Neutral meson mixing from lattice QCD

Felix Erben

Lattice meets Continuum, Siegen

30 September 2024

Overview

1) $\mathrm{K} - \bar{\mathrm{K}}$ mixing on the lattice
   1a) long-distance contribution
   1b) short-distance contribution
2) status of B_q mixing by RBC/UKQCD and JLQCD

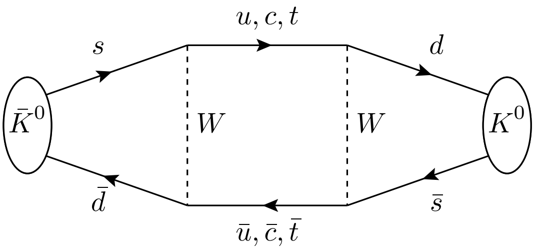
K meson mixing

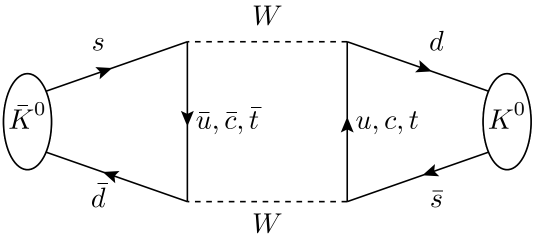
With CP symmetry, neutral kaons have eigenstates

$$|K_L\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$
$$|K_S\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

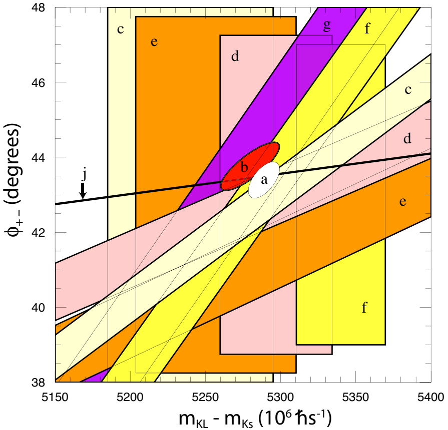
with a long-lived $|K_L\rangle$ and a short-lived $|K_S\rangle$.
Indirect CP violation parameter ϵ_K can be parameterized by mass and widths splittings

$$\Delta M_K = M_{K_L} - M_{K_S}, \qquad \Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$$
$$\phi_\epsilon = \frac{\Delta M_K}{\Delta\Gamma_K/2}$$
$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}\right)$$





K MESON MIXING



(a)–[PDG, PRD 24] $\chi^2 = 1$ contour of fit to experimental data:
(b)–FNAL KTeV '11, (c)–CERN CPLEAR '99, (d)–FNAL E773 '95,
(e)–FNAL E731 '93, (f)–CERN '74, (g)–CERN NA31 '90

- second-order weak transition
- sensitive to new physics
- precisely measured experimentally
  - $\Delta M_K = 3.484(6) \times 10^{-12}$ MeV
  - $\phi_\epsilon = 43.52(5)^\circ$

K MESON MIXING

$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{12}}{\Delta M_K} + \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}\right)$$

A_0 is the $K \to (\pi\pi)_{I=0}$ decay amplitude
M_{12} splits into

$$\begin{aligned} M_{12} = \langle K^0|\mathcal{H}_W^{eff}|\bar{K}^0\rangle &= \langle K^0|\mathcal{H}_W^{eff}|\bar{K}^0\rangle_{SD} + \langle K^0|\mathcal{H}_W^{eff}|\bar{K}^0\rangle_{LD} \\ &= \langle K^0|\mathcal{H}_W^{\Delta S=2}|\bar{K}^0\rangle + \sum_n \frac{\langle K^0|\mathcal{H}_W^{\Delta S=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta S=1}|\bar{K}^0\rangle}{M_K - E_n} \end{aligned}$$

On the lattice, we can compute both:

