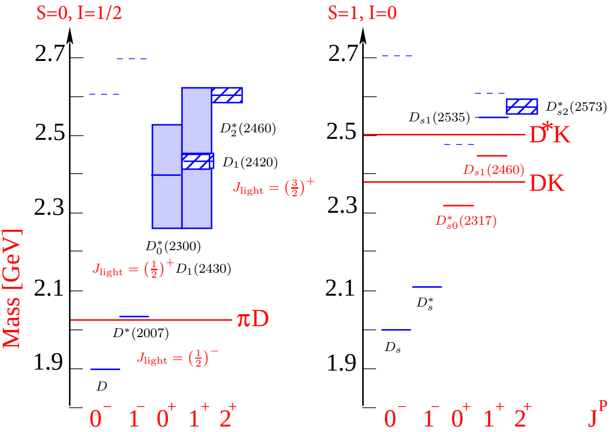
HADRON SPECTROSCOPY AND EXOTIC STATES IN THE CONTINUUM

on the example of pos. parity *D*-mesons

September 30, 2024 | Christoph Hanhart | IAS-4 Forschungszentrum Jülich

SETTING THE STAGE: *D*-MESONS



Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004

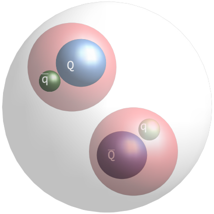
Puzzles: Why are/is

1. $M(D_{s1})\&M(D_{s0}^*)$ so light?
2. $M(D_{s1})-M(D_{s0}^*) \simeq M(D^*)-M(D)$?
3. $M(D_0^*) \simeq M(D_{s0}^*)$? $M(D_1) \simeq M(D_{s1})$?
4. Why do we have $M(D_0)^{\rm lat.} \ll M(D_0)^{\rm exp.}$?

All those puzzles disappear, if the states are hadronic molecules

HADRONIC MOLECULES

review article: Guo et al., Rev. Mod. Phys. 90(2018)015004



- are few-hadron states, bound by the strong force
- do exist: light nuclei.
  e.g. deuteron as pn & hypertriton as Λd bound state
- are located typically close to relevant continuum threshold;
  e.g., for $E_B = m_1 + m_2 - M$ ($\gamma = \sqrt{2\mu E_B}$; $\mu = m_1 m_2/(m_1 + m_2)$)
  - $E_B^{\rm deuteron} = 2.22$ MeV ($\gamma = 40$ MeV)
  - $E_B^{\rm hypertriton} = (0.13 \pm 0.05)$ MeV (to Λd) ($\gamma = 26$ MeV)
- can be identified in observables (Weinberg compositeness):

$$\frac{g_{\rm eff}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \;\rightarrow\; a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\;;\quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

$(1-\lambda^2)$=probability for molecular component in wave function

Corrections are $\mathcal{O}(\gamma R)$

Range corrections: Song, Dai, Oset (2022); Li, Guo, Pang, Wu (2022); Kinugawa, Hyodo (2022)

Are there mesonic molecules?

DISCLAIMERS AND OUTLINE

The method presented is 'diagnostic' — especially,

- it does not allow for conclusions on the binding force;
- it allows one only to study individual states;
- quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim R\gamma$)

In the rest of the talk I will present

- how unitarized chiral theory (UChPT) for GB-D-meson scattering solves all the mentioned puzzles of the pos. parity open flavor states
- how (lattice) data allow us to disentangle different scenarios

CHIRAL LAGRANGIAN (1)

We need a proper interaction $\Longrightarrow$ chiral perturbation theory

- The leading order Lagrangian (no free parameters)

$$\mathcal{L}^{(1)}_{\phi P} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger + \text{GB dynamics}$$

with $P = (D^0, D^+, D_s^+)$ for the D mesons, and the covariant derivative

$$\begin{aligned} D_\mu P &= \partial_\mu P + P\Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu)P^\dagger, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right), \quad u = \exp\left(i\lambda_a \phi_a/(2F_0)\right) \end{aligned}$$

where F_0 pion decay const.

