# $B\pi$ excited-state contamination in B-meson observables

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in collaboration with
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*Lattice meets continuum*
Siegen University
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# Outline

- Motivation: Nucleon-pion-state contamination in nucleon form factors
- …
- …
-

# MINERvA: Axial form factor of the nucleon

MINERvA experiment 2022: First measurements of neutrino-proton scattering

Cai *et al*, Nature Vol 614 (2023)

$$\overline{\nu}_\mu \, p \quad \rightarrow \quad \mu^+ \, n$$

direct handle on the proton’s axial form factor !

Previous measurements used nucleon bound states (e.g. Deuterium) involves “nuclear physics corrections”

Form factor (ff) decomposition:

$$\langle N(p')|A_\mu|N(p)\rangle$$

axial vector current

$G_{\rm A}(Q^2)$ axial ff

$\tilde{G}_{\rm P}(Q^2)$ induced pseudo scalar ff

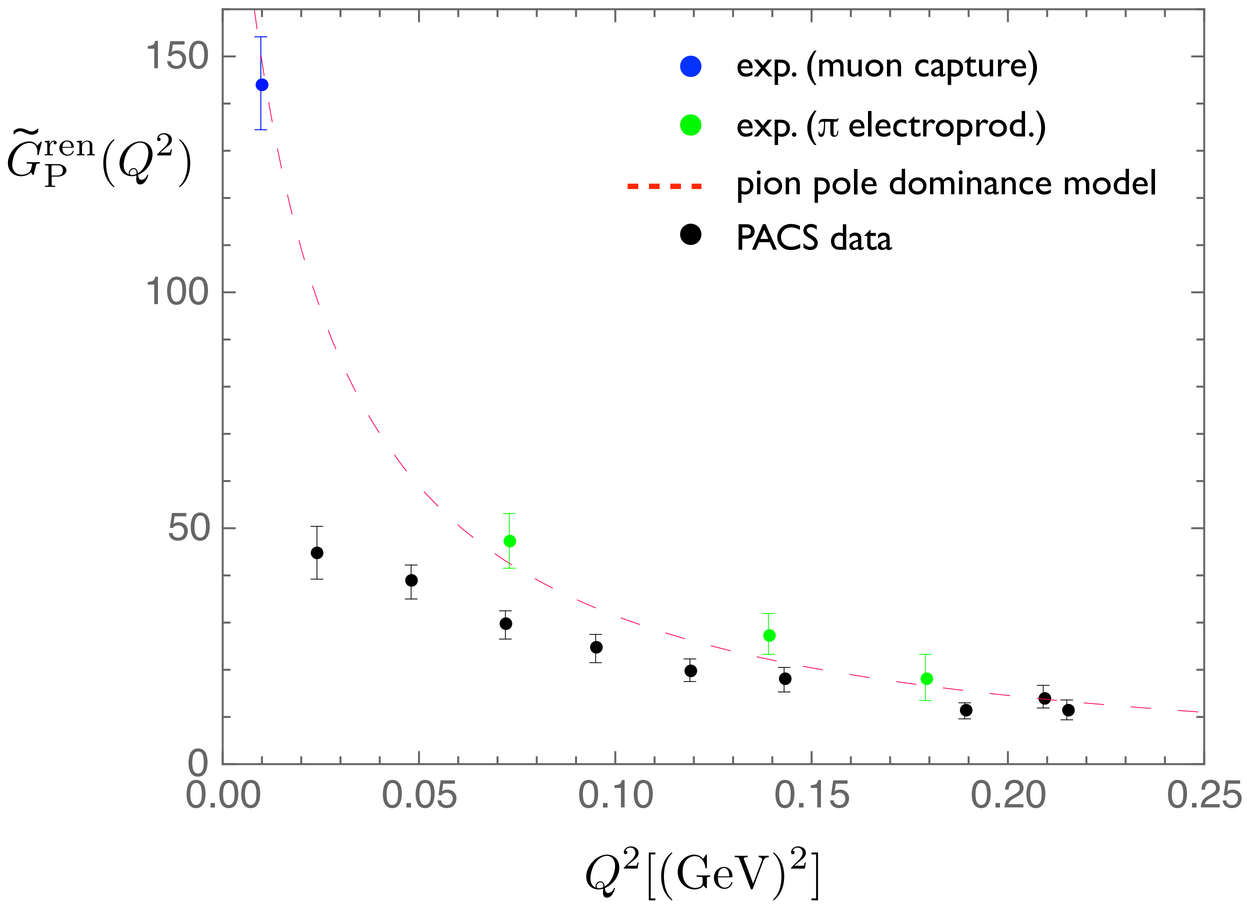
$(p' - p)^2 = -Q^2$ momentum transfer

Nucleon axial radius $r_A^2 \equiv -\frac{6}{G_{\rm A}(0)} \frac{dG_{\rm A}}{dQ^2}\Big|_{Q^2=0}$ ⇒ $r_A = 0.73 \pm 0.17\,\text{fm}$ MINERvA

# Induced pseudoscalar form factor - lattice data

Data by the PACS collaboration

Ishikawa *et al*, PRD 98 (2018)



The form factors can be computed in lattice QCD

Some lattice parameters:

$$a \approx 0.085\,\mathrm{fm}$$

$$M_\pi \approx 146\,\mathrm{MeV}$$

$$L \approx 8.1\,\mathrm{fm}$$

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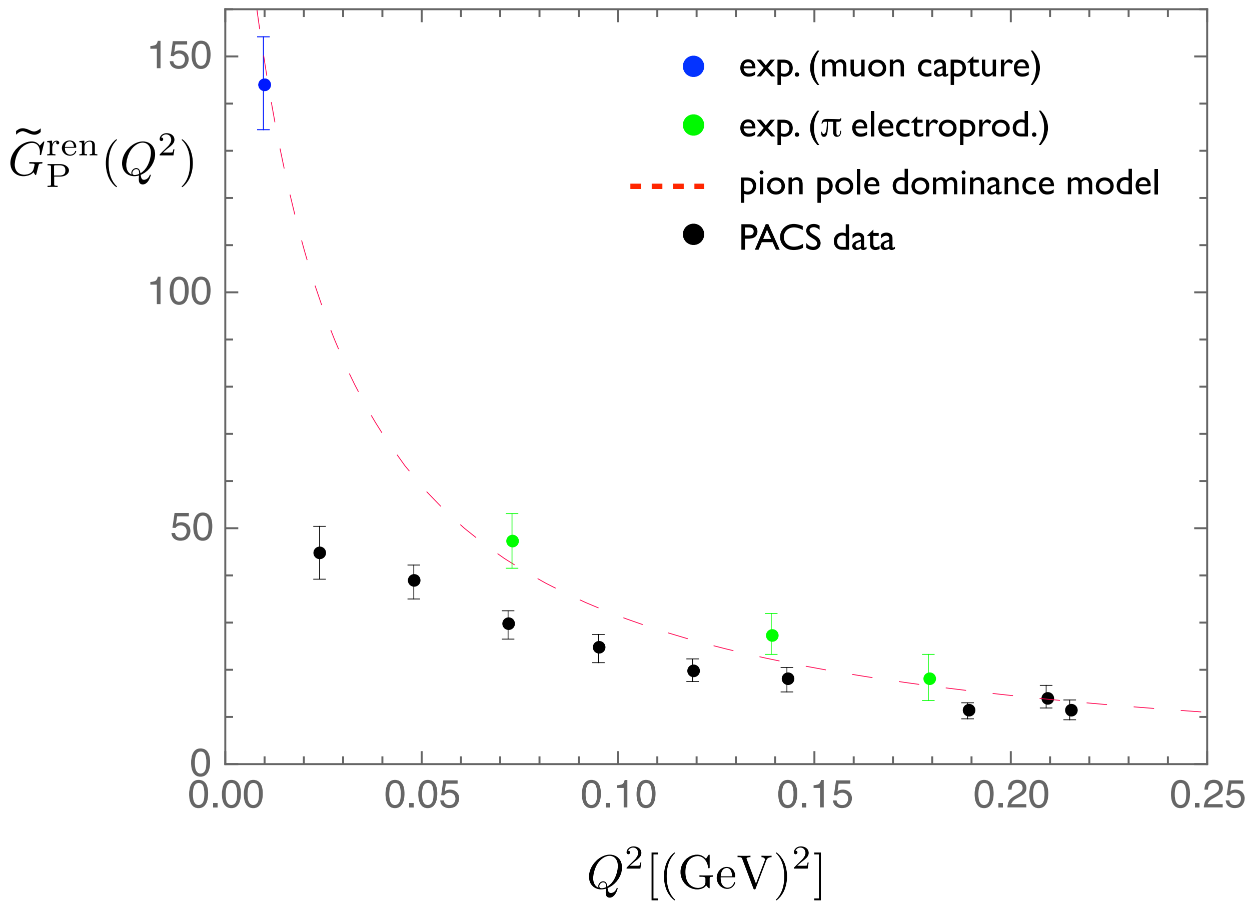

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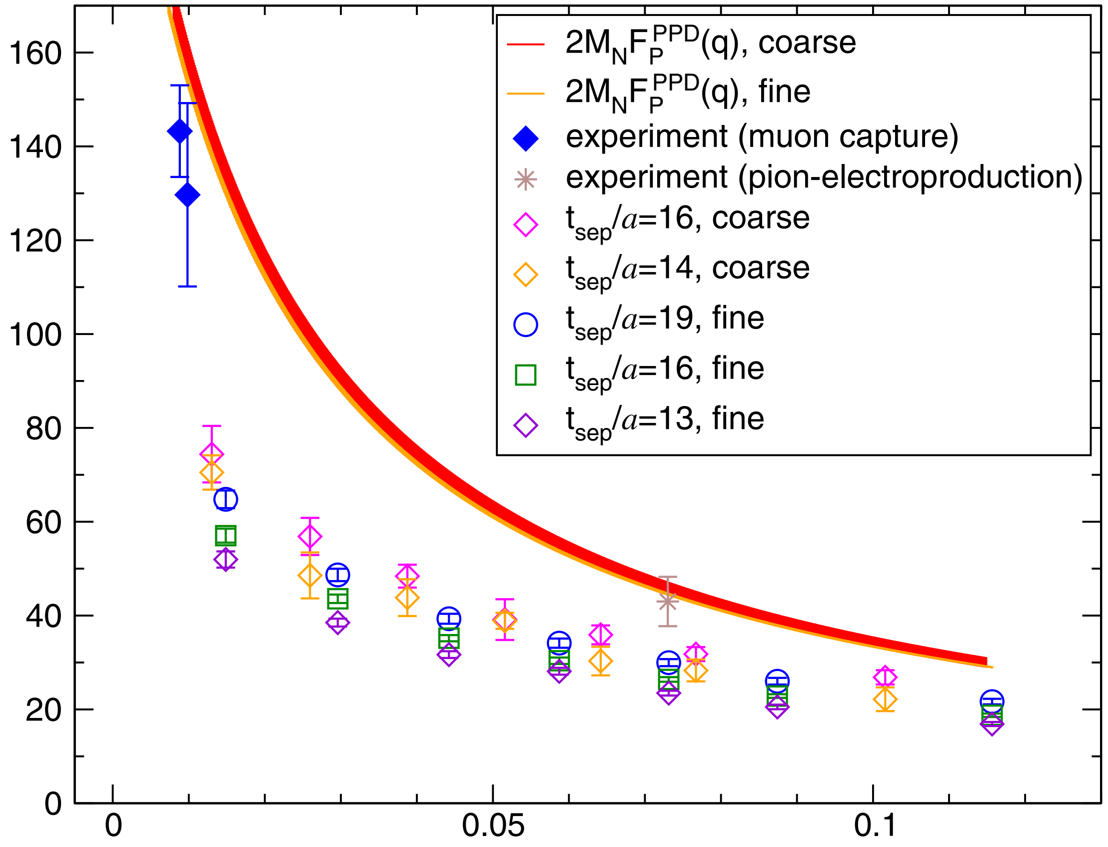

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Year 2018

$$a \approx 0.085\,\mathrm{fm}\,,\ \ 0.063\,\mathrm{fm}$$
$$M_\pi \approx 135\,\mathrm{MeV}$$
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Year 2024