- $\langle K^0|\mathcal{H}_W^{eff}|\bar{K}^0\rangle_{SD}$ [Kaon mixing beyond the standard model with physical masses; FE et al., PRD 24]
- $\langle K^0|\mathcal{H}_W^{eff}|\bar{K}^0\rangle_{LD}$ [Long-distance contribution to ϵ_K from lattice QCD; Bai et al., PRD 24]

Long-distance contribution to ϵ_K

EXTRACTING THE LONG-DISTANCE AMPLITUDE [CHRIST ET AL., PRD 13]

extracting the $K - \bar{K}$ mixing amplitude from finite-volume correlators [Christ et al., PRD 13]

- closest Euclidean correlation function: integrated 4pt correlator $\int dt_1 dt_2 \, \langle 0|T[\bar{K}^0(t_f) H_W(t_2) H_W(t_1) \bar{K}^0(t_i)]|0\rangle$
- on-shell intermediate states $|n\rangle\langle n|$ between H_W complicate calculation:

growing exponentials

- FV states E_n with mass $M_n < M_K$ lead to unphysical growing exponentials
- these must be removed explicity and then added back in later

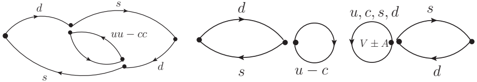
finite-volume effects

- consequently, FV estimator has poles at removed energies
- power-like volume effects are understood and described by $K \to \pi\pi$ and $\pi\pi \to \pi\pi$ scattering amplitudes

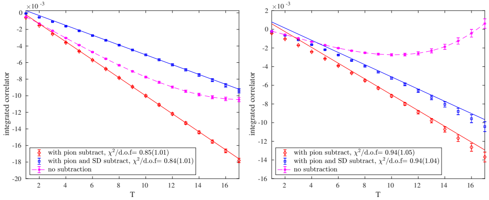
$\Rightarrow$ Precise knowledge of **excited-state spectrum** needed to extract long-distance amplitude from Euclidean finite-volume correlators

$\Rightarrow$ Details on these techniques [F. Romero-López, Wed 9:00]

- RBC/UKQCD Domain-Wall Fermion ensembles
- one coarse lattice spacing $a^{-1} = 1.78$ GeV
- 2 pion masses 339 MeV and 592 MeV
- non-perturbative renormalization
- result: $\epsilon_K^{LD} = 0.195(77) e^{i\phi_\epsilon} \times 10^{-3}$
- comparison: $\epsilon_K^{SD} = 1.360(154) e^{i\phi_\epsilon} \times 10^{-3}$
- smaller than experimental value: $|\epsilon_K| = 2.228(11) \times 10^{-3}$
- discrepancy not understood, but $|V_{cb}|$ contributes to ϵ_K determination, present uncertainty in incl. vs excl.



a selection of topologies to be computed



integrated 4pt-correlator, with subtractions

Calculation at physical pion mass underway, **progress report at this year's lattice conference** [Yikai Huo, Lattice 24]

Short-distance contribution of $\mathrm{K}-\bar{\mathrm{K}}$ mixing

- hadronic contribution to ϵ_K conventionally described by bag parameters $\mathcal{B}$

$$\mathcal{B}_i = \frac{\langle \bar{K}^0 | \mathcal{O}_i | K \rangle}{\langle \bar{K}^0 | \mathcal{O}_i | K \rangle_{\mathrm{VSA}}}$$

- can be computed from ratios of three-point and two-point functions
- knowledge of **ground states** suffices to extract them

$\Rightarrow$ less involved computation than for long-distance contribution

$\Rightarrow$ more rigourous extraction in terms of chiral-continuum limit & physical quark masses

- SM parameter $B_K = \mathcal{B}_1$
- 4 extra parameters encoding BSM physics $\mathcal{B}_{2/3/4/5}$
- BSM physics involve heavy, unobserved particles $\Rightarrow$ short-distance dominated

