



**Universität
Zürich^{UZH}**

Semileptonic decays in the continuum

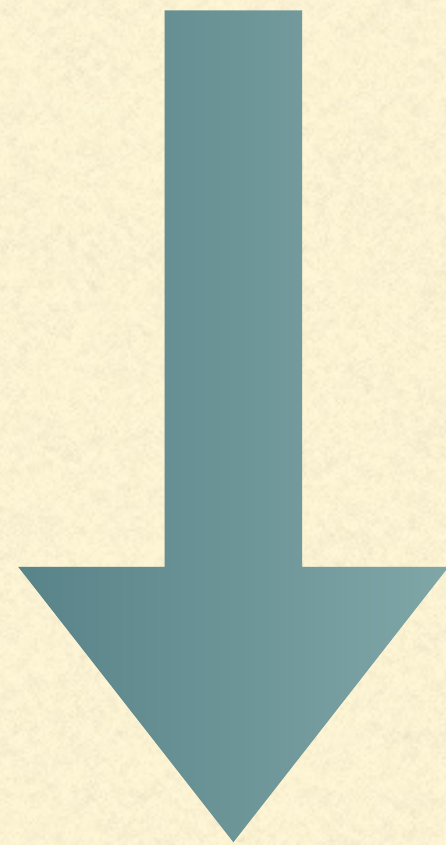
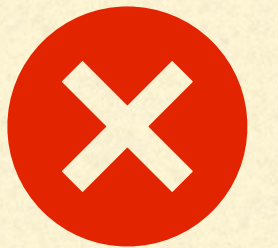
Florian Herren

What could I possibly talk about?

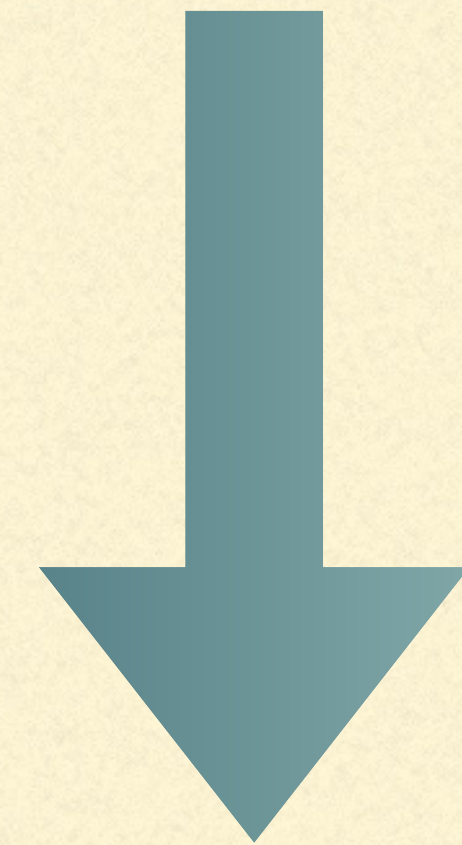
Experiment



Lattice



Semileptonic decays



Raynette's talk tomorrow

Many talks already

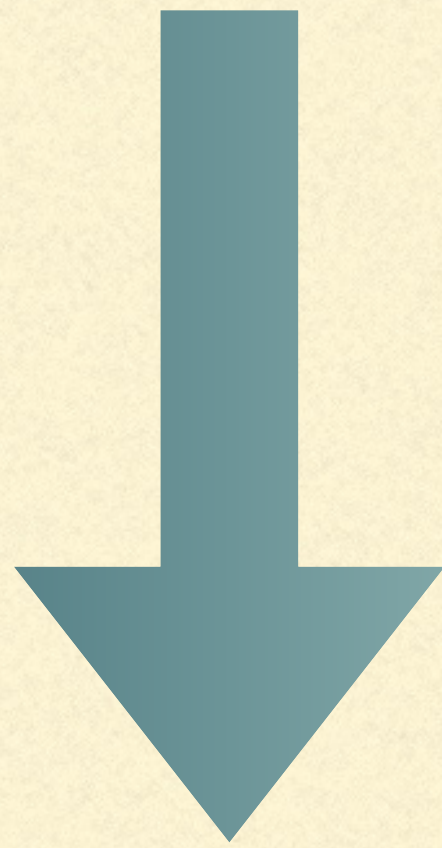
What could I possibly talk about?

Charged Current?

Semileptonic decays

What could I possibly talk about?

Charged Current?



Semileptonic decays

Several talks on the menu

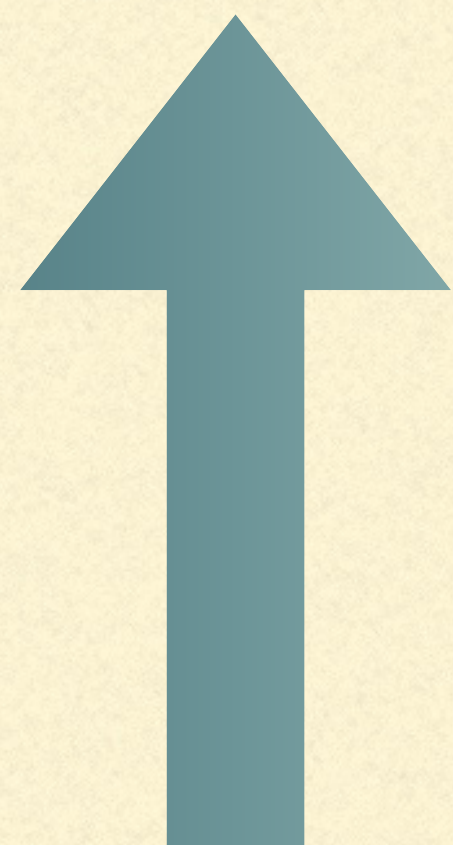
What could I possibly talk about?

Semileptonic decays

Neutral Current?

What could I possibly talk about?

Jack & Jaime



Semileptonic decays

Neutral Current?

What could I possibly talk about?

Inclusive decays?

Semileptonic decays

What could I possibly talk about?

Inclusive decays?

Semileptonic decays

Also covered...

What could I possibly talk about?

Semileptonic decays

Multihadron final states?

What could I possibly talk about?

Fernando...

Semileptonic decays

Multihadron final states?

What could I possibly talk about?

Charged Current

Semileptonic decays

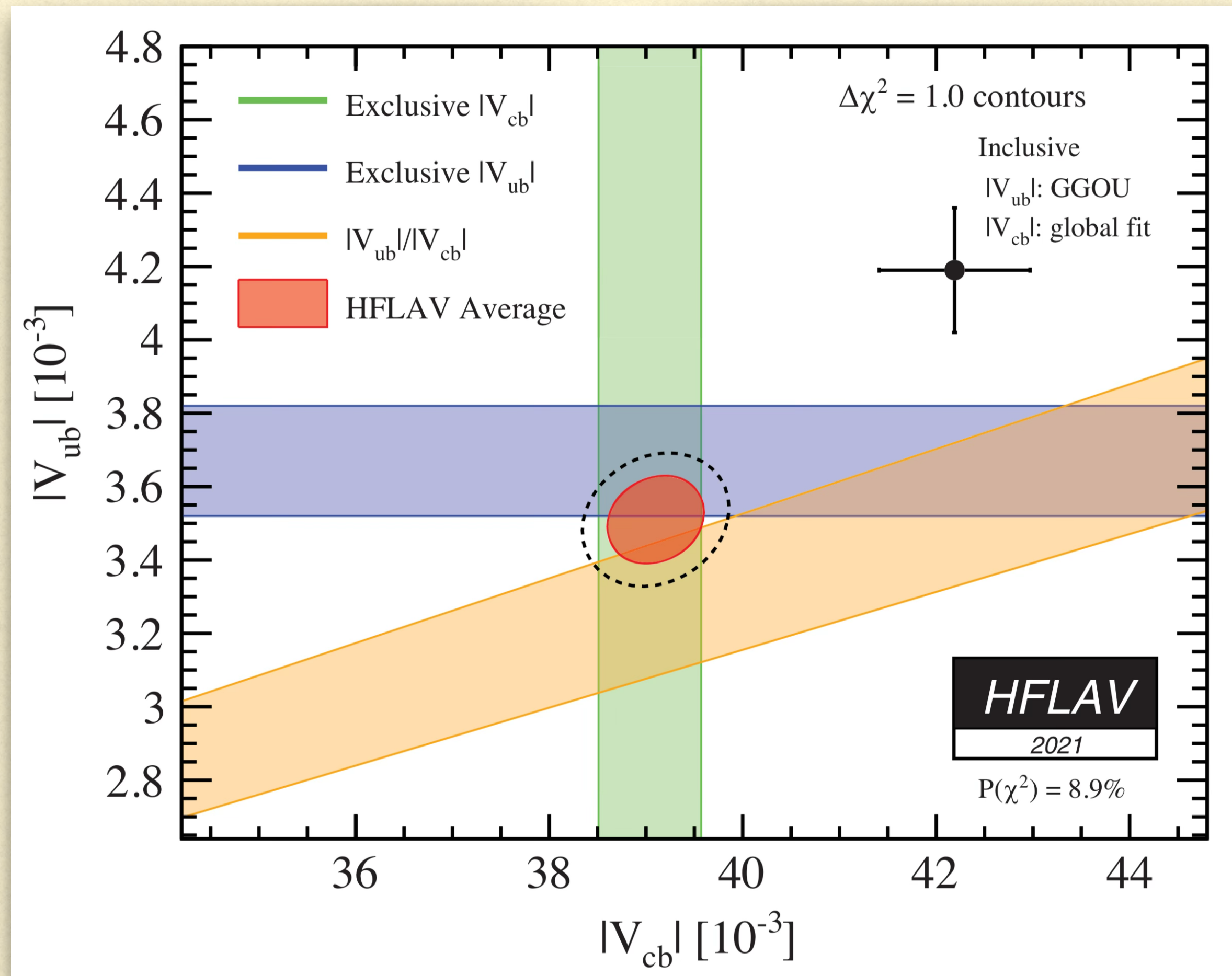
Multihadron final states

Outline

- Charged current semileptonic decays of heavy mesons
- How do form factors look?
- First phenomenological studies of decays with two final state hadrons

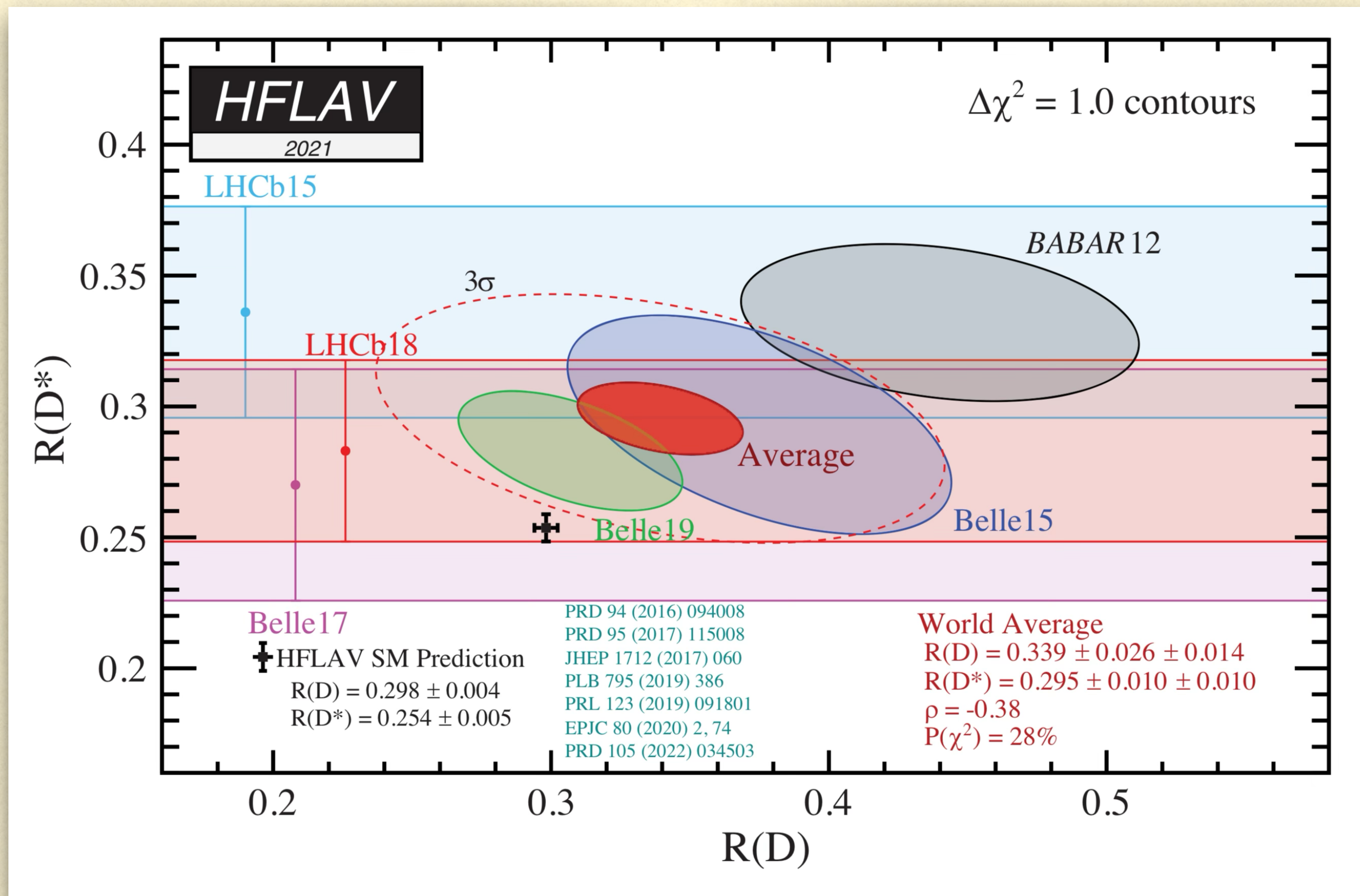
Charged current decays of heavy mesons

Why do we care?



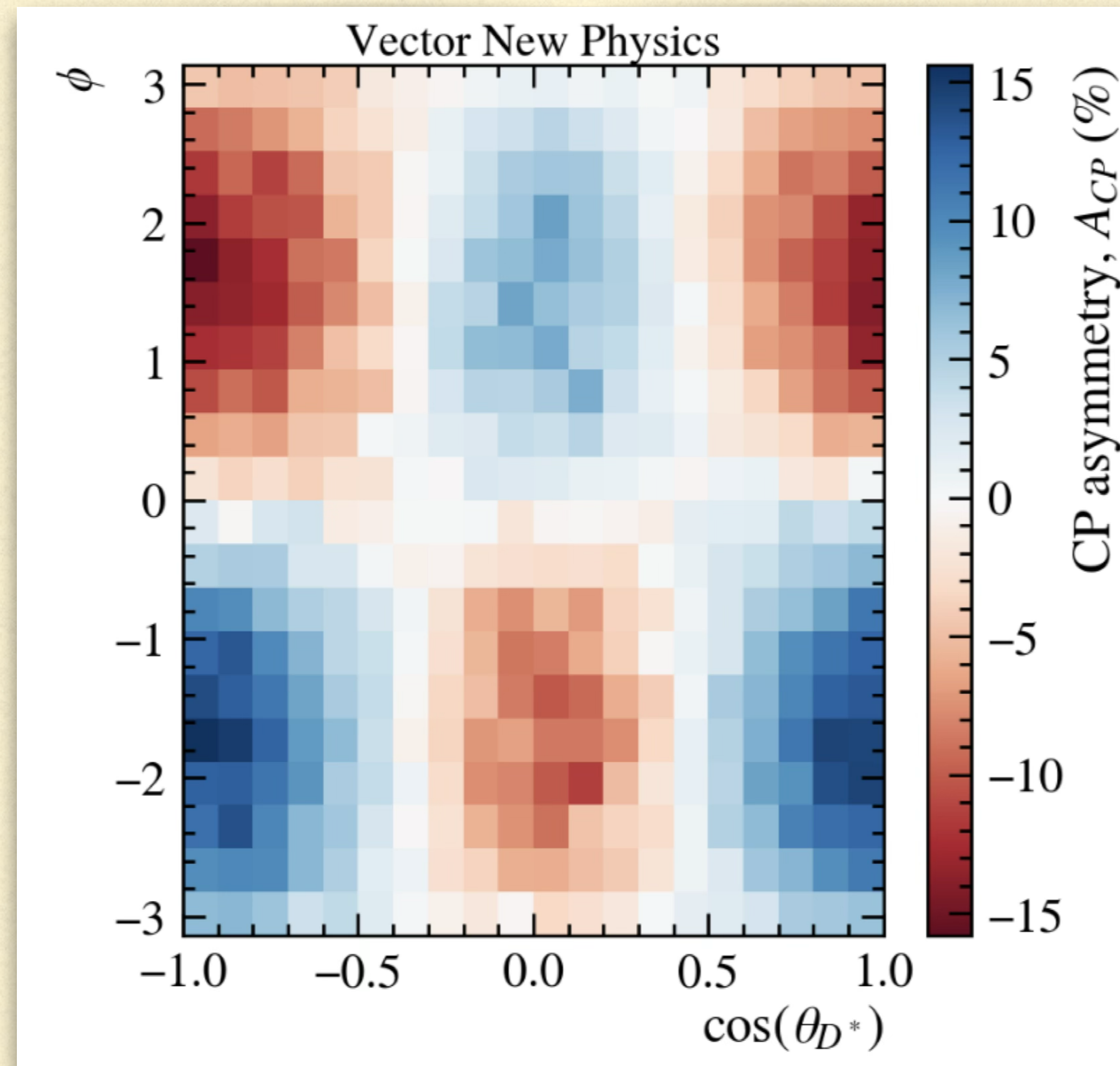
- Ideal laboratory for determinations of $|V_{cb}|$ and $|V_{ub}|$
- Tests of lepton flavour universality
- Tests of CP violation
-

Why do we care?



- Ideal laboratory for determinations of $|V_{cb}|$ and $|V_{ub}|$
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- ...

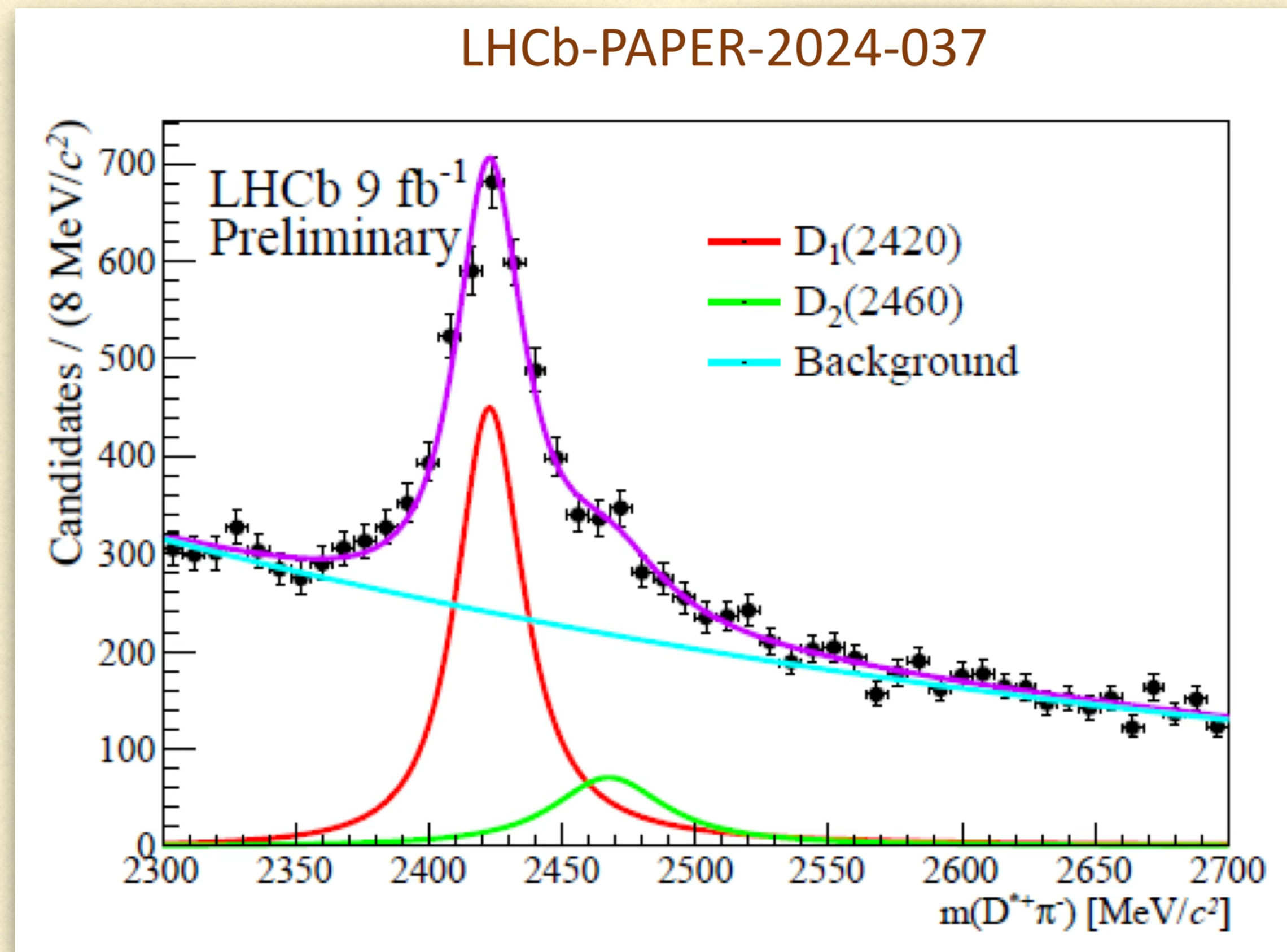
Why do we care?



Taken from Abhijit Mathad's [talk last week](#)

- Ideal laboratory for determinations of $|V_{cb}|$ and $|V_{ub}|$
- Tests of lepton flavour universality
- Tests of CP violation
-

What's the challenge with resonances?



Guy Wormser's [talk in Vienna](#)

- Lineshapes in use are generally just Breit-Wigner or Gounaris-Sakurai
- What on earth is nonresonant $B \rightarrow \pi\pi\ell\nu$?
- Rarely use the precision knowledge we have

What's the challenge with resonances?

