Light-cone sum rules and lattice QCD

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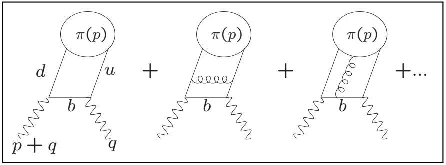
Outline

- Light-cone sum rules (LCSRs) in QCD: “a tale of two methods”:
  - with light-meson light-cone distribution ampltudes (DAs)
  - with heavy hadron DAs: B, Λ_b (HQET)
- How can lattice QCD help to improve LCSRs ?
- Hadron form factors accessible (so far) only with LCSRs
  - heavy-to-light dimeson form factors $(B \to 2\pi, K\pi)$
  - nonlocal effects in $B \to K^{(*)}\ell\ell$

“A guide on LCSRs” AK, B. Melic, Y.-M. Wang, 2311.08700

LCSR for $B \to \pi$ form factor with pion DAs



$\Leftarrow$ the correlator

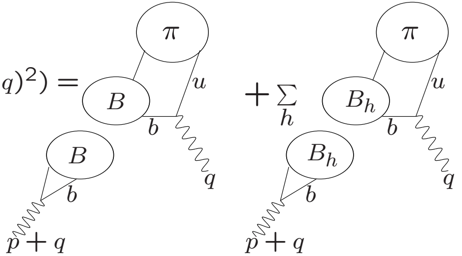
$\int d^4x\, e^{iqx} \langle \pi | \, T\{ j_W(x) j_B(0) \} | 0 \rangle$

calculated from OPE in terms of pion distribution amplitudes

at $(p+q)^2, q^2 \ll m_b^2$

$\Downarrow$

hadronic dispersion relation } $F(q^2,(p+q)^2) =$



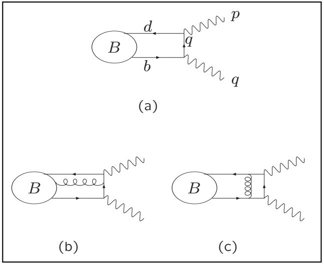
$f_B f_{B\pi}(q^2)$

$\Uparrow$ QCD 2-point SR

$$\sum_{B_h} \Rightarrow \int_{s_0^B}^{\infty} ds \, \frac{\mathrm{Im}F(q^2,s)_{OPE}}{s-(p+q)^2}$$

quark-hadron duality

LCSR for $B \to \pi$ form factors with B-meson DAs



$\Leftarrow$ the correlator

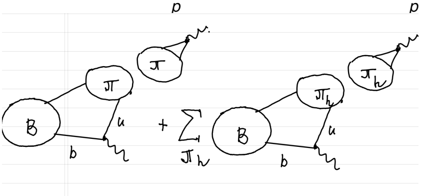
$$\int d^4x\, e^{iqx} \langle 0 |\, T\{ j_\pi(x) j_W(0)\} | B \rangle$$

calculated from OPE in terms of B distribution amplitudes

at $p^2 \ll 0$, $q^2 \ll m_b^2$

$\Downarrow$

hadronic dispersion relation $\Big\}$ $F(p^2, q^2) =$



$f_\pi f_{B\pi}(q^2)$

$$\sum_{\pi_h} \Rightarrow \int_{s_0^\pi}^{\infty} ds \frac{\mathrm{Im} F(q^2, s)_{OPE}}{s - p^2}$$

quark-hadron duality

LCSRs with light hadron DAs: input and accuracy

- employing m_b, m_c, m_s and α_s from PDG, - mainly provided by lattice QCD
- LCSRs combined with 2-point QCD sum rules, are “self-sufficient”
  but the accuracy of certain input paramaters (e.g. f_B, f_D) is limited
  $\Rightarrow$nowadays using FLAG averages
- pion and kaon DAs, twist $t = 2, 3, 4$, polynomial structure

$$\varphi_\pi^{(t)}(u,\mu) = f_\pi^{(t)}(\mu)\{C_0(u) + \sum_{n=1} a_n^{(t)}(\mu) C_n(u)\}$$

- Gegenbauer moments $a_n^{(2)}$ of π, K DAs calculated in lattice QCD
  e.g. V.M.Braun, et al. 1503.03656
- assessing the shape of the pion DA,a novel lattice technique:
  Lattice Parton Collaboration (LPC), 2201.09173
- hadronic dispersion relation, quark-hadron "semilocal" duality

for $B \to \pi$ neeed the spectrum of excited states with B quant.numbers ($J^P = 0^-$)

accessible for dispersive methods ?

