Inclusive q^2 moments in the Continuum

K. Keri Vos

Maastricht University & Nikhef

= ArXiv: 2409.15007 & preliminary =

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}$?)
- Extract power corrections from data (inputs to $B \to X_u$, $B \to X_s \ell\ell$ and lifetimes)
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive Decays: Heavy Quark Expansion

- b quark mass is large compared to Λ_{QCD}
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\rightarrow$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \dots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B\right\rangle$$

  - $C_i^{(k)}$ perturbative Wilson coefficients
  - $\langle B|\dots|B\rangle$ non-perturbative matrix elements $\rightarrow$ string of iD
  - operators contain chains of covariant derivatives

HQE elements: $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\dots(iD_{\mu_n})b_v|B\rangle$

- Currently extracted from data

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- Could be obtained from lattice? See e.g. Juetner et al. [2305.14092]

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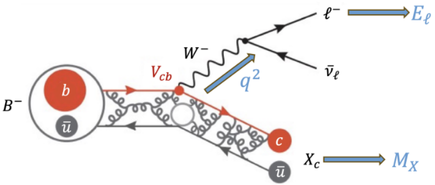
- Γ_2 :μ_π^2 and μ_G^2 at $1/m_b^2$
- Γ_3 :ρ_D^3 and ρ_{LS}^3 at $1/m_b^3$
- Many more at $1/m_b^{4,5}$ Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109 Mannel, Milutin, KKV [2311.12002]

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005. Pic from M. Fael

Non-perturbative matrix elements obtained from moments of differential rate



$$\langle O^n \rangle_{\rm cut} = \frac{\int_{\rm cut} dO\, O^n \frac{d\Gamma}{dO}}{\int_{\rm cut} dO \frac{d\Gamma}{dO}}$$

$M_X^2 = (p_B - q)^2$, $E_\ell = v_B \cdot p_\ell$ and $q^2 = (p_\nu + p_\ell)^2$

hadronic mass, lepton energy and q^2 moments

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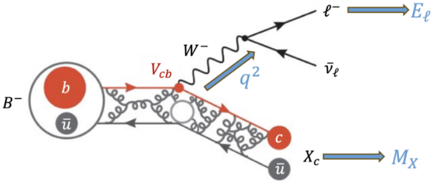

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hadronic mass, lepton energy and q^2 moments

- Different phase space cuts give additional (correlated) observables
- data $\to \mu_\pi^2, \mu_G^2, \rho_D^3 + \ldots \to$ total rate $\to |V_{cb}|$

Moments of the spectrum

Moments and total rate are double expansion in α_s and HQE parameters

$$
\begin{aligned}
L_i =& \frac{1}{\Gamma_0} \int_{E_l \geq E_{\rm cut}} \mathrm{d}E_l \, \mathrm{d}q_0 \, \mathrm{d}q^2 \, (E_l)^i \, \frac{\mathrm{d}^3\Gamma}{\mathrm{d}q^2 \, \mathrm{d}q_0 \, \mathrm{d}E_l} \\
=& (m_b)^i \Bigg[L_i^{(0)} + L_i^{(1)} \frac{\alpha_s(\mu_s)}{\pi} + L_i^{(2)} \left(\frac{\alpha_s(\mu_s)}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(L_{i,\pi}^{(0)} + L_{i,\pi}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\
& + \frac{\mu_G^2(\mu_b)}{m_b^2} \left(L_{i,G}^{(0)} + L_{i,G}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + \frac{\rho_D^3(\mu_b)}{m_b^3} \left(L_{i,D}^{(0)} + L_{i,D}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\
& + \frac{\rho_{LS}^3(\mu_b)}{m_b^3} \left(L_{i,LS}^{(0)} + L_{i,LS}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + O\left(\frac{1}{m_b^4} \right) \Bigg] \, ,
\end{aligned}
$$

Known QCD corrections

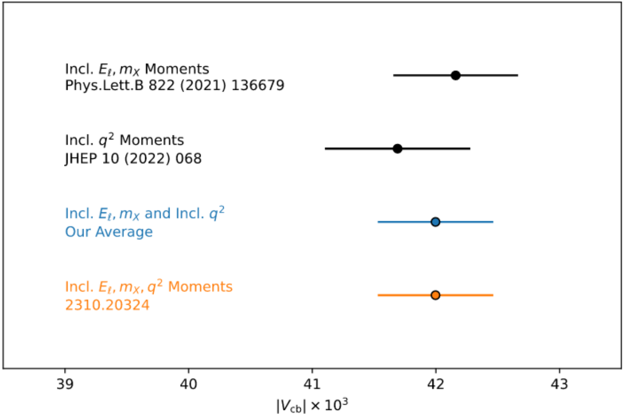
Nir, Pak, Downlin, Egner, Fael, Steinhauser, Gambino, Manohar, Block, Becher, Alberti, Mannel, Turczyk, Dassinger, Schonwald.

$\Gamma_{\rm sl}$	tree	α_s	α_s^2	α_s^3
Partonic				['21]
μ_π^2, μ_G^2		['06, '12,'13,'15]		
ρ_D^3, ρ_{LS}^3		['21]		
$1/m_b^4, 1/m_b^5$	['06,'10,'18,'23]			
$q_n(q^2_{\rm cut})$	tree	α_s	α_s^2	
Partonic			['24]	
μ_G^2, μ_π^2		['12,'13]		
ρ_D^3, ρ_{LS}		['21]		
$1/m_b^4, 1/m_b^5$	['18, '23]			
$\ell_n(E_{\rm cut}), h_n(E_{\rm cut})$	tree	α_s	$\alpha_s^2\beta_0$	α_s^2
Partonic			['05]	['08]*
μ_G^2, μ_π^2		['07,'13]		
ρ_D^3				
$1/m_b^4, 1/m_b^5$	['06, '10,'23]			

- Fael, Herren [2403.03976] Mannel, Moreno, Pivovarov [2112.03875]
- *only known for fixed m_c/m_b and lepton energy cuts

