Hadron spectroscopy and exotic states from Lattice QCD

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Outline

1. Introduction and Motivation
2. Example 1: Hadrons with beauty-quarks in a Lattice NRQCD setup
   - Approach and technical aspects
   - Positive-parity heavy-light hadrons
   - Doubly-heavy tetraquarks
3. Example 2: Coupled-channel scattering and the $\Lambda(1405)$
4. Conclusions and Outlook

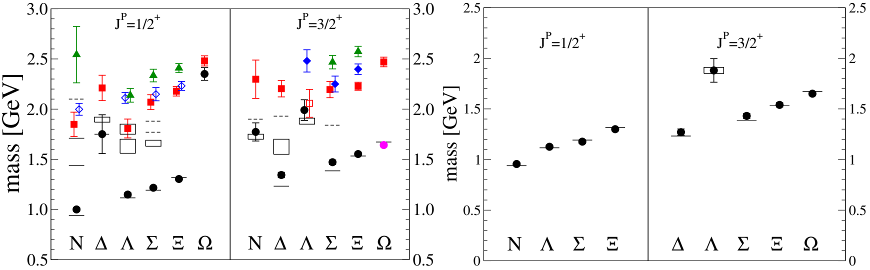
Lattice QCD and quark-model puzzles

- Various kind of exotic/unconventional states (examples)
  - light scalar resonances (σ and κ)
  - $D^*_{s0}(2317)$, $D_{s1}(2460)$ and b-quark cousins
  - XYZ *charmonium-like* QCD states
  - hybrid mesons
  - Roper resonance; $\Lambda(1405)$
  - Pentaquark states
  - Glueballs, . . .

- Considerable experimental hadron spectroscopy effort
  - Examples: BelleII, BESIII, COMPASS, GlueX, LHCb, $\bar{\text{P}}$ANDA
  - The $\bar{\text{P}}$ANDA experiment at the FAIR facility will be ideally suited to shed light on long-standing puzzles

Baryon bound-states and resonances: Ancient history

Engel, Lang, DM, Schäfer, PRD 87 074504 (2013)



- Lattice QCD calculations with $\bar{q}q$ and qqq interpolating fields often struggle to make contact to experiment; mostly no indications of multiparticle levels
  $\rightarrow$ Multi-hadron interpolators needed
- Spectra like these sensible for
  - getting an idea about the number of states
  - spectra at very heavy quark masses
  - some narrow states (i.e. high spin)

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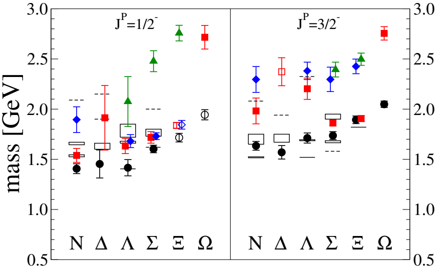

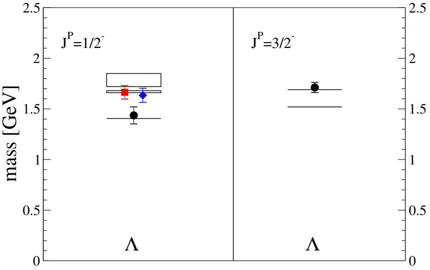

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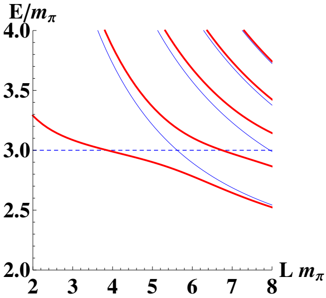
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite-volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \to \pi\gamma^*$)
- Significant progress for 3-particle scattering
  See Fernando Romero-López (Wednesday)



Challenges for bound-state/ resonance calculations

- Hierarchy of difficulties
  - Meson systems are simpler than baryons
    (exponentially degrading signal to noise)
  - Cost of correlation functions much larger for systems with baryons
  - Complicated scattering amplitudes need more data (volumes, frames)
    single two-hadron channel; coupled two-hadron channels; three-hadron scattering
- Hierarchy of projects:
  - Proof of principle
  - Explore quark mass dependence
  - Full spectroscopy calculation including continuum limit
  - Structure observables (transitions, form factors, . . .)
- This talk:
  - Two examples for exotic hadrons with beauty quarks
    Most systematics can be addressed
  - $\Lambda(1405)$ in coupled-channel $\pi\Sigma$–$\bar{K}N$-scattering
    Difficult but feasible with current methods

Projects and People

- Heavy-quark exotics ($ud\bar{b}\bar{b}$ and $us\bar{b}\bar{b}$) and positive-parity heavy-light mesons

  `R.J. Hudspith, DM, PRD 107, 114510 (2023)`

  `Editors' Suggestion`

  - TU Darmstadt/GSI: Jamie Hudspith, Daniel Mohler
- $\Lambda(1405)$ and meson-baryon scattering:

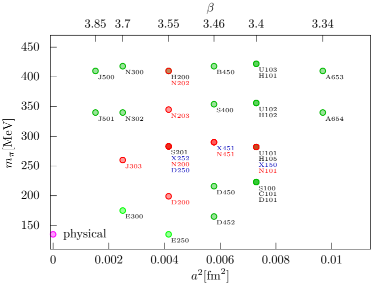
  `John Bulava, DM,` *`et al.`*`, PRD 109 014511 (2024)`
  `John Bulava, DM,` *`et al.`*`, PRL 132 051901 (2024)`