2pt-functions

$$\langle K(t)K^\dagger(0)\rangle_{L,a,m_l} \Rightarrow M_K(L,a,m_l),\, f_K(L,a,m_l)$$

3pt-functions

$$\langle K(\Delta T)\mathcal{O}_i(t)K^\dagger(0)\rangle_{L,a,m_l} \Rightarrow M_K(L,a,m_l),\, f_K(L,a,m_l),\, \mathcal{B}_i(L,a,m_l)$$

Leading to

$$\mathcal{B}_i = \lim_{a\to 0}\lim_{L\to\infty}\lim_{m_l\to m_l^p} \mathcal{B}_i(L,a,m_l)$$

or more precise values for

$$R_i = \lim_{a\to 0}\lim_{L\to\infty}\lim_{m_l\to m_l^p} \frac{\langle \bar{K}^0|O_i|K^0\rangle}{\langle \bar{K}^0|O_1|K^0\rangle}(L,a,m_l)$$

Non-perturbative renormalization

- bare lattice operators need to be properly renormalized
- We use the *Rome-Southampton method*, which completely avoids the use of lattice perturbation theory

⇒ Called ***Non-Perturbative Renormalization*** or ***NPR***

- First done for 2-fermion operators, which cannot be renormalized by solving the Ward identity [Martinelli et al., 1995]
- Idea is to fix renormalization conditions via tree-level matrix elements like

$$Z_\Gamma \langle p|O_\Gamma|p\rangle\Big|_{p^2=-\mu^2} = \langle p|O_\Gamma|p\rangle\Big|_{\mathrm{tree}}$$

⇒ Renormalization constants can be computed on the lattice

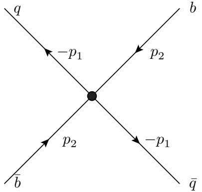
- ***NPR calculations of this project lead by Rajnandini Mukherjee***
- New directions explored using gradient flow [M. Black, Lattice24] [R. Harlander, Tue 14:30]

$$\langle \mathcal{O} \rangle_i^S(\mu) = \lim_{a^2 \to 0} \sum_{j=1}^{5} [Z_{\mathcal{O}}^S(a,\mu)]_{ij} \langle \mathcal{O} \rangle_j^{\text{bare}}(a)$$

for some regularisation independent scheme S at mass scale μ. Continuum perturbation theory can then match

$$\langle \mathcal{O} \rangle_i^{\overline{\text{MS}}}(\mu) = R^{\overline{\text{MS}} \leftarrow S} \langle \mathcal{O} \rangle_i^S(\mu)$$

We use the ”RI-SMOM” scheme. For mixing we need to compute four-quark vertices for $(\bar{b}q) \to (\bar{q}b)$. [Boyle et al., JHEP 10 (2017) 054]



RI-SMOM

Kinematics for fermion bilinears:

Original Rome-Southampton method **RI-MOM** [Martinelli et al., 1995]

$$p_1^2 = p_2^2 = -\mu^2, \;\; p_1 = p_2 \Rightarrow q = 0$$

which has *exceptional kinematics* $q^2 = 0 \ll \mu^2$, chiral symmetry breaking effects vanish with $1/p^2$

Non-exceptional kinematics **RI-SMOM** [Sturm et al., 2009]

$$p_1^2 = p_2^2 = q^2 = -\mu^2, \;\; q = p_1 - p_2$$

chiral symmetry breaking and infrared effects vanish with $1/p^6$

DOMAIN-WALL FERMIONS

- we use "Domain-Wall Fermions"
  - automatic $O(a)$ improvement in absence of odd powers in a

  $\Rightarrow$ reduced discretisation effects
  - chirally symmetric formulation

  $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}$$

$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{22} & \mathcal{O}_{23} \\ \mathcal{O}_{32} & \mathcal{O}_{33} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{44} & \mathcal{O}_{45} \\ \mathcal{O}_{54} & \mathcal{O}_{55} \end{pmatrix} \end{pmatrix}$$

