RUHR UNIVERSITÄT BOCHUM RUB

Spectroscopy of tetra-quarks and other exotic states

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Lattice meets Continuum

September 30- October 3, 2024

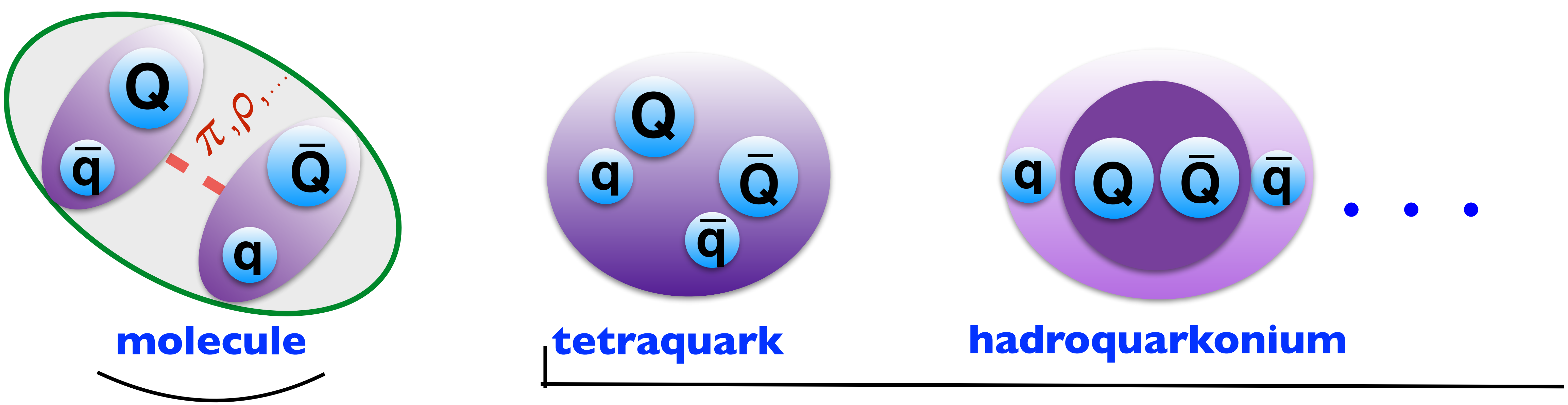
Outline:
- Introduction
- Weinberg's compositeness
- Identifying molecules in line shapes
- Tcc state: experiment, lattice, chiral EFT
- Summary and concluding remarks

Largely based on *PRD* 105, 014024(2022); *PRL* 131, 131903 (2023); *PRD* 109, L071506 (2024) and 2407.04649 *PRD 109, L111501 (2024); EPJA 57 (2021)*

in collaboration with: M. Abolnikov, X. Dong, M. Du, E. Epelbaum, A. Filin, A. Gasparyan, F.-K. Guo, C. Hanhart, I. Matuschek, L. Meng, A. Nefediev, J. Nieves and Q. Wang

Evidence for Exotic States near thresholds

- Heavy-light sector $D_{s0}(2317)$, $D_{s1}(2460)$, $X_{0/1}(2900)$,… Christoph Hanhart's Talk on Monday $cq q\bar{q}$
- XYZ
  - $X(3872)$, … ; $Z_c(3900)$, $Z_c(4020)$, $Z_{cs}(3982)$ … ; $Y(4230)$, $Y(4360)$, $Y(4660)$, … $c\bar{c}q\bar{q}$
  - $Z_b(10610)$, $Z_b(10650)$ $b\bar{b}q\bar{q}$
  - $X(6900)$ $cc\bar{c}\bar{c}$
- Pentaquarks $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, $P_{cs}(4459)$ $c\bar{c}qqq$
- double c-quark Tcc $ccq\bar{q}$



Size: $\sim 1/\sqrt{2\mu E_B} >> 1\mathrm{fm}$ $\sim 1/\Lambda_{\mathrm{QCD}} \sim 1\mathrm{fm}$

Weinberg compositeness

Weinberg 1963-65

Physical coupling and ERE parameters via probability of a molecular component X

$$a = -2\,\frac{X}{1-X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad r = -\frac{1-X}{X}\frac{1}{\gamma} + \mathcal{O}(1/\beta) \qquad g_R^2 = \frac{2\pi\gamma}{\mu^2} X + \mathcal{O}(1/\beta)$$

a < 0 — bound state

VB et al. 2004

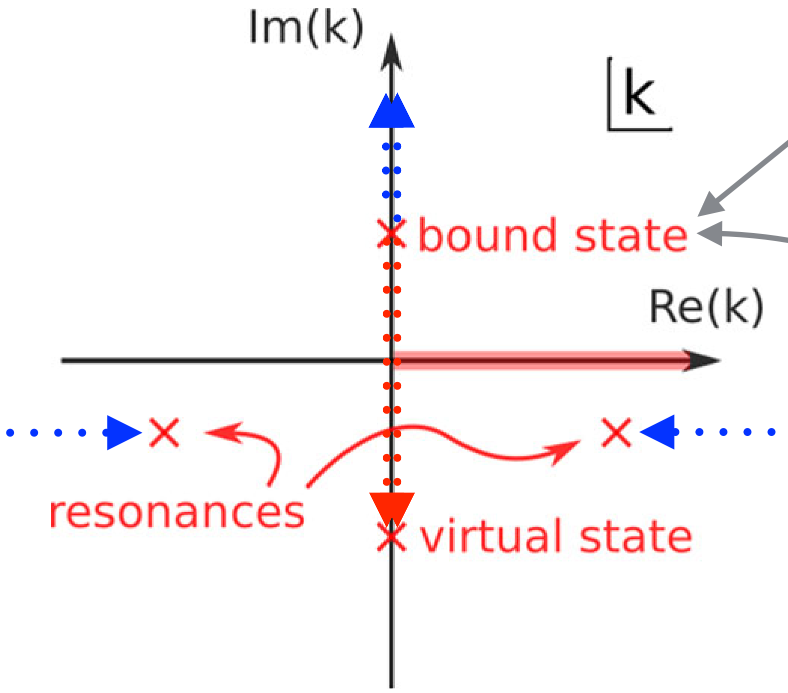
If $|a| \gg |r|,\ r \sim 1/\beta$ $\Longrightarrow$ $X \to 1$ $\Longrightarrow$ Molecule

If $|a| \ll |r|,\ r < 0$ $\Longrightarrow$ $X \to 0$ $\Longrightarrow$ Compact state

- Insights on range effects — Albaladejo, Nieves 2022, Li et al. 2022, Song et al 2022, Kinogawa, Hyodo 2022
- Extensions mostly for resonances by Jido, Kamai, Nieves, Oller, Oset, Sekihara,… — review Kamai and Hyodo 2017
- Recent generalisations to virtual states, coupled-channels, … — Matuschek et al. EPJA 57 (2021), VB et al., PLB 833 (2022)

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 57 (2021)

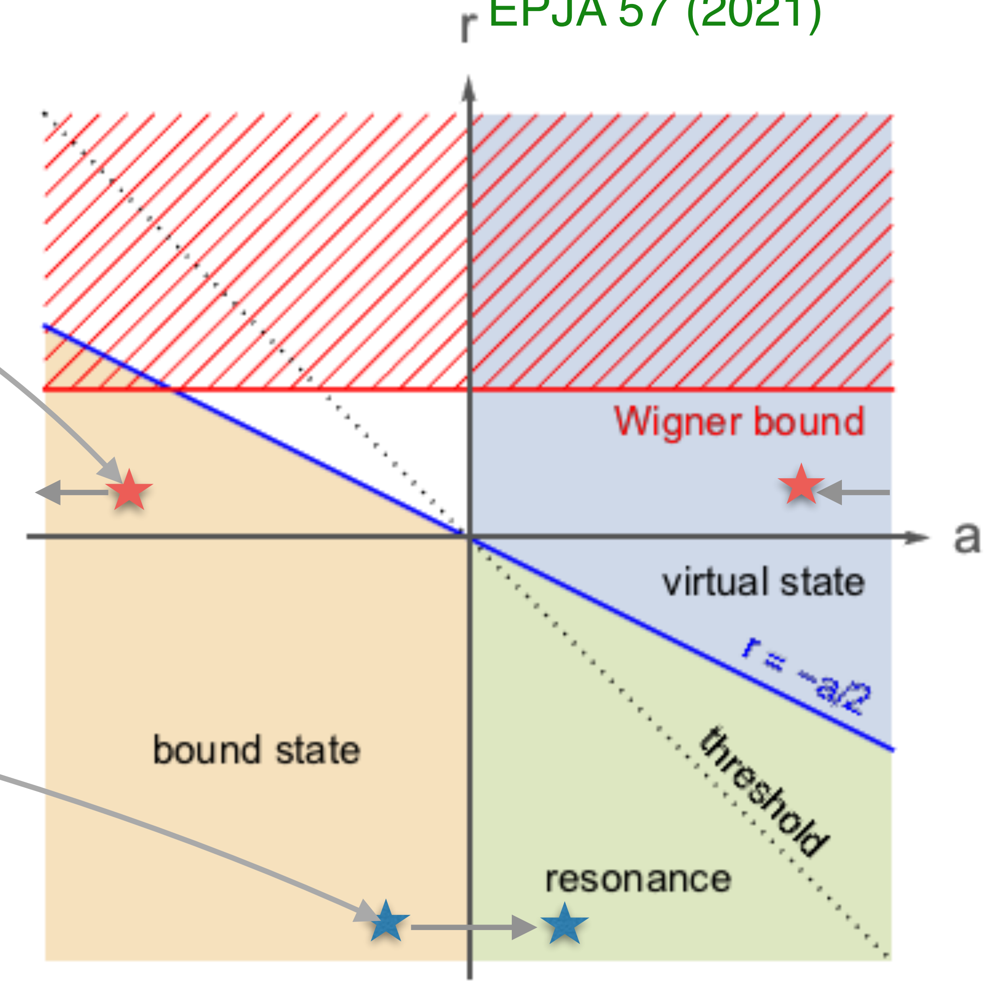


Molecular bound st.

$|a| \gg |r|, \ r \sim 1/\beta$

Compact bound st.

$|a| \ll |r|$ and $r < 0$



- Evolution of poles and analyticity → Extensions beyond bound states

Molecular pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ if sc. length changes sign → virtual state

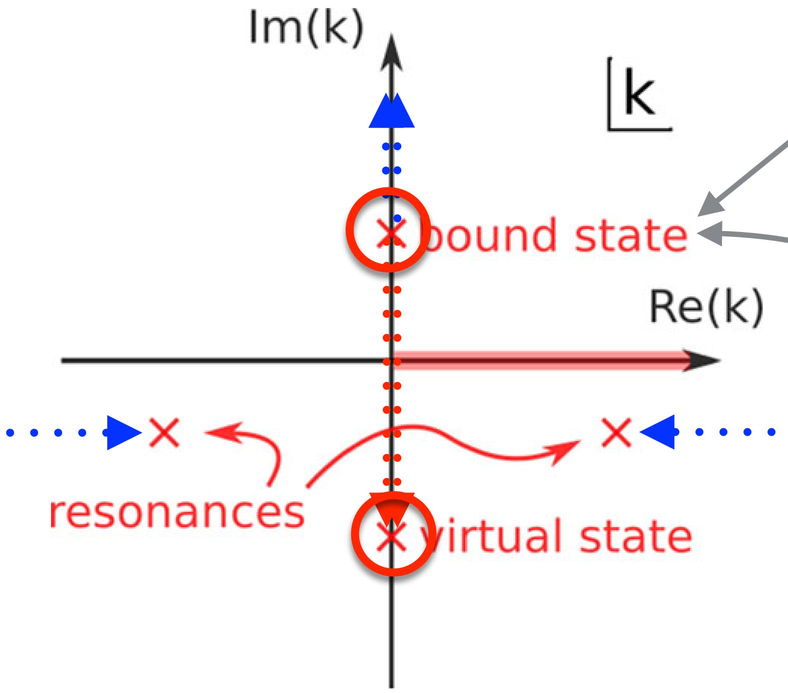
$|a| \gg |r|$

Compact pole: $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ if sc. length changes sign → turns to a resonance

$|a| \ll |r|$

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 57 (2021)



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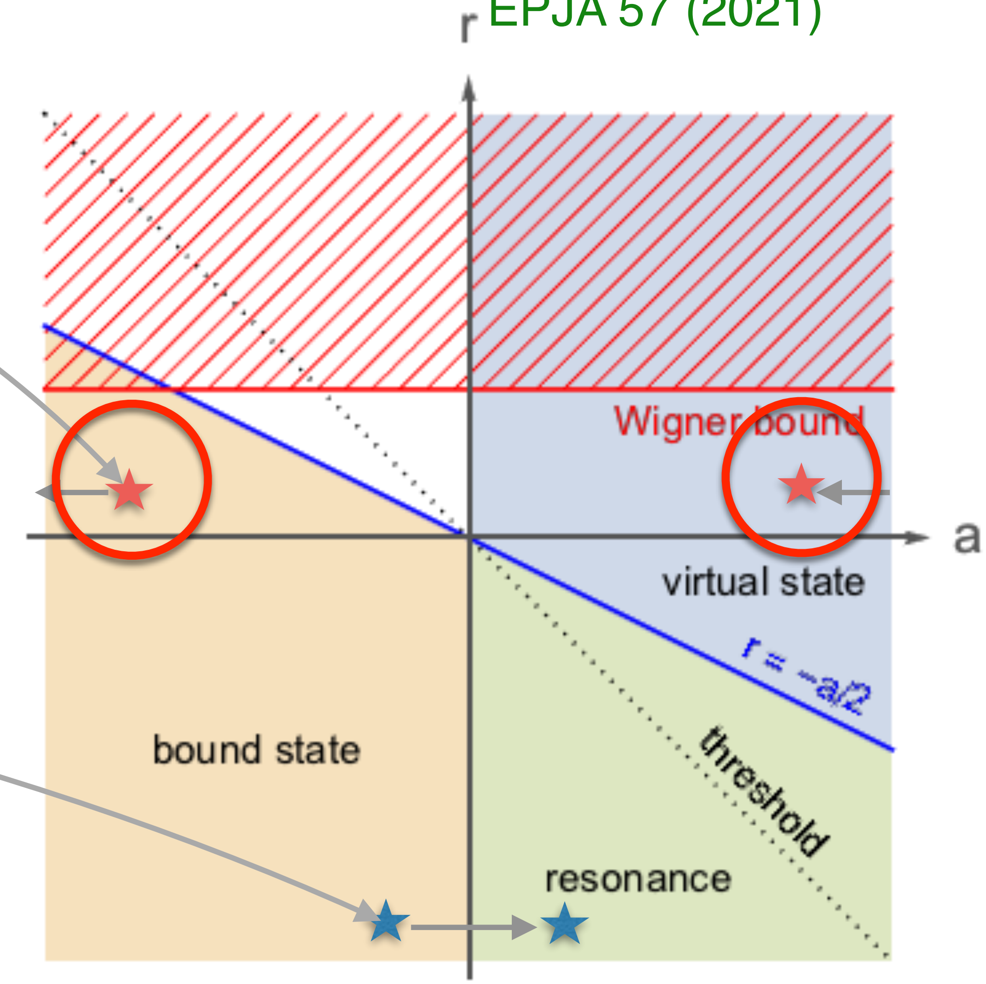

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Near thr. molecules

Compact pole: $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ if sc. length changes sign → turns to a resonance

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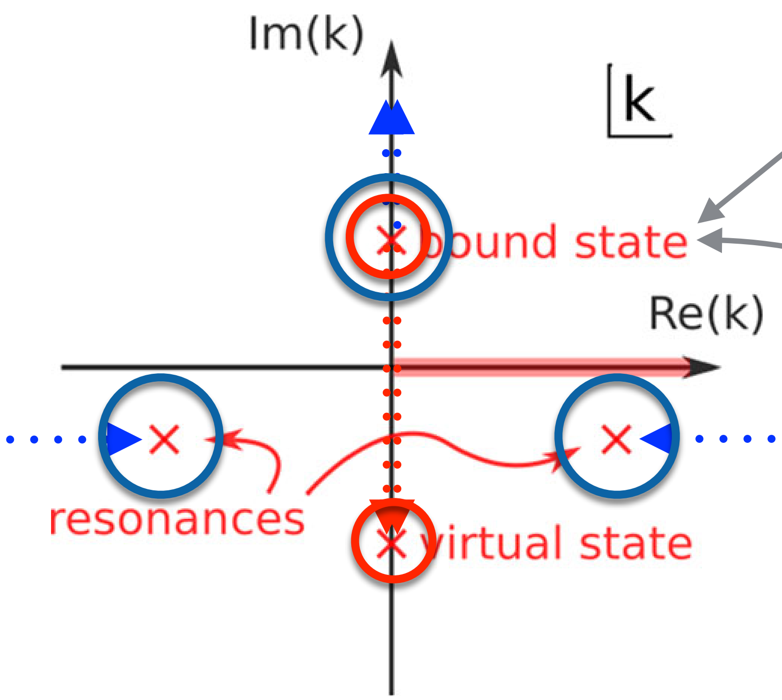
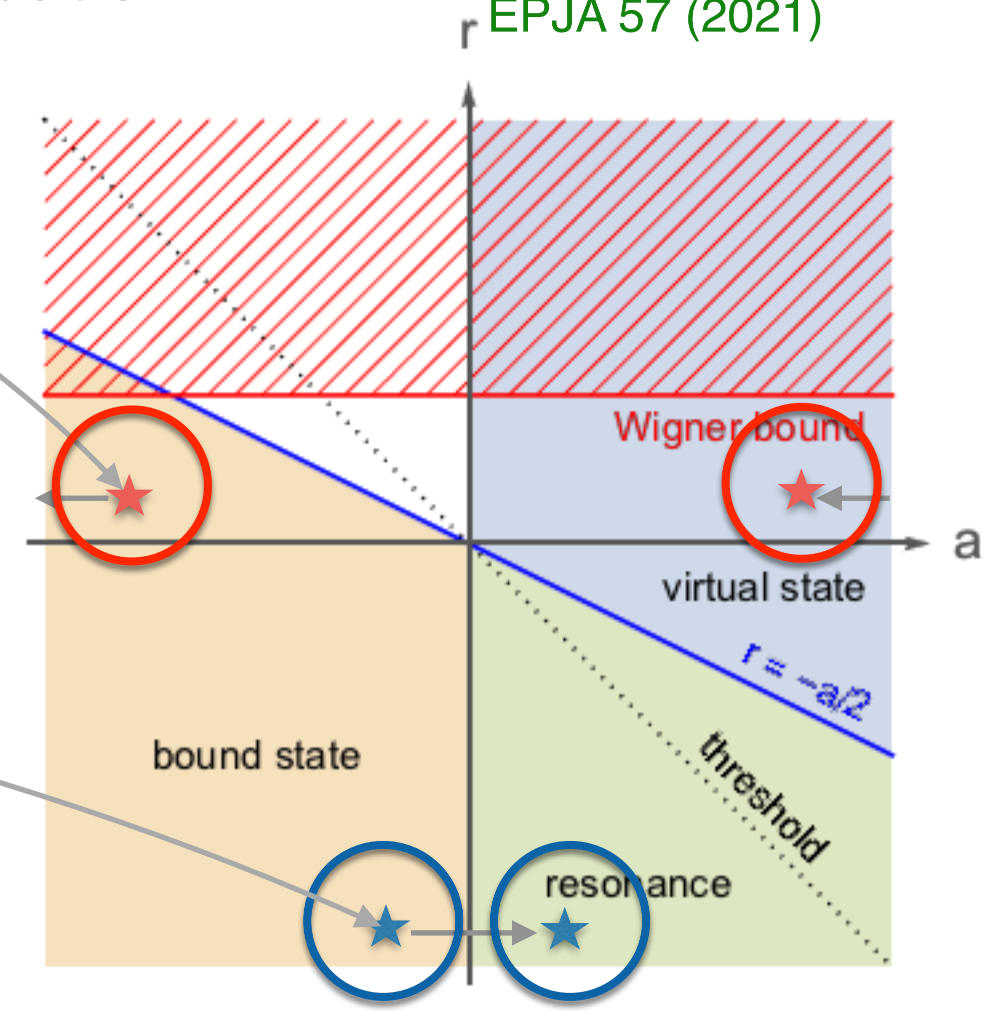


Molecular bound st.

$$|a| \gg |r|, \ r \sim 1/\beta$$

Compact bound st.

