Meson mixing and lifetimes

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Lattice meets Continuum

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Introduction

- The lifetime $\tau = \Gamma^{-1}$ and the mixing of neutral mesons
  - Constitute two fundamental properties of particles
  - Proceed via the interplay of the weak and strong interactions

Important playground to test our understanding of the SM

- Focus on heavy hadrons H_Q $m_Q \gg \Lambda_{QCD}$
  - Systematic theoretical framework available

Weak effective theory + Heavy Quark Expansion

e.g. [Buchalla, Buras, Lautenbacher '95; Shifman, Voloshin '85]

  - Consider both beauty and charm systems

$m_b \sim 4.5\,\text{GeV} \gg 0.5\,\text{GeV} \sim \Lambda_{\text{QCD}}$ $\quad$ $m_c \sim 1\,\text{GeV} \overset{?}{\gg} \Lambda_{\text{QCD}}$

Introduction

◇ Experimental precision already high for most observables

e.g. uncertainties of ‰ for B-meson lifetimes

[PDG, HFLAV]

* It will further significantly increase as more data is collected

[LHCb, ATLAS, CMS, BESIII, Belle II]

◇ Aim at competitive theoretical precision to both

* Test the SM and framework used
* Perform indirect BSM searches

$$\underbrace{\frac{\tau(B_s)}{\tau(B_d)}}_{\text{exp.}} = 1 - \underbrace{\tau(B_s)\left[\delta\Gamma(B_s) - \delta\Gamma(B_d)\right]^{\text{SM}}}_{\text{theory}} - \underbrace{\tau(B_s)\left[\delta\Gamma(B_s) - \delta\Gamma(B_d)\right]^{\text{BSM}}}_{\text{indirectly constrained}}$$

Lifetimes

The HQE

- Express $\Gamma(B)$ via the optical theorem [Shifman, Voloshin '85]

$$\Gamma(B) = \frac{1}{2m_B} \mathrm{Im} \langle B| \, i \int d^4x \, \mathrm{T}\left\{\mathcal{H}_{\text{eff}}(x) \, , \mathcal{H}_{\text{eff}}(0)\right\} |B\rangle$$

$\mathcal{H}_{\text{eff}}$ - weak effective Hamiltonian

- Compute the non-local amplitude using that
  - The b-quark carries most of the B-meson momentum

$$p_B^\mu = m_B v^\mu$$

  - It can be parametrised as

$$p_b^\mu = m_b v^\mu + k^\mu \qquad k \sim \Lambda_{QCD} \ll m_b$$

The HQE

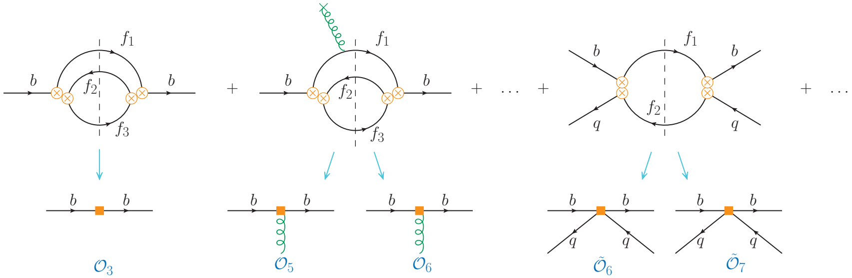
- Obtain systematic expansion

$$\Gamma(B) = \underbrace{\Gamma_3}_{\Gamma(b)} + \underbrace{\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]}_{\delta\Gamma(B)}$$

  - $\Gamma_d, \tilde{\Gamma}_d$ - short distance coefficients
  - $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local operators bilinear in the heavy quark field
  - $\Gamma(b)$ - total decay width of free b quark
  - $\delta\Gamma(B)$ - effects due to interaction with soft gluons and quarks

The HQE

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$



Very advanced framework thanks to huge effort of big community

Status of the HQE - as of August 2023

$$\Gamma_d = \Gamma_d^{(0)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)\Gamma_d^{(1)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)^2 \Gamma_d^{(2)} + \ldots$$

Semileptonic modes	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 Czakon, Czarnecki, Dowling '21
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi '13 Mannel, Pivovarov, Rosenthal '15
$\Gamma_6^{(1)}$	Mannel, Moreno, Pivovarov '19
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10
$\tilde{\Gamma}_6^{(1)}$	Lenz, Rauh '13