Block-structure:

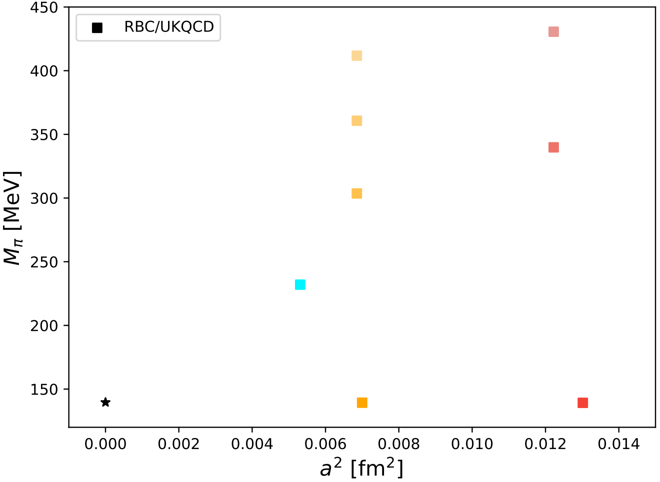
- $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix
- linearly independent from each other and from $\mathcal{O}_1$
- more complicated mixing pattern for other lattice fermions

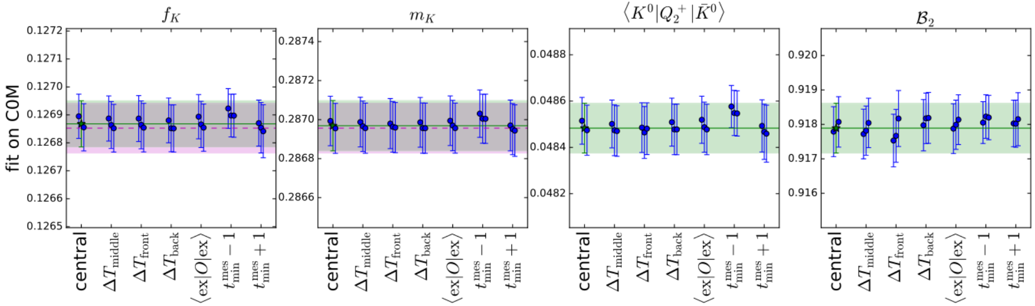
[Boyle et al., JHEP 10 (2017) 054]

RBC/UKQCD Domain-Wall Fermion ensembles

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

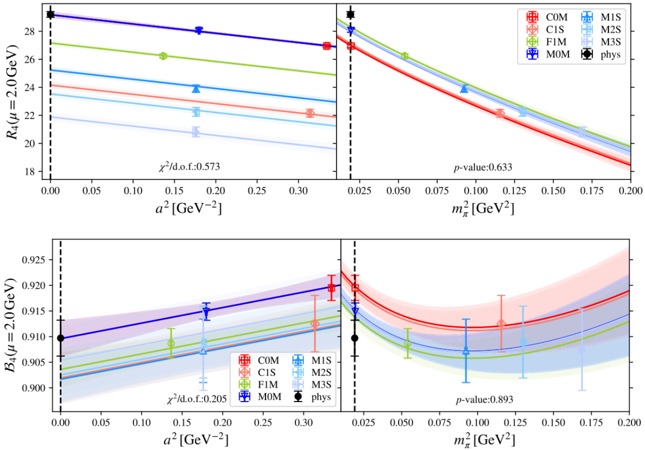




stability of correlation function fit variations on C0M ensemble for O_2

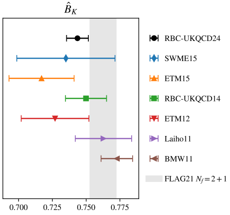
SHORT-DISTANCE CALCULATION [Kaon mixing beyond the standard model with physical masses; FE et al., PRD 24]