$|a| \ll |r|$ and $r < 0$



- Evolution of poles and analyticity → Extensions beyond bound states

Molecular pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ if sc. length changes sign → virtual state

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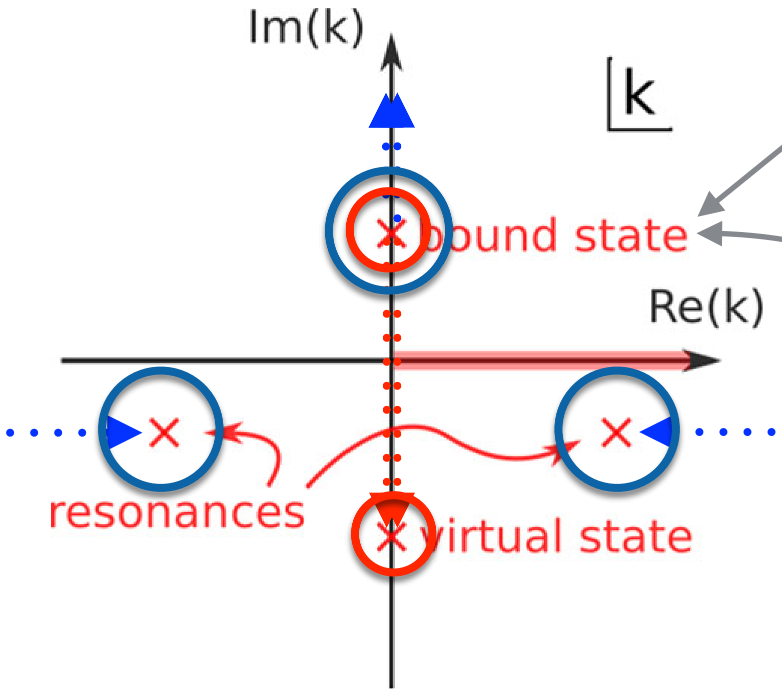
$|a| \ll |r|$

Near thr. compact states

Extensions beyond bound states

Matuschek, VB, Guo, Hanhart
EPJA 57 (2021)

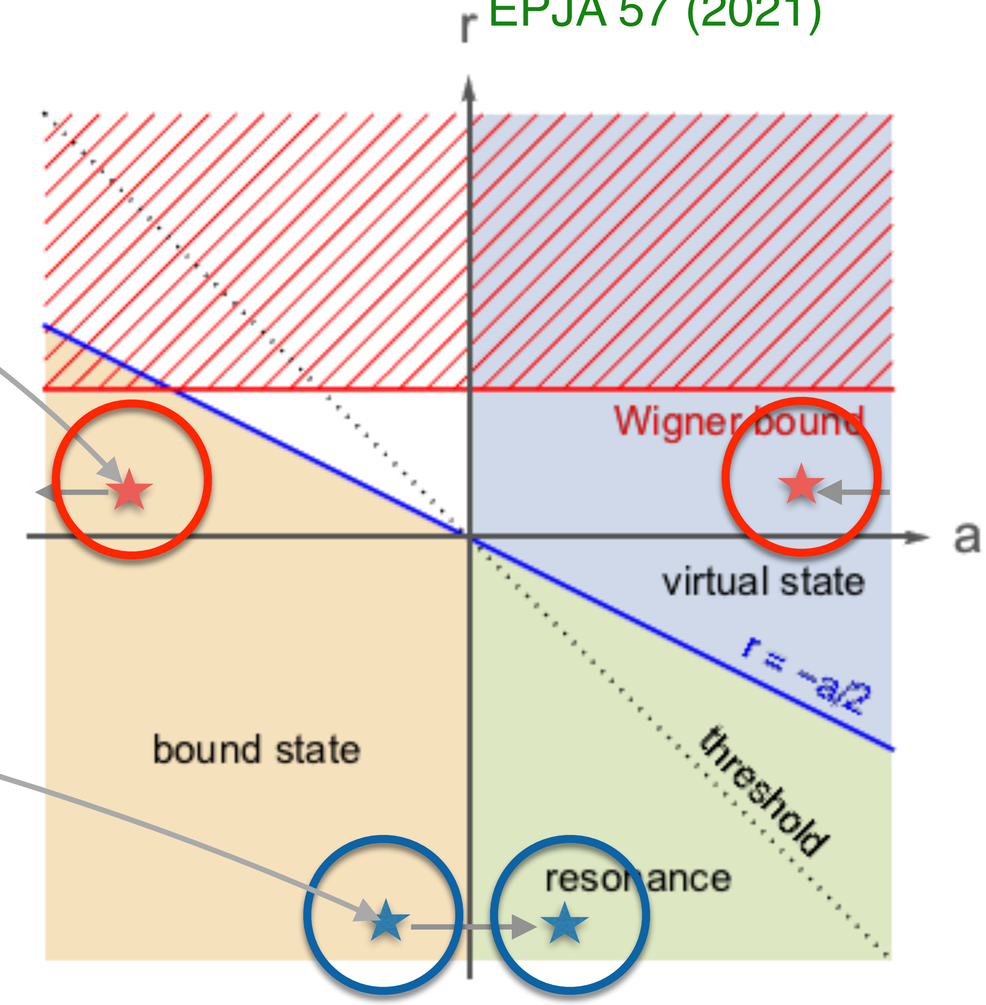


Molecular bound st.

$|a| \gg |r|, \;\; r \sim 1/\beta$

Compact bound st.

$|a| \ll |r|$ and $r < 0$



- Evolution of poles and analyticity → Extensions beyond bound states

Molecular pole: $k_1 = -\frac{i}{a}\left[1 + \mathcal{O}\left(\frac{r}{a}\right)\right]$ if sc. length changes sign → virtual state

$|a| \gg |r|$

Near thr. molecules

Compact pole: $k_{1,2} = \pm i\sqrt{\frac{2}{a\,r}} + \frac{i}{r} + \mathcal{O}\left(\sqrt{\frac{a}{r^3}}\right)$ if sc. length changes sign → turns to a resonance

$|a| \ll |r|$

Near thr. compact states

$$X_W = \sqrt{\frac{1}{1+2r/a}} \quad \Longrightarrow \quad \bar{X} = \sqrt{\frac{1}{1+|2r/a|}}$$

both cases subsumed here

- $\bar{X}$ allows one to test compositeness for bound/virtual states and resonances

Identifying a molecule in observables

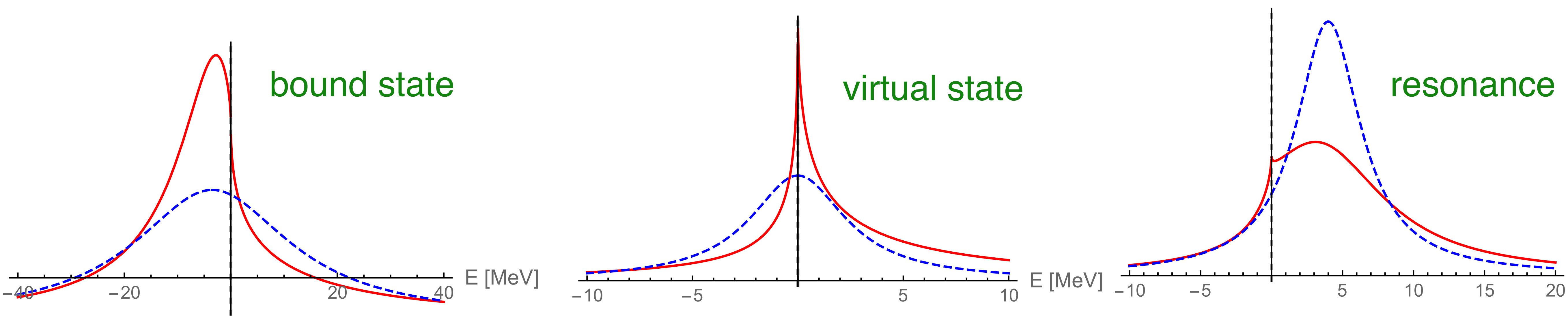
VB et al.(2004, 2005), Braaten et al.(2007), Hanhart et al.(2010), Oset et al.(2012), Oller et al.(2016), …

$$A_{\rm prod} = \frac{\rm const}{E + E_B + \frac{g_0^2\mu}{2\pi}(ik+\gamma) + i\frac{\Gamma_0}{2}}$$

large g0 $\Longrightarrow$ molecule

g0 =0 $\Longrightarrow$ (Breit-Wigner) compact state

Typical shape of the production rate in the inelastic channels: Molecule vs Compact



Molecular line shapes are strongly affected by threshold effects enhanced by nearby poles

Is a resonance always a peak?

Not necessarily!

two-body scattering:

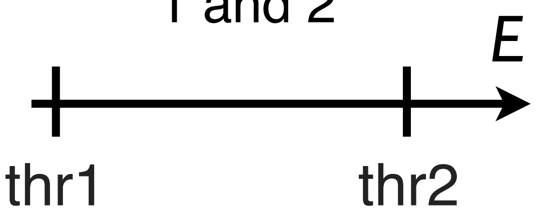
Single channel 2

$$T_2(E) = \frac{1}{(\frac{1}{a_{22}} - ik_2)}$$

T2 has a pole near thr2 if a22 is large

Two coupled channels

1 and 2



$$T_{11}(E) = \frac{\frac{1}{a_{22}} - ik_2}{(\frac{1}{a_{11}} - k_1)(\frac{1}{a_{22,eff}} - ik_2)}$$

$$T_{12}(E) = \frac{\frac{1}{a_{12}}}{(\frac{1}{a_{11}} - k_1)(\frac{1}{a_{22,eff}} - ik_2)}$$

a zero near thr2

a pole near thr2

$$a_{22,\mathrm{eff}}^{-1} = a_{22}^{-1} - a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-1}$$

Ex1: $\pi\pi$ amplitude has a dip near KK thanks to $f_0(980)$ — Kaminski et al. EPJ C4 (2002), VB et al. EPJ A23 (2005)

Ex2: $\pi\Sigma$ amplitude has a dip near KN thanks to $\Lambda(1405)$ — Bulava et al. PRL132 (2024), Daniel Mohler Talk on Monday

Is a resonance always a peak?

Not necessarily!

two-body scattering:

Single channel 2 $$T_2(E) = \frac{1}{(\frac{1}{a_{22}} - ik_2)}$$

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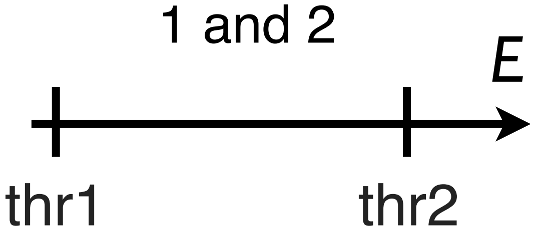
Two coupled channels 1 and 2



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$$T_{12}(E) = \frac{\frac{1}{a_{12}}}{(\frac{1}{a_{11}} - k_1)(\frac{1}{a_{22,eff}} - ik_2)}$$

→ a zero near thr2

→ a pole near thr2

$$a_{22,\mathrm{eff}}^{-1} = a_{22}^{-1} - a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-1}$$

Production process: Dong, Guo, Zou PRL 126 (2021)

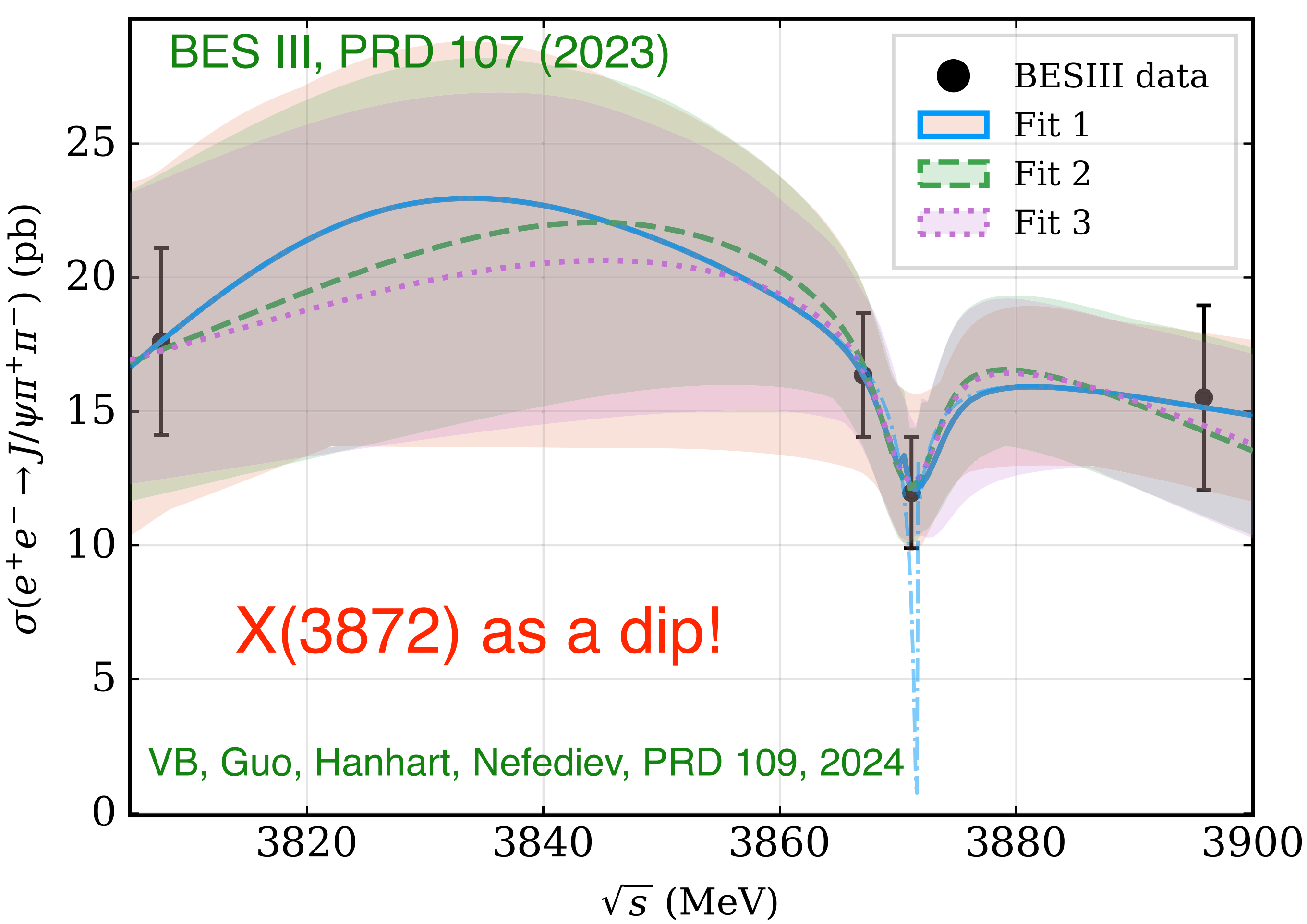
$$A_1(E) = P_0 + P_1 T_{11}(E) + P_2 T_{21}(E)$$

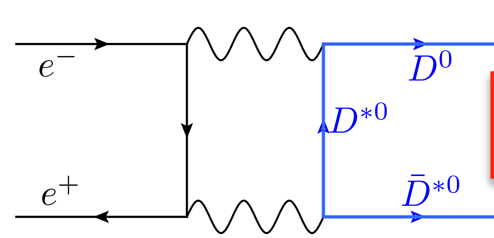
If $P_2 \ll P_1$ there will be a dip near thr 2

If $P_1 \ll P_2$ there will be a peak near thr 2

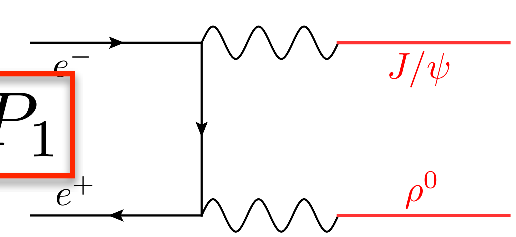
$$1 = \rho J/\Psi \quad 2 = D\bar{D}^*$$

$a_{22,\mathrm{eff}} = (-6.39 + i11.74)$ fm from VB et al. PLB 833, 2022





$P_2 \ll P_1$



$T_{cc}(3875)^+$ $cc\bar{u}\bar{d}$

— Tcc status: Experiment and Lattice

— Can we extract and understand Tcc properties from experiment and lattice systematically?