  `Both Editors' Suggestions`

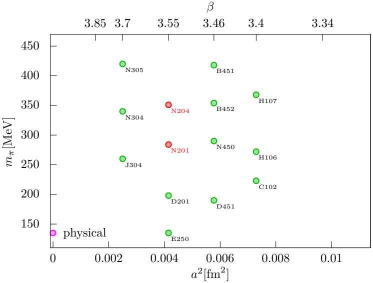
  - DESY Zeuthen → Bochum: John Bulava
  - BIN'S: Andrew Hanlon
  - Intel: Ben Hörz
  - North Carolina: Amy Nicholson, Joseph Moscoso
  - TU Darmstadt/GSI: Daniel Mohler, Barbara Cid Mora
  - CMU: Colin Morningstar, Sarah Skinner
  - MIT: Fernando Romero-López
  - LBNL: André Walker-Loud

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



plot style by Jakob Simeth, RQCD

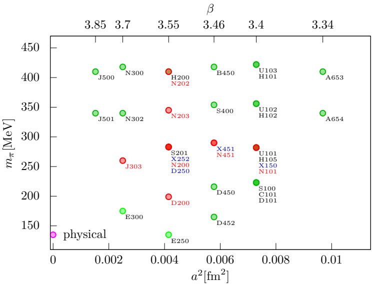
Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) physical quark masses

CLS gauge field ensembles

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

$Tr(M) = \text{const.}$



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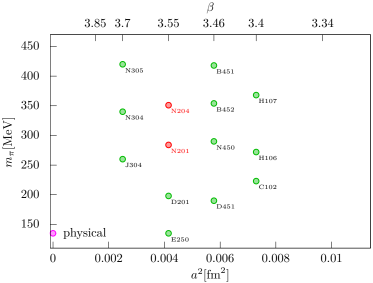


plot style by Jakob Simeth, RQCD

Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Want to exploit (power law) finite volume effects (keeping exponential effects small)
- Calculation at (or extrapolation to) physical quark masses

Outline

NRQCD action

Typical tadpole-improved NRQCD action (here we will use n=4)

Lepage *et al.*, PRD 46, 4052-4067 (1992)

$$
\begin{aligned}
H_0 &= -\frac{1}{2aM_0}\Delta^2, \\
H_I &= \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)(\Delta^2)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)} \\
H_D &= -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B} \\
\delta H &= H_I + H_D.
\end{aligned}
$$

Propagators generated through symmetric evolution equation

$$
G(x,t+1) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n \tilde{U}_t(x,t_0)^\dagger \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G(x,t).
$$

- We also tune a $\mathcal{O}(v^6)$ action with tree-level coefficients for the higher order terms

Neural net NRQCD tuning and setup

R.J. Hudspith, DM, PRD 106, 034508 (2022)
R.J. Hudspith, DM, PRD 107, 114510 (2023)

- Calculate runs with a random distribution for the action parameters
- Let the neural network make parameter predictions
- Due to additive mass we must only consider splittings $\rightarrow$ we subtract the η_B from all states
- Perform tuning at $SU(3)_f$-symmetric point
- Gauge-fixed wall sources
- Tuning precision is about 1%

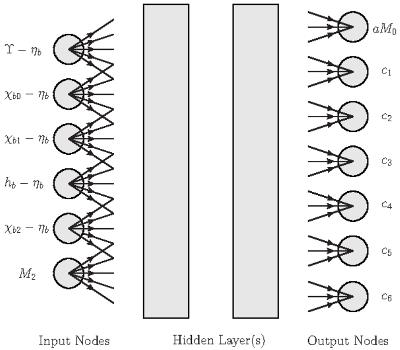


Figure: Schematic picture of our NRQCD setup

Input used for the tuning

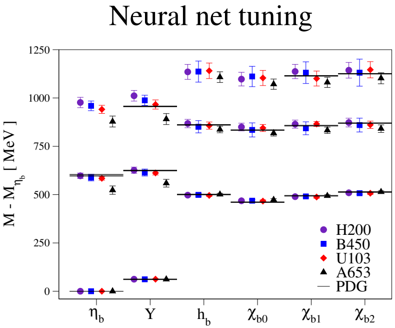
Consider only quark-line connected parts of simple meson operators

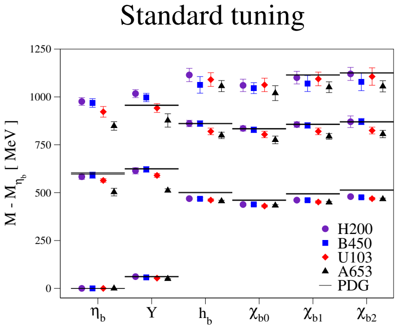
$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j \Delta_i - \sigma_i \Delta_j \ (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j \Delta_i + \sigma_i \Delta_j \ (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs.

NRQCD Neural Net Tuning: Stable s- and p-wave bottomonia





- Higher S- and P-wave states serve as a check whether our tuning leads to reasonable results
- Main results from the lattice spacing of U103; H200 used to estimate systematics

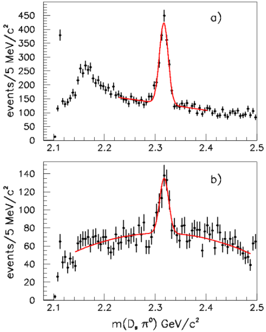
Exotic D_s and B_s candidates

Established s and p-wave hadrons:

D_s ($J^P = 0^-$) and D_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+),
$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)

B_s ($J^P = 0^-$) and B_s^* (1^-)
?
$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

$D_{s0}^*(2317)$:
PRL 90 242001 (2003)