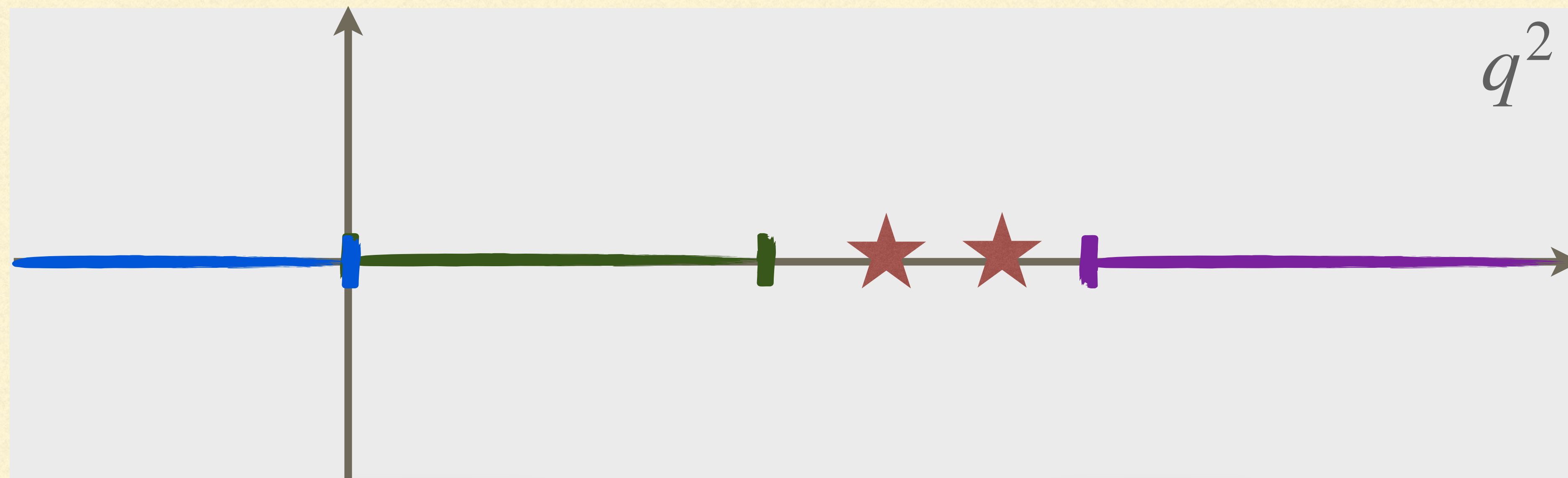
Source	$B^+ \rightarrow \rho^0 \ell^+ \nu_\ell$									
	$q1$	$q2$	$q3$	$q4$	$q5$	$q6$	$q7$	$q8$	$q9$	$q10$
Detector effects	2.8	2.0	1.6	1.1	1.7	1.9	2.4	1.4	1.4	1.6
Beam energy	2.1	1.9	1.9	1.5	1.3	1.1	1.0	0.9	0.8	0.5
Simulated sample size	14.1	7.8	7.4	6.3	6.3	5.2	6.4	5.6	6.2	7.3
BDT efficiency	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Physics constraints	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
Signal model	0.7	0.2	0.2	0.2	0.3	0.4	0.5	0.3	1.8	2.4
ρ lineshape	1.7	1.6	2.0	1.0	1.9	1.8	1.4	0.9	1.6	1.7
Nonresonant $B \rightarrow \pi\pi\ell\nu_\ell$	5.6	6.3	6.7	8.6	9.3	10.7	10.1	7.0	7.8	11.8
DFN parameters	3.6	5.5	4.1	3.5	1.1	1.2	2.7	1.7	1.9	2.3
$B \rightarrow X_u \ell \nu_\ell$ model	1.7	3.0	3.8	5.0	5.8	6.1	6.3	1.9	7.2	12.4
$B \rightarrow X_c \ell \nu_\ell$ model	1.8	1.9	1.7	1.1	1.4	1.7	0.9	0.9	1.9	2.6
Continuum	31.5	24.3	17.0	19.6	13.2	14.8	16.0	16.6	15.2	18.7
Total systematic	35.6	27.5	21.0	23.5	18.8	20.5	21.6	19.4	20.2	27.0
Statistical	30.0	17.5	20.8	14.4	12.4	13.6	14.1	10.4	12.2	11.8
Total	46.6	32.6	29.6	27.6	22.6	24.6	25.8	22.0	23.6	29.5

- Lineshapes in use are generally just Breit-Wigner or Gounaris-Sakurai
- What on earth is nonresonant $B \rightarrow \pi\pi\ell\nu$?
- Rarely use the precision knowledge we have

Belle II collaboration, [2407.17403](https://arxiv.org/abs/2407.17403)

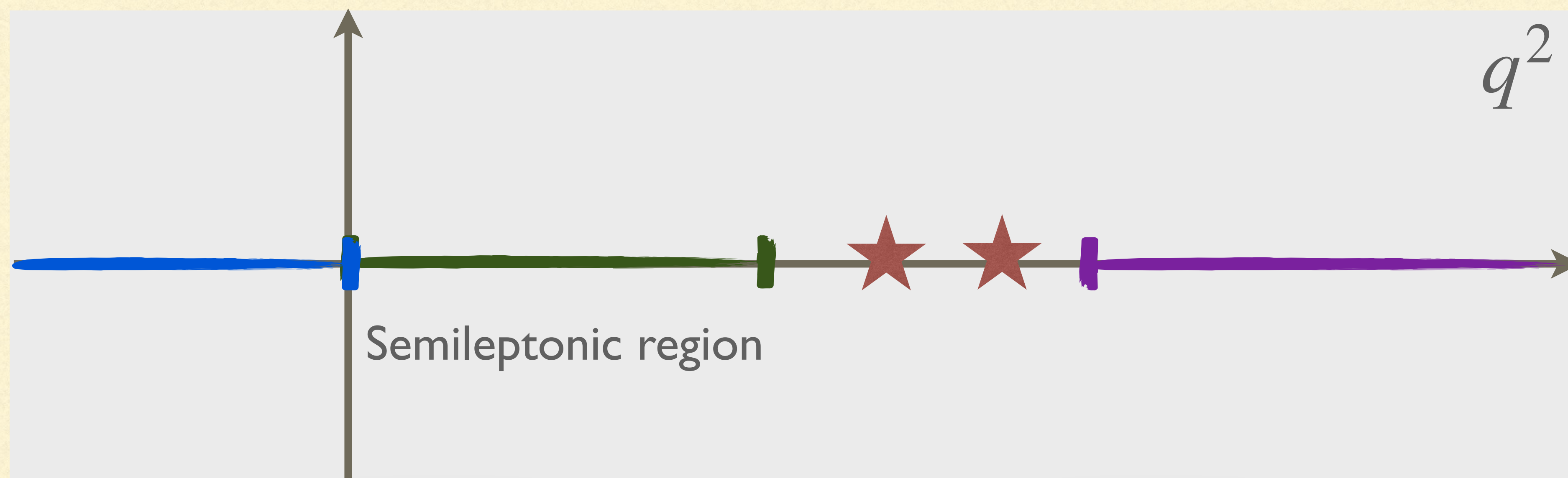
Form factors

Form factors in the Complex Plane



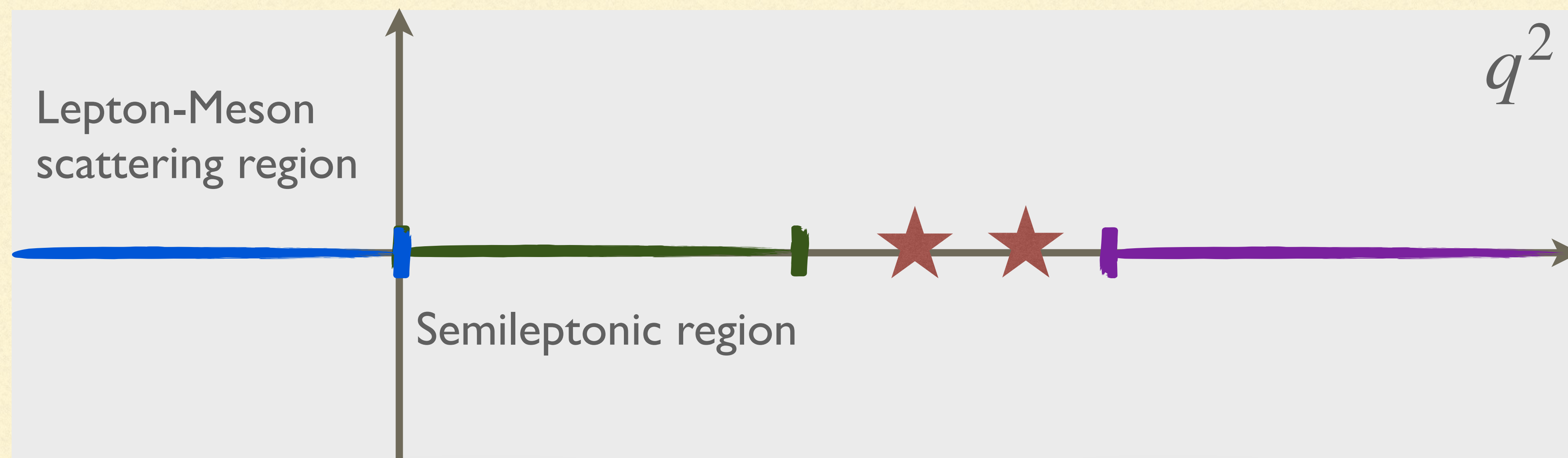
$$\langle M_1(p_1) | J^\mu | M_2(p_2) \rangle = \sum_i V_i^\mu(p_1, p_2) f_i(q^2)$$

Form factors in the Complex Plane



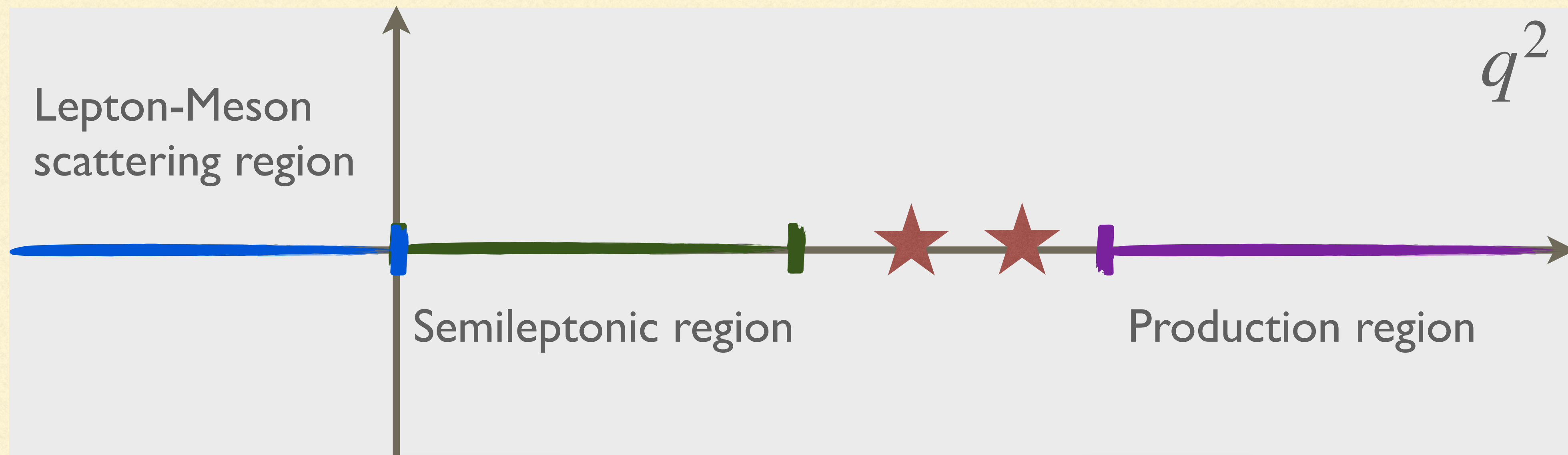
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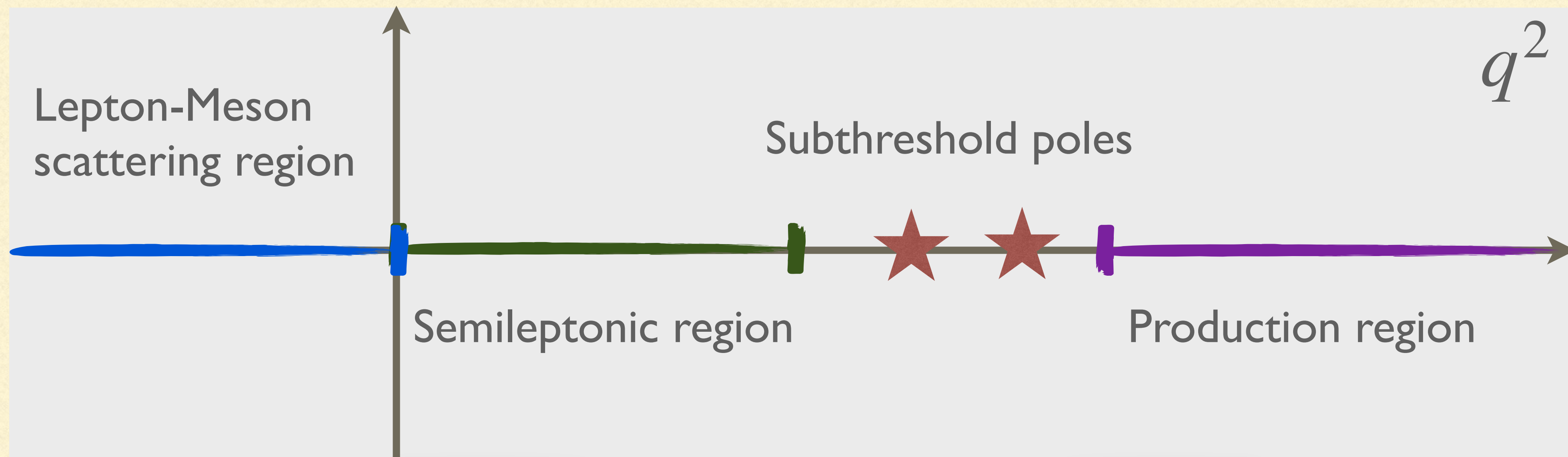
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Form factors: Unitarity bounds

$$\Pi_{(J)}^{L/T}(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | J^{L/T}(x) J^{L/T}(0) | 0 \rangle$$

$$\chi_{(J)}^L(Q^2) \equiv \left. \frac{\partial \Pi_{(J)}^L}{\partial q^2} \right|_{q^2=Q^2} = \frac{1}{\pi} \int_0^\infty dq^2 \frac{\text{Im} \Pi_{(J)}^L(q^2)}{(q^2 - Q^2)^2}$$

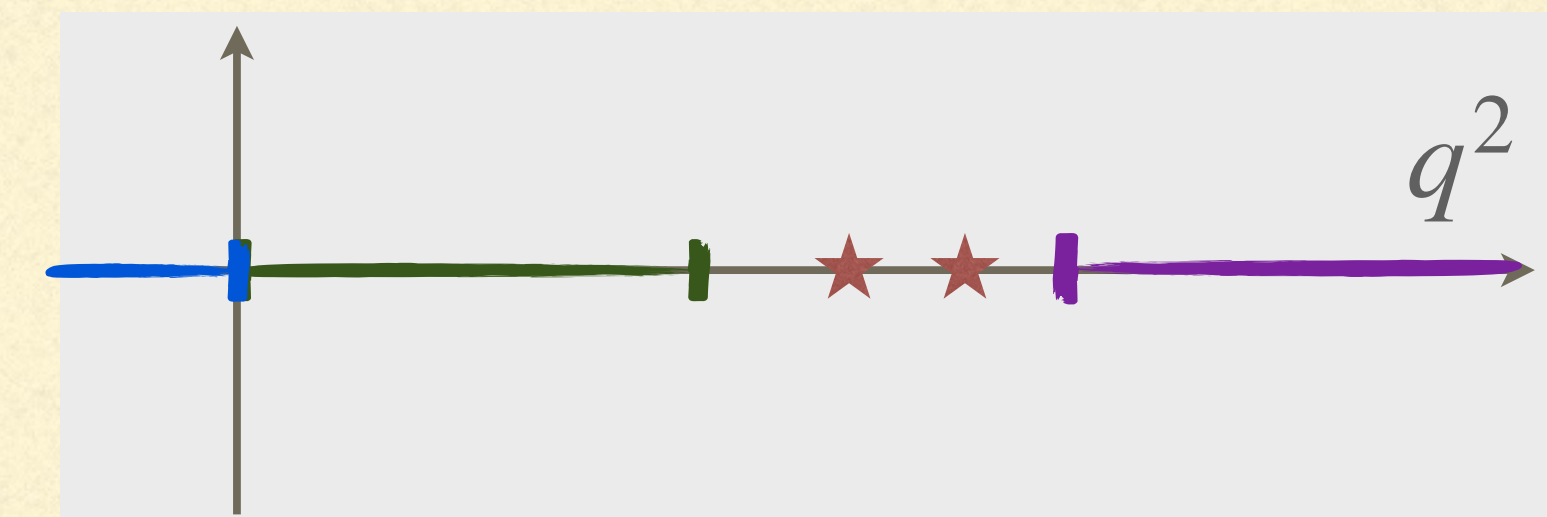
$$\chi_{(J)}^T(Q^2) \equiv \left. \frac{1}{2} \frac{\partial^2 \Pi_{(J)}^T}{\partial (q^2)^2} \right|_{q^2=Q^2} = \frac{1}{\pi} \int_0^\infty dq^2 \frac{\text{Im} \Pi_{(J)}^T(q^2)}{(q^2 - Q^2)^3}$$

- Starting point: once and twice subtracted dispersion relations [Boyd, Grinstein, Lebed; Caprini; ...]
- Susceptibilities perturbatively computable for large space-like Q^2 or at $Q^2 = 0$ if heavy quarks involved; also on the Lattice! (Martinelli, Simula, Vittorio; Harrison)
- Optical theorem allows to write the imaginary part as sum over all possible final states
- Neglecting a final state leads to an inequality

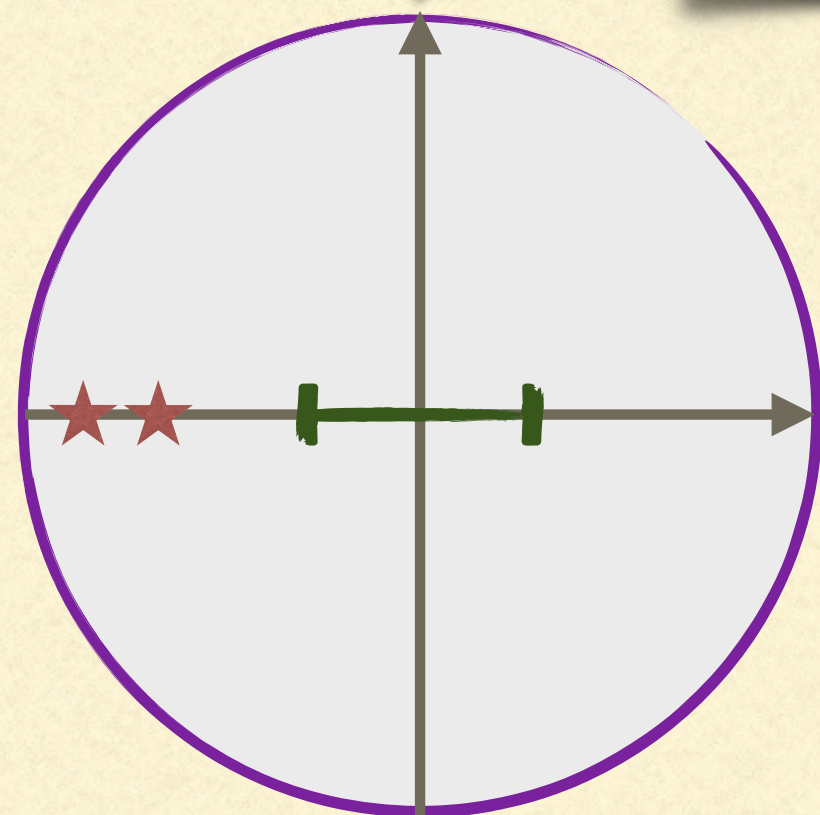
$$\text{Im} \Pi_{(J)}^{T/L} = \frac{1}{2} \sum_X \int d\text{PS} P_{T/L}^{\mu\nu} \langle 0 | J_\mu | X \rangle \langle X | J_\nu | 0 \rangle \delta^{(4)}(q - p_X)$$

$$\text{Im} \Pi_{(V)}^T |_{BD} = K(q^2) |f_+(q^2)|^2$$

Form factors: Function space



$$z(q^2, q_0^2) = \frac{\sqrt{q_+^2 - q^2} - \sqrt{q_+^2 - q_0^2}}{\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2}}$$

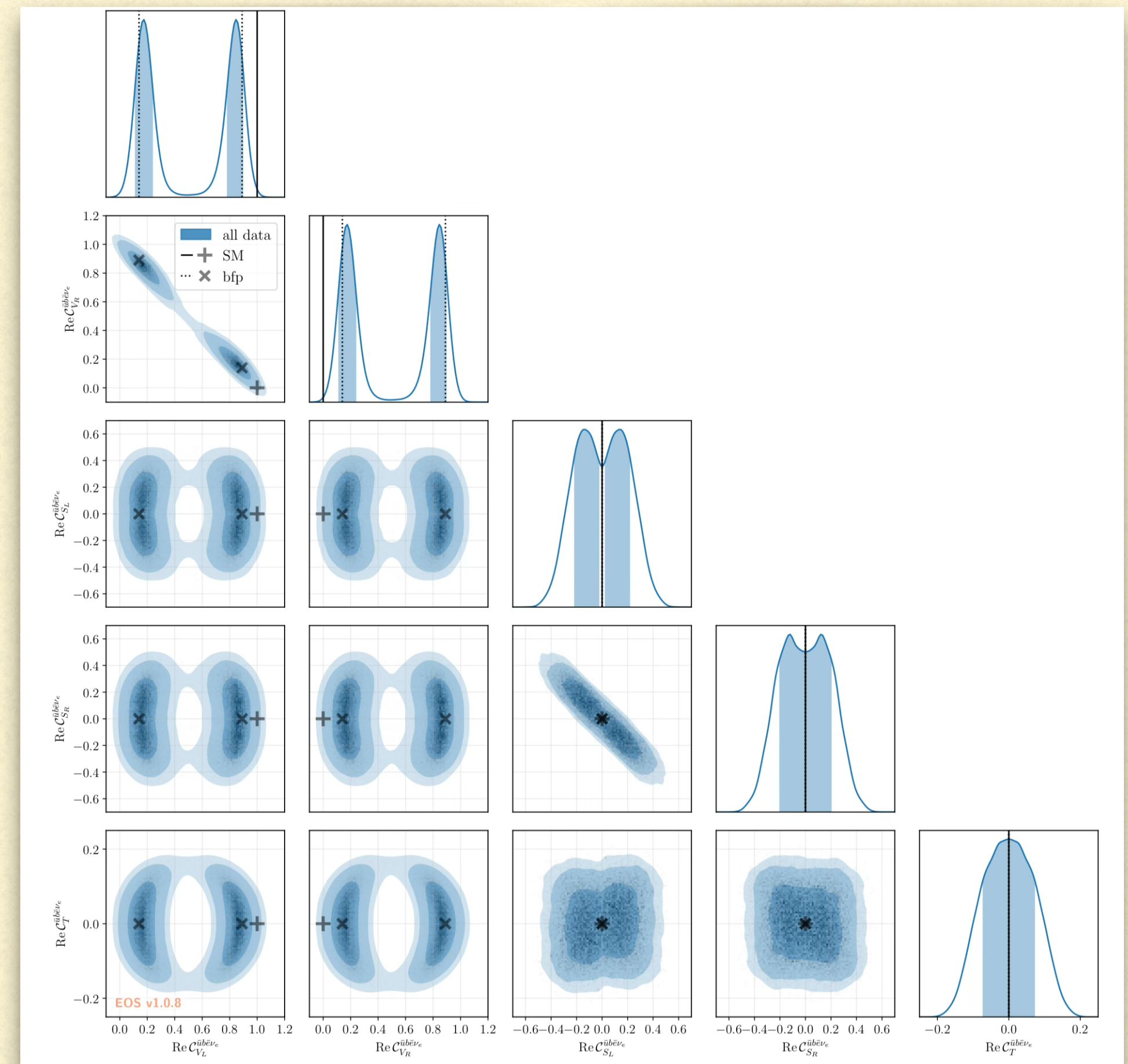
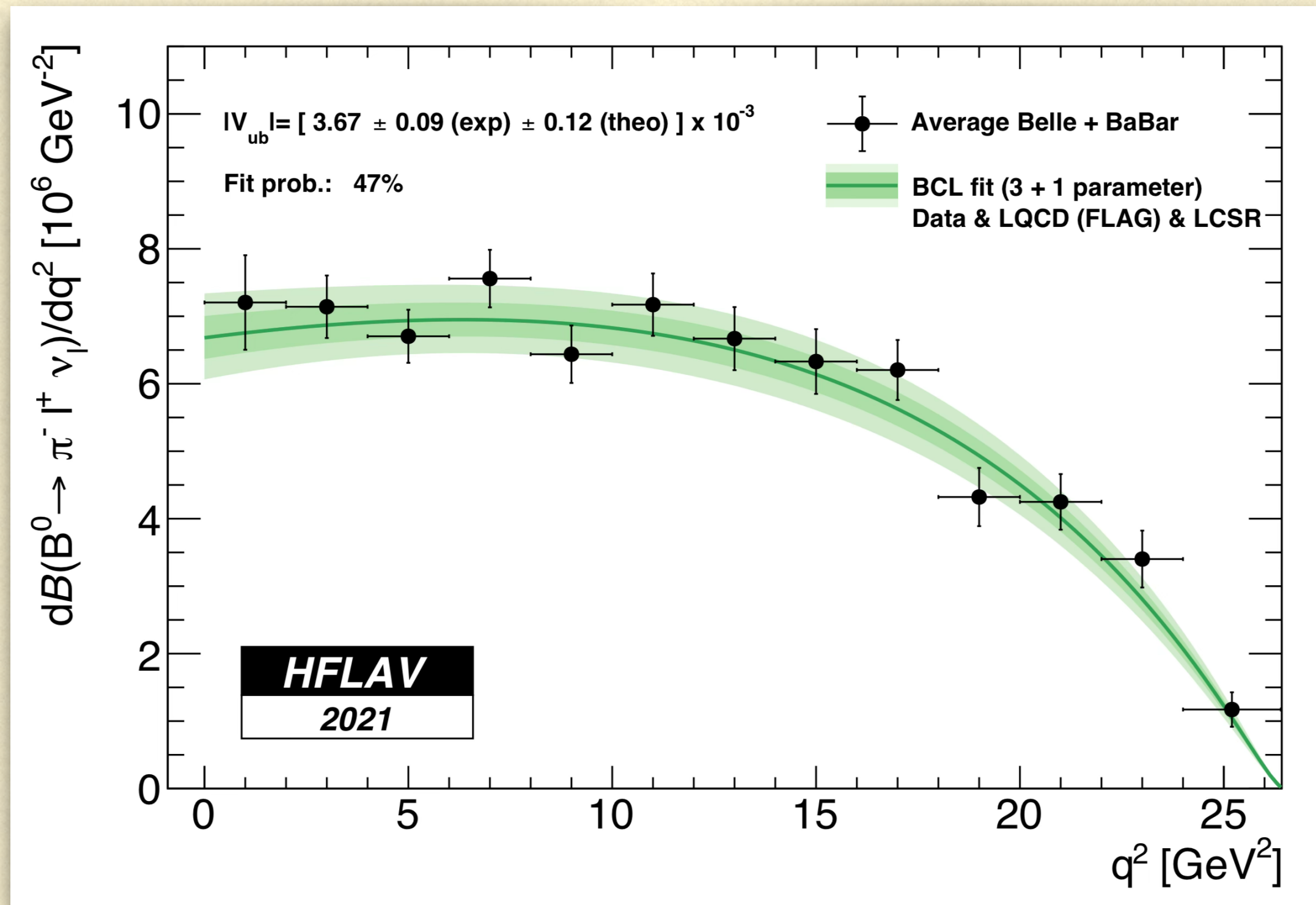


$$1 \geq \frac{1}{2\pi i} \oint \frac{dz}{z} |B(z)\Phi(z)f(z)|^2$$

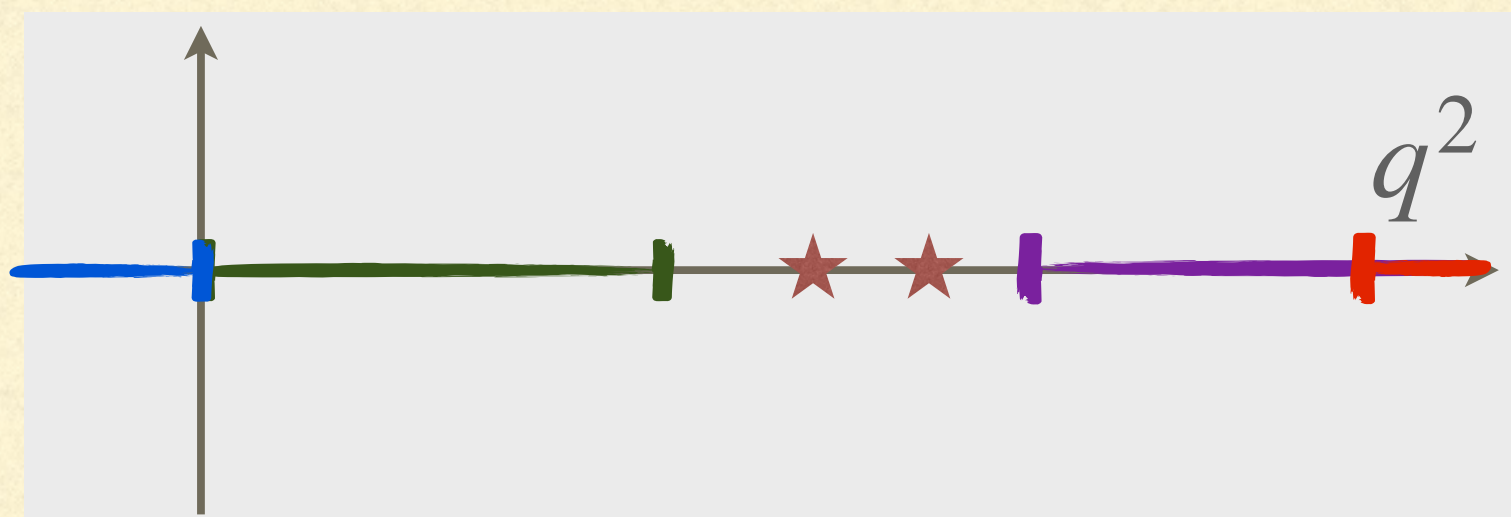
$$f(z) = \frac{1}{\Phi(z)B(z)} \sum_{i=0}^{\infty} a_i z^i \quad 1 \geq \sum_{i=0}^{\infty} |a_i|^2$$