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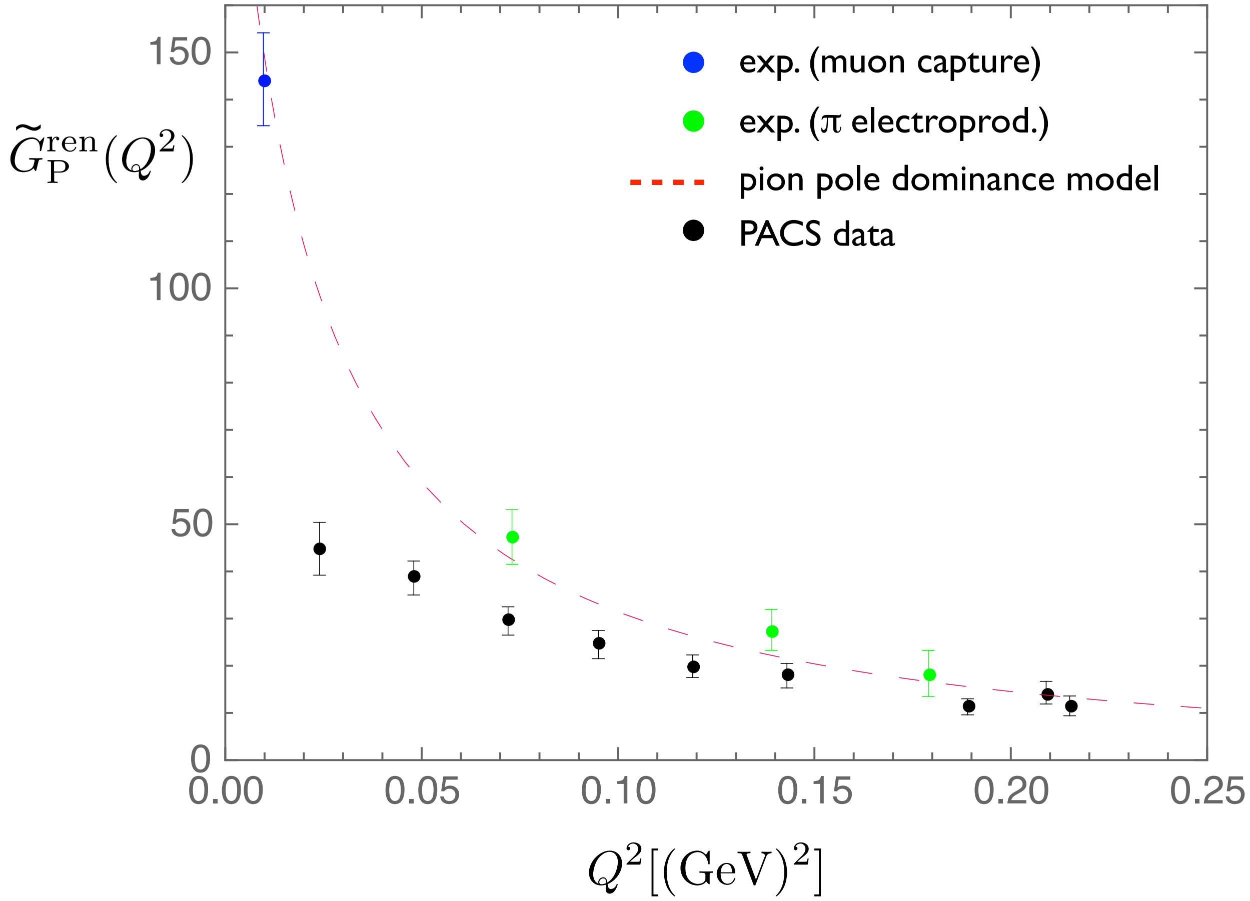

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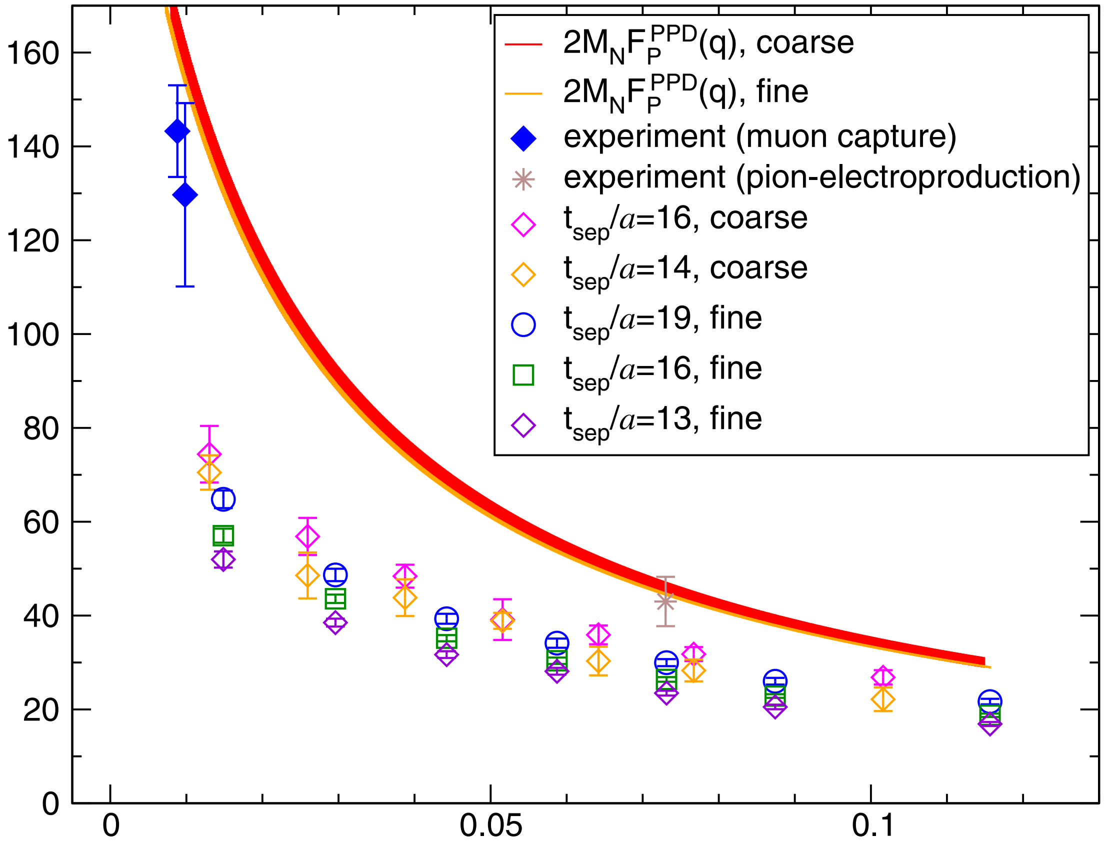

The lattice data underestimate the ff, in particular for small $Q^2$

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Year 2018

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Year 2024

# Induced pseudoscalar form factor - lattice data

Origin of the underestimation:
(wide agreement in the lattice community, after many years of discussion …)

(Large) excited-state contamination due to a two-particle nucleon-pion $(N\pi)$ state

Based on many studies

- employing lattice simulations
  Jang *et al*, PRD 109 (2024)
  Barca, Bali and Collins, PRD 107 (2023)
  Bali *et al*, JHEP 05 (2020)
  Jang *et al*, PRL 124 (2020)
  ….
- using Chiral Perturbation Theory (ChPT) OB, PRD 99 (2019)

ChPT ➡ predicts a large $N\pi$-state contamination

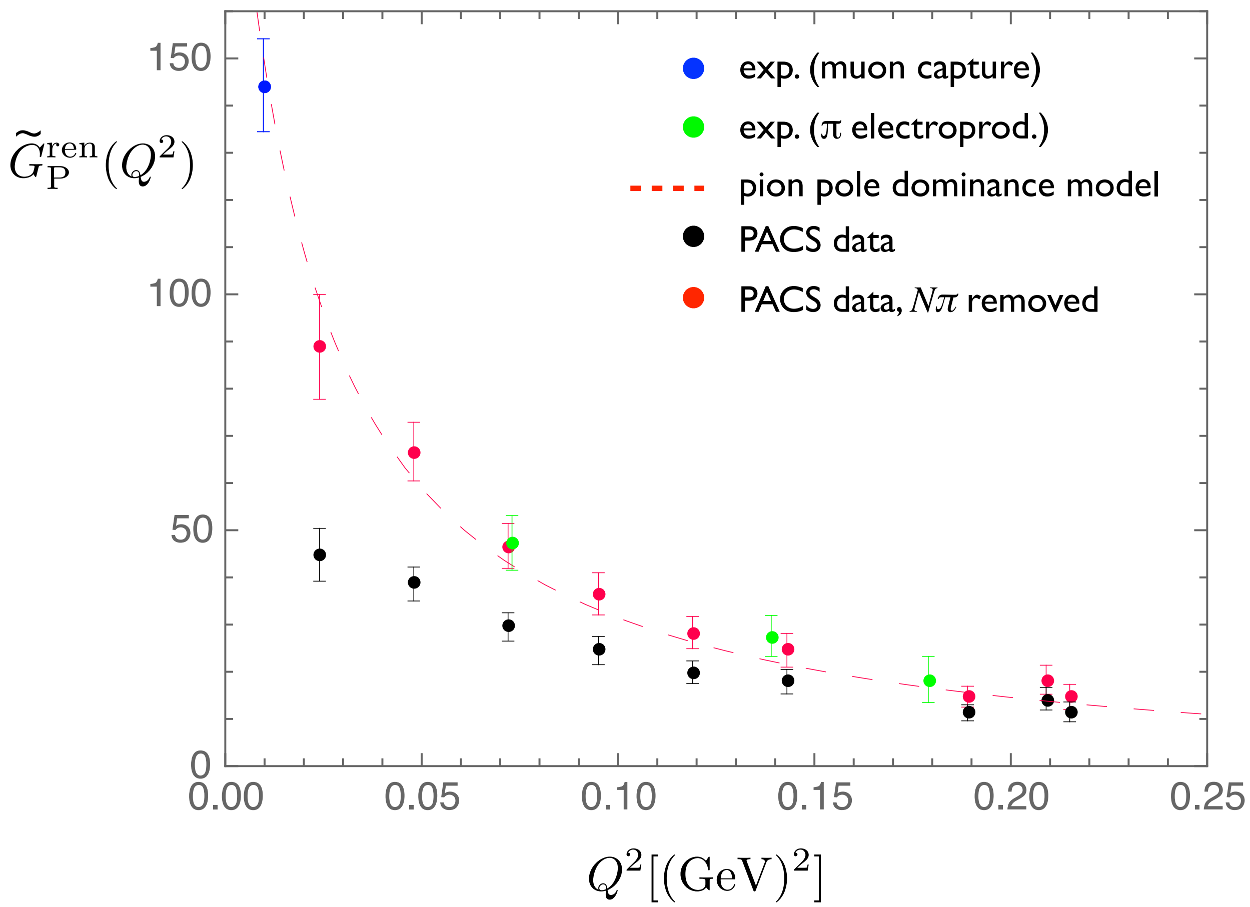
➡ predicts an underestimation

➡ provides a correction formula to "remove" the $N\pi$-state contamination from the lattice data

# Induced pseudoscalar form factor - lattice data

Data by the PACS collaboration

Ishikawa *et al*, PRD 98 (2018)



The “ChPT corrected” data agree much better with the experimental data / ppd model

# Outline

- Motivation: Nucleon-pion-state contamination in nucleon form factors
- Lattice basics: Excited-state contamination in B-meson 2pt function
- ChPT basics: Heavy meson Chiral Perturbation Theory (HM ChPT)
- Application: HM ChPT and the Bπ excited-state contamination in
  - B-meson decay constant
  - Vector current form factor for $B \to \pi l \overline{\nu}_l$
- Outlook

# Introduction: B-meson 2-pt function

- Consider the B-meson 2-pt function $$C_2(t) = \sum_{\vec{x}} \langle B(\vec{x},t) B^\dagger(\vec{0},0)\rangle$$
  - $\Sigma_x$ : projection to zero momentum
  - $B$ : interpolating field, quantum numbers of the B-meson

excited-state contribution

- Spectral decomposition → $$C_2(t) = b_0 e^{-M_B t} + b_1 e^{-E_1 t} + b_2 e^{-E_2 t} + \ldots$$
  finite spatial volume ⇒ discrete spectrum

$b_0 \propto |\langle 0|B(\vec{0},0)|B(\vec{p}=0)\rangle|^2$

$$M_{\text{eff}}(t) = -\partial_t \ln C_2(t) \quad \rightarrow \quad M_{\text{eff}}(t) = M_B + \frac{b_1}{b_0} e^{-\Delta E_1 t} + \frac{b_2}{b_0} e^{-\Delta E_2 t} + \ldots$$

effective mass

$$\Delta E_k = E_k - M_B$$

⇒ time separation needs to be sufficiently large for small excited-state corrections

Note: same statement for 3-pt functions with more than one time separation

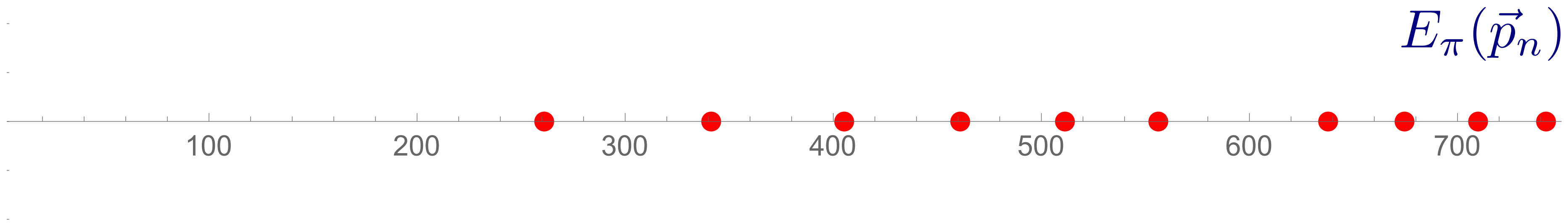
# Bπ and B*π states[1]