LCSRs with B-meson DAs: input and accuracy

- definition of two-particle DA in HQET: (1/m_B corrections?)

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$

$$= -\frac{if_B m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

$\oplus$ higher twists

- key input parameter: the inverse moment

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- possible to extract λ_B from $B\to\gamma\ell\nu_\ell$ using QCDF$\oplus$LCSR

M.Beneke, V.M. Braun, Y.Ji, Y.B. Wei (2018)

- current limit from Belle measurement (2018): $\lambda_B > 240$ MeV
- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

V.Braun, D.Ivanov, G.Korchemsky (2004)

- preliminary numerical results for lattice calculation of $D_s\to\gamma\ell\nu_\ell$,

C. Kane, C. Lehner, S. Meinel and A. Soni, [arXiv:1907.00279 [hep-lat]].

any perspectives for $B\to\gamma\ell\nu_\ell$?

$B \to \pi\pi\ell\nu_\ell$ decays

see also the talk by Florian Herren

- $\bar{B}^0 \to \pi^+\pi^0$ form factors, dipion with isospin 1:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle =$$

$$-\frac{4}{\sqrt{k^2\lambda_B}}\, i\epsilon^{\mu\alpha\beta\gamma}\, q_\alpha\, k_{1\beta}\, k_{2\gamma} F_\perp(q^2, k^2, \zeta) \oplus \text{axial current form factors}$$

$$(2\zeta - 1) = (1 - 4m_\pi^2/k^2)^{1/2} cos\theta_\pi, \text{ in dipion c.m.}$$

e.g., S. Faller, T. Feldmann, A.K., T. Mannel, D. van Dyk, 1310.6660

- expand in partial waves, isolate dipion P-wave:

$$F_\perp(q^2, k^2, \zeta) \Rightarrow F_\perp^{(\ell=1)}(q^2, k^2)$$

- two different LCSR methods:
  - with dipion distribution amplitudes (2π-DAs)

    Ch. Hambrock, AK, 1511.02509
  - with B-meson distribution amplitudes (B-DAs)

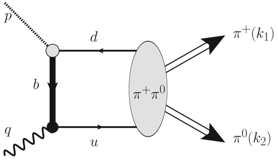
    S.Cheng, AK, J.Virto, 1701.01633

- questions to address:
  - how important are $\ell > 1$ partial waves of 2π states?
  - to what extent the ρ state dominates?

LCSRs with dipion distribution amplitudes

- The correlator:

OPE in terms of dipion DAs



- introduced for $\gamma^*\gamma \to 2\pi$ processes

M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998);

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994); M. V. Polyakov, (1999).

- twist-2 DAs:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_\perp^{I=1}(u,\zeta,k^2)\,,$$

- normalization conditions $\to$ pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{l} \Phi_\parallel^{I=1}(u,\zeta,k^2) \\ \Phi_\perp^{I=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta-1)\left\{ \begin{array}{l} F_\pi^{em}(k^2) \\ F_\pi^{t}(k^2) \end{array} \right.$$

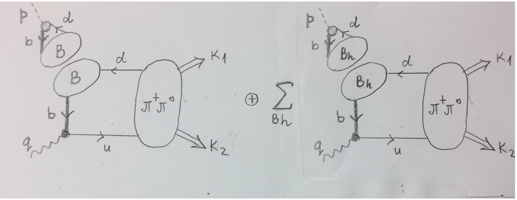
pion e.m. form factor

pion “tensor” form factor

- $F_\pi^{em}(0)=1$, • “tensor” charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^\perp$

lattice QCD , e.g. C. Alexandrou et al. [ETM], 2111.08135 [hep-lat].