Non-leptonic modes	
$\Gamma_3^{(2)}$	Czarnecki, Slusarczyk, Tkachov '05 *
$\Gamma_3^{(1)}$	Ho-Kim, Pham '83; Altarelli, Petrarca '91 Bagan et al. '94; Krinner, Lenz, Rauh '13 Lenz, Nierste, Ostermaier '97
$\Gamma_5^{(1)}$	Mannel, Moreno, Pivovarov '23 **
$\Gamma_6^{(0)}$	Lenz, MLP, Rusov '20 Mannel, Moreno, Pivovarov '20
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste '02 Franco, Lubicz, Mescia, Tarantino '02
$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03

* Only partial result

** Only massless final states

Status of the HQE - as of August 2023

$\langle \mathcal{O}_d \rangle \propto \langle B | \bar{b} \underbrace{(iD_\mu)\dots(iD_\nu)}_{d-3} b | B \rangle$

$\langle \tilde{\mathcal{O}}_d \rangle \propto \langle B | \left(\bar{b}\Gamma q\right) \left(\bar{q}\Gamma \underbrace{(iD_\mu)\dots}_{d-6} b\right) | B \rangle$

	B_d, B^+	B_s
$\langle \mathcal{O}_5 \rangle$	Fits to SL data ◇ Lattice QCD + HQET sum rules *	Spectroscopy relations **
$\langle \mathcal{O}_6 \rangle$	Fits to SL data ◇ EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$	Sum rules estimates ** EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET sum rules ‡	HQET sum rules ‡
$\langle \tilde{\mathcal{O}}_7 \rangle$	Vacuum insertion approximation	

◇ [Bordone, Capdevila, Gambino '21; Bernlochner, Fael et al. '22] * [Ball, Braun '94; Neubert '96]

+ [Gambino, Melis, Simula '17; Bazavov et al. '18] ** [Bigi, Mannel, Uraltsev '11]

‡ [Kirk, Lenz, Rauh '18; King, Lenz, Rauh '20]

also for D mesons

The observables

- Compute total decay widths

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

- And lifetime ratios

$$\tau(B_{(s)}^+)/\tau(B_d) = 1 + \left[\delta\Gamma(B_d)^{\mathrm{HQE}} - \delta\Gamma(B_{(s)}^+)^{\mathrm{HQE}} \right] \tau(B_{(s)}^+)^{\mathrm{exp}}$$

- No two-quark contributions for $\tau(B^+)/\tau(B_d)$ in isospin limit

- Crucial role of $\mathrm{SU}(3)_\mathrm{F}$ breaking effects for $\tau(B_s)/\tau(B_d)$

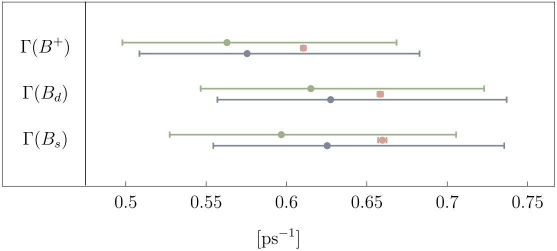
B-mesons lifetimes

Scenario A

- Larger inputs for B_d

[Bordone et al. '21]

- Larger $SU(3)_F$ breaking

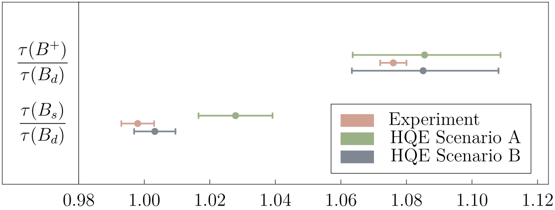


Scenario B

- Smaller inputs for B_d

[Bernlochner et al. '22]

- Smaller $SU(3)_F$ breaking



[Lenz, MLP, Rusov '22]

B-mesons lifetimes

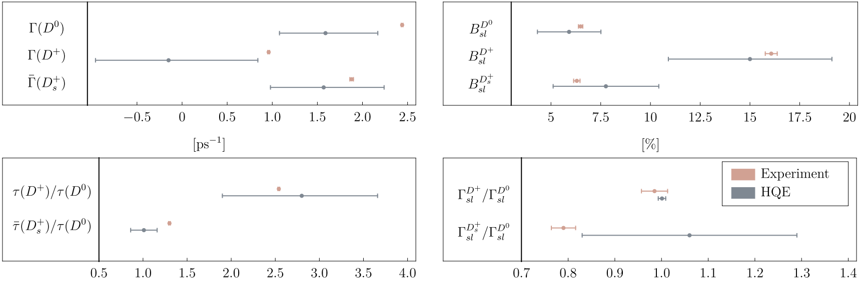
- Good agreement of HQE and data
  - For the total decay widths
    - Large uncertainties, dominated by scale variation in Γ_3

      Only NLO-QCD corrections included
  - For the ratio $\tau(B^+)/\tau(B_d)$
    - Dominant uncertainties due to four-quark matrix elements

      Important a cross-check from Lattice!
  - For the ratio $\tau(B_s)/\tau(B_d)$
    - Dominant uncertainties due to two-quark matrix elements
    - Tension with data in one scenario

      Need better control over size of non-pert inputs and $SU(3)_F$ break.