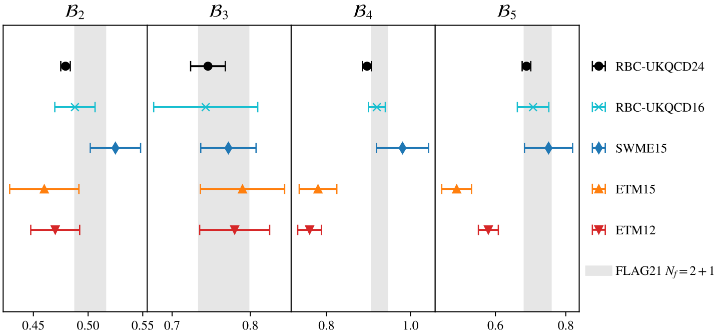
- chiral continuum fits of ratios $\mathcal{R}_i$ (top) and bag parameters $\mathcal{B}_i$ (bottom)
- two precise data points at M_π^{phys} render mass extrapolation very benign
- discretization effects $O(a), O(a^3), \ldots$ suppressed by DWF
- discretization effects $O(a^2)$ controlled by 3 lattice spacings
- remaining $O(a^4)$ estimated via variety of NPR estimators



TENSIONS IN BSM KAON MIXING [FE ET AL., PRD 24]

- BSM bag parameters $\mathcal{B}_4$, $\mathcal{B}_5$ are in tension between results using RI-MOM (with manually removed pion poles) and RI-SMOM
- tension confirmed by our calculation [FE et al., PRD 24]





B_q-meson mixing

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle = \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} + \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{LD}$$
$$= \langle B_q^0|\mathcal{H}_W^{\Delta B=2}|\bar{B}_q^0\rangle + \sum_n \frac{\langle B_q^0|\mathcal{H}_W^{\Delta B=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta B=1}|\bar{B}_q^0\rangle}{M_{B_q} - E_n}$$

short-distance contribution:

- t-loop enhancement (like for kaons)
- additional CKM hierachy enhancement

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} \sim \Big(\sum_{q'=u,c,t} V_{q'q}^* V_{q'b} S_0(m_{q'}^2/M_W^2)\Big)^2$$

long-distance contribution:

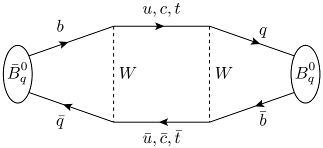
- CKM-suppressed

B_q-mixing dominated by short-distance contribution

B_q-MESON MIXING

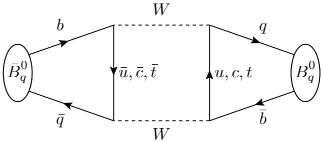
B-mesons B_d, B_s have mass eigenstates

$$|B^0_{qL}\rangle = p_q|B^0_q\rangle + q_q|\bar{B}^0_q\rangle$$
$$|B^0_{qH}\rangle = p_q|B^0_q\rangle - q_q|\bar{B}^0_q\rangle$$



with mass m_{qL} and total decay width Γ_{qL} for the lighter eigenstate.
Splittings:

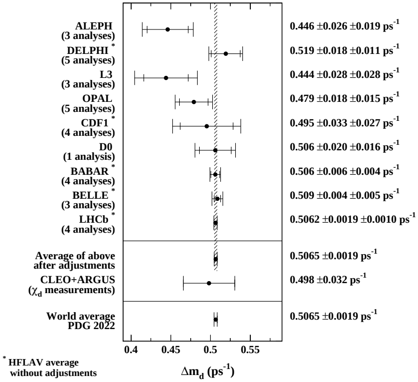
$$\Delta m_q = m_{qH} - m_{qL}$$
$$\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$$

