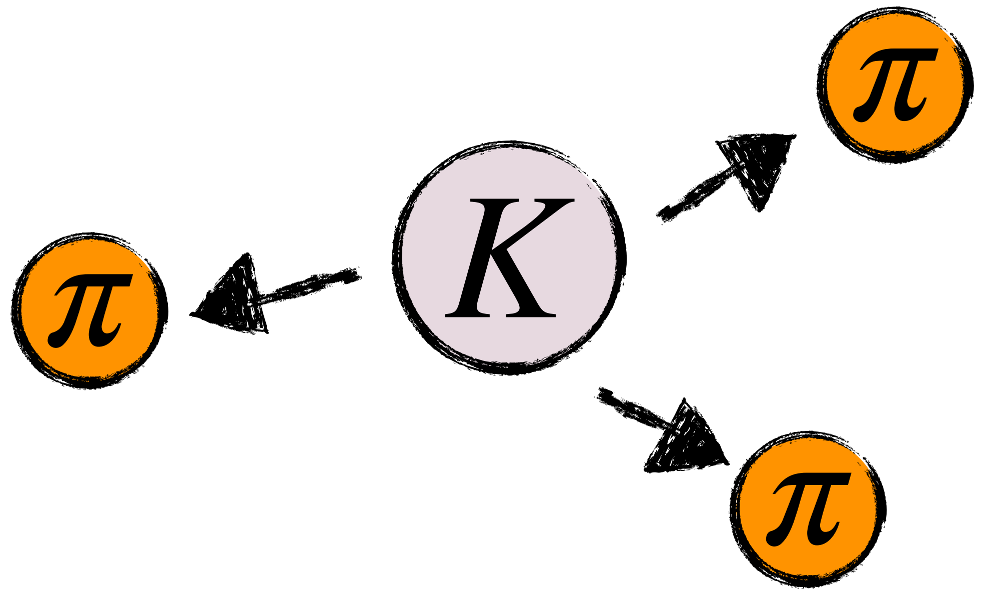
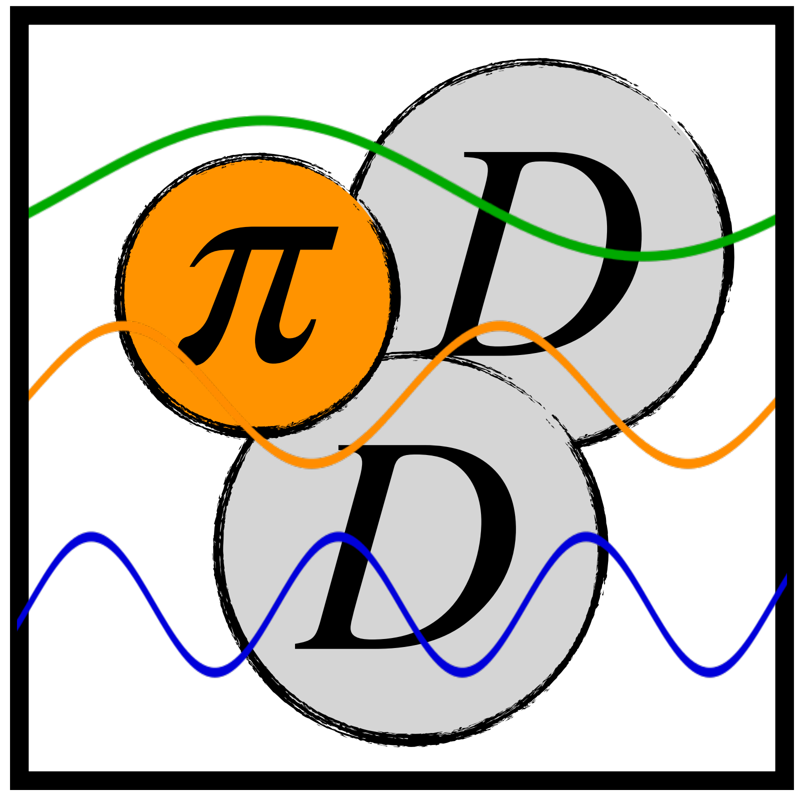
Multi-hadron decays from LQCD

Fernando Romero-López
Uni Bern
fernando.romero-lopez@unibe.ch

Siegen, October 2nd

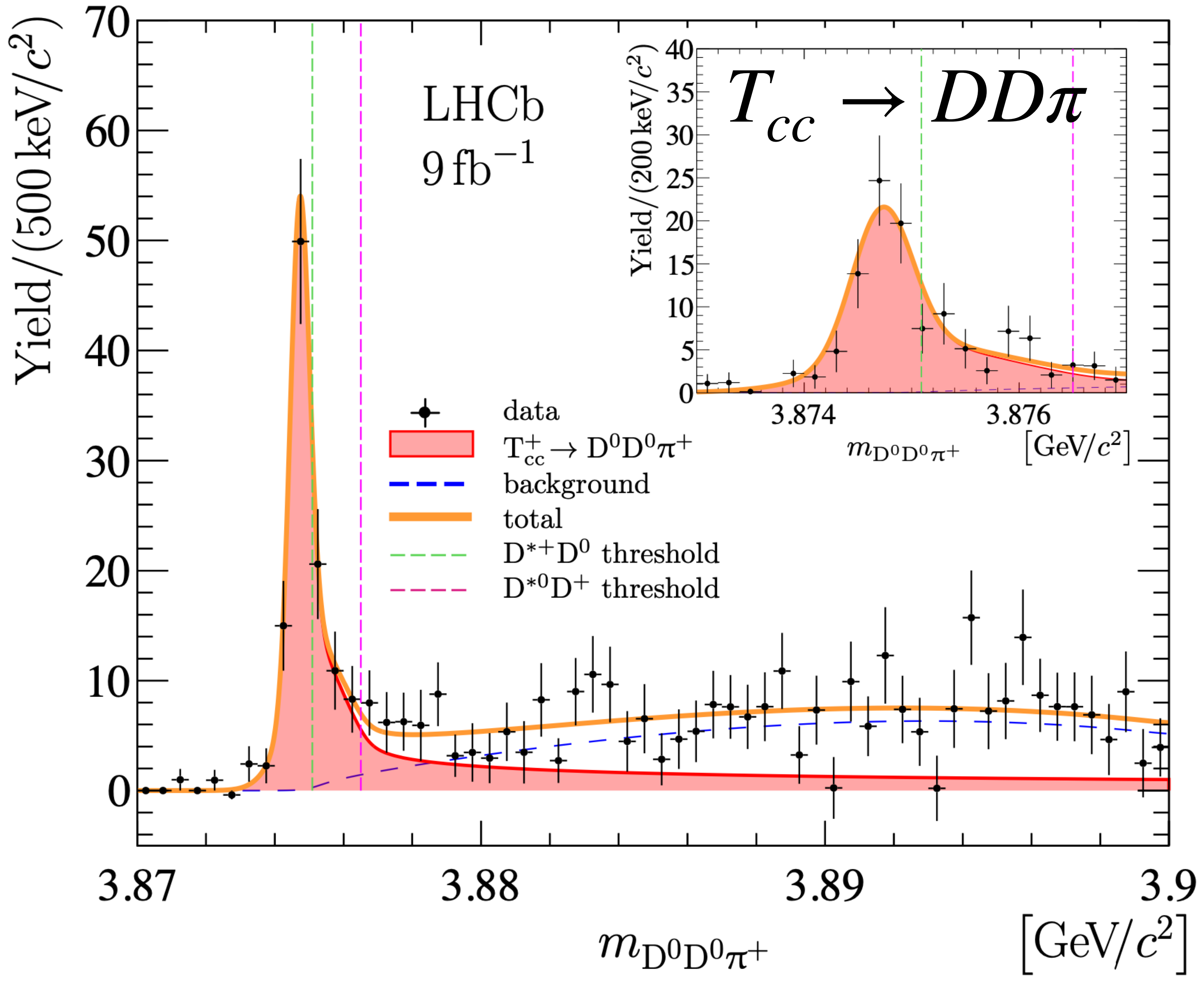




Multi-hadron decays

Many features of the Standard Model can be investigated through multi-hadron decays

- **Multi-hadron resonances and exotic hadrons**
  - D_0^* **[See talks by C. Hahnhart and D. Mohler]**
  - **Roper,** $\Lambda(1405)$

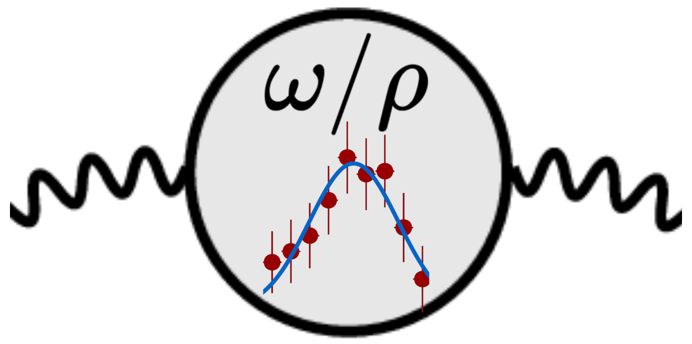


[See talk by V. Baru]

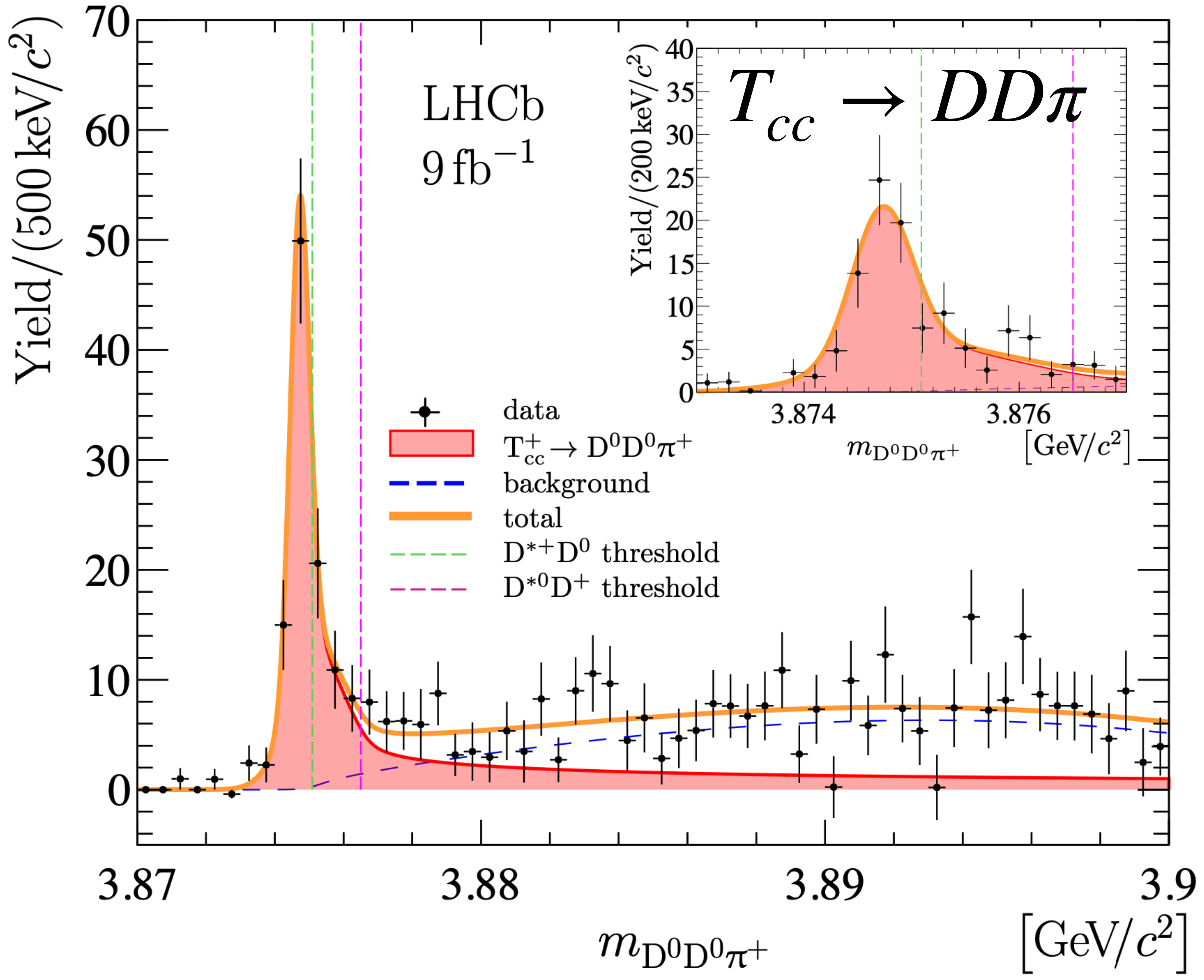
Multi-hadron decays

Many features of the Standard Model can be investigated through multi-hadron decays

- **Multi-hadron resonances and exotic hadrons**
  - D_0^* **[See talks by C. Hahnhart and D. Mohler]**
  - **Roper,** $\Lambda(1405)$
- **Electromagnetic transitions**
  - $\gamma^* \to 2\pi/3\pi$



- **Weak decays** **[See talks by F. Erben and F. Herren]**



[See talk by V. Baru]

CP violation in $D^0 \to K^+K^-/\pi^+\pi^-$ decays

$$\Delta a_{CP}^{\mathrm{dir}} = (-15.7 \pm 2.9) \times 10^{-4}$$

[LHCb, 2019]

CP violation in $K \to \pi\pi$ weak decays

$$(\varepsilon'/\varepsilon)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

[NA48 & KTeV, 2002 & 2009]

What are multi-hadron decays?

"Multi-hadron decays" is a general term that involves many different process

What are multi-hadron decays?

"Multi-hadron decays" is a general term that involves many different process

Strong decays of unstable hadrons

- Initial hadron is not a state in QCD Fock space
- Final state contains QCD stable hadrons
- These are decays of hadronic resonances

$$\rho(770) \to \pi\pi \qquad \Delta(1232) \to N\pi$$
$$N(1440) \to N\pi\pi \qquad T_{cc}(3875) \to DD\pi$$

What are multi-hadron decays?

"Multi-hadron decays" is a general term that involves many different process

Strong decays of unstable hadrons

- Initial hadron is not a state in QCD Fock space
- Final state contains QCD stable hadrons
- These are decays of hadronic resonances

$$\rho(770) \to \pi\pi \qquad \Delta(1232) \to N\pi$$
$$N(1440) \to N\pi\pi \quad T_{cc}(3875) \to DD\pi$$

Electroweak transitions of QCD-stable hadrons

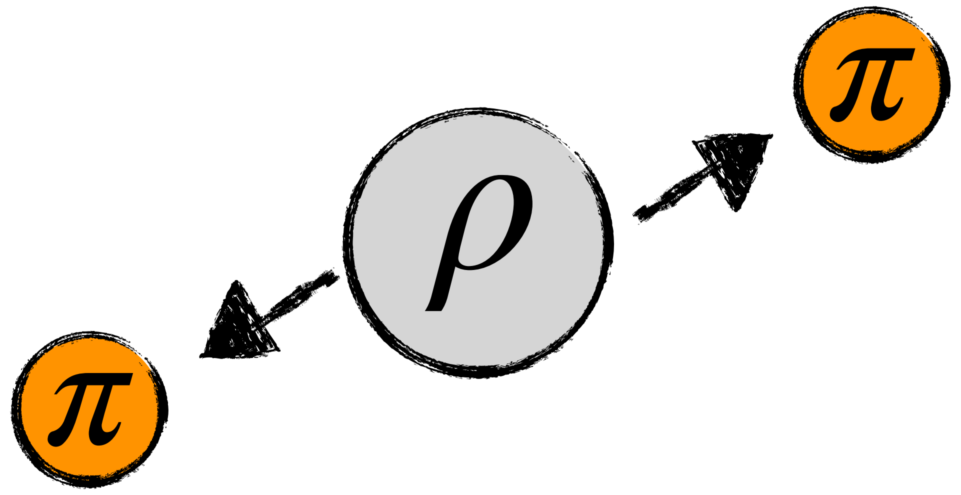
- Initial and final states include stable hadrons
- Also includes initial vacuum states
- Transition can be induced perturbatively

$$\gamma^* \to \pi\pi \qquad K \to \pi\pi$$
$$K \to \pi\pi\pi \qquad D \to K\bar{K}$$

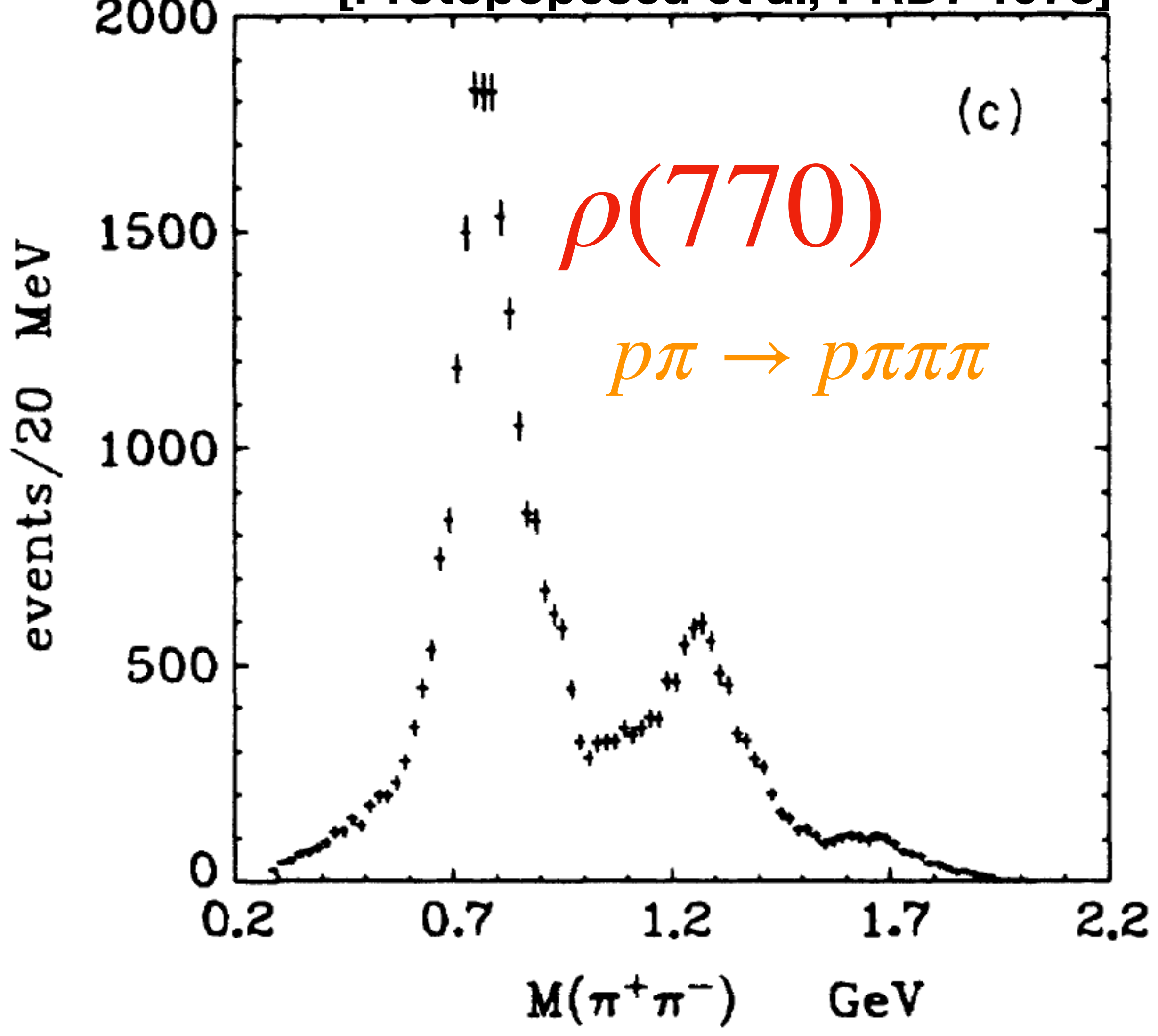
Fernando Romero-López, Uni Bern

Strong decays

- **Hadronic resonances typically manifest themselves as enhancements in cross-sections**



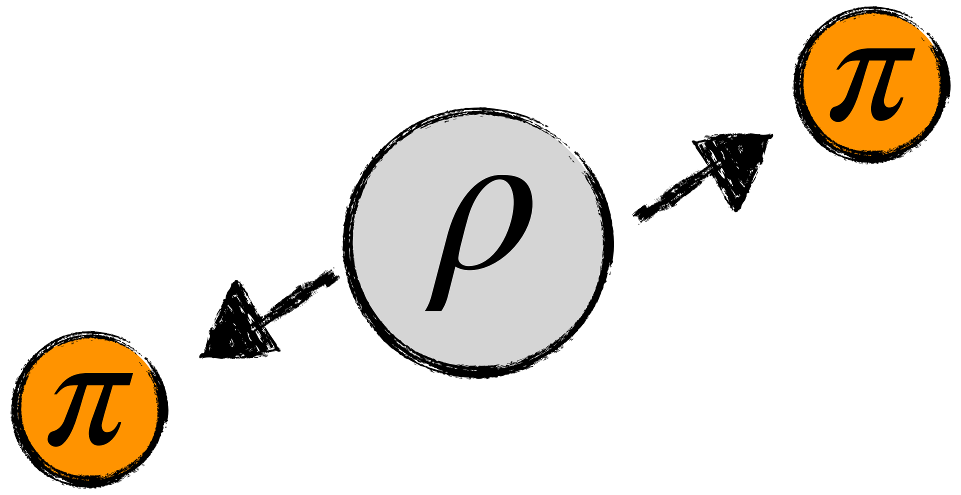
[Protopopescu et al, PRD7 1973]

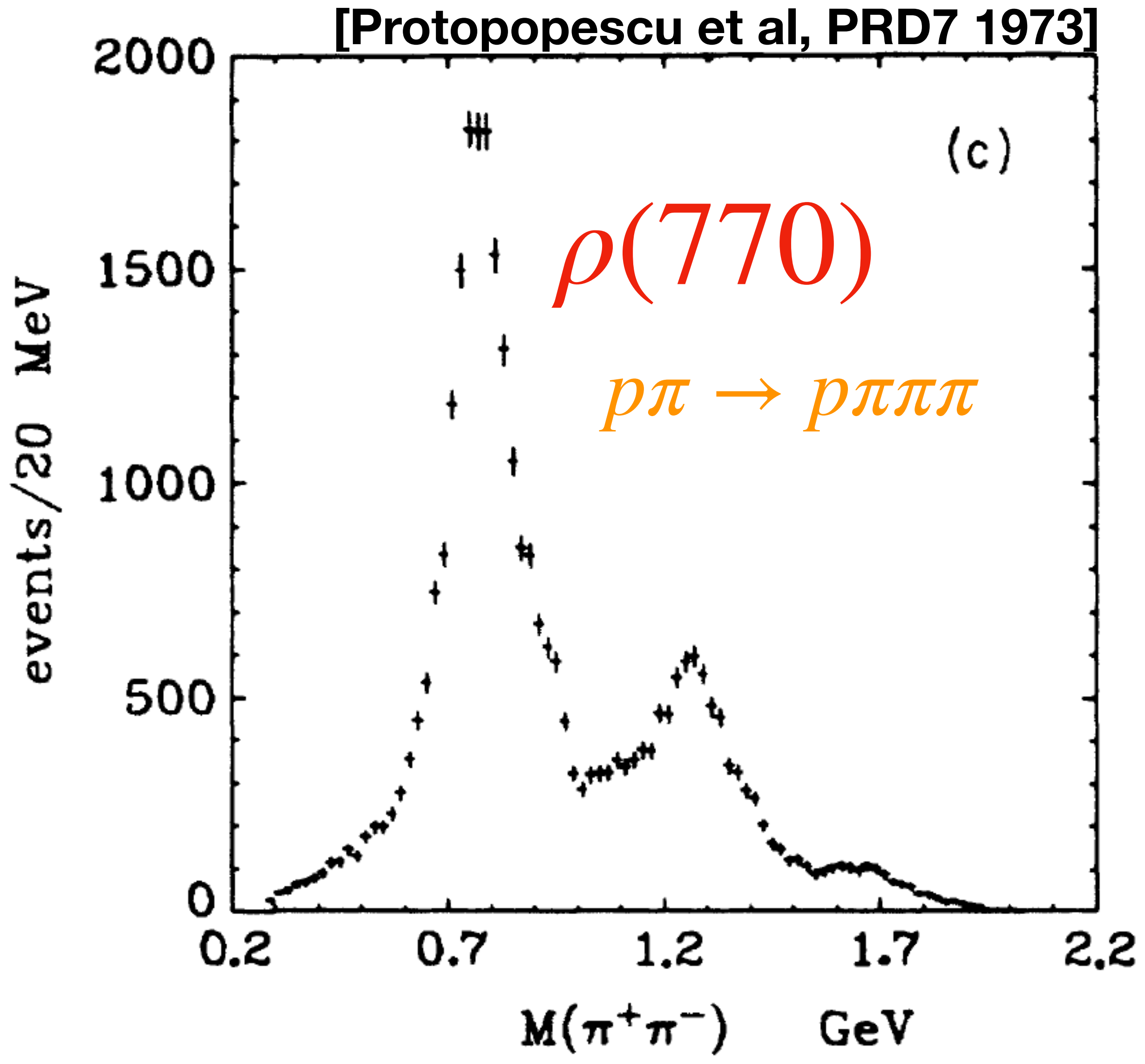


Fernando Romero-López, Uni Bern

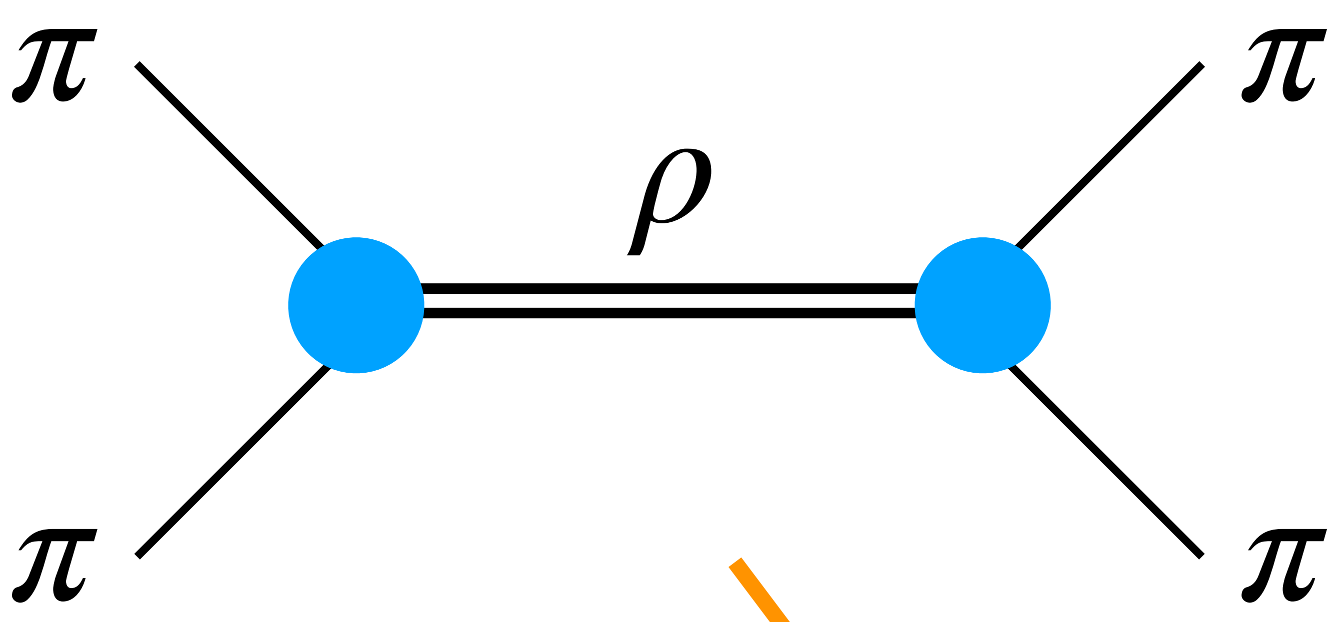
Strong decays

- Hadronic resonances typically manifest themselves as enhancements in cross-sections





- More rigorously, they are poles in scattering amplitudes



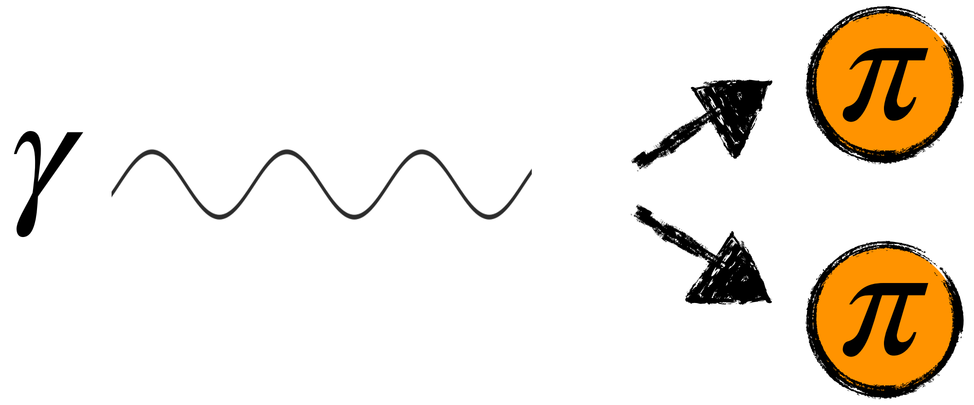
$$\mathcal{M}_\ell \sim -\frac{g^2}{s - s_R}$$

Strong decays of unstable hadrons need constraints on multi-hadron scattering amplitudes

Electroweak decays

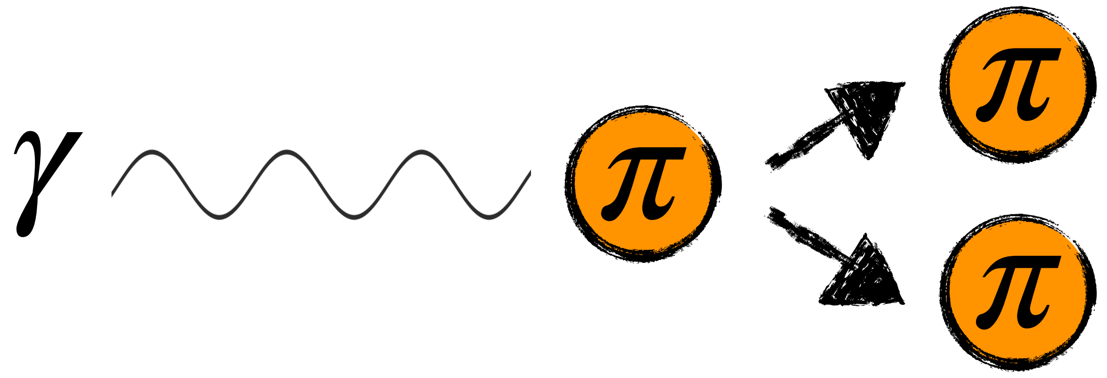
- EW decays (or transitions) involve only QCD stable hadrons in the initial and final states

$$\gamma \to \pi\pi$$



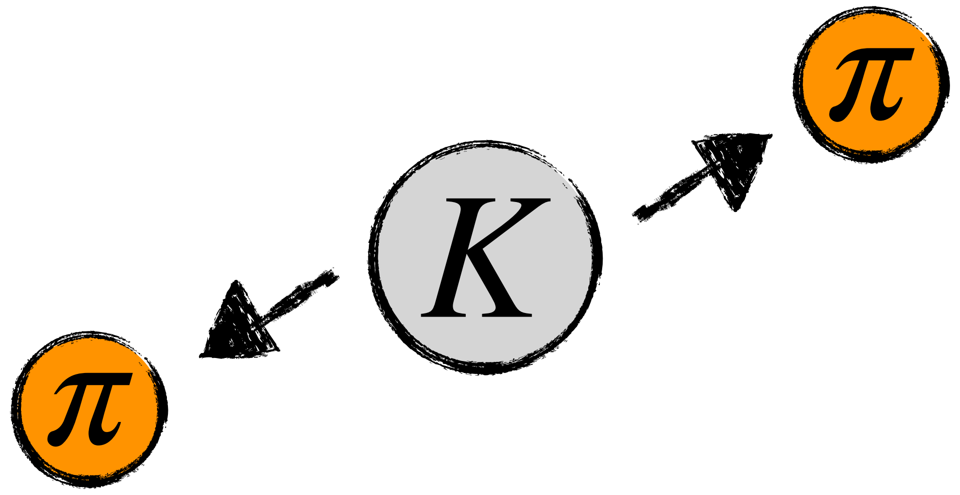
$$\langle \pi | V_\mu | \pi\pi \rangle$$

$$\gamma\pi \to \pi\pi$$



$$\langle 0 | V_\mu | \pi\pi \rangle$$

$$K \to \pi\pi$$



$$\langle K | \mathcal{H}_w^{\Delta S=1} | \pi\pi \rangle$$

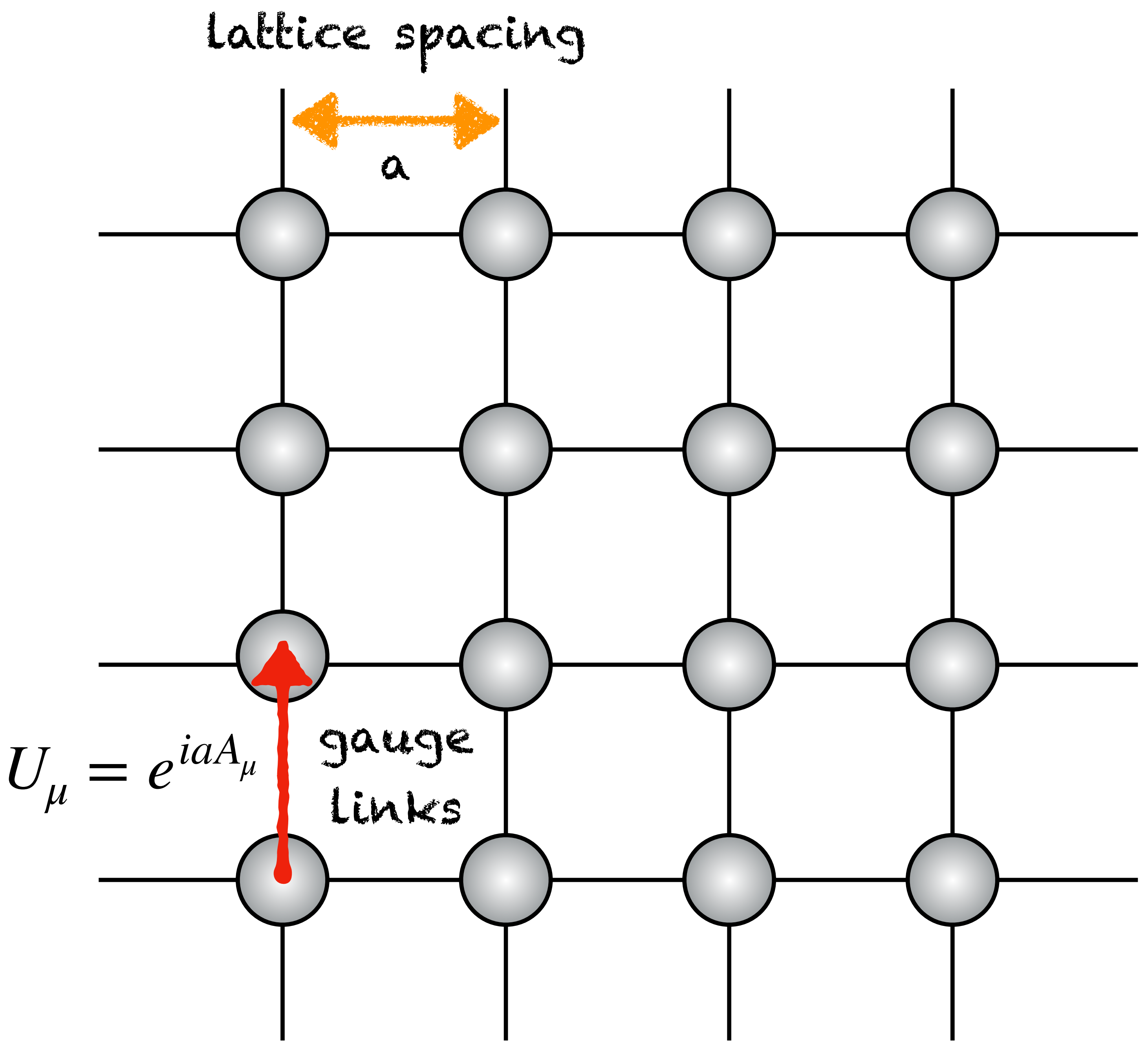
Electroweak operators are treated perturbatively

Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA \, \mathcal{O}(t)\mathcal{O}(0) e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



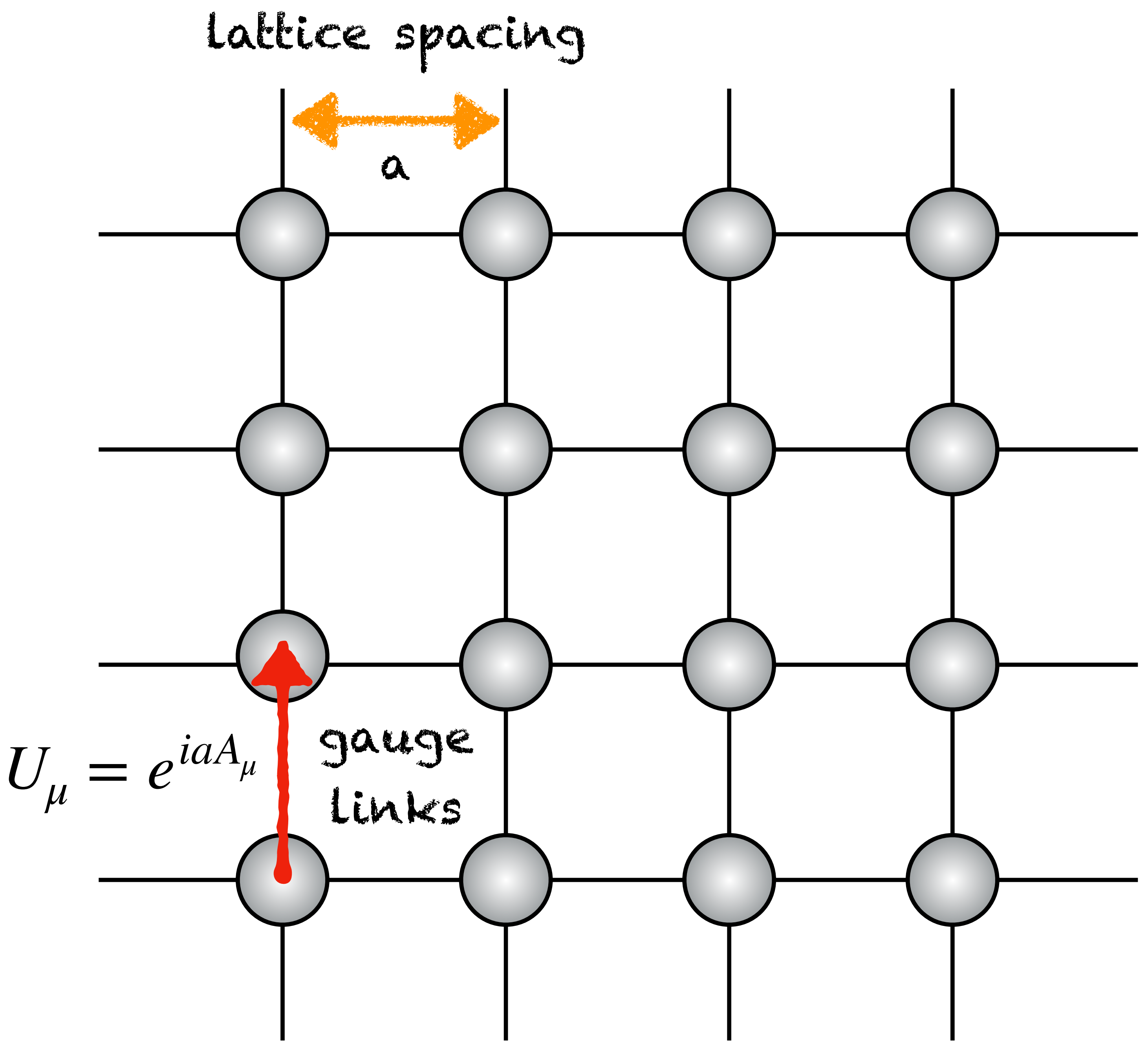
Fernando Romero-López, Uni Bern

Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA\ \mathcal{O}(t)\mathcal{O}(0)e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



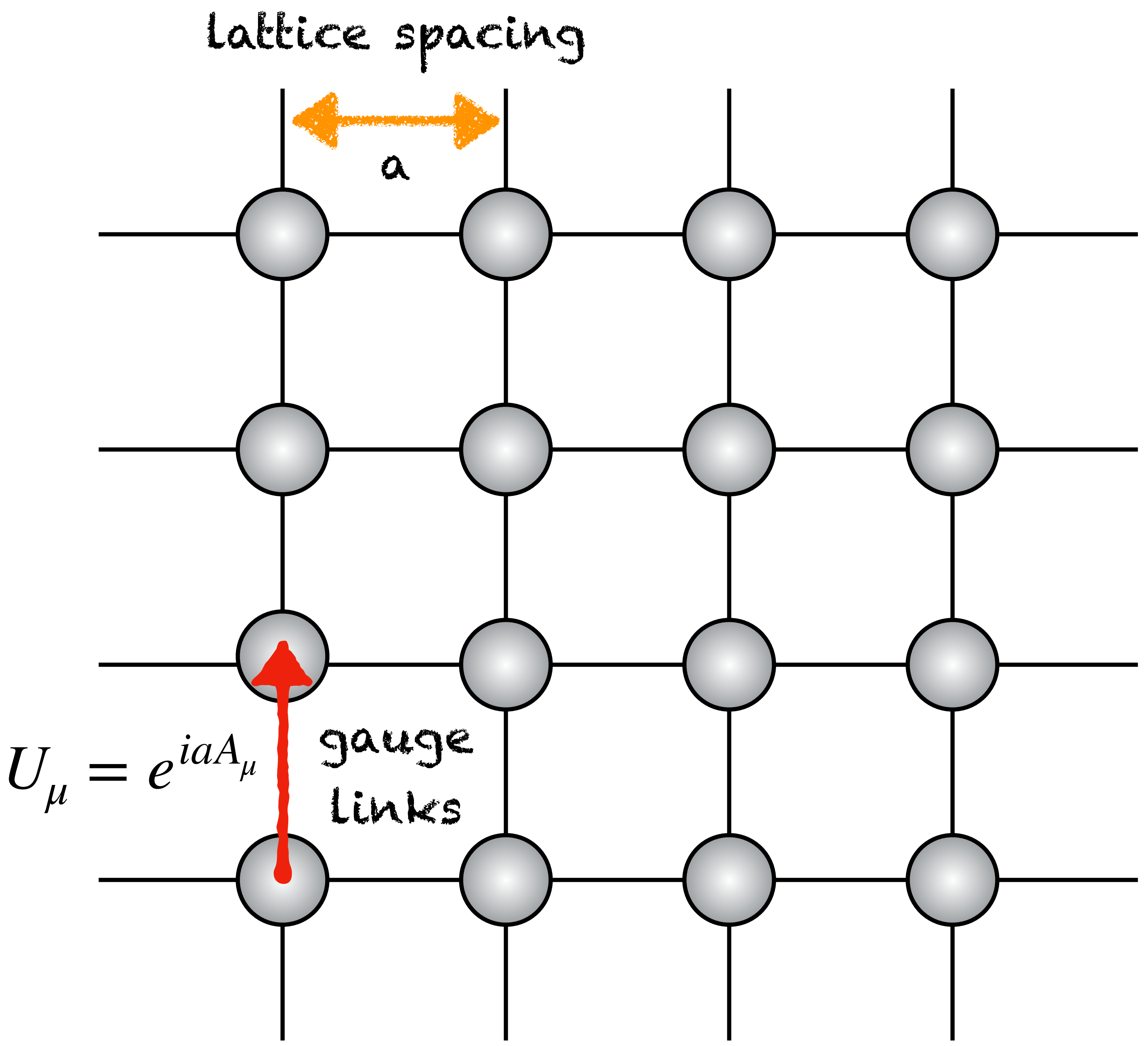
Can we compute multi-hadron decays from Euclidean correlation functions?

Lattice QCD

- Lattice QCD is a first-principles numerical approach to the strong interaction

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DA\ \mathcal{O}(t)\mathcal{O}(0)e^{-S_E(\psi,\bar{\psi},A_\mu)}$$

Euclidean action



Can we compute multi-hadron decays from Euclidean correlation functions?

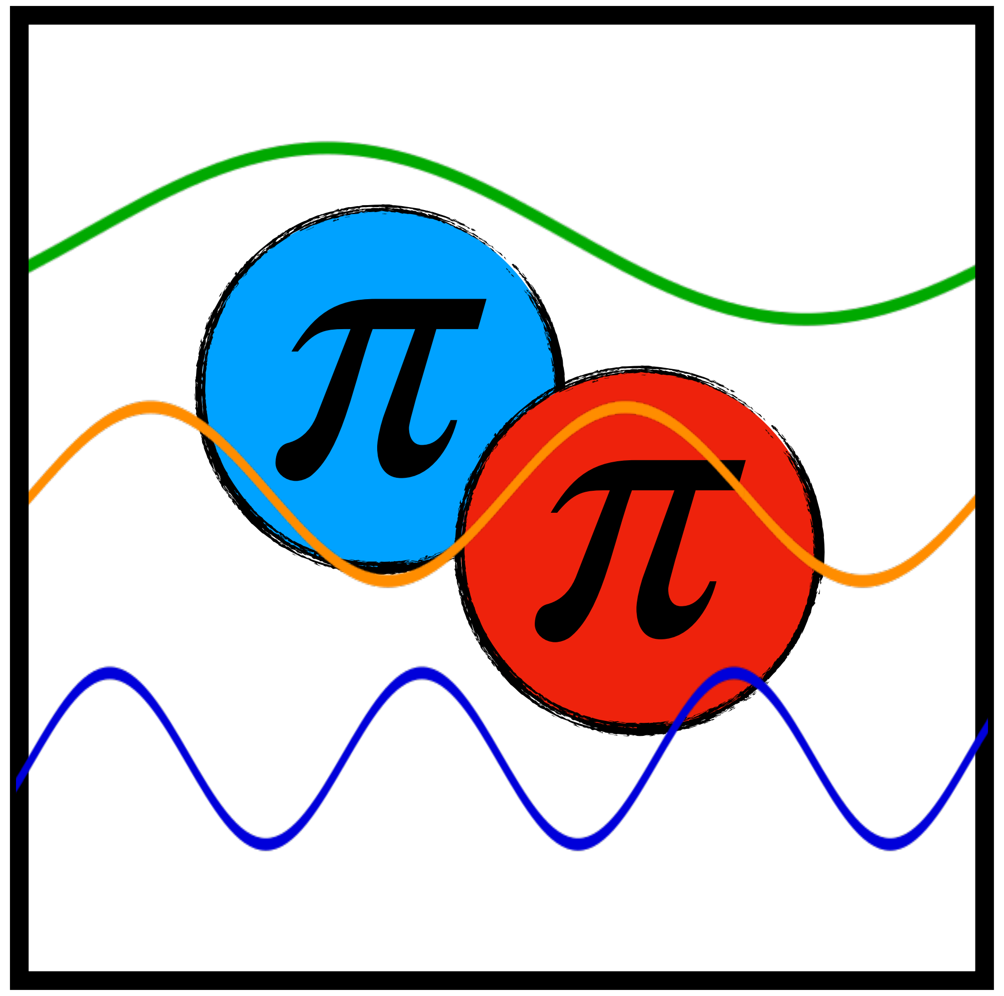
→ Yes, but not that simple!

Multi-hadron decays from LQCD

- The computation of multi-hadron decays faces two main complications in Lattice QCD

  - **Euclidean spacetime**
    - Scattering and decay is a real-time process
    - How can we define "incoming" and "outgoing" states?

  - **Finite volume**
    - Cannot define free asymptotic states
    - Only stationary finite-volume states

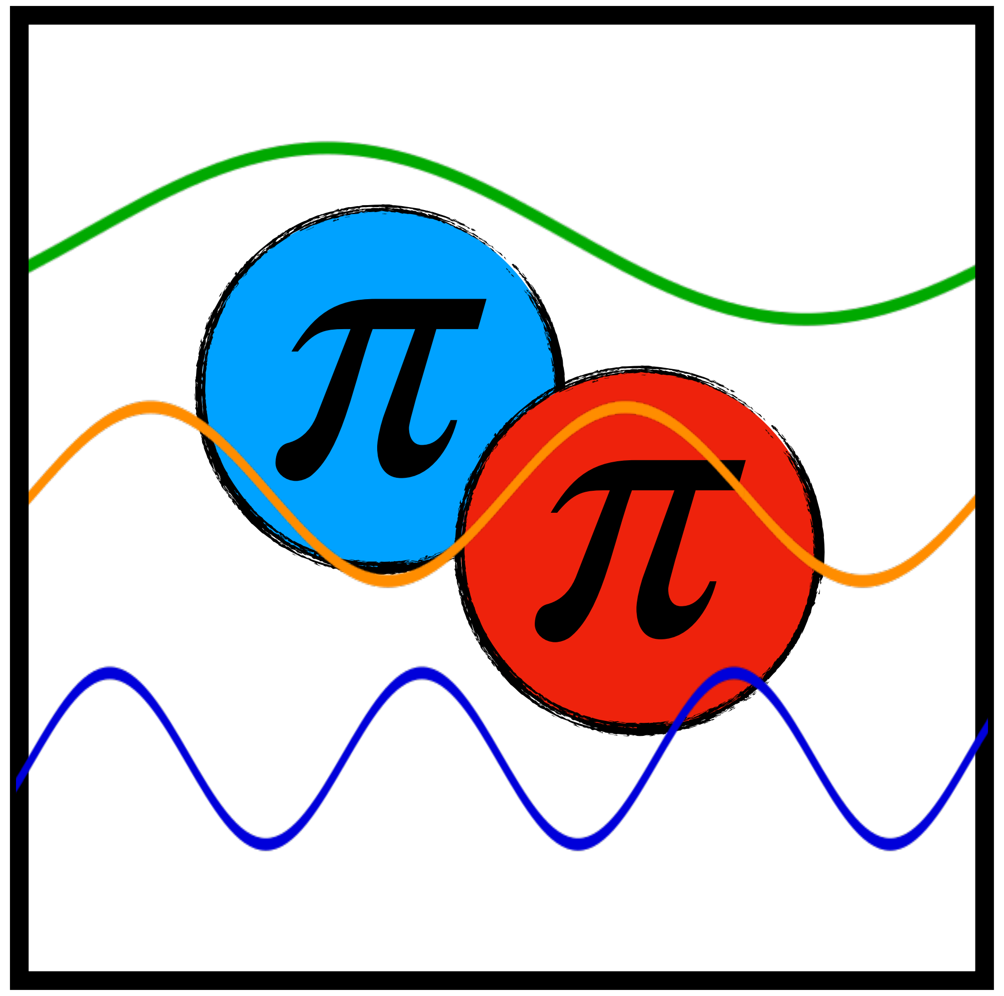


Multi-hadron decays from LQCD

- The computation of multi-hadron decays faces two main complications in Lattice QCD

  - **Euclidean spacetime**
    - Scattering and deca
    - How can we de
  - **Finite volume**
    - Cannot define free … mptotic states
    - Only stationary finite-volume states

Need “finite-volume formalism” to bridge the gap between LQCD and multi-hadron scattering and decay amplitudes



Fernando Romero-López, Uni Bern

Outline

1. Properties of unstable hadrons from LQCD

2. Multi-hadron electroweak transitions from LQCD

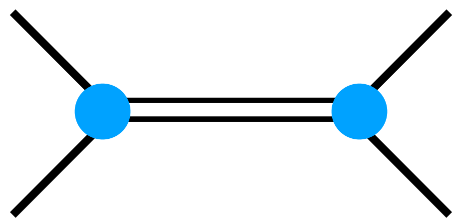
Properties of unstable hadrons from LQCD

Hadronic resonances

- The rigorous definition of a hadronic resonance is a pole in the complex plane

pole residue:
a.k.a coupling

$$\mathcal{M}_\ell \sim -\frac{g^2}{s - s_R}$$



$$\sqrt{s_R} = M_R - i\frac{\Gamma}{2}$$

width of
the resonance

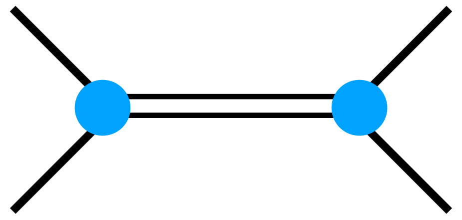
mass of
the resonance

Hadronic resonances

- The rigorous definition of a hadronic resonance is a pole in the complex plane

pole residue: a.k.a coupling

$$\mathcal{M}_\ell \sim -\frac{g^2}{s - s_R}$$

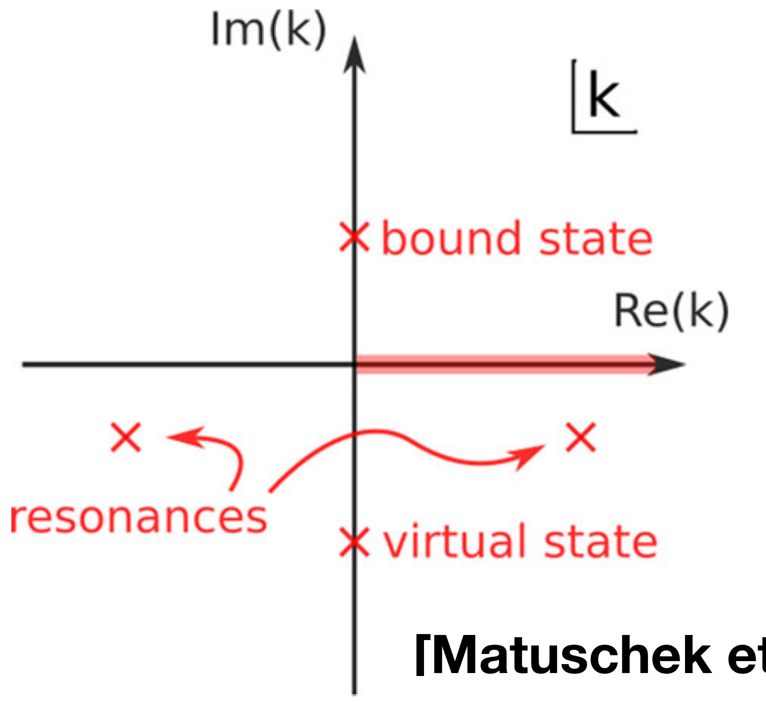


$$\sqrt{s_R} = M_R - i\frac{\Gamma}{2}$$

width of the resonance

mass of the resonance

- Based on the location of the poles, they receive different names
  - Bound states: stable particles, e.g. the deuteron is an NN bound state
  - Resonances: unstable hadrons, e.g. the rho resonance
  - Virtual states: “non-renormalizable QM states”, e.g. “dineutron”



[Matuschek et al, EPJA 2021]

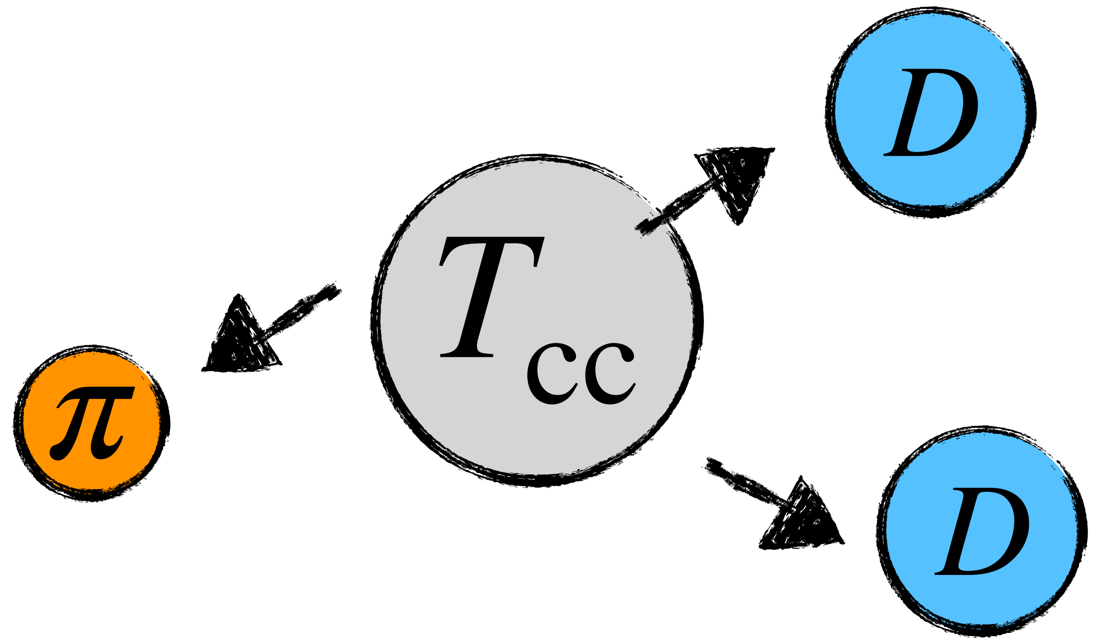
Fig. 1 Naming convention for the poles in the k-plane. The thick red line for positive real valued k marks the physical momenta in the scattering regime

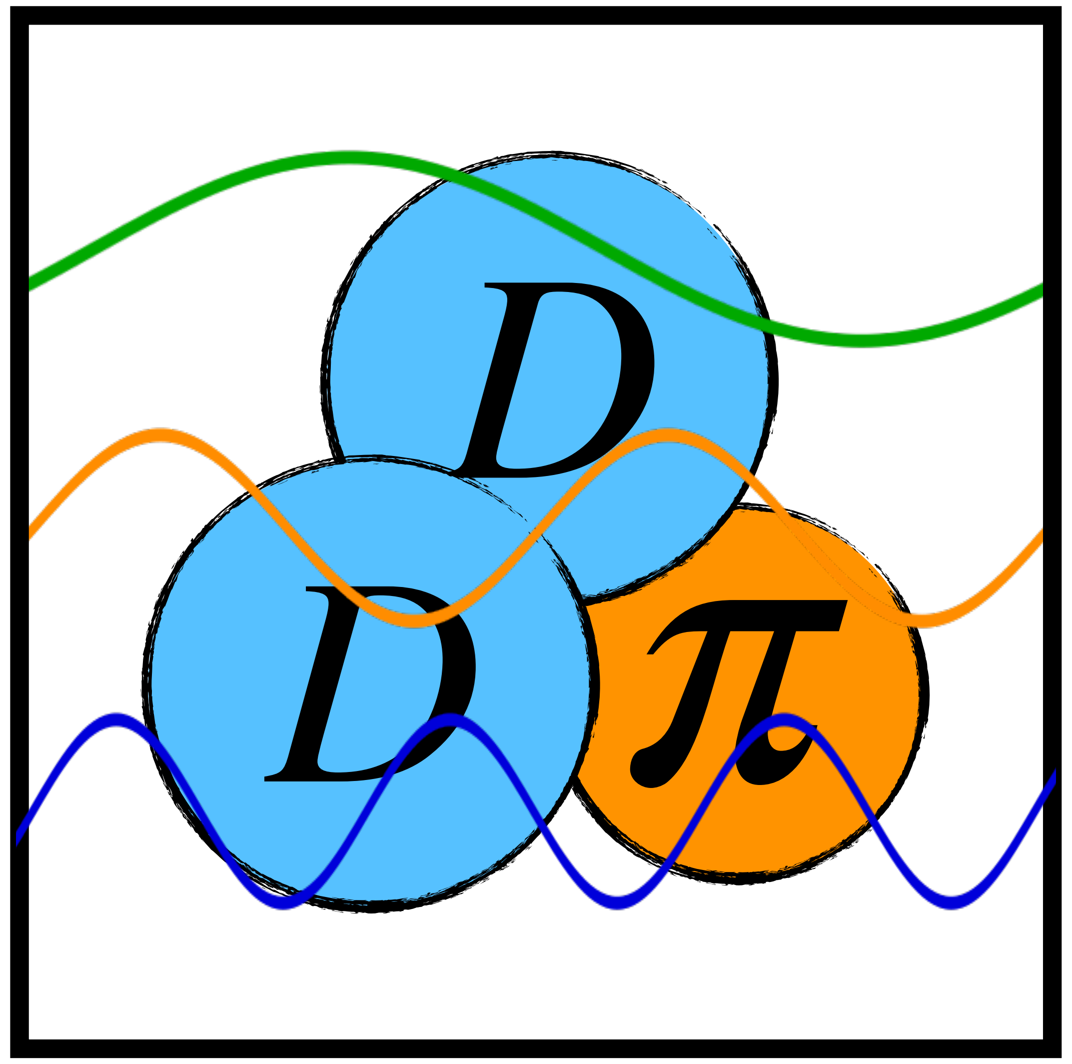
Hadronic resonances

Lattice QCD

- Euclidean time
- Stationary states in a box

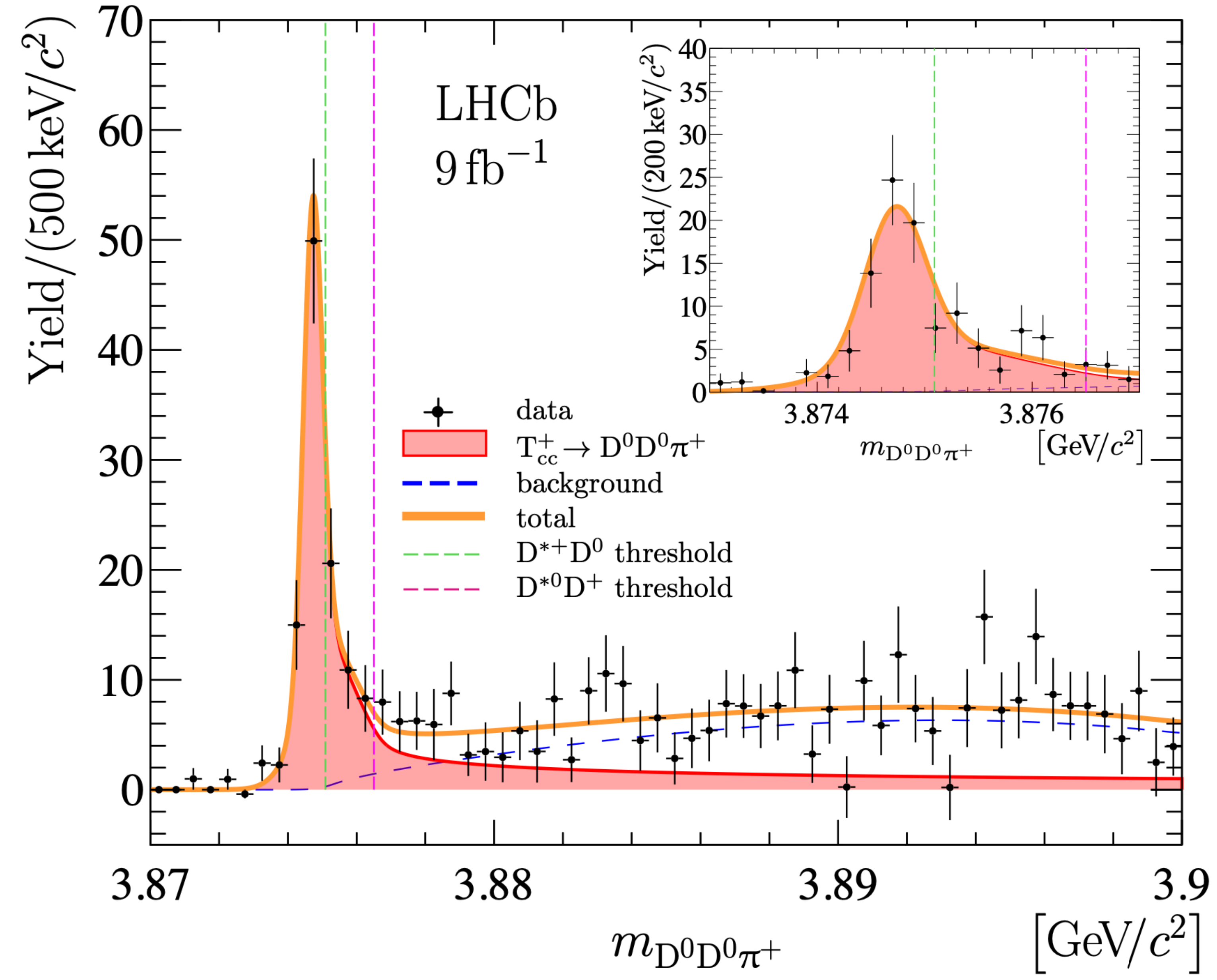
$$C(t) = \langle \mathcal{O}_{DD\pi}(t)\mathcal{O}^{\dagger}_{DD\pi}(0)\rangle = \sum_n \left| \langle 0 | \mathcal{O}_{DD\pi} | n \rangle \right|^2 e^{-E_n t}$$





Finite-volume formalism

[Lüscher 89']