- Mapping q^2 to the dimensionless variable z transforms integration region to unit circle
- In this form it is evident that our FFs live in the Hardy space H^2
- Insert Blaschke products to get rid of subthreshold poles and zeroes in kinematic factors
- Series expansion (or orthogonal polynomials)
- Semileptonic region: $|z| < 1$

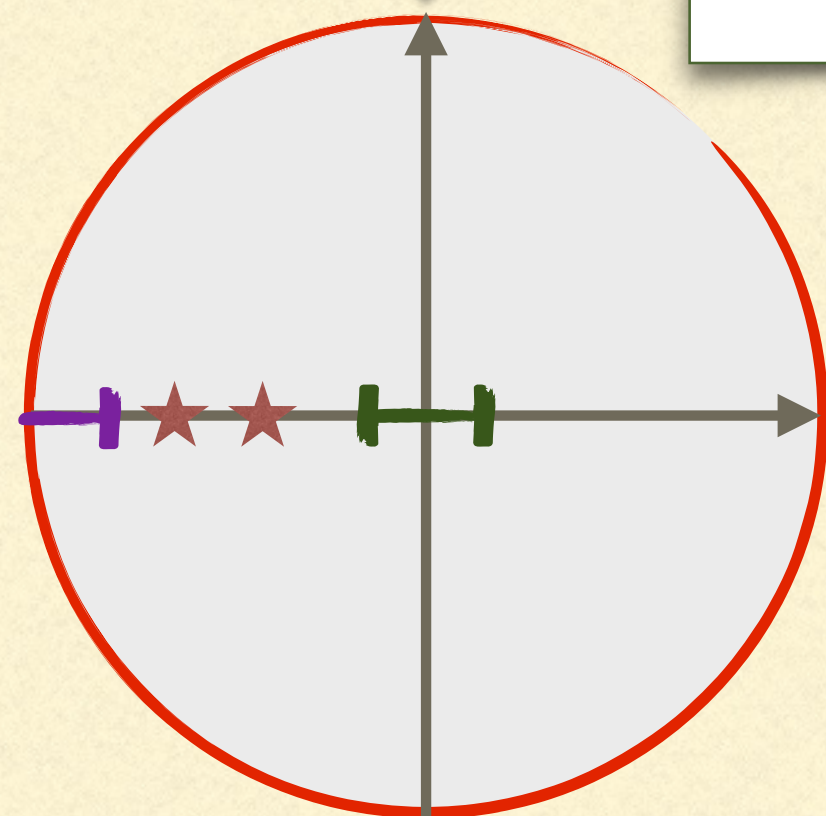
Form factors: Applications



Form factors: What is happening on the circle?



$$z(q^2, q_0^2) = \frac{\sqrt{q_{in}^2 - q^2} - \sqrt{q_{in}^2 - q_0^2}}{\sqrt{q_{in}^2 - q^2} + \sqrt{q_{in}^2 - q_0^2}}$$

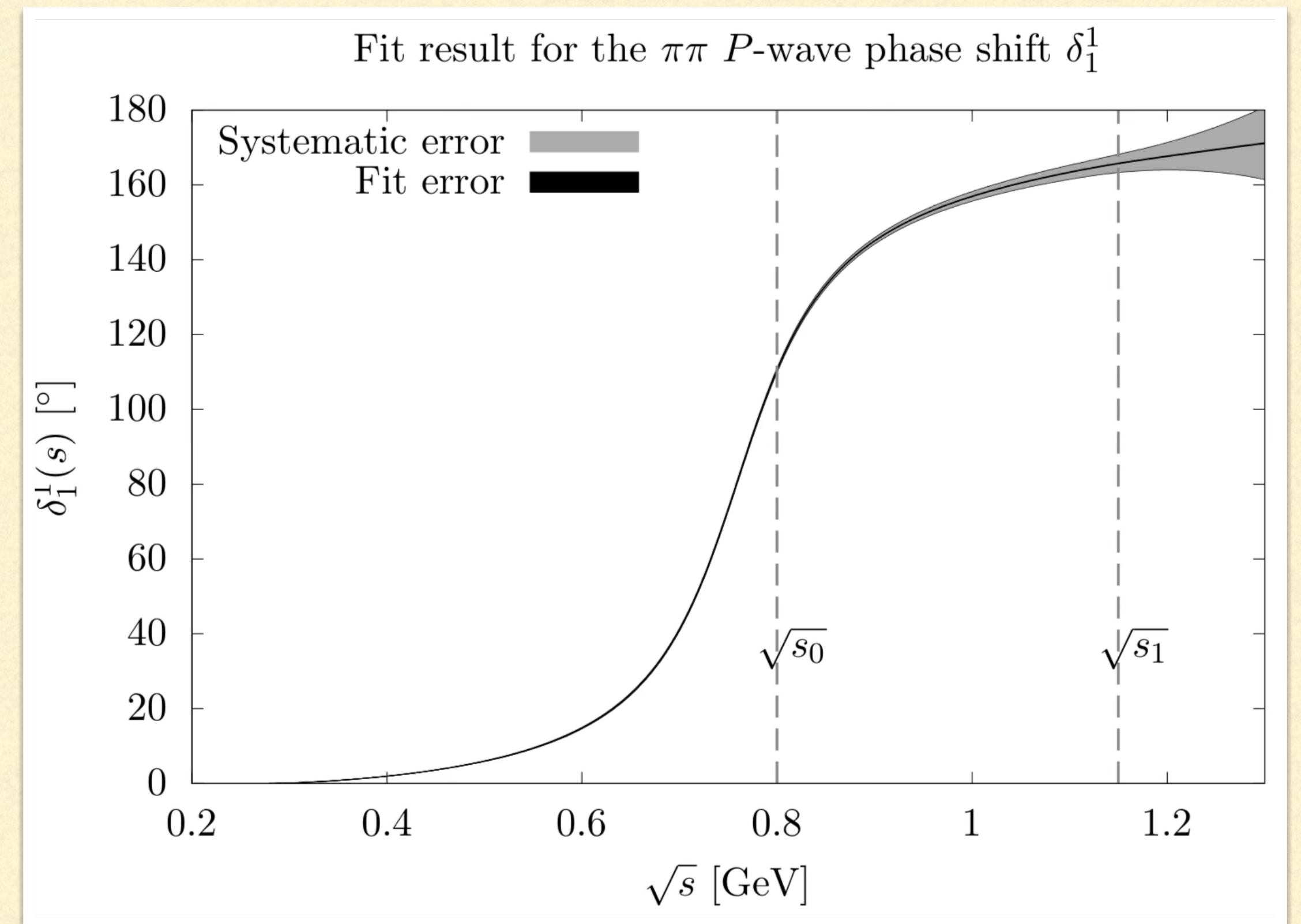
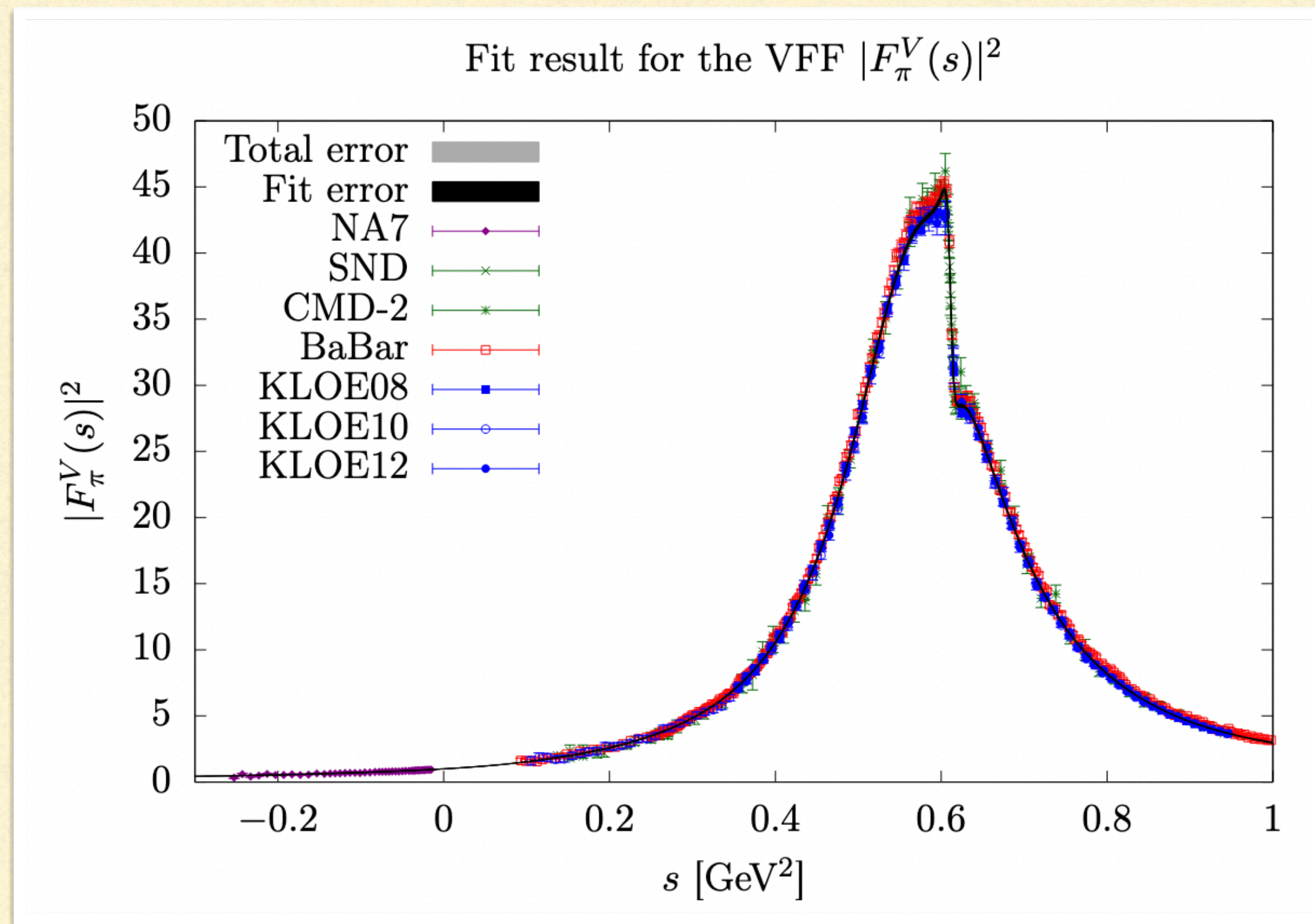


$$\ln \Omega_l(q^2) = \frac{q^2}{\pi} \int ds' \frac{\delta_l(s')}{s'(s' - q^2)}$$

$$f(z) = \frac{\Omega(q^2(z))}{\Phi(z)B(z)} \sum_{i=0}^{\infty} a_i z^i \quad 1 \geq \sum_{i=0}^{\infty} |a_i|^2$$

- Additional branch cuts open at higher q^2 as a consequence of other channels with the same quantum numbers
- We can use the first of these thresholds to map
- Iff we know the elastic scattering phase of the meson-meson system under question, we can properly describe the phase in $q_+^2 \leq q^2 \leq q_{in}^2$
- Omnès function provides model-independent way

Form factors: Applications



Colangelo, Hoferichter, Stoffer [JHEP 02 \(2019\) 006](#)

Two hadrons in the final state

Theoretical Fundamentals: $2 \rightarrow 2$ scattering

$$\langle p_3 p_4; b | \mathcal{S} - 1 | p_1 p_2; a \rangle = i(2\pi)^4 \delta^{(4)}\left(\sum p_i\right) \mathcal{M}_{ba}(\{p_i\})$$

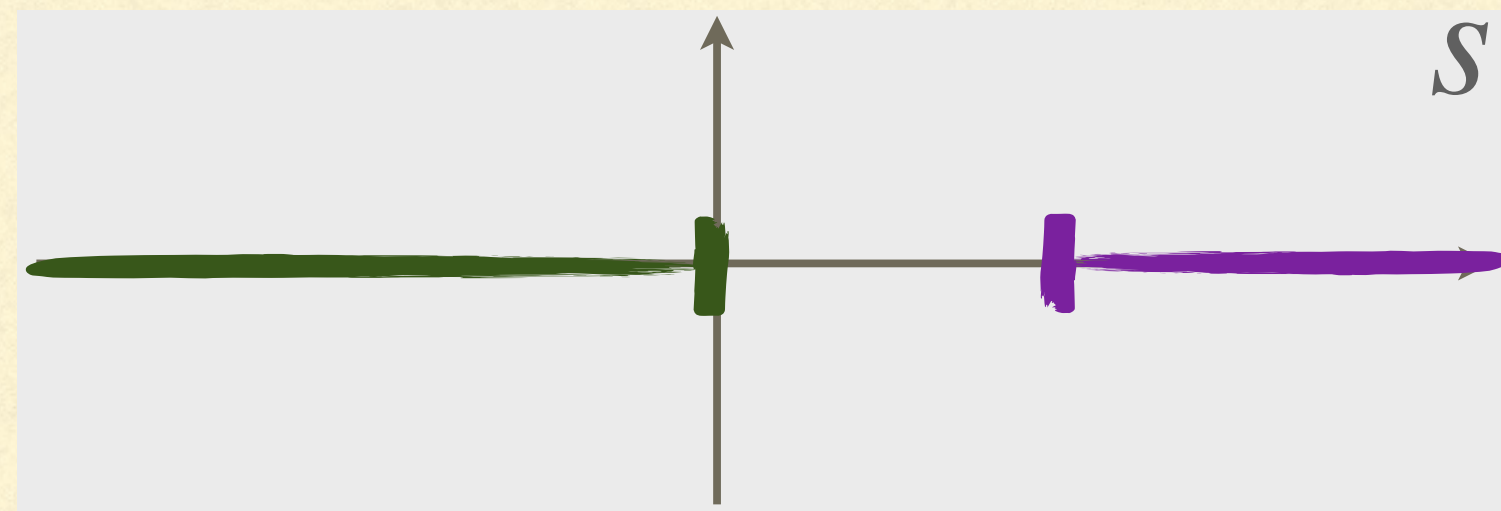
$$\mathcal{M}_{ab} - \mathcal{M}_{ba}^* = i(2\pi)^4 \sum_c \int d\Phi_c \mathcal{M}_{ca} \mathcal{M}_{cb}^*$$

$$\mathcal{A}_a - \mathcal{A}_a^* = i(2\pi)^4 \sum_c \int d\Phi_c \mathcal{M}_{ca}^* \mathcal{A}_c$$

- Simplest scattering process with nontrivial kinematic dependence
- Described by unitary operator \mathcal{S}
- Scattering amplitude \mathcal{M} depends on 2 independent Mandelstam variables
- \mathcal{M} real below lowest threshold, imaginary part constrained by Unitarity above
- Two-particle production amplitude \mathcal{A} shares phase with \mathcal{M} , e.g. pion production in lepton collisions

Partial-wave expansion for dummies

$$\mathcal{M}_{ba}(s, t) = \sum_l P_l(\cos \theta) \sqrt{\rho_b}^{-1} f_{ba}^l(s) \sqrt{\rho_a}^{-1}$$



$$f_{aa}^l(s) = \frac{\eta_l(s) e^{2i\delta_l(s)} - 1}{2i}$$

- Resonances have well-defined spin, their poles only occur in a specific partial wave of \mathcal{M}
- Partial-wave expansion conveniently separates different resonances, e.g. in pion scattering: $\rho, f_0(500), f_0(980), f_2(1270)$
- Partial-wave expanded amplitudes have left-handed branch cuts which are remnants of branch cuts in other Mandelstam variables
- Diagonal elements can be expressed through scattering phase δ_l and inelasticity η_l

Theoretical fundamentals: Three-body decays

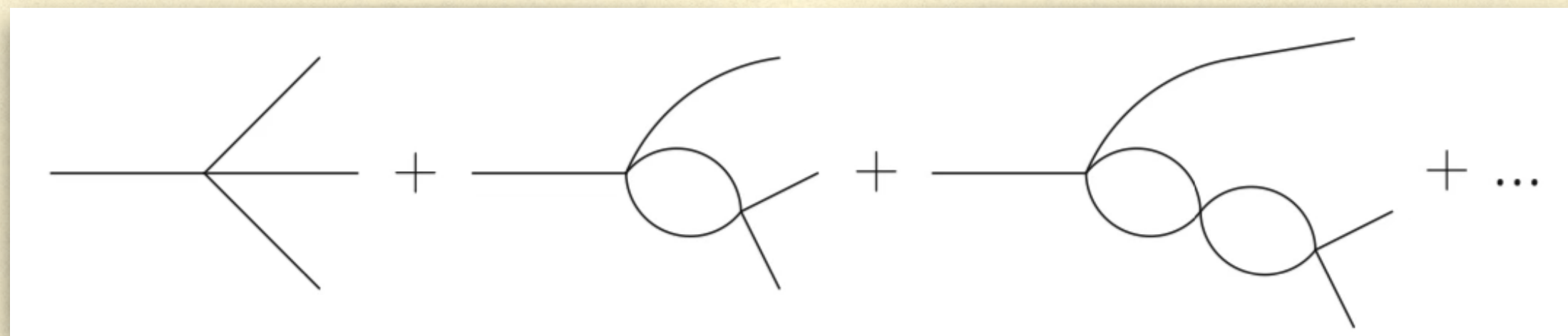
$$\text{Im}\Pi_{(J)}^{T/L} = \frac{1}{2} \sum_X \int \text{dPS} P_{T/L}^{\mu\nu} \langle 0 | J_\mu | X \rangle \langle X | J_\nu | 0 \rangle \delta^{(4)}(q - p_X)$$

$$\mathcal{F}(s, t, u) = \sum_{x \in \{s, t, u\}} \sum_l F_l^{(x)}(x) P_l(\cos \theta_x)$$

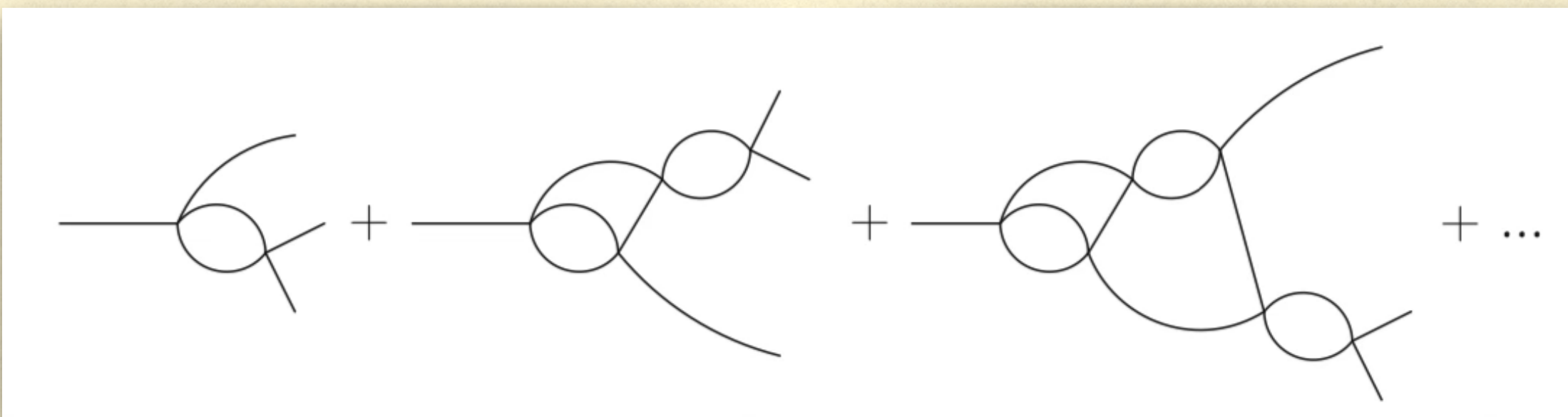
$$F_{(s)}^{(l)}(s) = \Omega_{(s)}^{(l)}(s) \left(Q_{(s)}^{(l)}(s) + \frac{s^n}{\pi} \int \frac{dx}{x^n} \frac{\sin \delta_{(s)}^{(l)}(s) \hat{F}_{(s)}^{(l)}(x)}{|\Omega_{(s)}^{(l)}(x)| (x - s)} \right)$$

- Amplitudes relevant for Unitarity bounds are $1 \rightarrow n$ amplitudes of particle with mass q^2
- Khuri-Treiman formalism already has 2 of our ingredients built in ([PR 119 1115-1121 \(1960\)](#))
- Write decay amplitude as sum of 3 partial-wave expanded amplitudes
- Fixed s, t & u dispersion-relations lead to coupled system of integral equations
- The two other channels enter via hat functions

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Taken from: [EPJC 83 \(2023\) 6, 510](#)

$$F_{(s)}^{(l)}(s) = \Omega_{(s)}^{(l)}(s) \left(Q_{(s)}^{(l)}(s) + \frac{s^n}{\pi} \int \frac{dx}{x^n} \frac{\sin \delta_{(s)}^{(l)}(s) \hat{F}_{(s)}^{(l)}(x)}{|\Omega_{(s)}^{(l)}(x)| (x - s)} \right)$$

A new parameterization

$$\text{Im}\Pi_{(J)}^{T/L} = \frac{1}{2} \sum_X \int \text{dPS} P_{T/L}^{\mu\nu} \langle 0 | J_\mu | X \rangle \langle X | J_\nu | 0 \rangle \delta^{(4)}(q - p_X)$$

$$\mathcal{F}(s, t, u) = \sum_{x \in \{s, t, u\}} \sum_l F_{(x)}^{(l)}(x, q^2) P_l(\cos \theta_x)$$

$$F_{(s)}^{(l)}(s, q^2) = \Omega_{(s)}^{(l)}(s) \left(f_{(s)}^{(l)}(s, q^2) + \frac{s^n}{\pi} \int \frac{dx}{x^n} \frac{\sin \delta_{(s)}^{(l)}(s) \hat{F}_{(s)}^{(l)}(x, q^2)}{|\Omega_l^{(s)}(x)| (x - s)} \right)$$