LCSRs with dipion distribution amplitudes



- the hadronic dispersion relation in B-meson channel:

$$\Pi_\mu^{(V)}(q,k_1,k_2) = \frac{\langle \pi^+\pi^0|\bar{u}\gamma_\mu b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2-p^2} + \underbrace{\sum_{B_h} \frac{\langle \pi^+\pi^0|\bar{u}\gamma_\mu b|B_h\rangle\langle B_h|im_b\bar{b}\gamma_5 d|0\rangle}{m_B^2-p^2}}_{\int_{s_0^B}^\infty ds \frac{\rho_\mu^{OPE,(V)}(s)}{s-p^2}}$$

quark-hadron duality approximation ⇒

- applying quark-hadron duality and Borel transformation:

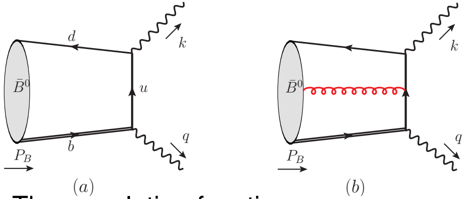
$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2(1-2\zeta)} \int_{u_0(s_0^B)}^{1} \frac{du}{u} \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_B^2}{M^2}-\frac{m_b^2-q^2\bar{u}+k^2 u\bar{u}}{uM^2}}$$

- applying the expansion in Legendre polynomials
  ⇒ LCSR for $\ell = 1, 3, 5, ...$- form factors
- Gegenbauer expansion for dipion DAs,

$B \to \pi\pi\ell\nu$ from LCSRs with B-meson DAs

- LCSRs with B-meson DA and $\bar{u}\gamma_\mu d$ interpolating current
- originally introduced to calculate $B \to \rho$ form factors,

A.K., N. Offen, Th. Mannel (2005),(2007);



- The correlation function:

$$F_{\mu\nu}(k,q) = i \int d^4x e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|\bar{B}^0(q+k)\rangle, \qquad (1)$$

Accessing $B \to \pi\pi$ form factors

- OPE diagrams $\Rightarrow$ invariant amplitudes $\Rightarrow$ dispersion form in k^2:

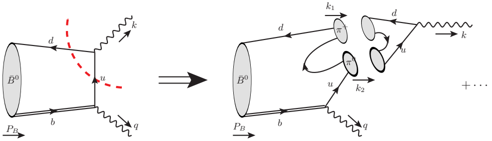
$$F^{\rm OPE}_{(\varepsilon)}(k^2,q^2) = f_B m_B \int_0^\infty d\sigma\, \frac{\phi_+^B(\sigma m_B)}{\bar\sigma(s-k^2)} + \{3-\textit{particle DAs}\}$$

$$s = s(\sigma,q^2) = \sigma m_B^2 - \sigma q^2/\bar\sigma\,, \qquad \bar\sigma \equiv 1-\sigma$$

- hadronic dispersion.relation and unitarity:

$$F_{(\varepsilon)}(k^2,q^2) = \frac{1}{\pi}\int\limits_{4m_\pi^2}^{\infty} ds\, \frac{{\rm Im}F_{(\varepsilon)}(s,q^2)}{s-k^2}\,.$$

$$2\,{\rm Im}F_{\mu\nu}(k,q) = \int d\tau_{2\pi}\, \underbrace{\langle 0|\bar d\gamma_\mu u\,|\pi^+\pi^0\rangle}_{F_\pi(s)}\, \underbrace{\langle \pi^+\pi^0|\bar u\gamma_\nu(1-\gamma_5)b|\bar B^0(q+k)\rangle}_{B\,\to\,2\pi\ (\ell\,=\,1)\ \text{form factors}} + \cdots\,,$$



Resulting sum rules

- e,g,. for the form factor $F_{\perp}^{(\ell=1)}$ of the vector current

$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds\; e^{-s/M^2} \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_{\perp}^{(\ell=1)}(s,q^2)$$

$$= \quad f_B m_B \left[\int_0^{\sigma_0^{2\pi}} d\sigma\; e^{-s(\sigma,q^2)/M^2}\, \frac{\phi_+^B(\sigma m_B)}{\bar\sigma} + m_B\, \Delta V^{BV}(q^2,\sigma_0^{2\pi},M^2) \right],$$

$\sigma_0^{2\pi}$ - the solution of $\sigma m_B^2 - \sigma q^2/\bar\sigma = s_0^{2\pi}$, three-particle DA contribution ΔV^{BV} cumbersome

- similar sum rules for all other P-wave $B \to 2\pi$ form factors
- not a direct calculation, given the shape of the $B \to 2\pi$ form factors, these sum rules can provide normalization
- the complex phases of $B \to 2\pi$ FFs and $F_\pi(s)$ equal at low s: a usual Watson theorem