D-mesons lifetimes



[King, Lenz, MLP, Rauh, Rusov, Vlahos ’21]

- HQE can accommodate observed pattern in the charm system
- Uncertainties still very large

  Mainly due to charm mass and non-perturbative inputs

- Results confirmed by other analyses, studied also baryons

  [Gratrex, Melić, Nišandžić ’22; Dulibič, Gratrex, Melić, Nišandžić ’23]

Current status of the HQE

Great progress has been made over the past year!

- Computation of $\Gamma_3^{(2)}$ for all non-leptonic modes

  [Egner, Fael, Schönwald, Steinhauser '24]

- Computation of $\Gamma_5^{(1)}$ with one internal massive quark

  [Mannel, Moreno, Pivovarov '24]

- Progress on determining dim-6 matrix elements from Lattice see talk F. Lange

  [Black, Harlander, Lange, Rago, Shindler, Witzel '23, '24]

- Updated SL fit including also partial α_s^2-corrections to q^2-moments

  [Finauri, Gambino '23]

  - Complete α_s^2-corrections to semileptonic q^2-spectrum now also known

    [Fael, Herren '24]

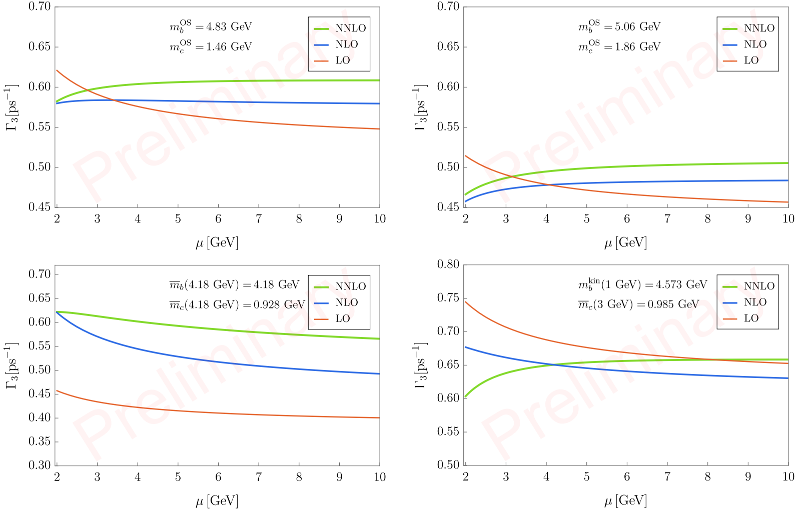
- First steps towards determination of $\langle \mathcal{O}_{5,6} \rangle$ for B_s from data

  [De Cian, Feliks, Rotondo, Vos '23]

- First steps towards measuring moments in inclusive SL D-decays

  [Bernlochner, Gilman, Malde, Prim, Vos, Wilkinson '24]

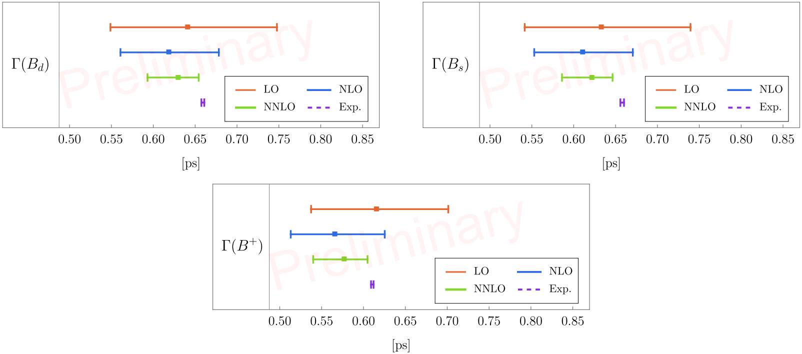
Updating the B-mesons lifetimes



[Egner, Fael, Lenz, MLP, Rusov, Schönwald, Steinhauser (in preparation)]

Updating the B-mesons lifetimes

[Egner, Fael, Lenz, MLP, Rusov, Schönwald, Steinhauser (in preparation)]