— Applications of chiral EFT

- Analysis of exp.data including three-body unitarity

  M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

- Alternative to Lüscher method: from E_{FV} to infinite volume pcot(δ) with left-hand cuts

  L. Meng, VB, A. Filin, E. Epelbaum and A. Gasparyan *PRD* 109, L071506 (2024)

  M. Du, A. Filin, VB, X. Dong, E. Epelbaum, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRL* 131, 131903 (2023)

- Light-quark mass dependence of the T_{cc} pole trajectory

  M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng *2407.04649*

T_{cc} : an ideal case for studying exotic properties

Aaij et al [LHCb] Nature Physics (2022)
Nature Comm.(2022)

— first exotic doubly charm state: $cc\bar{u}\bar{d}$

$I = 0,\ J^P = 1^+$

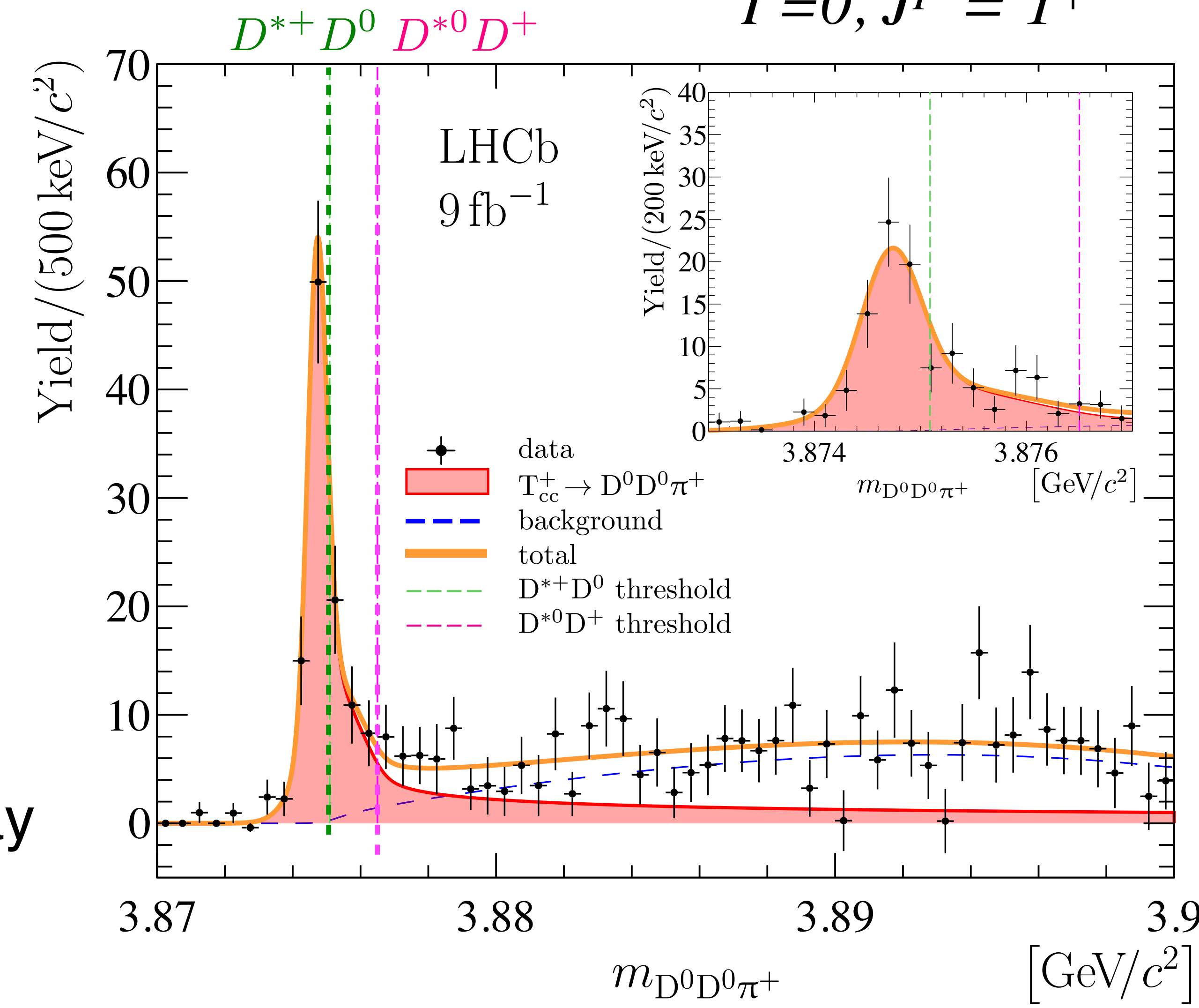
—Expansion in χEFT : $\chi = \frac{\sqrt{2\mu\Delta_M}}{\Lambda_\chi} < 0.1$

$\Delta_M = m(D^+D^{*0}) - m(D^0D^{*+})$

— No admixture of inelastic channels

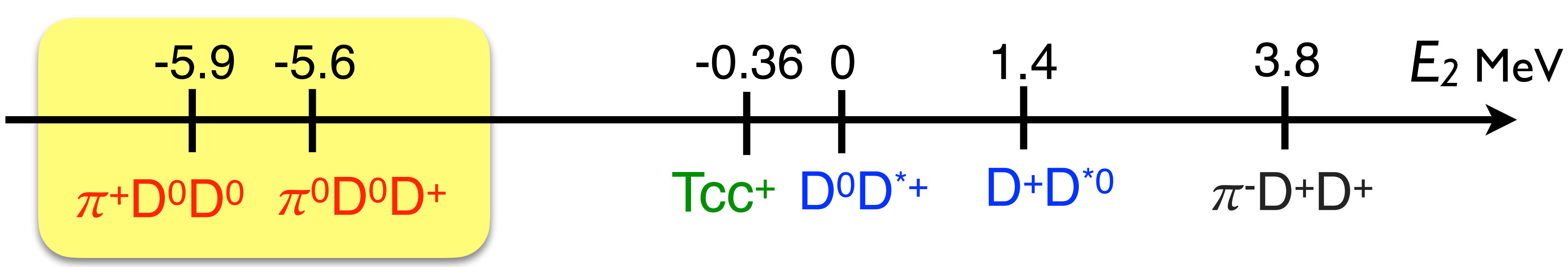
— Width: almost entirely from the only strong decay

$T^+_{cc} \to D^0D^{*+} \to D^0D^0\pi^+/D^0D^+\pi^0$



$\delta m_{BW} = -273 \pm 61 \pm 5^{+11}_{-14}$ keV

$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}$ keV



— $D\pi$ spectra: ~ 90% of the $D^0D^0\pi^+$ events contain a genuine D^{*+} meson

S-wave $\Rightarrow$ $J^P = 1^+$

DD^* framework is justified

T_{cc} on lattice

- HAL QCD Collaboration at mπ = 146 MeV: Lyu et al, *PRL* 131 161901 (2023)

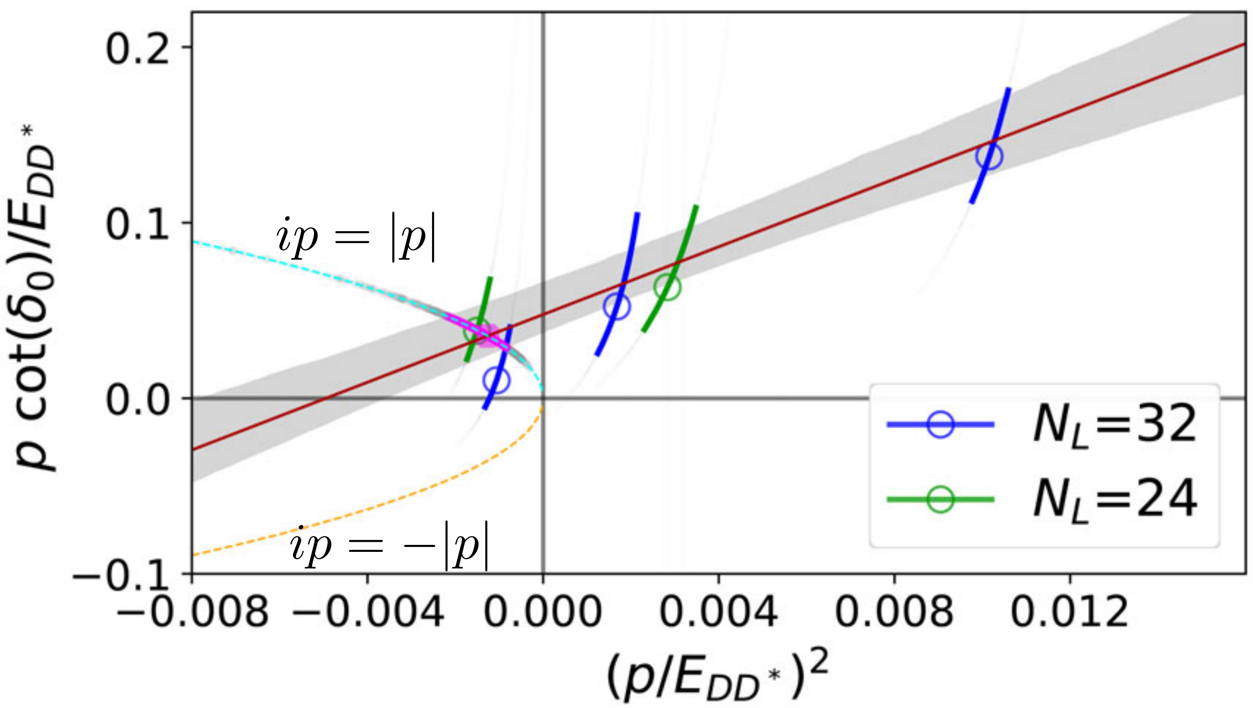
— calculate the DD* scattering potential ⇒ phase shifts above the two-body threshold

$$E_{\text{pole}} = -59^{+53\,+2}_{-99\,-67} \text{ keV}$$ DD* virtual state

- Lüscher method based analyses of FV energy levels

— DD* spectra at mπ = 280 MeV Padmanath and Prelovsek, PRL 129, 032002 (2022)

see also Collins et al *PRD* 109 (2024) 9, 094509



— DD* phase shifts parameterised using the ERE:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2}rp^2 + \mathcal{O}(p^4)$$

$$E_{\text{pole}} = -9.9^{+3.6}_{-7.2} \text{ MeV}$$ DD* virtual state

— DD*-D*D* coupled channel parameterization at mπ = 391 MeV T. Whyte, D. Wilson, and C. Thomas 2405.15741v1 (2024)

$E_{\text{pole}} = (-62 \pm 34)$ MeV DD* virtual state

$E_{\text{pole}} = (-49 \pm 35 + i(11 \pm 13)/2)$ MeV D*D* resonance

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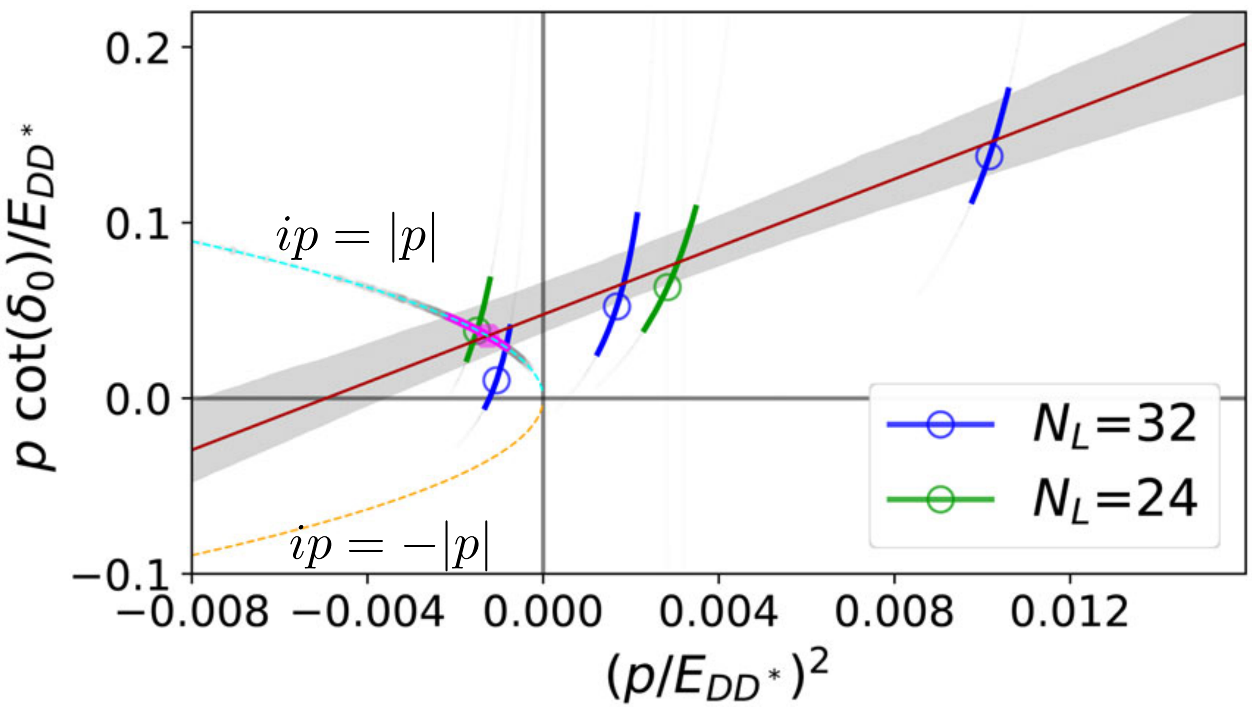

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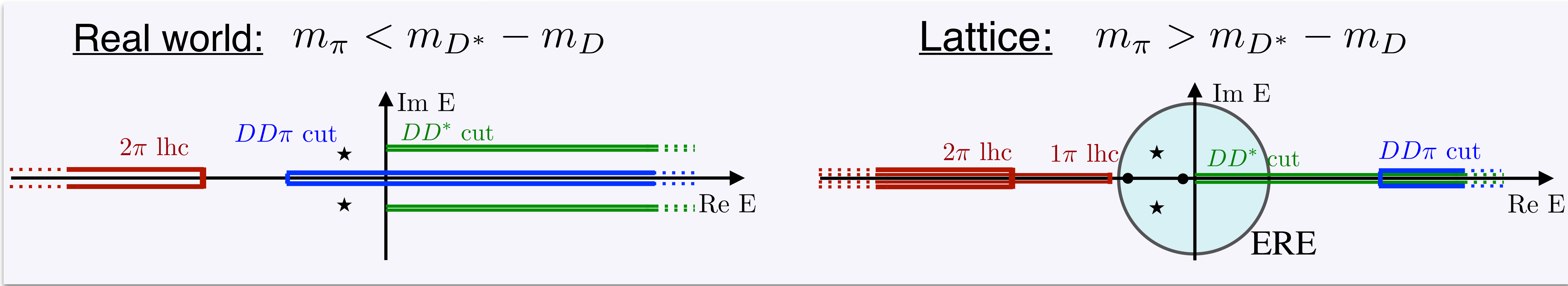
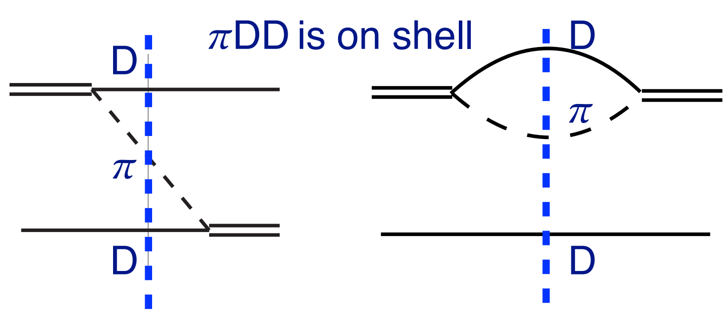
Main assumption for Lüscher method: no nearby left-hand cuts!

Analytic structure of the DD* scattering amplitude

– Cut structure depends on the (light-quark or) pion mass



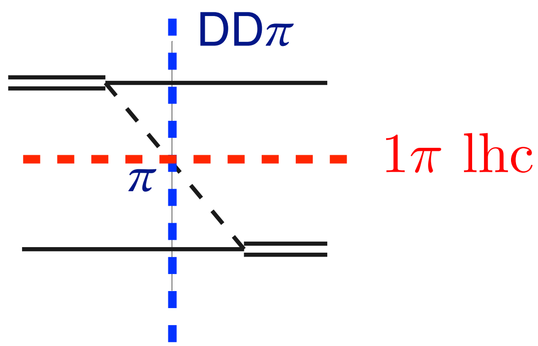
3-body DDπ cut



$\Rightarrow$ Prominent role for the T_{cc} width

M. Du, VB, et al *PRD* 105, 014024(2022)

Left-hand cuts



$\Rightarrow$ Constraints on the ERE applicability range

M. Du, VB et al, *PRL* 131, 131903 (2023)

$\Rightarrow$ Invalidate Lüscher's QC at least below lhc

Green et al (2021), Raposo and Hansen (2023),
Dawid et al. (2023), L. Meng, VB et al (2023), …

– The leading nearby cut is always associated with the one-pion exchange (OPE)

$\Rightarrow$ Theoretical framework has to include it!

Modified effective range expansion (MERE)

v.Haeringen and Kok PRA 26 (1982), Cohen and Hansen PRC 59 (1999), Steele and Furnstahl NPA645 (1999), …

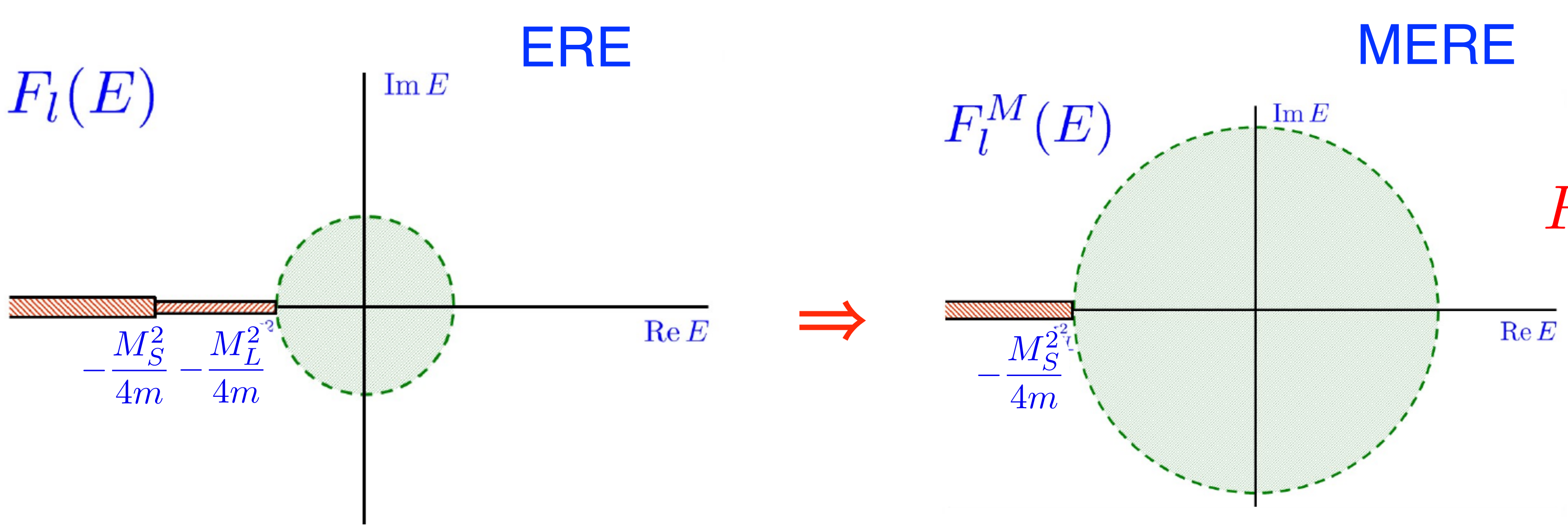
Low-energy theorems for NN scattering: VB, Epelbaum, Filin and Gegelia PRC 92 (2015), PRC94 (2016)

$$V = V_L + V_S \qquad r_L \sim M_L^{-1} \qquad r_S \sim M_S^{-1} \qquad M_L \ll M_S$$

$$F_l^M(k^2) \equiv R_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|} \cot[\delta_l(k) - \delta_l^L(k)]$$

- All long-range quantities — $f_l^L(k), R_l^L(k), \delta_l^L(k)$ — are known from the solution of the Schrödinger Eq. for V_L

- MERE expansion: $F_l^M(k^2) = -\frac{1}{a^M} + \frac{1}{2} r^M k^2 + v_2^M k^4 + v_3^M k^6 + v_4^M k^8 + \ldots$



$F_l^M(k^2)$ is meromorphic for $|k| < M_S/2$

- systematically parameterizes short-range physics: $1/M_S$ expansion

Example of MERE: Coulomb + strong interaction

$$V = V_C + V_S \qquad V_C = \frac{\alpha}{r}$$

v.Haeringen and Kok PRA 26 (1982)

— repulsive Coulomb potential, e.g. proton-proton scattering

— left hand cut starts from threshold ⇒ zero range of convergence for ERE

$$F_0^M(k^2) = 2k\,\eta\,h(\eta) + C_0^2(\eta)\,k\,\cot[\delta(k) - \delta^C(k)]$$

Coulomb phase

$$C_0^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \qquad h(\eta) = \mathrm{Re}\Big[\Psi(i\eta)\Big] - \ln(\eta), \quad \eta = \frac{m}{2k}\alpha, \quad \Psi(z) \equiv \Gamma'(z)/\Gamma(z)$$