Probing ρ-resonance models

- ansatz for the $B \to \pi\pi$ FF:
  inspired by experimental fit of $F_\pi(s)$

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - i m_\rho \Gamma_\rho(k^2)} \frac{V^{B\to\rho}(q^2)}{m_B + m_\rho}$$
$$+ \frac{g_{\rho'\pi\pi}}{m_{\rho'}^2 - k^2 - i m_{\rho'} \Gamma_{\rho'}(k^2)} \frac{V^{B\to\rho'}(q^2)}{m_B + m_{\rho'}} + \frac{g_{\rho''\pi\pi}}{m_{\rho''}^2 - k^2 - i m_{\rho''} \Gamma_{\rho''}(k^2)} \frac{V^{B\to\rho''}(q^2)}{m_B + m_{\rho''}}$$

- Model 1: • $V^{B\to\rho}(q^2)$ from LCSR with ρ-meson DAs (in which $\Gamma_\rho = 0$)

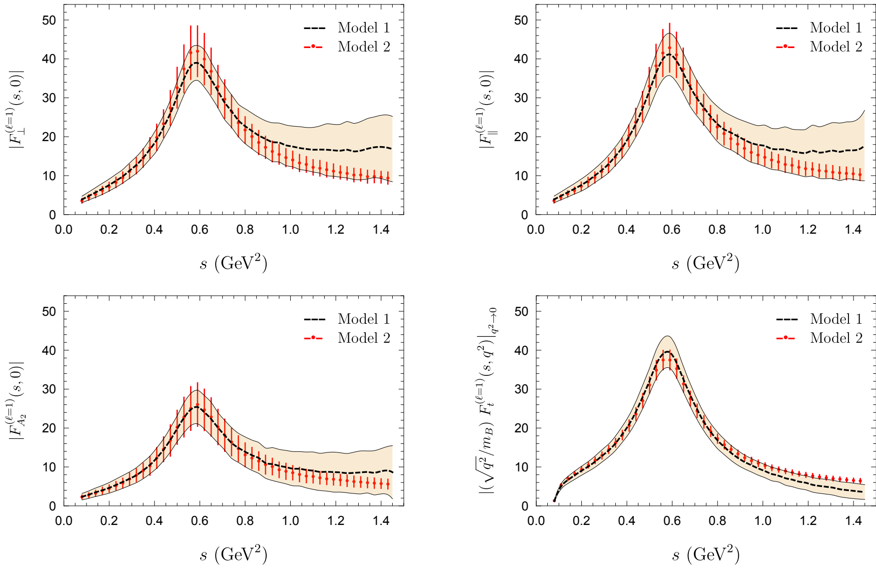
  taken from A.Bharucha, D.Straub and R.Zwicky,1503.05534

  • neglect ρ'' and substitute in LCSR

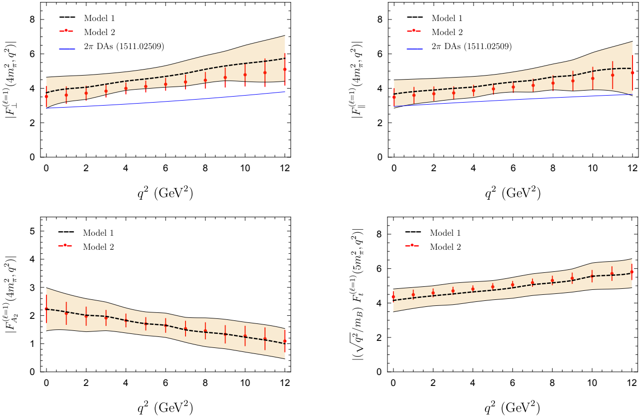
  $\Rightarrow$ an appreciable contribution of ρ' is consistent with the fit,

- Model 2 : • all three resonances taken into account
  • their proportion taken as in $F_\pi(s)$ Belle fit

$B \to 2\pi$ ($\ell = 1$) FFs: dipion mass dependence



$B \to 2\pi$ ($\ell = 1$) FFs: q^2-dependence at small k^2



- Applications to $B \to K\pi\ell^+\ell^-$ form factors

with $K\pi$ in P and S waves:

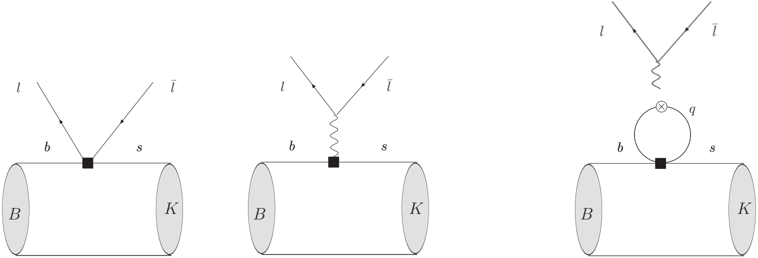
S. Descotes-Genon, A. Khodjamirian and J. Virto, 1908.02267;