Consider static B and B*-mesons:

$$\Delta E_n \approx E_\pi(\vec{p}_n) \qquad \vec{p}_n = \frac{2\pi}{L}\vec{n}$$

$$M_\pi L = 4$$

$$M_\pi = 140\,\mathrm{MeV}$$



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[1] I often won't distinguish between B and B* in the following …

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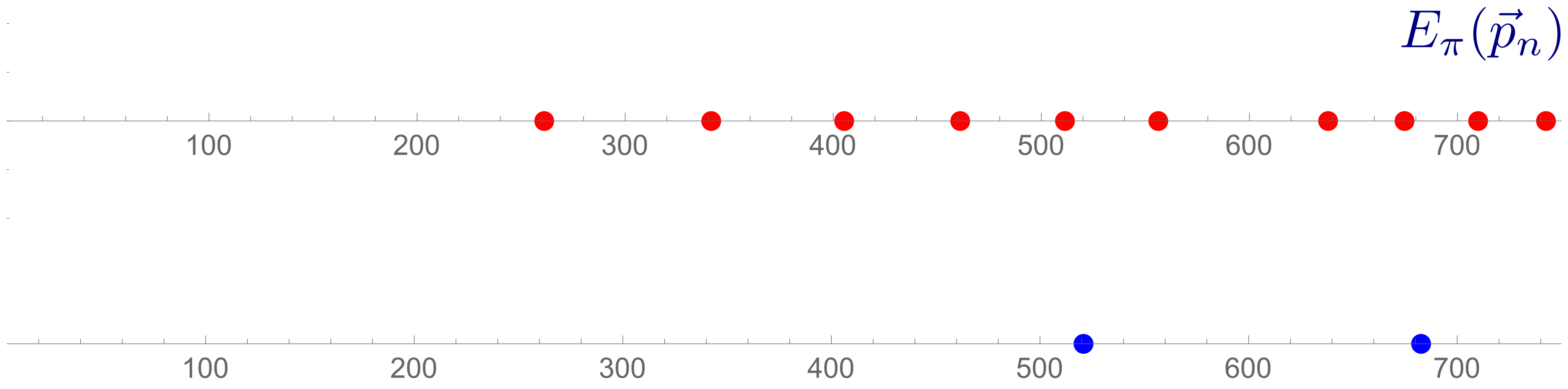

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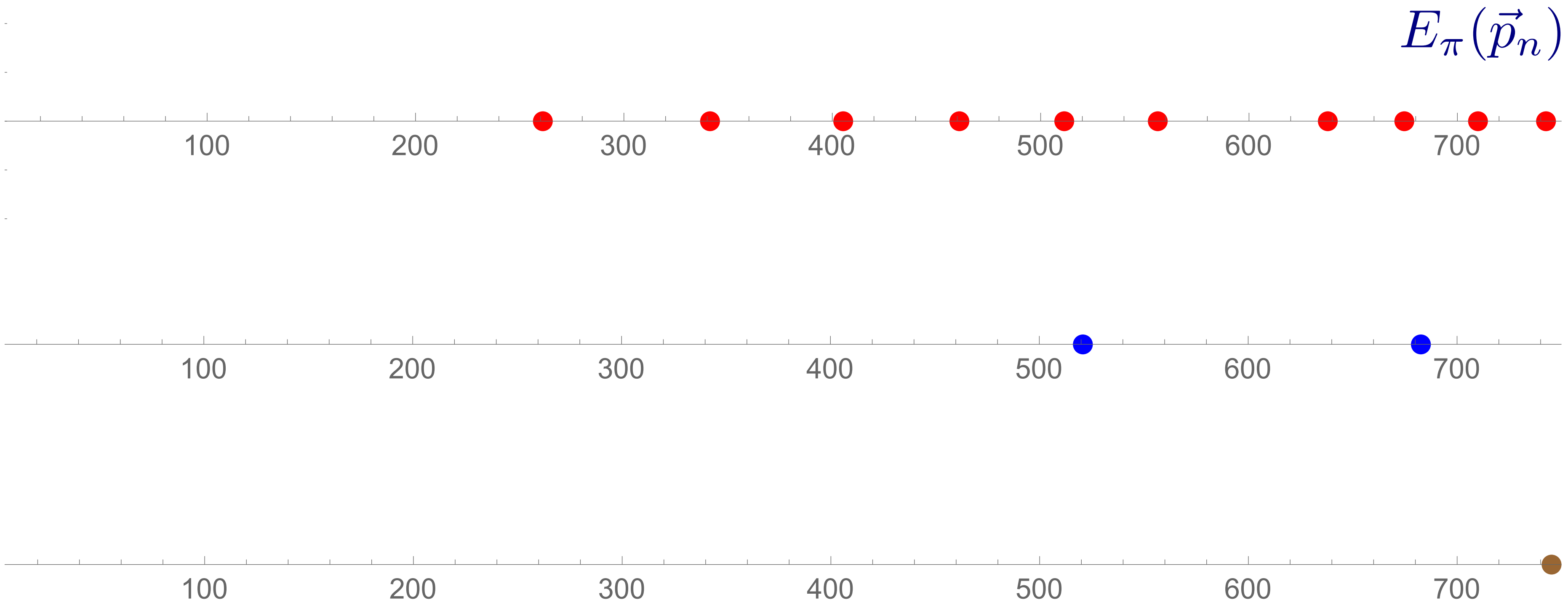
$M_\pi L = 4$

$M_\pi = 140\,\mathrm{MeV}$

$M_\pi = 280\,\mathrm{MeV}$

$M_\pi = 400\,\mathrm{MeV}$



The number of low-lying B*π states increases rapidly with decreasing pion mass

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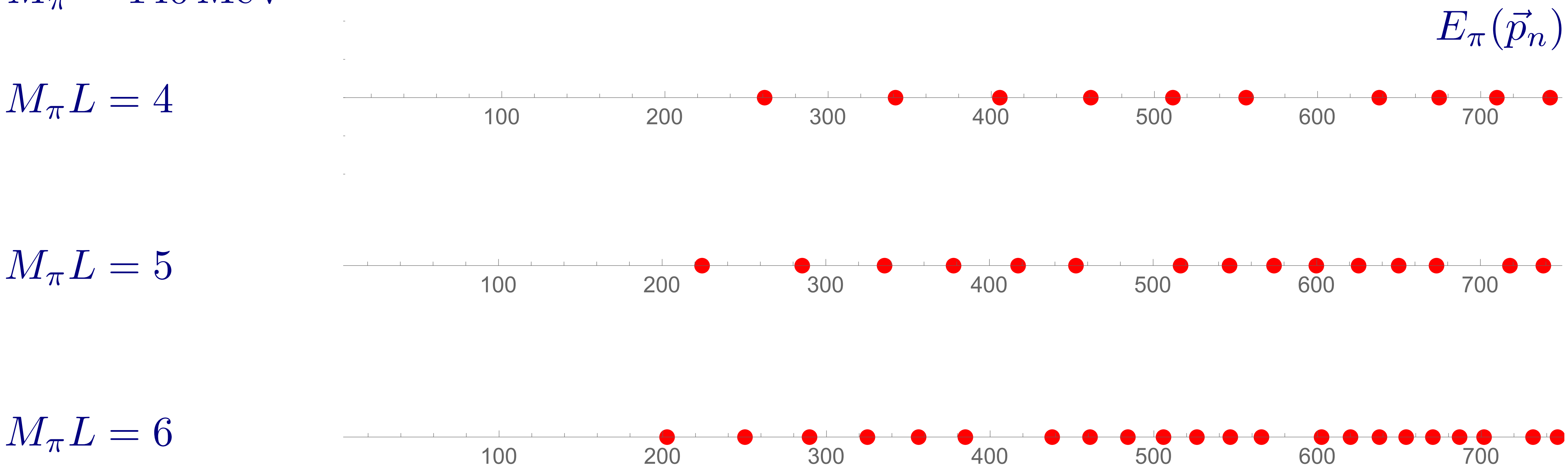

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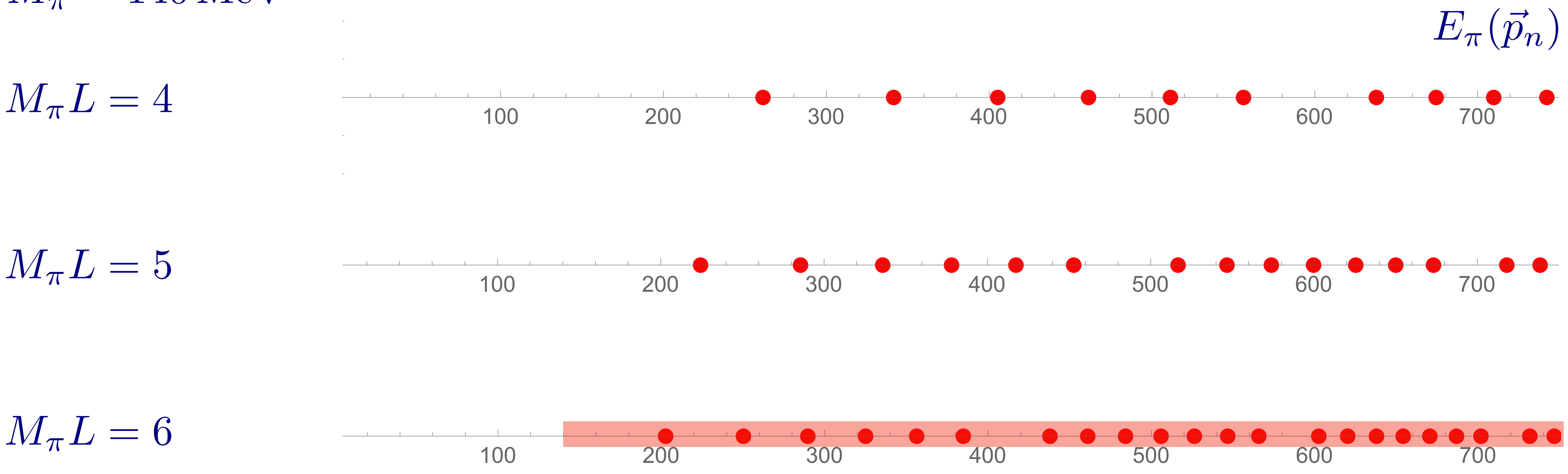

Infinite volume: continuous 2-particle spectrum, threshold = $M_B + M\pi$

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[1] I often won't distinguish between B and B* in the following …

# B$\pi$ excited states

$$M_{\text{eff}}(t) \approx M_B + \frac{b_1}{b_0} e^{-E_\pi(\vec{p}_1)} + \frac{b_2}{b_0} e^{-E_\pi(\vec{p}_2)} + \dots$$

- Expectation: Many B$\pi$ states contribute to the correlator and the effective mass
- However: Their impact depends also on the prefactors $b_k/b_0$

  Known: $b_k \propto 1/\,L^3$

  Again: not a finite volume (FV) effect, number of states increases for larger volumes
- Two questions
  1. How big is their impact ( = how big is $b_k/b_0$ ) ?
  2. If non-negligible: How to deal with them ?
- Concerning 1. → Use Chiral Perturbation theory to get estimates

# Chiral perturbation theory (ChPT)

- Well-established and widely used:

ChPT: low-energy effective theory of QCD

Weinberg 1979
Gasser, Leutwyler 1984
…

- Based on spontaneous (and small explicit) chiral symmetry breaking
  - Energy gap (the pions are fairly light)
  - Pion coupling is proportional to pion momenta (small at low energies) and pion mass
- Many applications in lattice QCD
  - Pion mass dependence of observables
  - FV effects due to pions
  - Taste-breaking effects with staggered fermions
  - $N\pi$-state contamination in nucleon observables (charges, form factors,…)
  - …
- In the following: $B\pi$-state contamination in B-meson observables

# QCD with a static b-quark

- Our starting point: QCD with a *heavy static b-quark*

E. Eichten and B. Hill (1990), …

$$\mathcal{L} = \sum_r \overline{q}_r(\gamma_\mu D_\mu + m_l)q_r + \overline{Q}(D_4 + m_b)Q + \mathcal{L}_{\mathrm{gauge}}$$

Note: LO in the *heavy quark expansion*, $1/\mathrm{m_b}$ corrections (not here)

- High degree of symmetry
  - Heavy quark spin symmetry → mass degenerate B and B* mesons
  - Local flavor number (LFN) symmetry: $Q(x) \to \exp[i\eta(\vec{x})]\, Q(x)$
  - Chiral symmetry (in the light quark sector)
  - Isospin symmetry ( if we assume $\mathrm{m_l = m_u = m_d}$) → 3 mass degenerate pions
- Corresponding chiral effective theory → Heavy Meson (HM) ChPT

  must respect all these symmetries

M. Wise (1992)
Burdman, Donoghue (1992)
…

- Note: We work in continuum QCD (and ignore Lorentz symmetry breaking at finite lattice spacing)

