathcal{M}_{\rm B} \quad \textit{with} \quad \gamma(\alpha)=\frac{11\pi}{36}\alpha^2$$

➥ $d\sigma \sim s^{2\gamma(\alpha)}$ *cross section enhancement ⇨ Froissart bound?*

V. Gribov, L. Lipatov, G. Frolov; H. Cheng and T. T. Wu (1970)

Forward annihilation $e^+e^- \to \mu^+\mu^-$

- Power suppressed amplitude
  - *soft fermions, eikonal photons*
  - *double logarithmic corrections in* $\alpha \ln^2(s/m_\mu^2)$



- Asymptotic result

$$d\sigma \sim s^{-1+2\gamma(\alpha)}, \quad \gamma(\alpha) = \sqrt{\frac{2\alpha}{\pi}}$$

"square root" of DL

V. G. Gorshkov, V. N. Gribov, L. N. Lipatov, G. V. Frolov (1968)

Power corrections

Current expansion

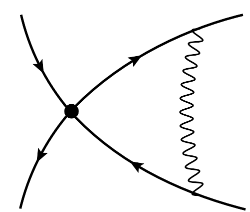
$$A^\mu j_\mu = A_- j_+ + \frac{\tilde{F}_{+-}}{4p_+} j_+^5 + \frac{iF_{+-}}{4p_+} j_+ + \mathcal{O}(|t/s|^{3/2})$$

induce local 4-fermion interaction

- *scaling:* $\tilde{F}_{+-} \propto B_{||} \sim q_\perp = \mathcal{O}(|t/s|^{1/2})$, $F_{+-} \propto E_{||} \sim q_\pm = \mathcal{O}(|t/s|)$

↪ *similar to* $\mathcal{O}(v)$ *Pauli and* $\mathcal{O}(v^2)$ *Darwin interaction in NRQED*

One-loop



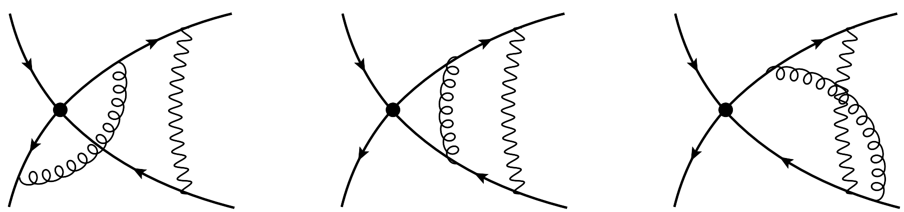
- *log interval* $l_\perp^2/\sqrt{s} < l_- < \sqrt{s}, \quad \sqrt{|t|} < l_\perp < \sqrt{s}$
- *no planar-nonplanar cancellation*
- *double log vs leading power single log*

$$z\left|\frac{t}{s}\right|\mathcal{M}_B, \quad z = \frac{\alpha}{2\pi}\ln^2\left|\frac{t}{s}\right|$$

Summing up double logs

- Summing up double logarithms
  - *additional Glauber photons*
  - *sum over vertex permutations*
  - ➥ *Glauber phase factorization*

- Remaining "non-factorizable" diagrams:



➥ *“soft” Glauber photons couple to an internal line*

Summing up double logarithms

- **Double logarithmic region**

  $q_\perp^2 < l'^2_{i\perp} < l_\perp^2 < s, \quad l_\perp^2/\sqrt{s} < l_- < l'_{1-} < \ldots < l'_{n-} < \sqrt{s},$

- **Double logarithmic result** $\mathcal{M}^{LL} = |t/s| z\, g(2z) \mathcal{M}_B$

- *Integral representation*

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta \;\; e^{2z\eta\xi}$$

Glauber gluon exponent

  - logarithmic variables $\xi = \ln(l_\perp^2/s)/\ln|t/s|,\ \eta = \ln(l_-/\sqrt{s})/\ln|t/s|,$

- *Exact formula and asymptotic behavior at* $z \to +\infty$

$$g(z) = {}_2F_2\left(1,1;3/2,2;z/2\right), \quad g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{1/2}$$

Magic #2

- Regge $e^+e^- \to e^+e^-$ NLP LL

$$\mathcal{M}^{LL} = |t/s| z\, g(2z) \mathcal{M}_B$$

$z = \frac{\alpha}{2\pi} \ln^2 |t/s|$

- Sudakov $gg \to H$ NLP LL

$$\mathcal{M}^{LL}_{gg\to H} \propto g(z)\, \mathcal{M}^{LO}_{gg\to H}$$

$z = (C_A - C_F) \frac{\alpha_s}{4\pi} \ln^2(m_q^2/s)$

↳ *same function $g(z)$ in both cases!*

Unitarity

- Cross section $d\sigma^{LL} \sim s^{2(-1+\frac{\alpha}{2\pi}\ln|s/t|)}$ ← *cf.* $\gamma(\alpha)$
  - *for* $\ln|s/t| \gg 1/\alpha$ *becomes dominant*
  - *breakdown of small-angle expansion*
  - *perurbative bound on energy* $|s/t| \approx e^{2\pi/\alpha}$
- *Unitarity problem does not appear below the bound!*

Summary II

- Regge limit at NLP in QED
  - *double logs v.s. leading power single logs*
  - *breakdown of small angle expansion*
  - *new scenario of unitarity restoration*

- *amazing relations between amplitudes and limits*