Summary of $|V_{cb}|$ inclusive

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w



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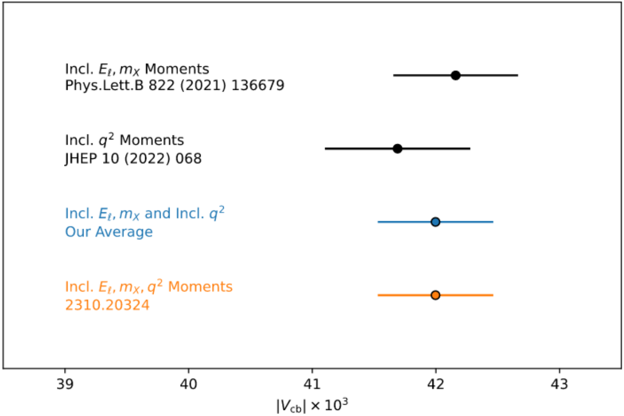

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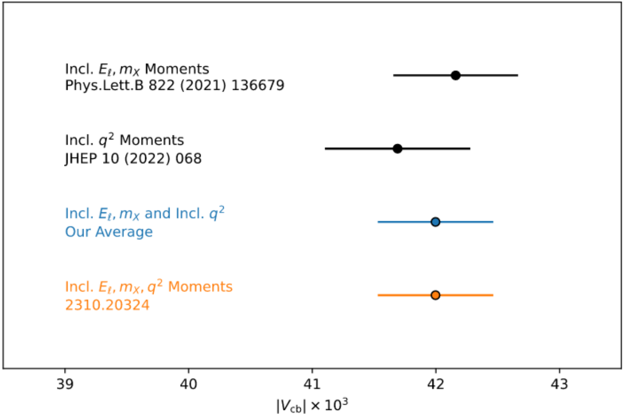

- Standard includes up to $1/m_b^3$
- Do not (yet) include NNLO to q^2 moments In progress

Getting to the bottom of $|V_{cb}|$

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

Vices and Virtues:

- Systematic framework for power-corrections
- Higher precision: Include higher-order $1/m_b$ and α_s corrections in rate and moments!
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109Mannel, Milutin, KKV [2311.1200]

Getting to the bottom of $|V_{cb}|$

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

Vices and Virtues:

- Systematic framework for power-corrections
- To be compared with Lattice?
- Higher precision: Include higher-order $1/m_b$ and α_s corrections in rate and moments!
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The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177, Mannel, Milutin, KKV [2311.1200]

- Standard lepton energy and hadronic mass moments are not RPI quantities
- Only RPI moments are q^2 moments
- Determinations from Belle and Belle II availabe Phys. Rev. D 104, 112011 (2022), Phys. Rev. D 107, 072002 (2023)

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Quirks:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)
- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)
- Only 18 parameters at $1/m_b^5$ (versus 31) Mannel, Milutin, KKV [2311.1200]

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- Only 18 parameters at $1/m_b^5$ (versus 31) Mannel, Milutin, KKV [2311.1200]
- q^2 moments could enable a full extraction up to $1/m_b^4$?

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- Agreement with BCG extraction (differs due to branching ratio inputs)
  Bordone,Capdevila, Gambino [2021]

$$|V_{cb}|^{\rm BCG}_{\rm incl} = (42.00 \pm 0.51) \times 10^{-3}$$

- Higher order terms reduce value by 0.25%.

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- Inputs for $B \to X_u \ell\nu$, B lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.

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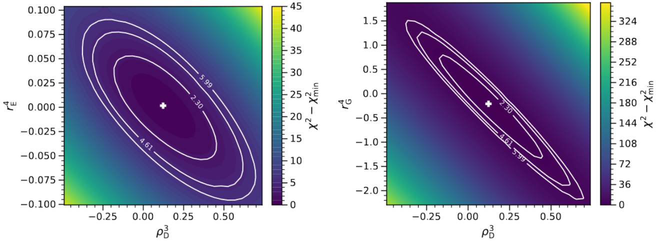

What if we combine everything?

Gambino, Finauri [2310.20324]

- Includes terms up to $1/m_b^3$

NEW! Calculation of BLM α_s^2 corrections to q^2 moments

NEW! Includes QED corrections to the lepton moments Bigi, Bordone, Gambino, Haisch, Piccione, JHEP 11 (2023) 163

Combined E_ℓ, M_X, q^2 moments:

$$|V_{cb}|^{\rm GF}_{\rm incl,all} = (41.95 \pm 0.27|_{\rm exp} \pm 0.31|_{\rm th} \pm 0.25|_{\Gamma}) \times 10^{-3} = (41.95 \pm 0.48) \times 10^{-3}$$

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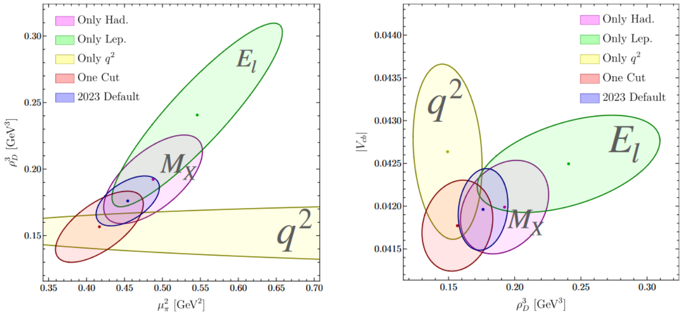
- Agrees with previous determinations, reduced uncertainty
- Agrees with determination from three points at fixed cut