- Amplitudes implicitly depend on mass
- s -dependence not polynomial above inelastic thresholds
- Find unitarity bound and parameterization for $f(s, q^2)$
- The hat functions now depend on $B^* \rightarrow \pi/D^{(*)}$ FFs

A new parameterization

$$\text{Im}\Pi(q^2) \Big|_{M_1 M_2 M_3} = \sum_x \int_{x_+}^{(\sqrt{q^2 - m_y})^2} dx \sum_l \frac{K_l(q^2, x)}{2l + 1} |F_{(x)}^{(l)}(x, q^2)|^2$$

$$\chi \geq \frac{1}{\pi} \int_0^\infty dq^2 \int_{s_+}^{s_-(q^2)} ds \frac{K(s, q^2)}{q^{2n}} |\Omega(s)f(s, q^2)|^2$$

$$\chi \geq \frac{1}{\pi} \int_{s_+}^\infty ds \hat{K}(s) \int_{q_+^2(s)}^\infty dq^2 \frac{\tilde{K}(s, q^2)}{q^{2n}} |f(s, q^2)|^2$$

- Unitarity bounds in general off-diagonal
- Off-diagonal terms small, ignore for derivation of parameterization
- Similar to KT treatment: ignore left-hand cuts and add them back later
- Crucial: change integration order!
- In NWA: $\hat{K}(s) \rightarrow \delta(s - M_R^2)$

A new parameterization

$$f(s, q^2) = \frac{1}{B(q^2)\phi(q^2; s)} \sum_i a_i(s) z^i(q^2, q_+^2(s))$$

$$\chi \geq \frac{1}{\pi} \sum_i \int_{s_+}^{\infty} ds \hat{K}(s) |a_i(s)|^2$$

$$a_i(s) = \frac{1}{\tilde{B}(s)\tilde{\phi}(s)} \sum_j b_{ij} y^j$$

- q^2 -integration as in standard BGL
- If $q_+^2(s_+)$ larger than lowest two-body threshold:
 $z^i \rightarrow p_i(z)$
- Now we can treat every a_i as an s -dependent FF
- Follow Caprini's treatment of pion VFF, ([EPJ C 13 471-484 \(2000\)](#))
- Alternative: BCL-like expansion

$$y = \frac{\sqrt{s_{in} - s} - \sqrt{s_{in}}}{\sqrt{s_{in} - s} + \sqrt{s_{in}}}$$

Putting it all together

- A model-independent parameterization of $1 \rightarrow 2$ decays is possible, building on 60+ years of dispersion theory
- Bound on $b_{ij,(x)}^{(l)}$ quadratic, but not diagonal
- In heavy-to-heavy decays the left-hand cuts are far from the semileptonic region, so we can ignore integrals over hat functions
- Powerful framework for many future phenomenological applications

$$F_{(s)}^{(l)}(s, q^2) = \Omega_{(s)}^{(l)}(s) \left(\frac{1}{B_{(s)}(q^2) \tilde{B}_{(s)}^{(l)}(s) \phi_{(s)}^{(l)}(q^2) \tilde{\phi}_{(s)}^{(l)}(s)} \sum_{i,j} b_{ij,(s)}^{(l)} z_{(s)}^i y_{(s)}^j + \frac{s^n}{\pi} \int \frac{dx}{x^n} \frac{\sin \delta_{(s)}^{(l)}(s) \hat{F}_{(s)}^{(l)}(x, q^2)}{|\Omega_l^{(s)}(x)| (x - s)} \right)$$

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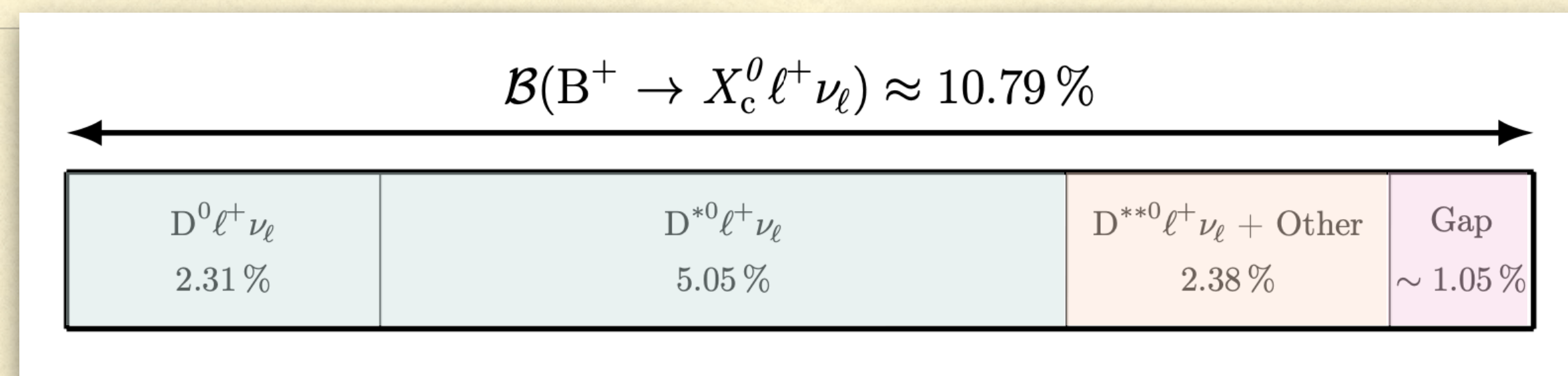
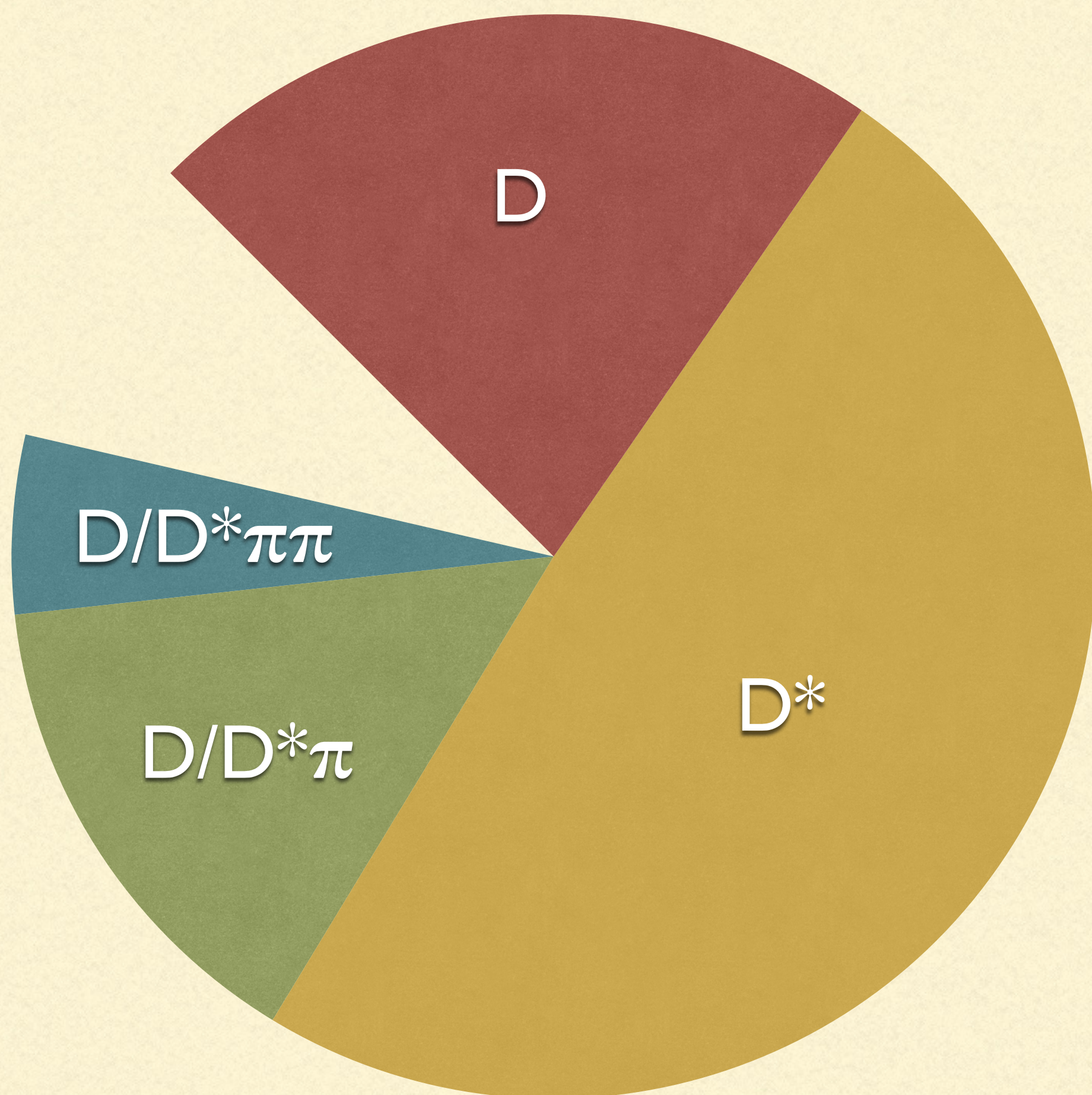
$$F_{(s)}^{(l)}(s, q^2) = \frac{\Omega_{(s)}^{(l)}(s)}{B_{(s)}(q^2) \tilde{B}_{(s)}^{(l)}(s) \phi_{(s)}^{(l)}(q^2) \tilde{\phi}_{(s)}^{(l)}(s)} \sum_{i,j} b_{ij,(s)}^{(l)} z_{(s)}^i y_{(s)}^j \rightarrow \frac{\Omega_{(s)}^{(l)}(s)}{B_{(s)}(q^2) \phi_{(s)}^{(l)}(q^2)} \sum_i b_i^{(l)} z_{(s)}^i$$

Putting it all together

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$$F_{(s)}^{(l)}(s, q^2) = \frac{g^{(l)} F^{(l)}(s, r_{BW})}{(s - M_{R,l}^2) + iM_{R,l}\Gamma_R(s)} \frac{1}{B_{(s)}(q^2)\phi_{(s)}^{(l)}(q^2)} \sum_i b_{i,(s)}^{(l)} z_{(s)}^i$$

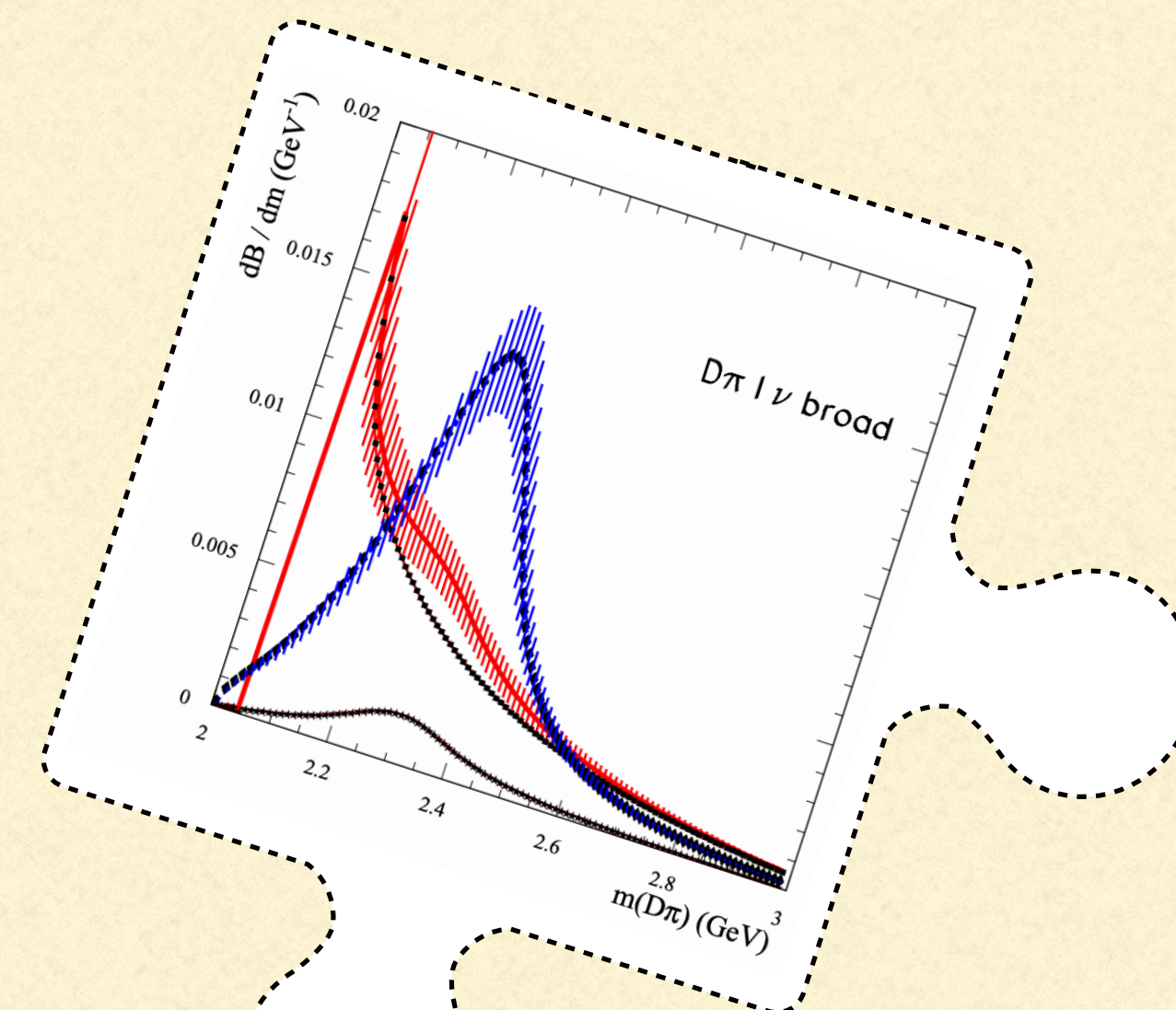
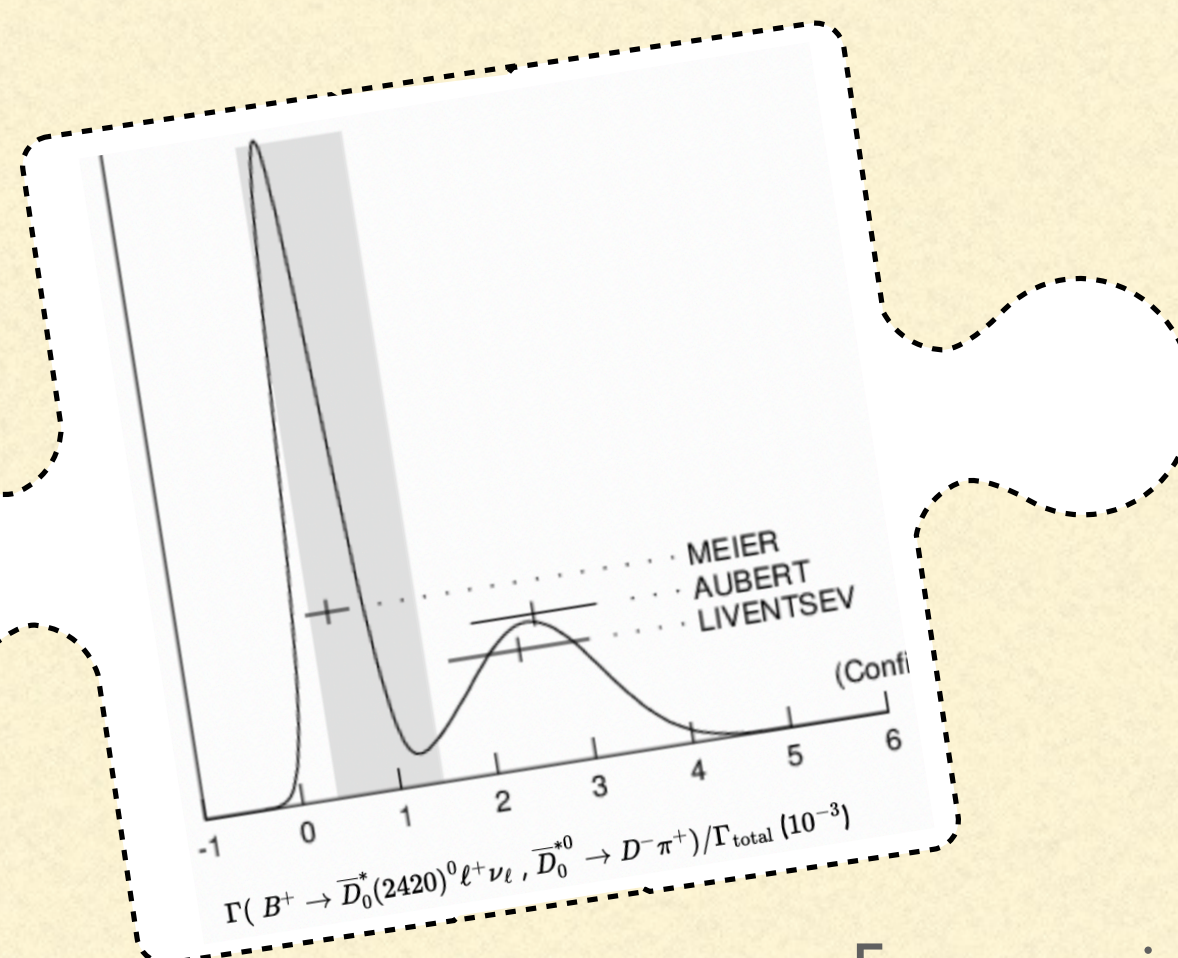
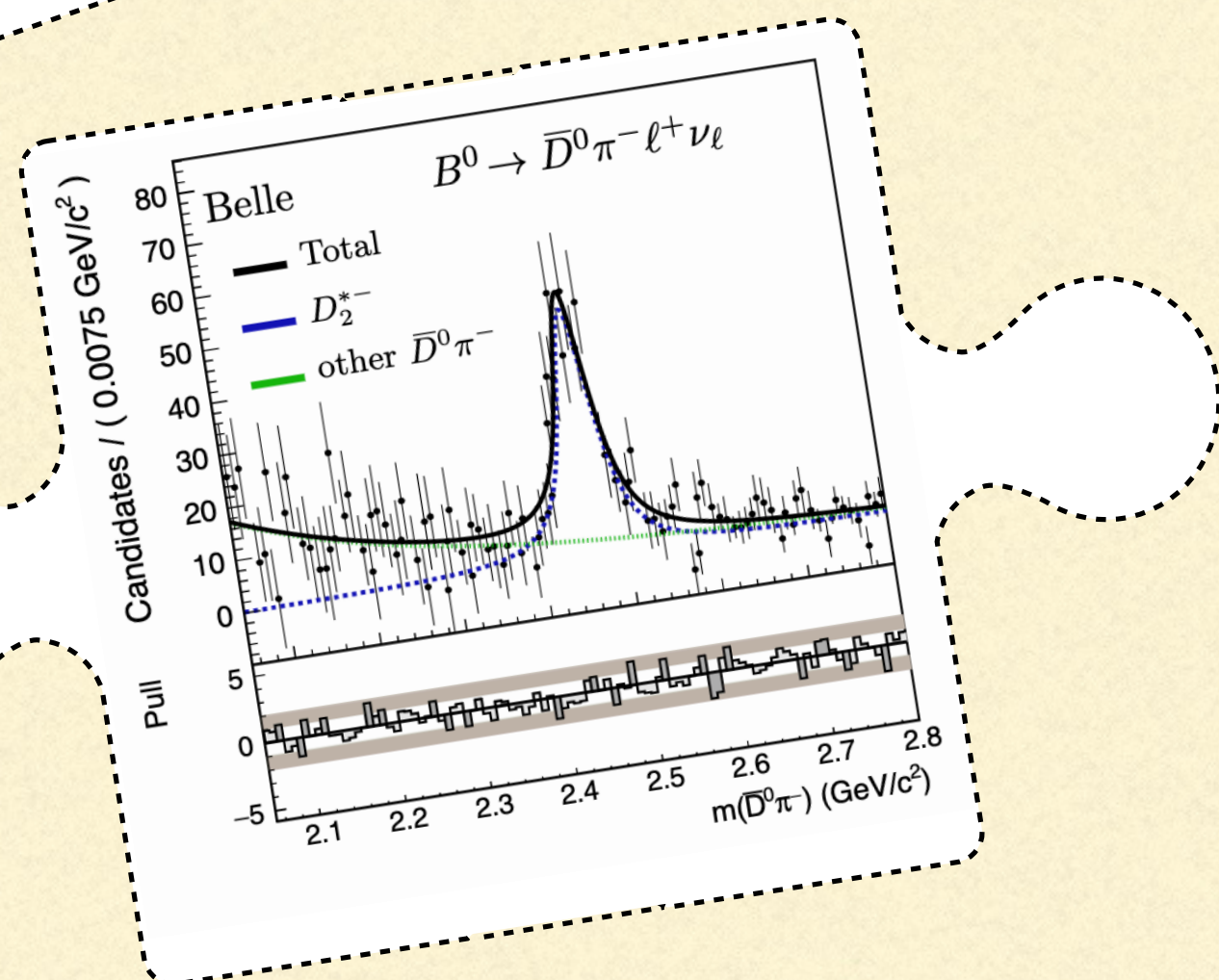
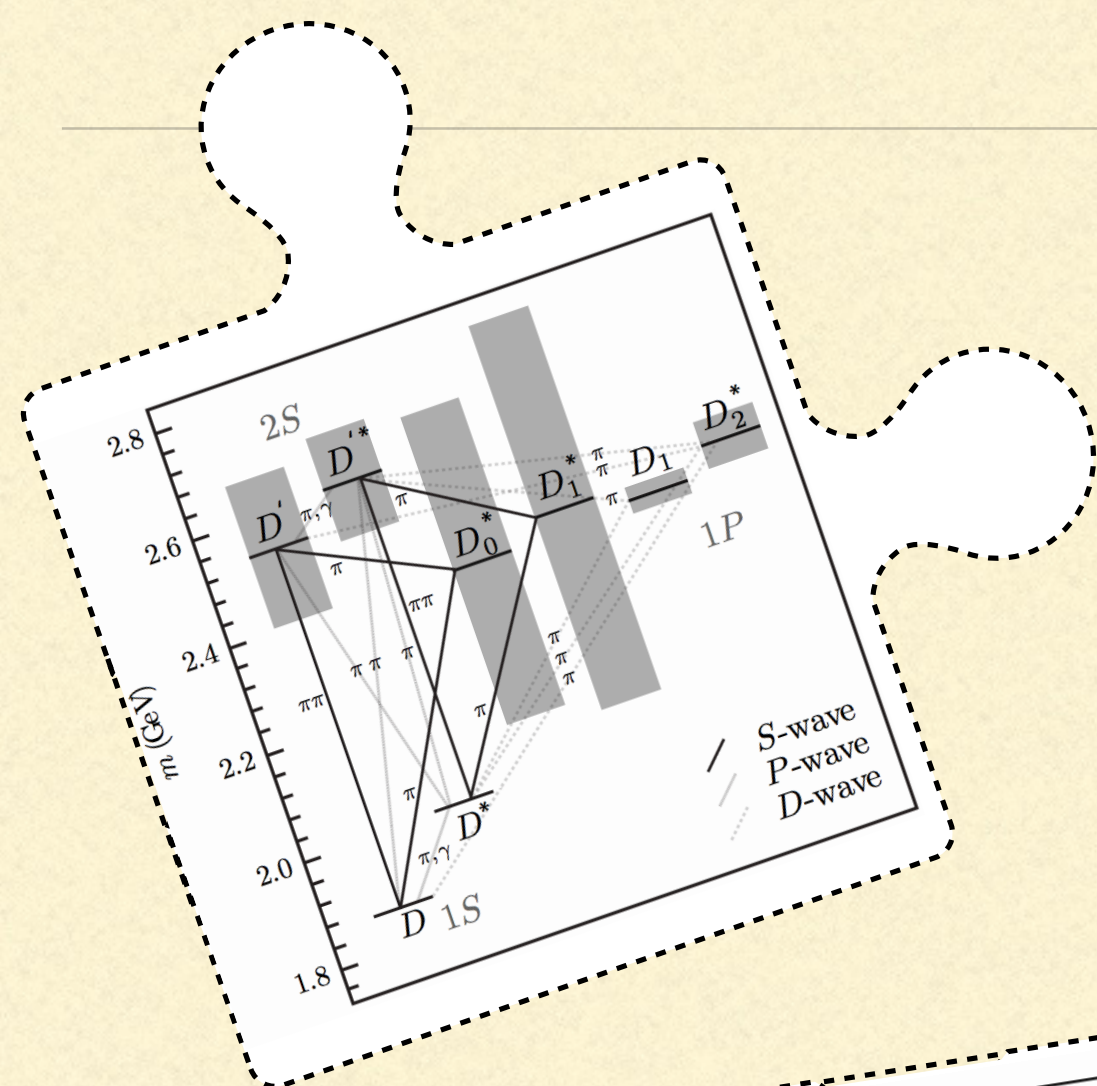
Application: $B \rightarrow D\pi\ell\nu$