Fernando Romero-López, Uni Bern

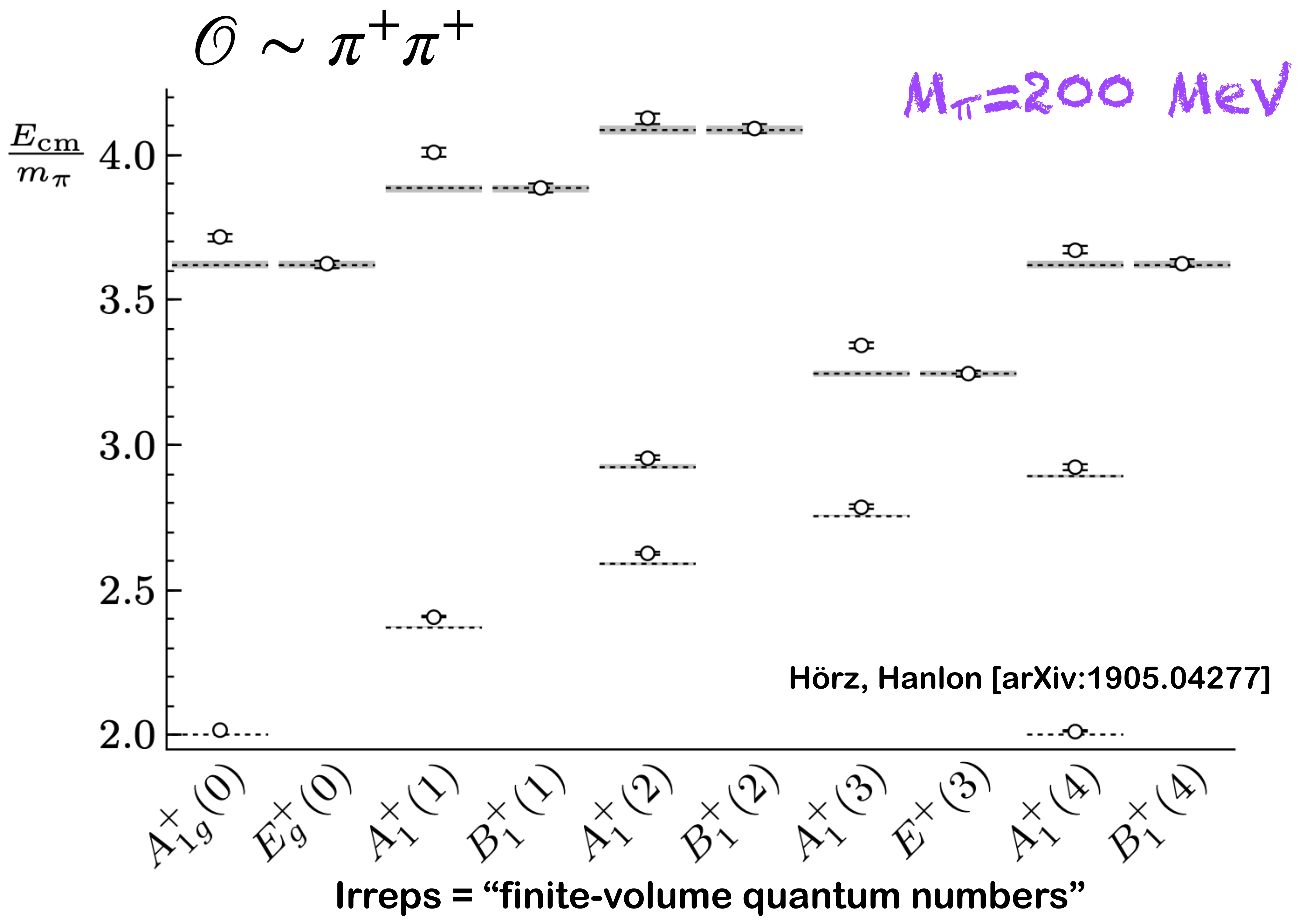
LQCD spectrum

- Compute matrix of Euclidean correlation functions using operators with the same quantum numbers

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$$

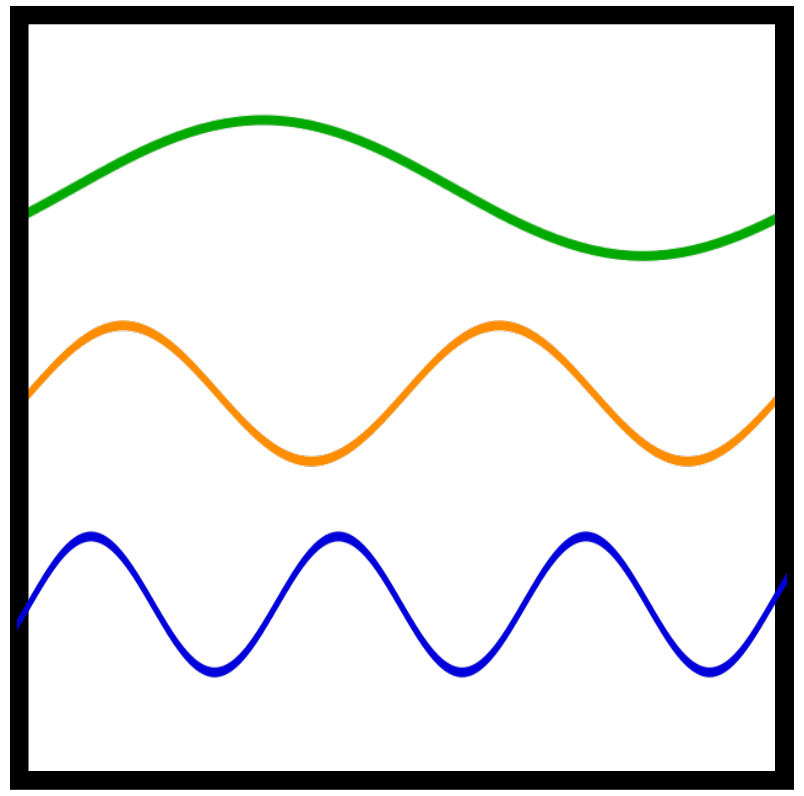
$$C_{ij}(t) = \langle 0|\mathcal{O}_i|n\rangle\langle n|\mathcal{O}_j|0\rangle^* e^{-E_n t}$$

- Variational techniques
  (Generalized EigenValue Problem, GEVP)
  - Extract (at most) as many levels as operators



Finite-volume spectrum

Free scalar particles in finite volume with periodic boundaries



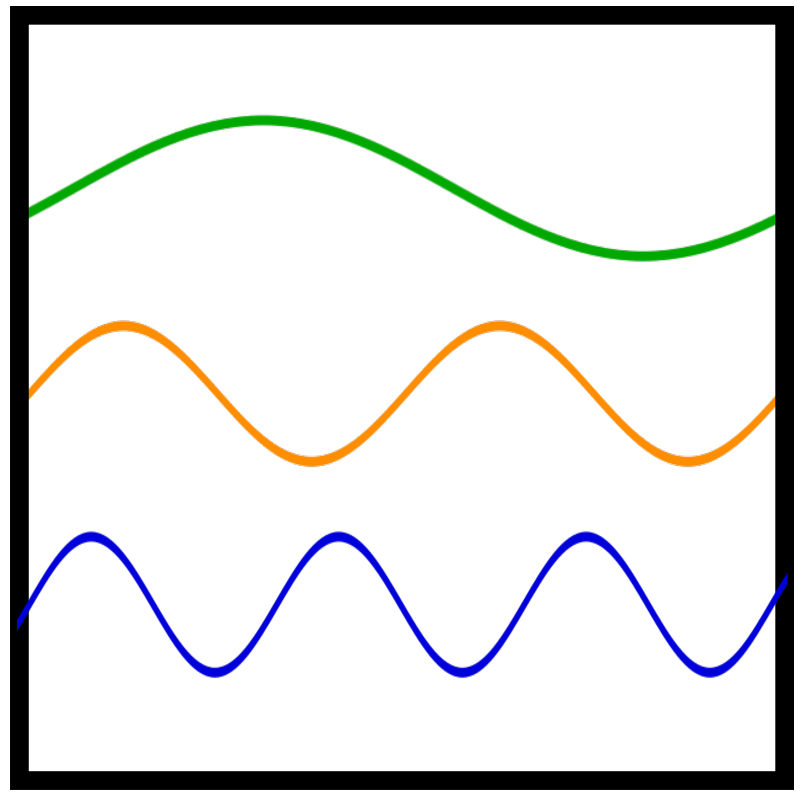
$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Two particles: $E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$

Fernando Romero-López, Uni Bern

Finite-volume spectrum

Free scalar particles in finite volume with periodic boundaries



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Two particles: $E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$

→

Interactions change the spectrum: it can be treated as a perturbation

Ground state to leading order

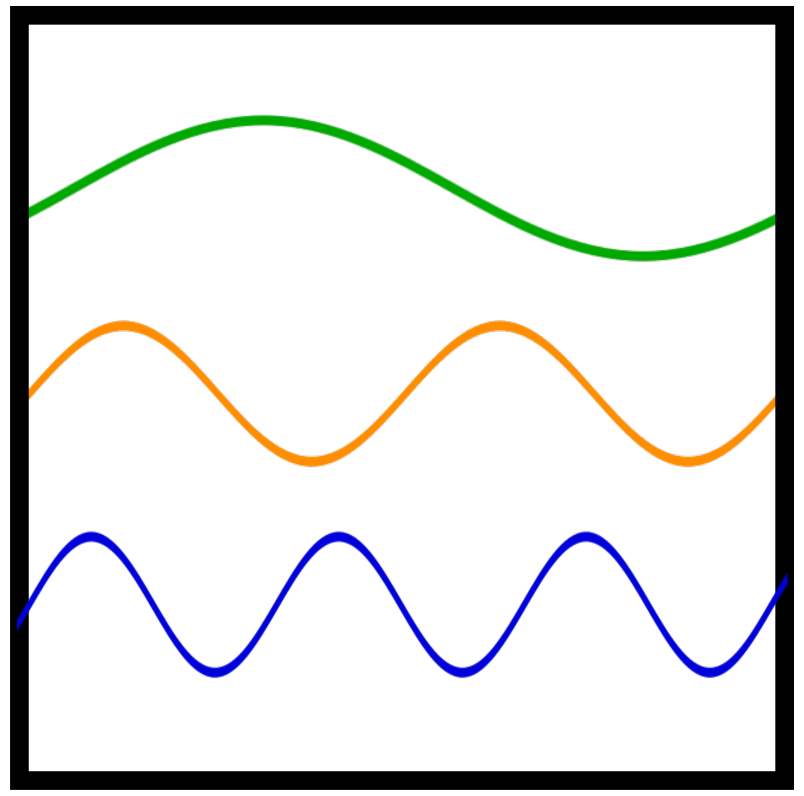
$$E_2 - 2m = \langle \phi(\vec{0})\phi(\vec{0}) \,|\, \mathbf{H_I} \,|\, \phi(\vec{0})\phi(\vec{0}) \rangle$$

$$\Delta E_2 = \frac{\mathcal{M}_2(E = 2m)}{8m^2L^3} + O(L^{-4})$$

[Huang, Yang, 1958]

Finite-volume spectrum

Free scalar particles in finite volume with periodic boundaries → Interactions change the spectrum: it can be treated as a perturbation



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Two particles: $E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$

Ground state to leading order

$$E_2 - 2m = \langle \phi(\vec{0})\phi(\vec{0}) \,|\, \mathbf{H_I} \,|\, \phi(\vec{0})\phi(\vec{0}) \rangle$$

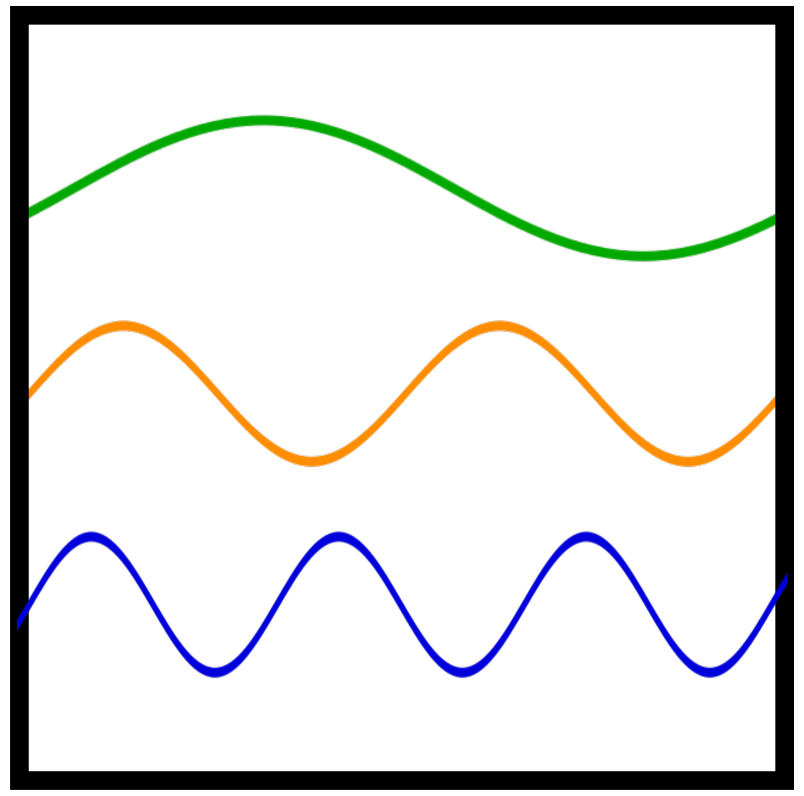
$$\Delta E_2 = \frac{\mathcal{M}_2(E = 2m)}{8m^2L^3} + O(L^{-4})$$

[Huang, Yang, 1958]

The energy shift of the two-particle ground state is related to the $2 \to 2$ scattering amplitude

Finite-volume spectrum

Free scalar particles in finite volume
with periodic boundaries



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Two particles: $E = 2\sqrt{m^2 + \frac{4\pi^2}{L^2}\vec{n}^2}$

Interactions change the spectrum:
it can be treated as a perturbation

In general a problem of
Quantum Field Theory
in finite volume

d state to leading order

$$2m = \langle \phi(\vec{0})\phi(\vec{0}) \,|\, \mathbf{H_I} \,|\, \phi(\vec{0})\phi(\vec{0}) \rangle$$

$$\Delta E_2 = \frac{\mathcal{M}_2(E = 2m)}{8m^2L^3} + O(L^{-4})$$

[Huang, Yang, 1958]

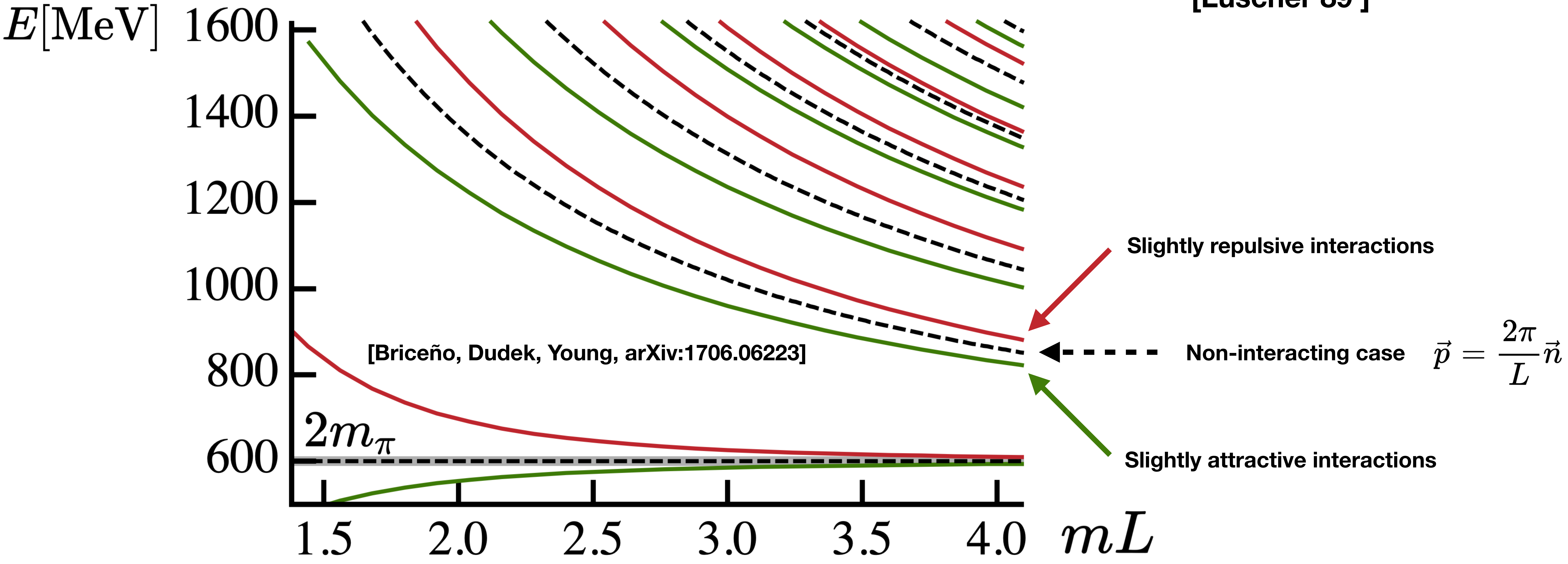
The **energy shift** of the two-particle ground state
is related to the $2 \to 2$ **scattering amplitude**

Fernando Romero-López, Uni Bern

Key insight

Volume dependence of finite-volume energy states contains scattering information

[Lüscher 89']



Finite-volume correlation

- Find finite-volume states by computing finite-volume correlation function

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle = \sum_n \frac{c_n}{P^2 - E_n^2}$$

Finite-volume states as poles
In finite-volume correlation

Spectrum in FV is discrete

Finite-volume correlation

- Find finite-volume states by computing finite-volume correlation function

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle = \sum_n \frac{c_n}{P^2 - E_n^2}$$

Finite-volume states as poles In finite-volume correlation

Spectrum in FV is discrete

Strategy:

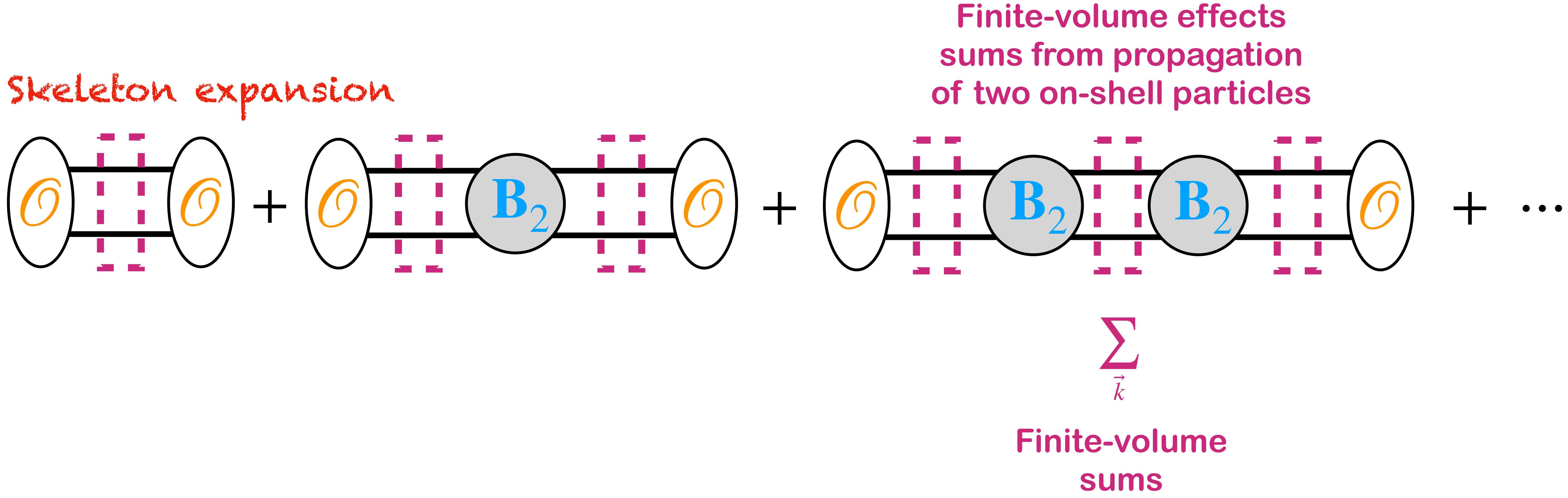
Note: $E < E_{\text{inelastic}}$

- Compute the FV correlation to all orders in a generic EFT
- Keep only power-like FV effects $\left[\sum_{\vec{k}} - \int d^3k \right] f(\vec{k})$
  - $f(\vec{k})$ is regular: $e^{-m_\pi L}$
  - $f(\vec{k})$ with poles: $1/L^n$
- Find location of poles in the finite-volume correlator

Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle =$$

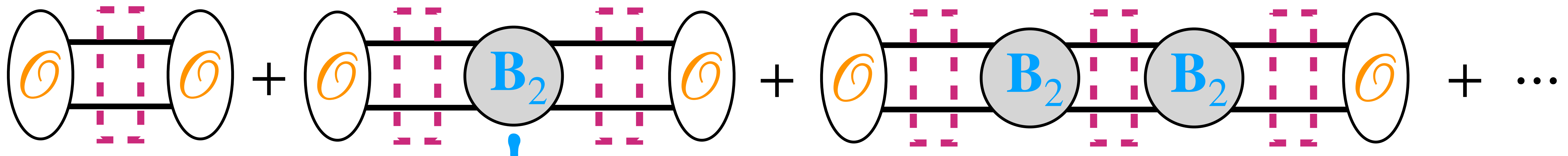


[à la Kim, Sachrajda, Sharpe]

Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle =$$



Skeleton expansion

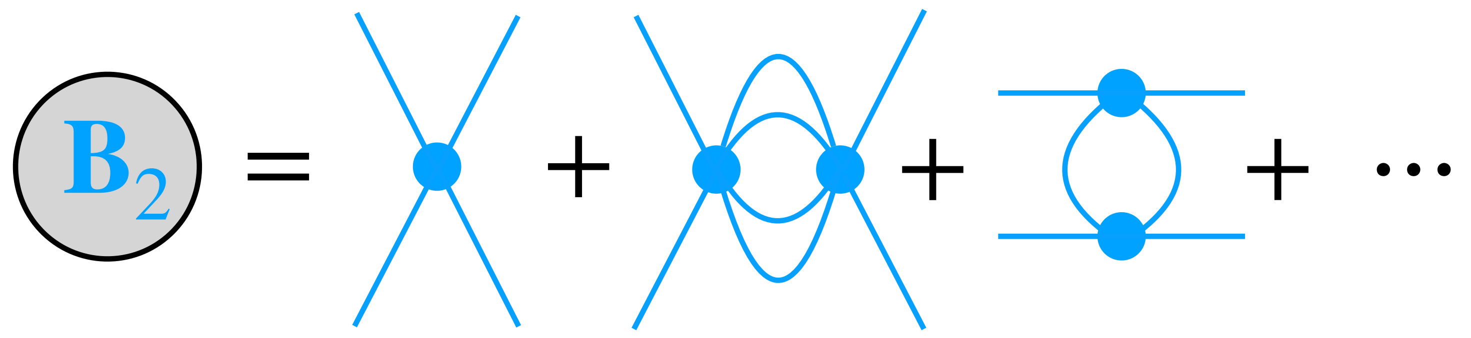
Finite-volume effects sums from propagation of two on-shell particles

[à la Kim, Sachrajda, Sharpe]

Bethe-Salpeter Kernels

$\sum_{\vec{k}}$

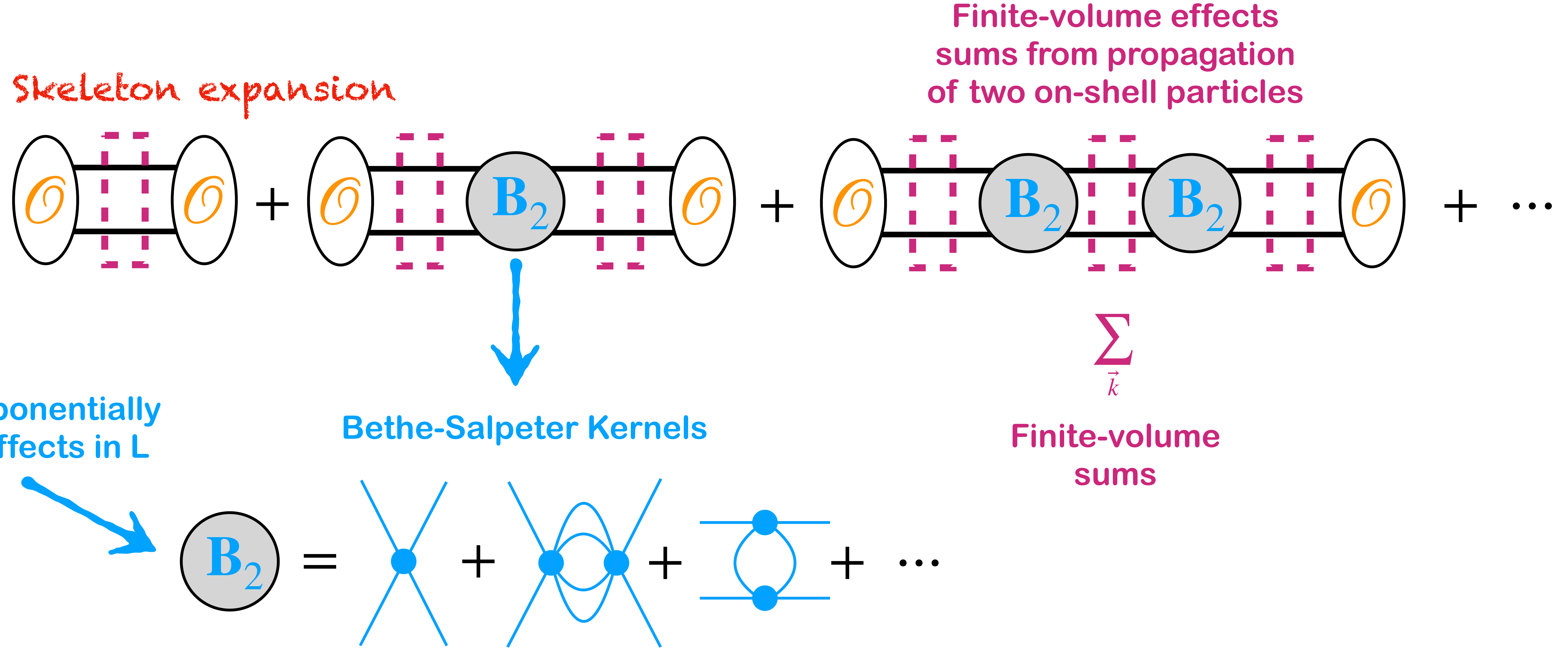
Finite-volume sums



Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle =$$



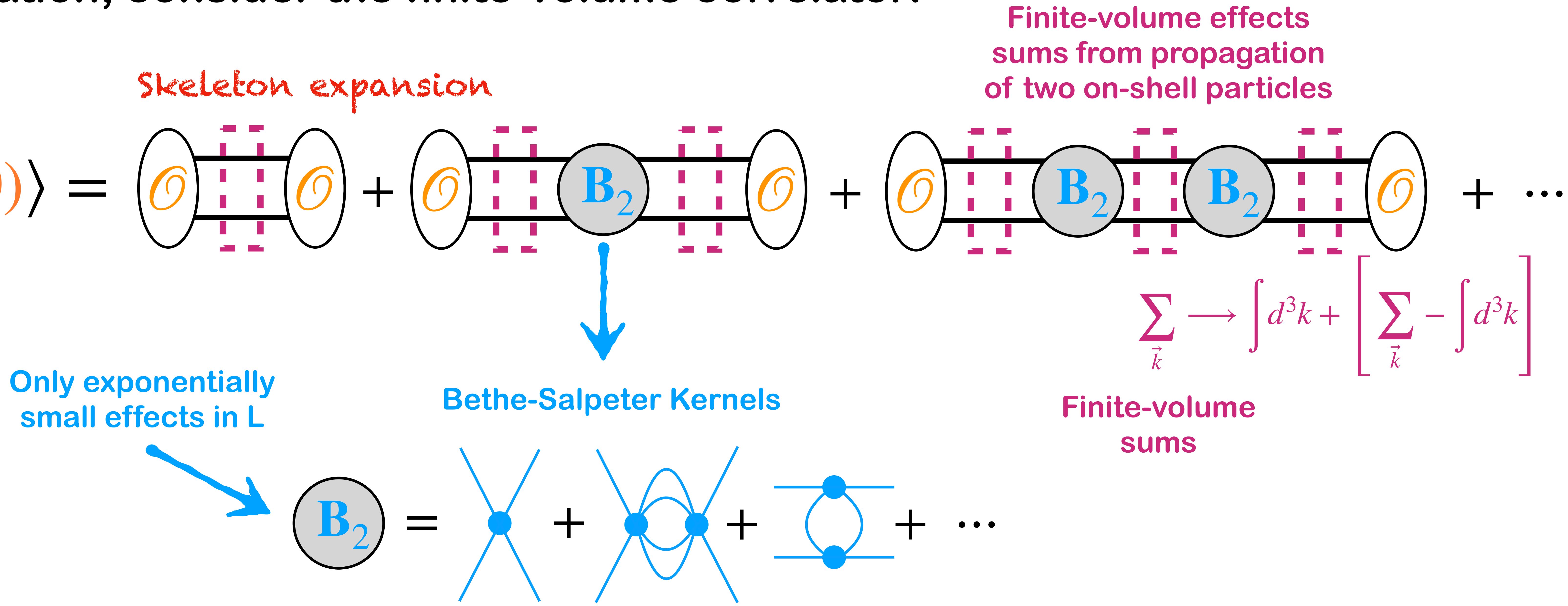
[à la Kim, Sachrajda, Sharpe]

Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle =$$

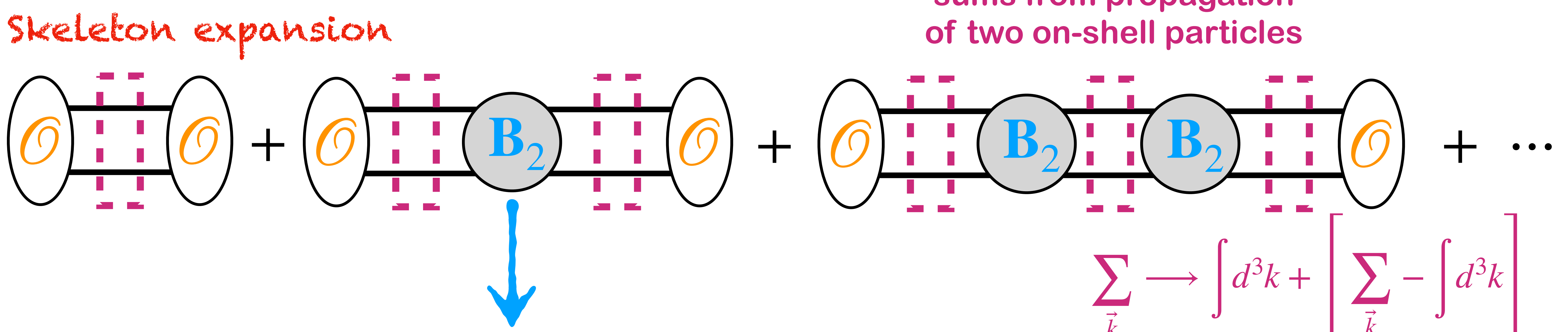
[à la Kim, Sachrajda, Sharpe]



Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E, \vec{P}) = \int_L e^{iPx} \langle \mathcal{O}(x) | \mathcal{O}(0) \rangle =$$

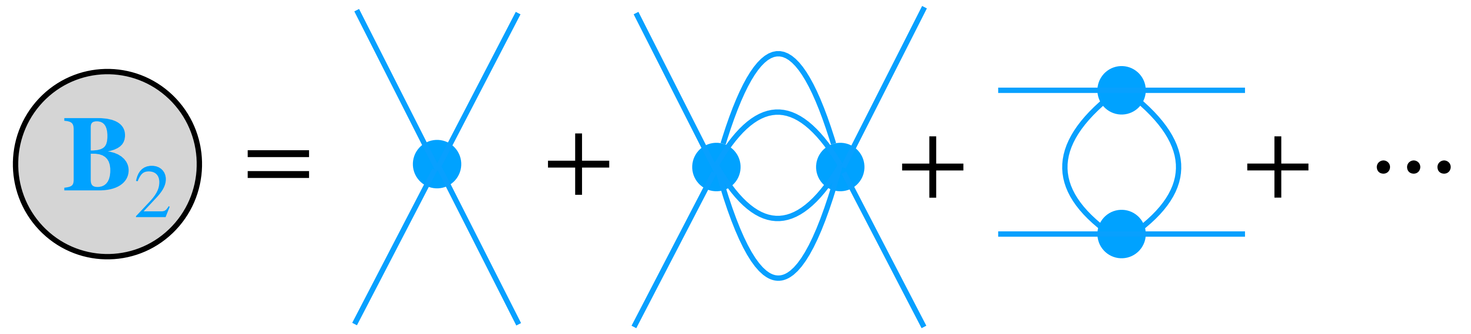


[à la Kim, Sachrajda, Sharpe]

$$\sum_{\vec{k}} \longrightarrow \int d^3k + \left[\sum_{\vec{k}} - \int d^3k \right]$$

Finite-volume sums

Only exponentially small effects in L

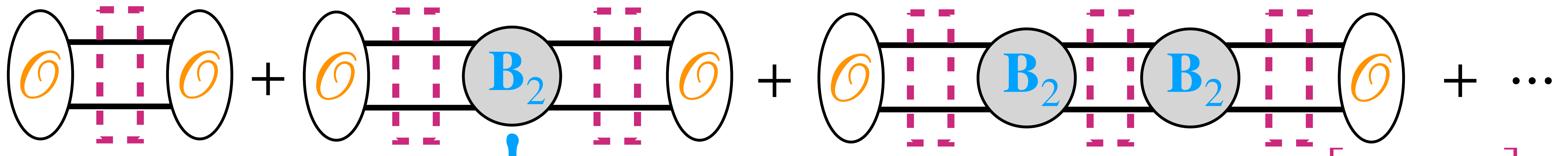


1. Separation of finite-volume effects
2. Resumation of diagrams

Finite-volume correlation

- In order to derive the full relation, consider the finite-volume correlator:

$$C_L(E,\vec{P}) = \int_L e^{iPx}\langle \mathcal{O}(x)\,|\,\mathcal{O}(0)\rangle =$$