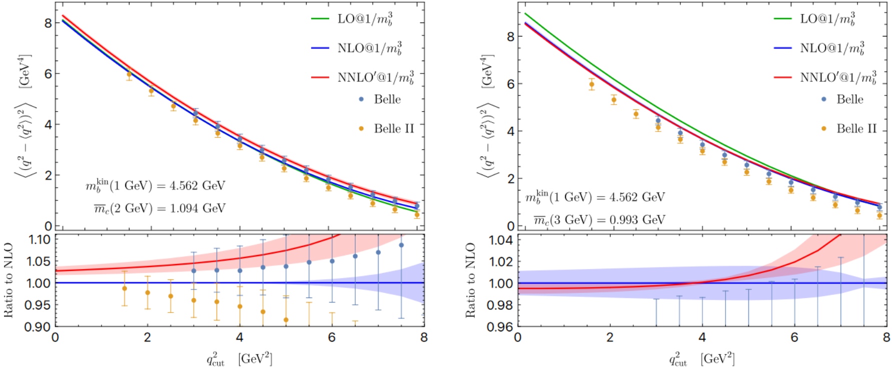
First combined Fit

Gambino, Finauri [2310.20324]



- Complementary between different measurements
- Extracted $\rho_D^3 = 0.176 \pm 0.019$ GeV^3
- Important input for lifetimes and $B \to X_u$, $B \to X_s$

Herren, Fael [2403.03976]



$\overline{m_c}(2\ \mathrm{GeV})$ not ideal choice

$\overline{m_c}(3\ \mathrm{GeV})$ better

NEW: NNLO corrections to q^2 momemts

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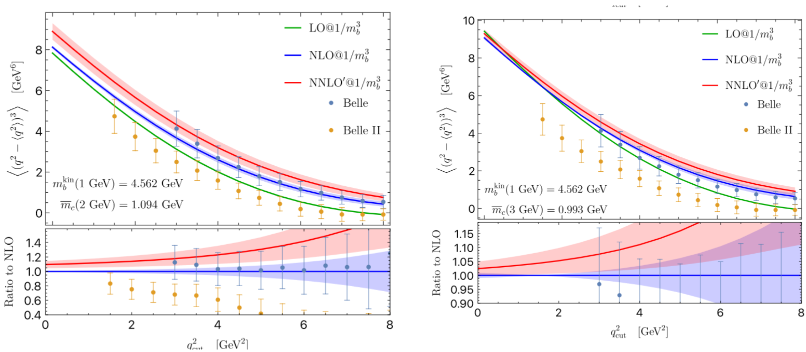

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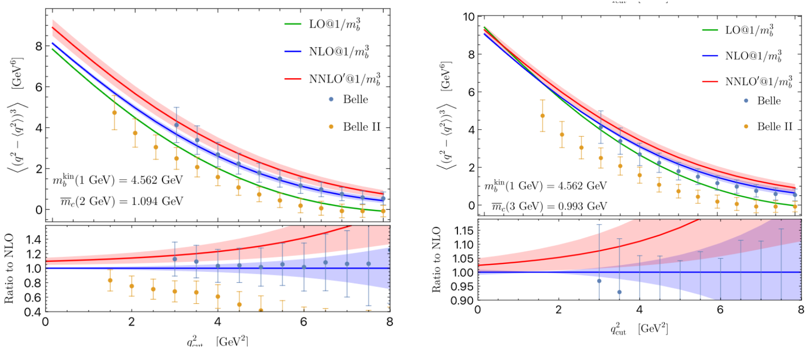

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Full combined analysis and updated q^2 fits in progress!

NEW: Inclusive decays: The Kolya package

Kolya package, Fael, Milutin, KKV [2409.15007]

Open source Python package:
https://gitlab.com/vcb-inclusive/kolya

- HQE predictions for several observables:
  - Centralized $\langle E_\ell \rangle$ moments
  - Centralized $\langle q^2 \rangle$ moments
  - Centralized $\langle M_X^2 \rangle$ moments
  - Total rate + branching ratio with kinematic cut

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Features:

- Includes power corrections up to $1/m_b^5$ Mannel, Milutin, KKV [2311.1200]
- Employs kinetic scheme for m_b and $\overline{\mathrm{MS}}$ for m_c
- Interface with CRunDec for automatic RGE evolution Chetyrkin, Kuhn, Steinhauser, Smidth, Herren
- Includes New Physics effects Fael, Rahimi, KKV [JHEP 02 (2023) 086]

Kolya: Installation and features

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- Provide example Jupyter notebooks (see also examples in backup)
- Several cross-checks with literature performed
- Default up to $1/m_b^3$. Higher orders included via flagmb4$=1$ and flagmb5$=1$
- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented

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- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented
- Only includes centralized moments

Outlook for Kolya

First version of Kolya available!

Plans to expand Kolya with:

- QED effects Bigi, Bordone, Gambino, Haisch, Piccione [2308.02849]
- Exact results for NNLO corrections to E_ℓ and M_X moments Herren, Fael [in progress]
- NLO corrections to HQE parameters for E_ℓ and M_X moments

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First version of Kolya available!

Plans to expand Kolya with:

- QED effects Bigi, Bordone, Gambino, Haisch, Piccione [2308.02849]
- Exact results for NNLO corrections to E_ℓ and M_X moments Herren, Fael [in progress]
- NLO corrections to HQE parameters for E_ℓ and M_X moments

And additional observables:

- Forward-backward asymmetry
- $R_X = \Gamma_{B\to X_c\tau\bar{\nu}_\tau}/\Gamma_{B\to X_c l\bar{\nu}_l}$
- Lifetimes
- Predictions for the decay into charmless final states $B \to X_u l\bar{\nu}_l$
- Inclusive D decays Mannel, Fael, KKV [1910.05234] Exploratory study for measurements at BES III [2408.10063]

Any other suggestions?

Constraining higher power corrections?

with Markus Prim, Florian Bernlochner, Ilija Milutin and Matteo Fael [in progress]

Inclusive fits with Kolya

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014], Finauri, Gambino [2023] Prim, Milutin, Fael, Bernlochner, KKV [in progress]

New:

Use Kolya + Experimental measurements → $|V_{cb}|$ and HQE parameters

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This talk:
Preliminary results using q^2 moments (with NNLO)

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Challenge: How to deal with theoretical uncertainties?

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This talk:
Preliminary results using q^2 moments (with NNLO)

Challenge: How to deal with theoretical uncertainties?