Taken from talks by Raynette

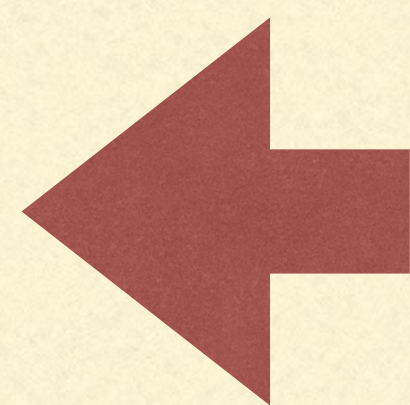
Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D\pi\pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^*\pi\pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \rightarrow D\eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow D^*\eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Application: $B \rightarrow D\pi\ell\nu$

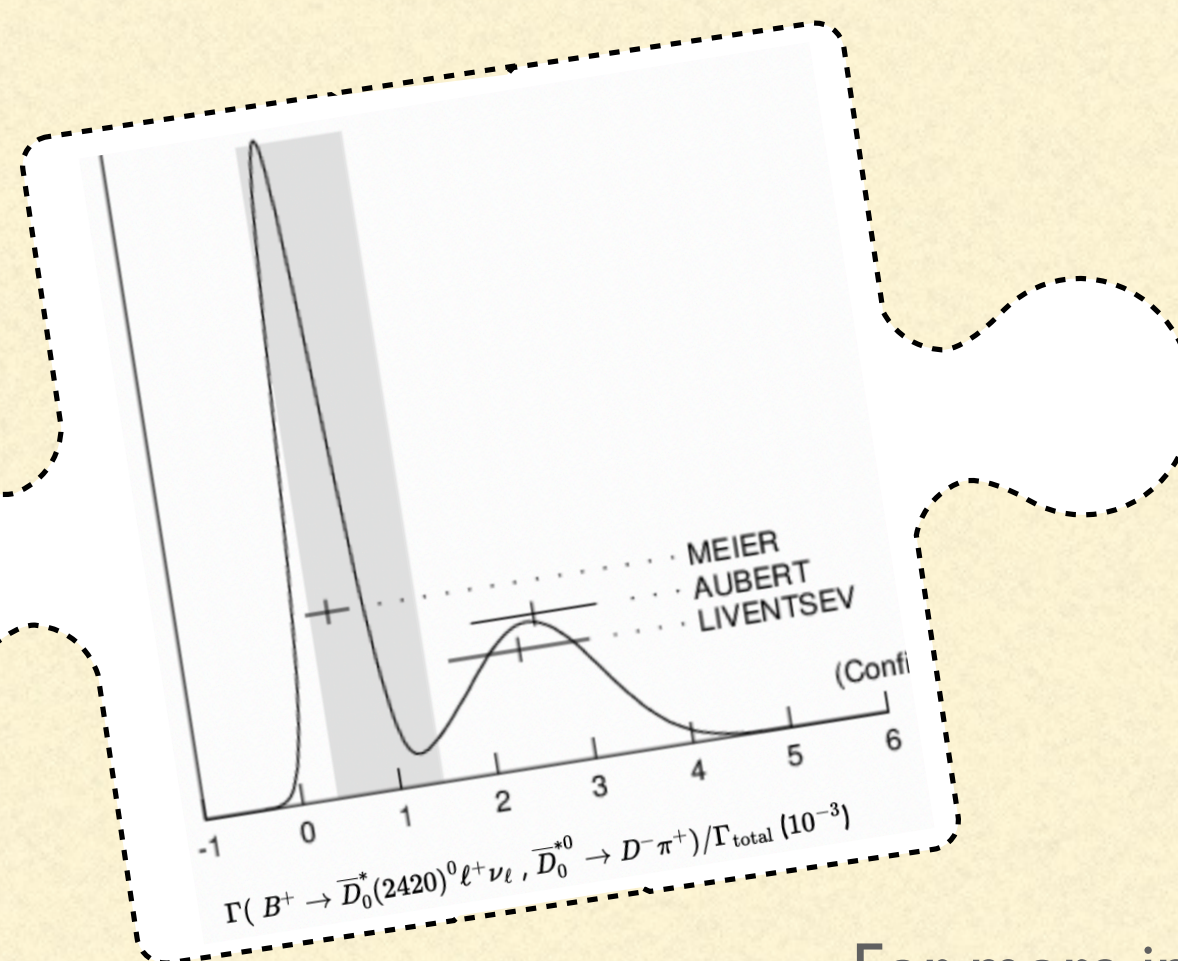
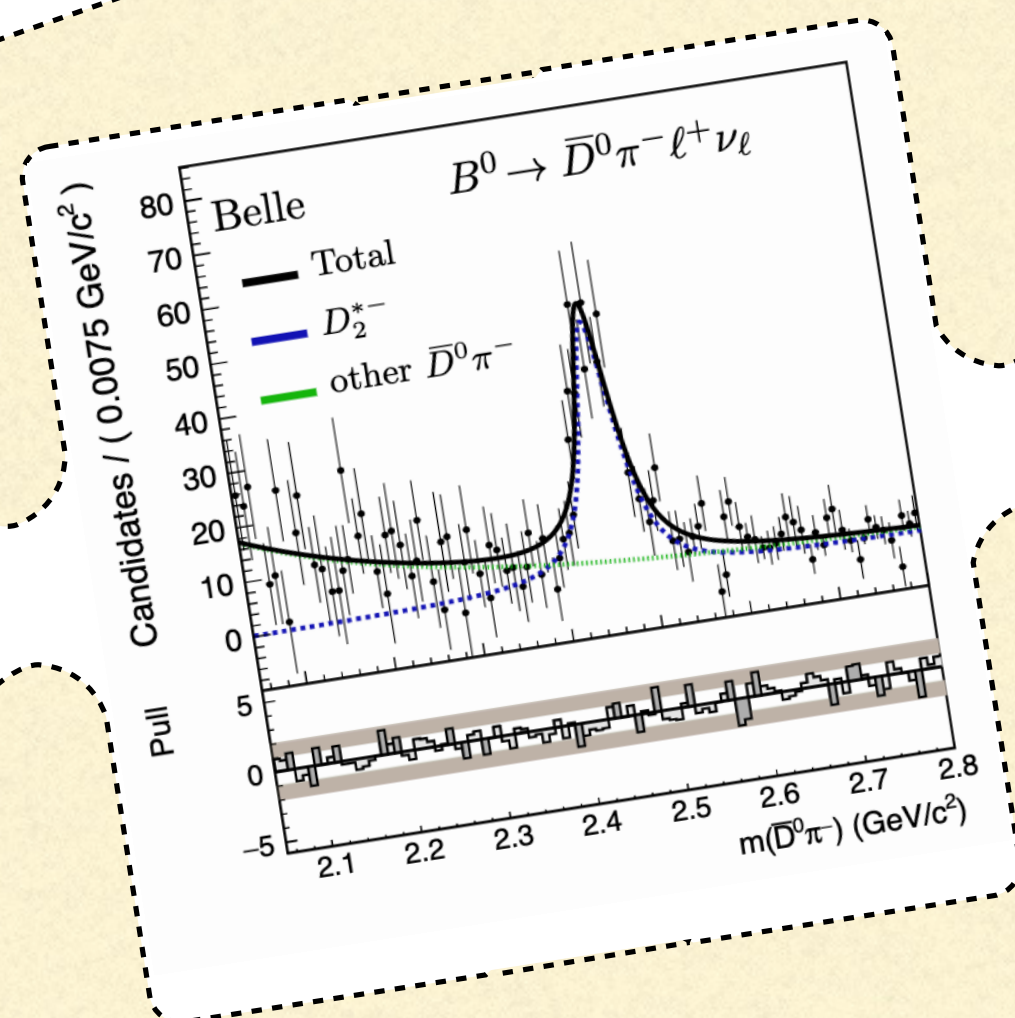
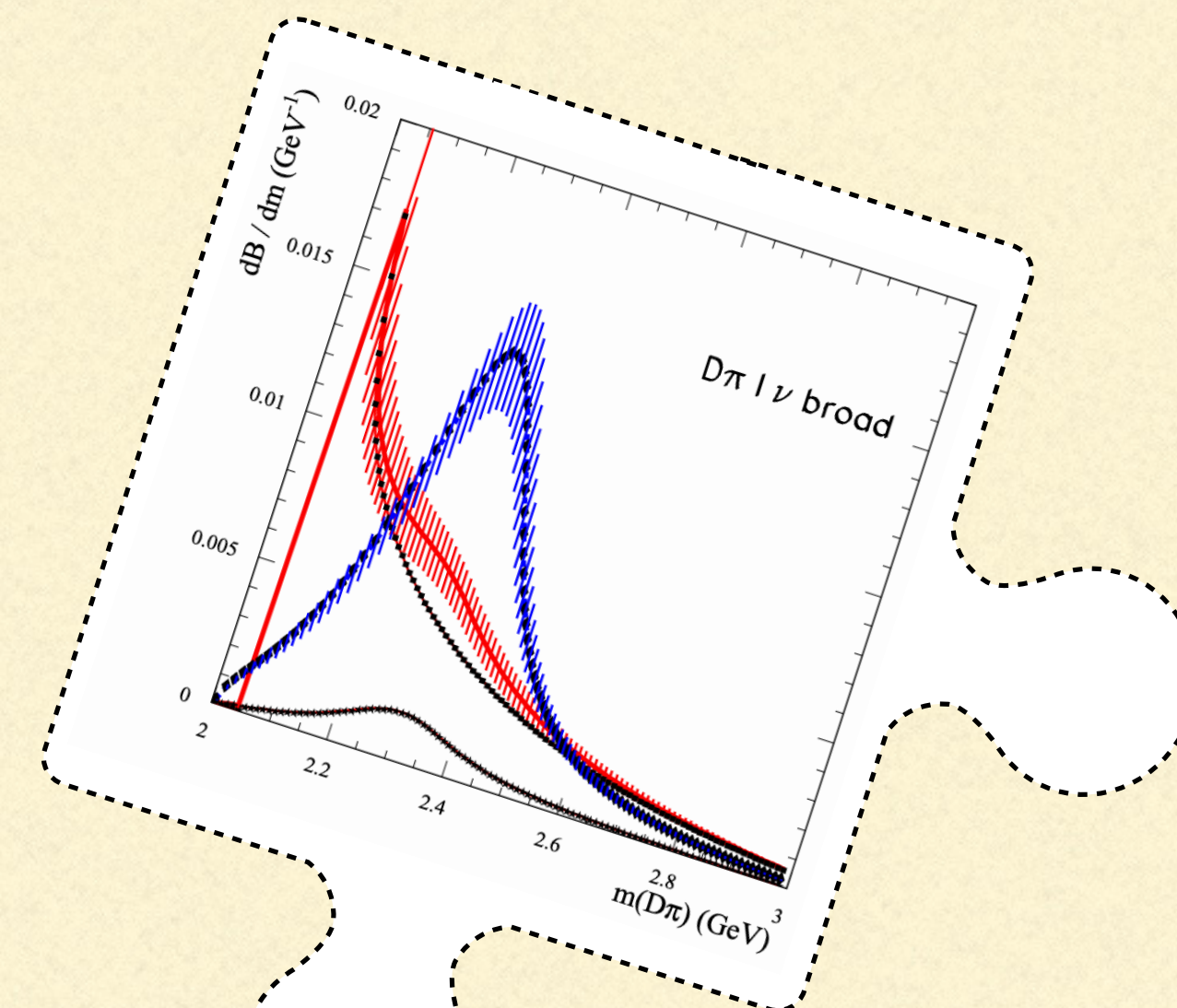


For more in-depth discussion see my talk at CKM 2023 and [Raynette's talk in Vienna](#) (with the original puzzle pieces)

Application: $B \rightarrow D\pi\ell\nu$



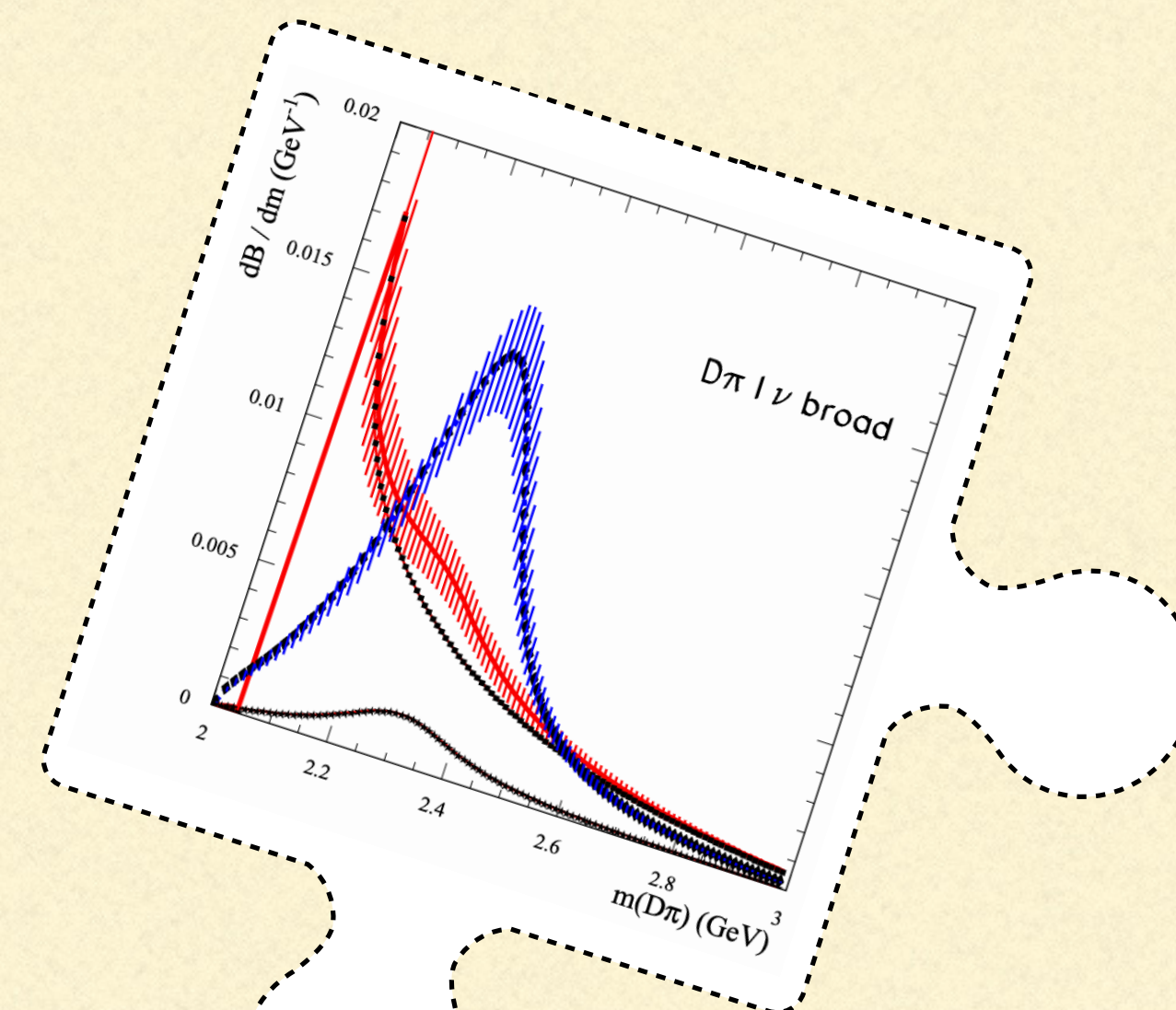
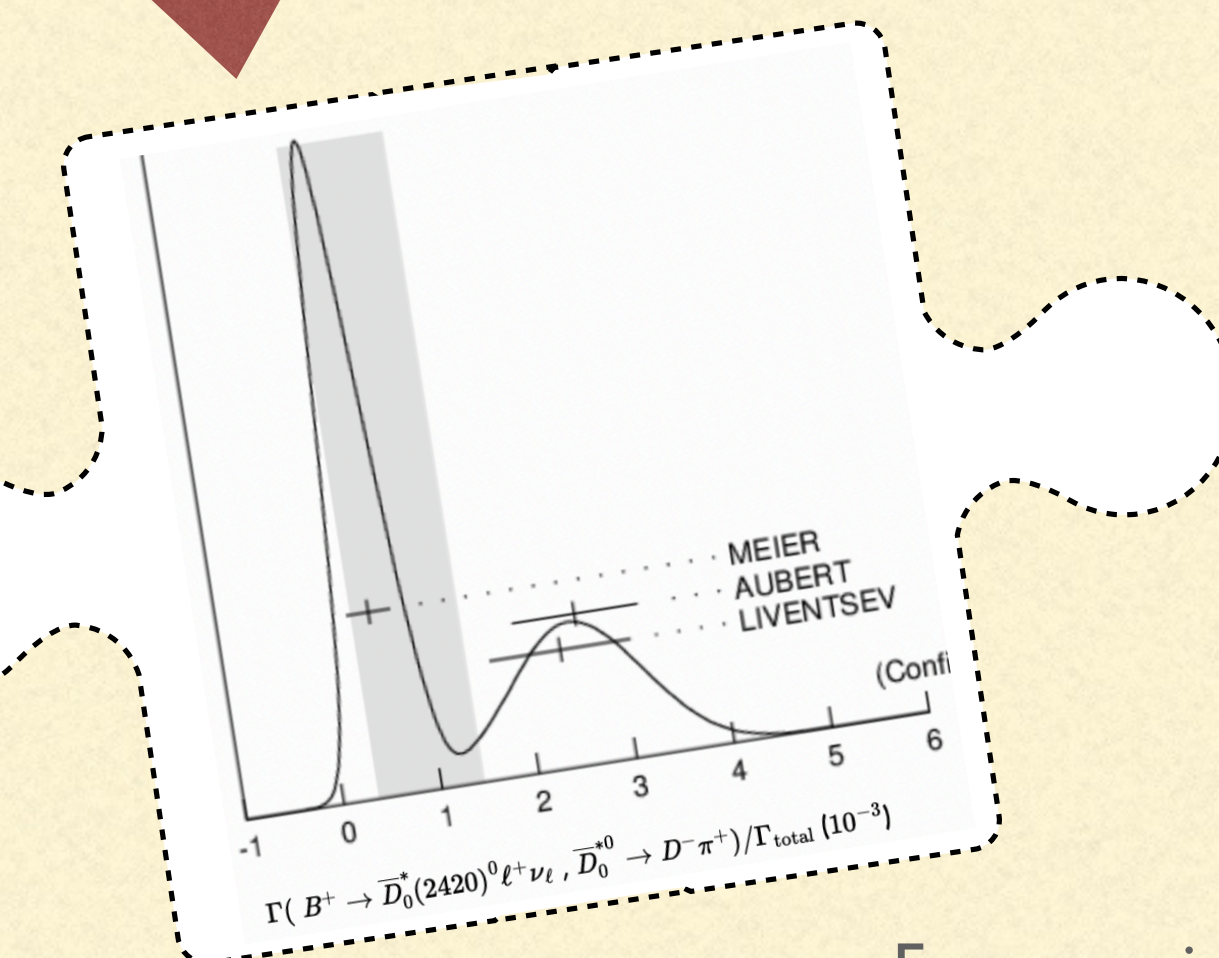
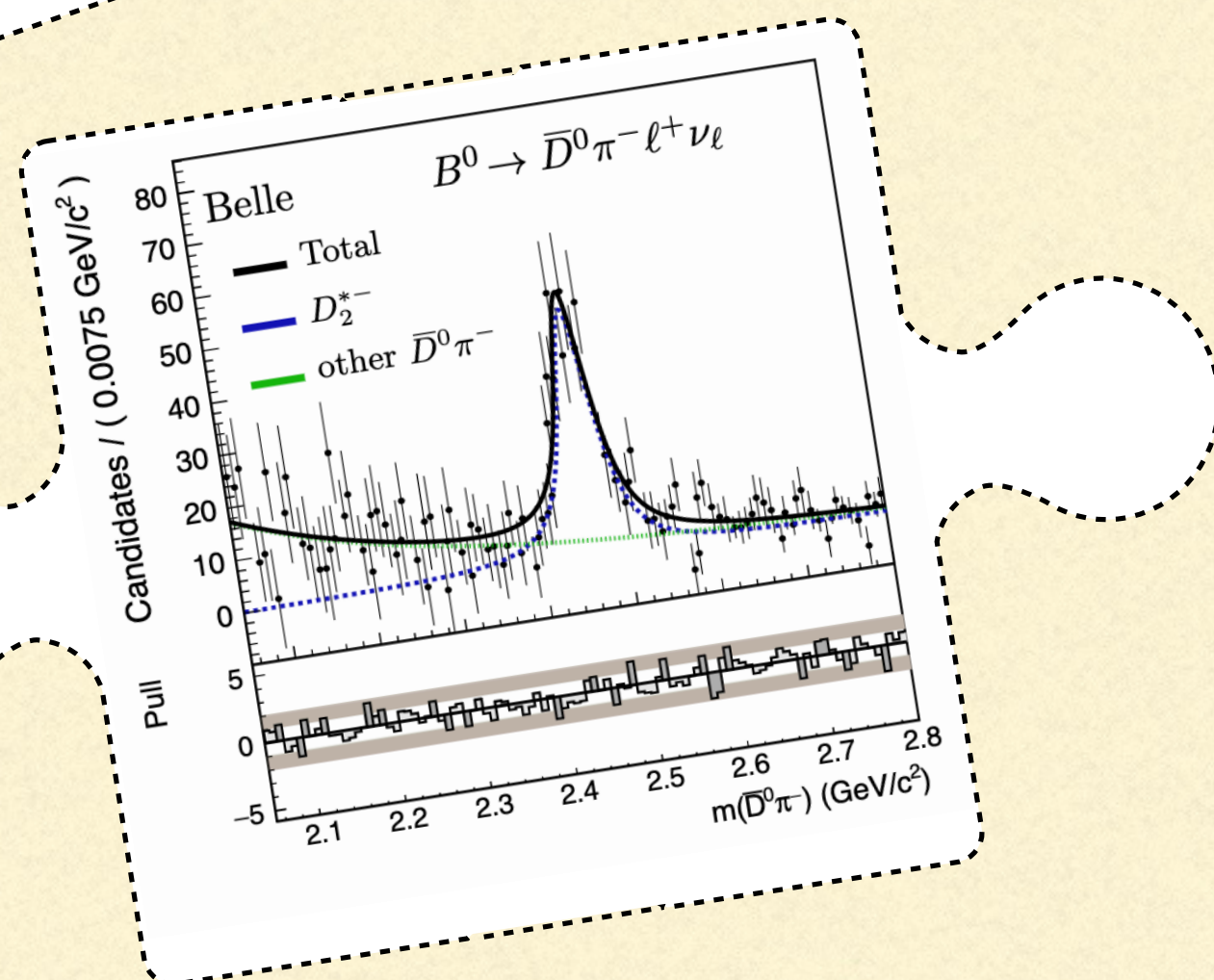
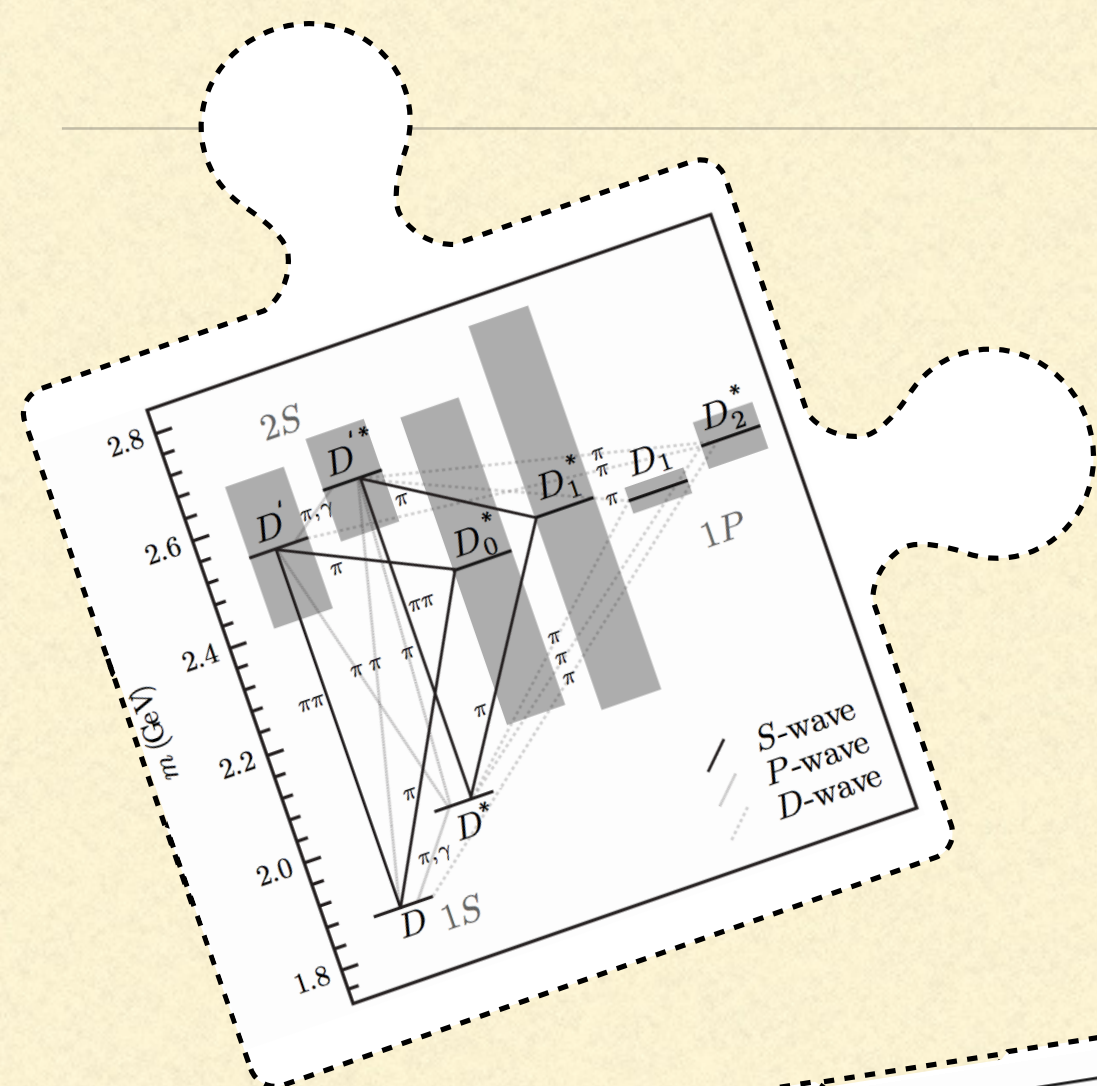
The D-meson spectrum is more complicated than originally assumed (see Christoph's talk yesterday)