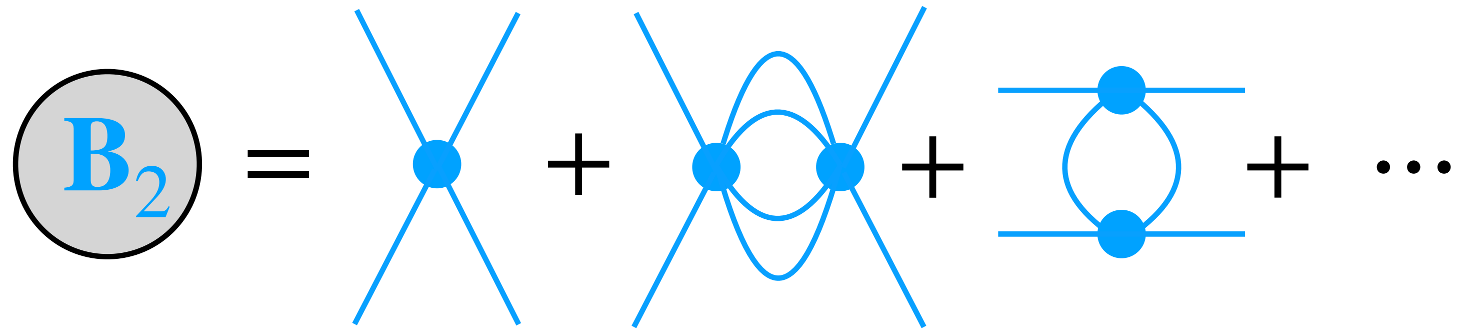
[à la Kim, Sachrajda, Sharpe]

$$\sum_{\vec{k}} \longrightarrow \int d^3k + \left[\sum_{\vec{k}} - \int d^3k\right]$$

Finite-volume sums

Bethe-Salpeter Kernels

Only exponentially small effects in L



Known kinematic function

1. Separation of finite-volume effects
2. Resumation of diagrams

$$C_L(E,\vec{P}) = C_\infty(E,\vec{P}) + A^\dagger \frac{1}{\mathcal{K}_2 + F^{-1}} A + O(e^{-mL})$$

$$\mathcal{M}_2^{-1} = \mathcal{K}_2^{-1} - i\sqrt{s-2m^2}$$

Fernando Romero-López, Uni Bern

Quantization Condition

$$C_L(E, \overrightarrow{P}) = \quad \text{some algebra ...} \quad = C_\infty(E, \overrightarrow{P}) + A^\dagger \frac{1}{\mathcal{K}_2 + F^{-1}} A + O(e^{-mL})$$

Quantization Condition

$$C_L(E,\vec{P}) = \quad \text{some algebra ...} \quad = C_\infty(E,\vec{P}) + A^\dagger \frac{1}{\mathcal{K}_2 + F^{-1}} A + O(e^{-mL})$$

K-matrix parametrized in terms of phase shift

$$\mathcal{K}_2^\ell = \frac{16\pi\sqrt{s}}{q^{2\ell+1}\cot\delta_\ell}$$

Two-particle Quantization Condition

$$\det_{\ell m}\left[\mathcal{K}_2(E_n) + F^{-1}(E_n,\vec{P},L)\right] = 0$$

Scattering K-Matrix

Known kinematic function

Finite-volume information

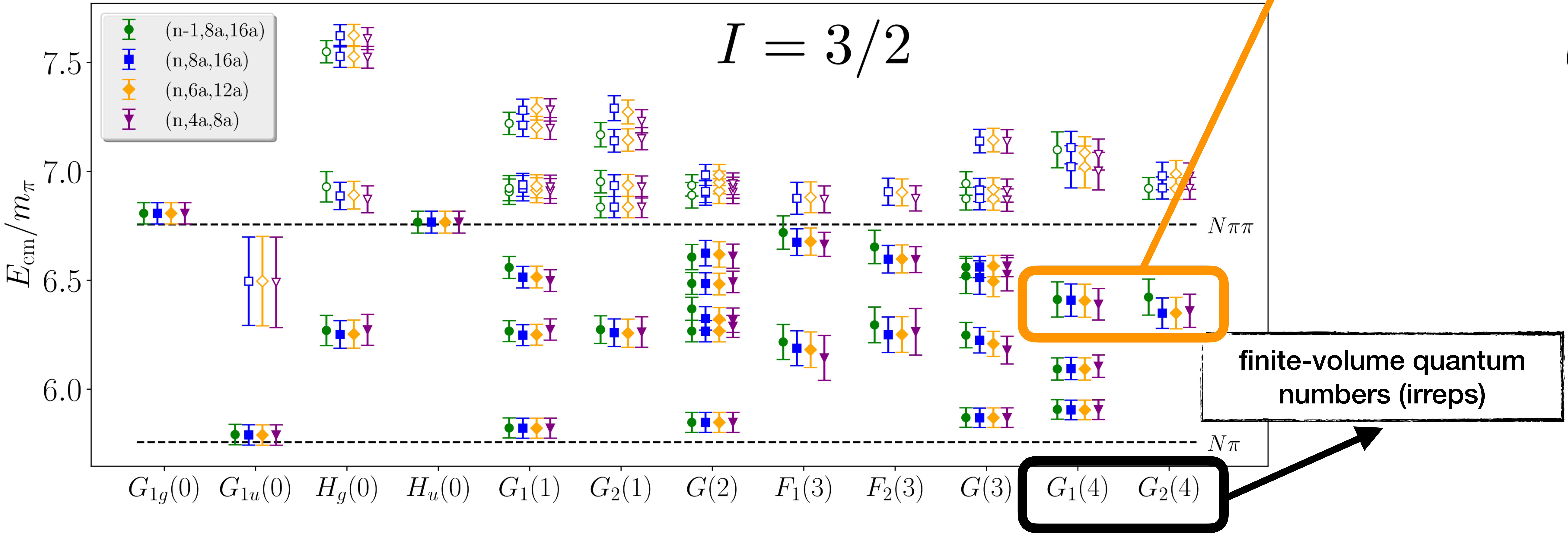
$$F_{00}(q^2) \sim \left[\frac{1}{L^3}\sum_{\vec{k}} - \int\frac{d^3k}{(2\pi)^3}\right]\frac{1}{k^2-q^2}$$

! **Note: only valid for two particles below inelastic thresholds.**

Example: πN scattering

- Key ingredient: reliable variational extractions of the lattice QCD energy levels: GEVP + stability

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]

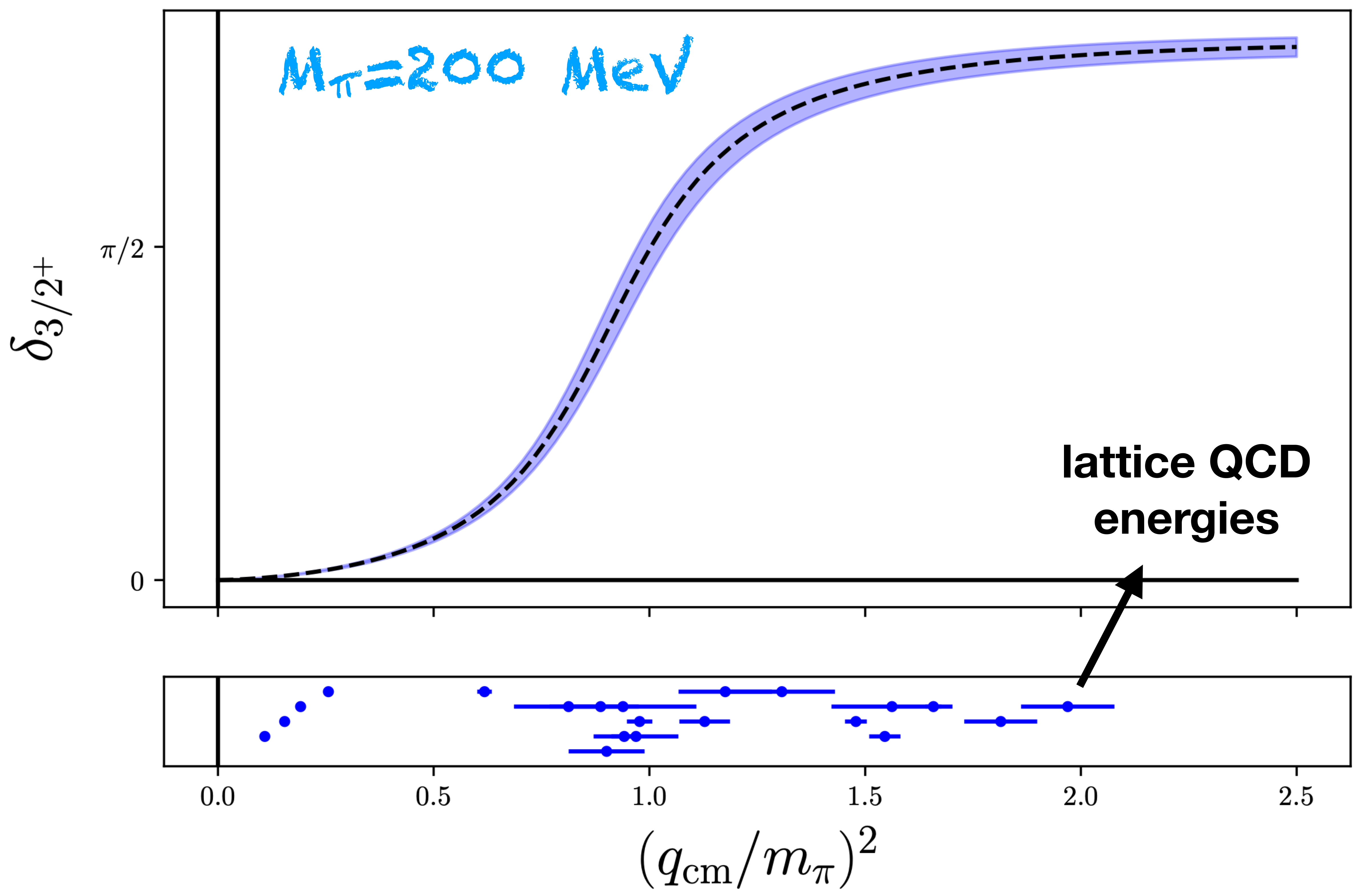


Fernando Romero-López, Uni Bern

The Δ(1232) from LQCD

Lattice QCD

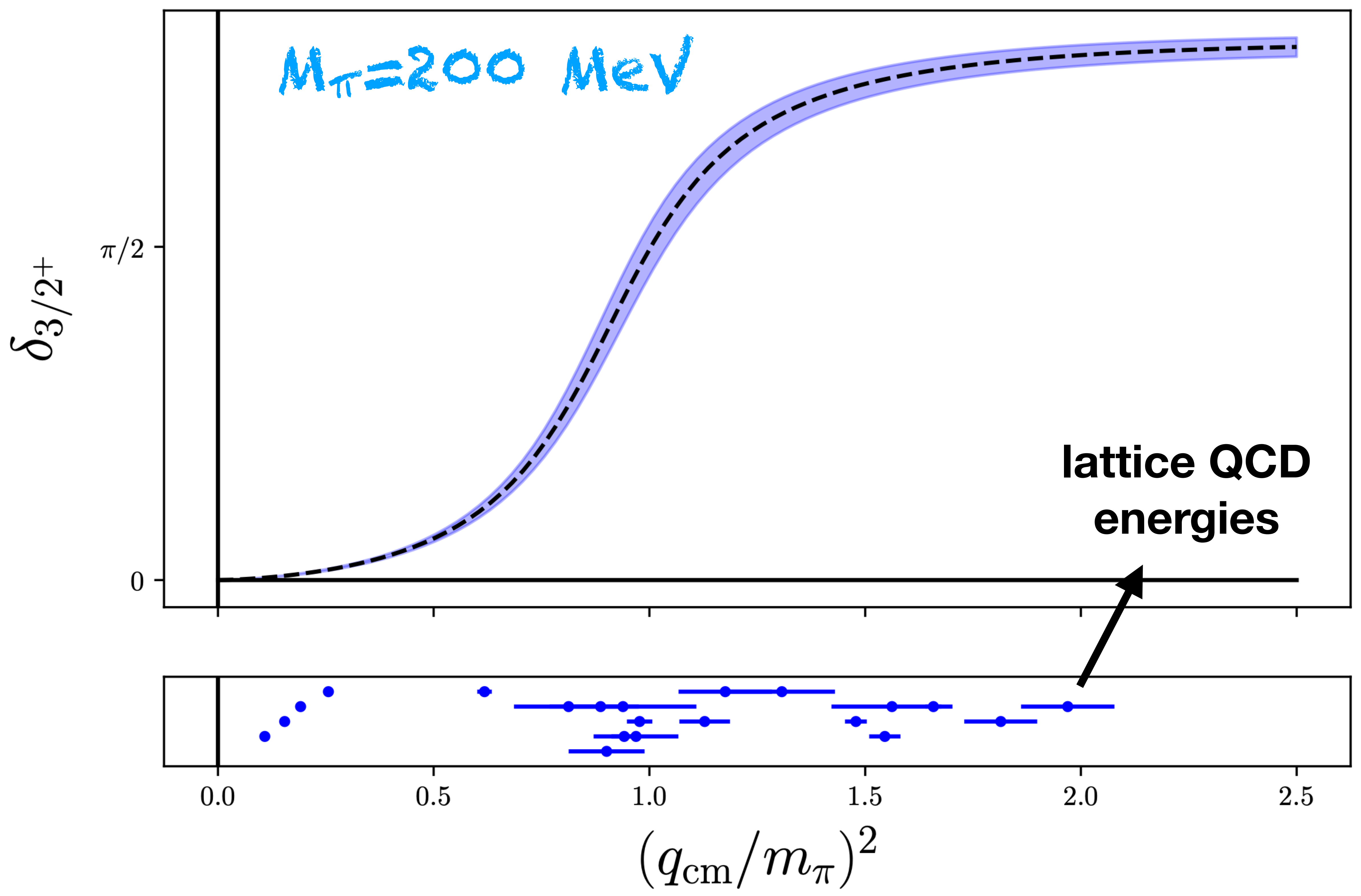
[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]



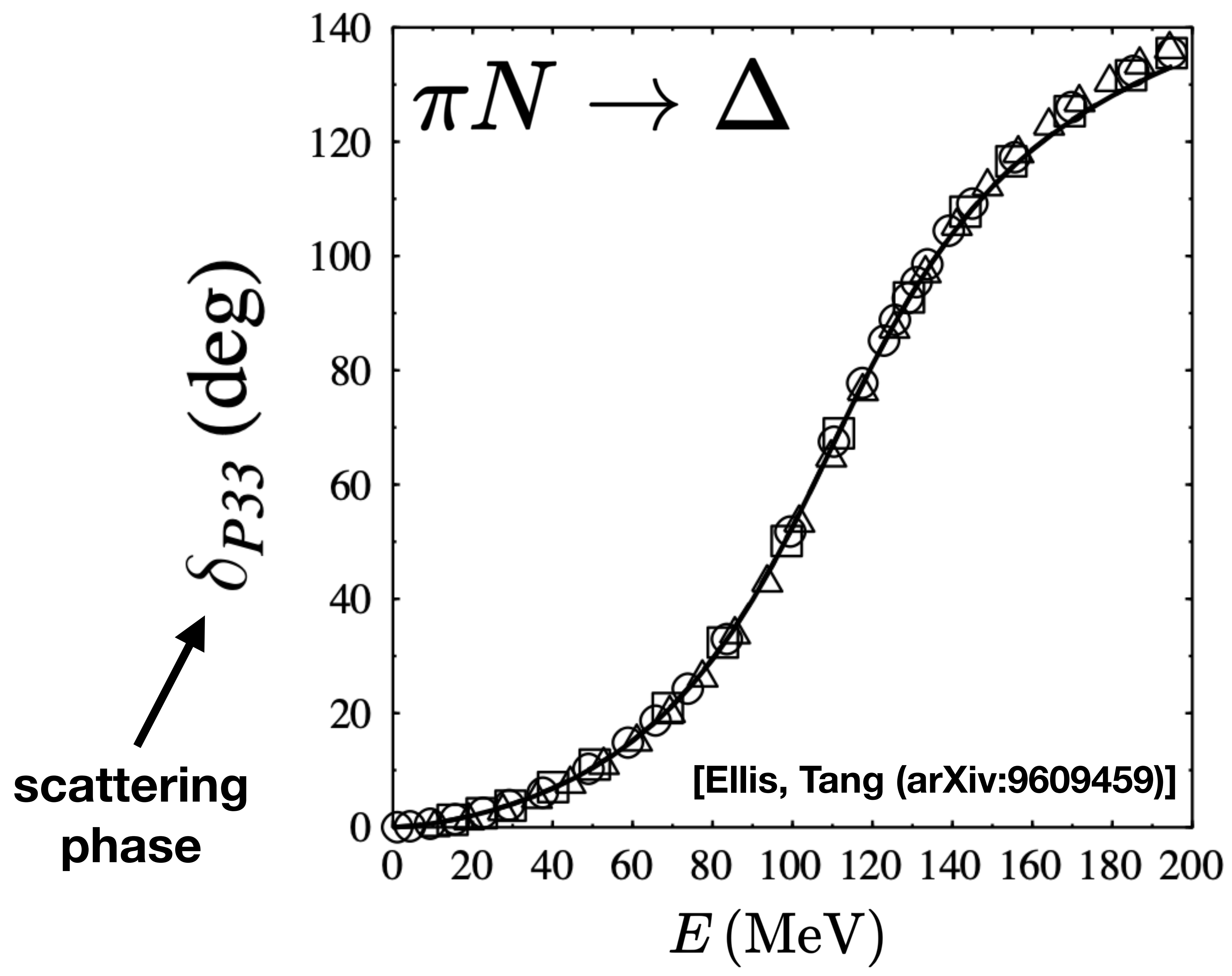
The Δ(1232) from LQCD

Lattice QCD

[Bulava, Hanlon, Hörz, Morningstar, Nicholson, FRL, Skinner, Vranas, Walker-Loud, 2208.03867]



Experiment



Fernando Romero-López, Uni Bern

The three-hadron frontier

- The two-body formalism is restricted to few interesting resonances
  - **Exotics:** $T_{cc} \to DD^*, DD\pi$
  - **Roper:** $N(1440) \to \Delta\pi \to N\pi\pi$
- Many-body nuclear physics: 3N force, tritium nucleus
- CP violation: $K \to 3\pi, \quad K^0 \leftrightarrow 3\pi \leftrightarrow \overline{K}^0$
- ☑ Major developments in the three-particle finite-volume formalism

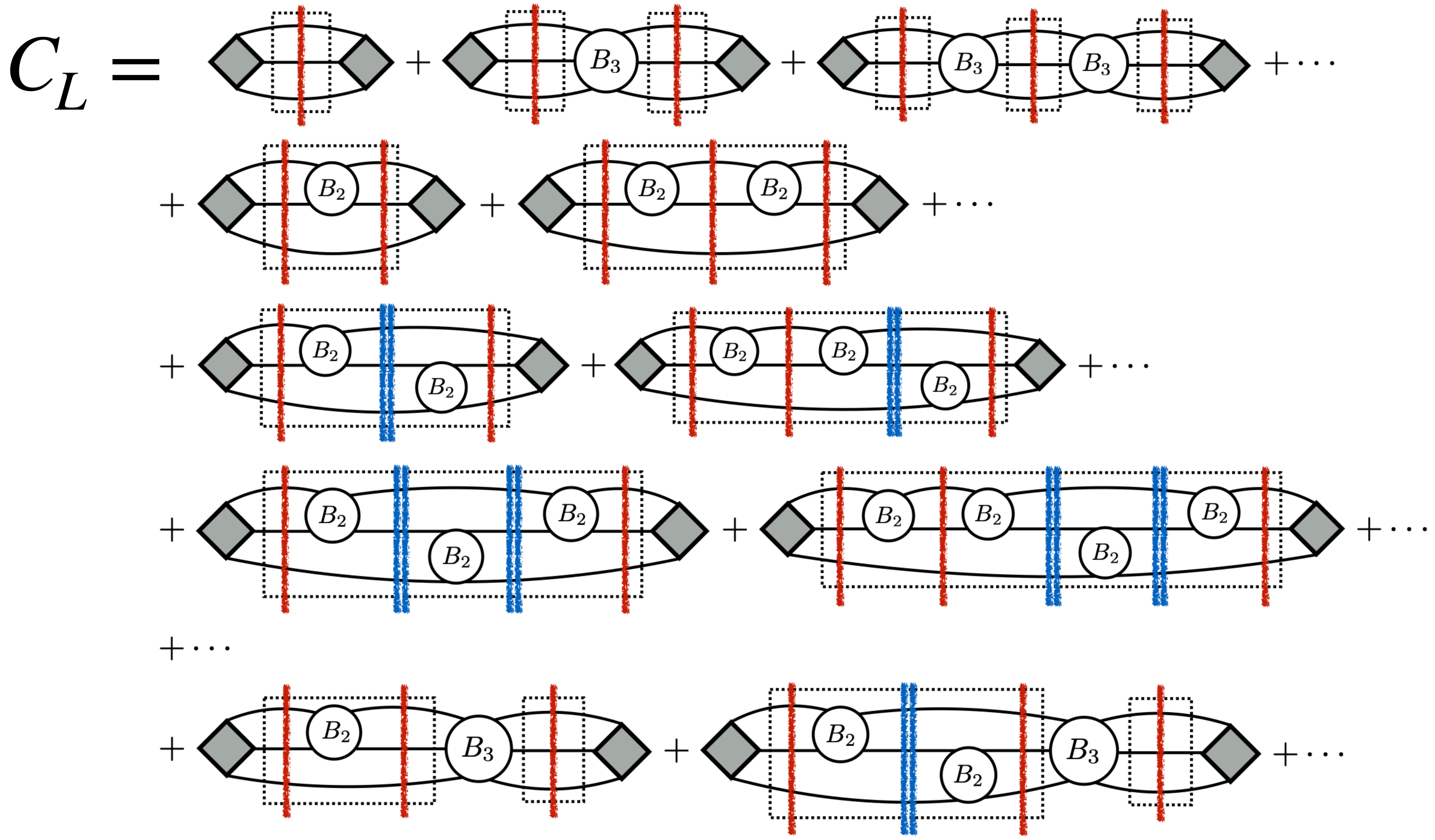
[Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
[Mai, Döring, EPJA 2017]
[...]
[Blanton, FRL, Sharpe, JHEP 2019], [Hansen, FRL, Sharpe, JHEP 2020]
[Hansen, FRL, Sharpe, JHEP 2021], [Blanton, FRL, Sharpe, JHEP 2022]
[Hansen, FRL, Sharpe, JHEP 2023]

Resonance	$I_{\pi\pi\pi}$	J^P
$\omega(782)$	0	1^-
$h_1(1170)$	0	1^+
$\omega_3(1670)$	0	3^-
$\pi(1300)$	1	0^-
$a_1(1260)$	1	1^+
$\pi_1(1400)$	1	1^-
$\pi_2(1670)$	1	2^-
$a_2(1320)$	1	2^+
$a_4(1970)$	1	4^+

(with $\geq 3\pi$ decay modes)

Quantization Condition

Skeleton expansion **[Hansen, Sharpe, PRD 2014 & 2015]**

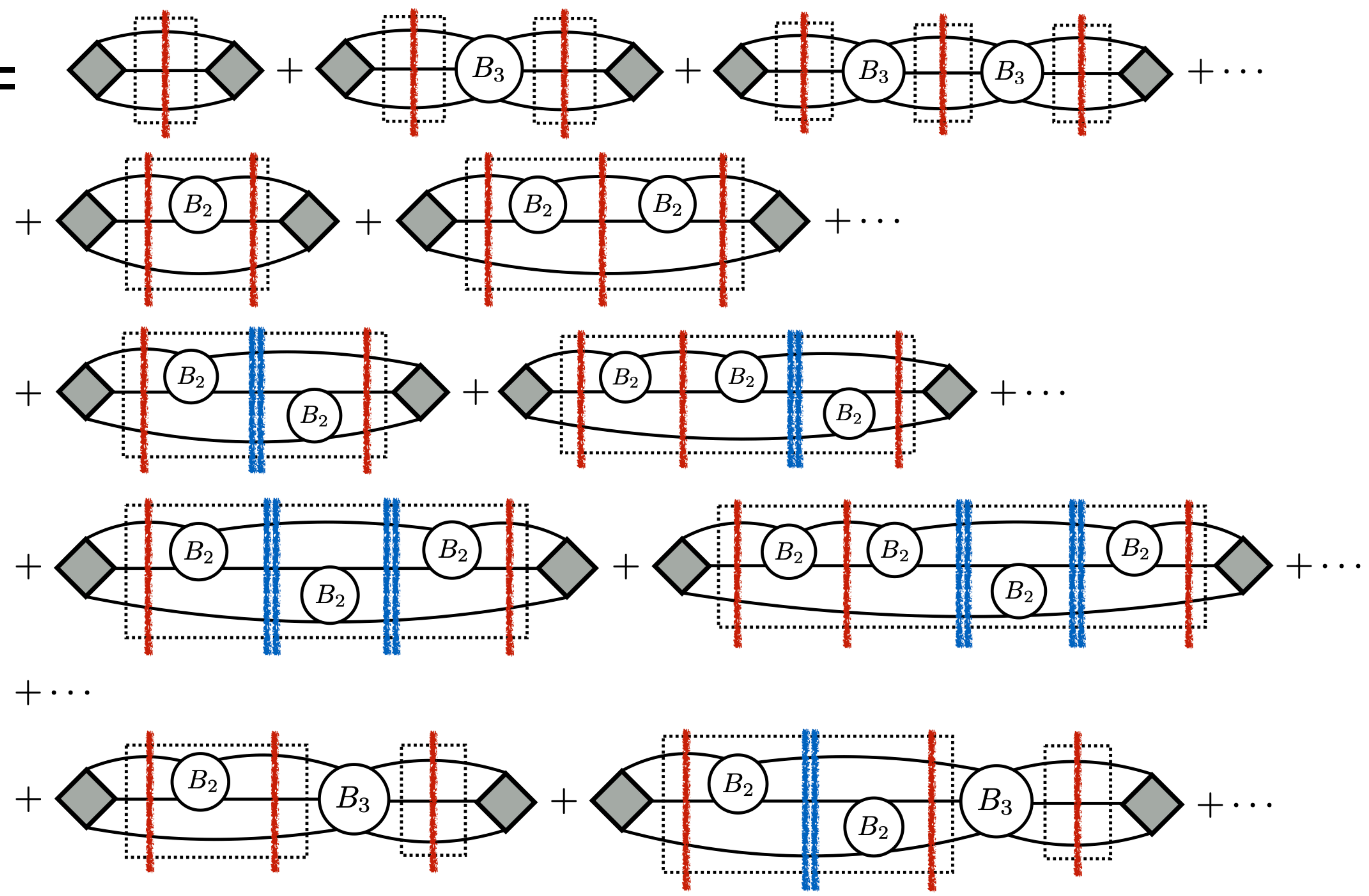


Fernando Romero-López, Uni Bern

Quantization Condition

Skeleton expansion **[Hansen, Sharpe, PRD 2014 & 2015]**

$C_L =$



Separation of finite and infinite volume terms:

$$= C_\infty(P) + A_3 \frac{1}{\mathcal{K}_{df,3} + F_3^{-1}} A'_3 + O(e^{-mL})$$

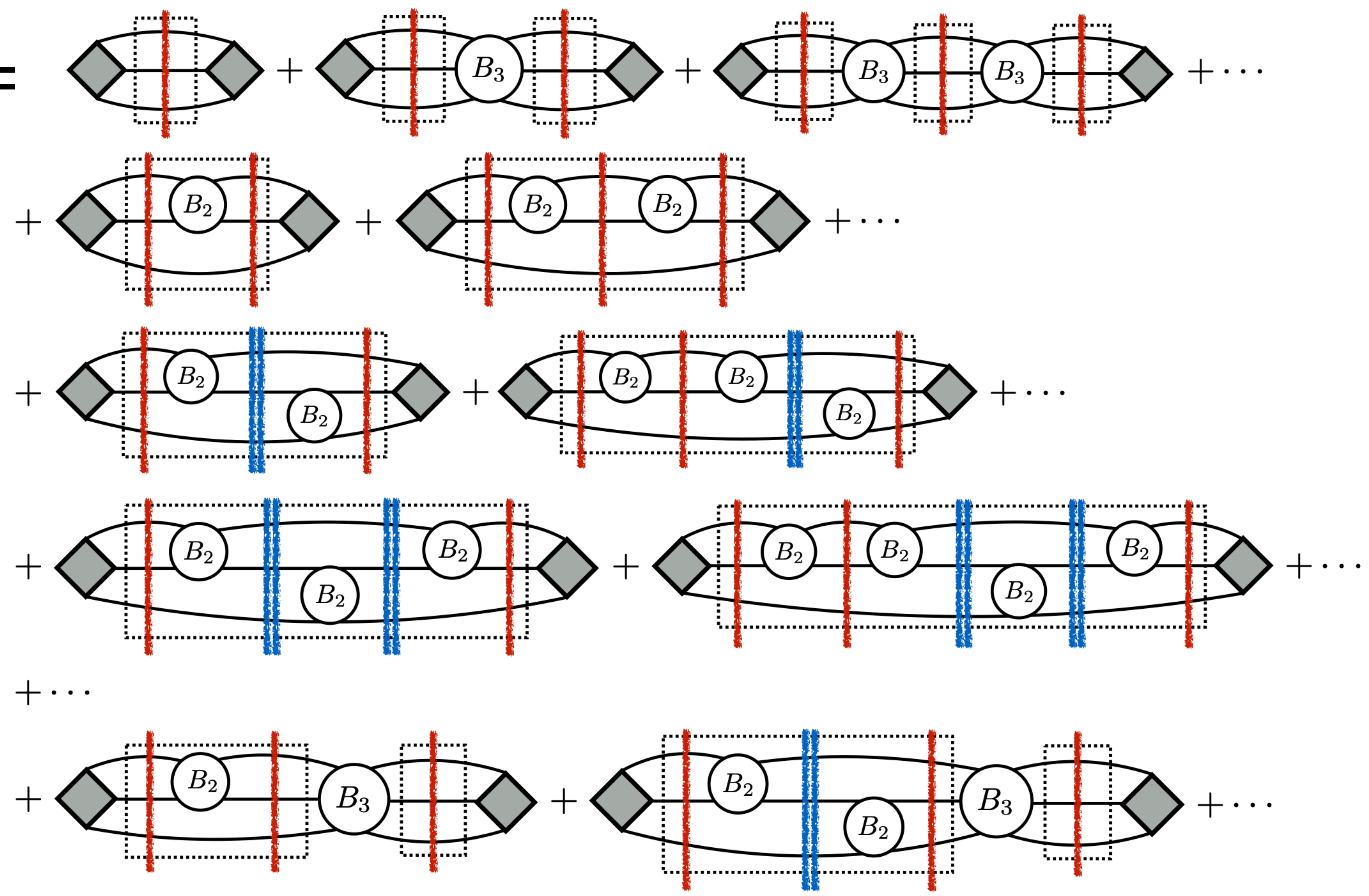
Easier derivation: Blanton, Sharpe [2007.16188]

Fernando Romero-López, Uni Bern

Quantization Condition

Skeleton expansion **[Hansen, Sharpe, PRD 2014 & 2015]**

$C_L =$



Easier derivation: Blanton, Sharpe [2007.16188]

Separation of finite and infinite volume terms:

$$= C_\infty(P) + A_3 \frac{1}{\mathcal{K}_{df,3} + F_3^{-1}} A_3' + O(e^{-mL})$$

Three-particle Quantization Condition for identical scalars with G-parity

$$\det\left[\mathcal{K}_3(E) + F_3^{-1}(E, \vec{P}, L)\right] = 0$$

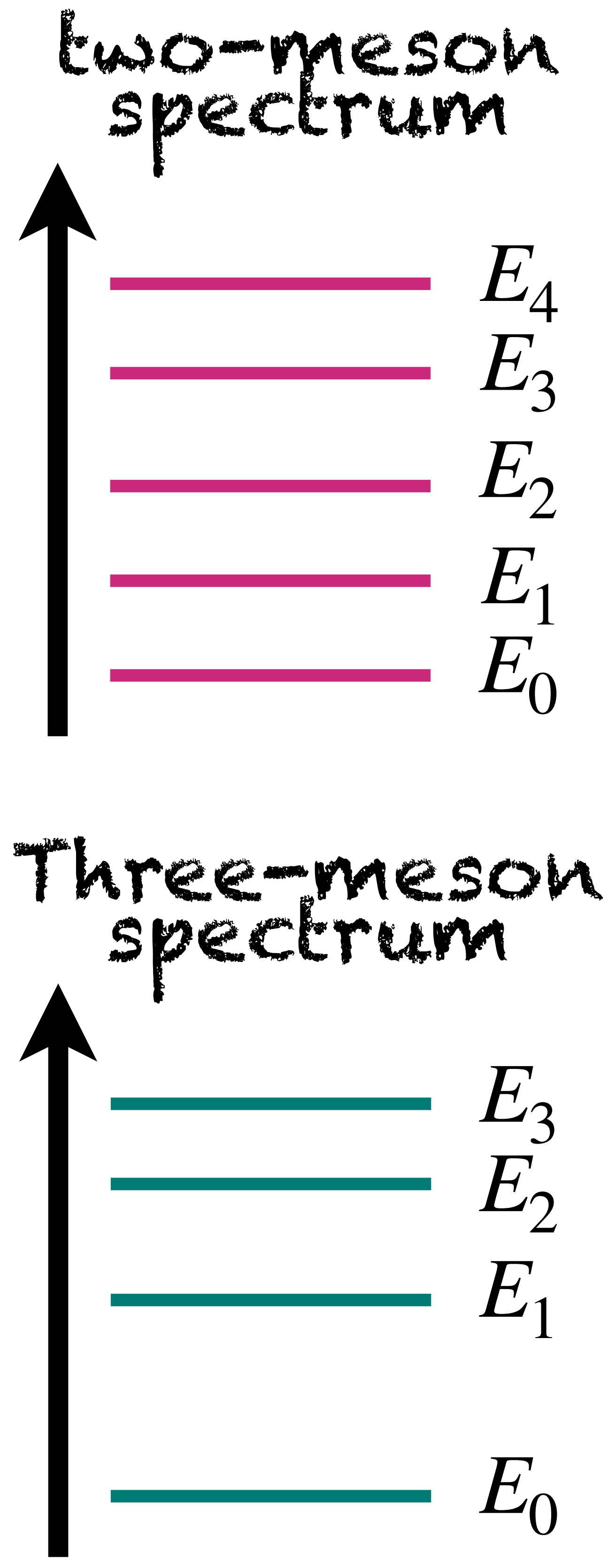
"QC3"