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the Weinberg–Tomozawa term for $P\phi$ scattering:

$$\propto E_\phi/F_0^2 + \mathcal{O}(1/M_D) \quad (S - \text{wave})$$

Interaction of kaons significantly stronger than that of pions

CHIRAL LAGRANGIAN (2)

F-K Guo, CH, S. Krewald, U.-G. Meißner, PLB**666**(2008)251

- At the next-to-leading order order p^2 (6 free parameters)

$$\mathcal{L}^{(2)}_{\phi P} = P\left[-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right]P^\dagger$$
$$+D_\mu P\left[h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right]D_\nu P^\dagger + \text{em. terms},$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u,\ \chi = 2B_0\,\mathrm{diag}(m_u, m_d, m_s),$$

$$u_\mu = i\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right]$$

- Low-energy constants:

$h_1 = 0.42$: from $M_{D_s} - M_D$

Same effective operator leads to strong isospin violation

$m_{D^+} - m_{D^0} = \Delta m^{\text{strong}} + \Delta m^{\text{e.m.}} = ((2.5 \pm 0.2) + (2.3 \pm 0.6))$ MeV

$\Longrightarrow$ fixes pertinent e.m. term

h_0: from quark mass dependence of charmed meson masses (lattice)

$h_{2,3,4,5}$: fixed from lattice results on scattering lengths

$\Longrightarrow$ some NLO terms large $\Longrightarrow$ call for unitarisation $\Longrightarrow$ UChPT

UNITARISATION

chiral perturbation theory only perturbatively consistent with unitarity

$\Longrightarrow$ Unitarisation; allows for generation of bound states and resonances

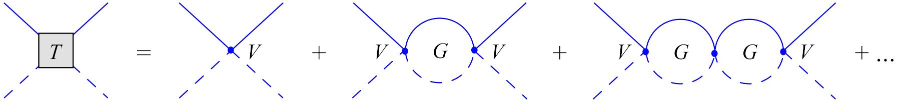
Truong, Dorado, Pelaez, Kaiser, Weise, Oller, Oset, Lutz, Kolomeitsev, Guo, Meißner, C.H., ...

Observe $\mathrm{Im}(t(s)) = \sigma(s)\,|t(s)|^2$ implies $\mathrm{Im}\left(t(s)^{-1}\right) = -\sigma(s)$

$\Longrightarrow$ write subtracted dispersion integral for $t(s)^{-1}$

$\Longrightarrow$ fix $\mathrm{Re}(t(s)^{-1})$ by matching to ChPT

Effectively this gives



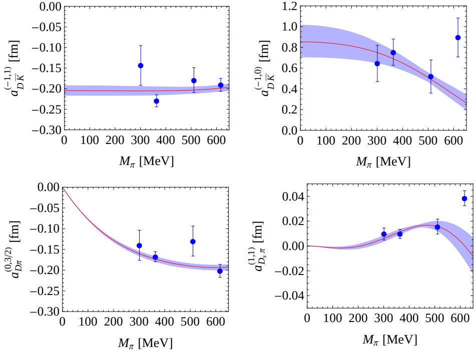
with ChPT expression for V ... and additional parameter $a(\mu)$ (from the loop)

Dependence on unitarization method needs to be clarified!

FIT TO LATTICE DATA

fit 4+1 para. to lattice data for $a_{D_x\phi}^{(S,I)}$ in selected channels

Liu et al. PRD87(2013)014508



controlled quark mass dependence

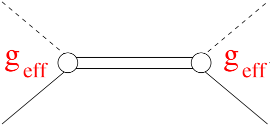
Fit range up to $M_\pi = 500$ MeV

- $\pi/K/\eta$–$D^{(*)}/D_s^{(*)}$ scattering fixed (chiral sym: πD int. weaker than KD)
- $D_{s0}^*(2317)$ emerges as a pole with $M_{D_{s0}^*} = 2315^{+18}_{-28}$ MeV ($E_b = 47^{+28}_{-18}$);

  since $E_b(D_{s0}) = E_b(D_{s1}^*) + \mathcal{O}(1/M_D) \Longrightarrow$ puzzel 2 solved