Use the $1/m_b^{4,5}$ corrections to better access the theory uncertainty

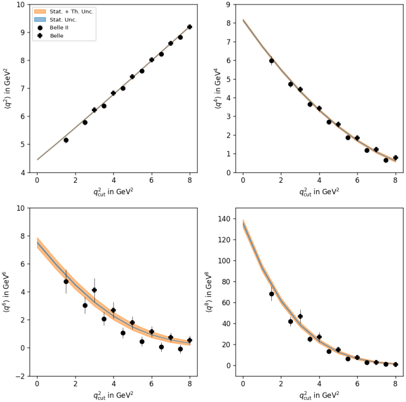
Preliminary fit results at $O3$

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

q^2 moments only, including NNLO

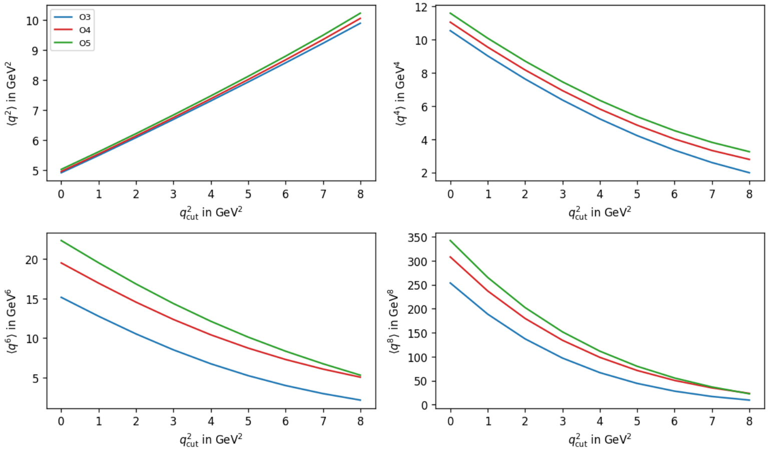
	Central	Fit Unc.	Theory Unc.	Tot Unc.	Precision [%]
μ_π^2	0.490	0.060	0.000	0.060	8.2
μ_G^2	0.360	0.008	0.050	0.050	7.2
ρ_D^3	0.089	0.000	0.018	0.018	5.0

- μ_π^2 does not enter at this order
- No fit uncertainty on ρ_D^3
- External constraint: $\mu_G^2 = 0.36 \pm 0.07$
- Very bad $\chi^2/d.o.f = 800/51$



Effect of higher-order corrections

Kolya



Using LLSA inputs for the HQE parameters and NNLO corrections

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4\ \text{GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- Gives central values but uncertainty challenging to estimate

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- Gives central values but uncertainty challenging to estimate
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60
  - LSSA estimated as priors (60% gaussian uncertainty)
  - -0.25% shift on $|V_{cb}|$ due to power corrections

Preliminary fit results at $O5$

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

	Central	Fit Unc.	Theory Unc.	Tot Unc.	Precision [%]
μ_π^2	0.463	0.058	0.004	0.059	7.9
μ_G^2	0.547	0.003	0.141	0.141	3.9
ρ_D^3	0.040	0.000	0.059	0.059	0.68
r_E^4	0.015	0.000	0.018	0.018	0.84
r_G^4	0.284	0.002	0.151	0.151	1.9
s_E^4	-0.03	0.005	0.045	0.045	0.7
s_B^4	-0.088	0.007	0.255	0.255	0.4
s_{qB}^4	-1.326	0.022	0.895	0.895	1.5

- $(\mu_G^2)\rho_D^3$ and $(\mu_\pi^2)\rho_D^3$ enter!
- Use LLSA ansatz for $1/m_b^{4,5}$ with 50% additional uncertainty and 0.05 GeV^4
- $\chi^2/d.o.f = 248/51$

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Seems to converge less fast as hoped?

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- Use LLSA ansatz for $1/m_b^{4,5}$ with 50% additional uncertainty and 0.05 GeV^4
- $\chi^2/d.o.f = 248/51$

$|V_{cb}|$ does not care! (shifts up by 0.8%)

Outlook: fits in inclusive decays

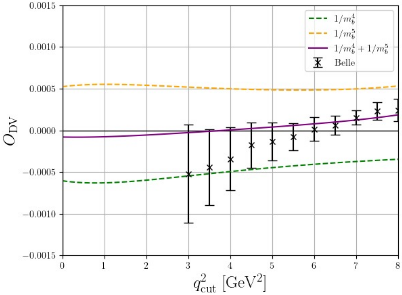
- First analysis of q^2 moments with NNLO corrections [preliminary!]
- Relax LLSA Ansatz for higher terms
- Full analysis of all moments ongoing
- Include uncertainty for missing higher orders
- Switch to sensitive observables or un-expanded versions!

Outlook: power-correction sensitive observables

Mannel, Milutin, Verkade, KKV [2407.01473]; Belle collaboration [2109.01685]

$$\bar{q}_i = C_i^{(0)} + \frac{\mu_G^2}{m_b^2} C_i^{(2)} + \frac{\tilde{\rho}_D^3}{m_b^3} C_i^{(3)} + R_i \ ,$$

- R_i contains higher order terms
- We can then construct an observable only sensitive to higher $k+1$ powers $O_{\rm DV}^{(k)} \sim \Lambda^{k+1}/m_n^{k+1}$
- First study using $O_{\rm DV}^{(3)} = \xi_1 \frac{\bar{q}_1}{m_b^2} + \xi_2 \frac{\bar{q}_2}{m_b^4} + \xi_3 \frac{\bar{q}_3}{m_b^6} + \xi_4 \frac{\bar{q}_4}{m_b^8}$



Alternative treatment of the heavy quark mass

with Anastacia and Thomas [in progress]

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_Q not observable $\rightarrow$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \rightarrow$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

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- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0|T[j_\mu(x) j_\nu(0)]|0\rangle = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)$$

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- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_Q^2}\right)$$

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- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_Q^2)^n M_n \quad \text{with} \quad M_n = \int \frac{ds}{s^{n+1}} R(s)$$