# Basics of Heavy Meson (HM) ChPT

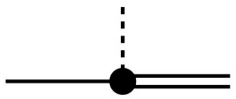
- Heavy quark spin symmetry → multiplet $H = P_+ \left(iB_k^* \gamma_k + iB\gamma^5\right) \qquad P_+ = (1+\gamma_4)/2$

  $\bar{H} = \gamma_4 H^\dagger \gamma_4$

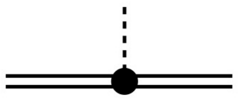
- Relevant interaction given by

$$\mathcal{L}_{\rm int} = i\frac{g}{f}\,{\rm tr}\left(H\gamma_5\gamma_\mu\partial_\mu\pi\bar{H}\right) + \ldots$$

→



$BB^*\pi$



$B^*B^*\pi$

Note:
- one pion derivative
- two LO LECs $f$: pion decay constant

  $g$: $BB^*\pi$ coupling

  chiral limit values

# Interpolating B-meson fields in HMChPT

- We are interested in correlation functions of interpolating fields for the B-mesons

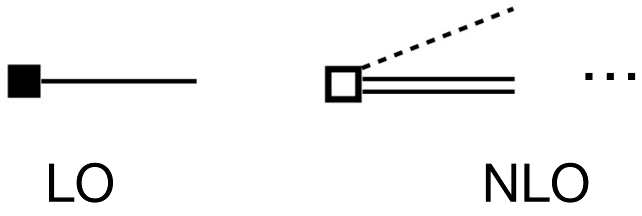
Quark level: $\overline{q}_r(x)\Gamma Q(x)$ $\Gamma$: Clifford algebra element e.g. $\gamma_5$ or $\gamma_k$

light u,d quark heavy (static) b-quark

- Are mapped to HMChPT as usual: most general term compatible with the symmetries

➥ LO 1 term
NLO 2 terms

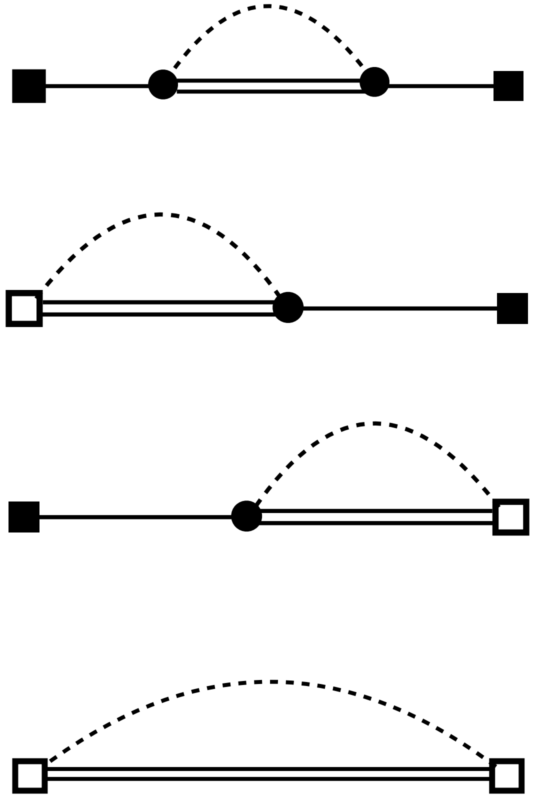
→ interpolating field for pseudoscalar B-meson



# Bπ contribution in HMChPT - 2pt function

- Calculation of the correlation functions is a standard task in PT
  (euclidean space time, finite spatial volume V=L³, L → ∞ in the end)

- E.g.: Feynman diagrams for the
  Bπ contribution in the 2-pt function



Extract contribution with exponential

$$\exp\left[-\left(M_B + E_{\pi,\vec{p}}\right)|t|\right] \qquad \vec{p} = \vec{p}_\pi = -\vec{p}_B$$

ignore others

- Obtain results for relative Bπ contribution:

$$\frac{C_2^{B\pi}(t)}{C_2^B(t)} \equiv \Delta C_2^{B\pi}(t) = \sum_{\vec{p}} c_{2\mathrm{pt}}(\vec{p})\, e^{-E_{\pi,\vec{p}}\,t}$$

non-trivial result
of the ChPT calculation

Recall Introduction

$$M_{\mathrm{eff}}(t) \approx M_B + \frac{b_1}{b_0} e^{-E_\pi(\vec{p}_1)} + \frac{b_2}{b_0} e^{-E_\pi(\vec{p}_2)} + \ldots$$

# Bπ contribution in HMChPT - 2pt function

$$\Delta C_2^{B\pi}(t) = \sum_{\vec{p}} c_{2\mathrm{pt}}(\vec{p})\, e^{-E_{\pi,\vec{p}}\,t}$$

$$\downarrow$$

$$c_{2\mathrm{pt}}(\vec{p}) \;=\; \frac{3}{8(fL)^2 E_{\pi,\vec{p}} L}\,\frac{p^2}{E_{\pi,\vec{p}}^2}\,\Big(\underset{\underset{\text{LO}}{\uparrow}}{g} + \underset{\underset{\text{NLO}}{\uparrow}}{\beta_1 E_{\pi,\vec{p}}}\Big)^2$$

Comments

- expected $1/L^3$ dependence (2-particle state!)
- $\Sigma_p$ → “tower of states” (recall: 1-loop diagrams)
- infinite volume limit can be taken ( → non-zero, involves modified Bessel and Struve functions )
- depends on LECs $f$,$g$ (LO) and $\beta_1$ (NLO) in case of local interpolating fields

Estimates:

$f \approx f_\pi \approx 93\,\mathrm{MeV}$ PDG

$g \approx 0.49$ Lattice: A. Gerardin *et al.* (2022)

$\beta_1 \approx 0.14(4)\,\mathrm{GeV}^{-1}$ Lattice: B. Colquhoun *et al.* (2022)

⇒ ChPT *prediction* for the Bπ contamination in effective mass and decay constant

# Bπ contribution in the effective decay constant

Example

$$\langle 0 | A_4^{\mathrm{RGI}}(0) | B(\vec{p}=0)\rangle \equiv \hat{f}$$

heavy light decay constant

$$\hat{f} = f_B \sqrt{M_B} \,/\, C_{\mathrm{PS}}$$

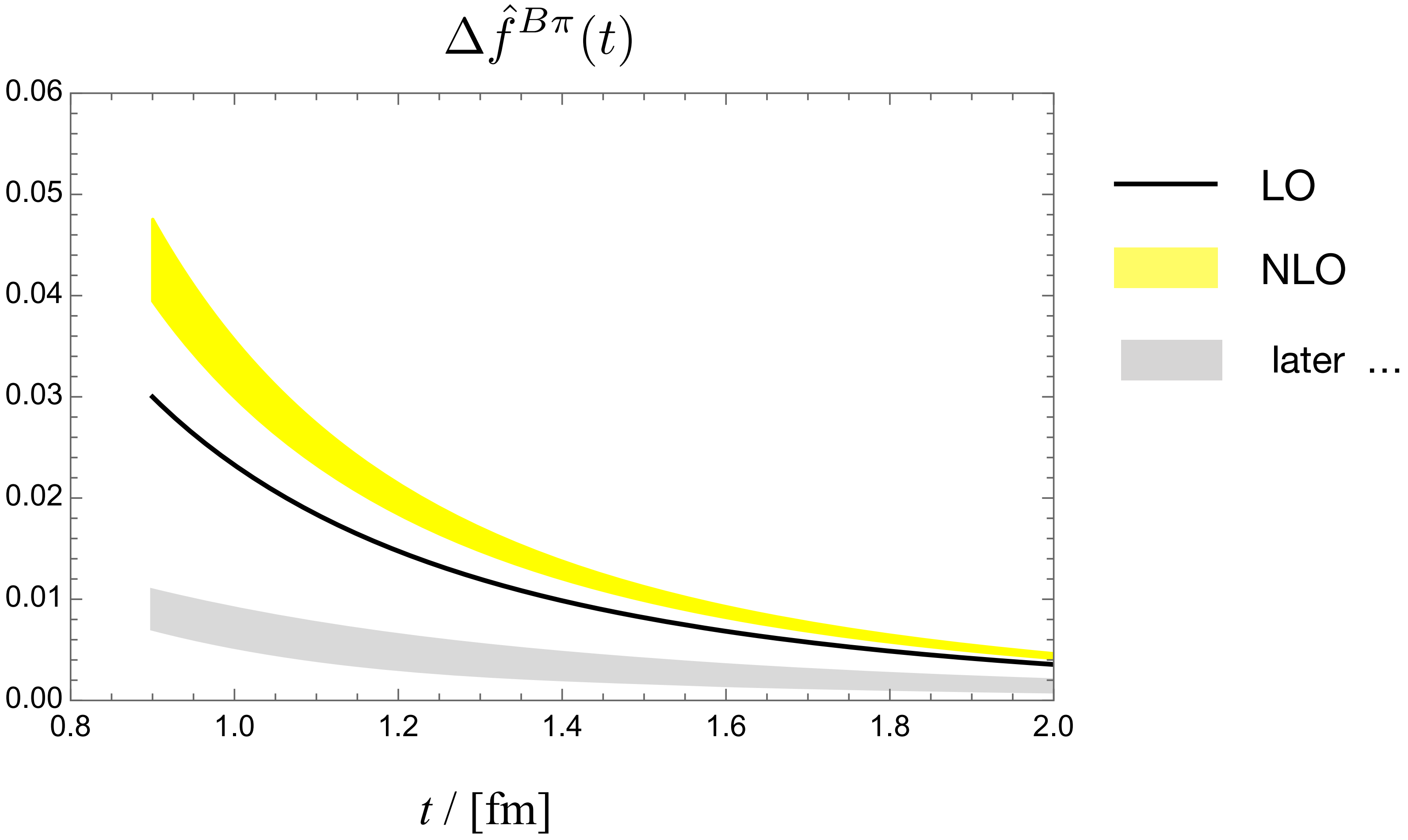
Estimator

not unique

$$\hat{f}_{\mathrm{eff}}(t) = \sqrt{2}\,\sqrt{C_2(t)}\,e^{\frac{1}{2}M_B^{\mathrm{eff}}(t)\,t}$$

$$= \hat{f}\left(1 + \Delta\hat{f}^{B\pi}(t)\right) \xrightarrow{t\,\to\,\infty} \hat{f}$$

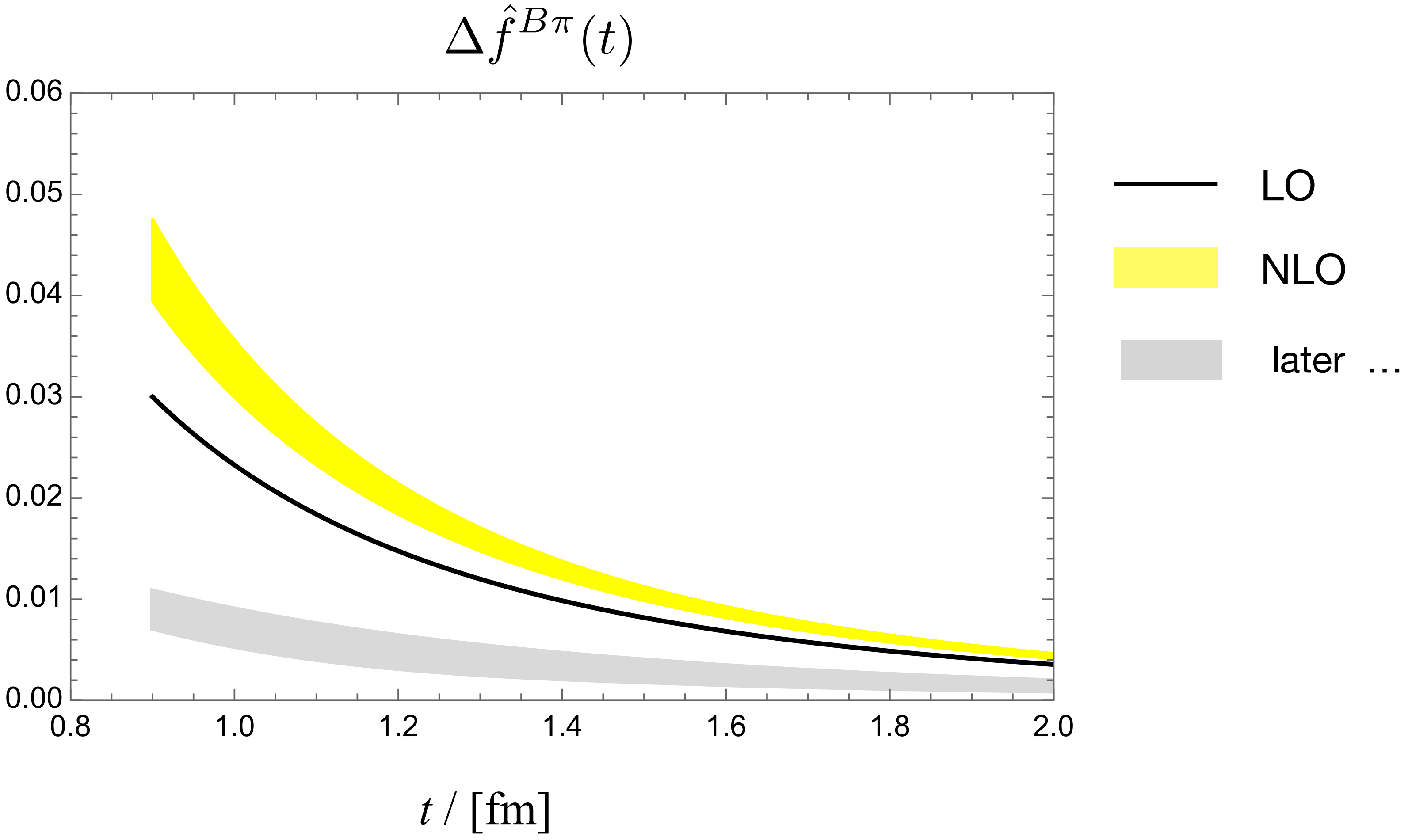
# Bπ contribution in the effective decay constant