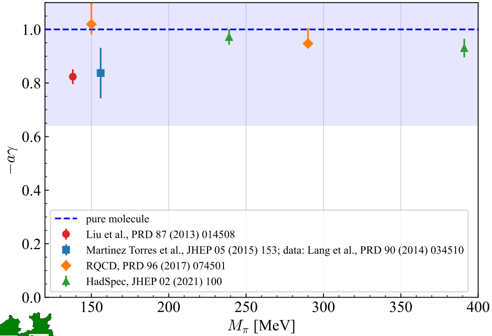
INTERPRETATION A LA WEINBERG

$D^*_{s0}(2317)$: $a = g_{\text{eff}}$



$g_{\text{eff}} + \mathcal{O}(1/\beta) \simeq -\left(\frac{2(1-\lambda^2)}{2-\lambda^2}\right)\frac{1}{\gamma}$

$\Longrightarrow a = -(1.05 \pm 0.36)\ \text{fm}$ for molecule ($\lambda^2 = 0$); smaller otherwise



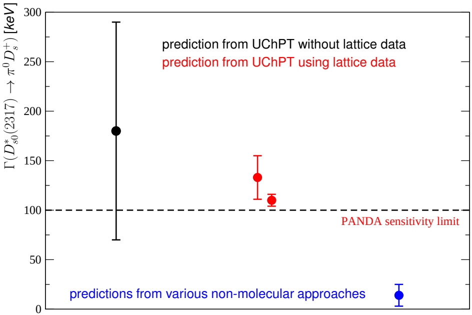
Various lattice studies show under binding $\Longrightarrow$

study $a\gamma$ (removes E_b dep.)

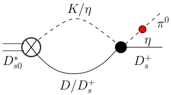
All analyses consistent with purely molecular $D^*_{s0}(2317)$ (analogous for $D_{s1}(2460)$)

$\Longrightarrow$ puzzel 1 solved

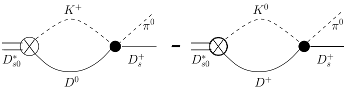
FURTHER TESTS



- Hadronic width:
  Genuine contribution:



Specific for molecules:



F.K. Guo et al., PLB666(2008)251; L. Liu et al. PRD87(2013)014508; X.Y. Guo et al., PRD98(2018)014510
and, e.g., P. Colangelo and F. De Fazio, PLB570(2003)180

Experiment needs high resolution $\to$ PANDA; from lattice QCD?

- Flavor symmetry: $M_{B^*_{s0}} = 5722 \pm 14$ MeV — Fu et al., EPJA58(2022)70

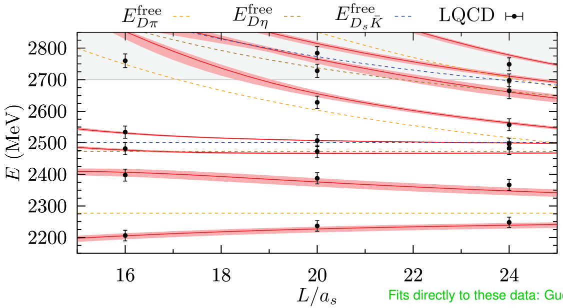
  Recent lattice result: $M_{B^*_{s0}} = 5699 \pm 14$ MeV — Hudspith & Mohler, PRD 107 (2023)114510 & next talk

Next: Study multiplet structure from GB-D-meson scattering

THE S = 0 SECTOR

Lattice: $@M_\pi = 391$ MeV pole at 2275.9 ± 0.9 MeV Had.Spec.Coll. JHEP10(2016)011
$@M_\pi = 239$ MeV pole at $2196 \pm 64 - i(210 \pm 110)$ MeV
HadSpec, JHEP07(2021)123