"Formally" similar to the two-particle case but several new features

Fernando Romero-López, Uni Bern

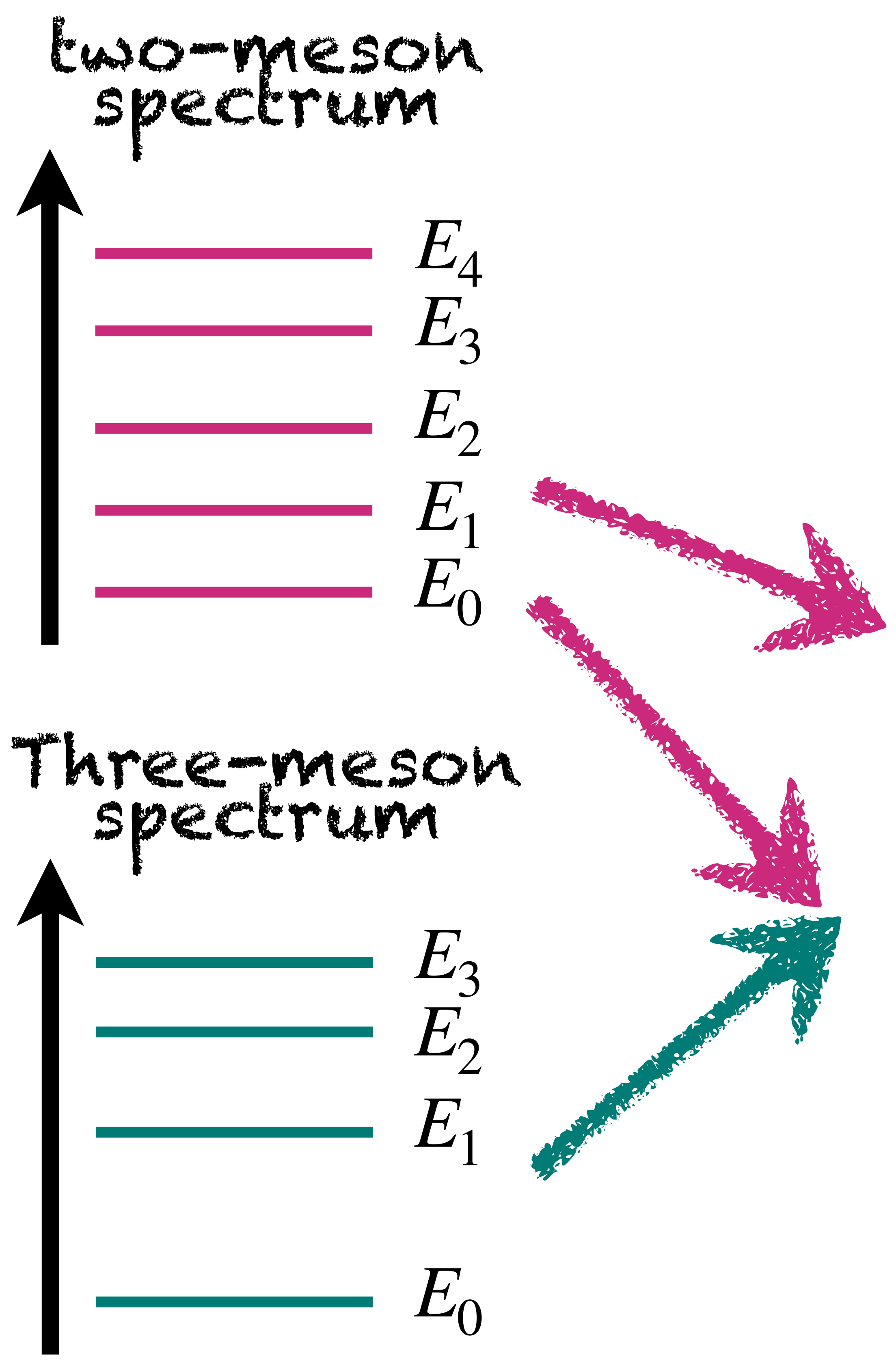
Formalism

[Hansen, Sharpe, PRD 2014 & 2015]



Formalism

[Hansen, Sharpe, PRD 2014 & 2015]



Quantization conditions

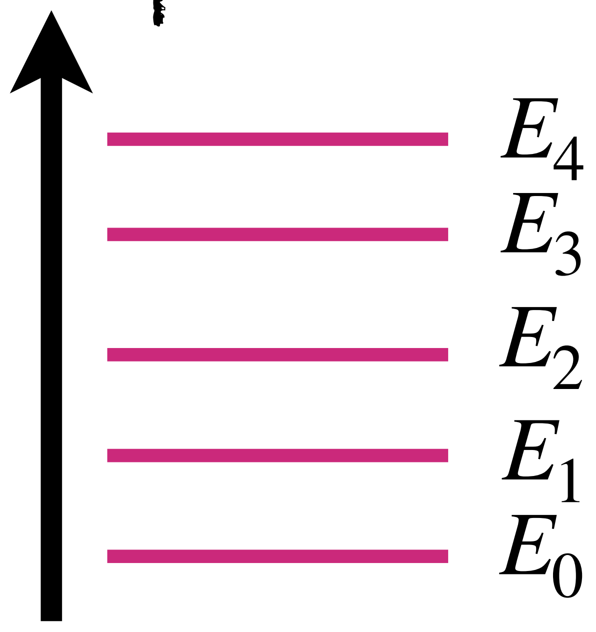
$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

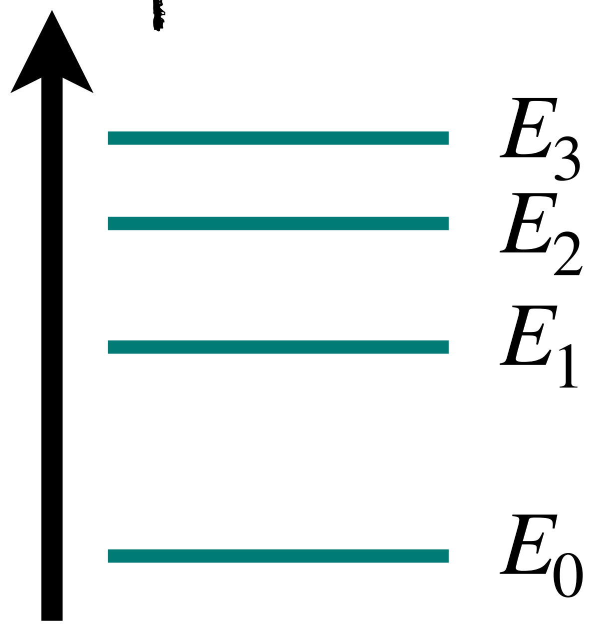
Formalism

[Hansen, Sharpe, PRD 2014 & 2015]

two-meson spectrum



Three-meson spectrum



Quantization conditions

$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

Fit

K-matrices

$$\mathcal{K}_2$$

$$\mathcal{K}_{\mathrm{df},3}$$

Parametrize:

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

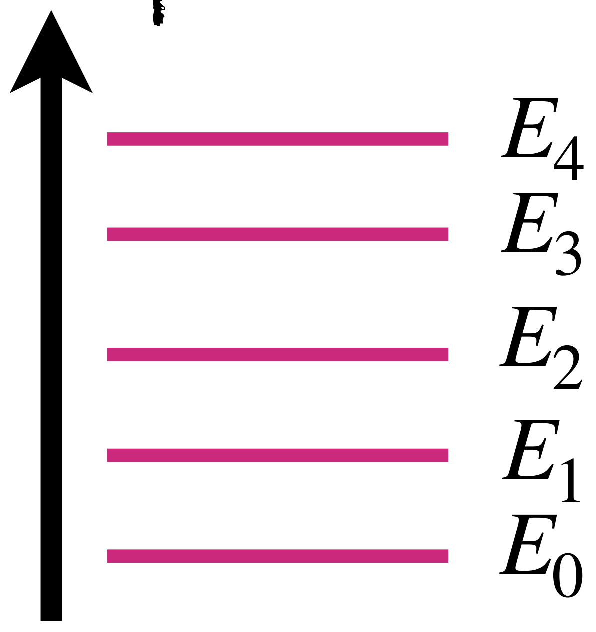
$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} + \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1}\left(\frac{s-9m^2}{9m^2}\right) + \ldots$$

[Blanton, FRL, Sharpe, JHEP 2019]

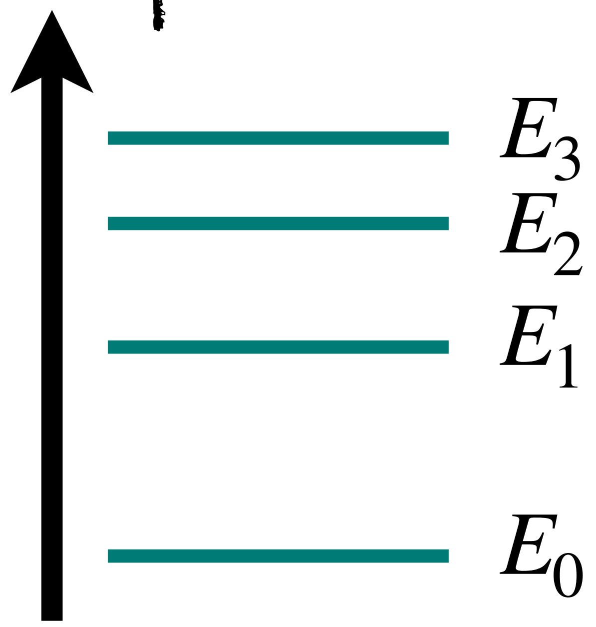
Formalism

[Hansen, Sharpe, PRD 2014 & 2015]

two-meson spectrum



Three-meson spectrum



Quantization conditions

$$\det_{\ell m}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

$$\det_{k\ell m}\left[\mathcal{K}_{\mathrm{df},3} + F_3^{-1}\right] = 0$$

Fit →

K-matrices

$$\mathcal{K}_2$$

$$\mathcal{K}_{\mathrm{df},3}$$

Parametrize:

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \ldots$$

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},0} + \mathcal{K}_{\mathrm{df},3}^{\mathrm{iso},1}\left(\frac{s-9m^2}{9m^2}\right) + \ldots$$

[Blanton, FRL, Sharpe, JHEP 2019]

Unitarity relations →

Integral equations

[Briceño et al., PRD 2018]
[Hansen et al., PRL 2021]
[Jackura et al., PRD 2021]
[Dawid et al., 2303.04394]
[See talk by S. Dawid @ LAT24]

Scattering amplitudes

$$\mathcal{M}_2$$

$$\mathcal{M}_3$$

3π+ K matrix

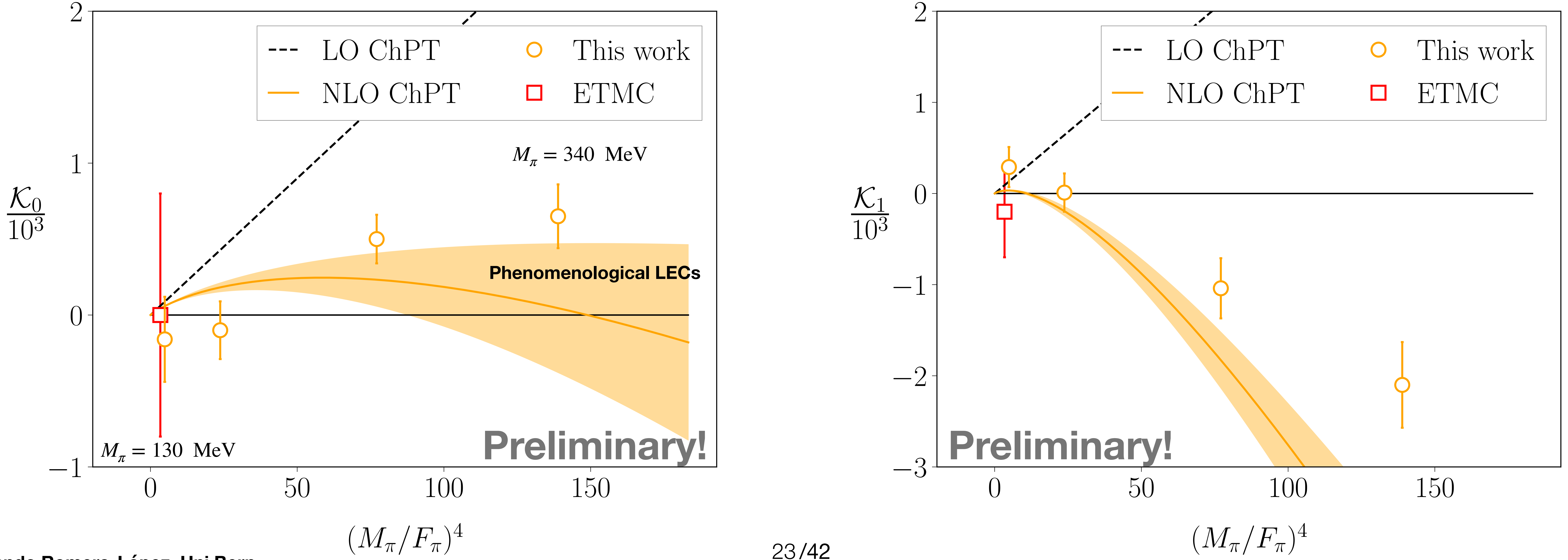
- **Compare to chiral perturbation theory**

NLO ChPT: [Baeza-Ballesteros, Bijnens, Husek, FRL, Sharpe, Sjö, JHEP 2023 [Talk by M. Sjö @ LAT24]

ETMC: [Fischer, Kostrzewa, Liu, FRL, Ueding, Urbach, EPJC 2021]

This work: [Dawid, Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, Skinner, JHEP 2023 + on-going work]

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \left(\frac{s - 9M_\pi^2}{9M_\pi^2} \right) + \cdots$$

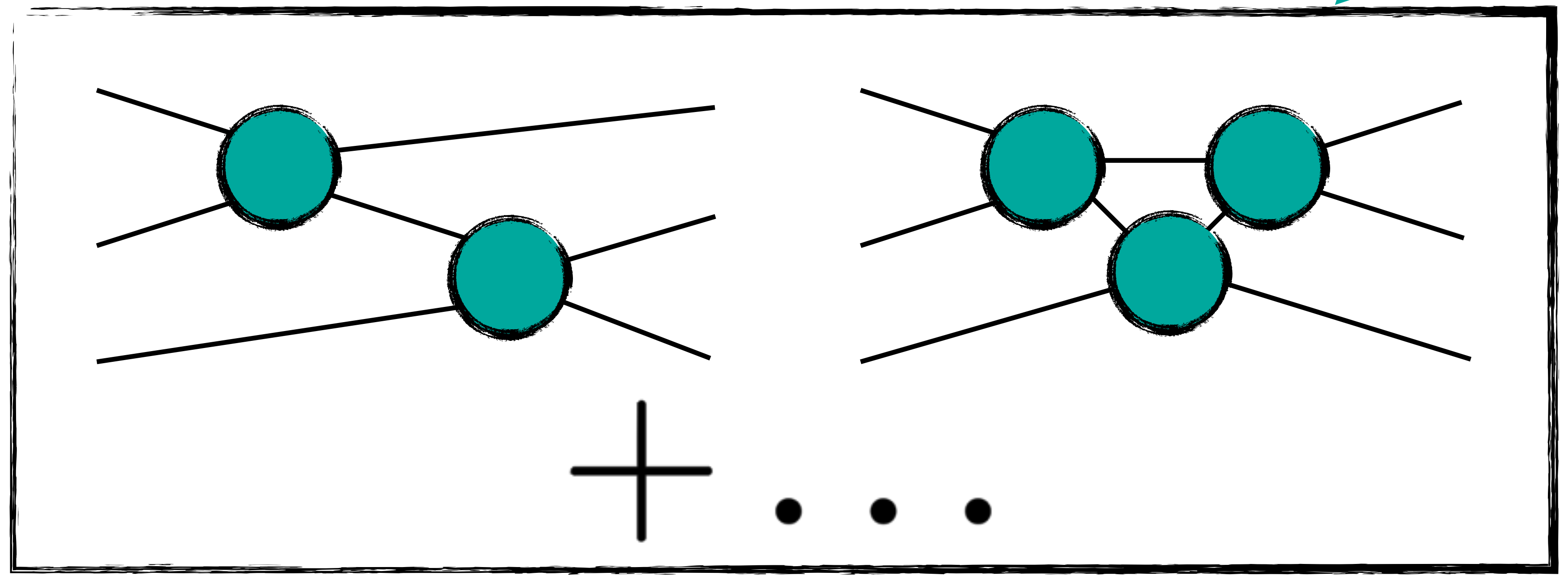


Scattering amplitudes

- Physical amplitudes that are consistent with unitary are obtained after solving integral equations:

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{\mathrm{df},3}$$

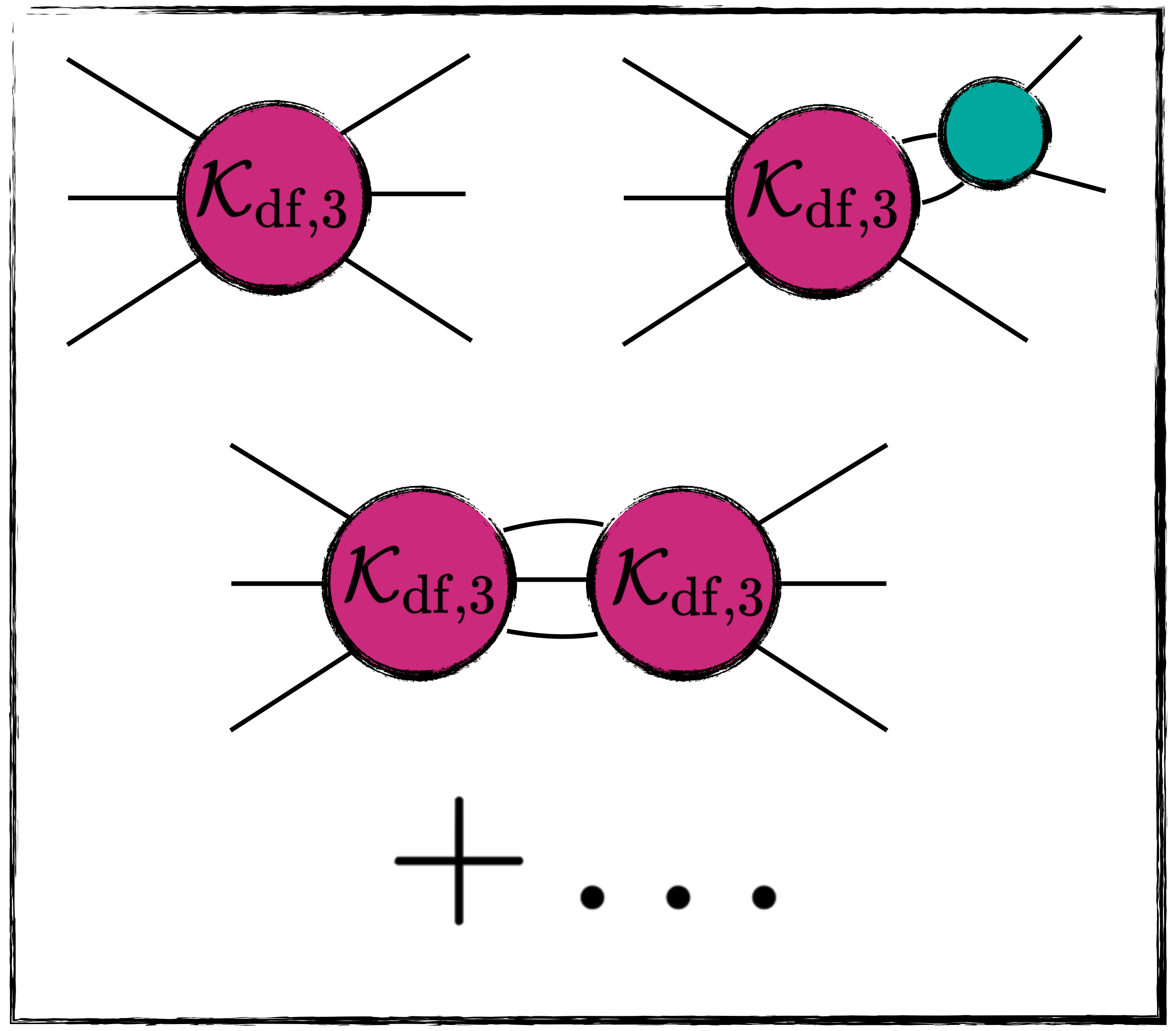
"ladder amplitude"



two-body rescattering

$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D}$$

"divergence-free amplitude"



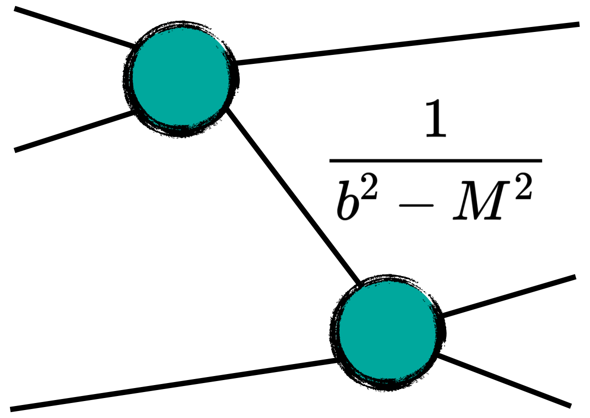
At least one
three-body interaction

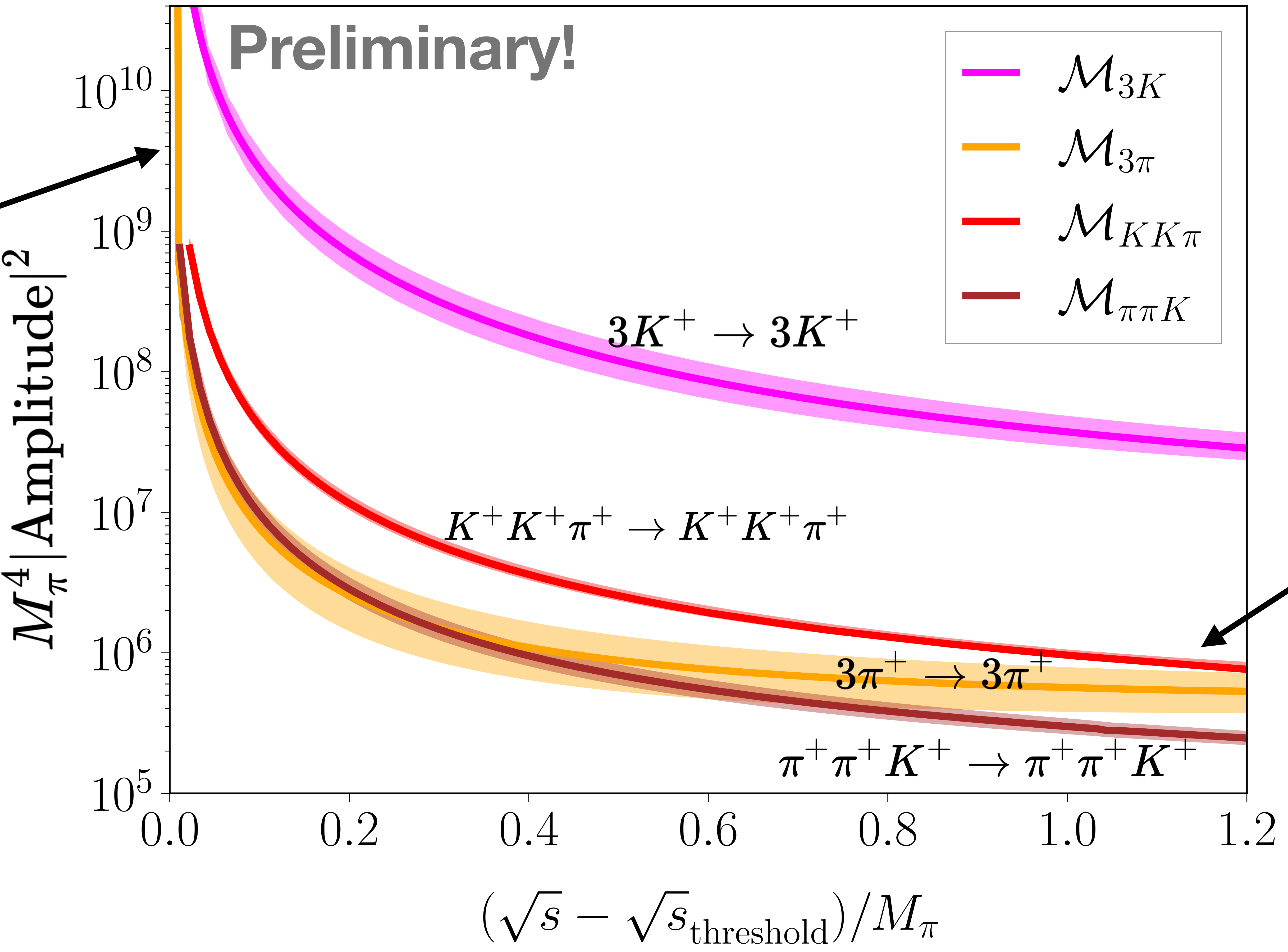
Three-meson amplitudes

Lattice QCD predictions for physical three-meson scattering amplitudes

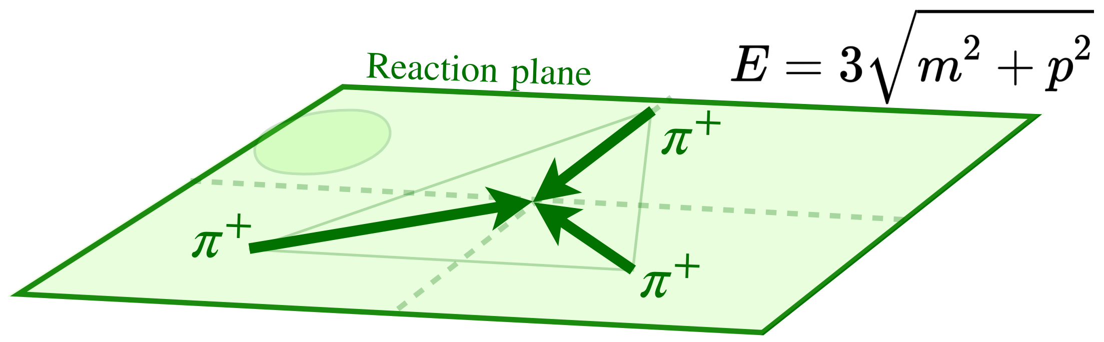
[Dawid, Draper, Hanlon, Hörz, Morningstar, FRL, Sharpe, Skinner, on-going work]

Divergent at threshold





Equilateral kinematic configuration

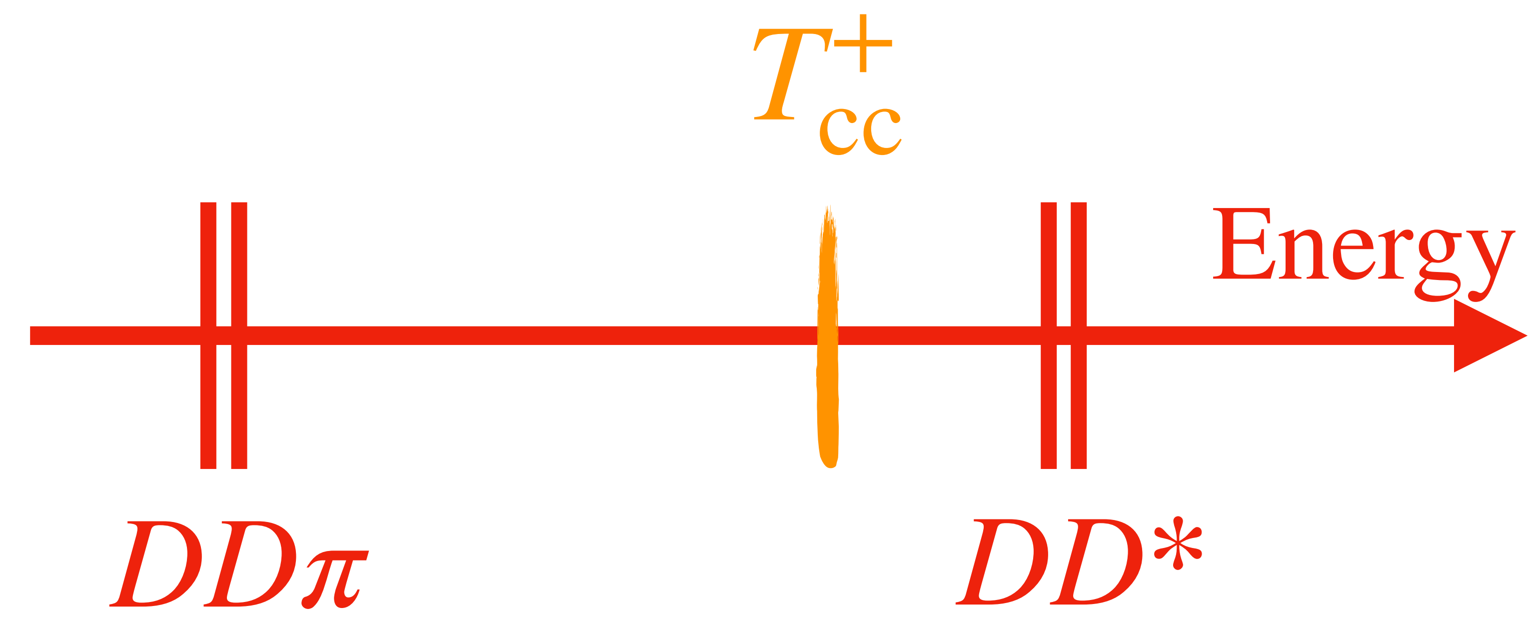


Pion interactions are chirally suppressed

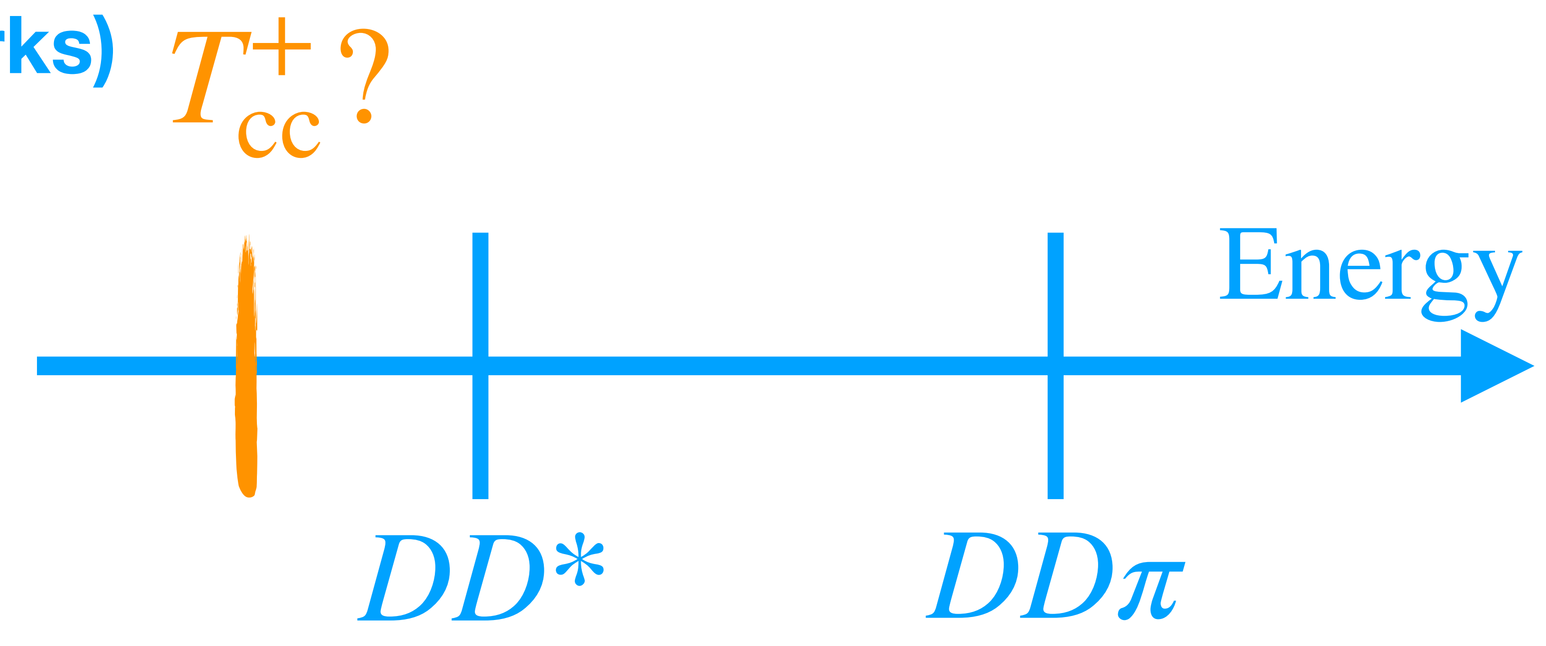
$$M_\pi^2 \mathcal{M}_3 = O(M_\pi^4 / F_\pi^4)$$

Doubly-charmed tetraquark

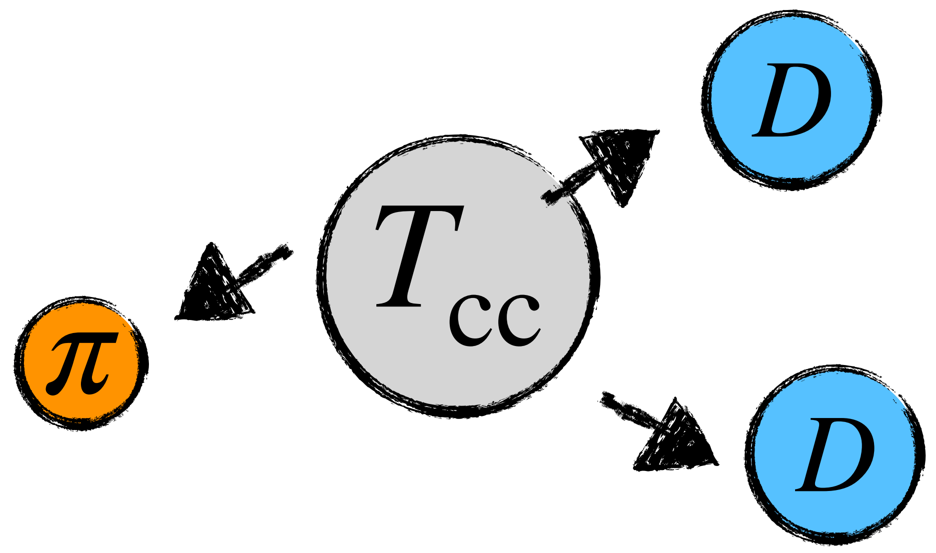
Experiment



N_f=2+1+1 QCD (heavier quarks)

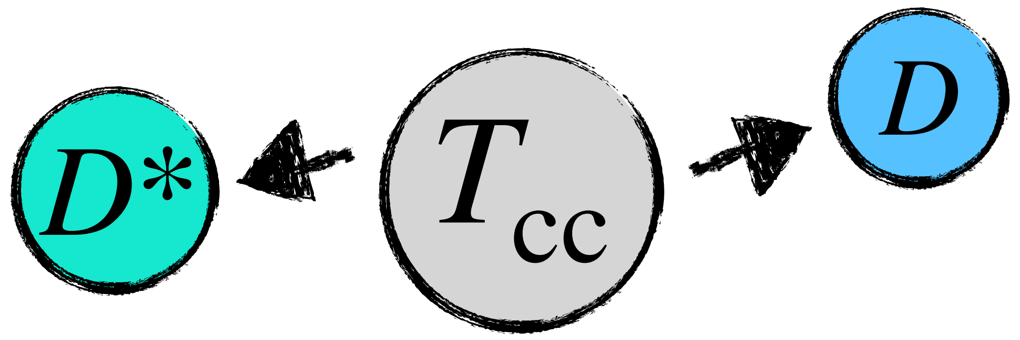


- **For physical quark masses is a three-body resonance**



need three-body formalism!