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- Replace m_Q:

$$m_Q = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344 Boushmelev, Mannel, KKV [2301.05607]

$$\begin{aligned}
\Gamma(b \to u\ell\nu) &\sim \left(\left(\frac{\bar{C}_n}{M_n}\right)^{1/2}\right)^5 \left(1 + \frac{\alpha_s(\mu)}{\pi} a_1 + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 a_2 + \cdots\right) \\
&\sim \left(\frac{\bar{C}_n^{(0)}}{M_n}\right)^{5/2} \left(1 + \frac{\alpha_s(\mu)}{\pi}\left[a_1 + \frac{5}{2n}\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}}\right]\right. \\
&\qquad \left. + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left[a_2 + \frac{5}{2n} a_1 \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} + \frac{5}{2n}\frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}} + \frac{5}{4n}\left(\frac{5}{4n} - 1\right)\left(\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}}\right)^2\right] + \cdots\right) \\
&\sim \left(\frac{\bar{C}_n^{(0)}}{M_n}\right)^{5/2} \left(1 + \frac{\alpha_s(\mu)}{\pi} d_n^{(1)} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left[d_n^{(2)} + \beta_0 d_n^{(1)} \ln\left(\frac{\mu^2}{m_Q^2}\right)\right] + \cdots\right)
\end{aligned}$$

- Conclusion: pert. series improves a bit
- In progress: Similar approach for the charm + power corrections

	n						
	1	2	3	4	5	6	7
$d_n^{(1)}$	10.24	7.29	5.85	4.94	4.29	3.80	3.41
$d_n^{(2)}$	70.41	49.45	39.69	33.70	29.52	26.40	23.93
$d_n^{(2)}/d_n^{(1)}$	6.87	6.79	6.78	6.81	6.89	6.95	7.03
μ/m_Q	0.167	0.170	0.170	0.169	0.166	0.163	0.160

Lattice and the Continuum Continued

Outlook: Lattice meets Continuum?

- HQE parameters are in kinetic scheme defined with QCD states
- Differ for B, B_s and D decays!
- Challenging to convert infinite mass parameters

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- New observables not accessible in experiment?

Backup

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

Total Rate

We define the total rate as

$$\Gamma_{\rm sl} = \frac{G_F^2 (m_b^{\rm kin})^5}{192\pi^3} |V_{cb}|^2 X$$

The coefficients X is a function of the quark masses, α_s, the HQE parameters and the Wilson coefficients. It is evaluated by the function `X_Gamma_KIN_MS(par, hqe, wc)`

[5]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
wc  = kolya.parameters.WCoefficients()
kolya.TotalRate.X_Gamma_KIN_MS(par,hqe,wc)
```

[5]: 0.539225163728085

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

[6]:
```
Vcb = 42.2e-2
kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)
```

[6]: 10.555834162102016

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git. Herren, Fael [2403.03976]

```
>>> kolya.Q2moments.moment_1_KIN_MS(8.0, par, hqe, wc, flag_DEBUG=1)
Q2moment n. 1 LO =  9.148659808170105
Q2moment n. 1 NLO = api * -1.319532010835962
Q2moment n. 1 NNLO = api^2 * -9.616956902561078
Q2moment n. 1 NLO pw = api * -0.7873907726673756
Q2moment n. 1 NNLO from NLO pw = api^2 * 8.39048437244325
```

- Includes new NNLO corrections
- NNLO and NLO to power corrections can be turned off

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

[1]:
```
import kolya
import numpy as np
```

Physical parameters

Physical parameters like quark masses like $m_b^{\rm kin}(\mu_{WC})$, $\overline{m}_c(\mu_c)$ and $\alpha_s(\mu_s)$ are declared in the class `parameters.physical_parameters`. Initialization set default values

[2]:
```
par = kolya.parameters.physical_parameters()
par.show()
```

```
bottom mass:       mbkin( 1.0  GeV)     =  4.563  GeV
charm mass:        mcMS( 3.0  GeV)      =  0.989  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.2182
```

In order to set the quark masses at scales different from the default ones in a consistent way, we include the method FLAG2023 which internally use CRunDec. For instance, we set the quark masses at a scale $\mu_{WC} = \mu_c = 2$ GeV in the following way:

[3]:
```
par = kolya.parameters.physical_parameters()
par.FLAG2023(scale_mcMS=2.0, scale_mbkin=2.0)
par.show()
```

```
bottom mass:       mbkin( 2.0  GeV)     =  4.295730717092438  GeV
charm mass:        mcMS( 2.0  GeV)      =  1.0940623249384822  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.21815198098622618
```

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

HQE parameters

Non-perturbative matrix elements in the HQE are declared in the class `parameters.HQE_parameters`. This class is defined in the historical basis of hep-ph/1307.4551. By default they are initalized to zero. We can set their values in the following way

```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
hqe.show()
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3
```

```
hqe.show(flagmb4=1)
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3

m1 =  0  GeV^4
m2 =  0  GeV^4
m3 =  0  GeV^4
m4 =  0  GeV^4
m5 =  0  GeV^4
m6 =  0  GeV^4
m7 =  0  GeV^4
m8 =  0  GeV^4
m9 =  0  GeV^4
```

Kolya: Installation and features

Fael, Milutin, KKV [2409.15007], https://gitlab.com/vcb-inclusive/kolya.git

- Implemented both "RPI" and "historical" (perp) basis
- LLSA predictions for the HQE elements implemented
- Gives "predictions" for moments and total rate

The classes `parameters.LSSA_HQE_parameters` and `parameters.LSSA_HQE_parameters_RPI` store the same HQE parameters as `parameters.HQE_parameters` and `parameters.HQE_parameters_RPI` in the "perp" and RPI basis respectively up to $1/m_b^5$. They are initialized to values predicted by the 'lowest-lying state saturation ansatz' (LSSA).

In [14]:

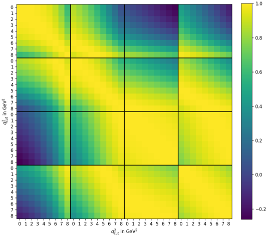
```
hqe_perp = kolya.parameters.LSSA_HQE_parameters()
hqe_RPI = kolya.parameters.LSSA_HQE_parameters_RPI()

print(r'LSSA prediction for rhoD in the perp basis: ', hqe_perp.rhoD)
print('LSSA prediction for m1 in the perp basis: ', hqe_perp.m1, '\n')

print('LSSA prediction for rhoD in the RPI basis: ', hqe_RPI.rhoD)
print('LSSA prediction for rEtilde in the RPI basis: ', hqe_RPI.rEtilde)
```

Out [14]:

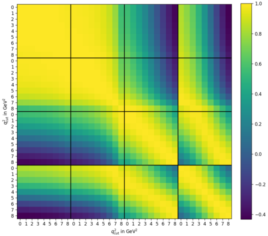
```
LSSA prediction for rhoD in the perp basis:  0.231
LSSA prediction for m1 in the perp basis:  0.126