- Corresponding $D_0^*(2400)$ and $D_1(2430)$ are broad resonances
- Perceived peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ (an old dispute; likely not the case)
- Additional exotic states are expected (in the sextet representation)

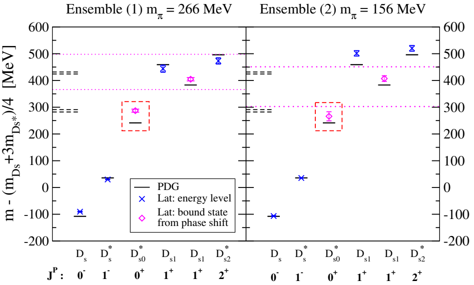
  See for example Kolomeitsev, Lutz, PLB 582, 39 (2004)

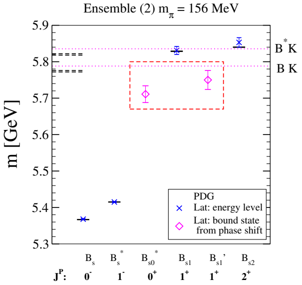
- B_s cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ not (yet) seen in experiment

Positive-parity states in the D_s and B_s spectrum

DM *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)





- Spectrum reliably extracted and agrees qualitatively with experiment
- Uncontrolled systematics sizable for the D_s states
- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD .

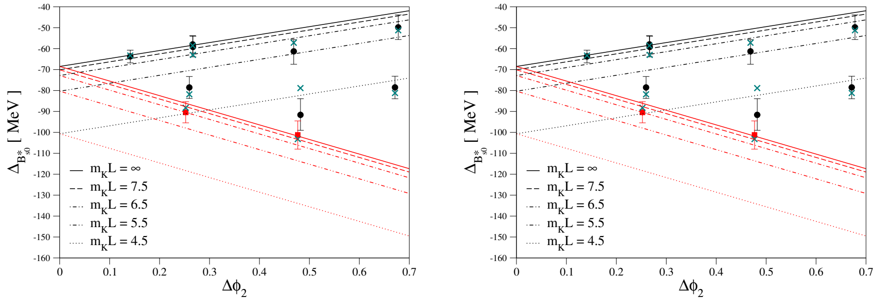
B_s: Chiral – infinite volume extrapolation

- We explore the previously predicted $J^P = 0^+$ and 1^+ bound states
- Mainly the CLS TrM = const trajectory and $2\,m_S$ = const ensembles

Combined extrapolation:

$$\Delta_{B^*_{s0}/B_{s1}}(\Delta\phi_2, m_K L, a) = \Delta_{B^*_{s0}/B_{s1}}(0,\infty,a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

$$\Delta\phi_2 = \phi_2^{\mathrm{Lat}} - \phi_2^{\mathrm{Phys}} \qquad ; \qquad \phi_2 = 8t_0 m_\pi^2$$



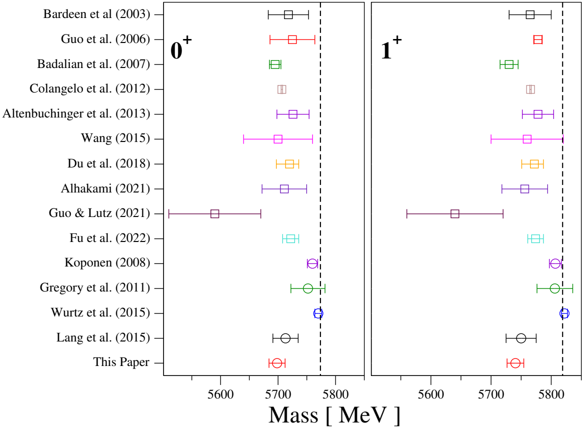
Systematic uncertainties and final result

Resulting binding energies:

$$\Delta_{B_{s0}^*}(0, \infty, 0) = -75.4(3.0)_{\text{Stat.}}(13.7)_{\text{a}}\ [\text{MeV}],$$
$$\Delta_{B_{s1}}(0, \infty, 0) = -78.7(3.7)_{\text{Stat.}}(13.4)_{\text{a}}\ [\text{MeV}].$$

- Small uncertainty from statistics + combined extrapolation
- Largest systematics from usage of NRQCD/discretization effects
- Central value shifted by applying half the mass difference between H200 and U103
- All other explored uncertainties (finite volume shapes, modified quark-mass dependence, etc.) small

Comparison to the literature



- Results agree well with models based on unitarized χPT
- Improved uncertainty estimate over older Lattice calculations

Tetraquarks - the T_{bb}

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark, T_{bb}, is the most concrete pure-tetraquark candidate phenomenologically and from the lattice in terms of being deeply-bound and strong-interaction-stable.
Cousin of the T_{cc} but likely has quite different physics,

T_{bb} **bound by** ≈ 100 **MeV**, T_{cc} by 360 KeV

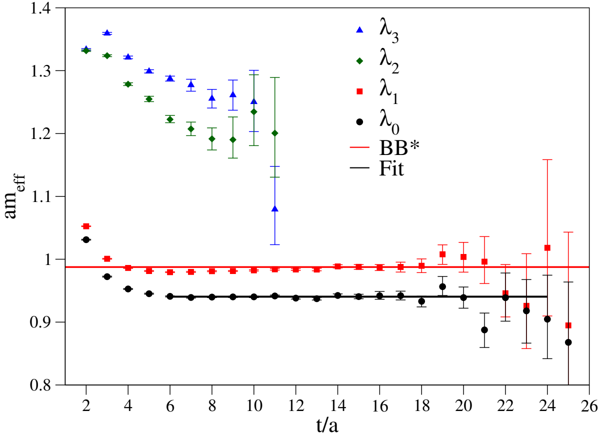
T_{bb} often described by the diquark picture:

- "Good" (attractive) light diquark $(u^T C\gamma_5 d)$ - lighter diquark increases binding
- Color-Coulomb heavy antidiquark $(\bar{b}C\gamma_i\bar{b}^T)$ - deeper binding as heavy mass gets heavier

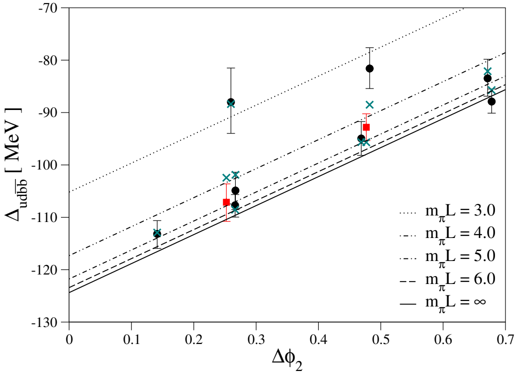
No Wick-contractions with annihilation $\rightarrow$ easy to compute on the lattice!

T_{bb} – Basis and effective masses (on N101)

$$D = ({u_a}^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T), \quad E = ({u_a}^T C\gamma_t\gamma_5 d_b)(\bar{b}_a C\gamma_i\gamma_t \bar{b}_b^T),$$
$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i d) - [u \leftrightarrow d], \quad N = (\bar{b} I u)(\bar{b}\gamma_5\gamma_i d) - [u \leftrightarrow d].$$



Combined mass and volume extrapolations

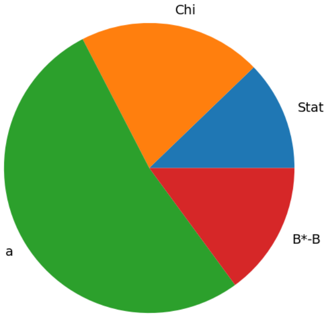


- Ansatz for a deeply-bound state:

$$\Delta_{ud\bar{b}\bar{b}}(\Delta\phi_2, m_\pi L, a) = \Delta_{ud\bar{b}\bar{b}}(0, \infty, a)(1 + A\Delta\phi_2 + Be^{-m_\pi L}).$$

- Strong $e^{-m_\pi L}$ volume effects and deeper binding at lighter pion mass.

T_{bb} – quantifying systematics

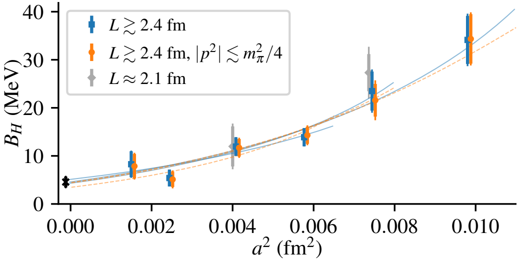


$$\Delta_{ud\bar{b}\bar{b}}(0,\infty,0) = -112.0(2.7)_{\text{Stat.}}(4.5)_{\chi}(11.6)_{a}(3.3)_{B^*-B}$$

- $(..)_a$ uncertainty from comparison of the results for two lattice spacings (H200 vs. U103)
- Two leading systematic uncertainties come from discretization effects/ the use of Lattice NRQCD!

Cautionary tale: The H-Dibaryon and discretization effects

Green, Hanlon, Junnarkar, Wittig, PRL 127 242003 (2021)

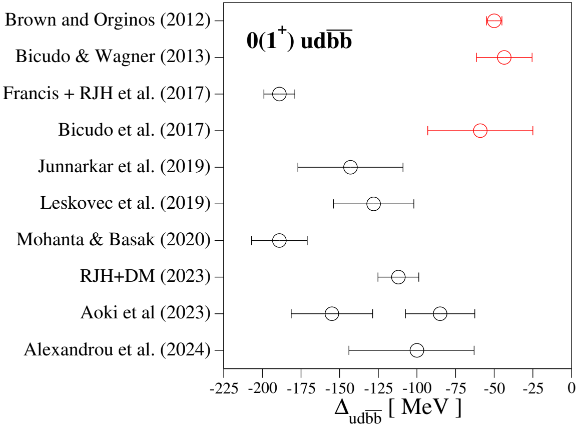


- First study of baryon-baryon scattering in the continuum limit
- Strategy: Global fits to the energy levels with parameterizations that account for discretization effects
- Binding energy at $SU(3)$ point with $m_\pi = 420$ MeV

$$B_H^{SU(3)_f} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$

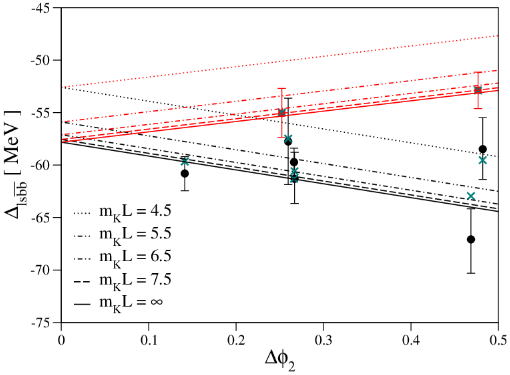
- Very large discretization effects in the binding energy!

Overview of Lattice $I(J^P) = 0(1^+)$ T_{bb} determinations



- Red: Static b-quarks; Black: Lattice NRQCD b quarks
- Interesting playground for understanding systematic uncertainties!