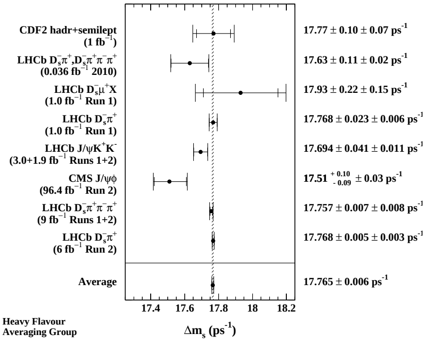


Experimentally, time dependent probabilities give access to the splittings, e.g.

$$\mathcal{P}(B^0_q \to \bar{B}^0_q) = \frac{1}{2}e^{-\Gamma_q t}[\cosh(\frac{1}{2}\Delta\Gamma_q t) - \cos(\Delta m_q t)]|q_q/p_q|^2$$

Experimental results, HFLAV 2021 [Phys.Rev.D 107 (2023) 5]

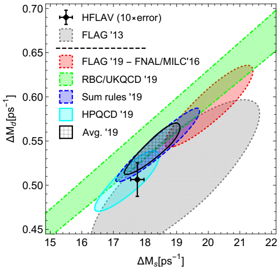




$$\Delta m_d = 0.5065(19)\text{ps}^{-1} \qquad \Delta m_s = 17.765(6)\text{ps}^{-1}$$

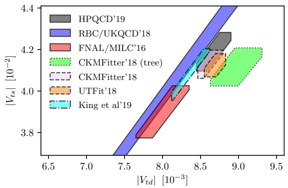
B_q MIXING - LATTICE

- current tension between Δm_d, Δm_s lattice determinations
  - FNAL/MILC [Bazavov et al., PRD 16] is in tension with experiment
  - HPQCD [Dowdall et al., PRD 19] is compatible with experiment
  - RBC/UKQCD [Boyle et al., arxiv 1812.08791] result still missing renormalization factors
  - theory uncertainty dominates experimental one



[Di Luzio et al., JHEP 19]

- similar picture in $|V_{td}|, |V_{ts}|$
  - lattice results in slight tension, but all compatible with sum-rules [King et al., JHEP 19]
  - unitarity-triangle fits favour HPQCD '19 result



[HPQCD, PRD 19]

CONTINUUM LIMIT (B-MIXING)

We need to control on each ensemble

- light-quark discretisation effects $\Rightarrow M_\pi L \gtrapprox 4$
- heavy-quark discretisation effects am_h

Two approaches for heavy quark:

effective theories

- allow expansion in $1/am_b$
- truncation at some order
- not easily improvable

method:

- Relativistic action (HQET, RHQ, Fermilab method)
- Nonrelativistic QCD (NRQCD)

fully relativistic

- $am_h \ll 1$ needed

$\Rightarrow$ fine lattice spacing for am_b^{phys}

- improvable with finer, larger boxes

method:

- extrapolation $am_h \to am_b$ for multiple $am_h < am_b$
- today impossible to reach am_l^{phys}, am_b^{phys} simultaneously

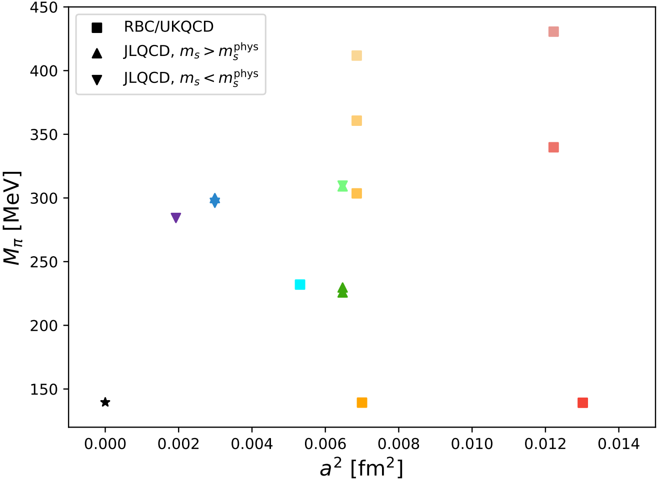
Joint project: RBC/UKQCD and JLQCD

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

JLQCD:

- 7 ensembles
- 3 lattice spacings $a = 0.044 - 0.081\text{fm}$
- one pair of ensembles with $M_\pi L \sim 3$ and $M_\pi L \sim 4$



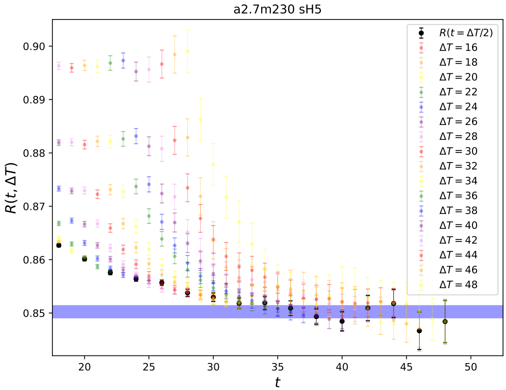
[PoS LATTICE2021 (2022) 224]

Fits to lattice correlation functions

- similar to K mixing, but much larger dataset:
- 15 ensembles
- 4-6 heavy-quark masses per ensemble
- heavy-light and heavy-strange sector
- 5 operators

$\Rightarrow$ over 700 combined fits

- multiple values for ΔT to control fits better
- two independent analyses by FE and J.T. Tsang

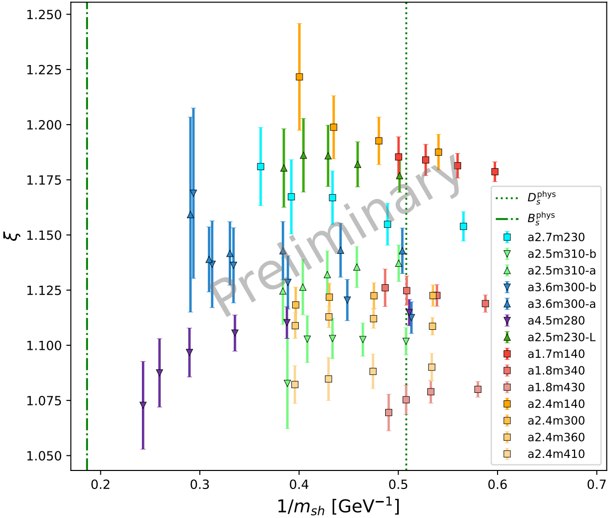


- Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble

Mixing ratios ξ

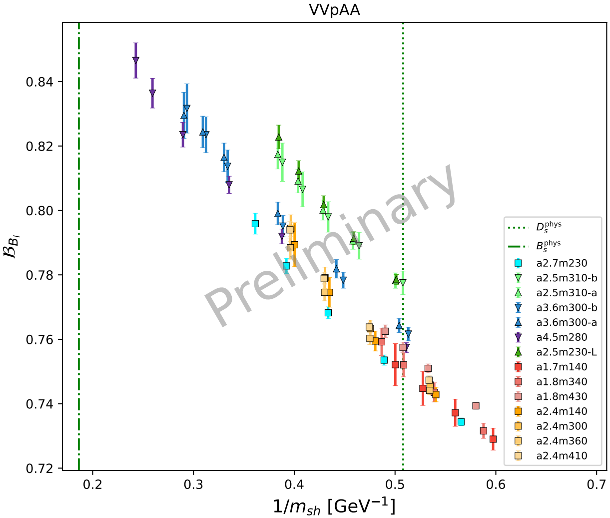
- update of RBC/UKQCD work

  [Boyle et al., arxiv 1812.08791]
- includes JLQCD ensembles
- completely new, fully correlated fitting strategy
- cancellation of renormalisation constants
- relatively flat $1/m_{sh}$ dependence with improved reach towards m_b^{phys}
- global fits on the data are investigated