LSSA prediction for rhoD in the RPI basis:  0.205
LSSA prediction for rEtilde in the RPI basis:  0.098
```

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

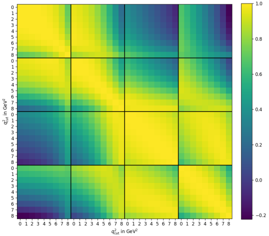
$O3$



$O4$



$O5$



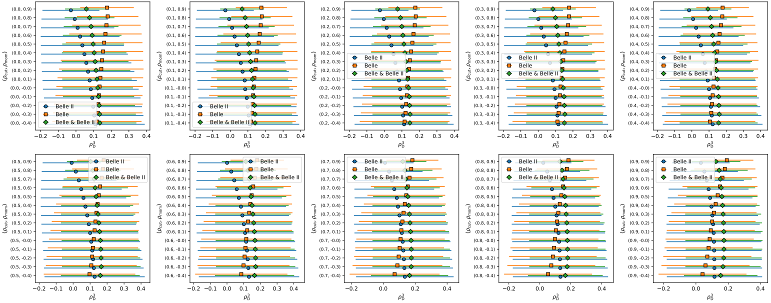
- Order of the analysis changes the correlations
- Large correlations between different cuts
- Correlations between different moments!

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$

$$\rho_n[q_n(q_A^2) - q_n(q_B^2)] = \rho_{\rm cut}^x \qquad x = \frac{|q_A^2 - q_B^2|}{0.5\mathrm{GeV}^2}$$

- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} stable w.r.t. theory correlations



What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

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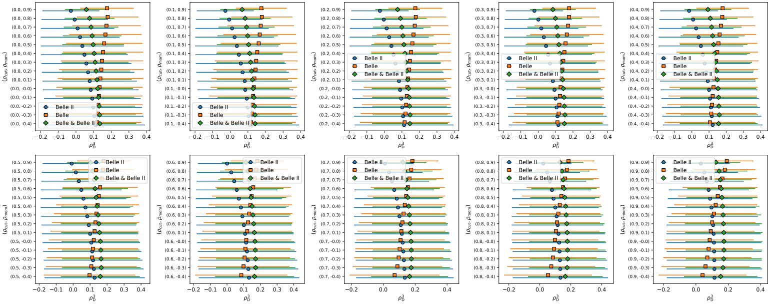
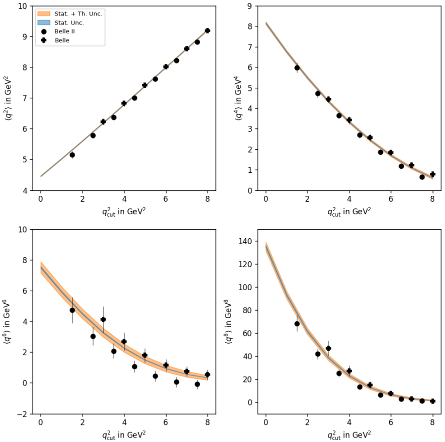
- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} uncertainty includes large range of correlations



Fit details

Preliminary!



Preliminary fit setup

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

- Nuisance parameters: scale α_s, μ_c and m_b^{kin}
- Include (μ_π^2), μ_G^2 and $\rho_D^3, \ldots$ in the likelihood
- Sample nuisance parameter from uniform distribution
- Refit with new sets of nuisance parameters
- Check how strongly the parameters of interest scatter

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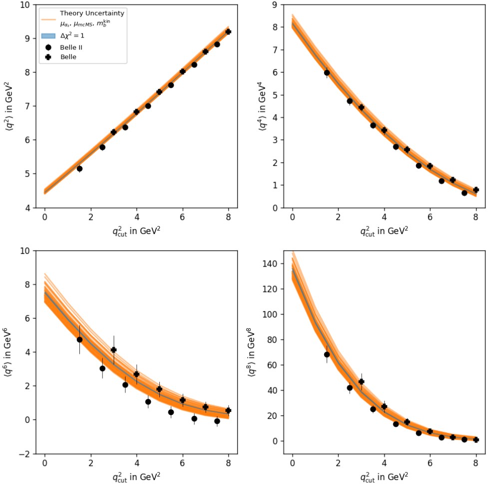
Do not include uncertainty on ρ_D^3 but check order by order if the fit improves

Refitting the data

Preliminary! Refitting at $O3$



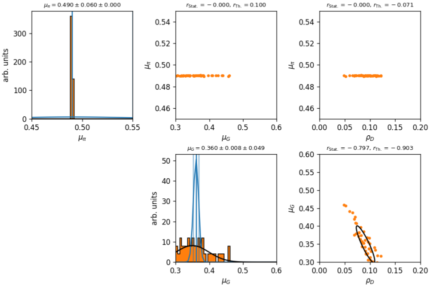
$$q^2_{\rm cut} = 0~{\rm GeV}^2\,, \qquad m_b^{\rm kin} = 4.573~{\rm GeV}\,, \qquad \bar{m}_c(2~{\rm GeV}) = 1.092~{\rm GeV}\,. \qquad ($$

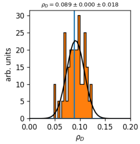
We then find[8]

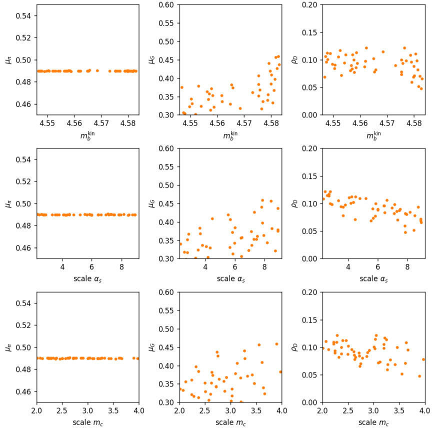
$$\begin{aligned}
q_1 = \frac{m_b^2}{\mu_3}\Big(& 0.22\mu_3 - 0.57\frac{\mu_G^2}{m_b^2} - 1.4\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 5.5\frac{\tilde\rho_D^3}{m_b^3} + 16\frac{\tilde r_E^4}{m_b^4} - 5.7\frac{r_G^4}{m_b^4} - 1.7\frac{\tilde s_E^4}{m_b^4} \\
&+ 0.097\frac{s_B^4}{m_b^4} - 0.064\frac{s_{qB}^4}{m_b^4} - 24\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 19\frac{X_1^5}{m_b^5} + 18\frac{X_2^5}{m_b^5} - 15\frac{X_3^5}{m_b^5} + 2.3\frac{X_4^5}{m_b^5} \\
&+ 6.5\frac{X_5^5}{m_b^5} + 0.91\frac{X_6^5}{m_b^5} - 7.0\frac{X_7^5}{m_b^5} + 8.0\frac{X_8^5}{m_b^5} + 5.2\frac{X_9^5}{m_b^5} - 4.4\frac{X_{10}^5}{m_b^5} + 0.047\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,, \\