T_{bbs} – chiral and infinite volume extrapolation

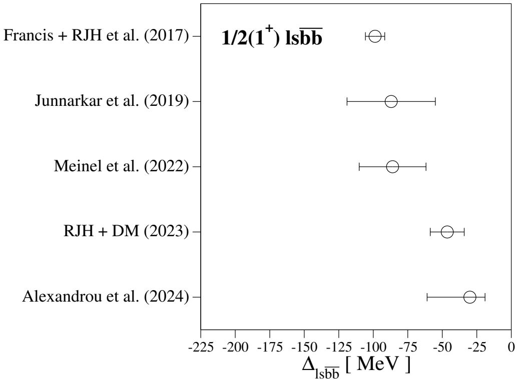


- Chiral/infinite-volume Ansatz:

$$\Delta_{\ell s\bar{b}\bar{b}}(\Delta\phi_2, m_K L, a) = \Delta_{\ell s\bar{b}\bar{b}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

- Large $e^{-m_K L}$ volume effects.
- Consistent with light-diquark picture.

Overview of lattice T_{bbs} determinations



- Close/overlapping EM threshold $BB_s\gamma$, still possible that it is narrow and decays weakly

Outline

An old puzzle: $\Lambda(1405)$, $J^P = \frac{1}{2}^-$

- PDG (4 star resonance)

$$M_\Lambda = 1405^{+1.3}_{-1.0} MeV \qquad \Gamma_\Lambda = 50.5 \pm 2.0$$

(Some) quark models struggled to accommodate this state.

- However
  - Unitarized χPT + Model input yields 2 poles with $\Re \approx 1400$ MeV
    $\rightarrow$ Now new PDG state $\Lambda(1380)$
  - CLAS observes different line shapes for $\Sigma^-\pi^+$, $\Sigma^+\pi^-$ and $\Sigma^0\pi^0$
    Interference between $I = 0$ and $I = 1$ amplitudes is the likely reason
  - Even the $\Sigma^0\pi^0$ is badly described by a single Breit-Wigner
  - CLAS data consistent with popular 2-pole picture
- Relevant channels: $\Sigma\pi$, $N\bar{K}$ (and maybe $\Lambda\eta$); simulation in isospin limit
- Goal: Explore coupled channel problem and extract scattering amplitudes from the low-lying energy spectrum

$\Lambda(1405)$ – Experimental developments

- Angular analysis of the process $\gamma + p \to K^+ + \Sigma + \pi$ by CLAS strongly favors the assignment of quantum numbers $J^P = \frac{1}{2}^-$

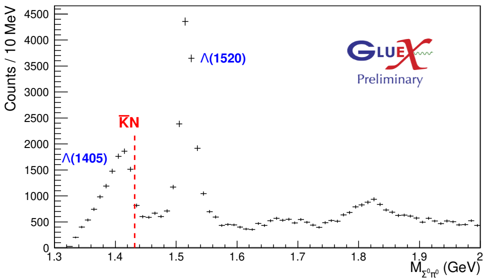
  Moriya *et al.*, PRC 87 035206 (2013)

- $K^- p$ scattering length determined by the SIDDHARTHA collaboration

  Bazzi *et al.*,PLB 704 (2011) 113

- A glimpse of the future: Preliminary analysis at GlueX

  Wickramaarachchi *et al.*, arXiv:2209.06230



Technical details: Ensemble and group theory

Current data on CLS Ensemble D200

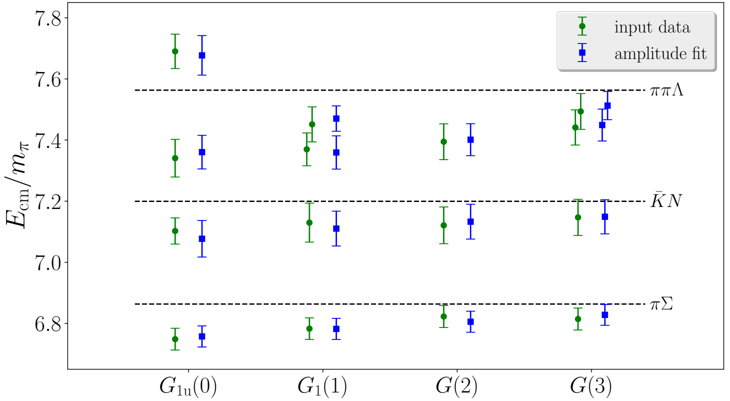
a [fm]	$T \times L^3$	m_π [MeV]	m_K [MeV]	$m_\pi L$	N_{cnfg}
0.0633(4)(6)	128×64^3	200	480	4.3	2000

Lattice irreducible representations for a given J^P

see Morningstar *et al.* arXiv:1303.6816

J^P	[000]	[00n]	[0nn]	[nnn]	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	Λ, $\Lambda(1600)$
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Lambda(1405)$, $\Lambda(1670)$
$\frac{3}{2}^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1690)$
$\frac{3}{2}^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1520)$, $\Lambda(1690)$

Finite-volume spectra



- Amplitude analysis uses ratios to extract energy differences with regard to non-interacting levels
- Blue squares indicate results from our preferred amplitude fit

Families of simple parameterizations (out of 6 total)

- An ERE in the K matrix:

$$\frac{E_{\rm cm}}{M_\pi}\tilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma},$$

$\Delta_{\pi\Sigma}$ measures the distance from the $\pi\Sigma$ threshold

- ERE in the inverse K matrix:

$$(\tilde{K}^{-1})_{ij} = \frac{E_{\rm cm}}{M_\pi}\left(\widetilde{A}_{ij} + \widetilde{B}_{ij}\Delta_{\pi\Sigma}\right),$$