UChPT for $M_\pi = 391$ (parameters fixed in 2013): Albaladejo et al., PLB767(2017)465



finds poles for

- $M_\pi \simeq 391$ MeV at
  (2264, 0) MeV [000] &
  (2468, 113) MeV [110]
- $M_\pi = 139$ MeV at
  (2105, 102) MeV [100] &
  (2451, 134) MeV [110]

Fits directly to these data: Guo et al., EPJC 79(2019)13; Lutz et al., PRD106(2022)114038

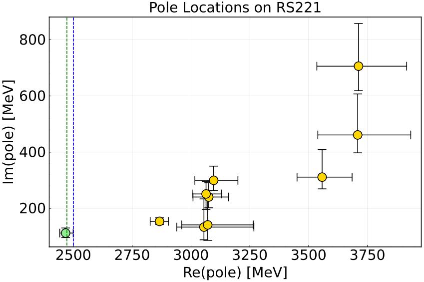
- Low mass incompatible with lowest 0^+ D at 2343 ± 10 MeV - why?
- Why do lattice analyses report only one pole but UChPT two?

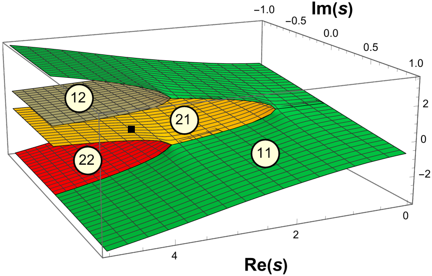
POLE STRUCTURE FROM LATTICE STUDY

Closer look @ $M_\pi = 391$ analysis

Moir et al. [Had.Spec.Coll.] JHEP10(2016)011

Second pole was present, but location depends on amplitude model





- Poles located on hidden on sheet

A. Asokan et al., EPJC83(2023)850

- Pole locations correlated; in line with pole from UChPT
- Distance to threshold balanced by size of residue

V. Baru et al.,EPJA23(2005)523

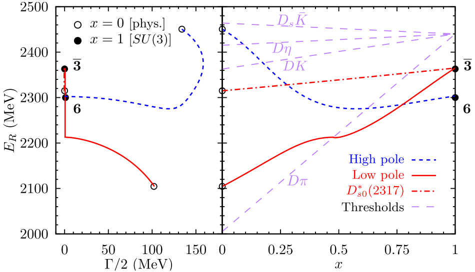
Explains correlation between Re(pole) and Im(pole)

SU(3) STRUCTURE FROM UCHPT

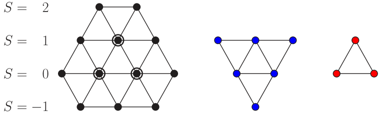
Albaladejo et al., PLB767(2017)465

$$m(x) = m^{\text{phy}} + x(m - m^{\text{phy}})$$

$m_\phi = 0.49$ GeV; $M_D = 1.95$ GeV



Multiplets: $[\bar{3}]\otimes[8]=[\overline{15}]\oplus[6]\oplus[\bar{3}]$

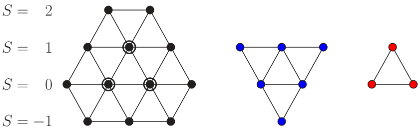


with $[\overline{15}]$ repulsive,
$[6]$ attractive,
$[\bar{3}]$ most attractive

- 3 poles give observable effect with SU(3)-breaking on
- At $SU(3)$ symmetric point $m_\phi \simeq 490$ MeV: 3 bound and 6 virtual states
- The light $D\pi$ state is the multiplet member of $D^*_{s0}(2317)$

$$\Longrightarrow \boxed{M_{D^*_{s0}(2317)} - M_{D^*_0(2100)} = 217 \text{ MeV}} \text{ puzzle 3 solved}$$

SU(3) STRUCTURE



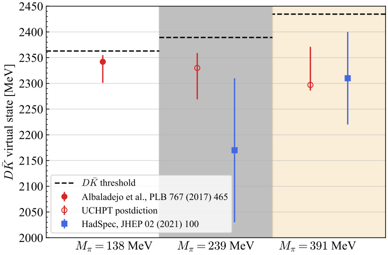
- Lattice shows repulsion in $[\overline{15}]$ as predicted in UChPT

Albaladejo et al., PLB767(2017)465

- States in [6] found in UChPT and lattice:

Hofmann and Lutz, NPA733(2004)142

  - $S = -1$



  - $S = 0$: Lattice finds virtual pole in [6] @$M_\pi \approx 600$ MeV in line with UChPT prediction

Gregory et al., [arXiv:2106.15391 [hep-ph]]+Lüscher analysis.