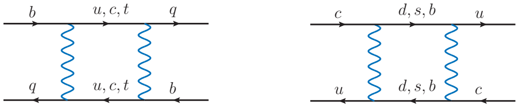
Using $m_b^{\text{kin}}(1\text{GeV}) = 4.573\text{GeV}$ and $\overline{m}_c(3\text{GeV}) = 0.985\text{GeV}$

- Significant improvement in scale and mass scheme dependence
  - A complete estimate of th. uncertainties is still in progress

Mixing

The theory framework

- Neutral mesons mix with their antiparticles via box diagrams



- Evolution described by 2×2 Hamiltonian matrix

$$i \frac{d}{dt} \begin{pmatrix} |\mathcal{M}^0(t)\rangle \\ |\overline{\mathcal{M}}^0(t)\rangle \end{pmatrix} = \left(\hat{M} - i \frac{\hat{\Gamma}}{2} \right) \begin{pmatrix} |\mathcal{M}^0(t)\rangle \\ |\overline{\mathcal{M}}^0(t)\rangle \end{pmatrix} \Rightarrow \begin{cases} |\mathcal{M}_L\rangle = p\,|\mathcal{M}^0\rangle + q\,|\overline{\mathcal{M}}^0\rangle \\ |\mathcal{M}_H\rangle = p\,|\mathcal{M}^0\rangle - q\,|\overline{\mathcal{M}}^0\rangle \end{cases}$$

- Define mixing observables

$$\Delta M = M_H - M_L \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$

- And the dimensionless ratios

$$x = \Delta M/\Gamma \qquad y = \Delta\Gamma/2\Gamma \qquad \text{with} \qquad \Gamma = (\Gamma_H + \Gamma_L)/2$$

The theory framework

- Compute mixing observables as

See e.g. reviews [Artuso, Borissov, Lenz '15; Lenz, Wilkinson '20]

$$\Delta M \approx 2|M_{12}| \qquad \Delta\Gamma \approx 2|\Gamma_{12}| \qquad \phi_{12} = \arg\left(-M_{12}/\Gamma_{12}\right)$$

Expanding in the small parameters $|\Gamma_{12}|/|M_{12}|$ and/or ϕ_{12}

- M_{12} - dispersive part of $\mathcal{M}^0 \to \overline{\mathcal{M}}^0$ amplitude

Directly sensitive to heavy BSM particles

- Γ_{12} - absorptive part of $\mathcal{M}^0 \to \overline{\mathcal{M}}^0$ amplitude

Directly sensitive to light BSM particles

- CP violation in mixing encoded in parameter $a_{\rm fs}$

$$\left|\frac{q}{p}\right| \approx 1 - \frac{a_{\rm fs}}{2} \qquad a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{12}$$

B-mixing

- Off-shell contributions to box diagrams

$$M_{12} = \lambda_u^2 \left(M_{cc} - 2M_{uc} + M_{uu}\right) + 2\lambda_u\lambda_t \left(M_{cc} - M_{uc} + M_{ut} - M_{ct}\right) + \lambda_t^2 \left(M_{cc} - 2M_{ct} + M_{tt}\right)$$

  - Described in terms of $\Delta B = 2$ effective Hamiltonian

- On-shell contributions to box diagrams

$$\Gamma_{12} = \lambda_u^2 \left(\Gamma_{cc} - 2\Gamma_{uc} + \Gamma_{uu}\right) + 2\lambda_u\lambda_t \left(\Gamma_{cc} - \Gamma_{uc}\right) + \lambda_t^2 \, \Gamma_{cc}$$

  - Described by double insertion of $\Delta B = 1$ effective Hamiltonian

Can be computed within the HQE

$\lambda_u^d \sim \lambda_c^d \sim \lambda_t^d$ $\quad$ $\lambda_u^s \ll \lambda_c^s \sim \lambda_t^s$ $\quad$ $\lambda_x^q = V_{xb} V_{xq}^*$

- GIM and CKM suppressions go in the same direction

B-mixing

- M_{12} known at NLO-QCD

$$M_{12}^q \propto G_F^2 \lambda_t^{q\,2} S_0(x_t) \hat{\eta}_B \langle \tilde{Q}_6 \rangle$$

[Buras, Jamin, Weisz '90]

- Determine Γ_{12} within the HQE

$$\Gamma_{12} = 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{Q}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{Q}_7 \rangle}{m_b^4} + \ldots \right]$$