- Blatt-Biederharn parameterization:

$$\tilde{K}_{ij} = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} f_0(E_{\rm cm}) & 0 \\ 0 & f_1(E_{\rm cm}) \end{pmatrix} \begin{pmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{pmatrix}$$

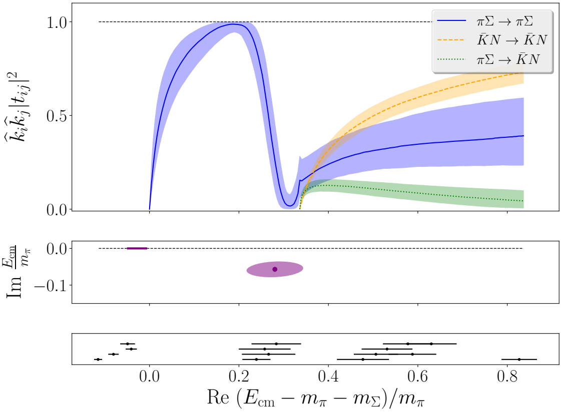
$$f_i(E_{\rm cm}) = \frac{M_\pi}{E_{\rm cm}}\frac{a_i + b_i\Delta_{\pi\Sigma}}{1 + c_i\Delta_{\pi\Sigma}}.$$

- Parameterization based on the leading-order Weinberg-Tomozawa term:

$$\widetilde{K}_{ij} = \widehat{C}_{ij}\left(2E_{\rm cm} - M_i - M_j\right),$$

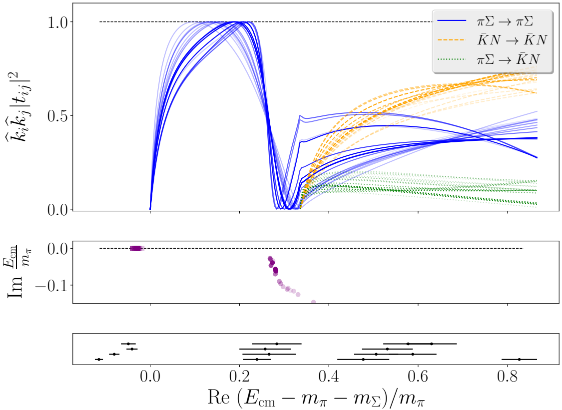
where $M_0 = m_\Sigma$ and $M_1 = m_N$.

Our preferred amplitude and resulting poles



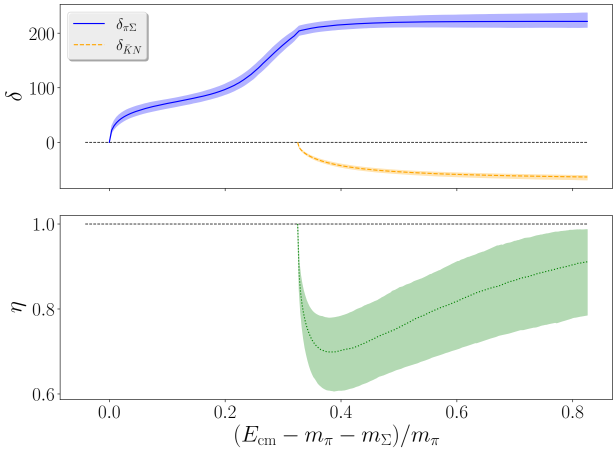
- Amplitudes evaluated with Akaike Information Criterion: $AIC = \chi^2 - 2\text{dof}$
- Sub-threshold levels pose strong constraints on the amplitude
- Limited data and therefore limited possibility to vary parameterizations

Some Variations of the used amplitude



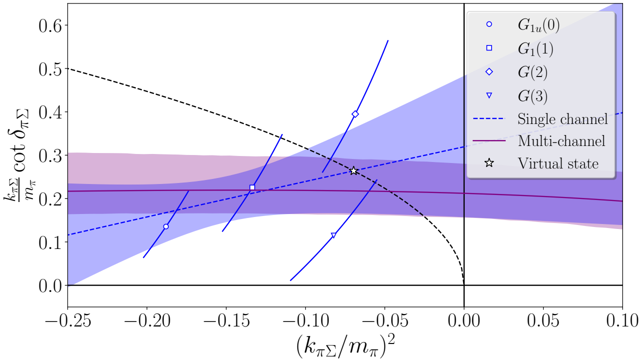
- Results from varying parameterization/ omitting highest data point
- Amplitudes that can accommodate 0,1, and 2 poles all yield 2 poles
- We also explored simple constraints for higher partial waves (negligible effect in range used)

Same thing different: Phases and inelasticity



- Alternative way of showing our results: 2 phases and inelasticity η

$\pi\Sigma$ scattering phase-shift close to threshold



- Lattice QCD provides unique constraints below the $\pi\Sigma$ threshold
- Single-channel treatment agrees qualitatively with our full coupled-channel parameterization using the larger energy range
- Results in a virtual bound-state on ensemble D200

Pole positions and expectations from the literature

- Poles labeled as $(\pm, \pm)$
  depending on the signs of the imaginary part of $(k_{KN}, k_{\pi\Sigma})$
- Two poles are found on the $(-, +)$ sheet,
  the closest to physical scattering between the thresholds
- Our final result for the poles is

$$\begin{aligned} \text{Pole II} \quad & 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\ \text{MeV} \\ \text{Pole I} \quad & 1455(13)_{\text{stat}}(2)_{\text{model}}(17)_{\text{a}}\ \text{MeV} \\ & - i \times 11.5(4.4)_{\text{stat}}(4)_{\text{model}}(0.1)_{\text{a}}\ \text{MeV} \end{aligned}$$