q_2 = \frac{m_b^4}{\mu_3}\Big(& 0.022\mu_3 - 0.12\frac{\mu_G^2}{m_b^2} - 0.61\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 1.6\frac{\tilde\rho_D^3}{m_b^3} + 7.7\frac{\tilde r_E^4}{m_b^4} - 2.1\frac{r_G^4}{m_b^4} - 0.66\frac{\tilde s_E^4}{m_b^4} \\
&+ 0.20\frac{s_B^4}{m_b^4} - 0.082\frac{s_{qB}^4}{m_b^4} - 12\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 20\frac{X_1^5}{m_b^5} + 15\frac{X_2^5}{m_b^5} - 22\frac{X_3^5}{m_b^5} + 3.2\frac{X_4^5}{m_b^5} \\
&+ 4.2\frac{X_5^5}{m_b^5} - 0.32\frac{X_6^5}{m_b^5} - 4.9\frac{X_7^5}{m_b^5} + 7.6\frac{X_8^5}{m_b^5} + 1.8\frac{X_9^5}{m_b^5} - 2.3\frac{X_{10}^5}{m_b^5} + 0.030\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,, \\
q_3 = \frac{m_b^6}{\mu_3}\Big(& 0.0012\mu_3 - 0.013\frac{\mu_G^2}{m_b^2} - 0.24\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 0.34\frac{\tilde\rho_D^3}{m_b^3} + 2.9\frac{\tilde r_E^4}{m_b^4} - 0.56\frac{r_G^4}{m_b^4} - 0.19\frac{\tilde s_E^4}{m_b^4} \\
&+ 0.093\frac{s_B^4}{m_b^4} - 0.035\frac{s_{qB}^4}{m_b^4} - 5.8\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 12\frac{X_1^5}{m_b^5} + 9.3\frac{X_2^5}{m_b^5} - 17\frac{X_3^5}{m_b^5} + 2.5\frac{X_4^5}{m_b^5} \\
&+ 2.0\frac{X_5^5}{m_b^5} - 0.42\frac{X_6^5}{m_b^5} - 2.8\frac{X_7^5}{m_b^5} + 5.1\frac{X_8^5}{m_b^5} + 0.10\frac{X_9^5}{m_b^5} - 1.0\frac{X_{10}^5}{m_b^5} + 0.016\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,, \\
q_4 = \frac{m_b^8}{\mu_3}\Big(& 0.0010\mu_3 - 0.012\frac{\mu_G^2}{m_b^2} - 0.10\frac{(\mu_G^2)^2}{m_b^4\mu_3} - 0.22\frac{\tilde\rho_D^3}{m_b^3} + 1.6\frac{\tilde r_E^4}{m_b^4} - 0.33\frac{r_G^4}{m_b^4} - 0.11\frac{\tilde s_E^4}{m_b^4} \\
&+ 0.047\frac{s_B^4}{m_b^4} - 0.018\frac{s_{qB}^4}{m_b^4} - 2.6\frac{\mu_G^2\tilde\rho_D^3}{m_b^5\mu_3} - 7.8\frac{X_1^5}{m_b^5} + 7.6\frac{X_2^5}{m_b^5} - 17\frac{X_3^5}{m_b^5} + 2.5\frac{X_4^5}{m_b^5} \\
&+ 1.2\frac{X_5^5}{m_b^5} - 0.26\frac{X_6^5}{m_b^5} - 2.5\frac{X_7^5}{m_b^5} + 4.7\frac{X_8^5}{m_b^5} - 0.40\frac{X_9^5}{m_b^5} - 1.0\frac{X_{10}^5}{m_b^5} + 0.015\frac{X_{\rm IC}^5}{m_b^3 m_c^2}\Big)\,. \qquad (
\end{aligned}$$

Correlations between parameters of interest

Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]







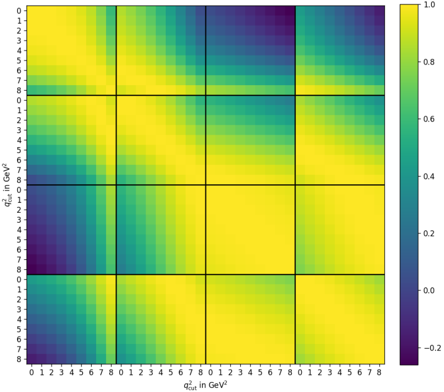
Best fit with 1σ band

Refitted with sampled nuisance parameters

Use central limits theorem to extract Gaussian uncertainty from flat priors

Theoretical correlations

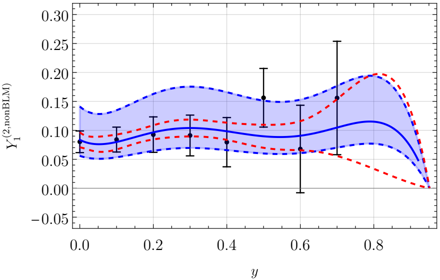
Preliminary! Prim, Milutin, Fael, Bernlochner, KKV [in progress]

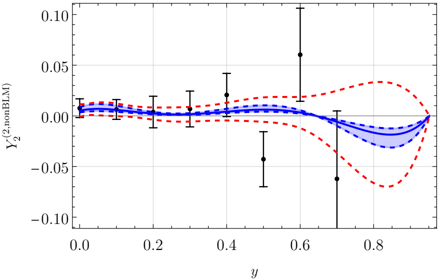