- **Stable D* at slightly heavier-than-physical quark mases**



suitable for the two-body finite-volume formalism?

D-D* scattering

- Several works study the T_{cc} channel in this setup

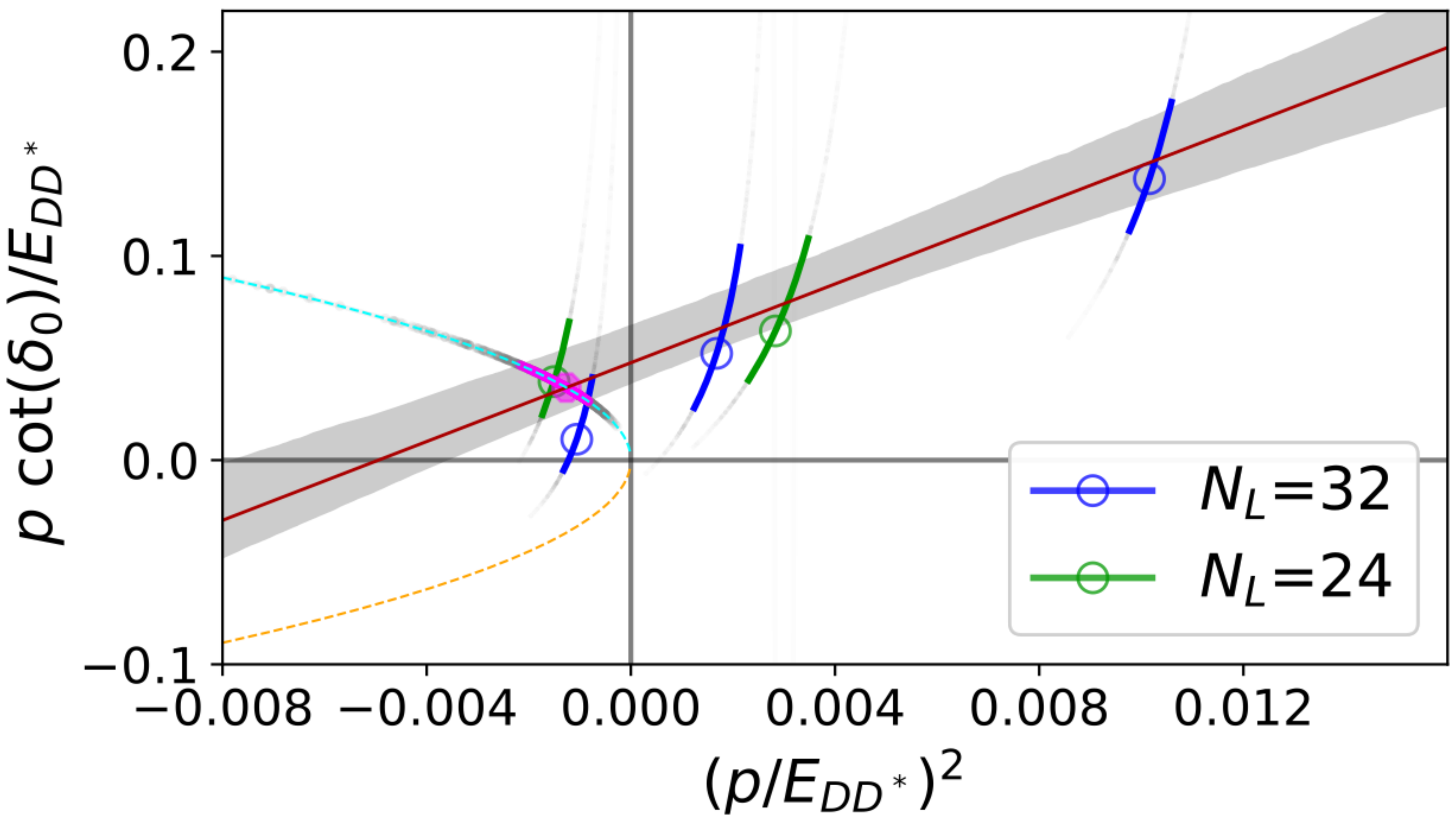
  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]

  [Padmanath & Prelovsek, 2202.10110]

  [Whyte, Thomas, Wilson, 2405.15741]

  - Signature of virtual bound state?

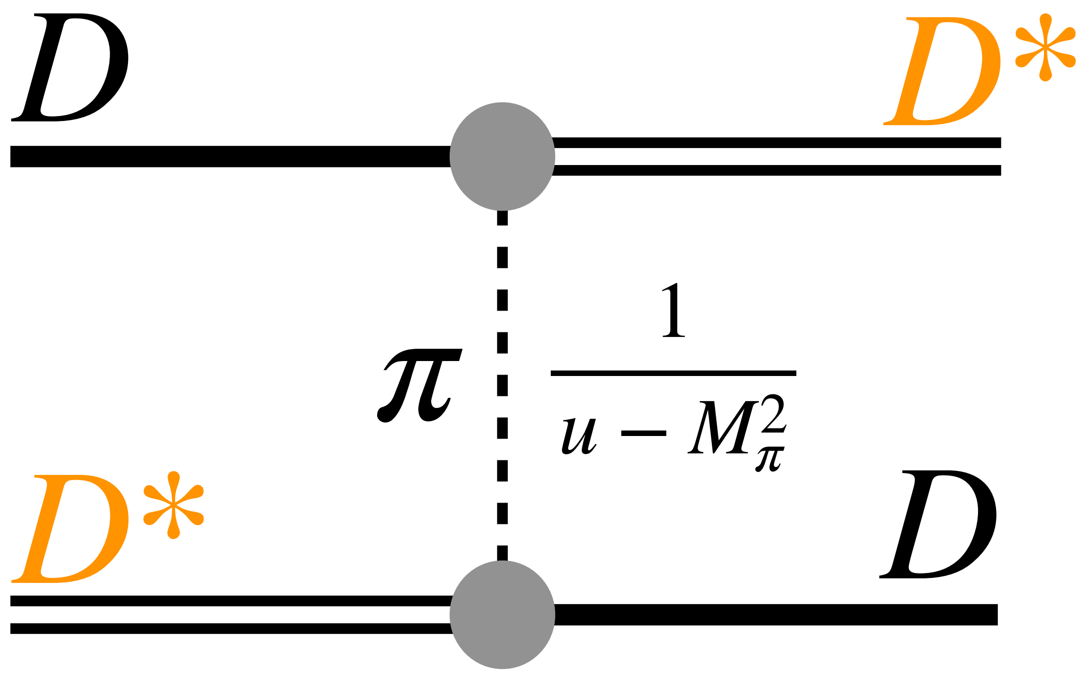
[Padmanath, Prelovsek, arXiv:2202.10110]



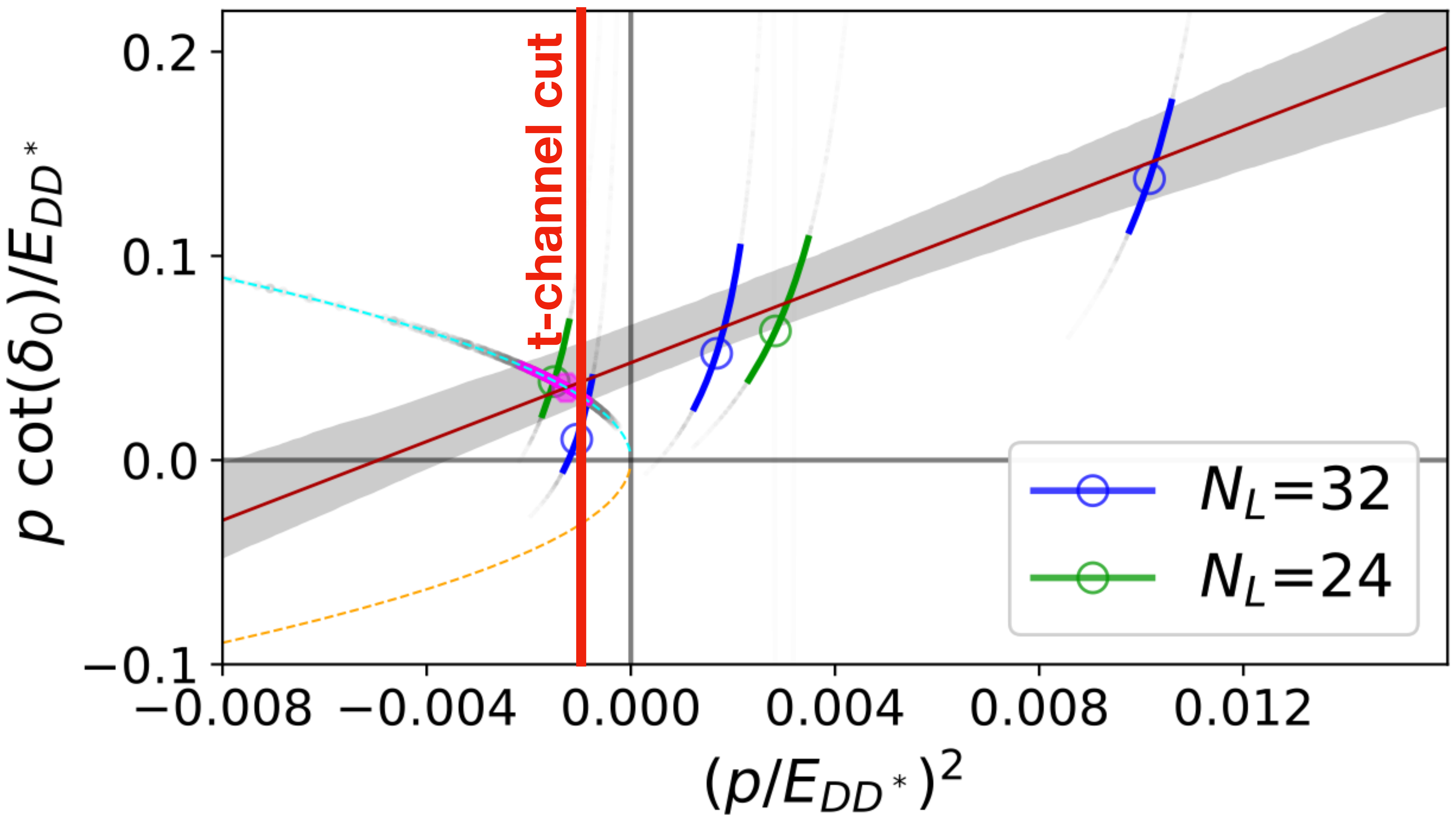
D-D* scattering

- Several works study the T_{cc} channel in this setup
  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]
  [Padmanath & Prelovsek, 2202.10110]
  [Whyte, Thomas, Wilson, 2405.15741]
  - Signature of virtual bound state?
  - But two-particle formalism breaks down
    i.e. complex phase shift

! one-pion exchange creates non-analytic behavior:



[Padmanath, Prelovsek, arXiv:2202.10110]



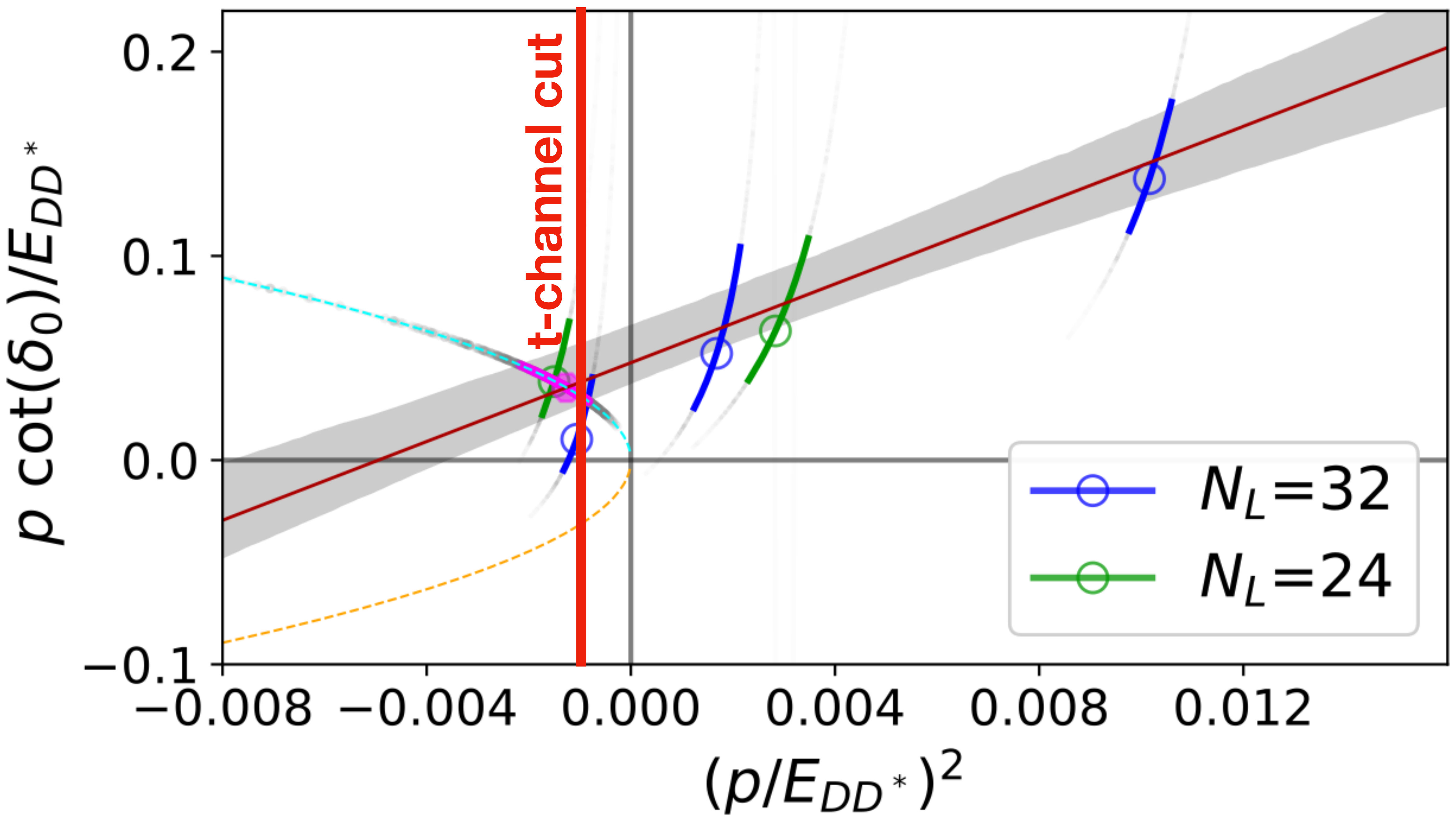
D-D* scattering

- Several works study the T_{cc} channel in this setup

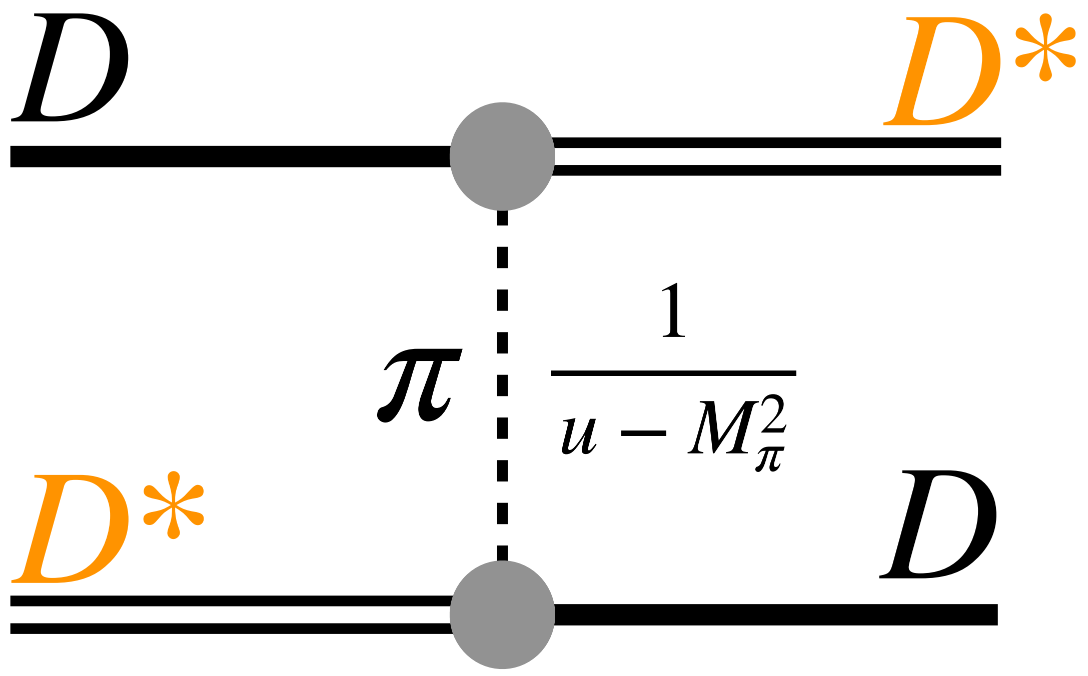
  [Chen et al., 2206.06185] [Lyu et al. (HALQCD), 2302.04505]
  [Padmanath & Prelovsek, 2202.10110]
  [Whyte, Thomas, Wilson, 2405.15741]

  - Signature of virtual bound state?
  - But two-particle formalism breaks down
    i.e. complex phase shift

[Padmanath, Prelovsek, arXiv:2202.10110]



! one-pion exchange creates non-analytic behavior:



Several solution have been proposed [See talk by V. Baru]

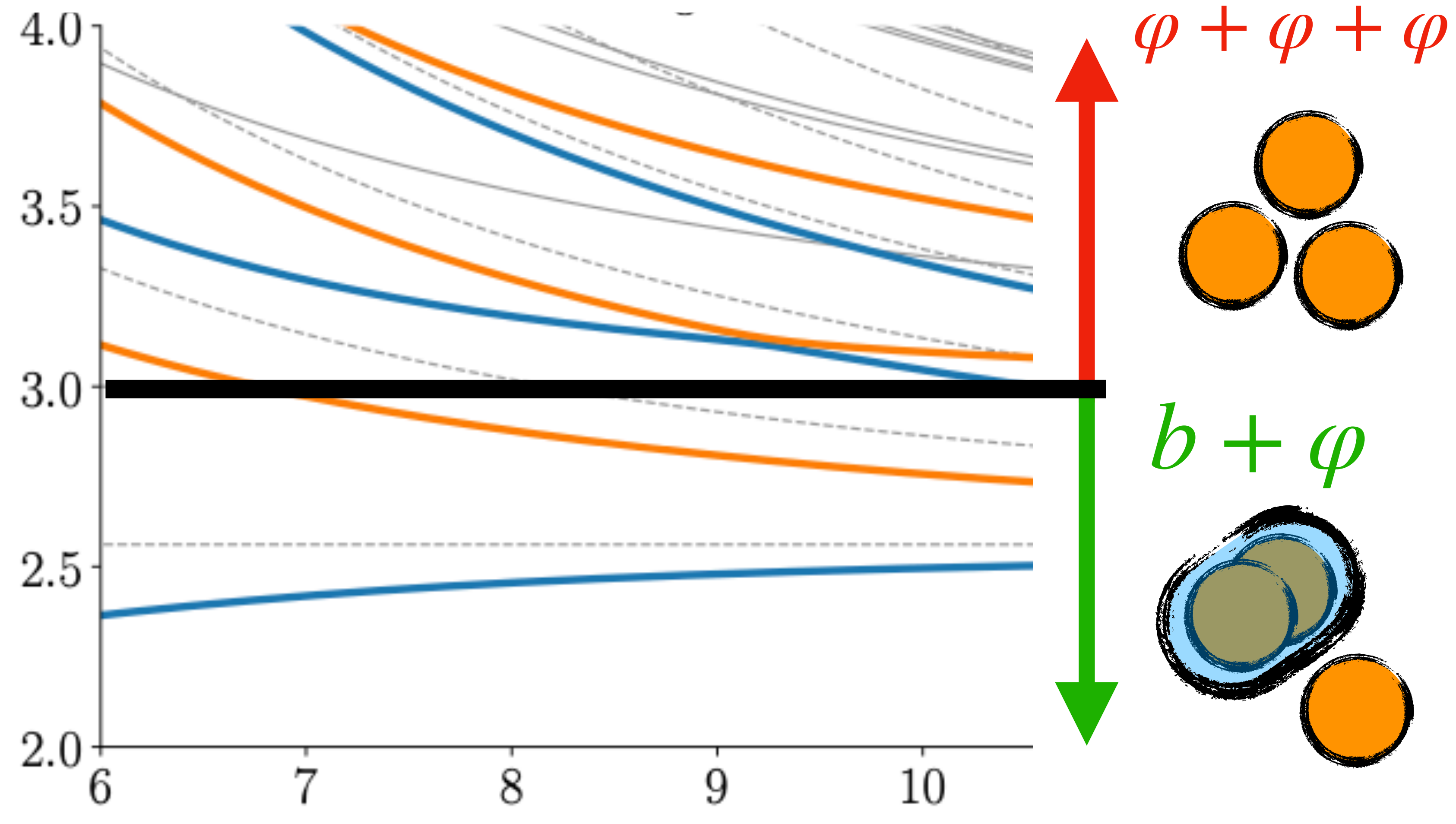
[Du et al (2408.09375), Abolnikov et al. (2407.04649), Bubna et al. (2402.12985), Meng et al. (2312.01930), Raposo, Hansen (2311.18793)]

A three-body solution

- In the presence of a two-body bound state:
  - Below the three-particle threshold, effective “particle-dimer”

[FRL et al 2302.04505] [Jackura et al 2010.09820]
[Dawid, Islam, Briceño, 2303.04394]
[Briceño, Jackura, Pefkou, FRL 2402.12167]

[FRL, Sharpe, Blanton, Briceño, Hansen 1908.02411]

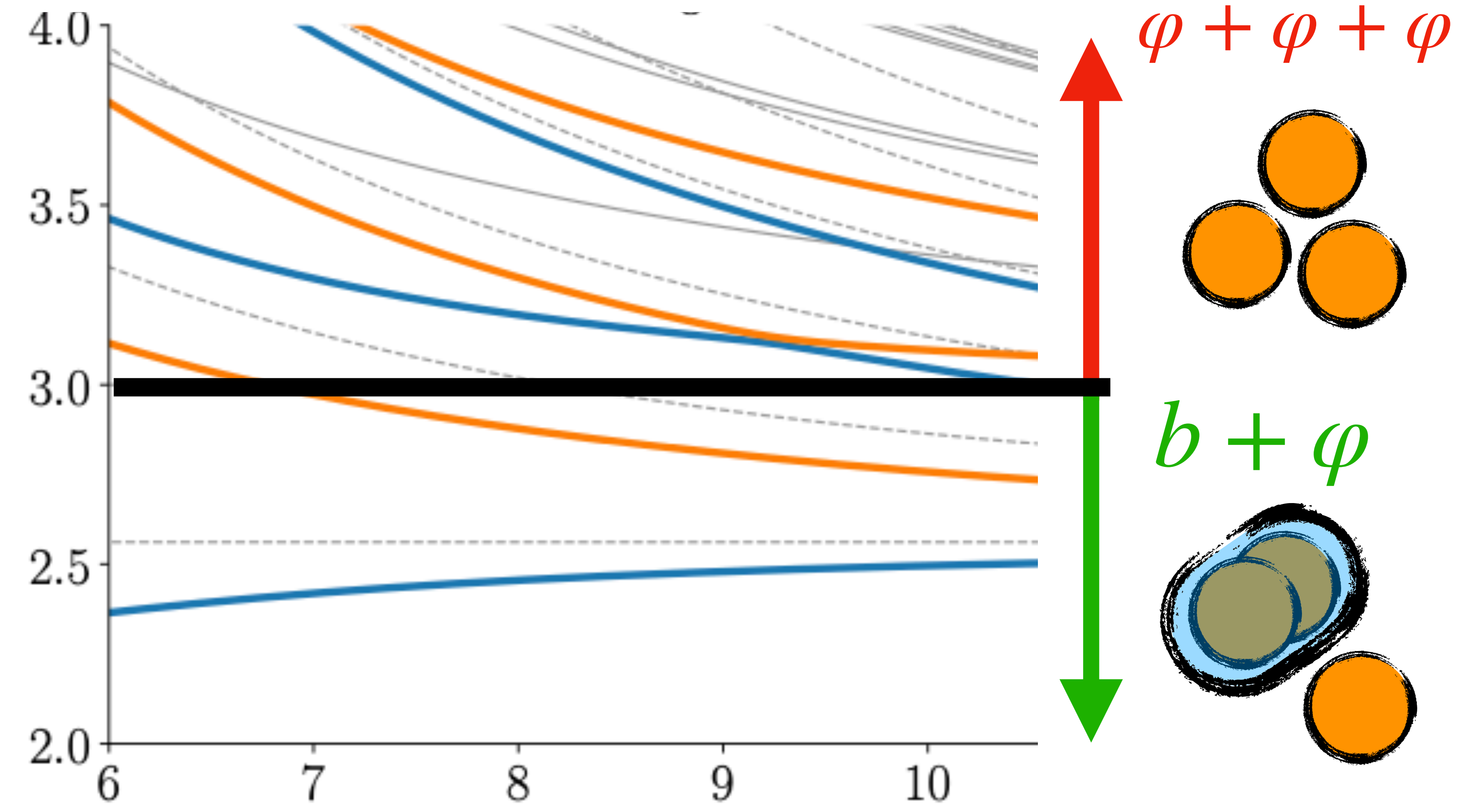


A three-body solution

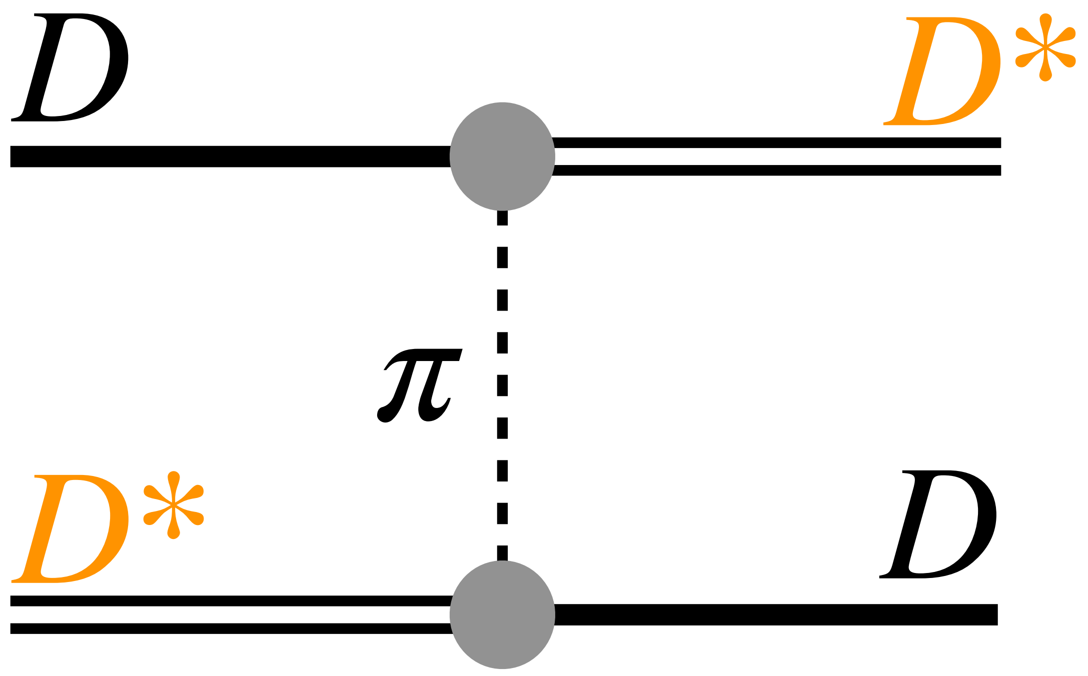
- In the presence of a two-body bound state:
  - Below the three-particle threshold, effective "particle-dimer"

    [FRL et al 2302.04505] [Jackura et al 2010.09820]
    [Dawid, Islam, Briceño, 2303.04394]
    [Briceño, Jackura, Pefkou, FRL 2402.12167]