- Examples from the PDG review

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i\ 81^{+19}_{-8}$
Ref. [17], Fit II	$1421^{+3}_{-2} - i\ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i\ 114^{+24}_{-25}$
Ref. [18], solution #2	$1434^{+2}_{-2} - i\ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i\ 56^{+17}_{-11}$
Ref. [18], solution #4	$1429^{+8}_{-7} - i\ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i\ 90^{+12}_{-18}$

Outline

Conclusions and Outlook

- Positive-parity heavy-light mesons
  - NRQCD calculation with full uncertainty estimate for B_0^* and B_{s1} $\rightarrow$ refined predictions for LHCb, BelleII
  - Calculation could be further improved with RHQ action
- Doubly-heavy tetraquarks
  - Results for the T_{bb} and T_{bbs} from different groups mostly agree
  - Situation less clear for other flavor combinations/ quantum numbers
- Coupled-channel scattering and the $\Lambda(1405)$
  - First coupled-channel LQCD calculation in the baryon sector
  - Suitable K-matrix parameterizations suggest two poles at our m_π
  - We will explore the quark-mass dependence and calculate more comprehensive spectra
  - We pan to explore other channels with strangeness
  - Inconvenient things: discretization effects, chiral extrapolation, better parameterizations

Backup slides

CLS ensembles used for these studies

R.J. Hudspith, DM, PRD 107, 114510 (2023)

Ensemble	Mass trajectory	$L^3 \times L_T$	$N_{\text{Conf}} \times N_{\text{Prop}}$
U103	$\text{Tr}[M] = C$	$24^3 \times 128$	1000×23
H101	$\text{Tr}[M] = C$	$32^3 \times 96$	500×12
U102	$\text{Tr}[M] = C$	$24^3 \times 128$	732×18
H102	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
U101	$\text{Tr}[M] = C$	$24^3 \times 128$	600×18
H105	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
N101	$\text{Tr}[M] = C$	$48^3 \times 128$	537×18
C101	$\text{Tr}[M] = C$	$48^3 \times 96$	400×16
H107	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H106	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H200	$\text{Tr}[M] = C$	$32^3 \times 96$	500×28

Varying the NRQCD tuning

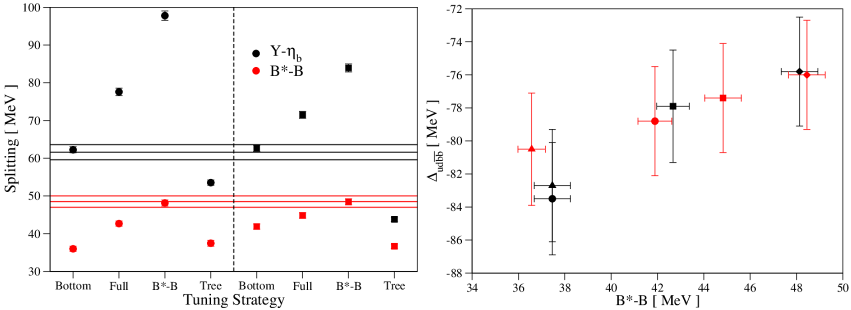
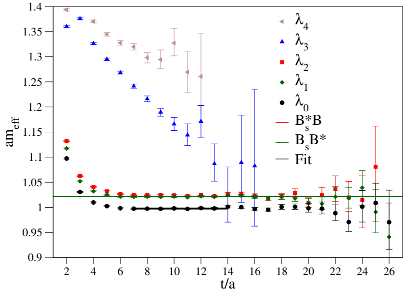


Figure: Alternative tuning strategies with/without B-mesons and higher-order terms (left). Clear correlation of the $B^* - B$ splitting with the T_{bb} binding. (right)

- Simultaneously reproducing both hyperfine splittings seems impossible
- Tree-level performs poor; For our strategies higher order terms help.
- Shallower T_{bb} binding, with increased $B^* - B$ splitting.

T_{bbs} – Basis and effective masses

$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i s), \quad N = (\bar{b}Iu)(\bar{b}\gamma_5\gamma_i s)$$
$$O = (\bar{b}\gamma_5 s)(\bar{b}\gamma_i u), \quad P = (\bar{b}Is)(\bar{b}\gamma_5\gamma_i u)$$
$$Q = \epsilon_{ijk}(\bar{b}\gamma_j u)(\bar{b}\gamma_k s).$$



Sad prospects for T_{bb}: Difficult to see at the LHC

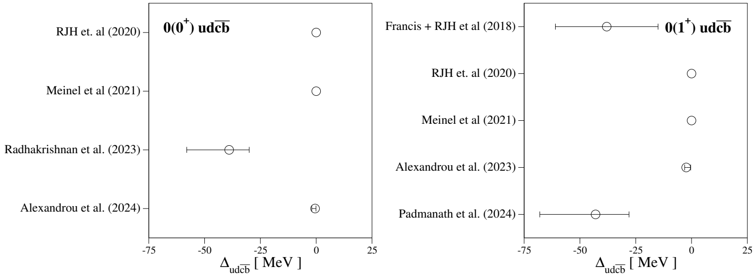
- T_{bb} is very heavy (≈ 10.5 GeV) and decays weakly
- A possible exemplary decay channel could be
  see Phys.Rev.Lett. 118 (2017) 14, 142001 - A. Francis, RJH et al.:

$$T_{bb} \to B^+ \bar{D}^0$$

- It is unlikely to be found anytime soon at the LHC

- Obvious next candidate 0^+ or 1^+ $ud\bar{c}\bar{b}$ "T_{cb}"
  potentially unbound or very weakly bound, due to the reduction of binding from the heavy antidiquark.
- Further exotic states $ud\bar{s}\bar{b}$ or $us\bar{c}\bar{b}$
  seem to be unlikely by diquark picture but worth investigating as some models predict these being deeply bound (mostly Chiral Quark models)