S. Descotes-Genon, A. Khodjamirian, J. Virto and K. K. Vos, 2304.02973

Nonlocal effects in $B \to K^{(*)}\ell^+\ell^-$

- the simplest $b \to s\ell^+\ell^-$ decay mode is $B \to K\ell^+\ell^-$,

  (schematic) hadronic diagrams:



- Decay amplitude:

  effective operators sandwiched between the initial B and final $K\ell\ell$ states

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \, \langle K\ell^+\ell^- \mid O_i \mid B \rangle$$

Hadronic matrix elements in $B \to K\ell^+\ell^-$

- factorizing the lepton pair and intermediate photon,
  isolating hadronic matrix elements (form factors)

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Big[(\bar{\ell}\gamma^\mu\gamma_5\ell)\, C_{10} \langle K|\bar{s}\gamma_\mu b|B\rangle$$

$$+(\bar{\ell}\gamma^\mu\ell)\Big(C_9 \langle K|\bar{s}\gamma_\mu b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K|\bar{s} i\sigma_{\nu\mu}(1+\gamma_5) b|B\rangle$$

$$+\frac{8\pi^2}{q^2} \sum_{i=1,2,\ldots,6,8} C_i \langle K(p)|i\int d^4x\, e^{iqx}\, T\{j^\mu_{em}(x), O_i(0)\}|B(p+q)\rangle \Big)\Big]$$

- the local $B \to K$ form factor $\quad \langle K(p)|\bar{s}\gamma_\mu b|B(p+q)\rangle \Rightarrow f^+_{BK}(q^2)$
- the nonlocal $B \to K$ form factors

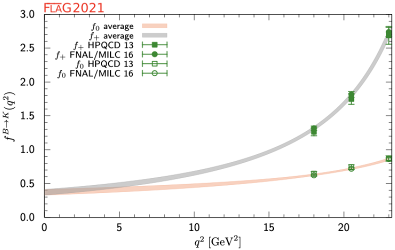
$$\langle K(p)|i\int d^4x\, e^{iqx}\, T\{j_{em}(x), O_i(0)\}|B(p+q)\rangle \Rightarrow H^i_{BK}(q^2),$$

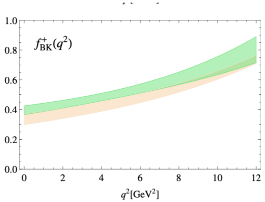
- need a nonperturbative QCD method to calculate the form factors
  characteristic scale $\bar{\Lambda} = m_B - m_b$

$B \to K$ form factor, the results

- QCD Light-cone sum rules (see a review 2311.08700, AK, B.Melic,YM Wang)

$f^+_{BK}(q^2=0)$	method	Ref.
0.395 ± 0.033	K DAs	AK, A.Rusov , 1703.04765
0.27 ± 0.08	B DAs	N. Gubernari, A,Kokulu, D.van Dyk, 1811.00983
0.325 ± 0.085	B DAs $\oplus$ SCET	C.-D. Lü,Y.-L.Shen, Y.-M.Wang, Y.-B.Wei,1810.00819





LCSR [1703.04765], (green, vs Fermilab MILC, 1503.07839

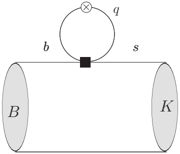
- lattice QCD: (see FLAG review for < 2021 results)

$$f^+_{BK}(q^2=0) = 0.332 \pm 0.012$$

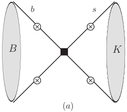
HPQCD, 2207.12468 (used in their $B \to K\nu\bar{\nu}$)

extrapolation error ?

Anatomy of nonlocal matrix elements in $B \to K\ell\ell$

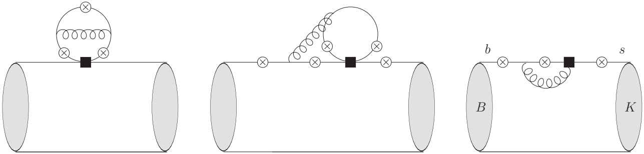


LO (factor.)