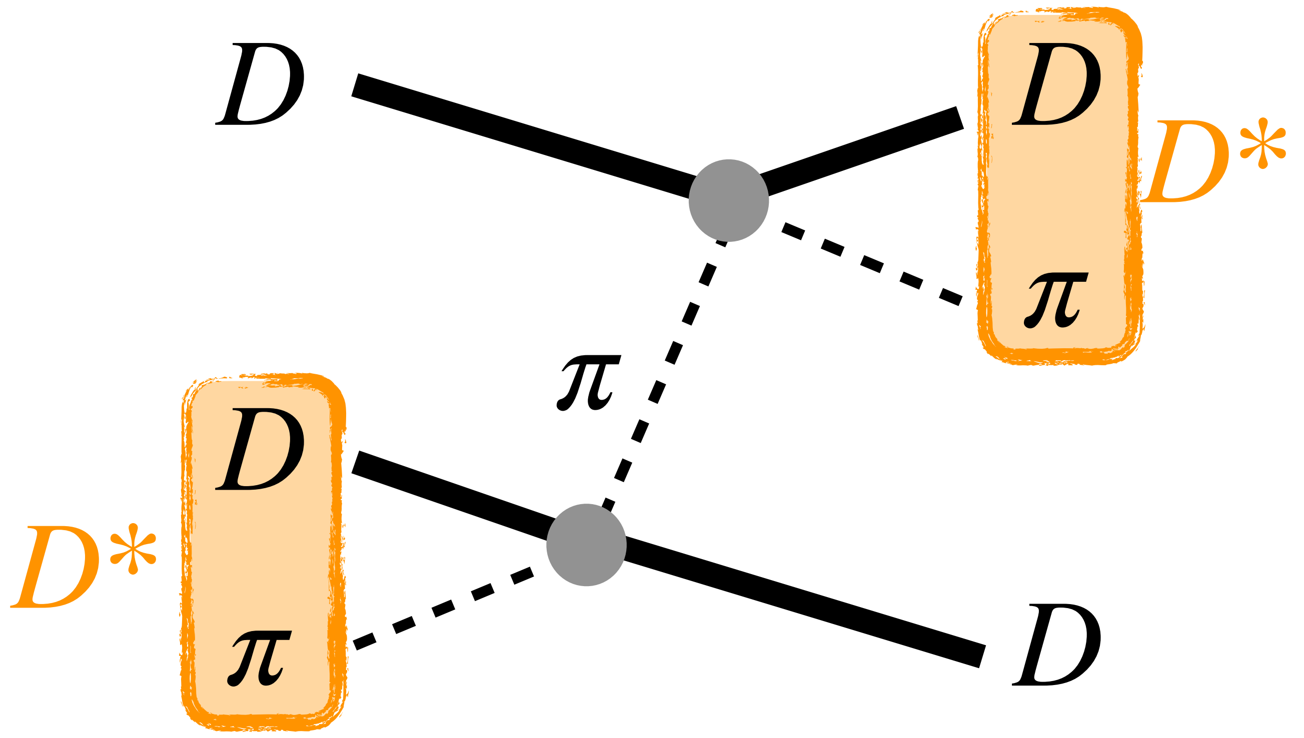
[FRL, Sharpe, Blanton, Briceño, Hansen 1908.02411]



- This solves the left-hand cut problem:
  - Finite-volume effects from one-pion exchange naturally incorporated



[Hansen, FRL, Sharpe, arXiv:2401.06609]

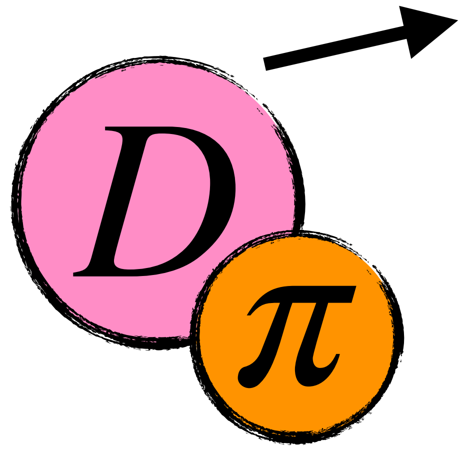


The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]

Two-meson spectra

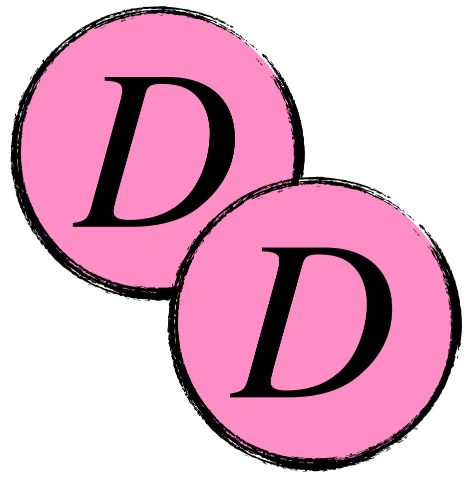
$I_{D\pi} = 1/2$



D^* as a bound state or resonance

s and p waves

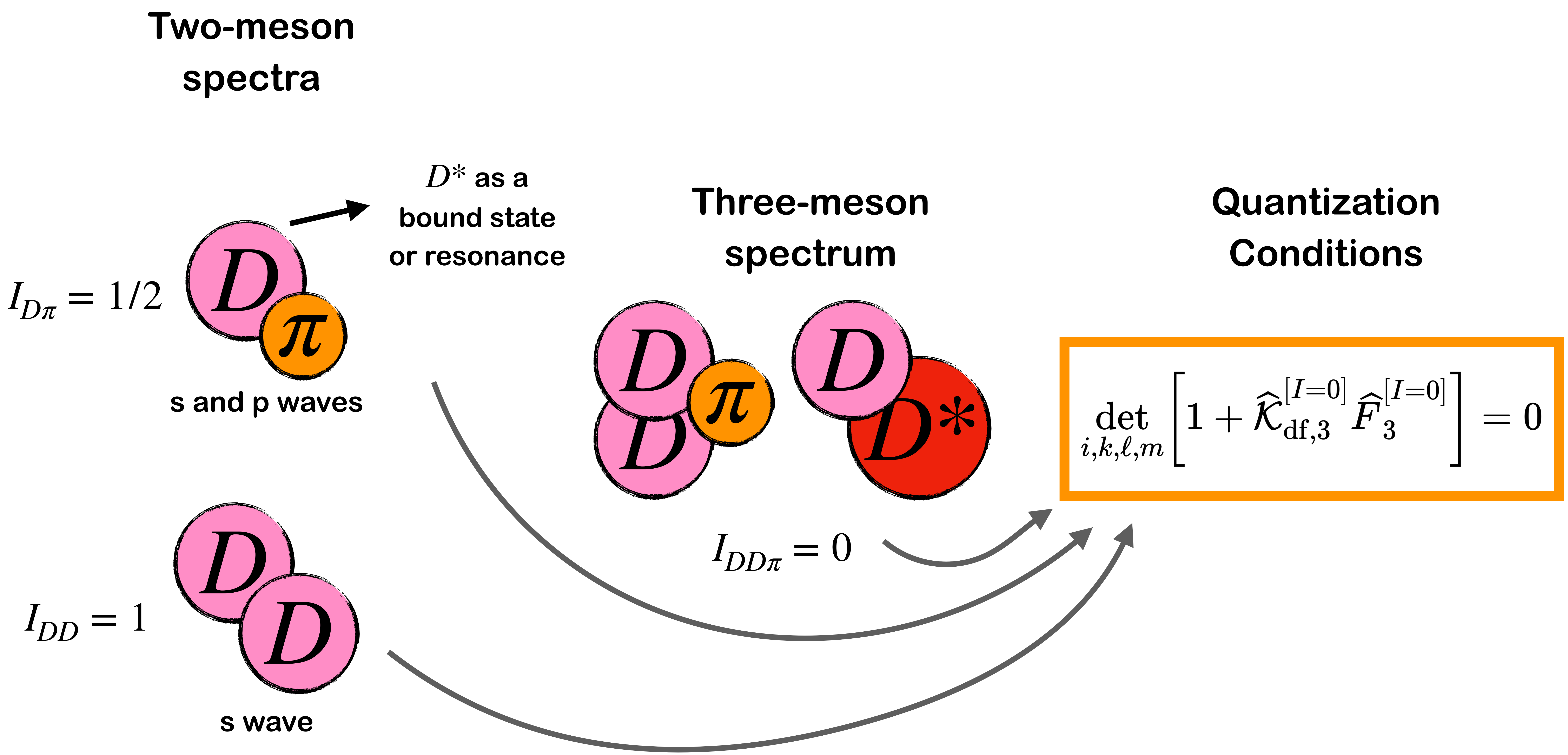
$I_{DD} = 1$



s wave

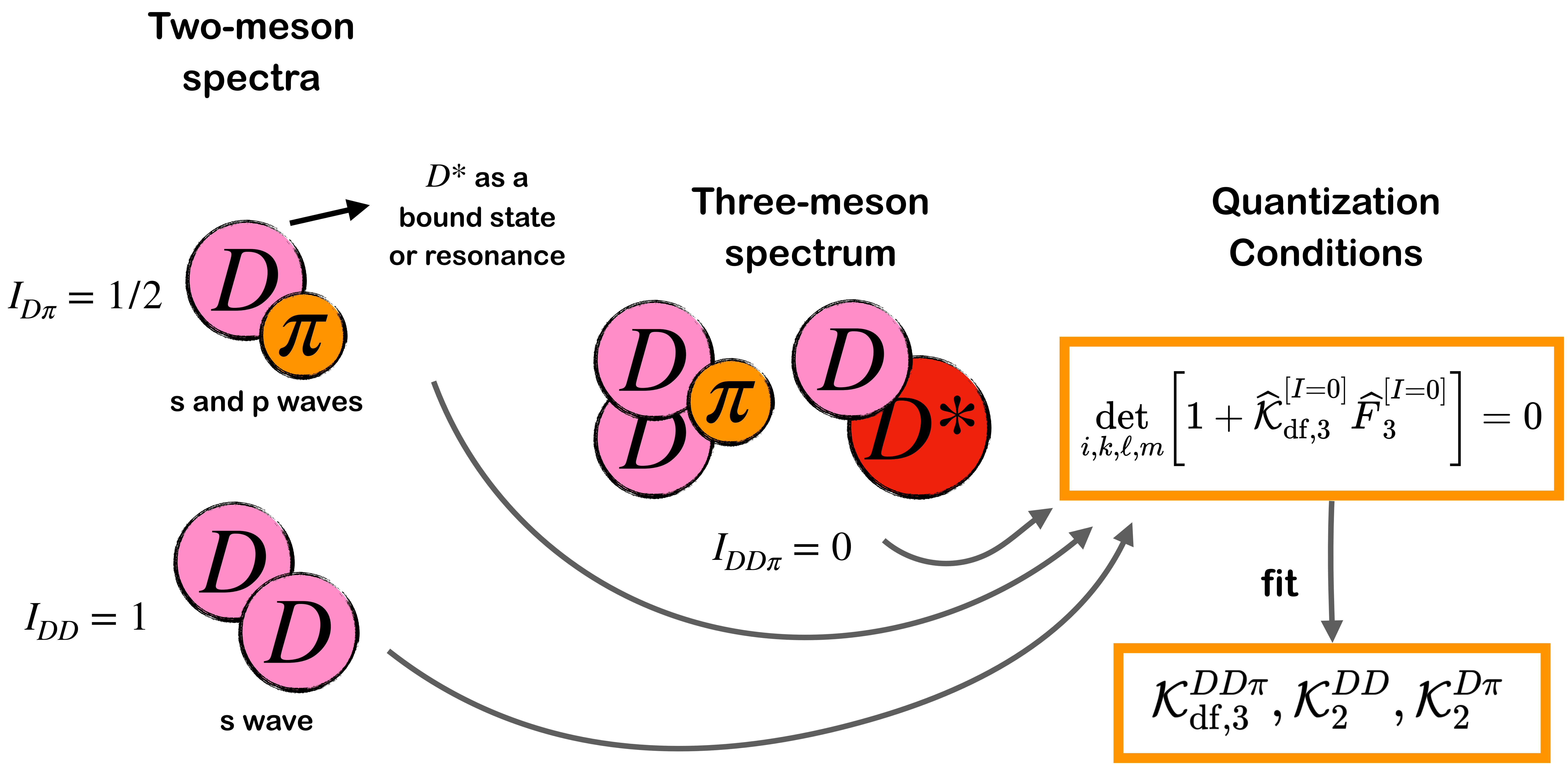
The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]



The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]

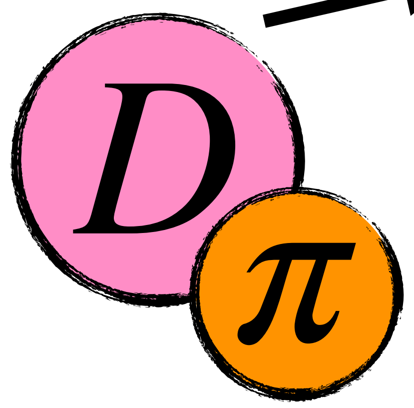


The strategy for the T_{cc}

[Hansen, FRL, Sharpe, arXiv:2401.06609]

Two-meson spectra

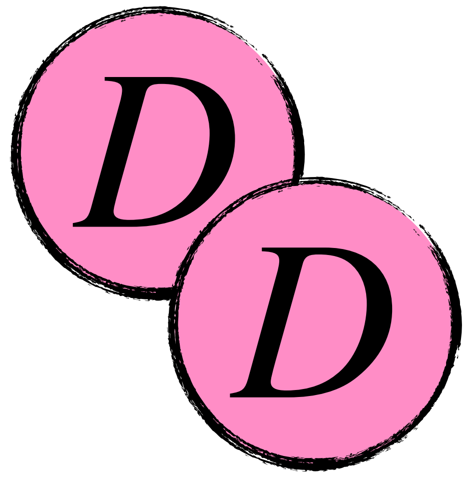
$I_{D\pi} = 1/2$



D^* as a bound state or resonance

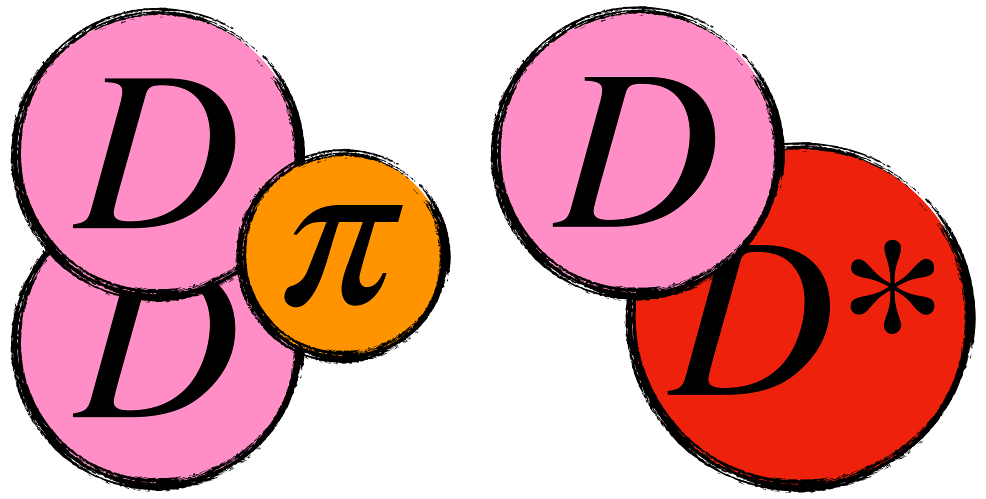
s and p waves

$I_{DD} = 1$



s wave

Three-meson spectrum



$I_{DD\pi} = 0$

Quantization Conditions

$$\det_{i,k,\ell,m}\left[1 + \widehat{\mathcal{K}}_{\mathrm{df},3}^{[I=0]}\widehat{F}_3^{[I=0]}\right] = 0$$

fit

$$\mathcal{K}_{\mathrm{df},3}^{DD\pi}, \mathcal{K}_2^{DD}, \mathcal{K}_2^{D\pi}$$

Tetraquark properties

$$\mathcal{M}_3 \sim \frac{-g^2}{s - M_{T_{cc}}^2}$$

Integral equations

[Dawid, FRL, Sharpe (in prep)]

Analyzing D-D* data

[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]

- Published data only provides DD^* energies
  [Padmanath, Prelovsek, 2202.10110]

- $D\pi$ and DD interactions from "educated guesses"
  - HChPT and lattice results
  - Neglect DD interactions

- Only "free" parameter in the three-body K matrix

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_E (p_\pi - p'_\pi)^2$$

Analyzing D-D* data

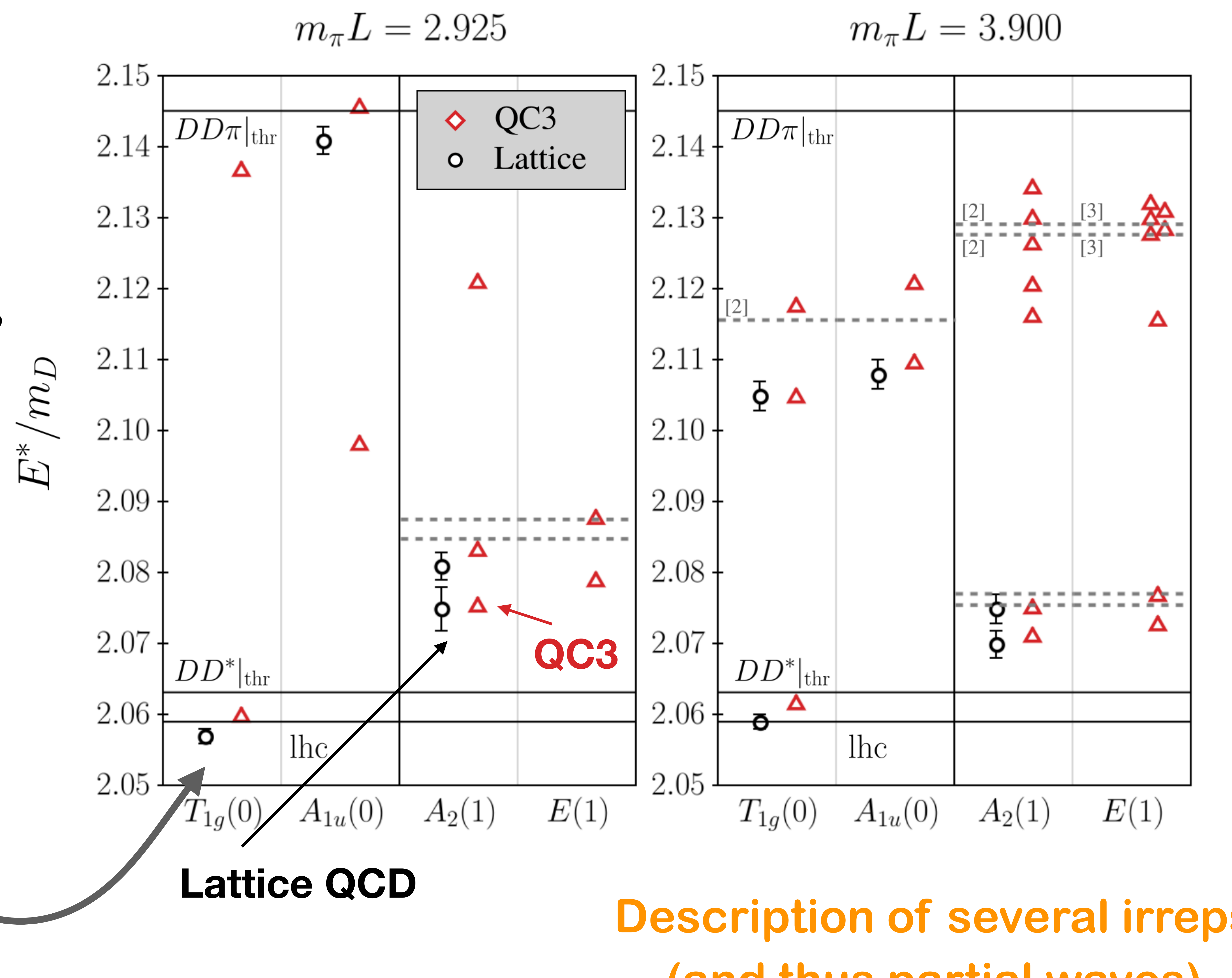
[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]

- Published data only provides DD* energies
  [Padmanath, Prelovsek, 2202.10110]

- Dπ and DD interactions from "educated guesses"
  - HChPT and lattice results
  - Neglect DD interactions

- Only "free" parameter in the three-body K matrix

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_E (p_\pi - p'_\pi)^2$$



Description of several irreps (and thus partial waves)

Analyzing D-D* data

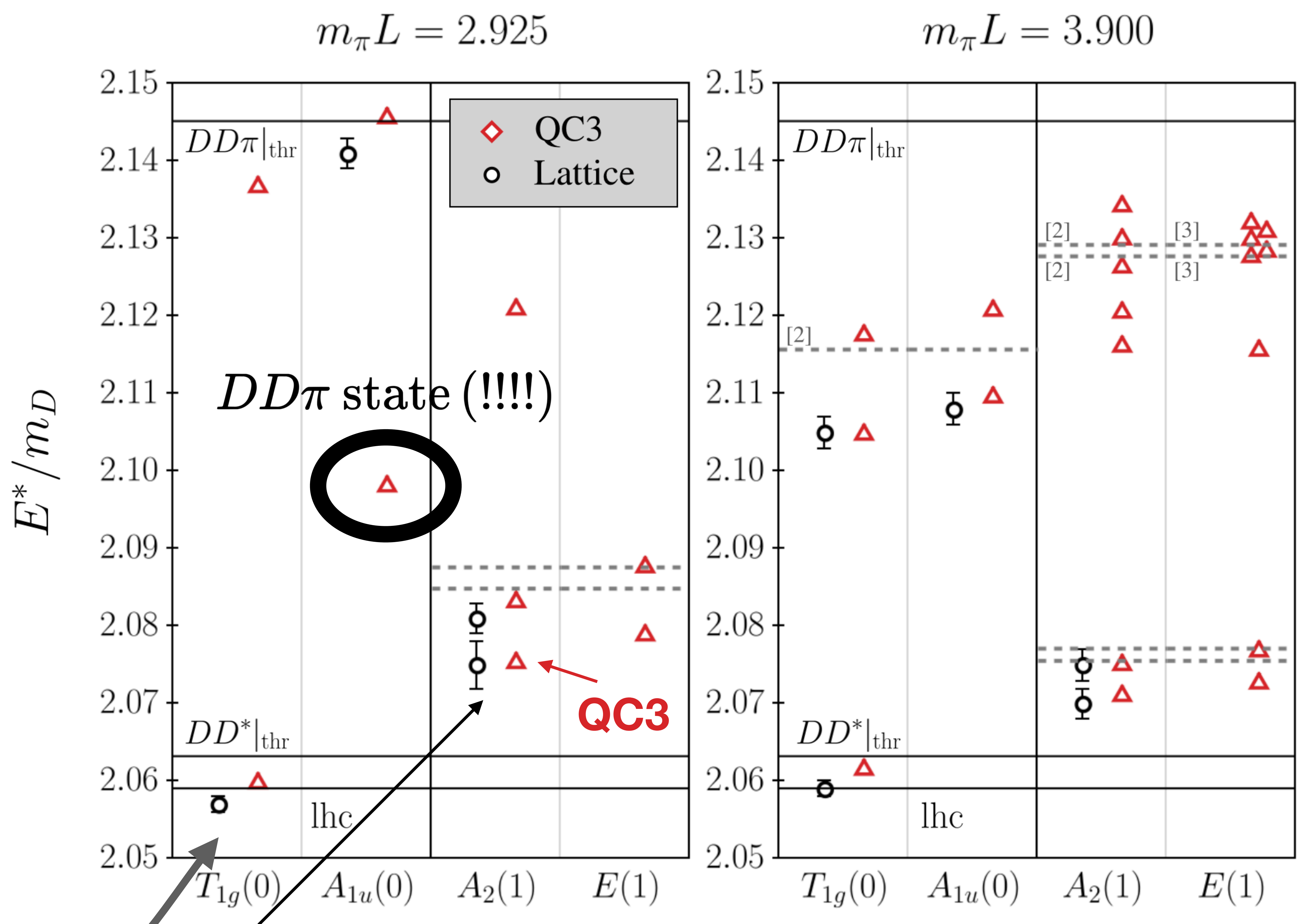
[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]

- Published data only provides DD* energies
  [Padmanath, Prelovsek, 2202.10110]

- Dπ and DD interactions from "educated guesses"
  - HChPT and lattice results
  - Neglect DD interactions

- Only "free" parameter in the three-body K matrix

$$\mathcal{K}_{\mathrm{df},3} = \mathcal{K}_E (p_\pi - p'_\pi)^2$$



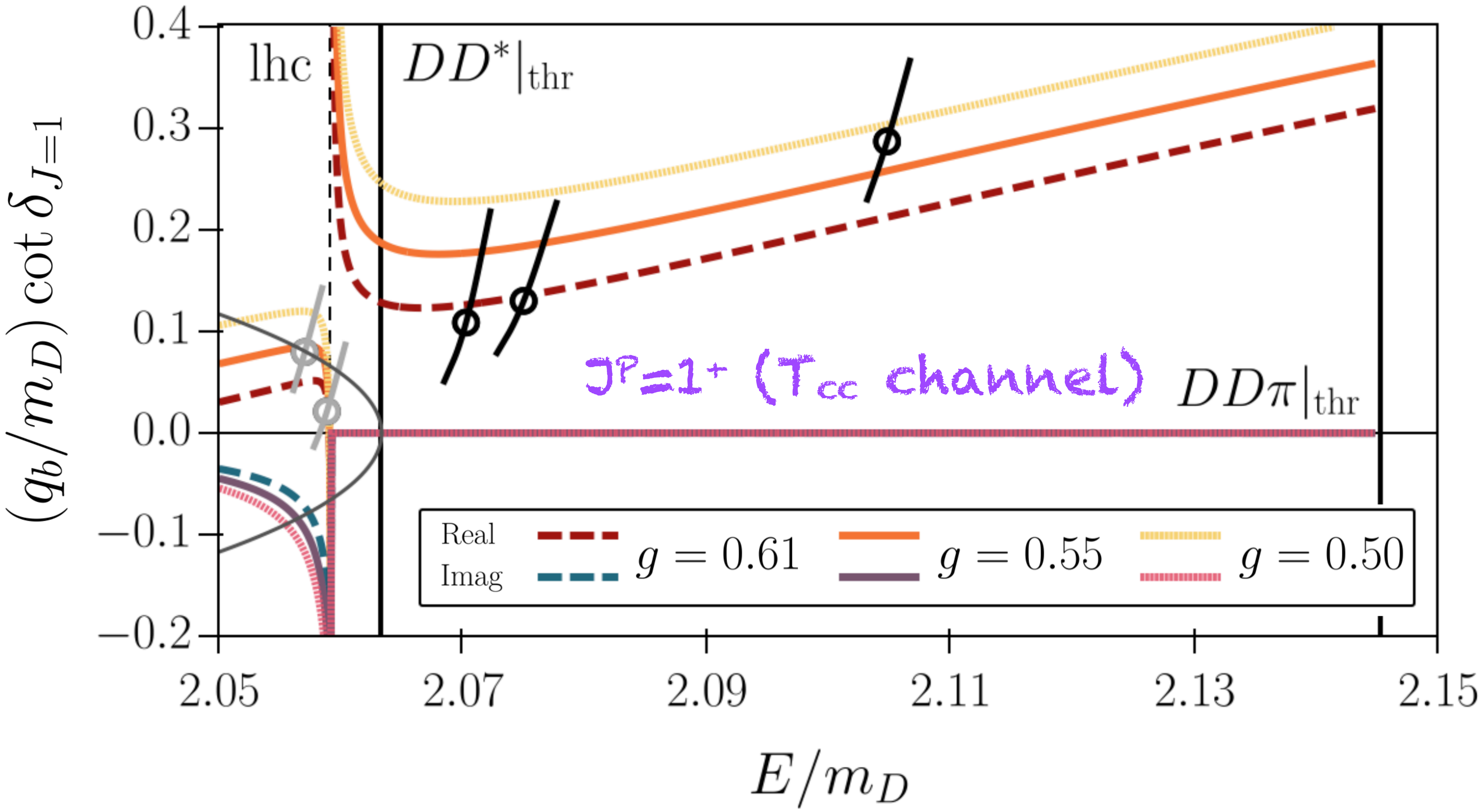
Finite-volume energies near the left-hand cut

Description of several irreps (and thus partial waves)

Analyzing D-D* data

- With simple parametrizations we are able to reproduce lattice QCD energies.

[S. Dawid, FRL, S. Sharpe, arXiv:2409.17059]   Data from: [Padmanath, Prelovsek, arXiv:2202.10110]



Need a genuine three-body study of the T_{cc} !

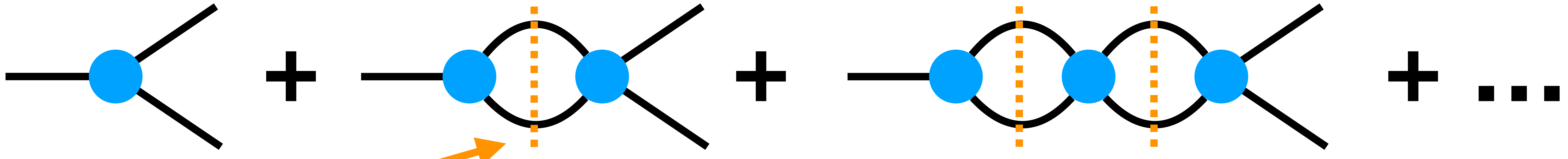
- DD and Dπ calculation on the same ensemble
- DDπ operators in lattice QCD calculations

Multi-hadron electroweak transitions from LQCD

Decays in Finite Volume

- From three-point function, it is possible to extract the finite-volume matrix elements

$$C(t) = \langle \mathcal{O}_K^\dagger(0)\, H_w(t_1)\, \mathcal{O}_{\pi\pi}(t_2) \rangle \longrightarrow \langle K | H_w | \pi\pi \rangle_L$$

$\langle K | H_w | \pi\pi \rangle_L =$ 

$$\left[\frac{1}{L^3} \sum_{\vec{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{E - 2\omega_q}$$

Finite-volume effects from on-shell propagation of two-body states

- **Final-state interactions induce finite-volume effects**
- **Volume-dependent corrections are calculable!**

[Lellouch, Lüscher, hep-lat/0003023]

Decays in Finite Volume

- Need volume-dependent factors to correct for final state interactions

$$T(K \to \pi\pi) = F(L) \times \langle K | H_w | \pi\pi \rangle_L$$

Lellouch-Lüscher Factor

Valid when only two-hadron final states are possible!

$$F(L)^2 = \frac{4\pi m_K E_{\pi\pi}^2}{k^3} \left(k \frac{\mathrm{d}\delta_0}{\mathrm{d}k} + q \frac{\mathrm{d}\phi}{\mathrm{d}q} \right)$$

Depends on two-body scattering

Kinematic and volume-dependent function

Decays in Finite Volume

- Need volume-dependent factors to correct for final state interactions

$$T(K \to \pi\pi) = F(L) \times \langle K | H_w | \pi\pi \rangle_L$$

Lellouch-Lüscher Factor

Valid when only two-hadron final states are possible!

$$F(L)^2 = \frac{4\pi m_K E_{\pi\pi}^2}{k^3}\left(k\frac{\mathrm{d}\delta_0}{\mathrm{d}k} + q\frac{\mathrm{d}\phi}{\mathrm{d}q}\right)$$

Computation of multi-hadron electroweak transitions require a dedicated scattering calculation