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Application: $B \rightarrow D\pi\ell\nu$

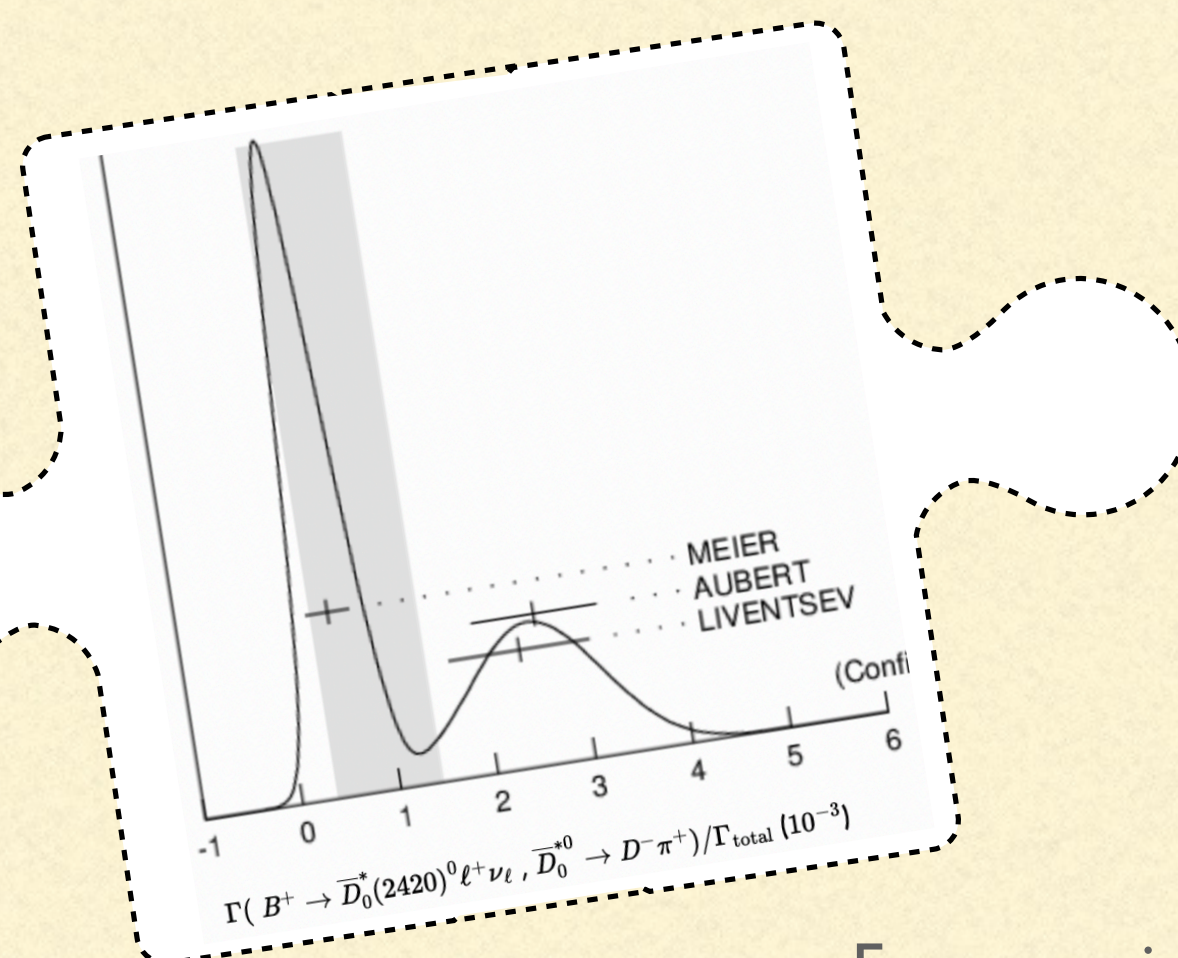
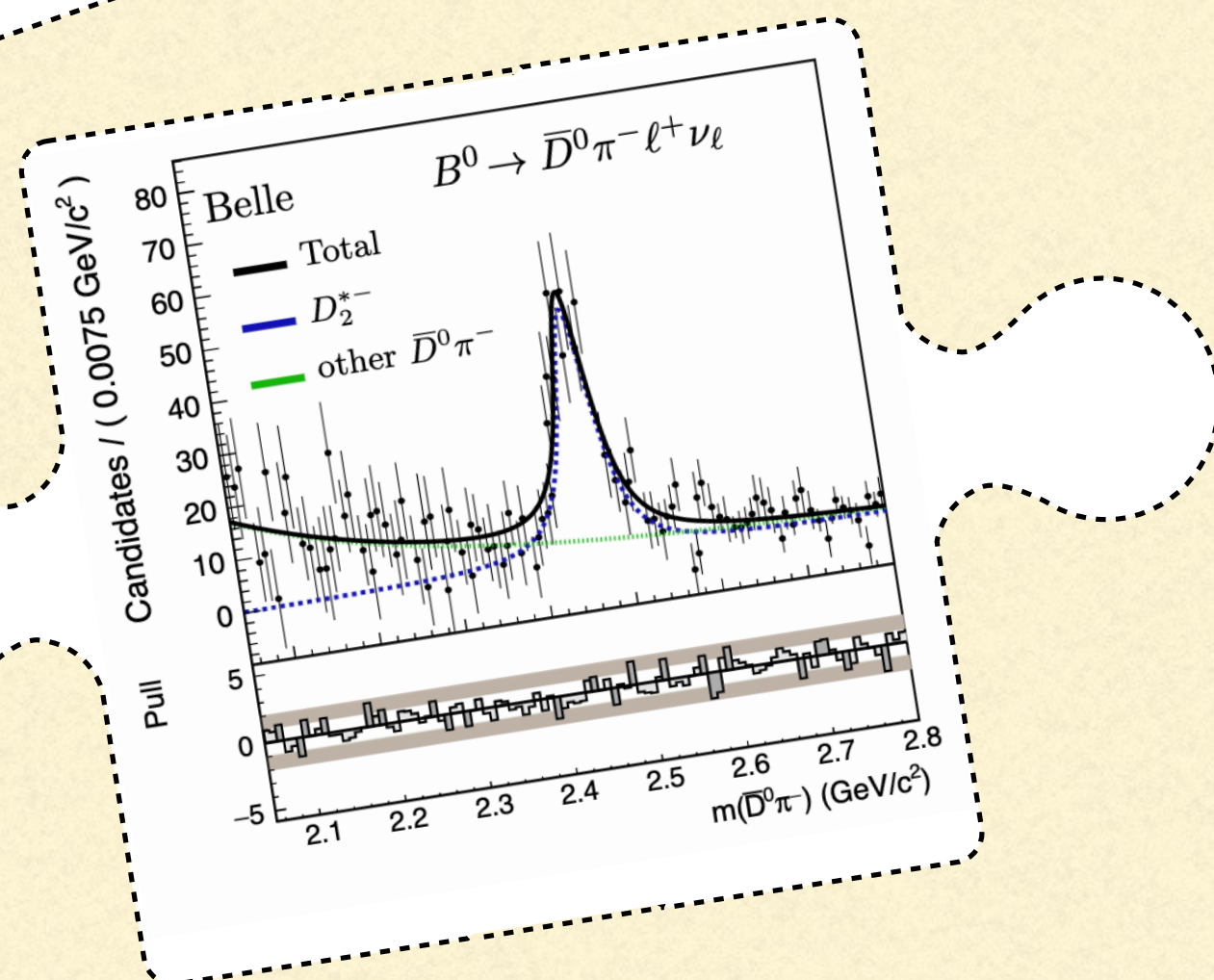
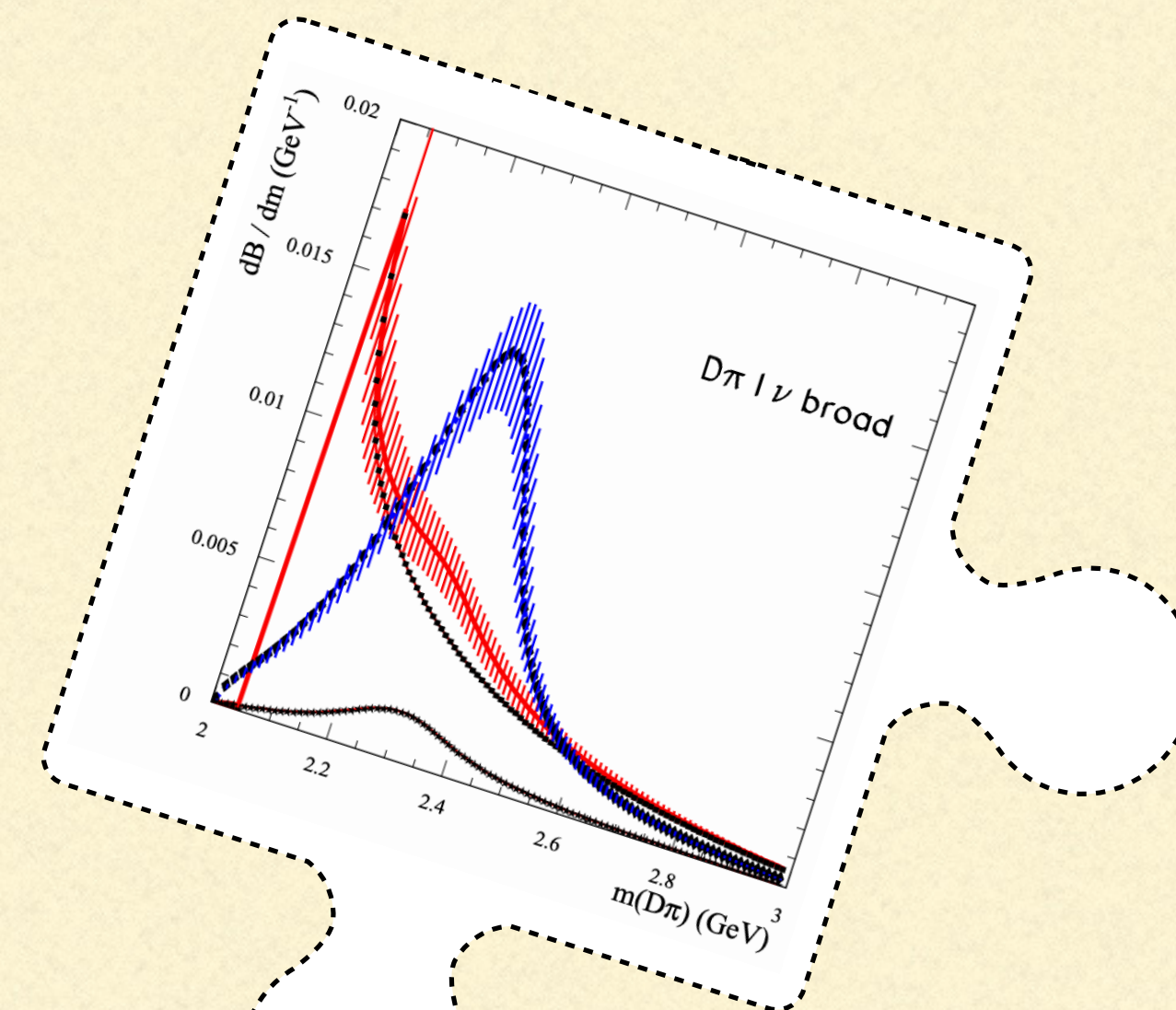
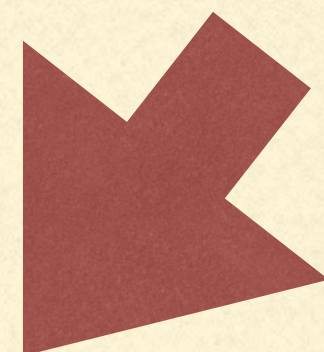
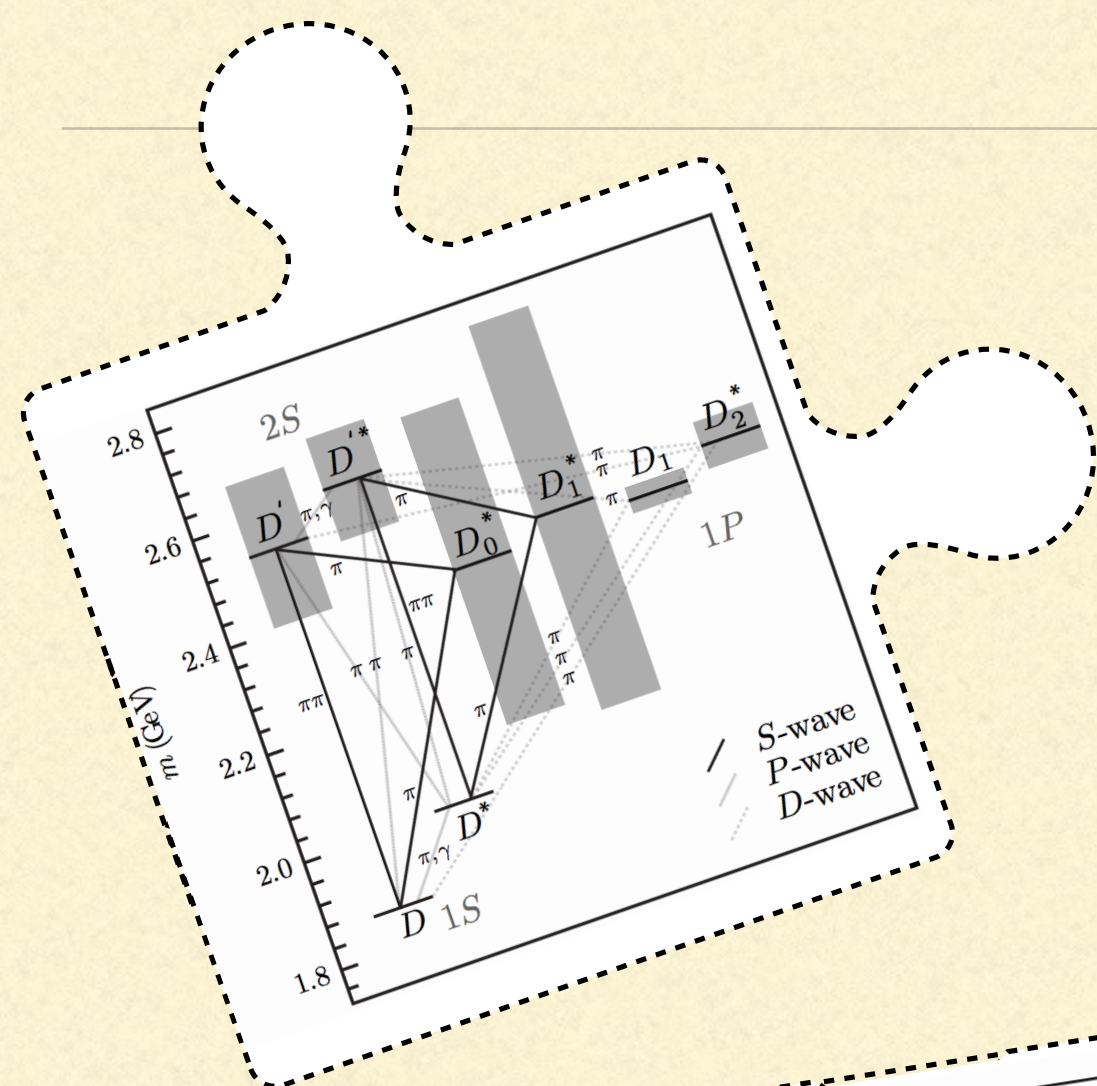
In a recent Belle analysis, much lower BFs to the S-wave where reported



For more in-depth discussion see my talk at CKM 2023 and [Raynette's talk in Vienna](#) (with the original puzzle pieces)

Application: $B \rightarrow D\pi\ell\nu$

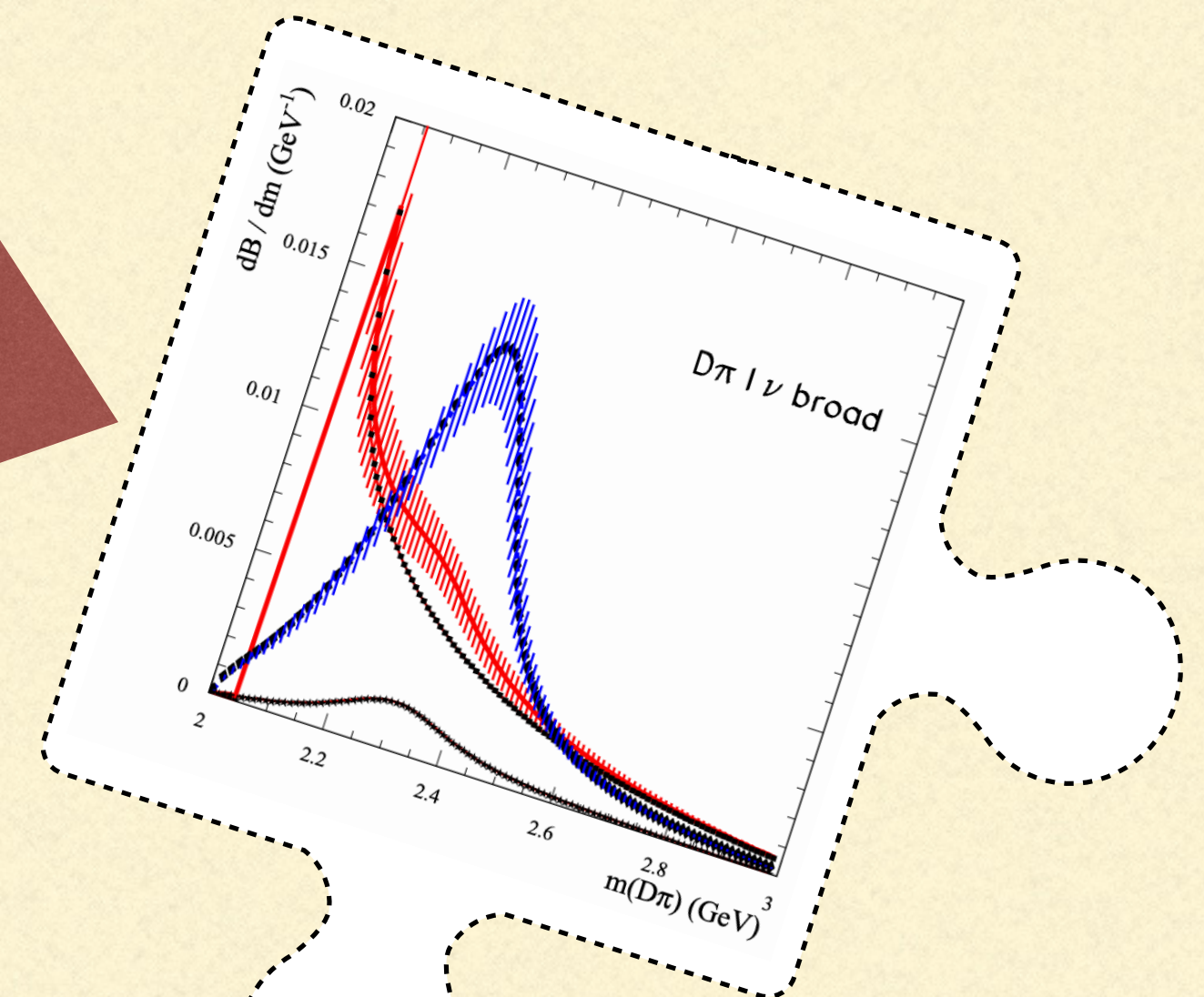
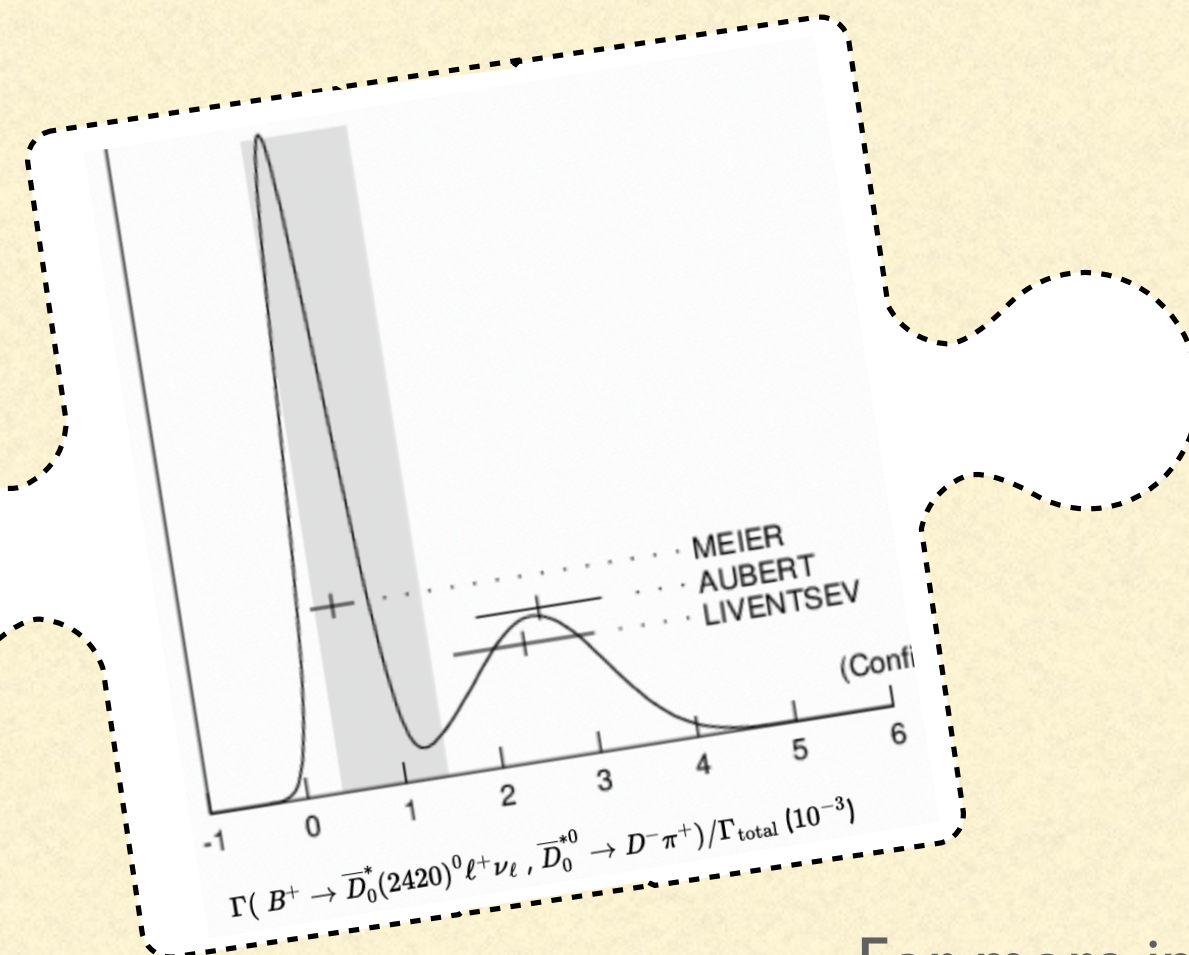
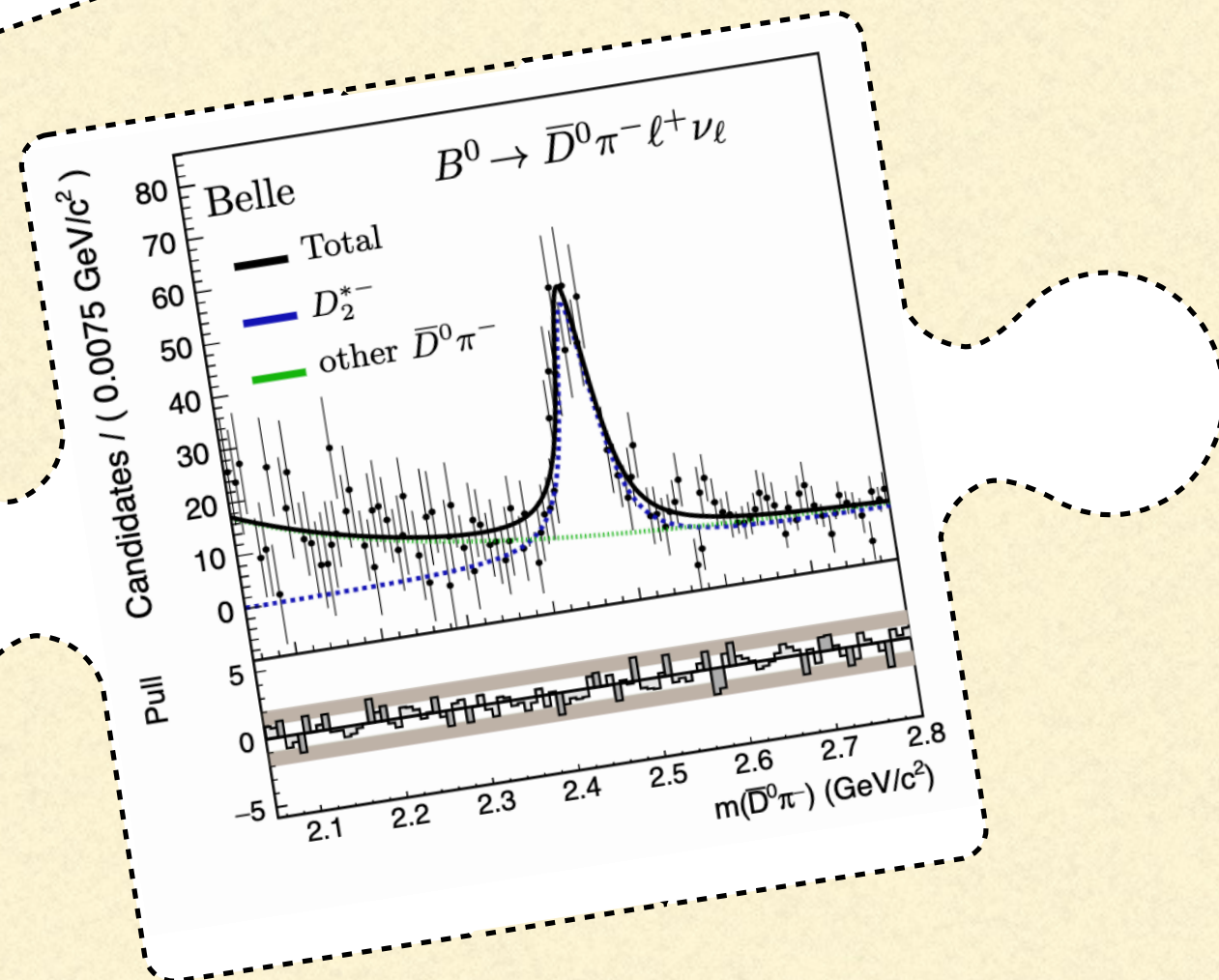
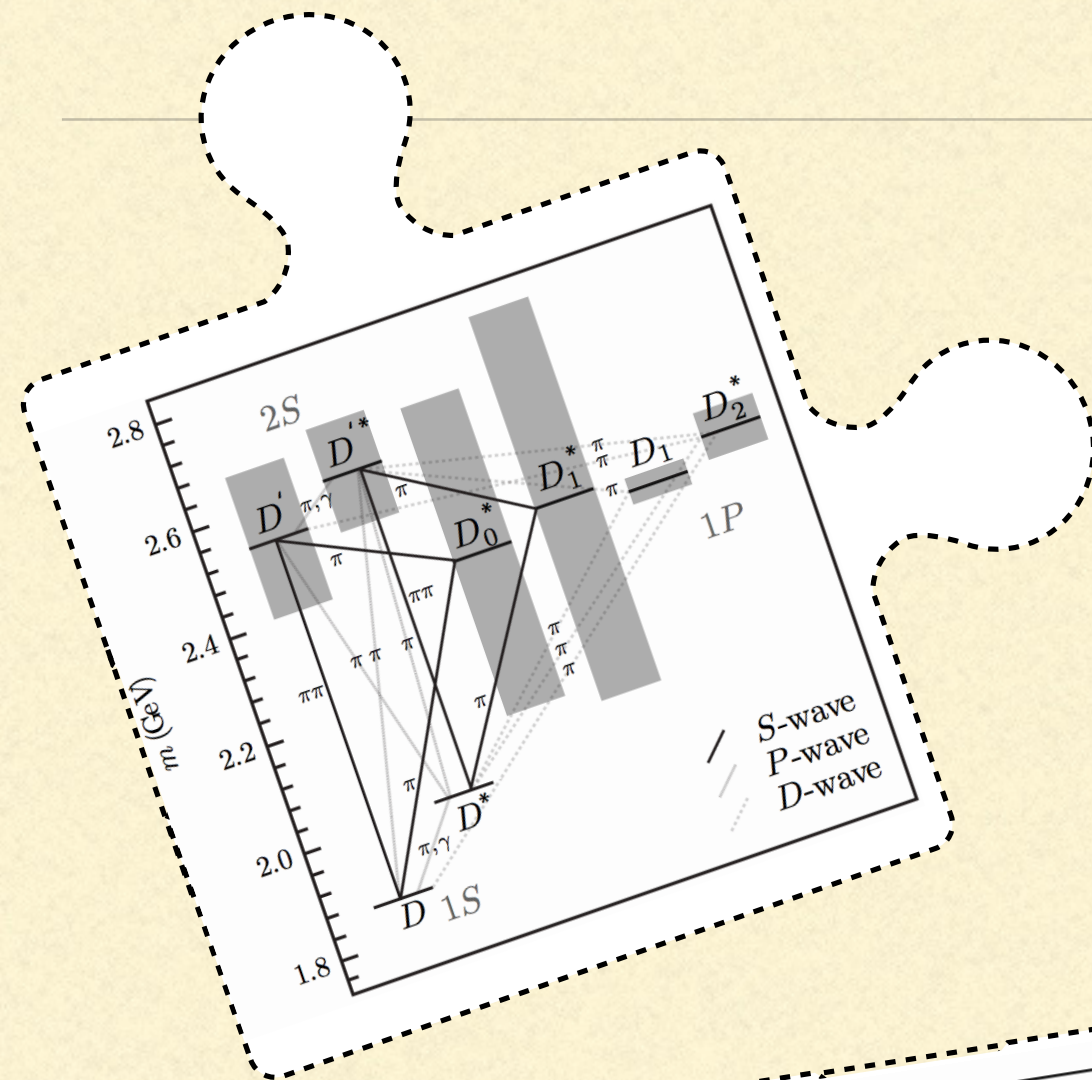
A falling component is necessary to describe the data.