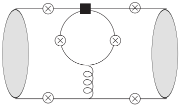


weak annihilation

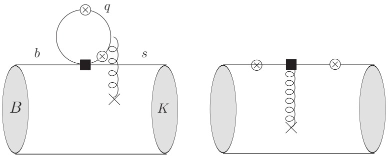
$\otimes$ -virtual photon



NLO (factor.) H. Asatryan, C. Greub , J.Virto. [hep-ph/0109140]



spectator (nonfactor.)



soft (low virtuality) gluons

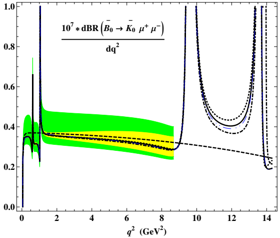
M.Beneke, Th.Feldmann, D.Seidel (2001)

A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang,

Non-lattice QCD results for $B \to K\mu^+\mu^-$

- the method: LCSR $\oplus$ QCDF ($q^2 < 0$)$\oplus$ dispersion relation

  AK, T. Mannel, A.Pivovarov, Y.-M . Wang, 1006.4945 [hep-ph]



- AK, T. Mannel, Y.M . Wang 1211.0234 [hep-ph]]
  - the long-dashed line - without nonlocal contributions.
  - the green (yellow) area - with (without) the uncertainties of $f_{BK}(q^2)$

- AK, A.Rusov 1703.04765 – more accurate LCSR for $f_{BK}(q^2)$

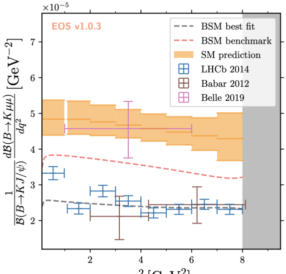
$$BR(B^+ \to K^+\mu^+\mu^-)_{[1.0-6.0\ GeV^2]} = (2.19 \pm 0.33) \times 10^{-7}$$

- N. Gubernari, M. Reboud, D. van Dyk, J. Virto, 2206.03797

– improved NLO factor.diags, smaller soft gluon effct

– Bayesian (EOS) analysis of the uncertainties

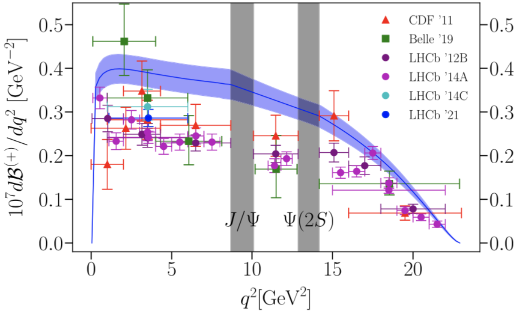
$$BR(B^+ \to K^+\mu^+\mu^-)_{[1.1-6.0\ GeV^2]} = (2.3 \pm 0.2) \times 10^{-7}$$



$$BR(B^\pm \to J/\psi K^\pm) = (1.02 \pm 0.02) * 10^{-3}$$

$B \to K\ell^+\ell^-$ from lattice QCD

- HPQCD result for differential width vs experiment:



HPQCD: $BR(B^+ \to K^+\mu^+\mu^-)_{[1.0-6.0\ GeV^2]} = (1.91 \pm 0.19) \times 10^{-7}$

- the (subdominant) nonlocal effects are modelled with non-lattice technique

- there is definitely a tension compared to the measurements !

LHCb-2021: $BR(B^+ \to K^+\mu^+\mu^-)_{[1.0-6.0\ GeV^2]} = (1.401 \pm 0.09) \times 10^{-7}$

CMS-2024: $BR(B^+ \to K^+\mu^+\mu^-)_{[1.1-6.0\ GeV^2]} = (1.242 \pm 0.068) \times 10^{-7}$

Summary

- Normalization and shape parameters of light meson DAs calculated on the lattice will enable us to get more acccuracy in LCSRs
- inverse moment of B-meson DAs on the lattice
- combining LCSRs and hadronic amplitude analysis to study dimeson form factors with both types of LCSRs
- nonlocal effects are so far only accessible in the continuum framework LCSR $\oplus$ QCDF$\oplus$ hadronic dispersion relation
- new applications of LCSRs to $D \to \pi \ell^+ \ell^-$ decay

  A.Bansal, AK, Th.Mannel, work in progress