Confirmed by J.D.E. Yeo, C.E. Thomas and D.J. Wilson, [arXiv:2403.10498 [hep-lat]].

- Quark Model: $[\bar{3}] \otimes [1] = [\bar{3}]$ — the [6] is absent

COMPACT TETRAQUARKS

The heavy-light diquarks, *cq* of spin 0 and spin 1, in the flavor [3]

line up with diquarks of light anti-quarks $\bar{q}\bar{q}$: $[\bar{3}] \otimes [\bar{3}] = \underbrace{[3]}_{anti-sym.} \oplus \underbrace{[\bar{6}]}_{sym.}$

Imposing Fermi symmetry: anti-sym. in color $\Longrightarrow$

- spin 0 (anti-sym.) $\rightarrow$ flavor anti-sym. $\longrightarrow$ flavor [3]

Combining with the *cq* diquark: $[3] \otimes [3] = [\bar{3}] \oplus [6]$

L. Maiani, A. D. Polosa and V. Riquer, PRD110 (2024) 3

But there should also be

- spin 1 (sym.) $\rightarrow$ flavor sym. $\longrightarrow$ flavor $[\bar{6}]$

Combining with the *cq* diquark: $[3] \otimes [\bar{6}] = [\bar{3}] \oplus [\overline{15}]$

't Hooft int. pushes $[\bar{3}]$ up

Dmitrasinovic, PRD70(2004)096011

Mass estimates:

$$M_{cq}[S=1] - M_{cq}[S=0] \approx M_{D^*_{s1}(2460)} - M_{D_{s0}(2317)} \approx 140 \text{ MeV}$$
$$M_{qq}[S=1] - M_{qq}[S=0] \approx M_{\Sigma_c} - M_{\Lambda_c} \approx \underline{170 \text{ MeV}}$$
$$\approx 300 \text{ MeV}$$

There should be a $[\overline{15}]$-state about 300 MeV above 2.1 GeV

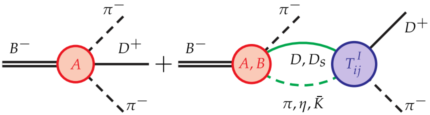
Guo, C.H. in preparation

Absent in lattice data $\Longrightarrow$ Tetraquark picture falsified (!?)

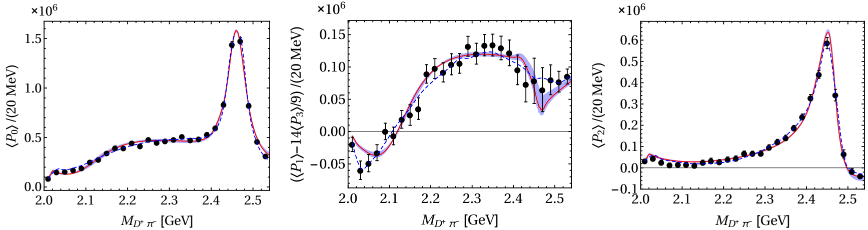
EXPERIMENTAL ACCESS: $B^- \to D^+\pi^-\pi^-$

With ϕD amplitude fixed we can calculate production reactions:

Du et al., PRD98(2018)094018; for more results see Du et al., PRD99(2019)114002



for the *S*-wave (two free para.);
other partial waves from BW-fit

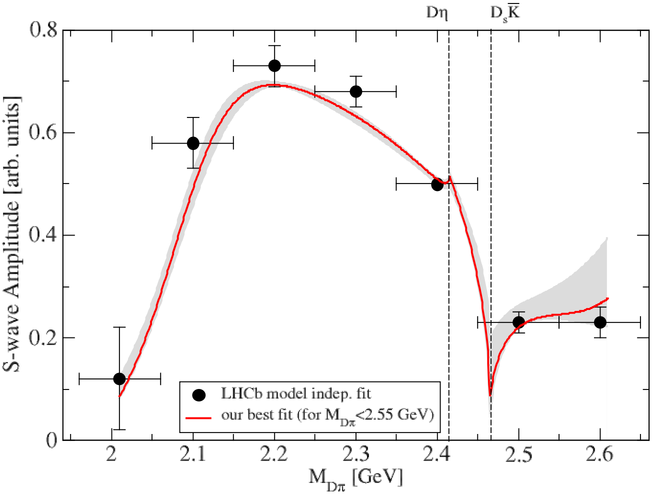


LHCb, PRD94(2016)072001

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2, \quad \langle P_2 \rangle \propto \tfrac{2}{5}|\mathcal{A}_1|^2 + \tfrac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2 - \delta_0)$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1 - \delta_0)$$

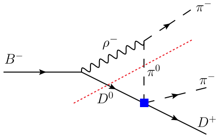
$D\pi$ S-WAVE FROM $B^- \to D^+\pi^-\pi^-$

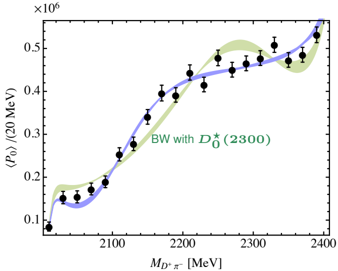


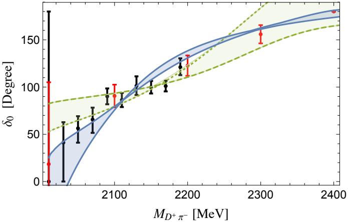
Effect of higher thresholds enhanced, by pole at $\sqrt{s_p} \sim (2451 - i134)$ MeV

on nearby unphysical sheet

LIGHTEST CHARMED SCALAR







Mass of lightest charmed $J^P = 0^+$ state:

Analogous pattern for $J^P = 1^+$

- BW with $m = 2300$ MeV incompatible with data
- UChPT with $(2105 \pm 8 - i(102 \pm 11))$ MeV is compatible

Du et al., PRL126(2021)192001

Uncertainties from loop!

Direct access to πD phases via $B \to D\pi l\nu$

talk Florian Herren, tomorrow

CHARMED STATES

Puzzles: posed by $D_{s0}(2317)$ & $D^*_{s1}(2460)$ and our Solution

Why are $M(D_{s1})\&M(D^*_{s0})$ so light?	Since they are are DK and D^*K bound states (=hadronic molecules)
Why is $M(D_{s1})-M(D^*_{s0}) \simeq M(D^*)-M(D)$?	Since spin symmetry gives equal binding
Why is $M(D^*_0) \simeq M(D^*_{s0})$? and $M(D_1) \simeq M(D_{s1})$?	Since listings need to be corrected: Lightest D_0@2100 MeV & D_1@2240 MeV
Why do we have $M(D_0)^{\mathrm{lat.}} \ll M(D_0)^{\mathrm{exp.}}$?	Since structure at 2300 MeV is made of two poles

... role of left-hand cuts needs to be clarified

Lutz et al., PRD106(2022)114038; Korpa et al., PRD107(2023)L031505

SUMMARY AND CONCLUSION

- For near threshold states Weinberg criterion provides proper diagnostics
- View extended by studying the $SU(3)_f$ multiplet structure
  - what kinds of multiplets are there?
  - pattern of spin and flavor symmetry breaking important to disentangle different scenarios
- Interplay of different poles leads to
  - non-trivial line shapes
  - non-trivial phase motions

We are on a good path to identify the hadronic molecules in the spectrum

... and to exploit their imprint on various observables

Thanks a lot for your attention