⇒ relation between the Coulomb removed scattering length a_M and the full one a_{CS}

$$-\frac{1}{a_M} = -\frac{1}{a_{CS}} - \frac{2}{a_B}\log[\mu a_B] - \gamma$$

γ - Euler constant

a_B - Bohr radius

μ - mass scale

Example: for proton-proton scattering a_{CS}=-7.82 fm ⇒ $a_M \simeq$ -19 fm

MERE explicit vs implicit

- OPE is singular at small r and requires *regularization* and *renormalization* ⇒ QFT

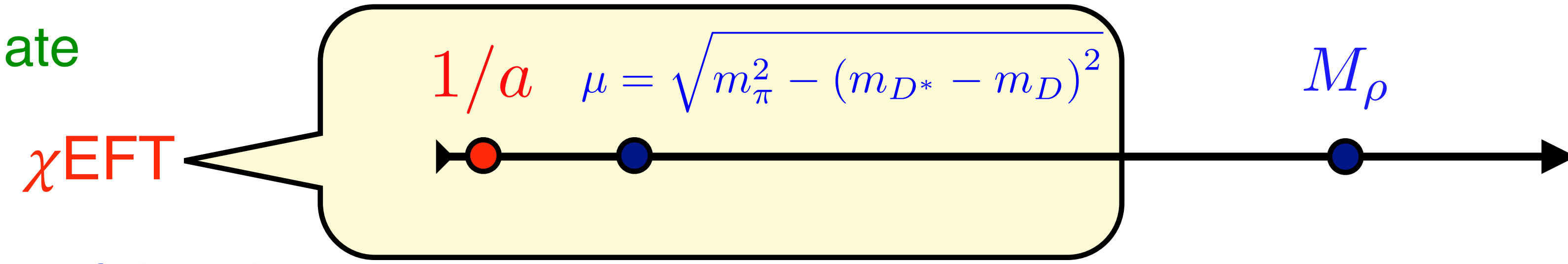
- Implicit implementation : MERE ≡ Chiral EFT for the given pion mass

→ see also explicit realization of the MERE in the finite volume: Bubna et al. JHEP 05 (2024)

Hierarchy of scales

— Systematic expansion with an error estimate

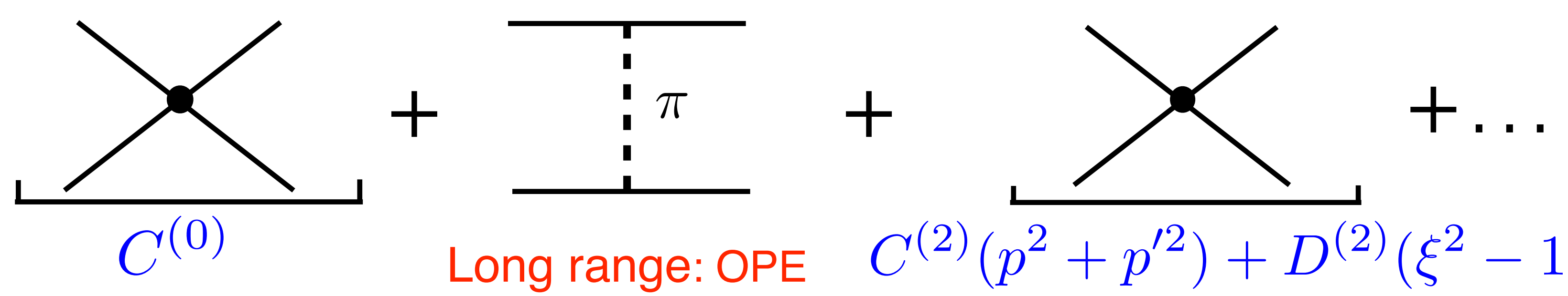
— Keep track of relevant scales



— Incorporate the relevant cuts: 3-body cuts, left-hand cuts

— Applicable both in the infinite and finite volumes

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + \ldots =$$



$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

- Amplitudes are solutions of the integral equations

$$T_{\alpha\beta} = V^{\rm eff}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

G - Green functions

→ Consistent with Unitarity, analyticity and renormalizability

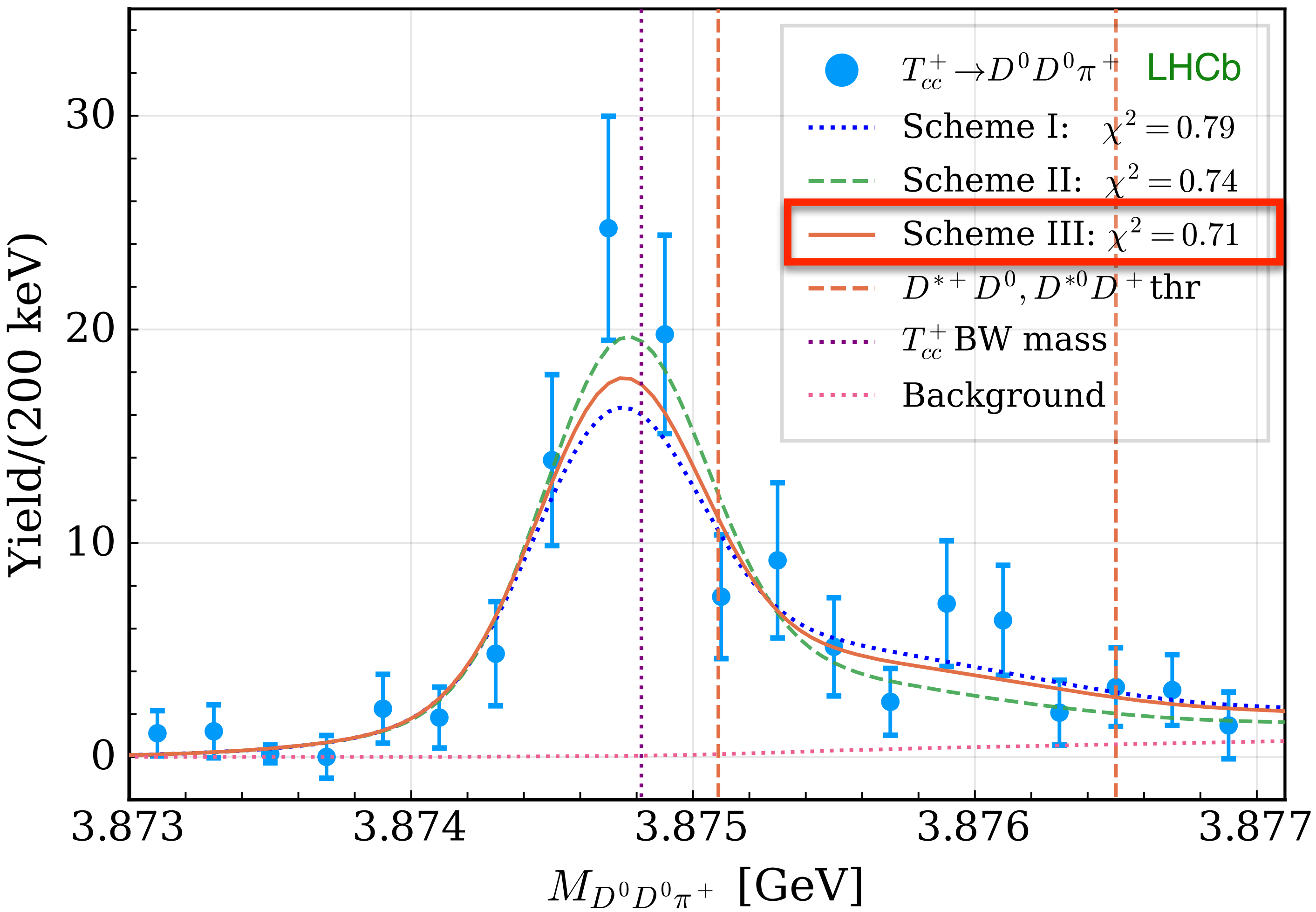
— Regularize $V_{\rm OPE}$ and make sure that amplitudes are cutoff independent

Applications

App I: LO χEFT analysis of $D^0D^0\pi^+$ data by LHCb

M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

with resolution



The pole

III
full 3-body unitarity: OPE + dynamical D^* width
$-356^{+39}_{-38} - i(28 \pm 1)$
0.71

— Coupled $D^0D^{*+} - D^+D^{*0}$ scattering

— 1 parameter + overal normalization

Re part of the T_{cc} pole: Inconclusive about the role of 3-body effects with current exp. precision

Im part of the T_{cc} pole: Controlled by 3-body effects

$$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$$

a_0 [fm]	r_0' [fm]	$\bar{X}_A$
$\left(-6.72^{+0.36}_{-0.45} \pm 0.27\right) - i\left(0.10^{+0.03}_{-0.03} \pm 0.03\right)$	$1.38 \pm 0.01 \pm 0.85$	$0.84 \pm 0.01 \pm 0.06$

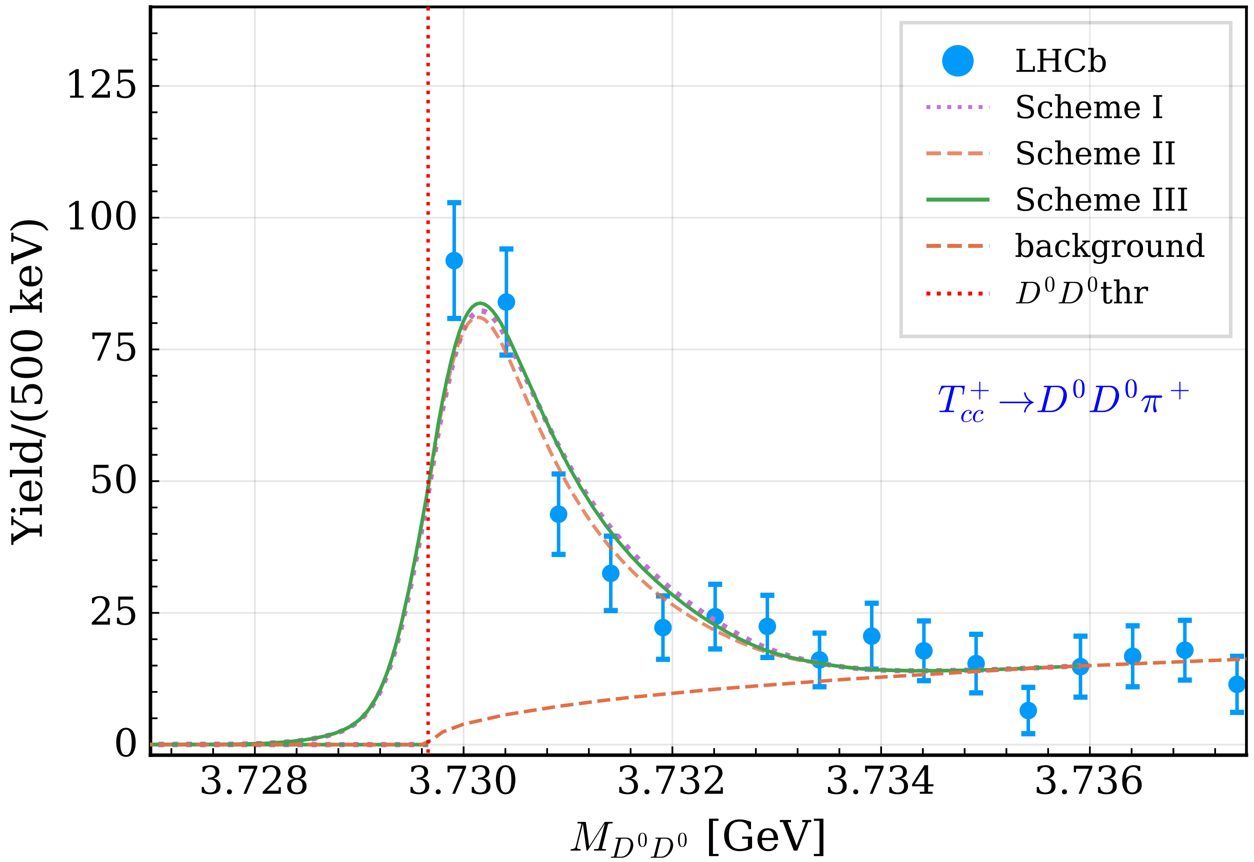
$$r_0' \ll |a_0|$$

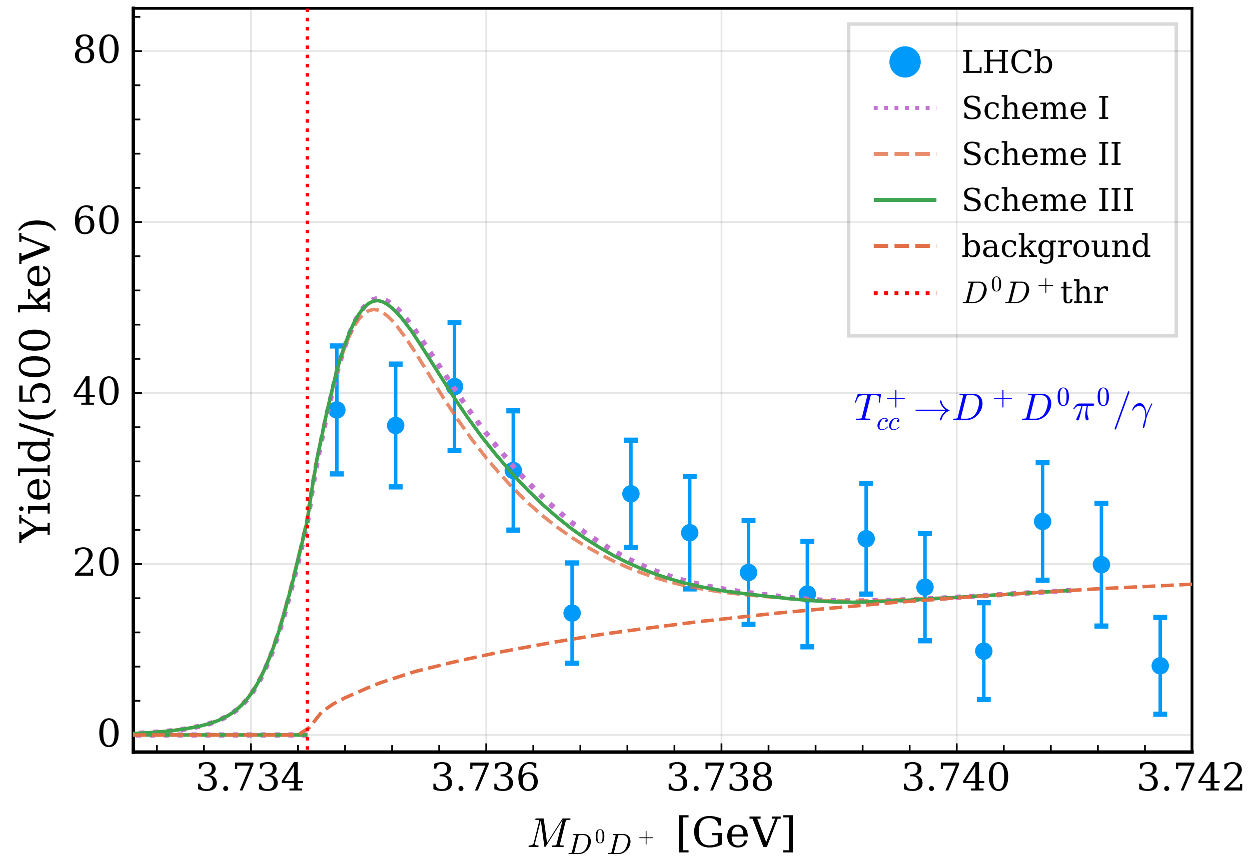
T_{cc} is consistent with an isoscalar molecule!

Parameter-free predictions

D⁰D⁰ and D⁰D⁺ spectra

with resolution





Heavy quark spin partners

see also Albaladejo PLB 829 (2022) in contact EFT

$$V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+)$$

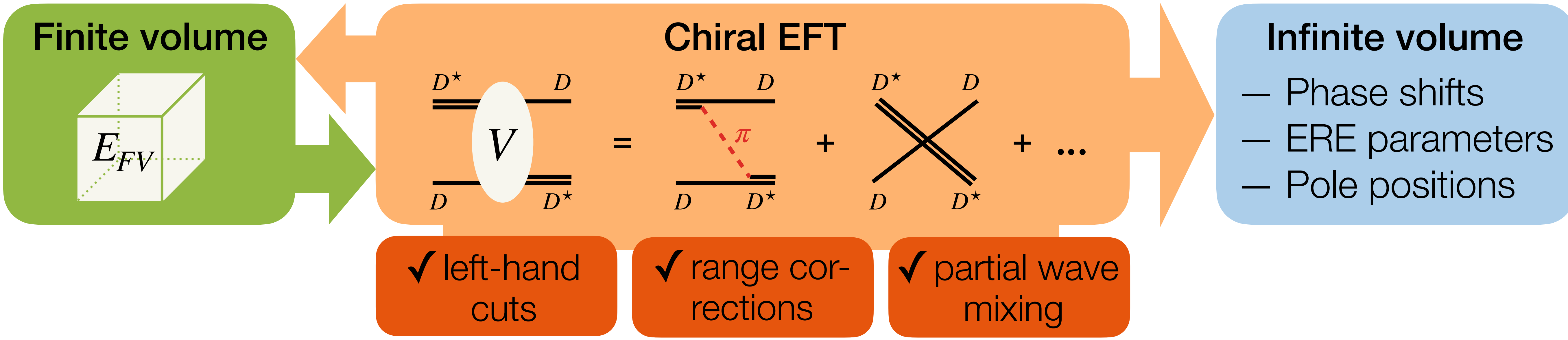
$$\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^* = -503(40)\ \text{keV}$$

⇒ (quasi)bound D*D* state ~ 0.5 MeV below the threshold

M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

App II: χEFT as an alternative to Lüscher

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)



— Construct regularized effective potential truncated to a given order

$$V = V^{(0)}_{\text{OPE}} + V^{(0)}_{\text{cont}} + V^{(2)}_{\text{cont}} + \ldots$$

$$V^{(0)+(2)}_{\text{cont}}[{}^3S_1] = \left(C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2)\right)(\vec{\epsilon}\cdot\vec{\epsilon}'^{*})$$

$$V^{(2)}_{\text{cont}}[{}^3P_0] = C^{(2)}_{{}^3P_0}\,(\vec{p}'\cdot\vec{\epsilon}'^{*})(\vec{p}\cdot\vec{\epsilon})$$