$\tilde{\Gamma}_6^{(2)}$	Gerlach, Nierste, Stabovenko, Steinhauser '22 Asatrian, Hovhannisyan, Nierste, Yeghiazaryan '17
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste '98 Beneke, Buchalla, Lenz, Nierste '03 Ciuchini, Franco, Lubicz, Mescia, Tarantino '03 Lenz, Nierste '06
$\tilde{\Gamma}_7^{(0)}$	Beneke, Buchalla, Dunietz '96 Dighe, Hürth, Kim, Yoshikawa '01

$\langle \tilde{Q}_6 \rangle$	King, Lenz, Rauh '19 Kirk, Lenz, Rauh '17 HPQCD 1907.0102 RBC/UKQCD 1812.08791 Fermilab/MILC 1602.03560
$\langle \tilde{Q}_7 \rangle$	Vacuum insert. approx. HPQCD 1910.00970

see talk by F. Erben

B-mixing

- The theoretical status from review [Albrecht, Bernlochner, Lenz, Rusov '24]

$\Delta\Gamma_d^{SM} = (2.7 \pm 0.4) \cdot 10^{-3} \text{ps}^{-1}$ $\Delta M_d^{SM} = (0.535 \pm 0.021) \text{ps}^{-1}$ $a_{\text{fs,d}}^{SM} = (-5.1 \pm 0.5) \cdot 10^{-4}$

$\Delta\Gamma_s^{SM} = (9.1 \pm 1.5) \cdot 10^{-2} \text{ps}^{-1}$ $\Delta M_s^{SM} = (18.23 \pm 0.63) \text{ps}^{-1}$ $a_{\text{fs,s}}^{SM} = (2.2 \pm 0.2) \cdot 10^{-5}$

New results for $\tilde{\Gamma}_6^{(2)}$ not yet included

- Compare with experimental status [HFLAV '24]

$(\Delta\Gamma_d/\Gamma_d)^{\text{exp.}} = 0.001 \pm 0.010$ $\Delta M_d^{\text{exp.}} = (0.5069 \pm 0.0019) \text{ps}^{-1}$ $a_{\text{fs,d}}^{\text{exp.}} = (-21 \pm 17) \cdot 10^{-4}$

$\Delta\Gamma_s^{\text{exp.}} = (8.3 \pm 0.5) \cdot 10^{-2} \text{ps}^{-1}$ $\Delta M_s^{\text{exp.}} = (17.765 \pm 0.006) \text{ps}^{-1}$ $a_{\text{fs,s}}^{\text{exp.}} = (-60 \pm 280) \cdot 10^{-5}$

D-mixing

- Strong interplay of CKM and GIM suppression!

$\lambda_d \sim \lambda_s \sim \lambda \gg \lambda_b \sim \lambda^5$ $\lambda_x = V_{cx} V_{ux}^*$

$$M_{12} = \lambda_s^2 \left(M_{ss} - 2M_{ds} + M_{dd} \right) + 2\lambda_s \lambda_b \left(M_{bs} - M_{bd} + M_{dd} - M_{sd} \right) + \lambda_b^2 \left(M_{bb} - 2M_{bd} + M_{dd} \right)$$

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{ds} + \Gamma_{dd} \right) + 2\lambda_s \lambda_b \left(\Gamma_{sd} - \Gamma_{dd} \right) - \lambda_b^2 \Gamma_{dd}$$

- All terms are of similar size, pronounced cancellations
- Dominated by double insertion of $\Delta C = 1$ effective Hamiltonian
  - Can be computed within the HQE?

D-mixing

◇ Experimentally both x and y are measured quite precisely

[HFLAV '23]

$$x^{\text{exp.}} = (0.407 \pm 0.044)\,\% \qquad y^{\text{exp.}} = \left(0.645^{+0.024}_{-0.023}\right)\%$$

◇ Determine y within the HQE

$$y^{\text{HQE}} \sim 10^{-4}\, y^{\text{exp.}}\,!$$

◇ Complete failure of HQE for charm mixing?

However HQE successful for charm lifetimes

- ⋆ Alternative renormalisation scale setting can lift GIM cancellation

  [Lenz, MLP, Vlahos '20]

- ⋆ Potential large impact of higher dimensional operators

  [Georgi '92; Ohl et al. '93; Bigi, Uraltsev '00]

◇ Use different method: lattice determination, exclusive approach

[Hansen, Sharpe '12] see e.g. [Falk et al. '01]

Conclusions

- The HQE is a powerful framework for inclusive B- and D-decays
- Update of B-meson lifetimes shows very promising results

  Later, extend also to charmed mesons
- Th. uncertainties are in most cases larger than expetimental ones

  However, significant progress is being made
- D-mixing puzzle still on-going

  Higher order perturbative and power corrections needed
- Cooperation between theory and exp. communities crucial

Thanks for the attention