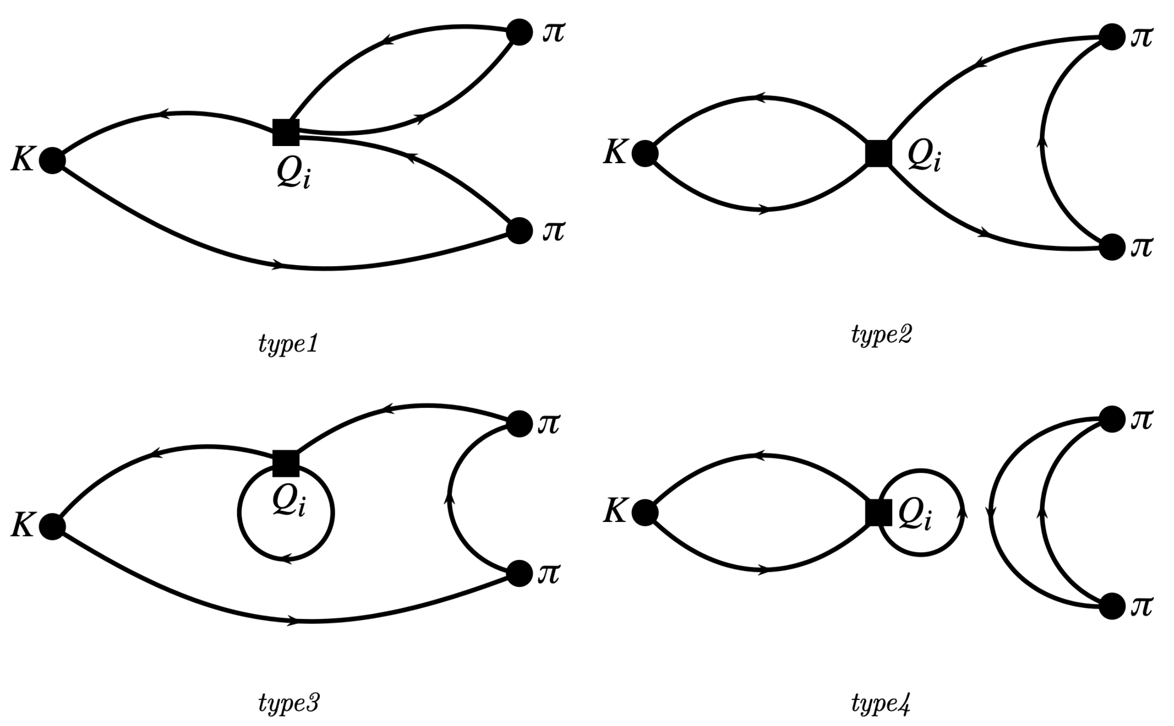
Depends on two-body scattering

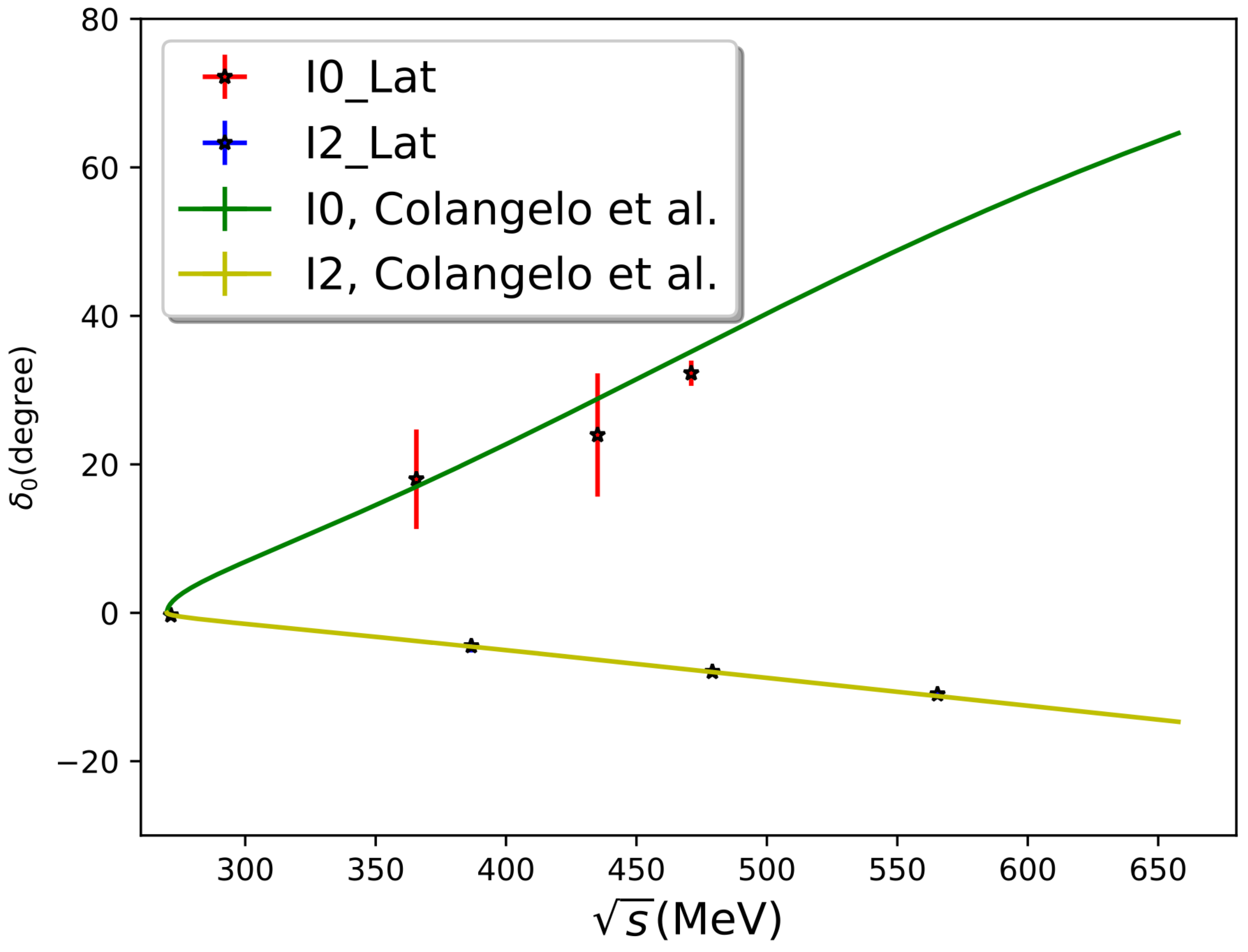
Kinematic and volume-dependent function

Kaon decays

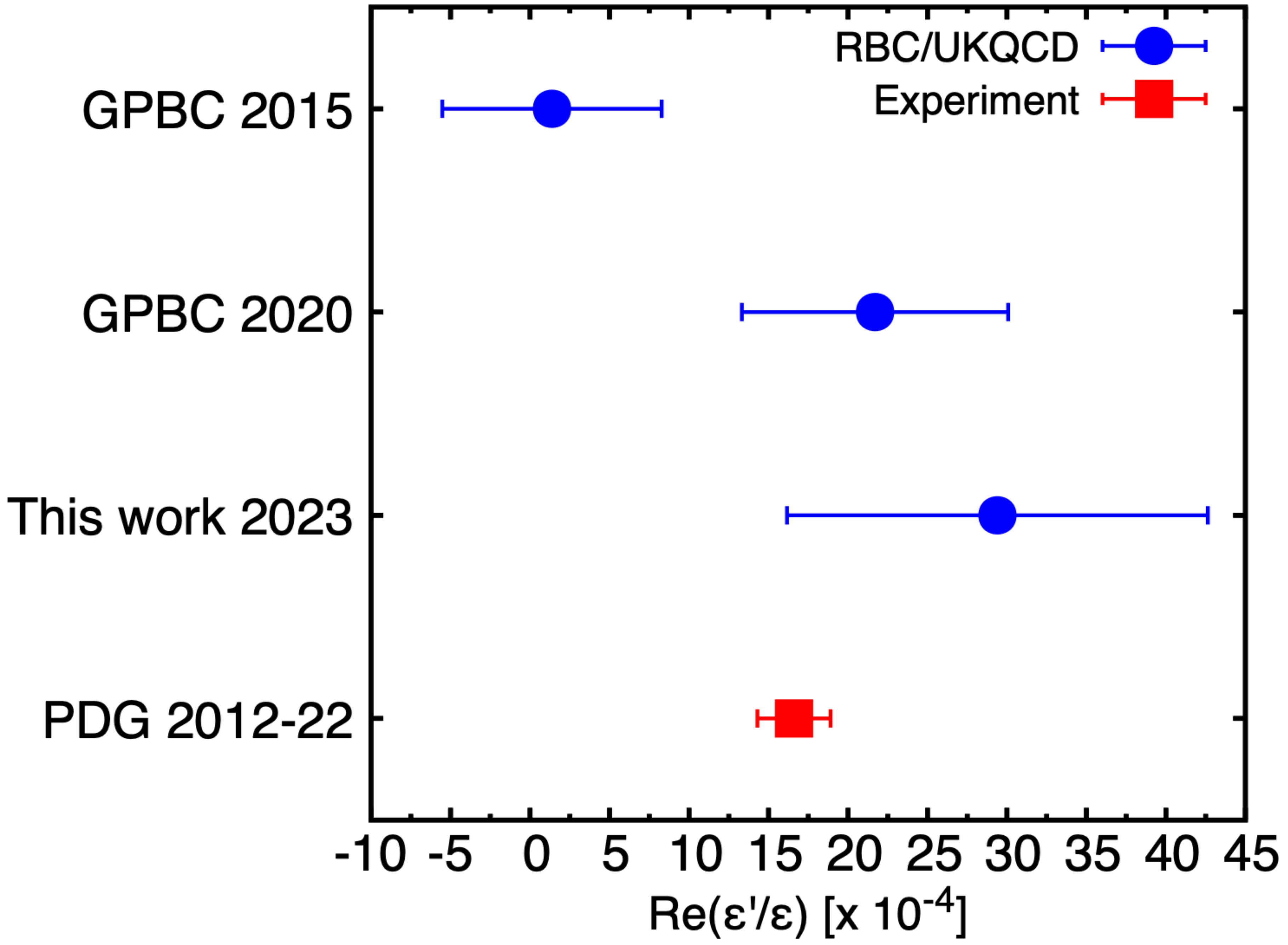
- CP violation in kaon decays for precision tests of the Standard Model
  - Computation of matrix elements
  - Requires dedicated scattering calculation!

$$A_I = \langle K|\mathcal{H}_w|(\pi\pi)_I\rangle$$





[Blum et al, RBC/UKQCD 2103.15131]

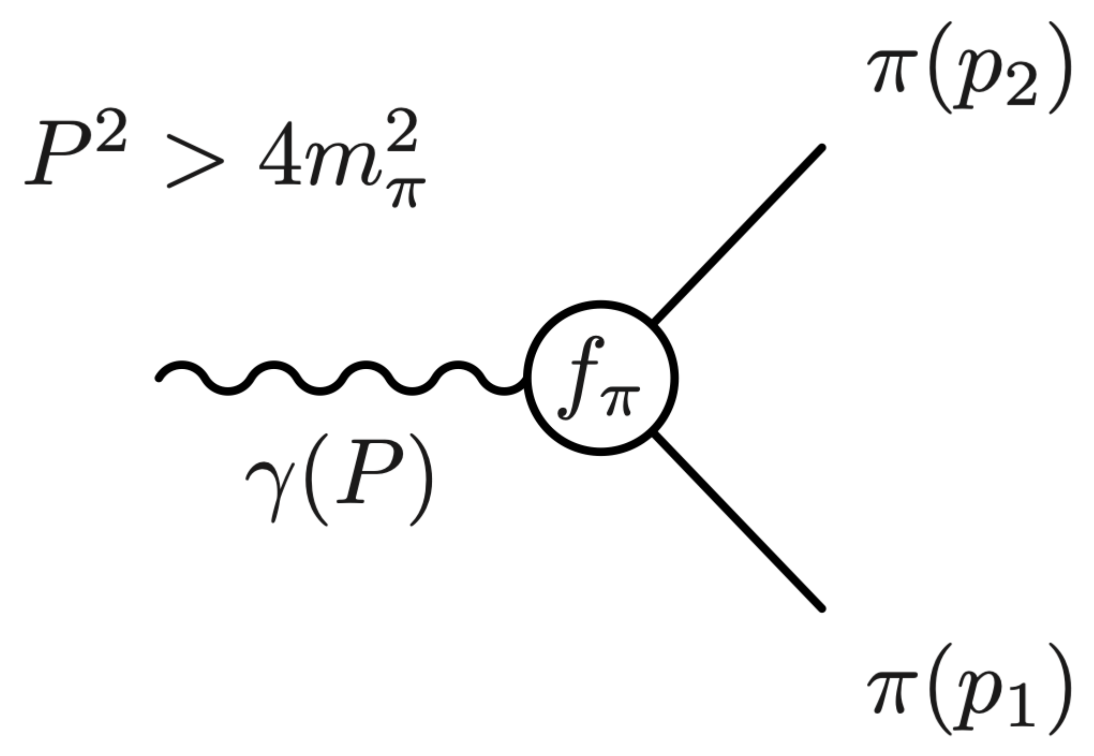


[Blum et al, RBC/UKQCD 2306.06781]

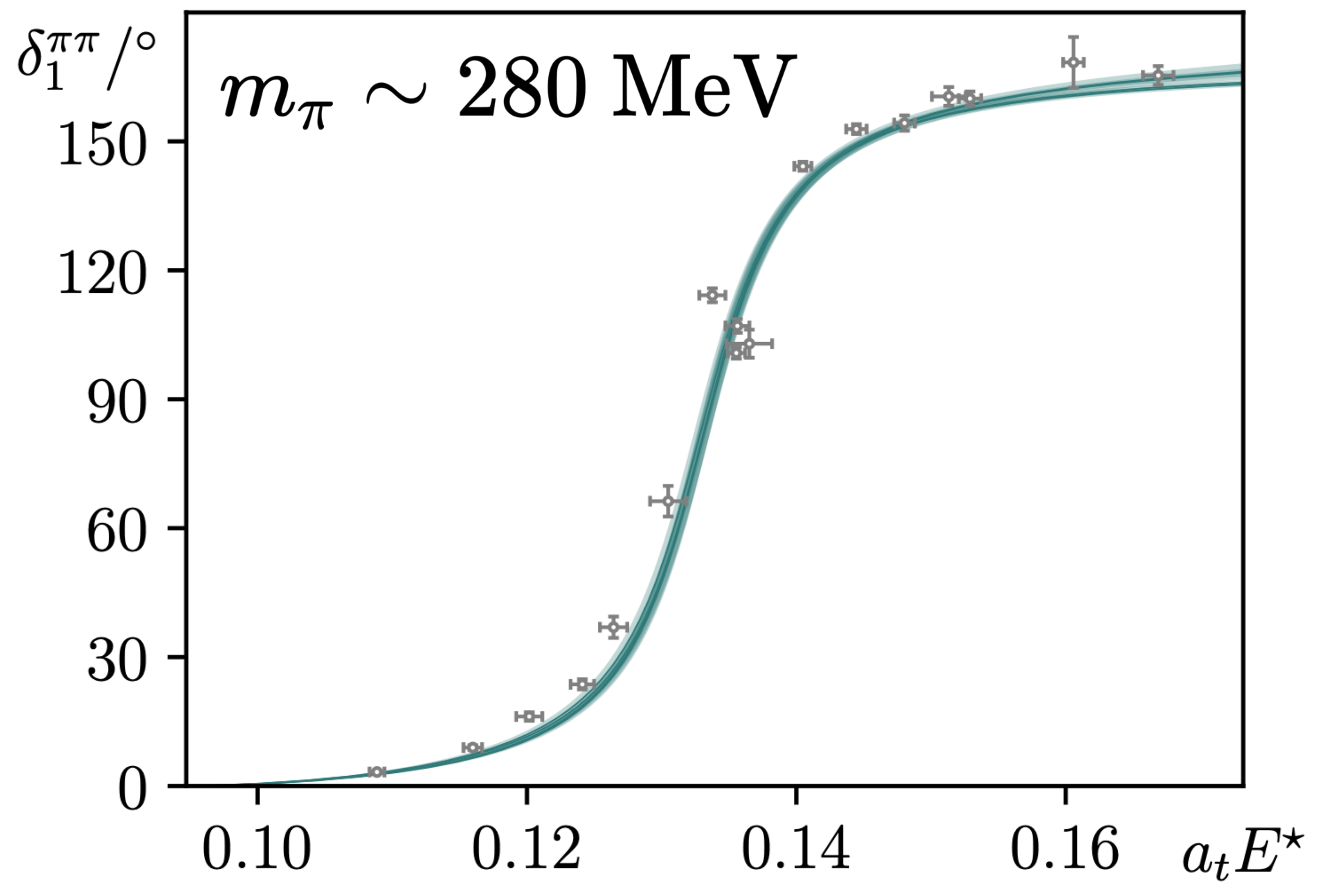
Pion timelike form factor

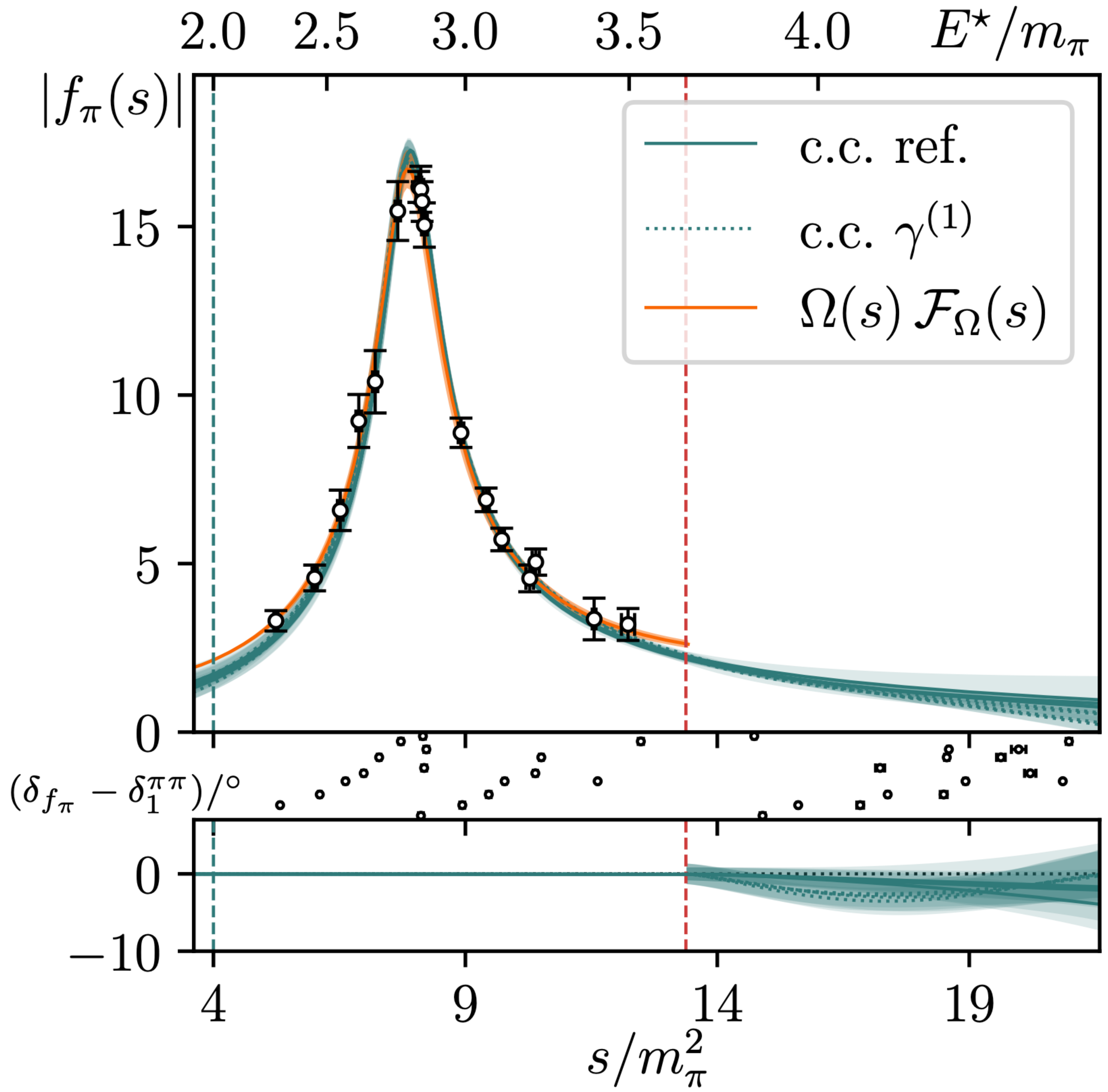
- Related to $e^+e^- \to \pi^+\pi^-$ transitions [Ortega-Gama et al, HadSpec 2407.20617]

$$\langle 0|V_\mu|\pi(p_1)\pi(p_2)\rangle = (p_1 - p_2)^\mu f_\pi(s)$$



▶ Need $\rho(770)$ phase shift





Fernando Romero-López, Uni Bern

Three-hadron decays

- Decays to three-hadrons also have phenomenological interest

Isospin = 0,1,2

$$K \to \pi\pi\pi$$

CP violation

Isospin = 0

$$\gamma^* \to \pi\pi\pi$$

related to HPV in g-2

Isospin 1

$$\eta \to \pi\pi\pi$$

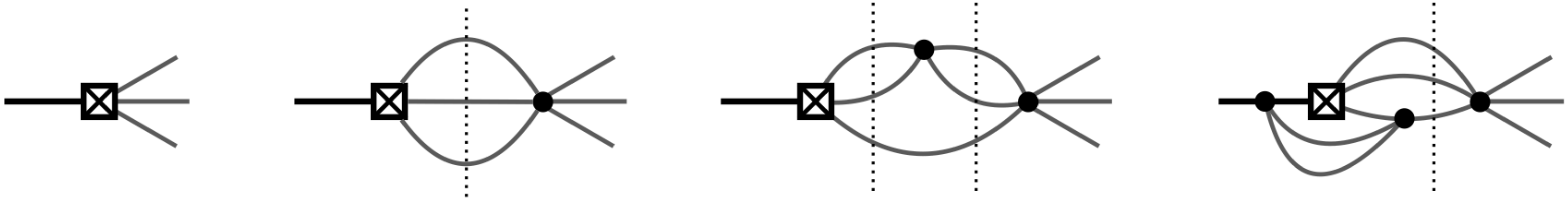
isospin-breaking effect

Three-hadron decays

- Decays to three-hadrons also have phenomenological interest

Isospin = 0,1,2	Isospin = 0	Isospin 1
$K \to \pi\pi\pi$	$\gamma^* \to \pi\pi\pi$	$\eta \to \pi\pi\pi$
CP violation	related to HPV in g-2	isospin-breaking effect

- Treating finite-volume effects needs accounting for intermediate three-hadron states
  [Hansen, FRL, Sharpe, arXiv:2101.10246] [Pang et al, arXiv:2312.04391] [Müller, Rusetsky, arXiv:2012.13957]



Three-hadron decays

- Infinite-volume amplitude can be obtained via "generalized Lellouch-Lüscher Factor"

$$|T_{K\to 3\pi}(E^*, m_{12}^2, m_{23}^2)|^2 = F_{3\pi}^2 |\langle 3\pi, L|\mathcal{H}_w|K\rangle|^2$$

Three-hadron decays

- Infinite-volume amplitude can be obtained via "generalized Lellouch-Lüscher Factor"

$$|T_{K\to 3\pi}(E^*, m_{12}^2, m_{23}^2)|^2 = F_{3\pi}^2 |\langle 3\pi, L|\mathcal{H}_w|K\rangle|^2$$

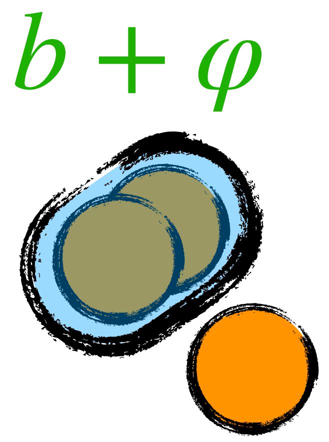
Volume dependence

$$F_{3\pi}^2 = 2E_K(\boldsymbol{P})L^6 \left| \mathcal{L}(E^*, m_{12}^2, m_{23}^2) \frac{1}{1 + F_3^\infty(E^*)\mathcal{K}_3(E^*)} \right|^2 \left(\frac{\partial F_3(E, \boldsymbol{P}, L)^{-1}}{\partial E} + \frac{\partial \mathcal{K}_3(E^*)}{\partial E} \right)$$

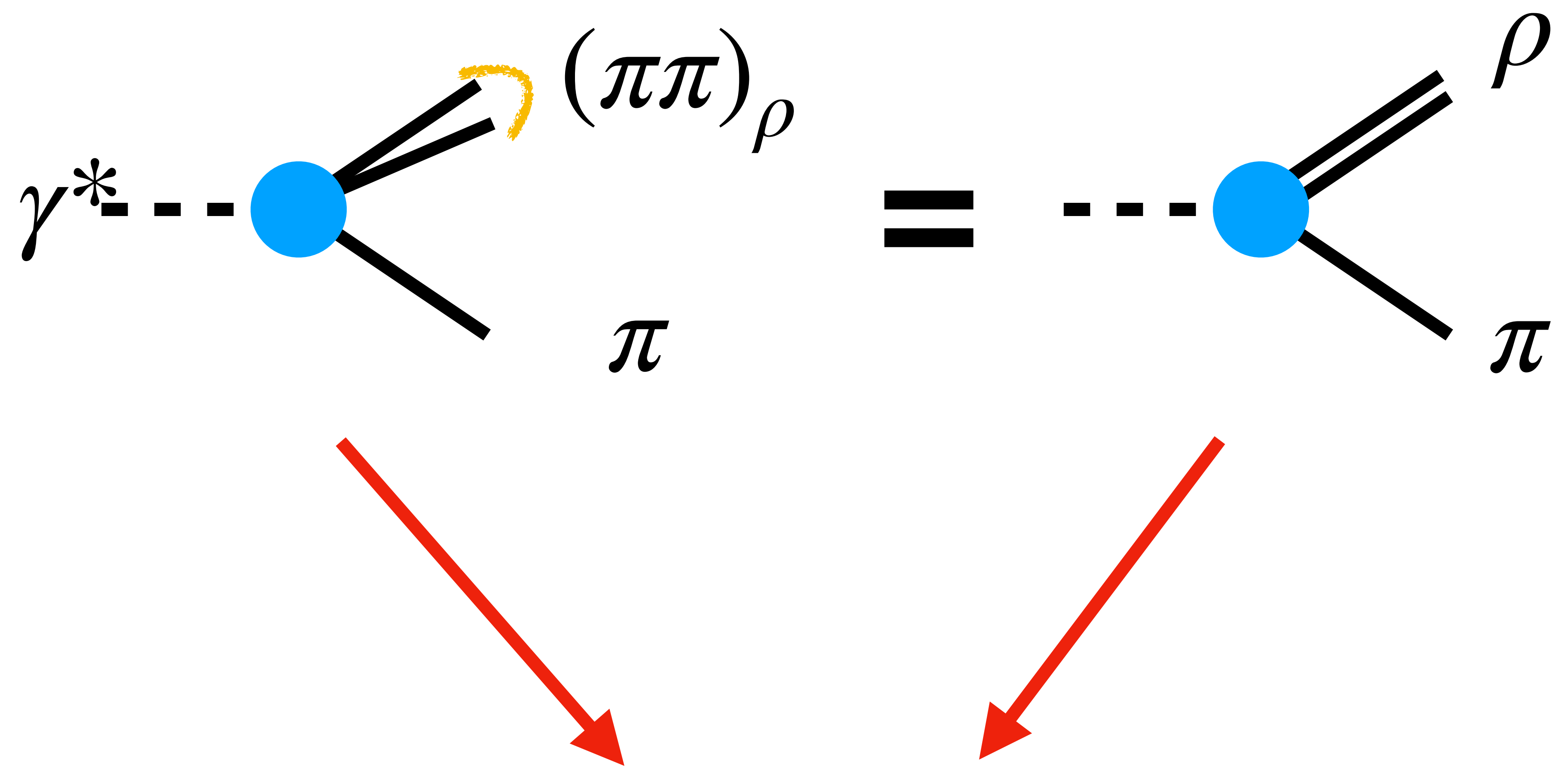
Two-body rescattering

Three-body Interactions

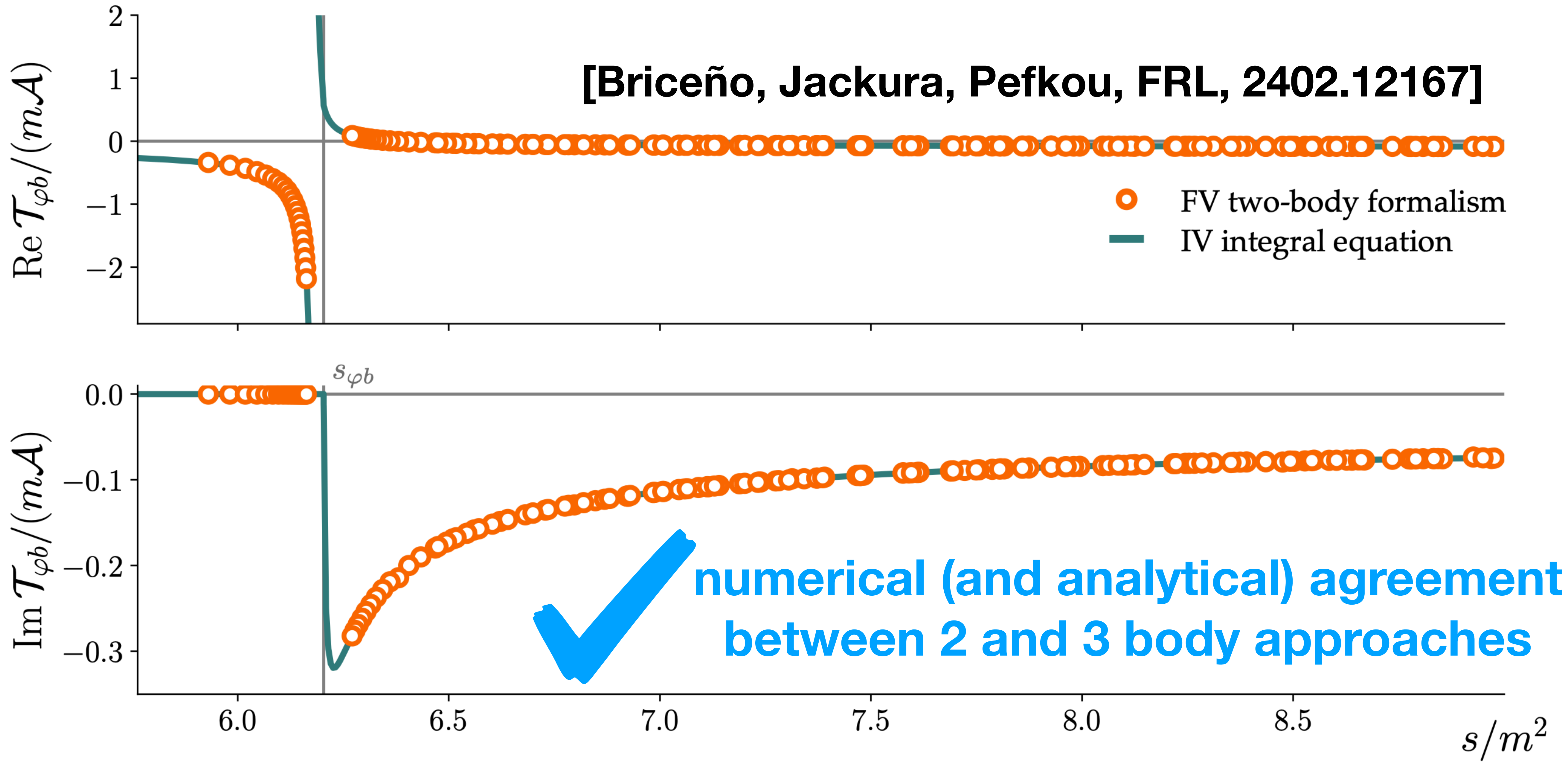
Consistency checks



- Consider a three-particle system where two particles can form bound states
  - For instance, $\rho\pi$ at heavier than physical pion masses
- Formalism for three-body decays should reduce to two-body Lellouch-Lüscher



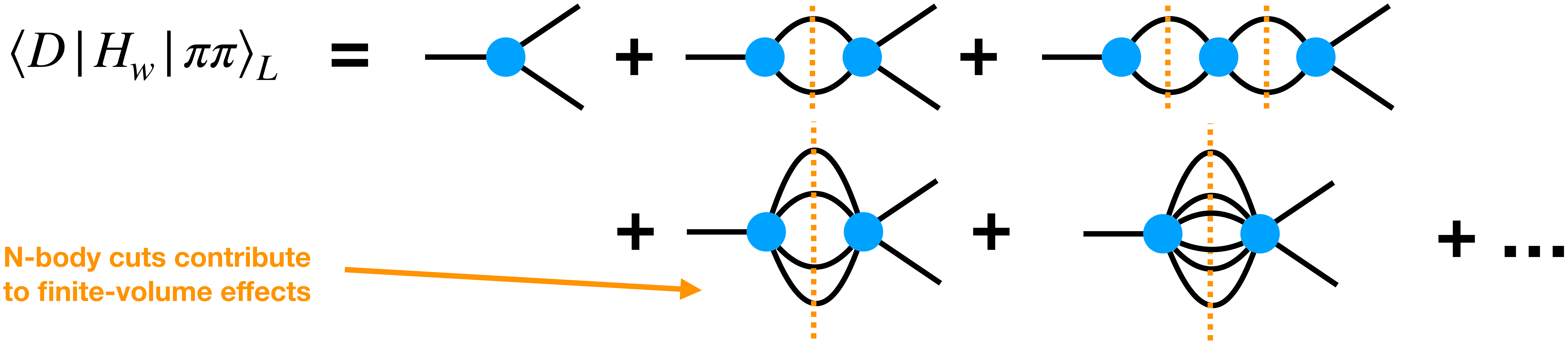
Descriptions of finite-volume effects should match!



Beyond three-hadron decays

- In principle, a similar analysis could could be possible for D decays

$$\langle D | H_w | \pi\pi \rangle_L =$$



- Main complication is the mixing between $2\pi \leftrightarrow 4\pi \leftrightarrow 6\pi$ states in finite volume
  - Needs knowledge of four, six and higher scattering amplitudes.
  - Thus, Lellouch-Lüscher approach might not be feasible beyond 3 or 4 hadron.

Summary and Outlook

Summary and Outlook

- Finite-volume scattering formalism allows to treat two and three-body amplitudes
  - Connection between energy levels and scattering amplitudes
  - Resonances (a.k.a. strong multi-hadron decays) obtained via poles in scattering amplitudes
- Many applications:
  - πN scattering, 3π scattering, T_{cc}
- Formalism for two and three-hadron decays is known
  - Volume-dependent corrections to finite-volume matrix elements
  - Electroweak hadronic decays need a dedicated calculation of final-state scattering amplitudes
- It is yet not know how to do scattering or decays beyond three hadrons from lattice QCD
  - Unclear if (Lellouch-)Lüscher-like approaches will be the optimal way forward

Summary and Outlook

- Finite-volume scattering formalism allows to treat two and three-body amplitudes
  - Connection between energy levels and scattering amplitudes
  - Resonances (a.k.a. strong multi-hadron decays) obtained via poles in scattering amplitudes
- Many applications:
  - πN scattering, 3π scattering, T_{cc}
- Formalism for two and three-hadron decays is known
  - Volume-dependent corrections to finite-volume matrix elements
  - Electroweak hadronic decays need a dedicated calculation of final-state scattering amplitudes
- It is yet not know how to do scattering or decays beyond three hadrons from lattice QCD
  - Unclear if (Lellouch-)Lüscher-like approaches will be the optimal way forward

Thanks!