The $0^+/1^+$ T_{cb} - confusing results



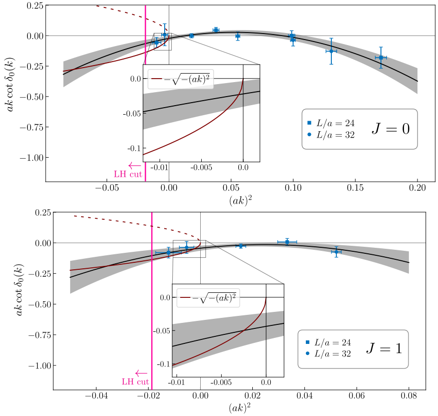
- New results:

  `Alexandrou et al., PRL 132 151902 (2024)`
  `Radhakrishnan et al., arXiv:2404.08109`
  `Padmanath et al., PRL 132 201902 (2024)`

- Close to threshold state could also be a virtual bound state
- Results are more or less incompatible

Shallow bound states and broad resonances in a scattering study

Alexandrou *et al.*, PRL 132 151902 (2024)



	Δm_{GBS} [MeV]	Δm_{R} [MeV]
$J=0$	$-0.5^{+0.4}_{-1.5}$	138(13)
$J=1$	$-2.4^{+2.0}_{-0.7}$	67(24)

- Obtained resonance poles just outside the radius of convergence of the ERE

The “Distillation” method

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Peardon et al. PRD 80, 054506 (2009)
Morningstar et al. PRD 83, 114505 (2011)
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- Idea: Construct separable quark smearing operator using low modes of the 3D lattice Laplacian
  Spectral decomposition for an $N \times N$ matrix:

$$f(A) = \sum_{k=1}^{N} f(\lambda^{(k)})\, v^{(k)} v^{(k)\dagger}.$$

  With $f(\nabla^2) = \Theta(\sigma_s^2 + \nabla^2)$ (Laplacian-Heaviside (LapH) smearing):

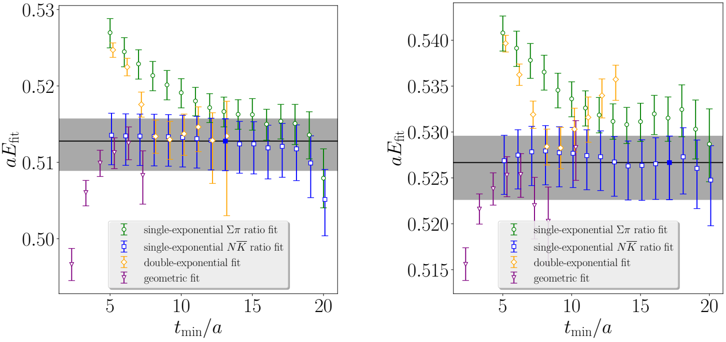
$$q_s \equiv \sum_{k=1}^{N} \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger}\, q \;=\; \sum_{k=1}^{N_v} v^{(k)} v^{(k)\dagger}\, q\,.$$

- Advantages: momentum projection at source; large interpolator freedom, small storage
- Disadvantages: expensive; unfavorable volume scaling
- Stochastic approach (partly) eliminates bad volume scaling

Specific setup on D200

- Combined basis of simple 3-quark structures and 2 hadron interpolators with the lowest few momentum combinations in each irrep
- Distillation setup:
  - $n_{ev} = 448$ eigenmodes of the Lattice Laplacian
  - Quark lines connecting source and sink:
    Noise dilution scheme with $(TF, SF, LI16)$ and 6 noises
  - Lines starting end ending on the same time slice:
    Noise dilution scheme with $(TI8, SF, LI16)$ and 2 noises
  - Four source time slices
  - Lattice Laplacian constructed on stout smeared links with $(\rho, n) = (0.1, 36)$

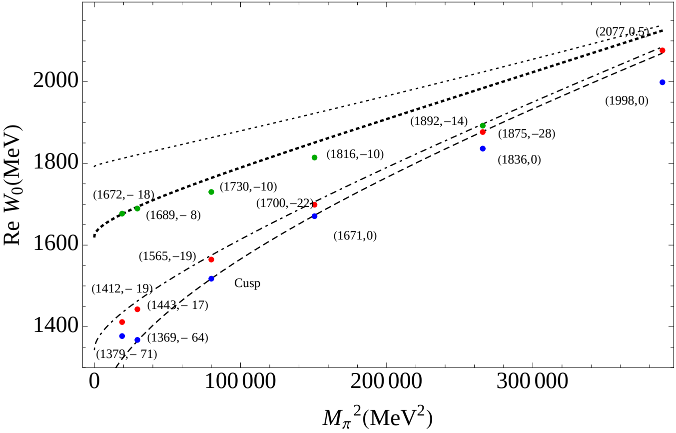
Extracting the spectrum (examples)



- We used various methods/cross checks
- Geometric series fit: $C(t) = \frac{A e^{-E_0 t}}{1 - B e^{-\Delta E t}}$
- Two students with two slightly different analysis methods

Expected quark-mass dependence

Molina, Döring, PRD 94 056010 (2016)



- Plots shows expected behavior for PACS-CS ensembles
- Qualitative agreement with regard to expected behavior