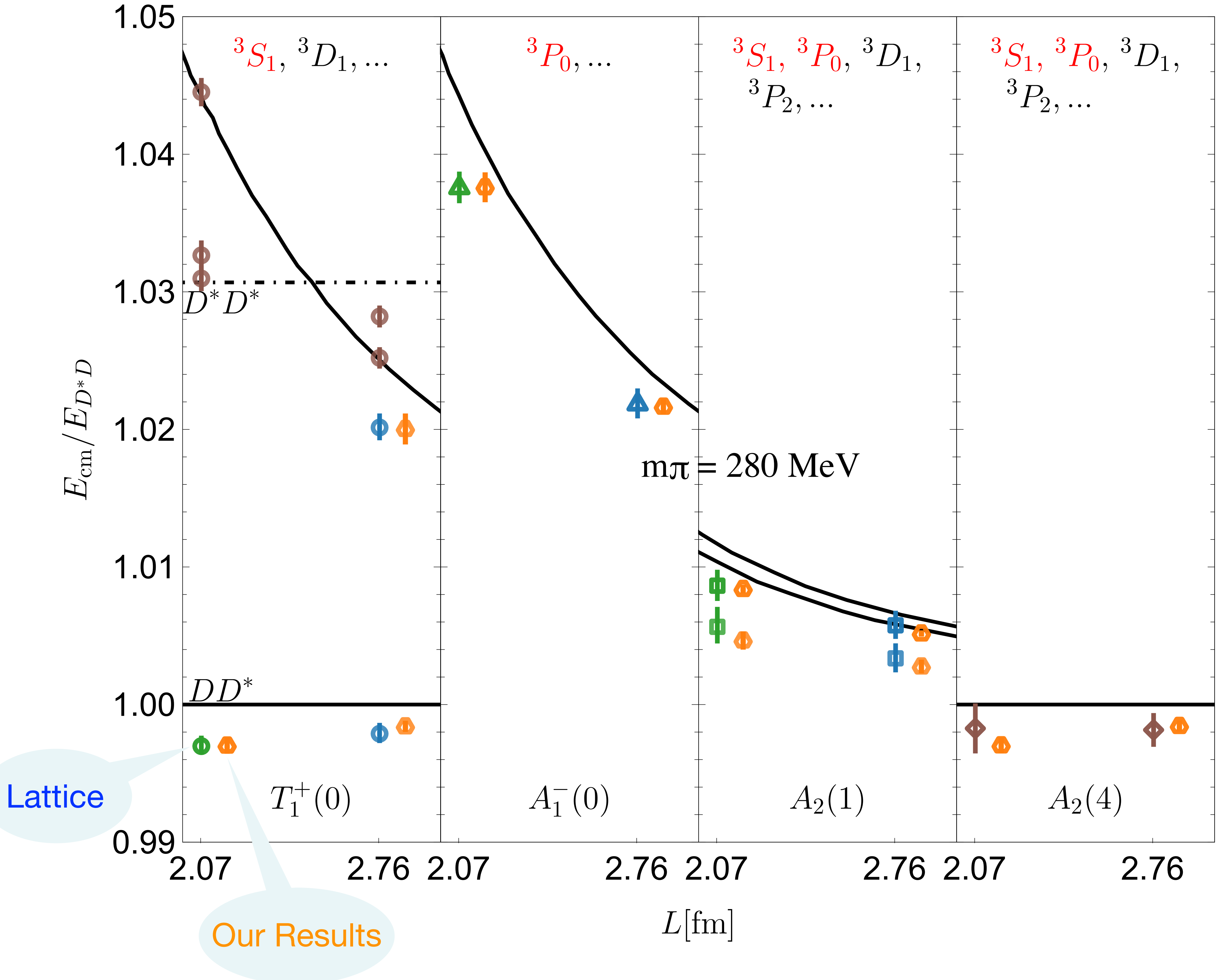
— Calculate E_{FV} in each irrep as a solution of the eigenvalue problem

$$\det\left[\mathbb{G}^{-1}(E) - \mathbb{V}(E)\right] = 0 \qquad \mathbb{G}_{\mathbf{n},\mathbf{n}'} = \mathcal{J}\frac{\delta_{\mathbf{n}',\mathbf{n}}}{L^3}\frac{1}{4E_D(\tilde{p}_{\mathbf{n}})E_{D^*}(\tilde{p}_{\mathbf{n}})}\,\frac{1}{E - E_D(\tilde{p}_{\mathbf{n}}) - E_{D^*}(\tilde{p}_{\mathbf{n}})}$$

— Adjust LEC's C's from best fits to E_{FV}: C's are independent of the volume size L

— Employ the EFT potential to calculate infinite volume amplitudes using LSE

DD* Finite Volume Energy Levels at $m_\pi = 280$ MeV



Lattice
Padmanath and Prelovsek, PRL 129, 032002 (2022)

Our Results at NLO χEFT

3 Param's:

2 in $^3S_1$

1 in $^3P_0$

— All PW's in OPE are included

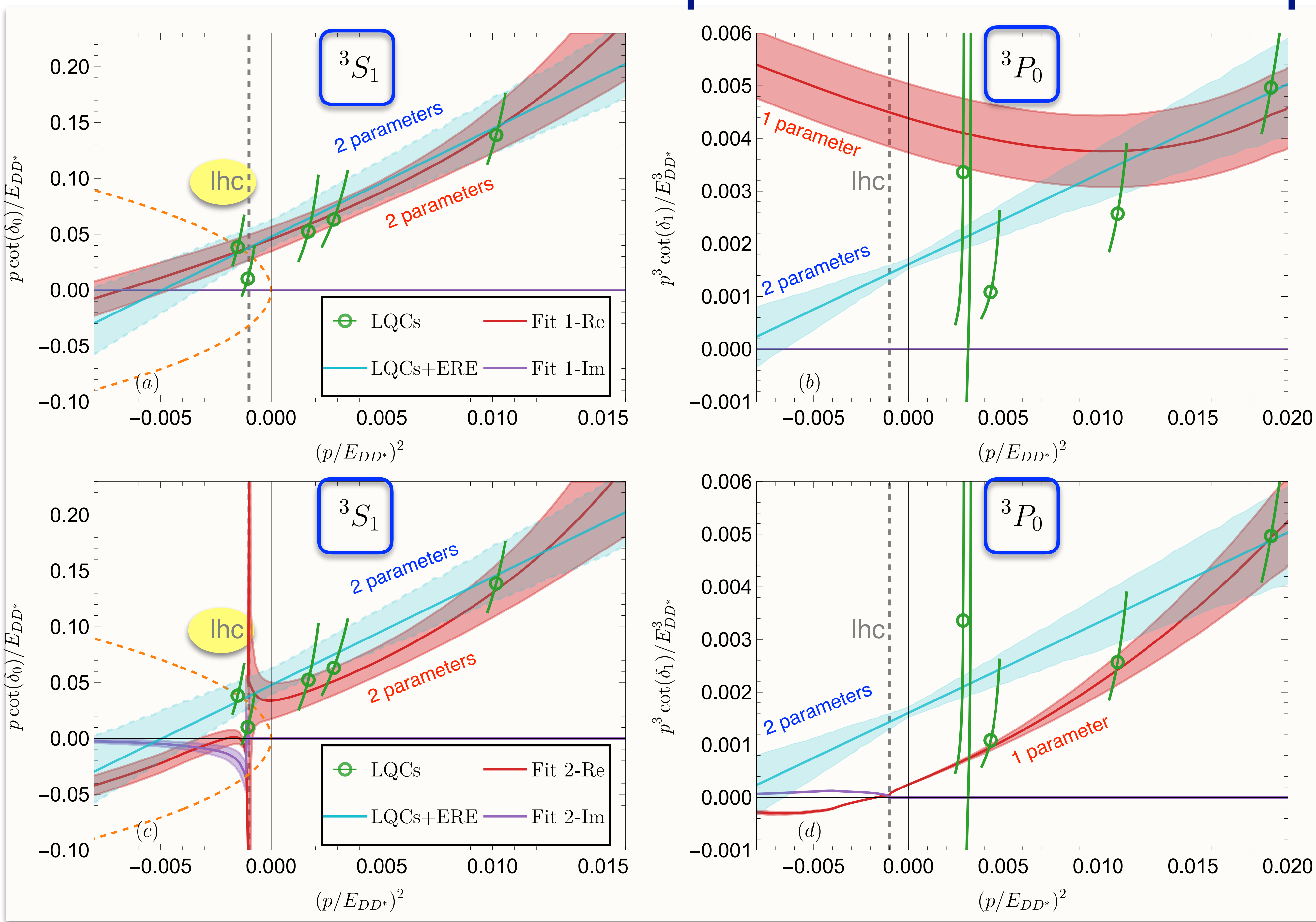
— mπ dependence of $g_{D^*D\pi}$, $m_{D^{(*)}}$ and f_π is included

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

Predict infinite volume phase shifts and Tcc pole

No OPE

With OPE



- $^3P_0$ shape controlled by OPE
- $^3S_1$ near lhc controlled by OPE

	$a_{^3S_1}$ [fm]	$r_{^3S_1}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	$a_{^3P_0}$ [fm^3]	$r_{^3P_0}$ [fm^{-1}]	χ^2/dof	# of param's
LQCs+ERE fit [23]	1.04 ± 0.29	$0.96^{+0.18}_{-0.20}$	$-9.9^{+3.6}_{-7.2}$	$0.076^{+0.008}_{-0.009}$	6.9 ± 2.1	3.7/5	4
Fit 1: cont.	1.09 ± 0.35	0.75 ± 0.14	-10.6 ± 4.4	0.028 ± 0.004	-4.3 ± 0.05	5.52/6	3
Fit 2: cont.+OPE	1.46 ± 0.57	0.096 ± 0.53	$-6.6(\pm1.5) - i4.0(\pm3.7)$	0.497 ± 0.007	5.63 ± 0.19	2.95/6	3

[23] Padmanath and Prelovsek, PRL 129, 032002 (2022)

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

T_{cc} at m_π = 280 MeV is a resonance state with 85% probability

App III: Pion-mass dependence of the Tcc pole

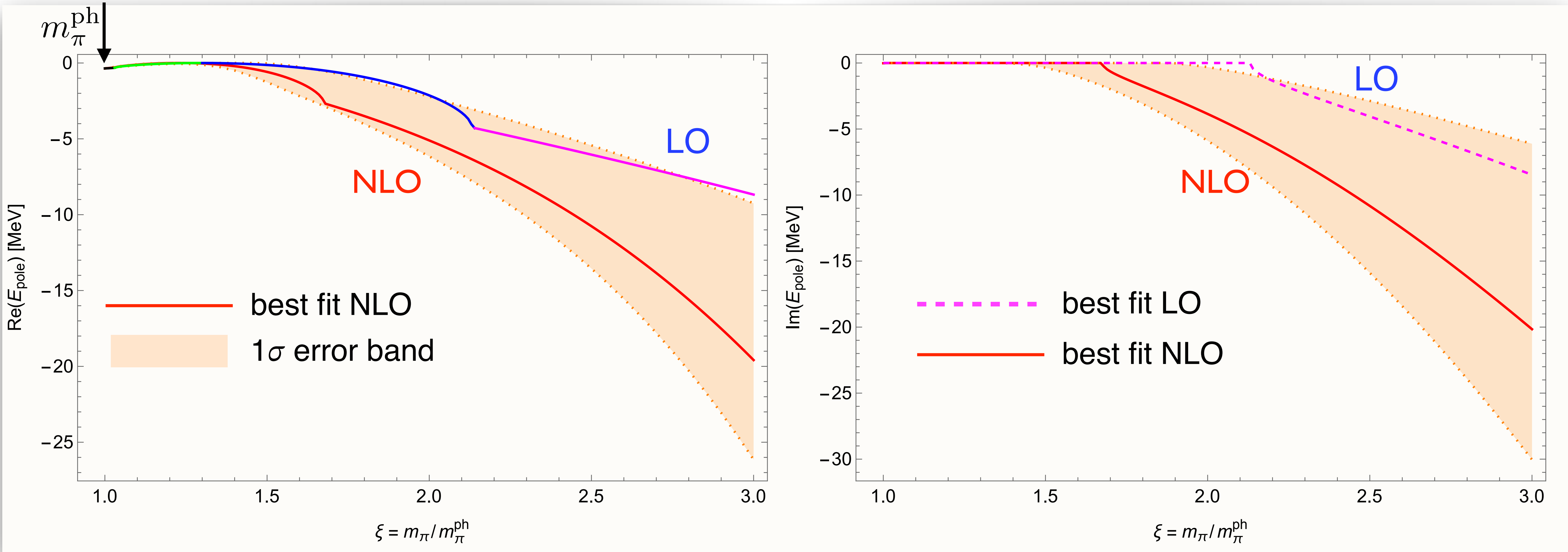
M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

- Employ knowledge of the NLO potential at $m_\pi = m_\pi^{\rm ph}$ and $m_\pi = 280~\text{MeV}$

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} \qquad V^{(0)+(2)}_{\rm cont}[^3S_1] = C^{(0)}_{^3S_1} + C^{(2)}_{^3S_1}(p^2+p'^2) + D^{(2)}_{^3S_1}(\xi^2-1)$$

$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

- Calculate scattering amplitude for any m_π



- Tcc pole transitions: quasi-bound → bound → virtual → resonance

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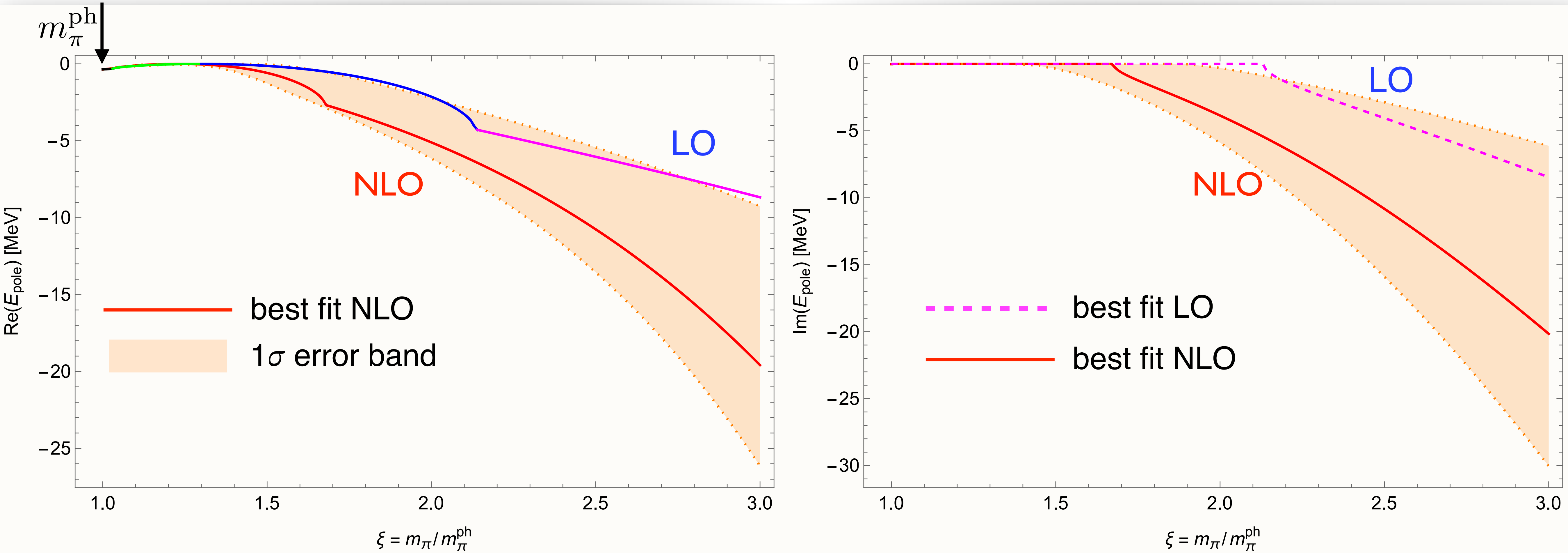


- Tcc pole transitions: quasi-bound → bound → virtual → resonance

- NLO is qualitatively consistent to LO; resonance is formed at smaller m_π

⇒ Trajectory consistent with hadronic molecule Matuschek, VB, Guo, Hanhart, EPJA 57, 101 (2021)

Truncation uncertainty of chiral expansion

Add higher-order interactions + naturalness

$$V = {\color{red}V^{(0)}_{\rm OPE}} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + V^{(4)}_{\rm cont}$$

$$V^{(0)+(2)}_{\rm cont}[{}^3S_1] = C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2) + D^{(2)}_{{}^3S_1}(\xi^2 - 1)$$

Fits

$$C_2 = \frac{\alpha_2}{F_\pi^2}\frac{1}{\Lambda_\chi^2}; \quad \alpha_2 = 0.2$$

$$D_2 = \frac{\tilde{\alpha}_2}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^2; \quad \tilde{\alpha}_2 = 0.4$$

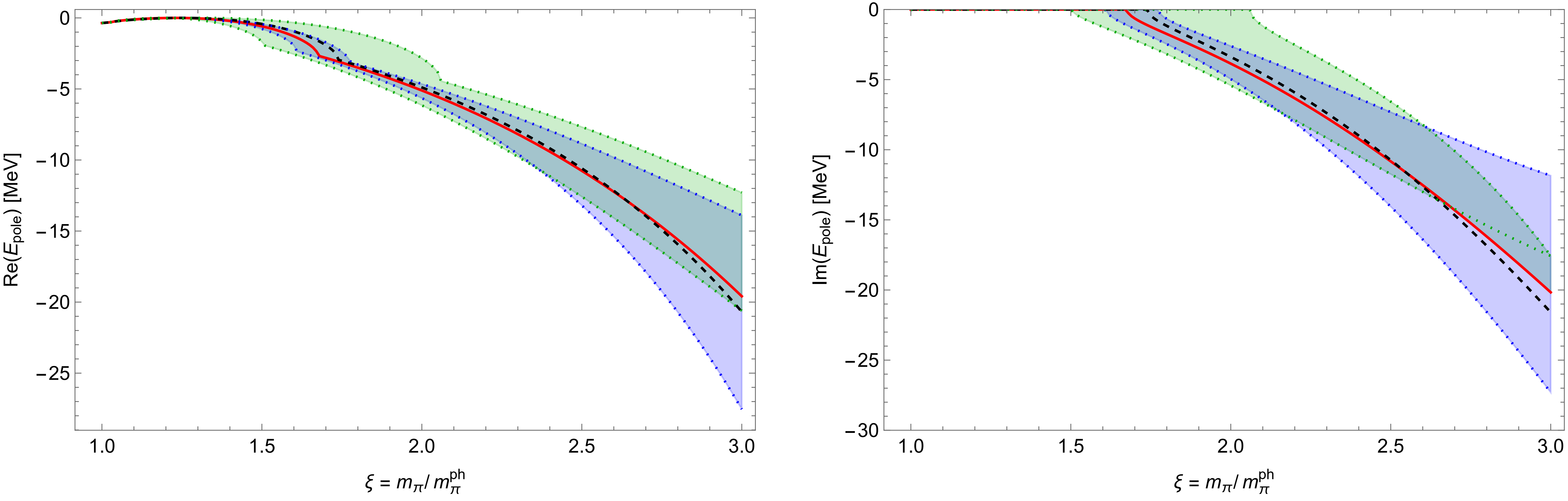
Naturalness

$$V^{(4)}_{\rm cont} = {\color{green}D_4(\xi^2 - 1)(p^2 + p'^2)} + {\color{blue}\tilde{D}_4(\xi^4 - 1)}$$

$${\color{green}D_4 = \frac{\alpha_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi^2}\right)^2; \quad \alpha_4 \in [-1, 1]}$$

$${\color{blue}\tilde{D}_4 = \frac{\tilde{\alpha}_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^4; \quad \tilde{\alpha}_4 \in [-1, 1]}$$