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 2%

small effect but relevant for % precision

# Bπ contribution in the effective decay constant



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 2%

small effect but relevant for % precision

Analogous results for B-meson mass, BB*π coupling $g_\pi$ → Bπ contamination ≈ a few percent

A. Broll, OB and R. Sommer
Eur. Phys. J C 83 (2023) 757

# Semileptonic B decay

- Semileptonic B decay $B \to \pi l \overline{\nu}_l$

➥ matrix element $\langle \pi(p_\pi) \,|\, V^\mu \,|\, B(p_B) \rangle$

$p_\pi^k \, h_\perp(E_\pi) \qquad \mu = k$

$h_\parallel(E_\pi) \qquad \mu = 0$

$\vec{p}_B = 0$

heavy-light vector current

form factor decomposition

- Extract form factors from suitably defined ratio of 3pt and 2pt functions

➥ effective form factors

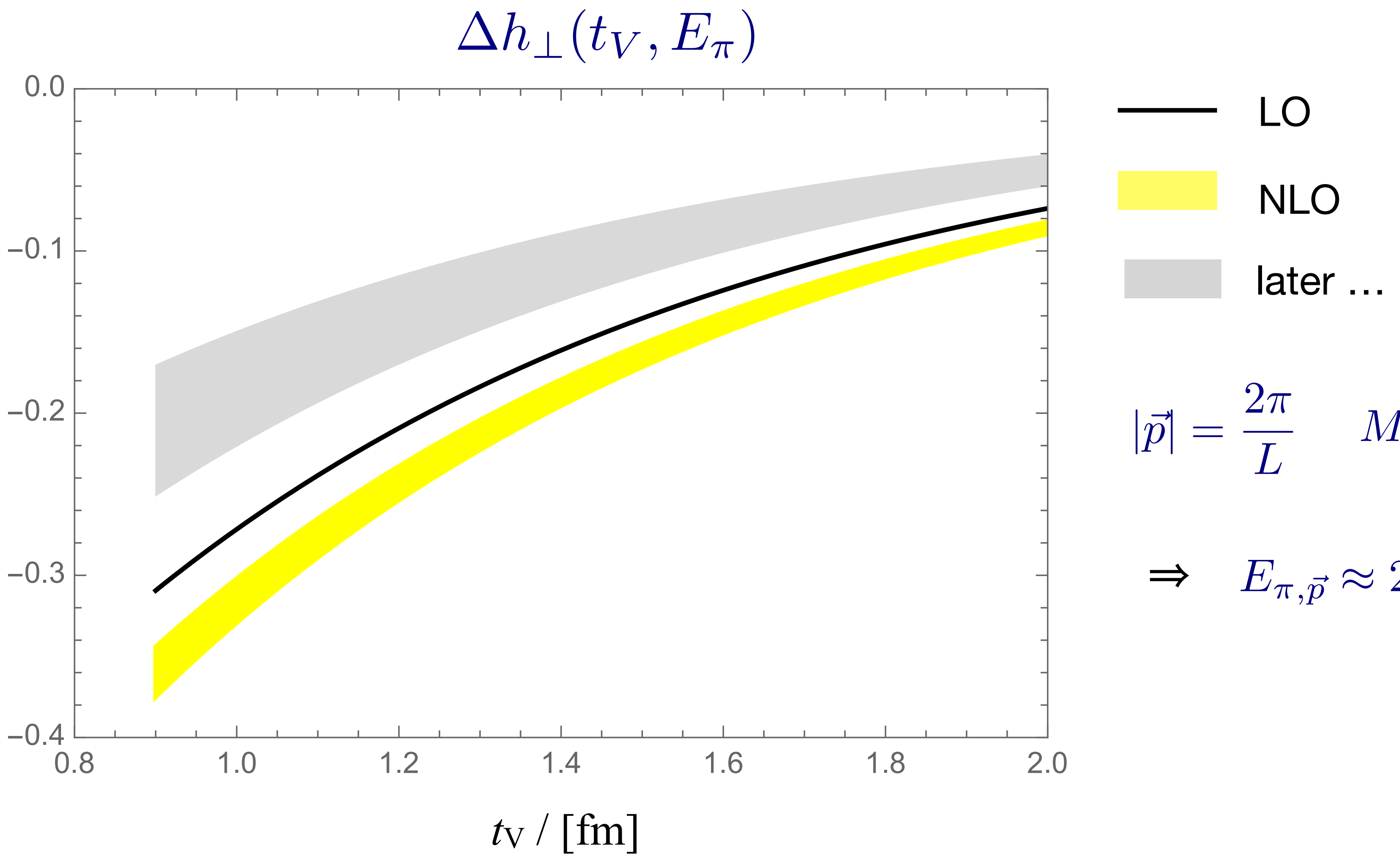
$$h_\perp^{\text{eff}}(t_V, E_\pi) \;=\; h_\perp(E_\pi)\Big[1 + \Delta h_\perp(t_V, E_\pi)\Big]$$

current insertion time

Bπ-state contamination

stemming from $\langle \pi \,|\, V^k \,|\, B^* \pi \rangle$

# Bπ contamination $\Delta h_\perp$



$$|\vec{p}| = \frac{2\pi}{L} \qquad M_\pi L = 4$$

$$\Rightarrow \quad E_{\pi,\vec{p}} \approx 260\,\mathrm{MeV}$$

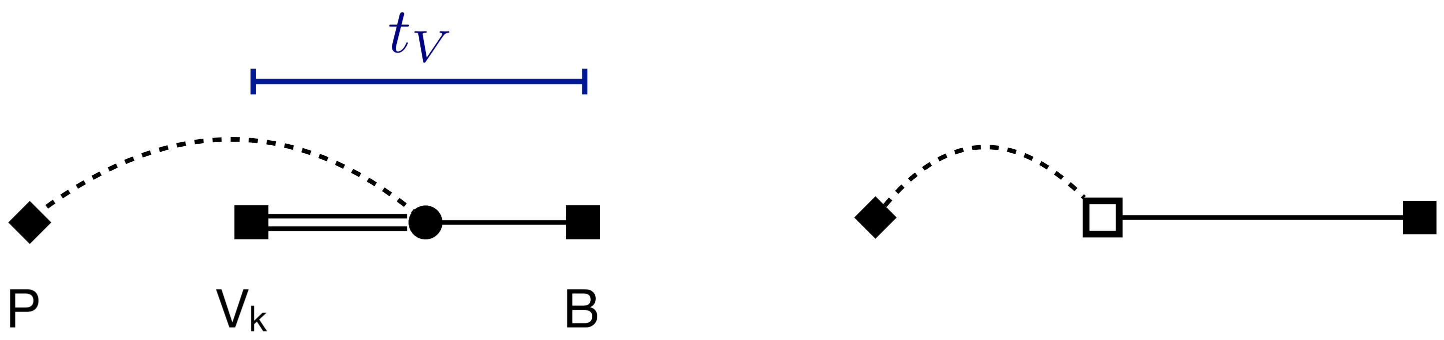
The B*π contamination in the 3-pt function …

- leads to an *underestimation of ~ 20%* of the form factor
- Why so much bigger than in the 2-pt function ?

# Why is the Bπ contamination in $\Delta h_\perp$ so big?

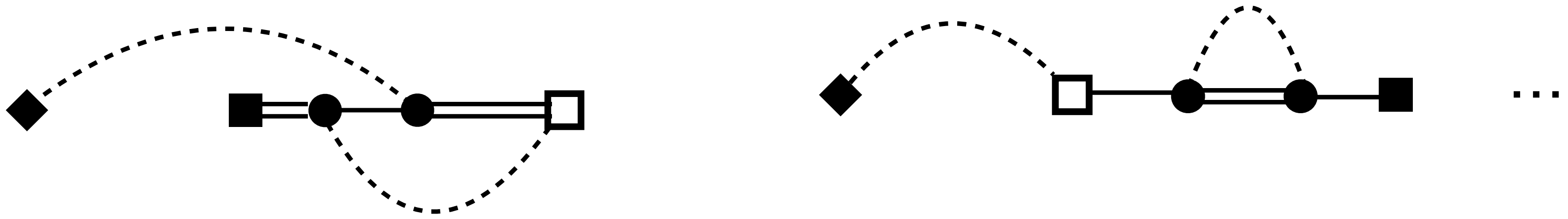
Feynman diagrams for

$C_3^B(t, t_V)$



$C_3^{B\pi}(t, t_V)$

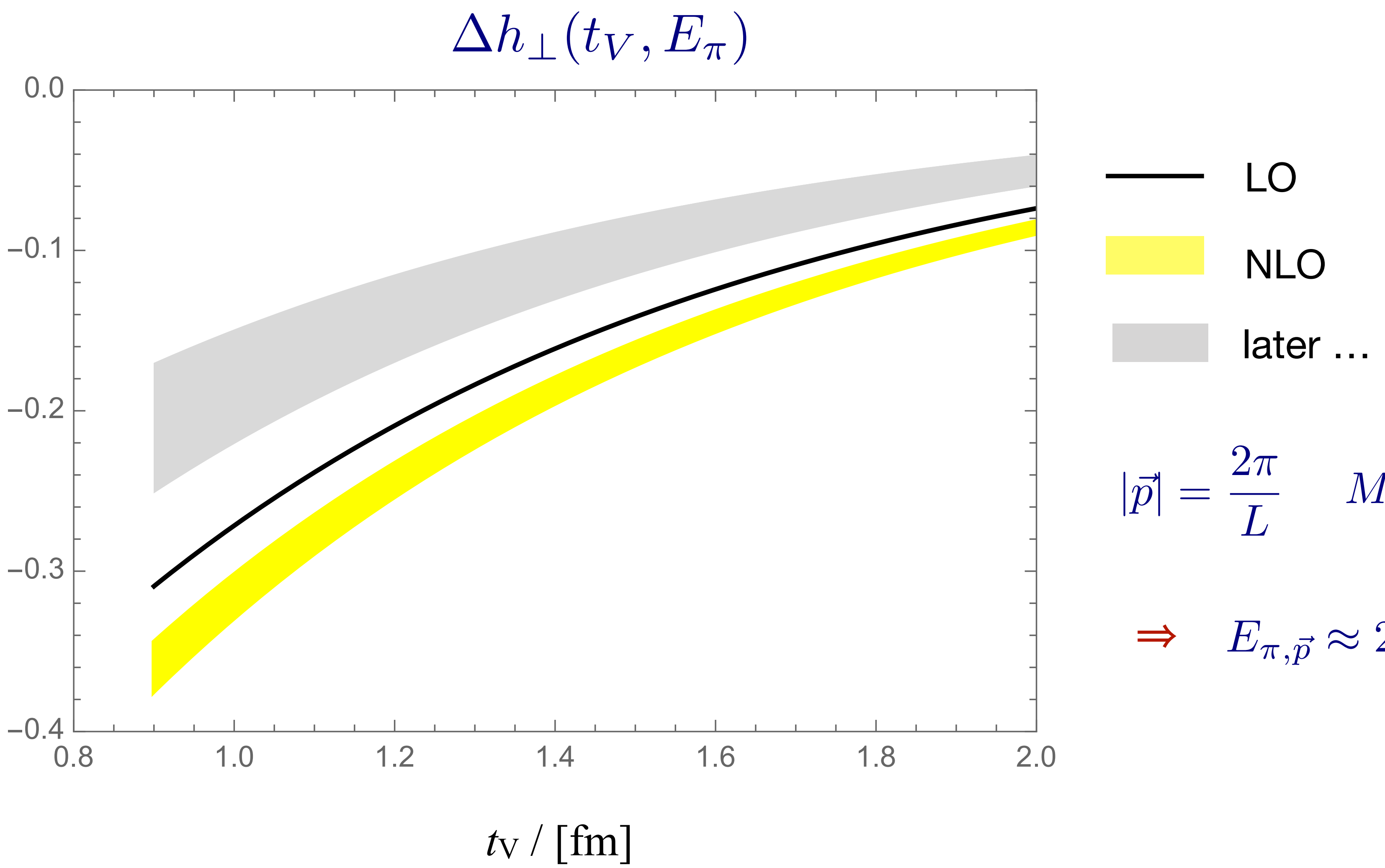
loop diagrams



tree diagrams

$$\Delta h_\perp^{B\pi} = \Delta h_\perp^{B\pi,\text{tree}} + \Delta h_\perp^{B\pi,\text{loop}} \approx \Delta h_\perp^{B\pi,\text{tree}}$$