For more in-depth discussion see my talk at CKM 2023 and [Raynette's talk in Vienna](#) (with the original puzzle pieces)

Application: $B \rightarrow D\pi\ell\nu$

Previously predicted in analogy to
 nonleptonic decays! Le Yaouanc, Leroy, Roudeau
 PRD 105 (2022) 1, 013004

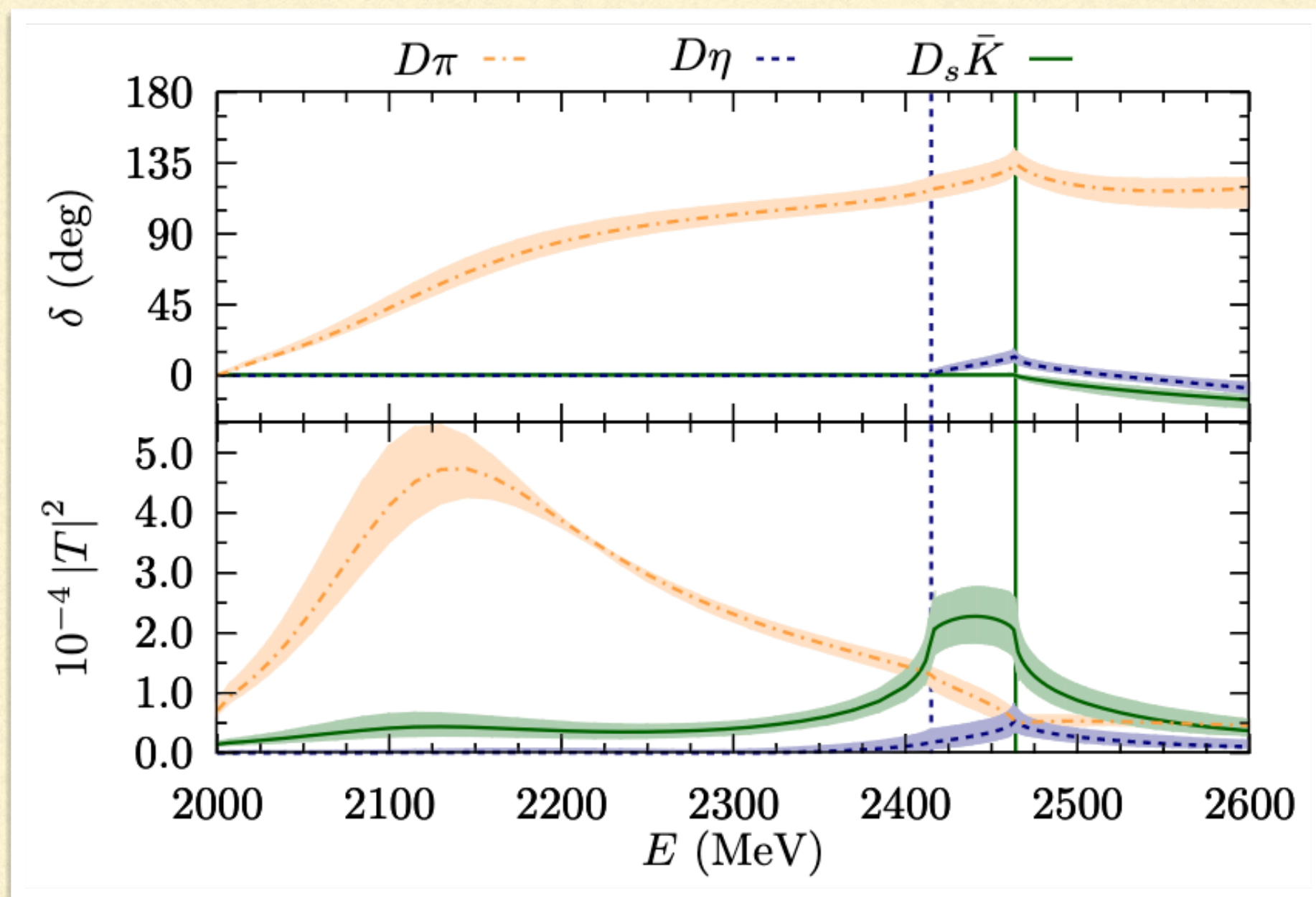


For more in-depth discussion see my talk at CKM 2023 and [Raynette's talk in Vienna](#) (with the original puzzle pieces)

Application: $B \rightarrow D\pi\ell\nu$

arXiv:2311.00864 Erik Gustafson, FH, Ruth Van der Water, Raynette van Tonder, Mike Wagman

$$\text{Im } \Omega(s + i\epsilon) = \frac{1}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{T^*(s')\Sigma(s')\Omega(s')}{s' - s - i\epsilon} ds'$$

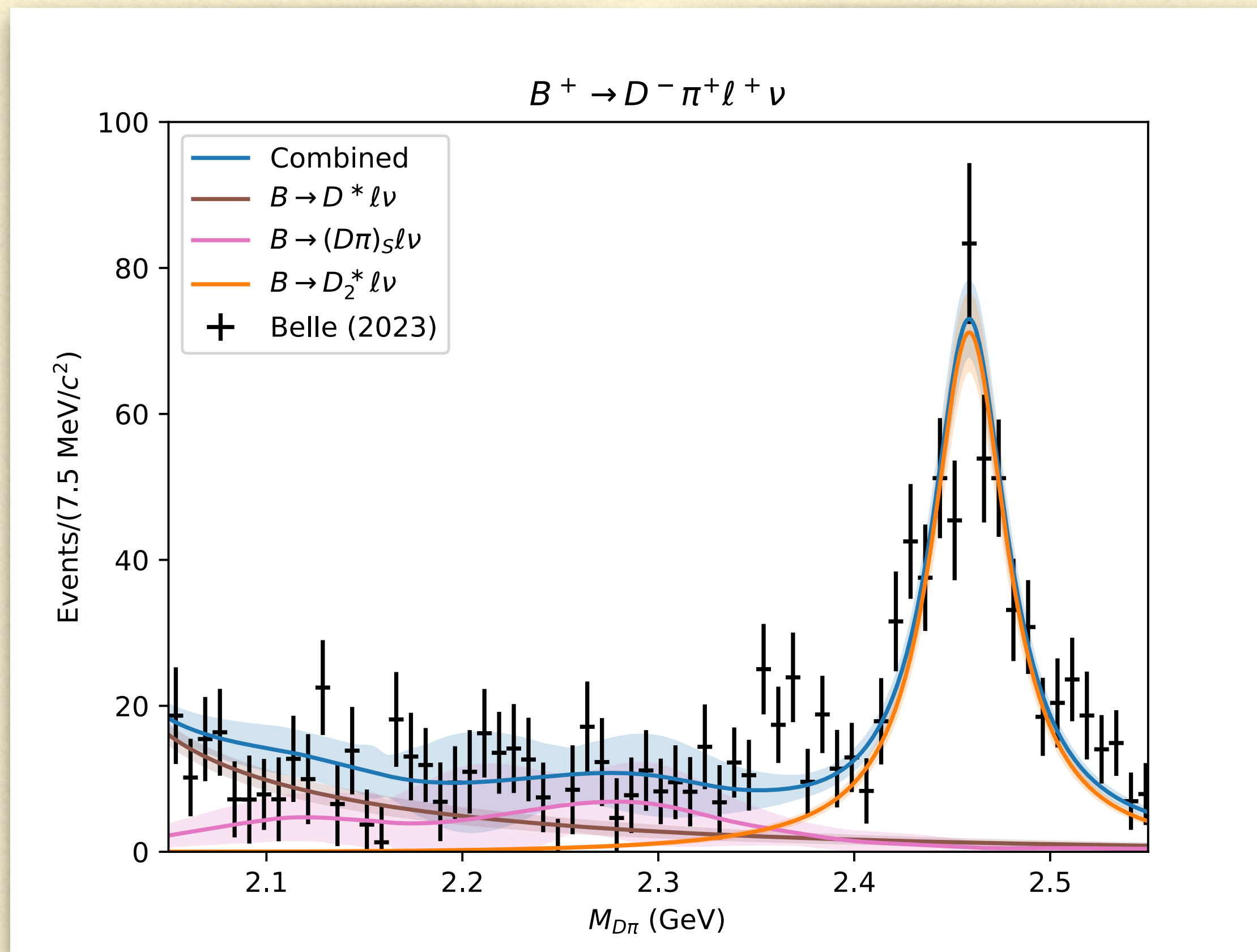


Plot from Albaladejo et al. [PLB 767 \(2017\) 465-469](#)

- To describe the S-wave, we decided to put all the work Christoph talked about yesterday into action
- Solve the coupled channel Muskhelishvili-Omnès equation to get lineshapes
- Nontrivial phase motion and interplay between the different channels
- As mentioned yesterday, Kaons and the Eta lead to nontrivial structures around 2.45 GeV

Application: $B \rightarrow D\pi\ell\nu$

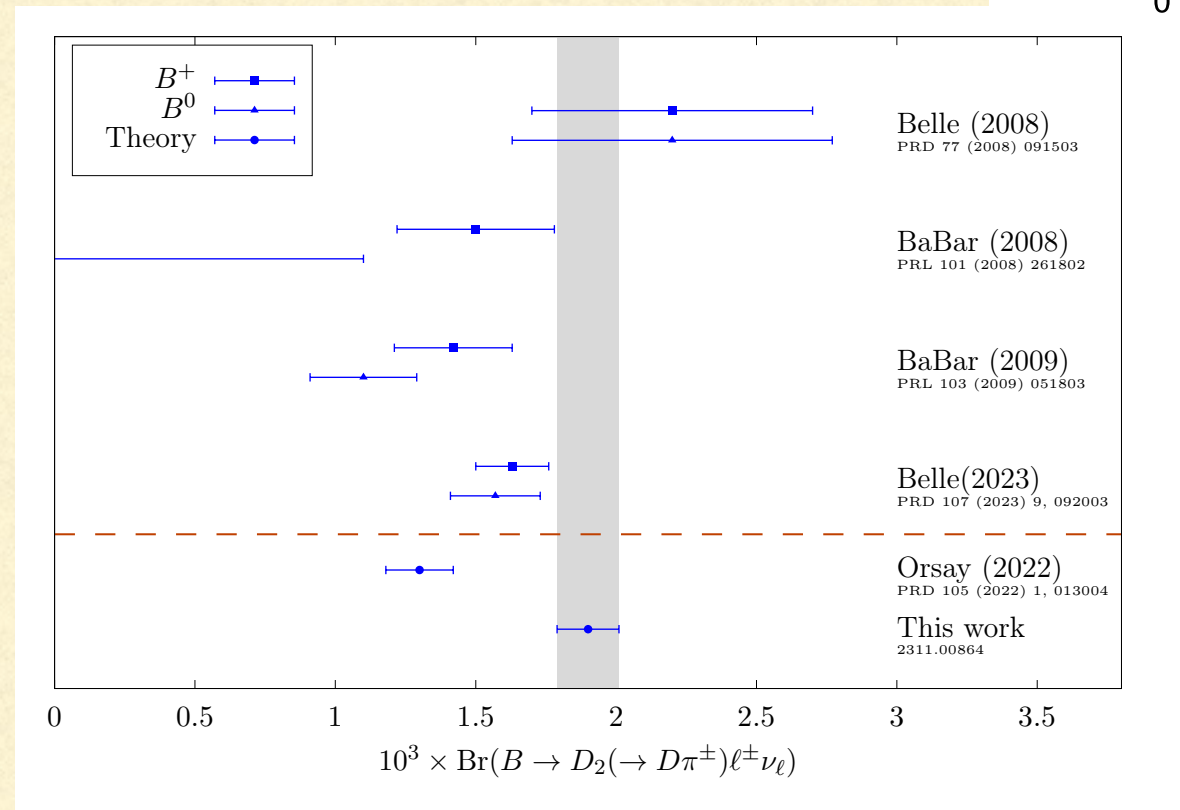
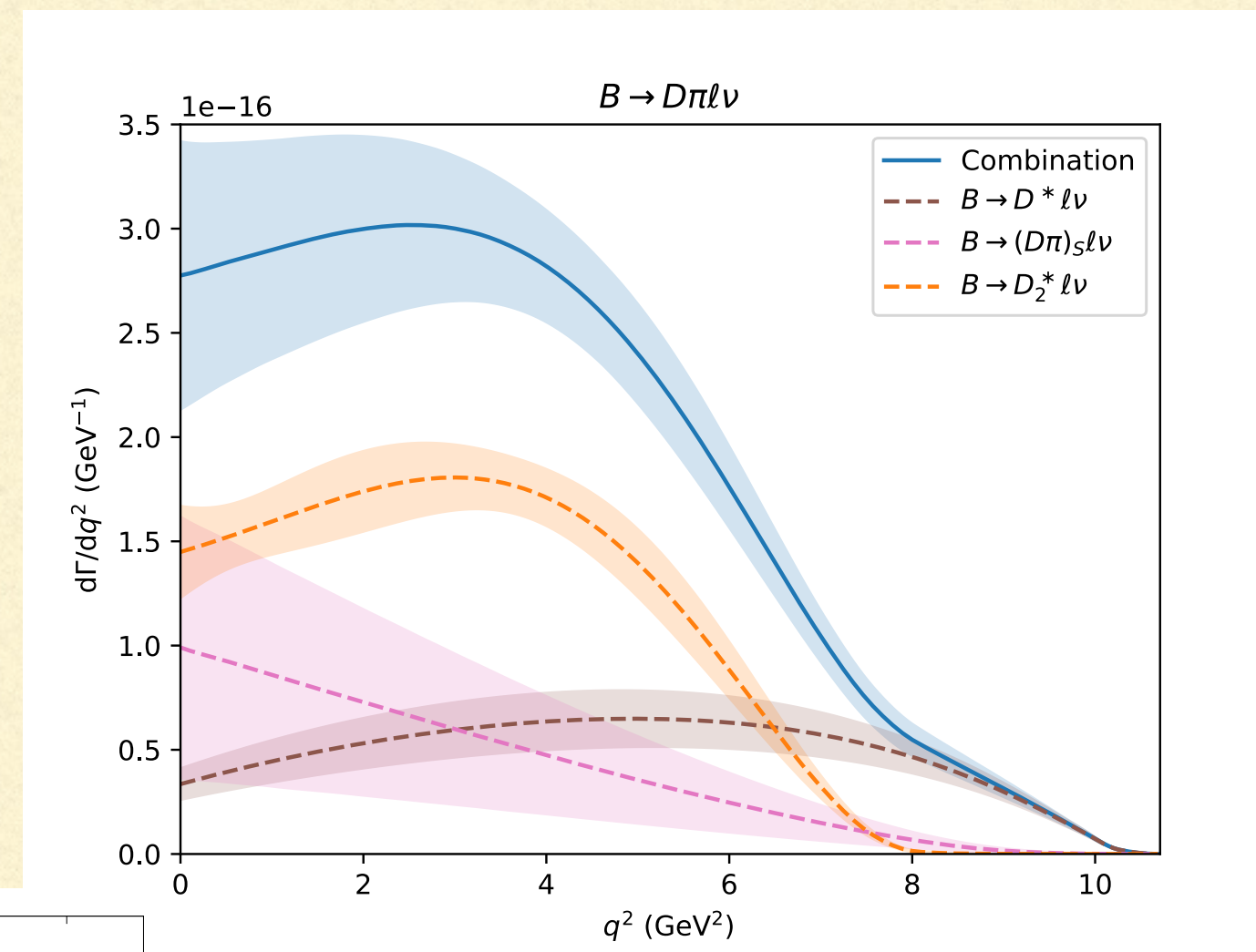
arXiv:2311.00864 Erik Gustafson, FH, Ruth Van der Water, Raynette van Tonder, Mike Wagman



- Combining S-wave with P-wave and D-wave resonance (mass and width from RPP)
- We fit to the q^2 spectrum for the D_2^* from Belle
- We take the D^* FFs from FNAL/MILC
- We do not impose any constraint on the 3 S-wave channels \rightarrow 3 independent FFs

Application: $B \rightarrow D\pi\ell\nu$

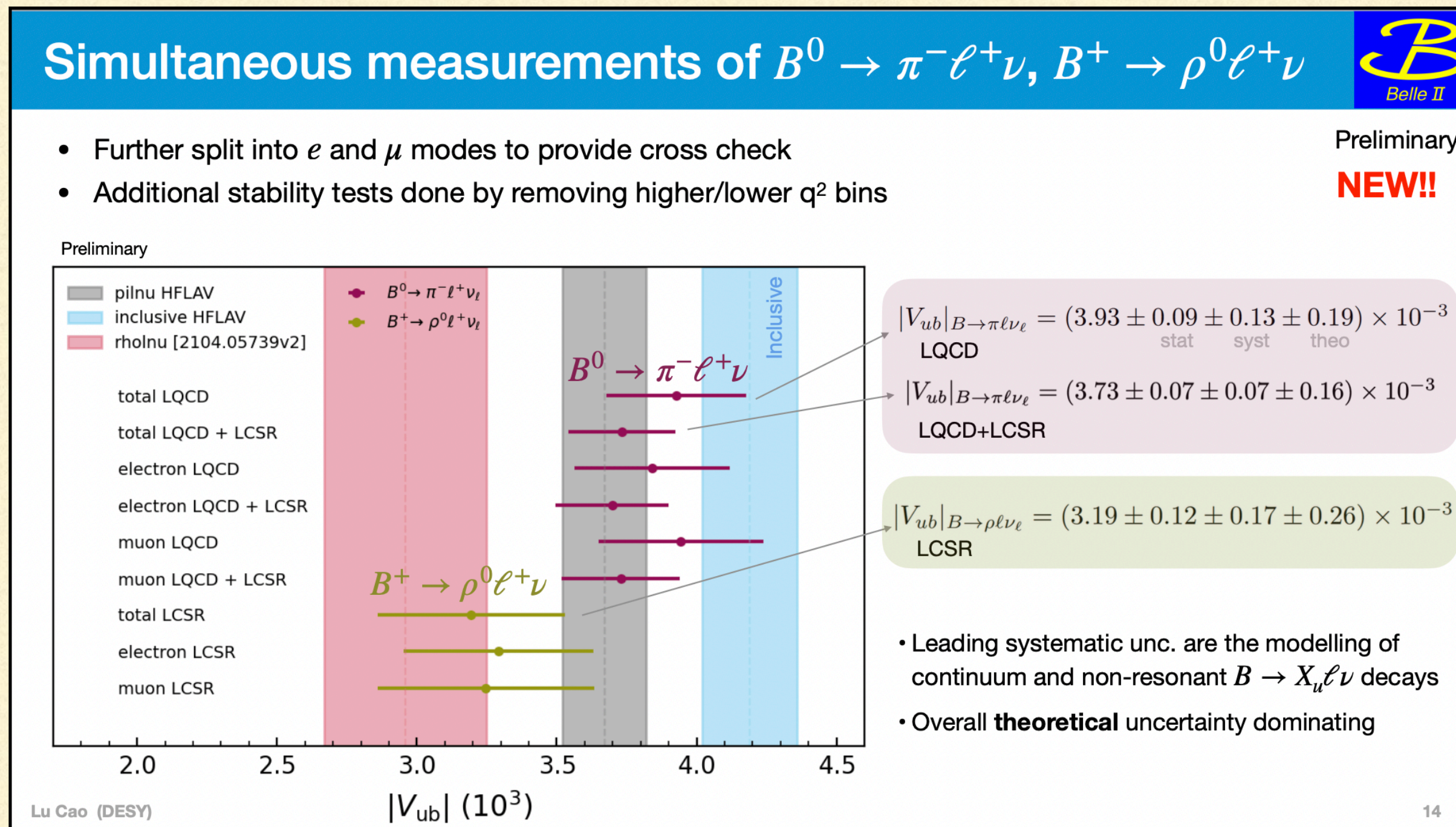
arXiv:2311.00864 Erik Gustafson, FH, Ruth Van der Water, Raynette van Tonder, Mike Wagman