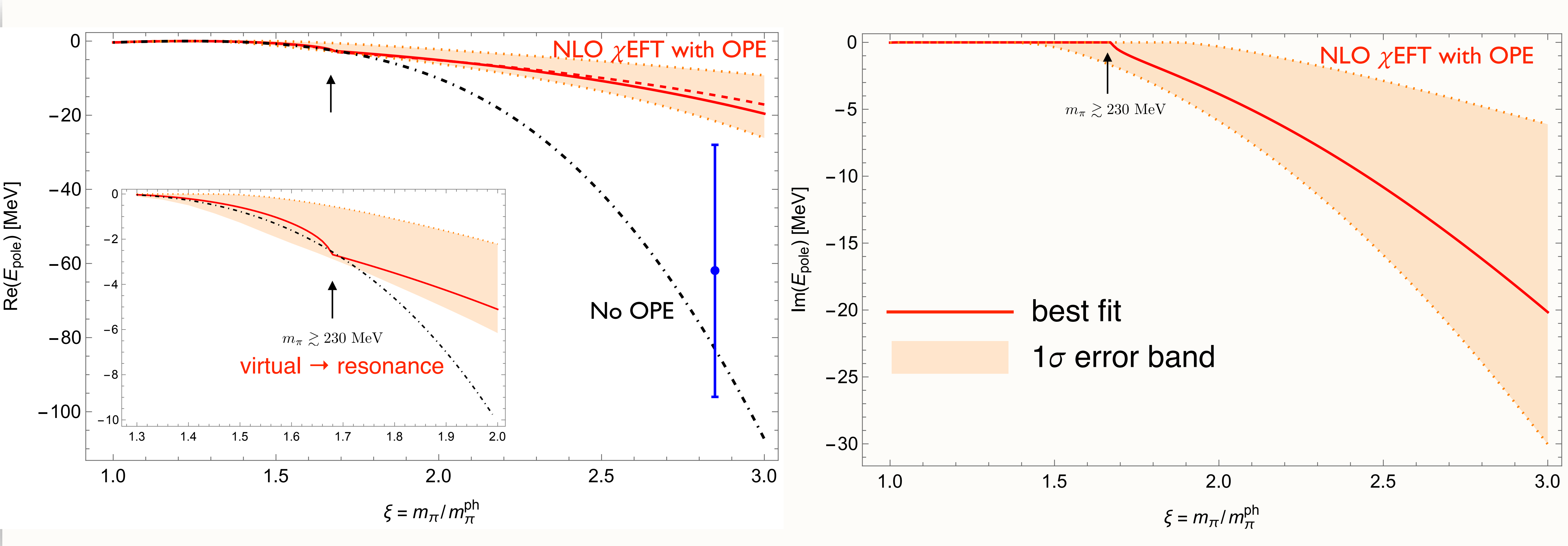
⇒ Comparable with statistical uncertainty



T_{cc} pole vs m_π: Contact vs Pionful

M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

$$V = V_{\rm OPE}^{(0)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} \qquad V_{\rm cont}^{(0)+(2)}[^3S_1] = C_{^3S_1}^{(0)} + C_{^3S_1}^{(2)}(p^2 + p'^2) + D_{^3S_1}^{(2)}(\xi^2 - 1)$$



- Our pionless trajectory is consistent with new lattice data at $m_\pi = 391$ MeV
  - — Virtual state extracted using Lüscher + amplitude parameterization without pions

T. Whyte, D. Wilson, and C. Thomas arXiv:2405.15741v1

- But due to repulsion from the OPE, NLO χEFT yields a resonance for $m_\pi > 230$ MeV

⇒ Long-range physics significantly changes the pole trajectory

Summary and Conclusions

- Properties of exotic state: Line-shape analyses preserving unitarity

- χEFT is well suited for analysing data in the infinite and finite volume:

  — Incorporates long range dynamics with relevant cuts

  — Control of systematics: truncation of the chiral EFT

  — Plain wave basis in the finite volume: partial wave mixing   Meng, Epelbaum JHEP 10 (2021)

  — Statistical errors using bootstrap

- All Tcc properties: ERE parameters, Compositeness, and the pole trajectory

  quasi-bound → bound → virtual → resonance   consistent with a molecule

- 1π exchange: Important role in low-energy DD* scattering

- Other finite-volume methods developed   Fernando Romero-López talk on Wednesday

  Dawid, Romero-Lopez, Sharpe 2409.17059;   Raposo, Hansen *JHEP* 08 (2024); Bubna et al. JHEP 05 (2024); Hansen et al. JHEP 06 (2024)

  — All based on MERE: explicit long-range + low-energy parameterization of short range effects

Summary and Conclusions

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Many applications and interesting physics still to come…   Thank you!

Backup

χEFT for Tcc

- LO effective Lag consistent with chiral and heavy-quark spin symmetries (HQSS)

Mehen and Powell, PRD 84,114013(2011)
AlFiky et al., PLB640,238(2006)

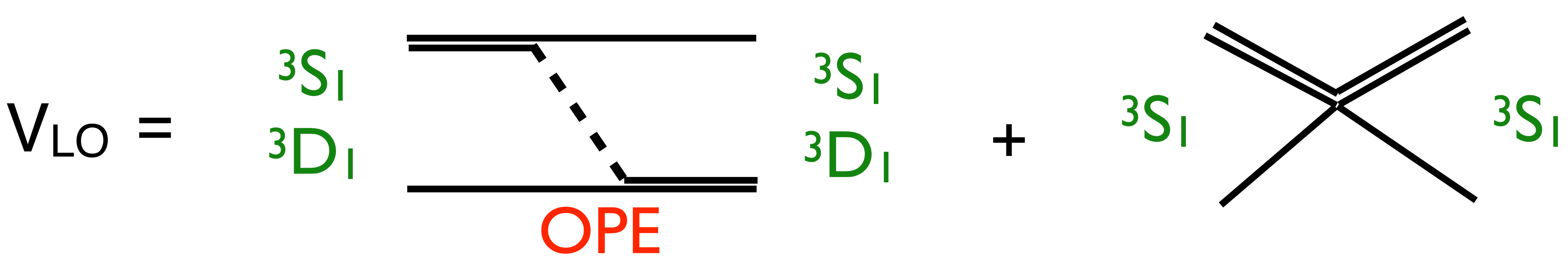
$$\mathcal{L}_{\rm LO} = -\frac{D_{10}}{8}\mathrm{Tr}\left(\tau^A_{aa'}H^\dagger_{a'}H_b\tau^A_{bb'}H^\dagger_{b'}H_a\right) - \frac{D_{11}}{8}\mathrm{Tr}\left(\tau^A_{aa'}\sigma^i H^\dagger_{a'}H_b\tau^A_{bb'}\sigma^i H^\dagger_{b'}H_a\right)$$
$$-\frac{D_{00}}{8}\mathrm{Tr}\left(H^\dagger_a H_b H^\dagger_b H_a\right) - \frac{D_{01}}{8}\mathrm{Tr}\left(\sigma^i H^\dagger_a H_b \sigma^i H^\dagger_b H_a\right) + \frac{1}{4}g\,\mathrm{Tr}\left(\boldsymbol{\sigma}\cdot\boldsymbol{u}_{ab}H_b H^\dagger_a\right)$$

$$H_a = P_a + \boldsymbol{V}_a\cdot\boldsymbol{\sigma}, \qquad P_a = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}_a, \quad \boldsymbol{V}_a = \begin{pmatrix} \boldsymbol{D}^{*0} \\ \boldsymbol{D}^{*+} \end{pmatrix}_a \qquad \boldsymbol{u} = -\nabla\Phi/f_\pi \qquad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

— HQSS and isospin constrain the # of param's to just one: $v_0 \equiv -2(D_{01} - 3D_{11})$

— g is known from $D^* \to D\pi$

- LO coupled-channel isoscalar potential in the particle basis $\{D^{*+}D^0, D^{*0}D^+\}$



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$$V_{\rm CT}(D^*D \to D^*D; 1^+) = \frac{1}{2}\begin{pmatrix} v_0 & -v_0 \\ -v_0 & v_0 \end{pmatrix}$$

χEFT for Tcc

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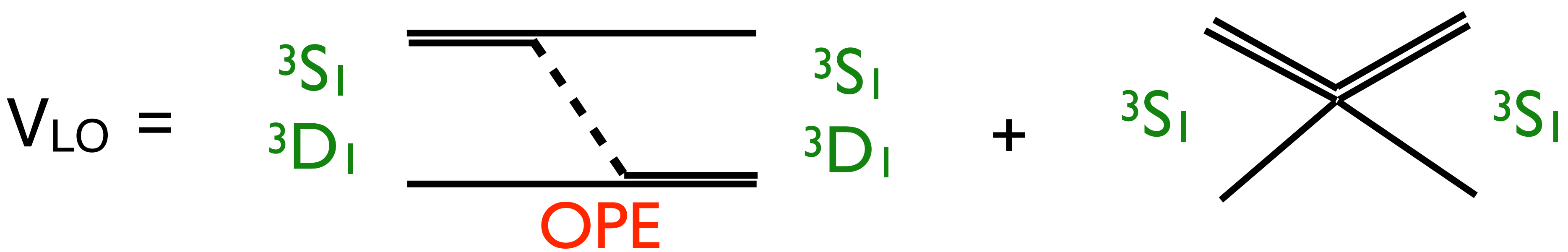

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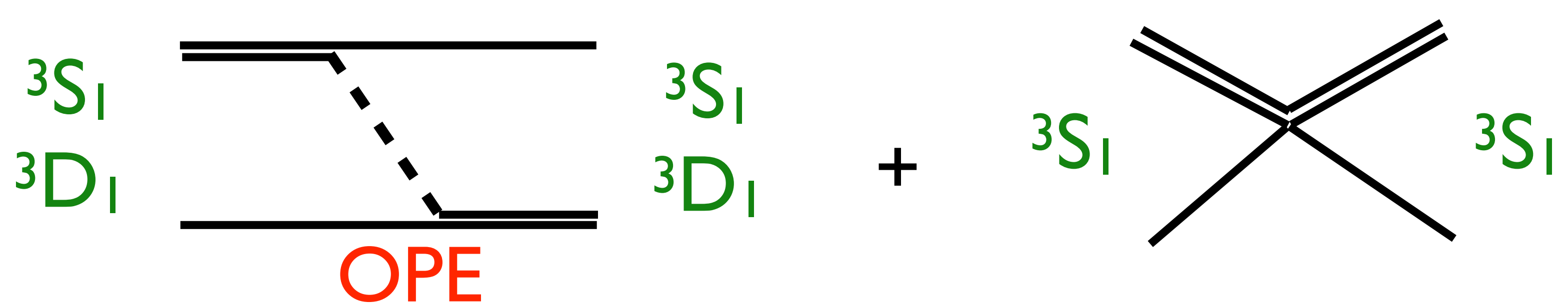
nonperturbative

V_{LO} =



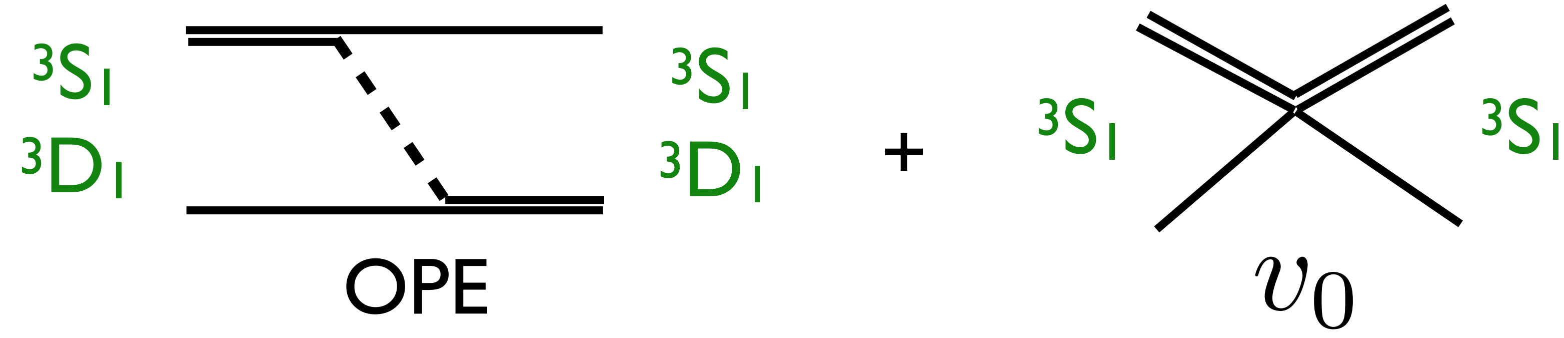
→ $T_{\alpha\beta}$

$$T_{\alpha\beta} = V^{\rm eff}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

Pointlike production source

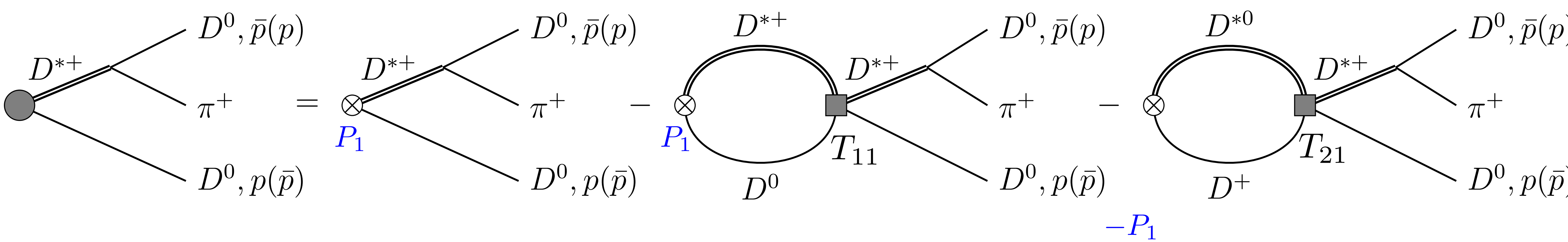
- LO isoscalar potential:

V_{LO} =



$\rightarrow \; T_{\alpha\beta}$

- Production amplitude:



- Only two parameters to be fitted to the $D^0D^0\pi^+$ spectrum: v_0 and overall Norm ~ P_1^2

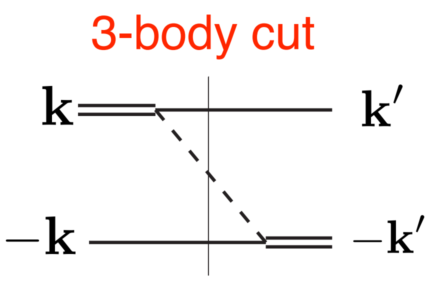
3-body DDπ cut

- OPE potential:

$$V_{\mathrm{DD^*\to DD^*}}(\mathbf{k},\mathbf{k}',E) \propto \frac{g_c^2}{(4\pi f_\pi)^2}\,\tau_1\cdot\tau_2\,\frac{(\epsilon_1\cdot\vec{q})\,(\epsilon_2'^{\,*}\cdot\vec{q})}{2E_\pi(\mathbf{k}-\mathbf{k}')}\left(\frac{1}{D_{DD\pi}(\mathbf{k},\mathbf{k}',E)}+\frac{1}{D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}',E)}\right)$$

$$D_{DD\pi}(k,k',E) = E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E \quad \Rightarrow$$

goes on shell!



$$D_{DD\pi}(k,k',E)\to i\pi\delta(E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E)\to \text{Im part}$$

3-body cut condition

- ☛ For each $E \geq E_{\mathrm{thr}} \equiv 2m+m_\pi$, there are real values of k and k$'$ such that $D_{DD\pi}(k,k',E)=0$

  3-body branch point is (k=k$'$=0): $E \equiv 2m+m_\pi$

3-body DDπ cut

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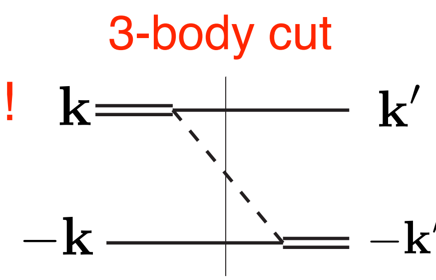

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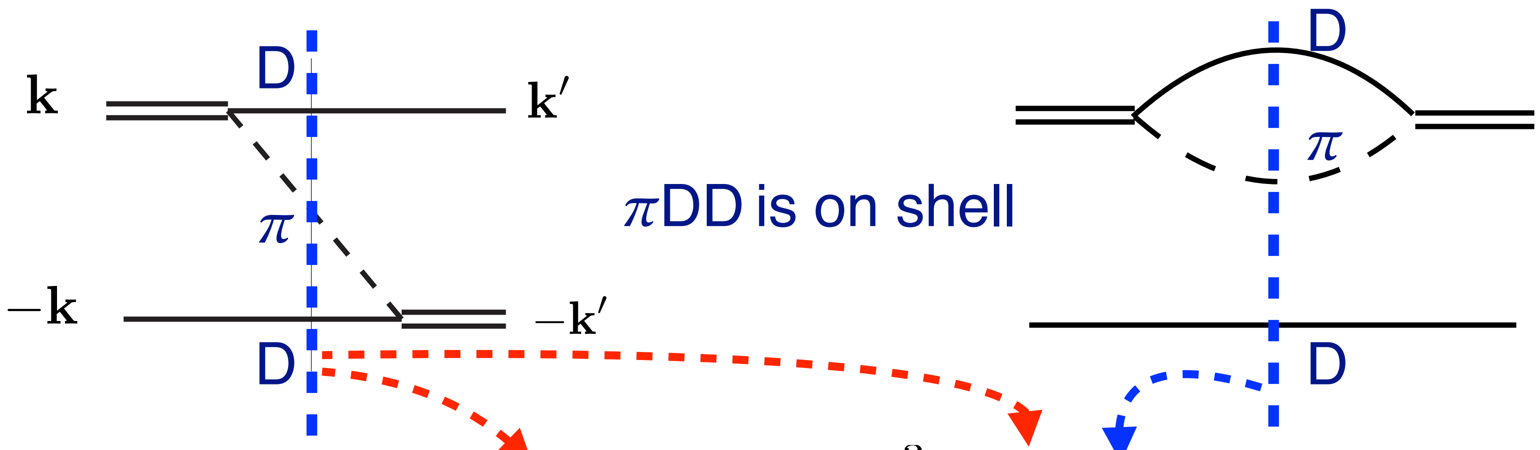
3-body branch point is (k=k′=0): $E \equiv 2m+m_\pi$

- 3-body cut stems from OPE potential and self energies in the Green function



$$T_{\alpha\beta} = V^{\mathrm{eff}}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\mathrm{eff}}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

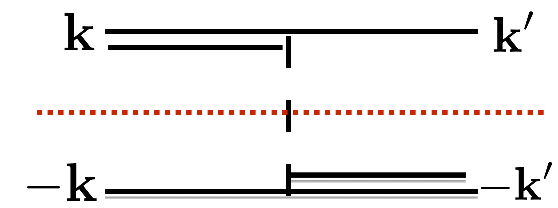
Bose statistics for DD requires both cont's to appear together

Full analogy to the X(3872)

VB et al. PRD84 (2011)

Left-hand cut

— Leading singularity is from the on shell one-pion exchange



PWD of the static potential:

$$V_{l=0}(k,k') \propto \int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2+\mu^2} = \frac{1}{2kk'} \log\frac{(k+k')^2+\mu^2}{(k-k')^2+\mu^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2}\log\frac{4p^2+\mu^2}{\mu^2}$$