# Bπ contamination $\Delta h_\perp$



$$|\vec p| = \frac{2\pi}{L} \qquad M_\pi L = 4$$

$$\Rightarrow \quad E_{\pi,\vec p} \approx 260\,\mathrm{MeV}$$

$$\Delta h_\perp^{B\pi}(t_V, E_{\pi,\vec p}) \approx \Delta h_\perp^{B\pi,\mathrm{tree}}(t_V, E_{\pi,\vec p}) = -1 \times e^{-E_{\pi,\vec p}\, t_V} \quad + \text{ NLO}$$

Note:

- no factor $1/L^3$ $\Rightarrow$ sometimes called “*volume enhanced*” contribution
- no sum over pion momenta (i.e. no tower of states), *one fixed pion momentum only*
- exactly the same result as in $\tilde{G}_\mathrm{P}$ in the nucleon sector (recall *Motivation*)

# Questions / Outlook

- HM ChPT predicts a non-negligible and in some cases a significant Bπ contamination
    - How reliable are these NLO ChPT results?
      *needs to be checked with lattice data* → first promising results: A. Gerardin, Lattice 2024

- How to deal with the Bπ contamination ?
    - Apply HMChPT results to subtract (some of the) excited-state contamination?
      What about remaining uncertainties?
    - Smeared interpolating fields?
      Is smearing helpful/effective in suppressing Bπ excited states ?
      *needs to be checked with lattice data* → first results: A. Gerardin, Lattice 2024
    - GEVP including two-hadron (Bπ) interpolating fields ?
      *Currently under investigation …*

# Interpolating B-meson fields in HMChPT

- We are interested in correlation functions of interpolating fields for the B-mesons

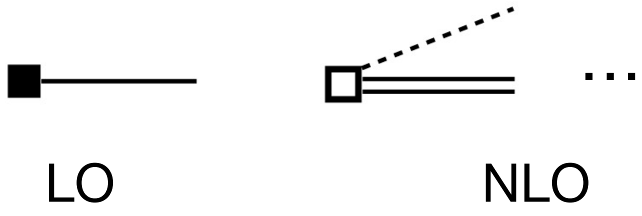
Quark level: $\bar{q}_r(x)\Gamma Q(x)$ $\Gamma$: Clifford algebra element e.g. $\gamma_5$ or $\gamma_k$

light u,d quark heavy (static) b-quark

- Are mapped to HMChPT as usual: most general term compatible with the symmetries

➥ LO 1 term
NLO 2 terms

*local*
→ interpolating field for pseudoscalar B-meson



# *Smeared* interpolating B-meson fields in HMChPT

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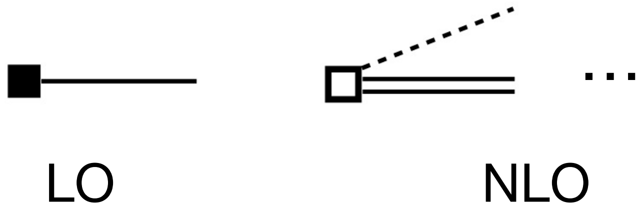
*local* → interpolating field for pseudoscalar B-meson



- Holds also for *smeared interpolating fields* with provided $\bar{q}_r(x) \rightarrow \bar{q}_r^{\,\mathrm{sm}}(x) = \int \mathrm{d}^4y\, \bar{q}_r(y) K(y,x)$

$$R_{\text{smear}} \ll \frac{1}{M_\pi}$$

$R_{\text{smear}} = 0.3 \ldots 0.4\ \mathrm{fm}$ seem okay

- Difference between local and smeared interpolators: different values for the LECs

# Impact of smearing (?)

Interpolating B-meson field

local

smeared

$\beta_1 \approx 0.14(4)\,\mathrm{GeV}^{-1}$

B. Colquhoun *et al.* [JLQCD], 2022

$\tilde{\beta}_1 \approx$ ?

Expectation:

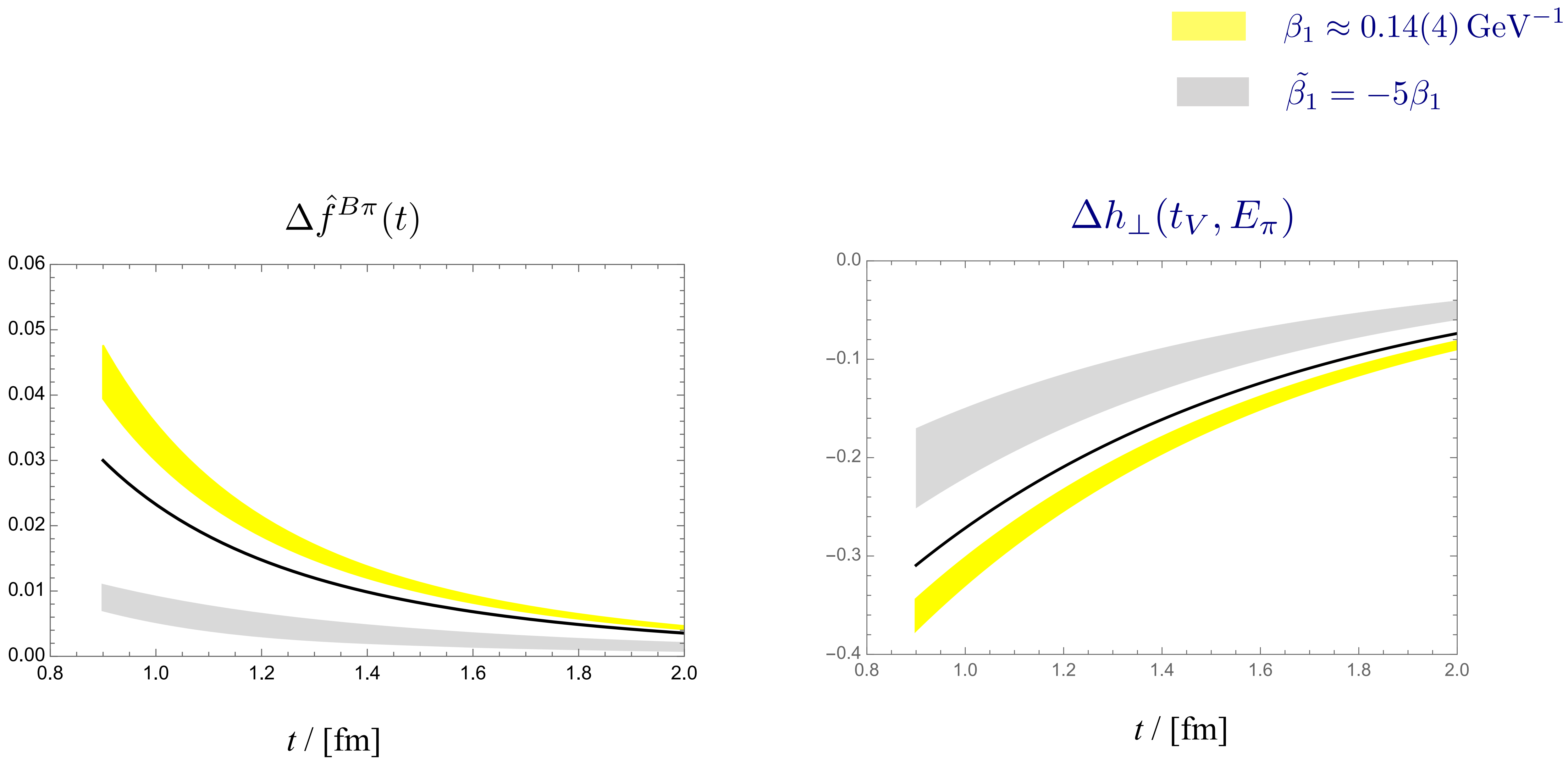
Smearing

increases overlap with the ground state

decreases overlap with the excited states

Justified for B$\pi$ states?

# Impact of smearing (?)



Smearing has the potential to significantly reduce the Bπ contamination

But: Is there a smearing procedure that causes $\tilde{\beta}_1 \approx -5\beta_1$ ?

# Lattice determination of LECs β₁ and β̃₁

$\mathbf{p}^*$: reference momentum

HM ChPT prediction to NLO:

A. Broll, R. Sommer, OB, (2023)

$$R(\mathbf{p}) \equiv \frac{\langle \pi(\mathbf{p})|V_k|B\rangle}{\langle \pi(\mathbf{p}^*)|V_k|B\rangle} = \frac{1-\beta_1\, E_\pi(\mathbf{p})/g}{1-\beta_1\, E_\pi(\mathbf{p}^*)/g} \times \frac{E_\pi(\mathbf{p}^*)}{E_\pi(\mathbf{p})} \times \frac{\mathbf{p}_k}{(\mathbf{p}^*)_k}$$

➡ extract $\beta_1$ from the pion energy dependence

Lattice estimator: $R^{\mathrm{eff}}(\mathbf{p}, t_V)$

suitably defined ratio of 3pt and 2pt functions

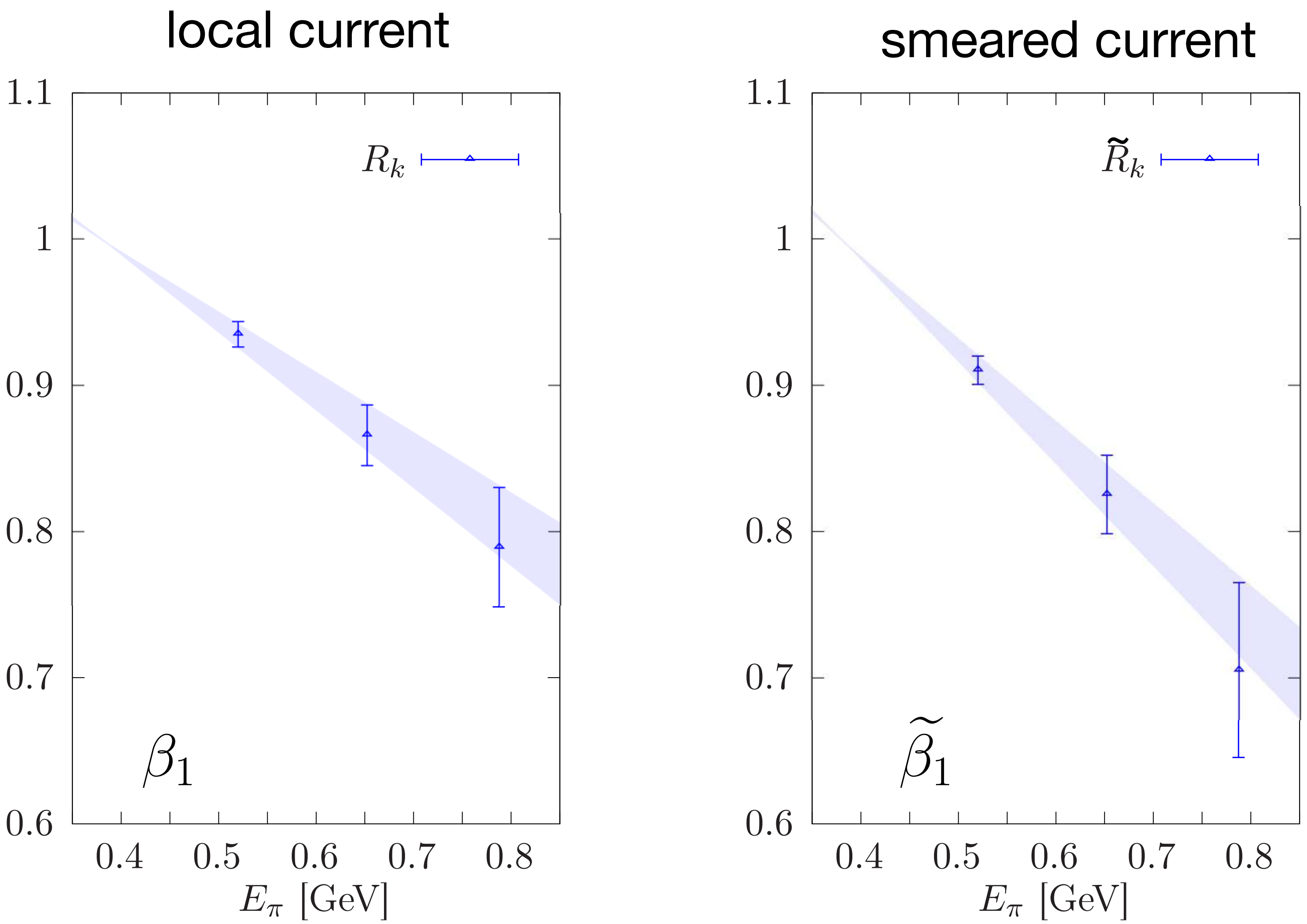
Smearing of the vector current: $V_k \rightarrow \tilde{V}_k$ $\Rightarrow$ $R(\mathbf{p}) \rightarrow \tilde{R}(\mathbf{p})$

$\beta_1 \rightarrow \tilde{\beta}_1$

➡ extract $\tilde{\beta}_1$

# Lattice determination of LECs β₁ and $\tilde{\beta}_1$

Lattice results (preliminary) ➞ contribution by Antoine Gerardin, *Lattice 2024*



Gaussian smearing

$r_{\text{smear}} \approx 0.45\,\text{fm}$

$\beta_1 \approx 0.20(2)\text{GeV}^{-1}$

$\tilde{\beta}_1 \approx 0.23(3)\text{GeV}^{-1}$

$\beta_1 \approx \tilde{\beta}_1$ ⇒ small impact of Gaussian smearing on Bπ excited states !