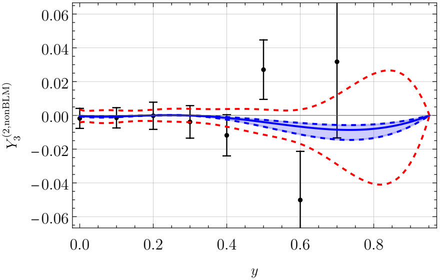
- Large correlations between different cuts
- Correlations between different moments!

NNLO contributions to lepton energy moments

Biswas, Melnikov [0911.4142], Gambino [1107.3100]. Fael, Milutin, KKV [2409.15007]

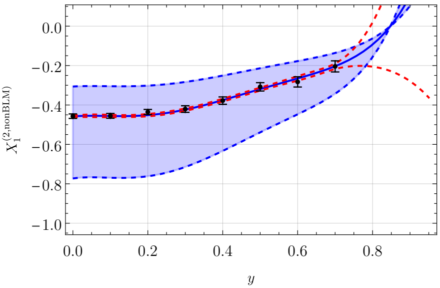


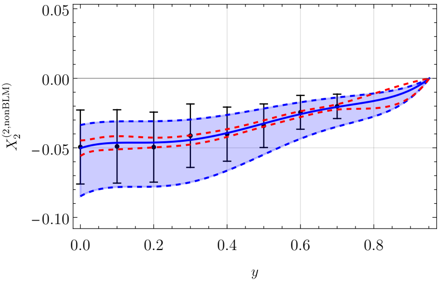


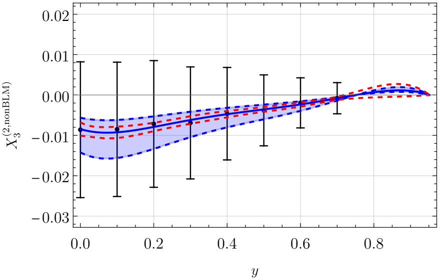


NNLO contributions to M_X moments

Biswas, Melnikov [0911.4142], Gambino [1107.3100]. Fael, Milutin, KKV [2409.15007]







State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)}\frac{\alpha_s}{\pi} + \Gamma_0^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)}\left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)\right.$$
$$\left. + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\left(\Gamma^{(D,0)} + \Gamma_0^{(1)}\left(\frac{\alpha_s}{\pi}\right)\right) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

- Include terms up to $1/m_b^4$* see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

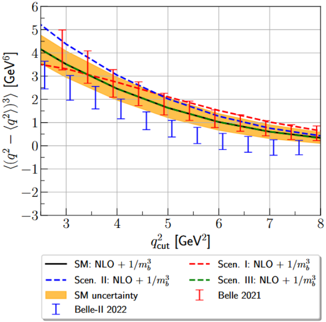
$$|V_{cb}|^{\rm BCG}_{\rm incl} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

Fael, Rahimi, KKV [2208.04282]



- NP would also influence the moments of the spectrum [Never tested!]
- Requires a simultaneous fit of hadronic parameters and NP

Fael, Rahimi, KKV [2208.04282]

$P_{L(R)} = 1/2\,(1 \mp \gamma_5)$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F V_{cb}}{\sqrt{2}} \left[(1 + C_{V_L})\, O_{V_L} + \sum_{i = V_R, S_L, S_R, T} C_i\, O_i \right],$$

$$\begin{aligned} O_{V_{L(R)}} &= \left(\bar{c}\gamma_\mu P_{L(R)} b\right) \left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right) , \\ O_{S_{L(R)}} &= \left(\bar{c} P_{L(R)} b\right) \left(\bar{\ell} P_L \nu_\ell\right) \\ O_T &= \left(\bar{c}\,\sigma_{\mu\nu} P_L b\right) \left(\bar{\ell}\,\sigma^{\mu\nu} P_L \nu_\ell\right) . \end{aligned}$$

$$\begin{aligned} \langle \mathcal{M} \rangle = \xi_{\text{SM}} &+ |C_{V_R}|^2\, \xi_{\text{NP}}^{\langle V_R, V_R \rangle} + |C_{S_L}|^2\, \xi_{\text{NP}}^{\langle S_L, S_L \rangle} + |C_{S_R}|^2\, \xi_{\text{NP}}^{\langle S_R, S_R \rangle} + |C_T|^2\, \xi_{\text{NP}}^{\langle T, T \rangle} \\ &+ \text{Re}((C_{V_L} - 1) C_{V_R}^*)\, \xi_{\text{NP}}^{\langle V_L, V_R \rangle} + \text{Re}(C_{S_L} C_{S_R}^*)\, \xi_{\text{NP}}^{\langle S_L, S_R \rangle} + \text{Re}(C_{S_L} C_T^*)\, \xi_{\text{NP}}^{\langle S_L, T \rangle} \\ &+ \text{Re}(C_{S_R} C_T^*)\, \xi_{\text{NP}}^{\langle S_R, T \rangle} , \end{aligned}$$

Expanded moments in terms of C_i