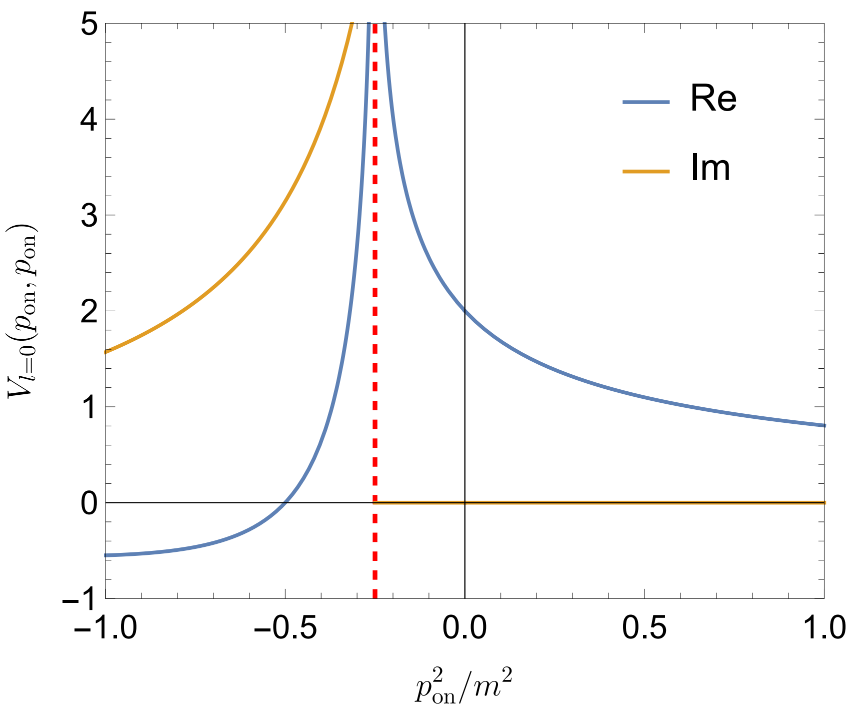
$\Rightarrow$ left-hand cut (lhc) branch point is at

$$(p_{\rm lhc}^{1\pi})^2 = -\frac{\mu^2}{4}$$

For DD*

$\mu^2 = m_\pi^2 - \Delta M^2$

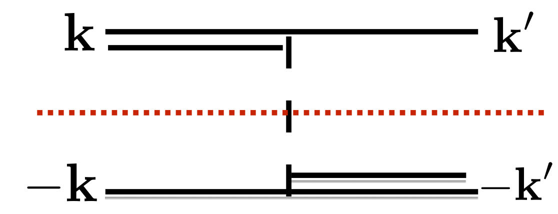
$\Delta M = m_{D^*} - m_D$



Sc. amplitude is complex for E below the lhc
$\Rightarrow$ Lüscher's method breaks down

Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021) , Meng et al, *PRD* 109 (2024)

Left-hand cut



— Leading singularity is from the on shell one-pion exchange

PWD of the static potential:

$$V_{l=0}(k,k') \propto \int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2+\mu^2} = \frac{1}{2kk'}\log\frac{(k+k')^2+\mu^2}{(k-k')^2+\mu^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2}\log\frac{4p^2+\mu^2}{\mu^2}$$

$\Rightarrow$ left-hand cut (lhc) branch point is at $(p_{\rm lhc}^{1\pi})^2 = -\frac{\mu^2}{4}$

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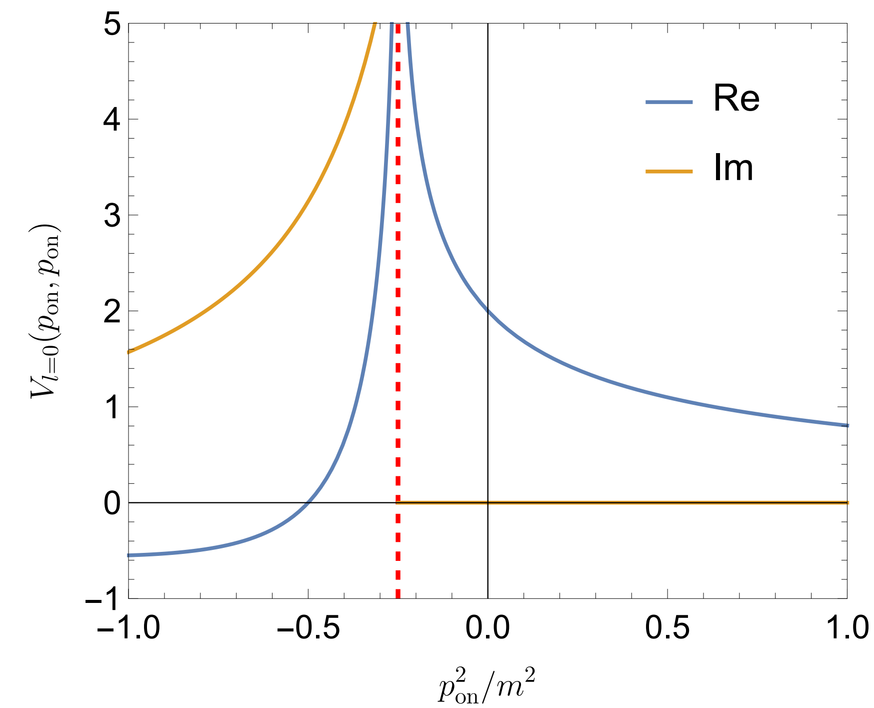
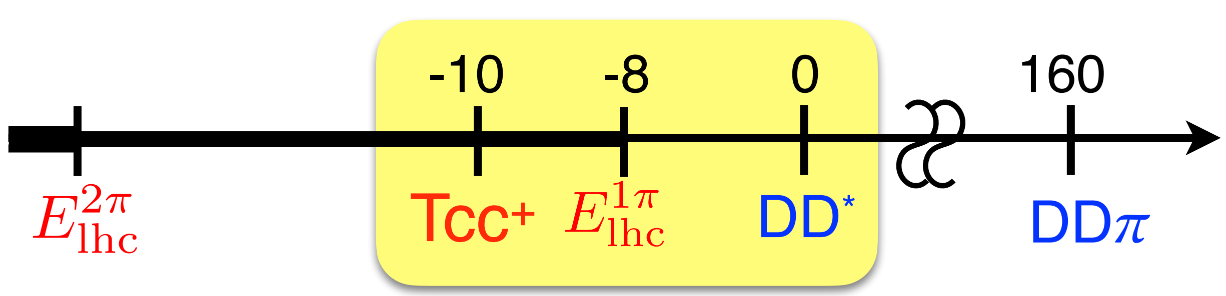


Sc. amplitude is complex for E below the lhc
$\Rightarrow$ Lüscher's method breaks down

Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021) , Meng et al, *PRD* 109 (2024)

At mπ = 280 MeV

$$E_{\rm lhc}^{1\pi} = \frac{(p_{\rm lhc}^{1\pi})^2}{2\mu_{DD^*}} = -8\,\text{MeV} \Rightarrow E_{\rm lhc}^{1\pi}$$ sets the range of convergence of the ERE: $E \ll |E_{\rm lhc}^{1\pi}|$

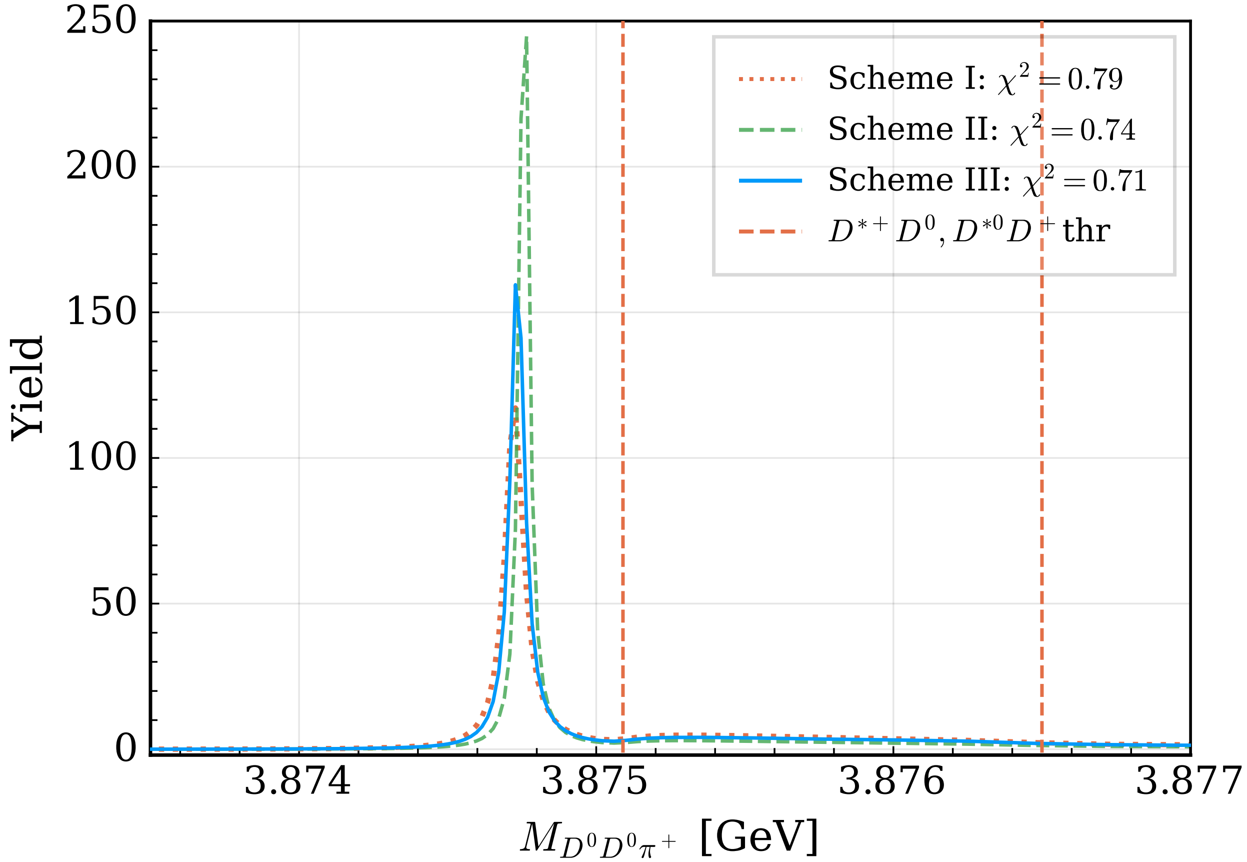


$\Rightarrow$ ERE is not applicable

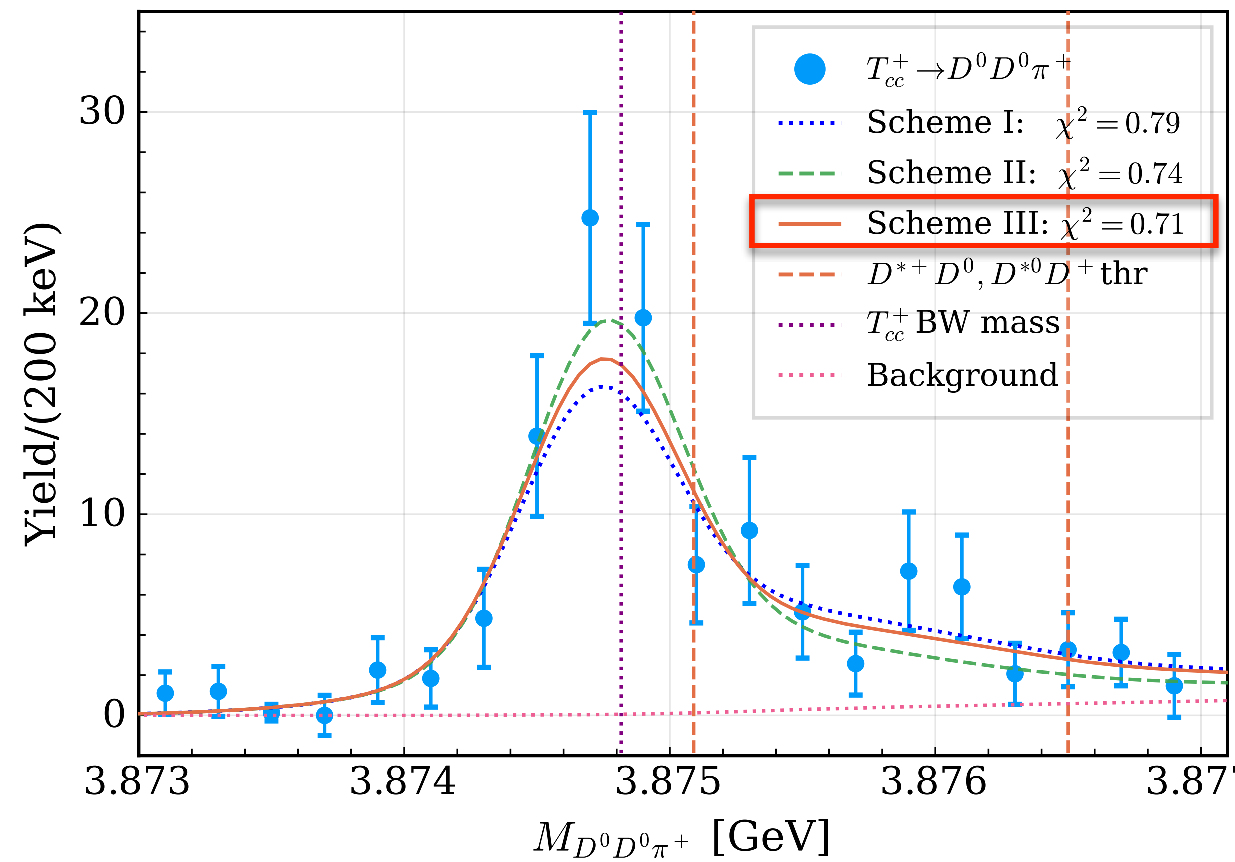
M. Du et al, *PRL* 131, 131903 (2023)

App I: Fits to the $D^0D^0\pi^+$ mass spectrum

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

Real part of the pole: all Fits are consistent within 1σ —more precise data are needed

Width of Tcc+ : Accuracy requires 3-body effects

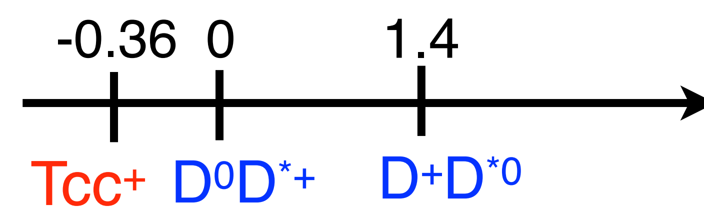
$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$ —remove OPE→ 36 keV —remove dynam.width→ 74 keV

Low-energy parameters

Du et al. PRD 105, 014024 (2022)

Scattering amplitude in the 1st (close to the pole) channel :

$$T_{D^{*+}D^0 \to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0} + \frac{1}{2}r_0 k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$



$$r_0' = r_0 - \Delta r$$

Eff. range in the 1st channel

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2\delta_2}} \simeq -3.8 \text{ fm}$$

Negative "correction" from 2nd $D^{*0}D^+$ channel caused by isospin breaking δ_2

$$\delta_2 = m_{\text{thr2}} - m_{\text{thr1}}$$

VB et al., PLB 833 (2022)

a_0 [fm]	r_0 [fm]	r_0' [fm]	X_A
$\left(-6.72^{+0.36}_{-0.45} \pm 0.27\right) - i\left(0.10^{+0.03}_{-0.03} \pm 0.03\right)$	$-2.40 \pm 0.01 \pm 0.85$	$1.38 \pm 0.01 \pm 0.85$	$0.84 \pm 0.01 \pm 0.06$

$$r_0' \ll |a_0|$$

– r_0' positive and is of natural size

– Contrib. to r_0' from OPE is ~ 0.4 fm

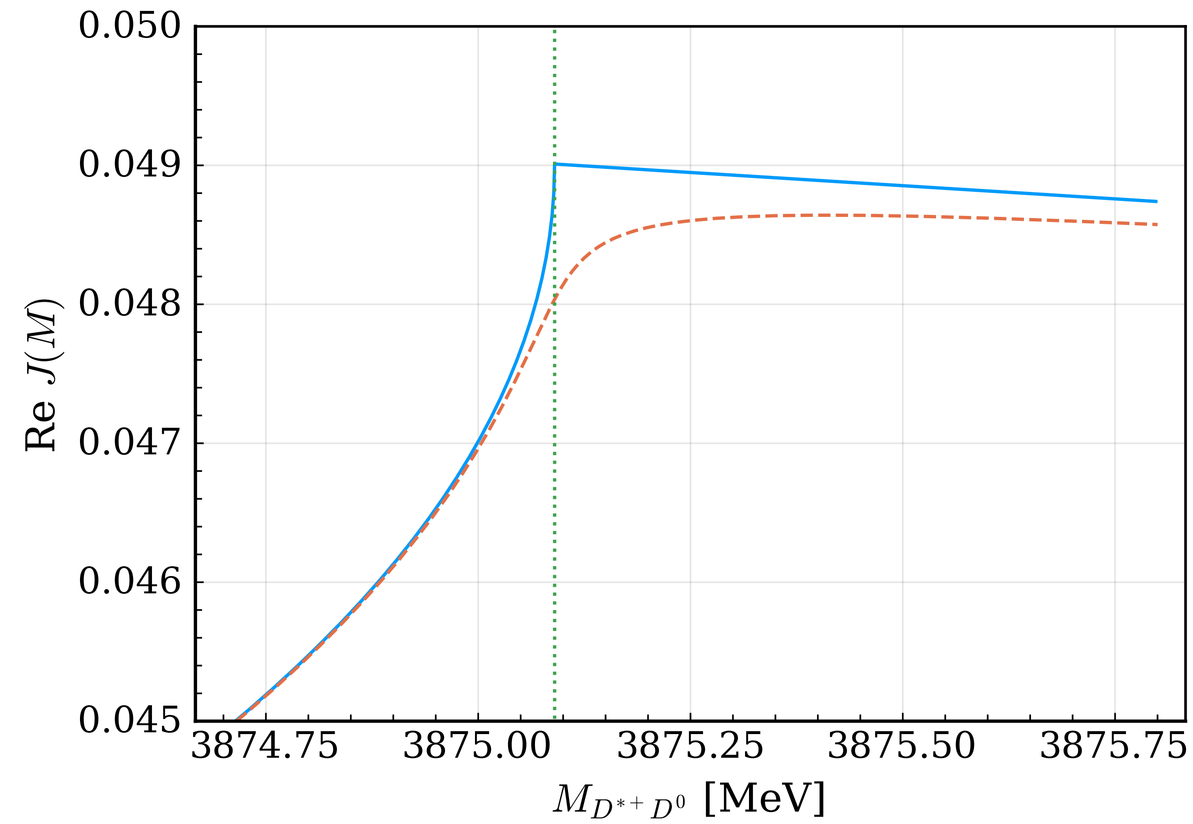
T_{cc} is consistent with a pure isoscalar molecule!