Recall: $\beta_1 \approx 0.14(4)\text{GeV}^{-1}$ by JLQCD

# Questions / Outlook

- HM ChPT predicts a non-negligible and in some cases a significant Bπ contamination
  - How reliable are these NLO ChPT results?
    *needs to be checked with lattice data* → first promising results: A. Gerardin, Lattice 2024

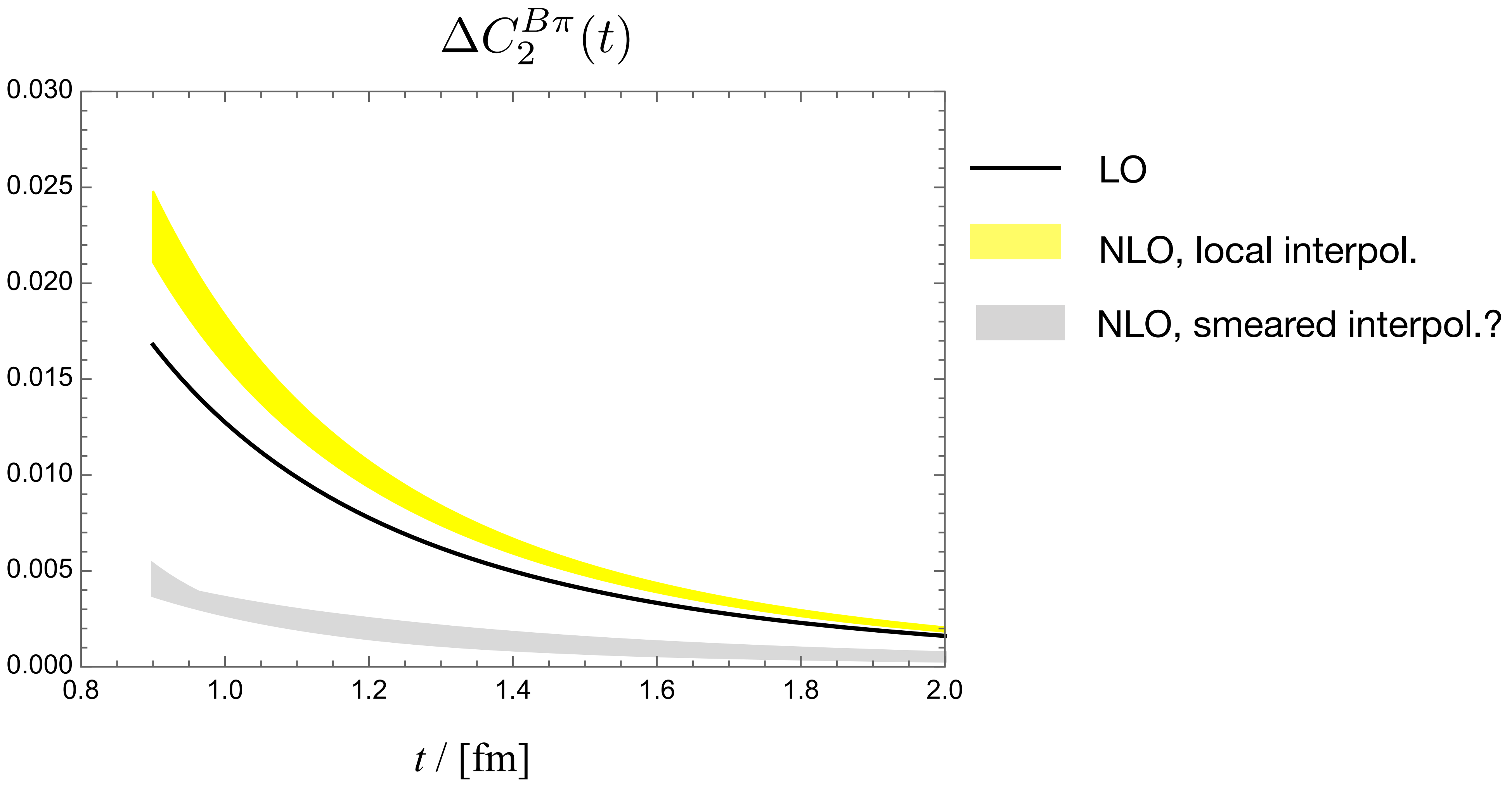
- How to deal with the Bπ contamination ?
  - Apply HMChPT results to subtract (some of the) excited-state contamination?
    What about remaining uncertainties?
  - Smeared interpolating fields?
    Is smearing helpful/effective in suppressing Bπ excited states ?
    *needs to be checked with lattice data* → first results: A. Gerardin, Lattice 2024
  - GEVP including two-hadron (Bπ) interpolating fields ?
    *Currently under investigation …*

Preliminary answer for
Gaussian smearing: No

But: Distillation looks better
work in progress …

# Backup slides

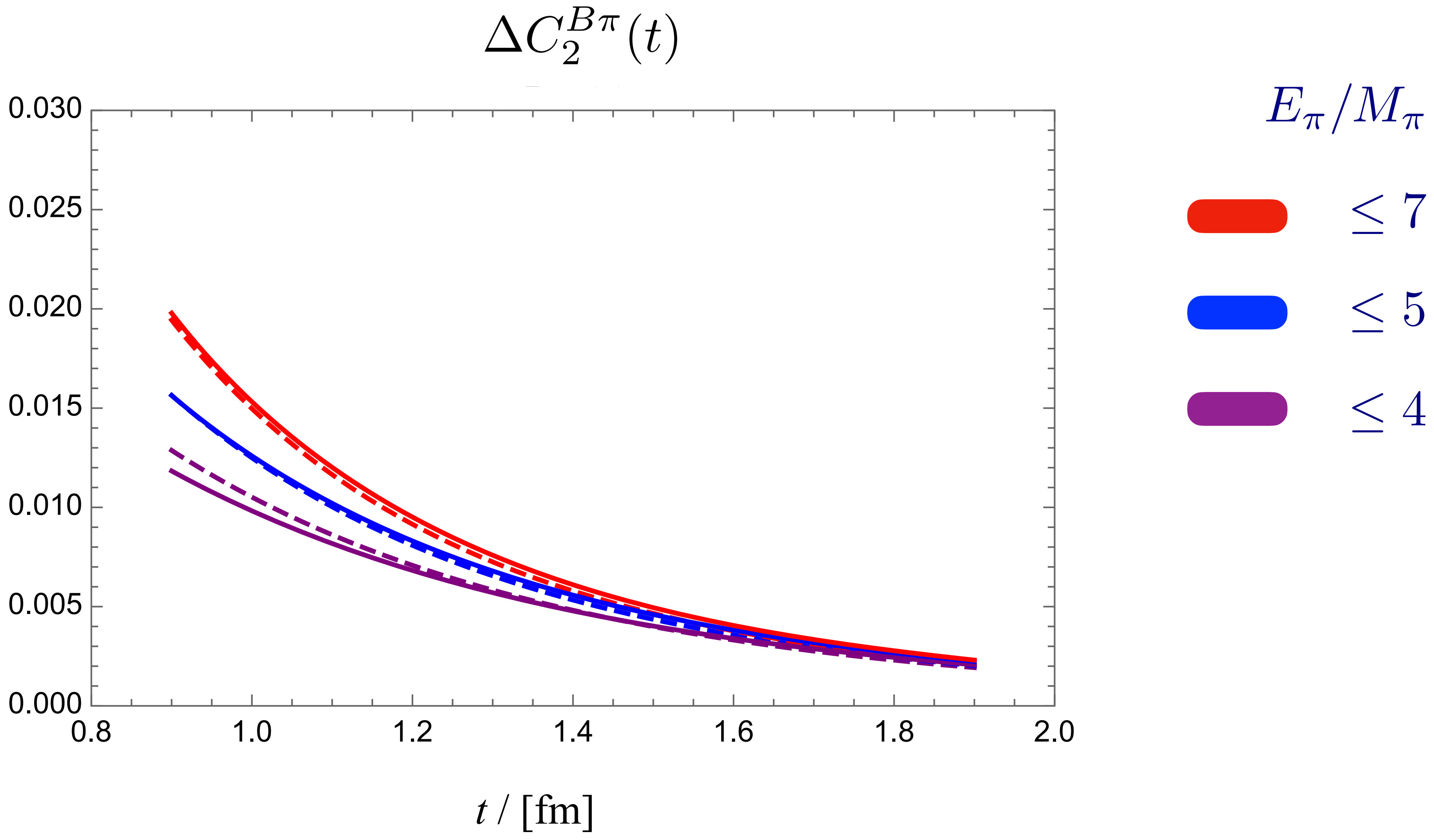
# Bπ contribution in the 2pt function



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 1%

if local interpolating fields are used

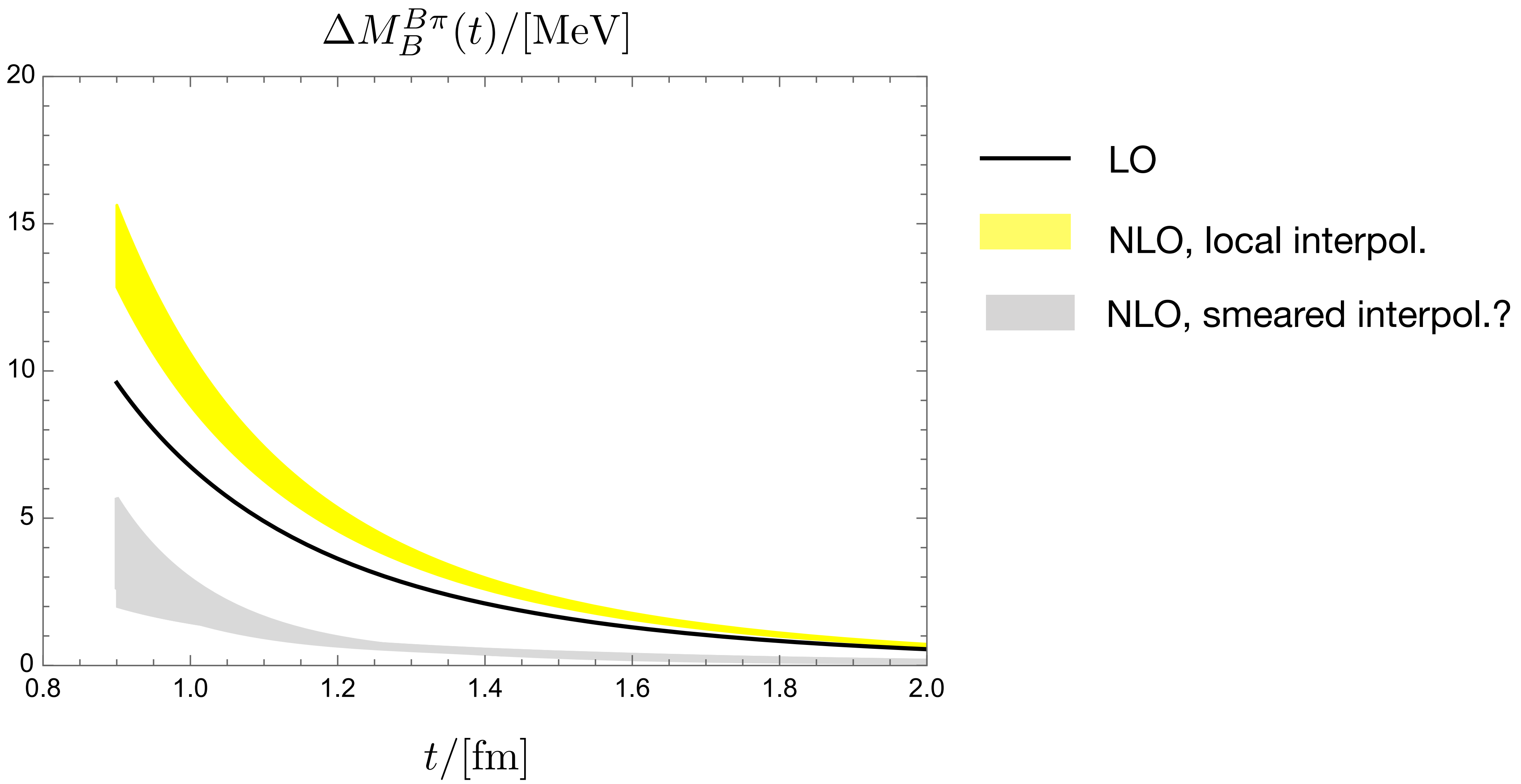
# Bπ contribution in the 2pt function



dashed lines: $M_\pi L = 4$

solid lines: infinite volume limit

# Bπ contribution in the effective mass



$t = 1.3$ fm: Bπ-state contamination leads to an *overestimation* of ≈ 5 MeV

if local interpolating fields are used

# Antoine Gerardin, talk at *Lattice 2024*

## Calculation of the LECs : $\beta_1$ and $\widetilde{\beta}_1$

- HM$\chi$PT prediction (at NLO) for the form factor $h_\perp$ [ref still missing]

$$\frac{\langle \pi(\mathbf{p})|V_k|B\rangle}{\langle \pi(\mathbf{p}^\star)|V_k|B\rangle} = \frac{1-\beta_1/g\,E_\pi(\mathbf{p})}{1-\beta_1/g\,E_\pi(\mathbf{p}^\star)} \times \frac{E_\pi(\mathbf{p}^\star)}{E_\pi(\mathbf{p})} \times \frac{p_k}{(p^\star)_k}$$