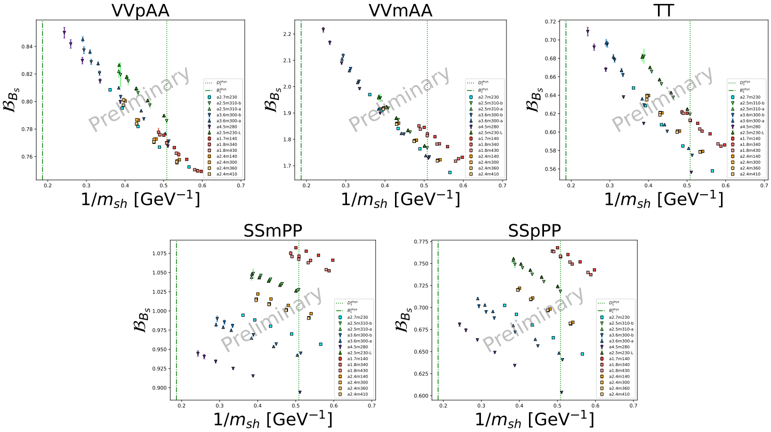


Bag parameter $\mathcal{B}_{hl}$ - $VV + AA$

- heavy-light bag parameters, renormalised at mass scale μ
- not yet matched to continuum scheme
- discretisation effects for O_1 are small
- global fits to renormalised bag parameters are investigated



- heavy-strange bag parameters, renormalised at mass scale μ
- O_1, O_2: mild a^2 dependence
- O_3, O_4: strong a^2 dependence
- O_5: medium a^2 dependence and curvature in $1/m_{sh}$
- very similar for heavy-light sector



Conclusions

$K - \bar{K}$ mixng

- short and long distance contributions using lattice QCD
- recent result for SD contribution, controlling all limits [FE et al., PRD 24]
- renormalization coefficients & strategy finalized and published
- LD harder, exploratory done with 40% errors, physical-point calculation in progress [Yikai Huo, Lattice 24]

$B_q - \bar{B}_q$ mixng

- 15 ensembles, 6 lattice spacings from 2 collaborations, including two ensembles at M_π^{phys}
- simple renormalisation for chiral Domain-Wall Fermions
- fully relativistic treatment of heavy-quark
- very fine lattice spacings
- all limits will be under control

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Backup

LATTICE SETUP

- RBC-UKQCD's 2+1 flavour domain wall fermions [Blum et al. Phys.Rev.D 93 (2016) 7]
  - pion masses from $M_\pi = 139$ MeV to $M_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5 m_b$, using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ [Boyle et al. arxiv:1812.08791]
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at M_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions [Kaneko et al. EPJ Web Conf. 175 (2018) 13007]
  - pion masses from $M_\pi = 226$ MeV to $M_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

LATTICE SETUP

	L/a	T/a	a^{-1} [GeV]	M_π [MeV]	$M_\pi L$	hits $\times N_{conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48 × 90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32 × 100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32 × 101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64 × 82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32 × 83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32 × 76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32 × 81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24 × 100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16 × 100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16 × 100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16 × 100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48 × 72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24 × 50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24 × 50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32 × 50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last

Other neutral meson mixings

For other neutral mesons $M^0 \in \{K, D, B_q\}$

$$\begin{aligned}\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle &= \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} + \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD} \\ &= \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle + \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}\end{aligned}$$

short-distance contribution:

- t enhancement for $K, B_{(s)}$
- additional CKM hierachy enhancement for $B_{(s)}$
- sub-dominant for D, but ok to describe CP-violating contributions

long-distance contribution:

- relevant but smaller than short-distance for K
- dominant for D
- CKM-suppressed for $B_{(s)}$