Extraction of the effective range

Scattering amplitude in the 1st channel:

$$T_{D^{*+}D^0\to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0}+\frac{1}{2}r_0k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$

$$T^{-1}(M) = V_{\rm CT}^{-1} + J(M), \quad J(M) = \int \frac{{\rm d}^3\boldsymbol{p}}{(2\pi)^3} G(M,p)$$



no width:

$$r_0 \propto -\mathfrak{Re}\frac{{\rm d}J(M)}{{\rm d}M}\bigg|_{M=M_{\rm thr}+0^+}$$

finite width: ERE has a small radius of convergence

$$k \leqslant \sqrt{\mu_{c0}\Gamma_{D^{*+}}} \approx 9 \text{ MeV}$$

Approximate Solution: expand around the pole of the Green function

Braten and Stapleton (2010)

$$M = m_c^* - i\Gamma_c/2 + m_0 + \frac{k^2}{2\mu_{c0}}$$

Corrections scale as $\frac{1}{2}\frac{\Gamma_{D^*}}{M_{\rm thr2}-M_{\rm thr3}}$ ⟶ tiny for the problem at hand

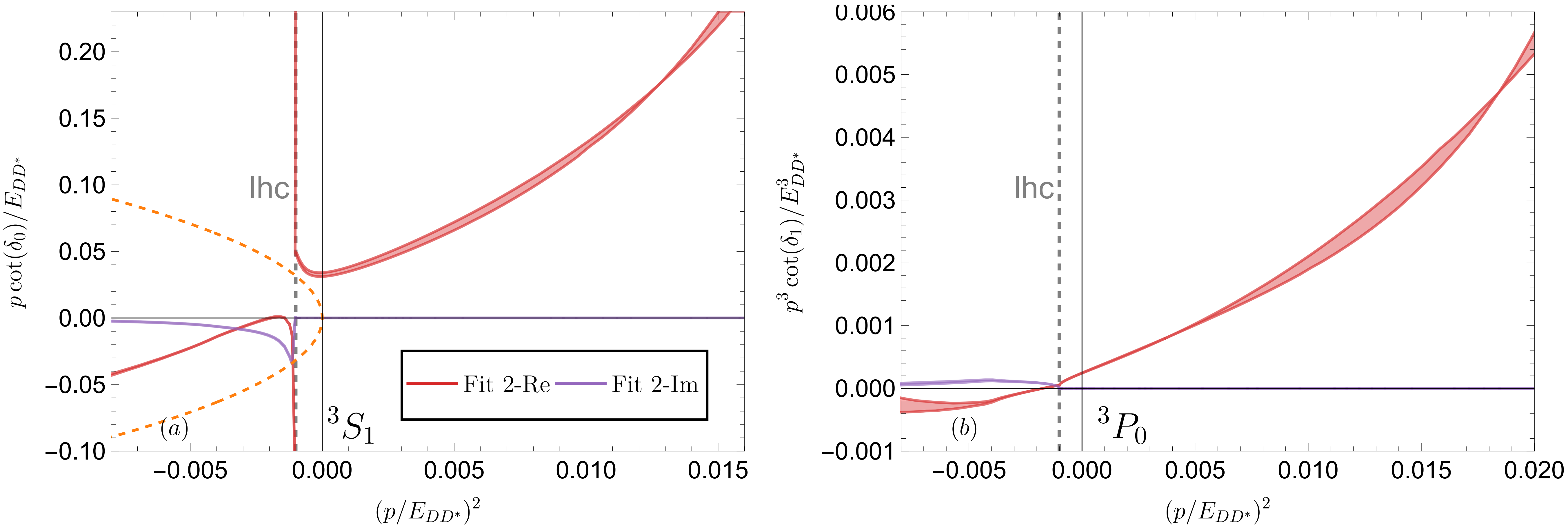
Du et al. 2110.13765

Hanhart et al (2010)

App II: Residual cutoff dependence

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

- Cutoff variation from 0.7 to 1.2 GeV



$\Rightarrow$ very small

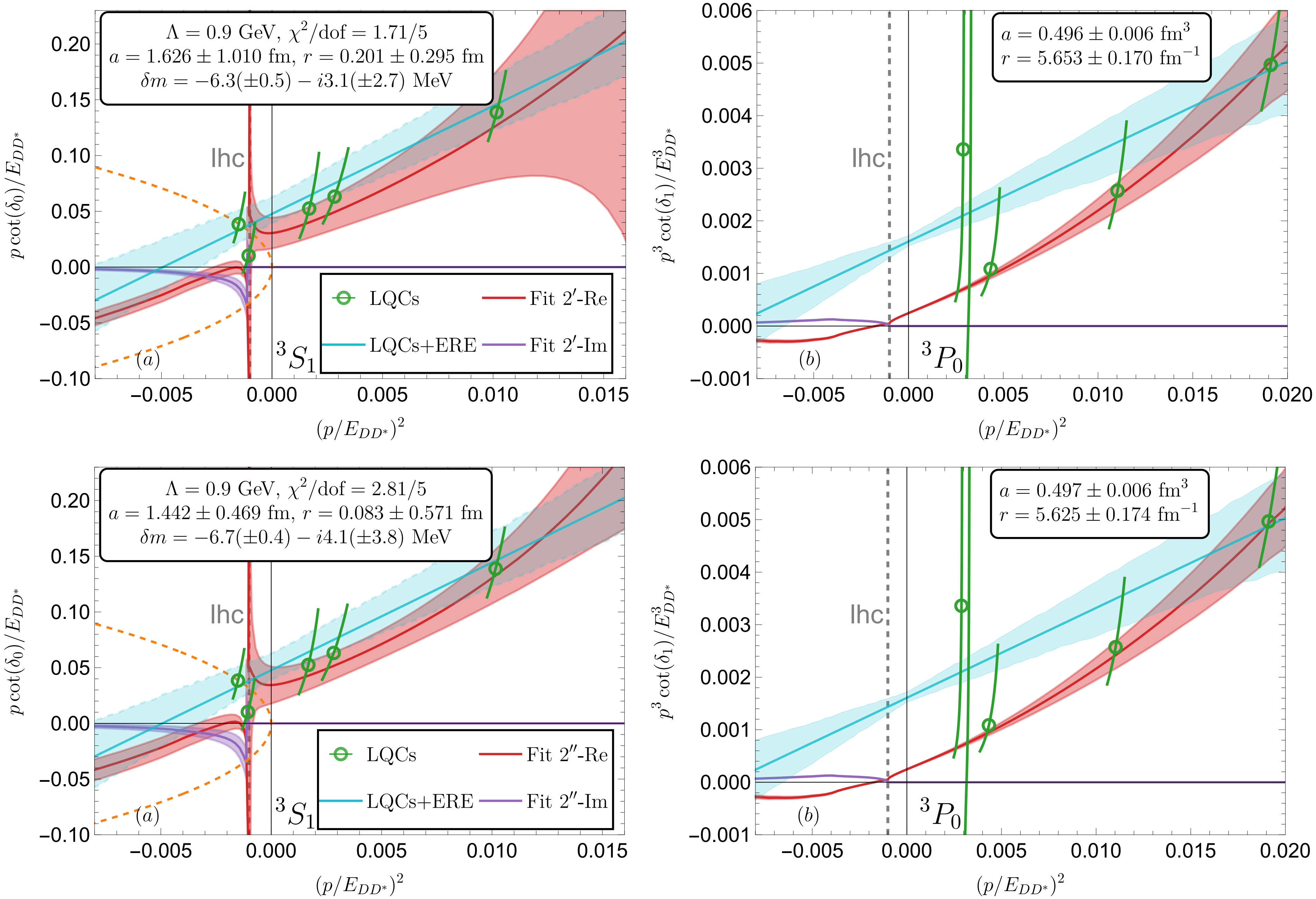
Testing chiral truncation uncertainty at $m_\pi = 280$ MeV

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

- Additional contact terms

$$V^{(2)}_{\text{cont}}[{}^3S_1 - {}^3D_1] = C^{(2)}_{SD}\, p'^2$$

$$V^{(2)}_{\text{cont}}[{}^3P_2] = C^{(2)}_{{}^3P_2}$$



⇒ Some effect of the S-D term on phase shifts at larger momenta

⇒ The impact near the threshold and on the pole is minor

The pion coupling from fits to data

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

Two parameter fits to lattice + physical value:

- Linear extrapolation:

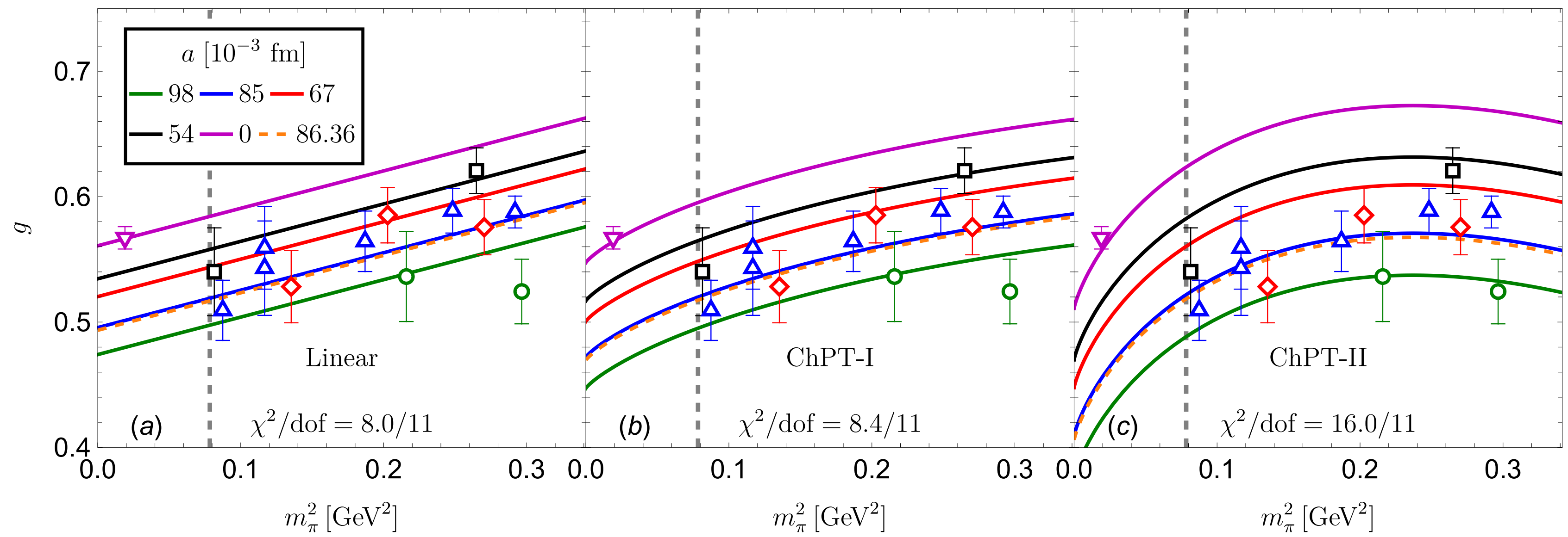
$$g(a, m_\pi) = g_0(1 + \alpha m_\pi^2 + \beta a^2)\,,$$

- ChPT-I:

$$g(a, m_\pi) = g_0\left(1 - \frac{2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$

- ChPT-II:

$$g(a, m_\pi) = g_0\left(1 - \frac{1 + 2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$



Lattice data: Becirevic and Sanfilippo Phys. Lett. B 721, 94 (2013)

	g_0	α [GeV^{-2}]	β [fm^{-2}]	g
Linear	0.561(9)	0.53(13)	−16.1(44)	0.517(15)
ChPT-I	0.547(8)	0.24(14)	−19.1(45)	0.517(15)
ChPT-II	0.511(8)	−0.59(15)	−27.6(48)	0.519(15)

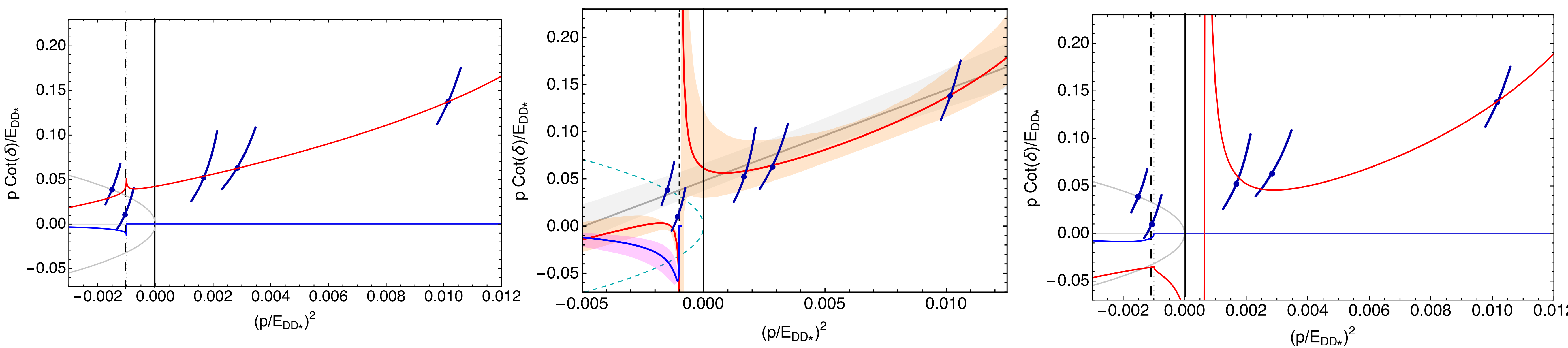
$g \equiv g(0.08636, 0.280)$

Dependence on the pion coupling

M. Du, A. Filin, VB, X. Dong, E. Epelbaum, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRL* 131, 131903 (2023)

- Importance of lhc is controlled by its position and strength (residue)

$\frac{1}{10} V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280{\rm MeV})$ $\quad V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280{\rm MeV})$ $\quad 10\ V^{\rm OPE}_{\rm DD^* \to DD^*}(m_\pi = 280{\rm MeV})$



- The smaller the coupling, the closer the fit is to the ERE

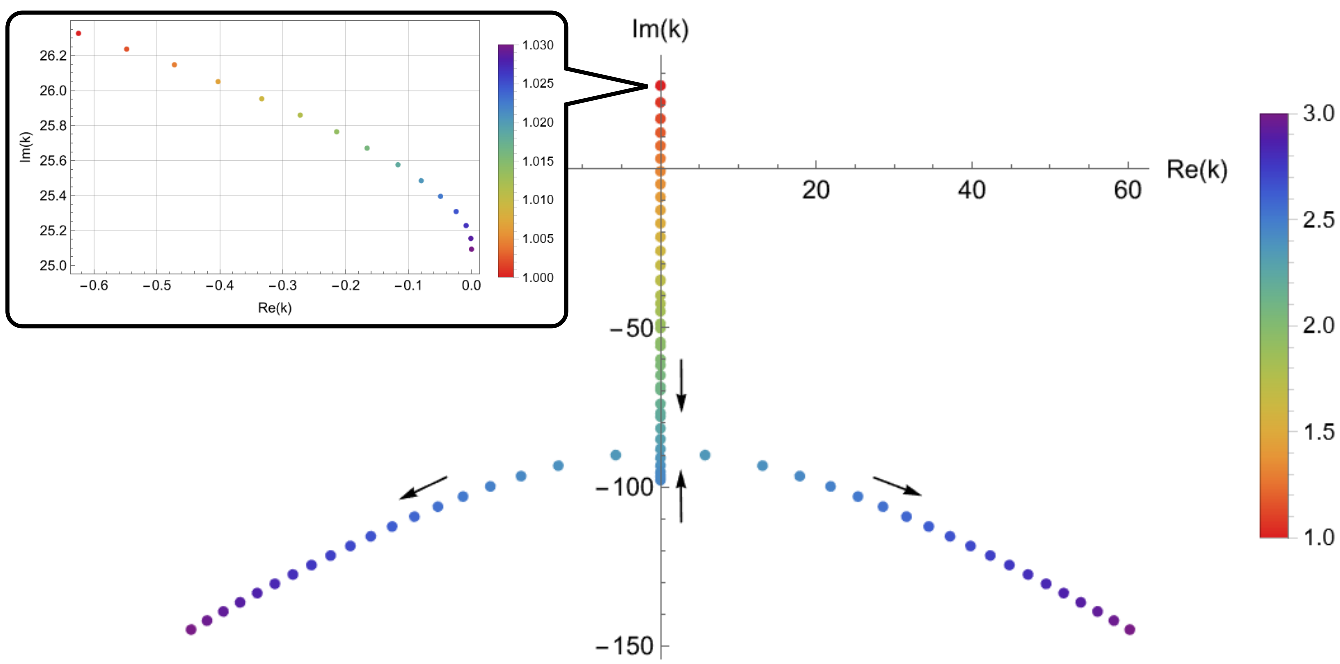
App III: Pion-mass dependence of the Tcc pole at LO

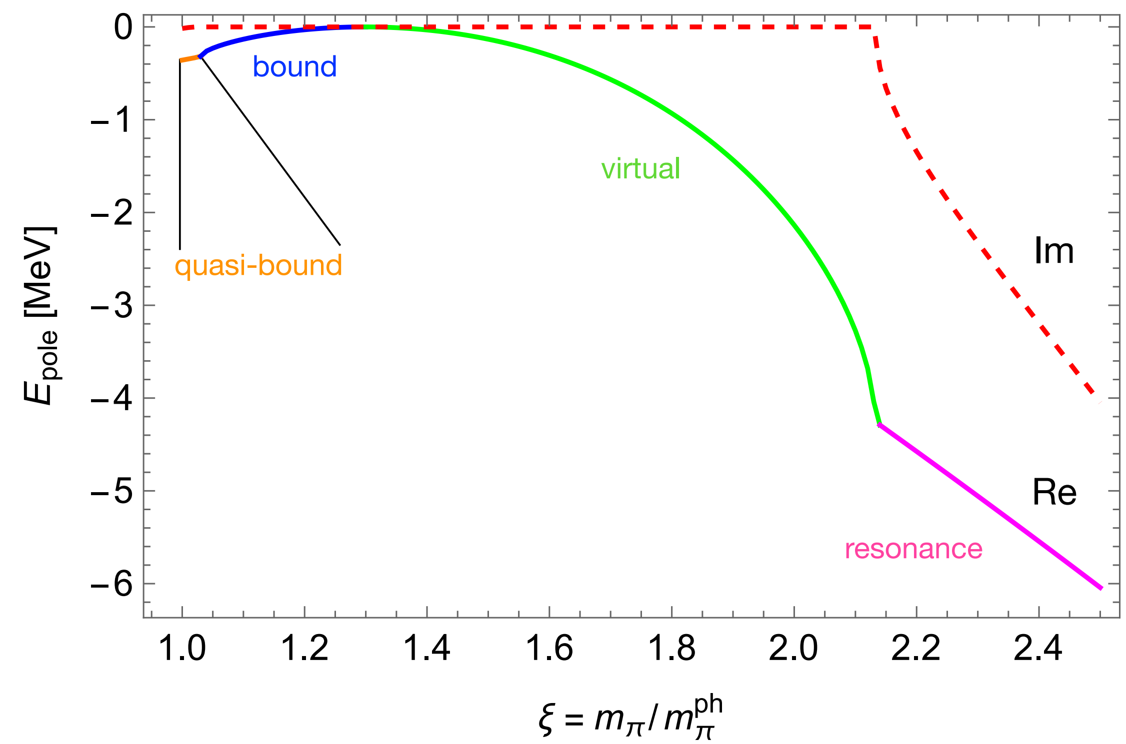
M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

Fixed from the Tcc pole at physical $m_\pi^{\rm ph}$

LO results

$$V = V_{\rm OPE}^{(0)} + C_{^3S_1}^{(0)}$$





Tcc pole transitions: quasi-bound → bound → virtual → resonance

Consistent with hadronic molecule

I. Matuschek, VB, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A 57, 101 (2021)

Ex: proton-neutron bound and virtual states

Matuschek, VB, Guo, Hanhart
EPJA 2021

Deuteron

- $a = -5.41\ \text{fm}$, $r = +1.75\ \text{fm}$ $\Longrightarrow$ large a: $|a| \gg |r|$, $r \sim O(1/M_\pi)$

$\Longrightarrow$ Clear molecule

- But $X = \sqrt{\dfrac{1}{1+2r/a}} \simeq 1.7 \gg 1$

— X was derived in the zero-range approximation and has a pole when r/a is negative

- Meanwhile, $\bar{X} = \sqrt{\dfrac{1}{1+|2r/a|}} \approx 0.8$

1S0 pn virtual state

- $a = 23.74\ \text{fm}$, $r = +2.75\ \text{fm}$ (Dumbrajs et al 1983) $\Longrightarrow$ large a: $|a| \gg |r|$, $r \sim O(1/M_\pi)$

$\Longrightarrow$ Clear molecule

— both a and r changed the sign $\Longrightarrow$ no pole

- $X = \bar{X} \approx 0.9$

$\bar{X} \simeq 1$, as expected for a molecule up to the range corrections!