→ extract $\beta_1$ from the pion energy dependence

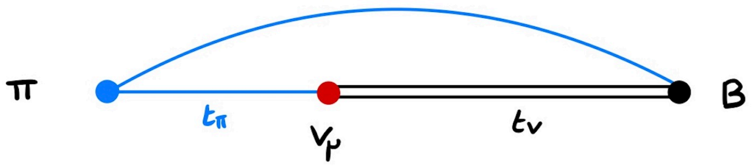
→ smearing of the vector current ($V_\mu \to \widetilde{V}_\mu$) : gives access to $\widetilde{\beta}_1$ (LECs for smeared $B$ operators)

[O. Bär, A. Broll, R. Sommer '23]

- Matrix element obtained from 3-point functions in the static limit

$$C^{(3)}_\mu(t_\pi, t_v; \mathbf{p}) = \frac{a^9}{V^2} \sum_{\mathbf{x_f},\mathbf{y},\mathbf{x_i}} \langle \overline{\mathcal{O}}_\pi(\mathbf{x_f}, t_v+t_\pi)\, V_\mu(\mathbf{y}, t_v)\, \mathcal{O}_B(\mathbf{x_i}, 0)\rangle \; e^{-i\mathbf{p}(\mathbf{x_f}-\mathbf{y})}$$

Replace local by smeared vector current : $\widetilde{V}_\mu \longrightarrow \widetilde{C}^{(3)}_\mu$



- Lattice estimator :

$$\boxed{R^{\mathrm{eff}}(t, t_v; \mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)}\,\frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p})}{\widetilde{C}^{(3)}_k(t_\pi, t_v; \mathbf{p}^\star)}\,\frac{C^{(2)}_\pi(t_\pi, \mathbf{p}^\star)}{C^{(2)}_\pi(t_\pi, \mathbf{p})}}$$

→ this estimator is itself affected by excited states : can be used to correct our data

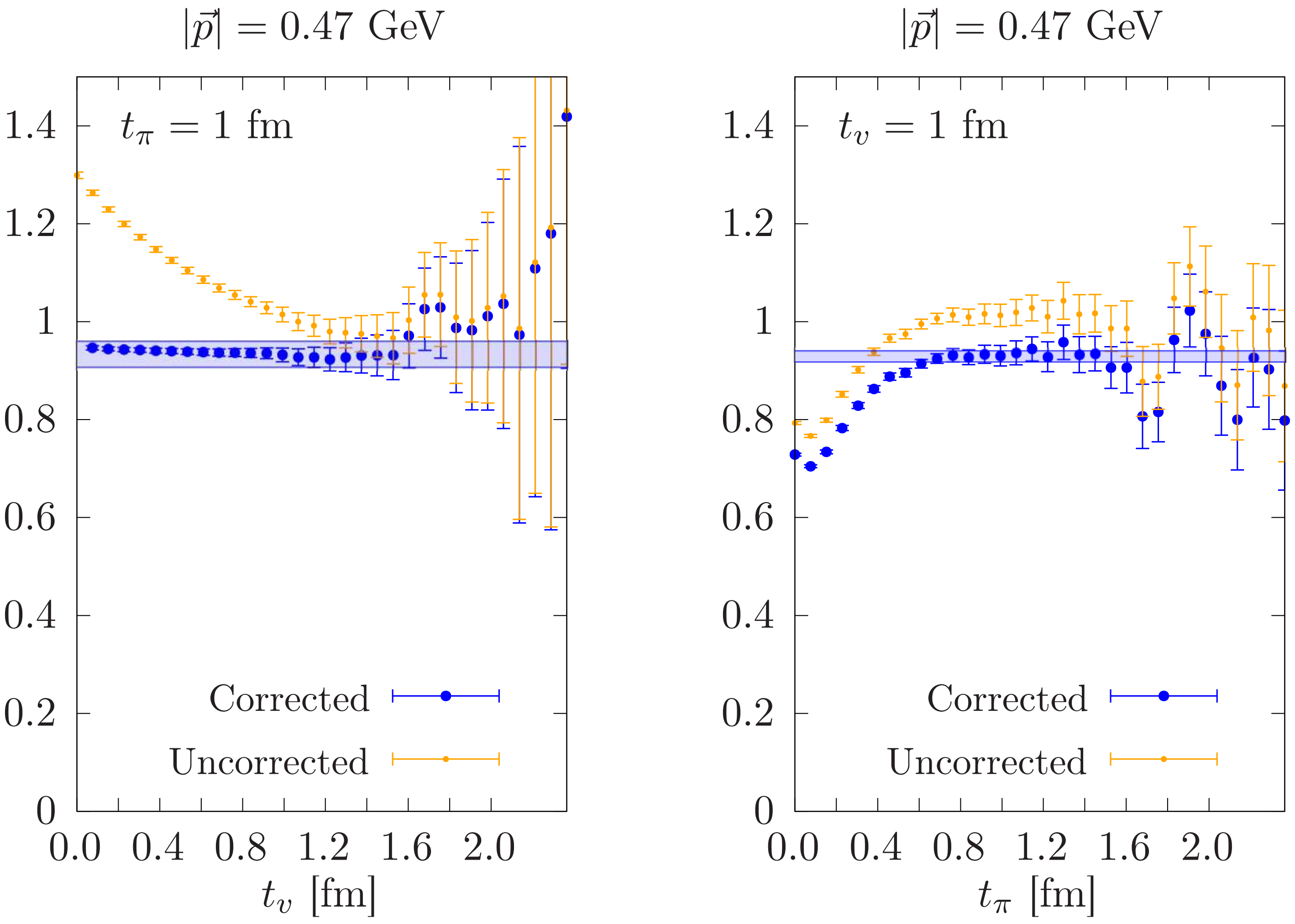
$$1+\delta_{B\pi}(t_v;\mathbf{p}) \;=\; \frac{1+\Delta h_\perp(t_v;\mathbf{p})}{1+\Delta h_\perp(t_v;\mathbf{p}^\star)} \;\approx\; 1 + e^{-(E_\pi(\mathbf{p})-E_\pi(\mathbf{p}^\star))t_v} + \frac{\beta_1+\tilde{\beta}_1}{g}\left(E_\pi(\mathbf{p})e^{-E_\pi(\mathbf{p})t} - E_\pi(\mathbf{p}^\star)e^{-E_\pi(\mathbf{p}^\star)t}\right) + \cdots$$

# Antoine Gerardin, talk at *Lattice 2024*

## Preliminary results : $\beta_1$ and $\tilde{\beta}_1$

$$R^{\mathrm{eff}}(t,t_v;\mathbf{p}) \equiv \frac{E_\pi(\mathbf{p})}{E_\pi(\mathbf{p}^\star)}\,\frac{(p^\star)_k}{p_k} \times \frac{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p})}{\widetilde{C}_k^{(3)}(t,t_v;\mathbf{p}^\star)}\,\frac{C_\pi^{(2)}(t-t_v,\mathbf{p}^\star)}{C_\pi^{(2)}(t-t_v,\mathbf{p})}$$

- Plateaus at fixed $t_\pi$ (left) or at fixed $t_v$ (right)



$\approx 10\%$ correction ($B^*\pi$)

- Repeat the analysis for different values of $E_\pi$ in the range [0.29 : 0.85] GeV

# Antoine Gerardin, talk at *Lattice 2024*

## Test of HM$\chi$PT : two-point heavy-light function with distillation

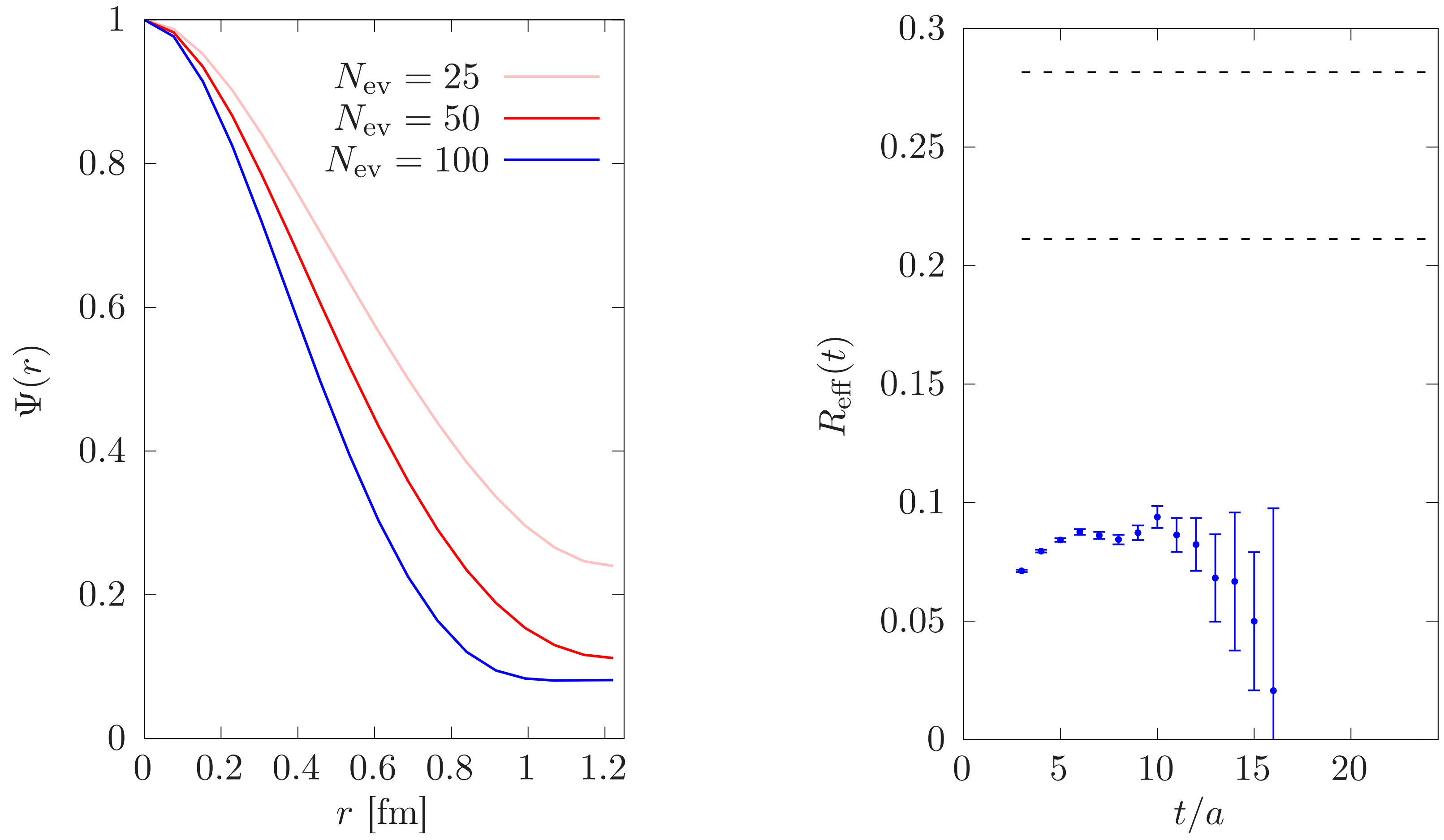
- $\widetilde{\beta}_1$ depends on the detail of the smearing operator

  $\rightarrow$ other smearings may yield better results ( ?)

- Preliminary results obtained with distillation [M Peardon et al. '09]

Profiles

Ratio of overlap factors



$\rightarrow$ preliminary results suggest smaller overlap with $B^*\pi$ states as compared to gauss smearing

$\rightarrow$ next step : compute $\widetilde{\beta}_1$ in distillation (applicability of HLChPT ?)

# More results

- Analogous results for
  - B-meson mass
  - BB*$\pi$ coupling $g_\pi$

  with B$\pi$ contamination ≈ a few percent

A. Broll, OB and R. Sommer
Eur. Phys. J C 83 (2023) 757

- But: The B$\pi$ contamination can be significantly larger in some cases

  Example:  Vector current form factor