- The q^2 spectrum we obtain is harder than what Belle & Belle II assume in their MC, in line with observed mismodelling
- Our $B \rightarrow D_2^*(\rightarrow D\pi)\ell\nu$ BF is larger than other extractions, does it make sense to quote this number?
- However, our fit is only mildly better than with a broad Breit-Wigner resonance, just the invariant mass spectrum is not enough
- Finally: $\text{Br}(B \rightarrow D\eta\ell\nu) = (1.9 \pm 1.7) \times 10^{-5}$

Application: $B \rightarrow \pi\pi\ell\nu$

WIP with Bastian Kubis, Ruth Van der Water, Raynette van Tonder

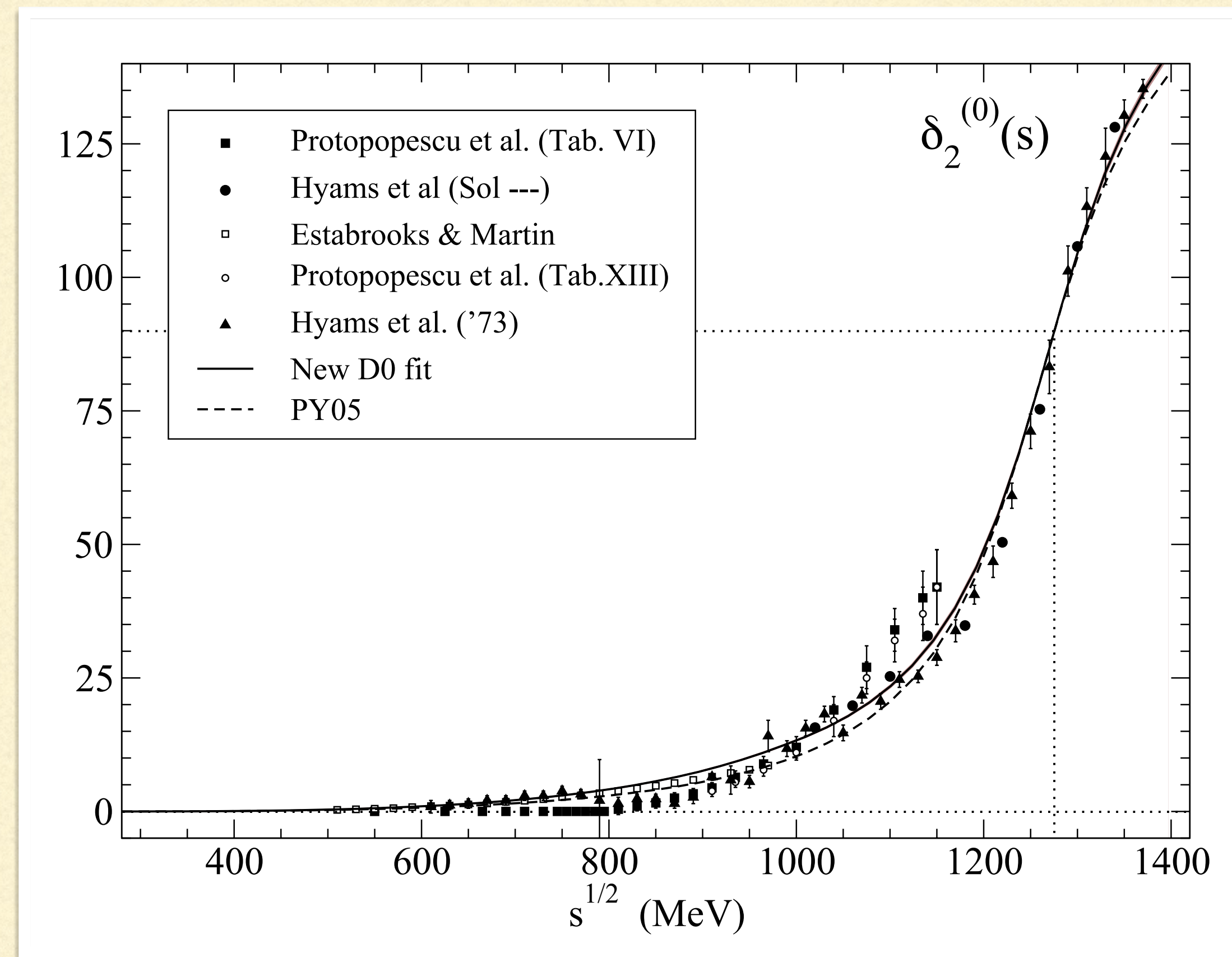


Lu Cao's talk at [Moriond EWK 2024](#)

- Next, we wanted to look at a similar process with more data available
- Phenomenologically relevant and LCSR & LQCD calculations of FFs possible (see talks by Fernando and Alex)
- P-wave phase shift well understood
- D-wave sufficiently understood
- S-wave funny
- Data from Belle

Application: $B \rightarrow \pi\pi\ell\nu$

WIP with Bastian Kubis, Ruth Van der Water, Raynette van Tonder

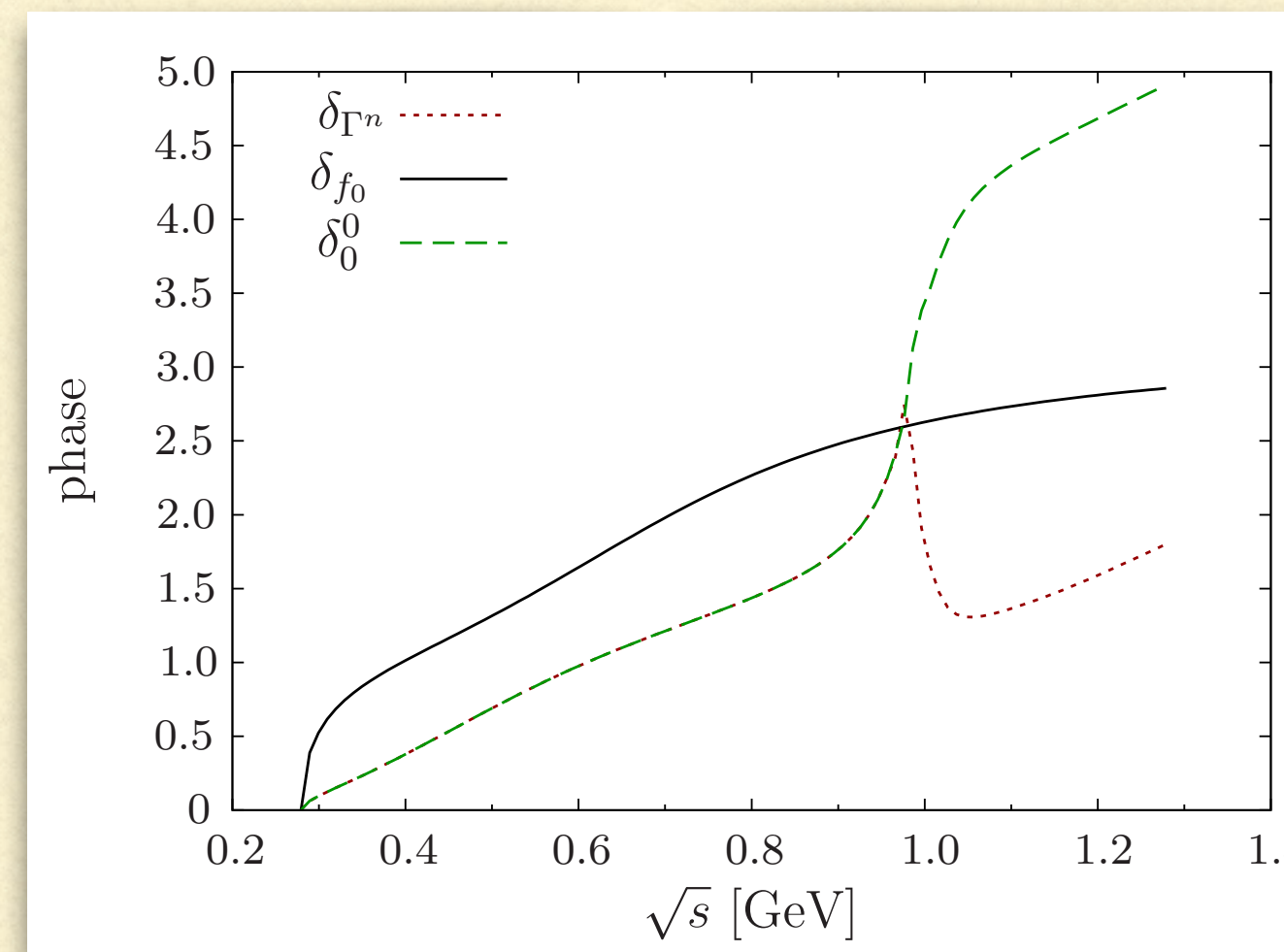


Taken from: Kaminski, Pelaez, Yndurain [PRD 74 \(2006\) 014001](#)

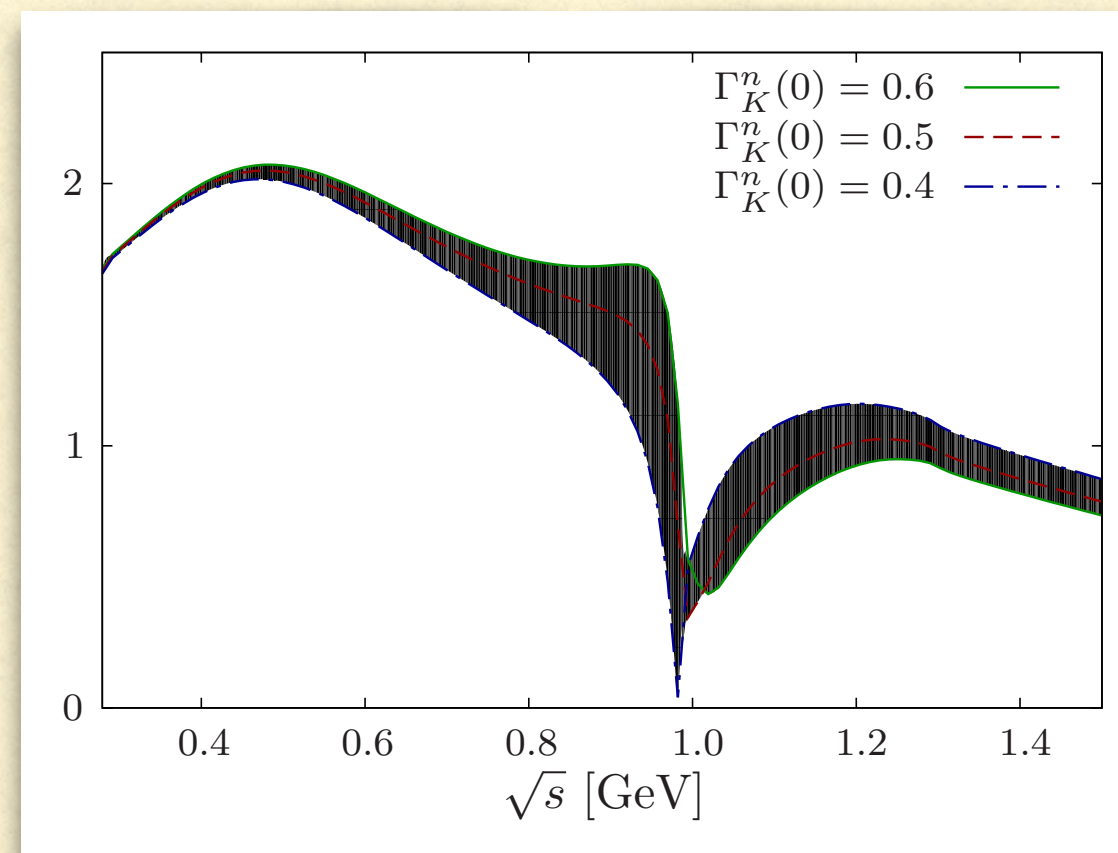
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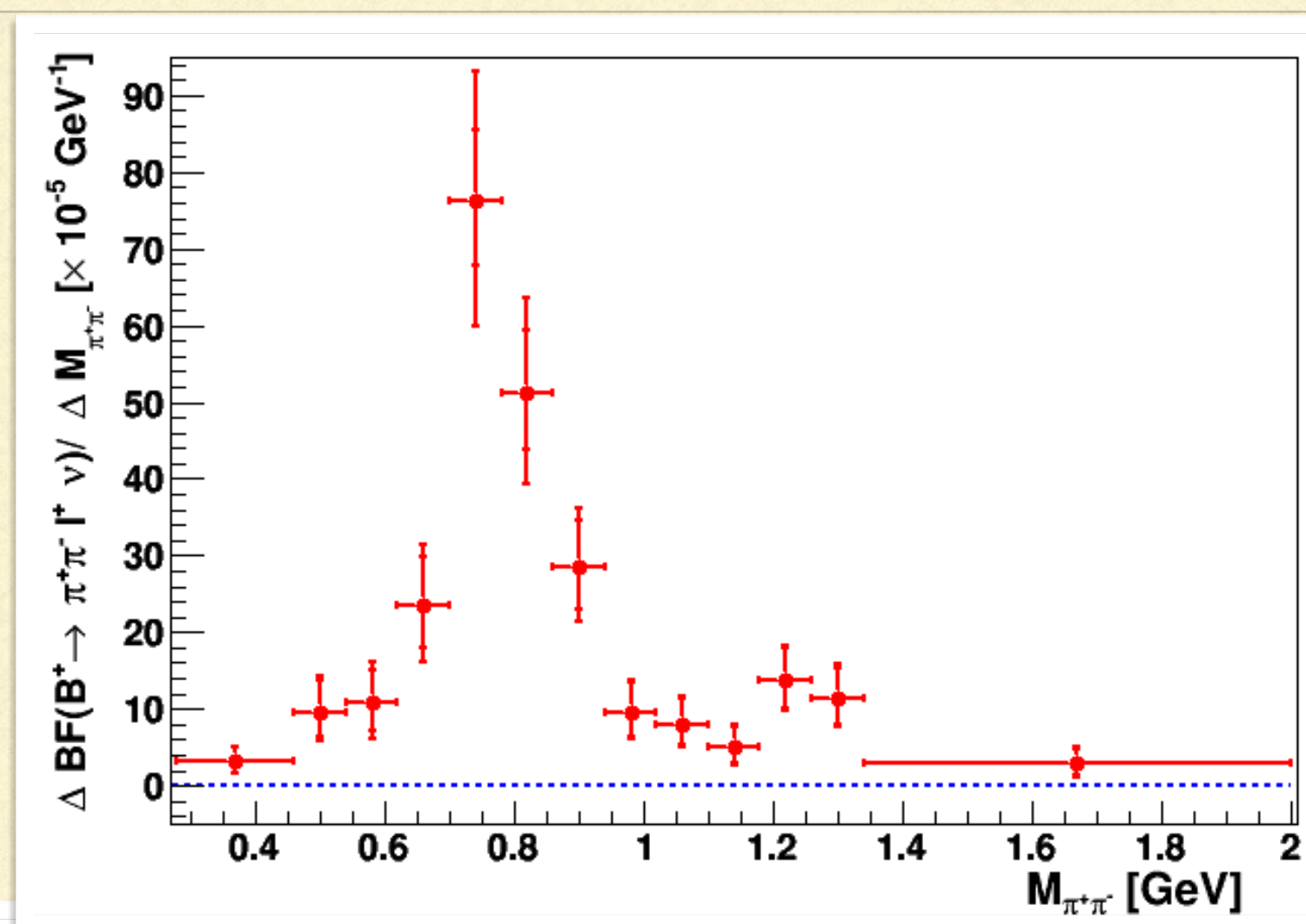
Taken from: Daub, Hanhart, Kubis [JHEP](#)
[02 \(2016\) 009](#)



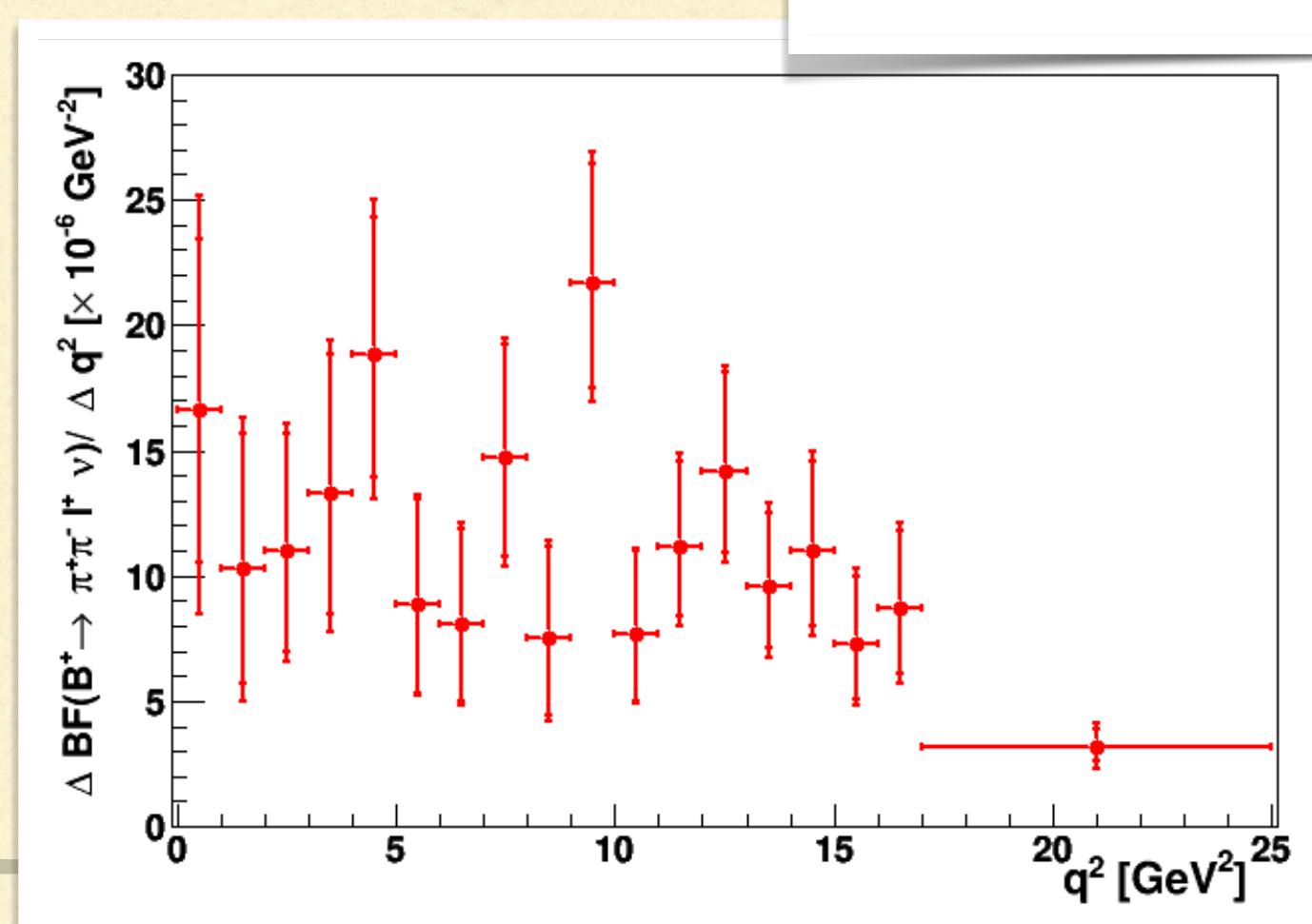
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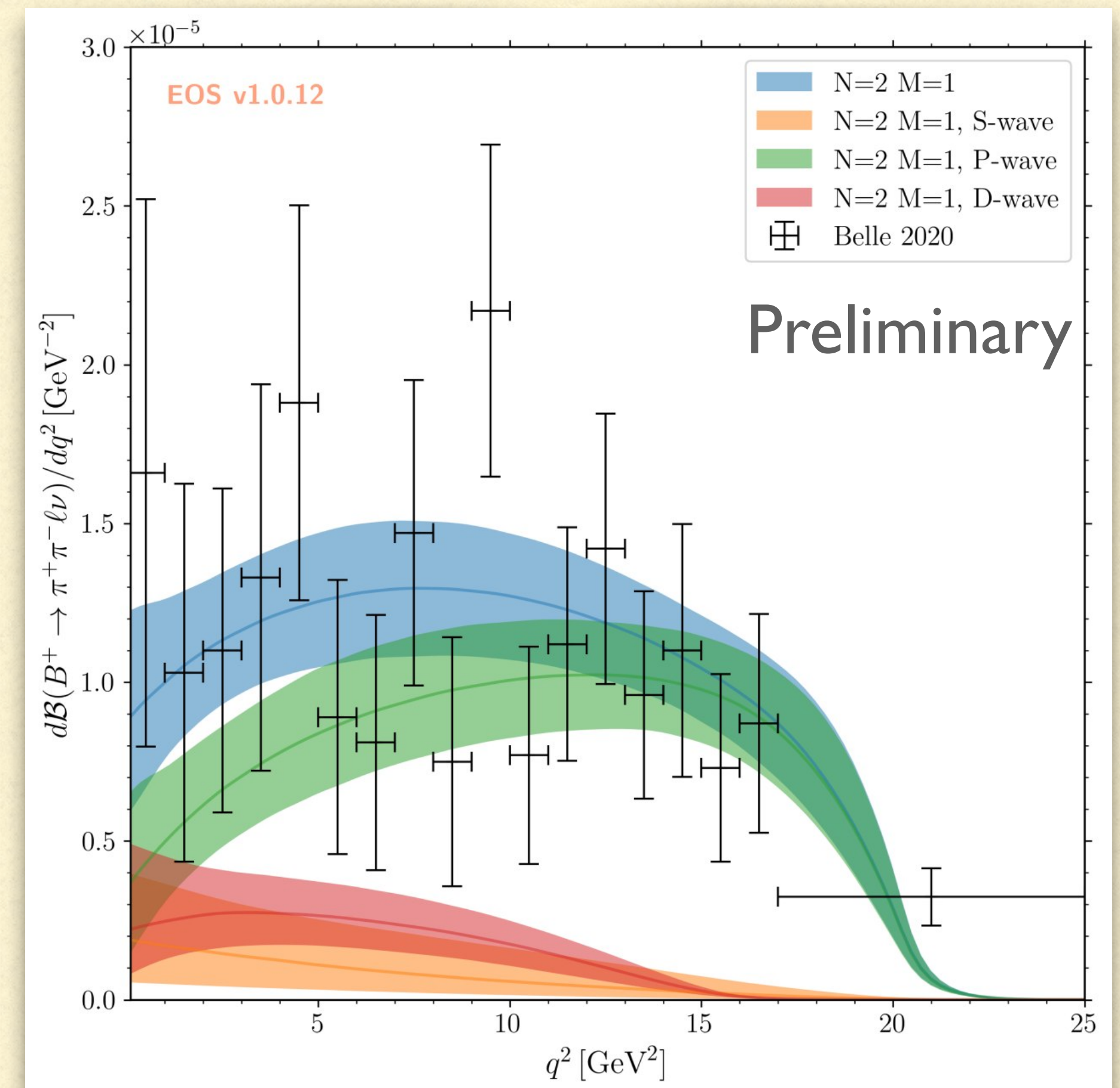
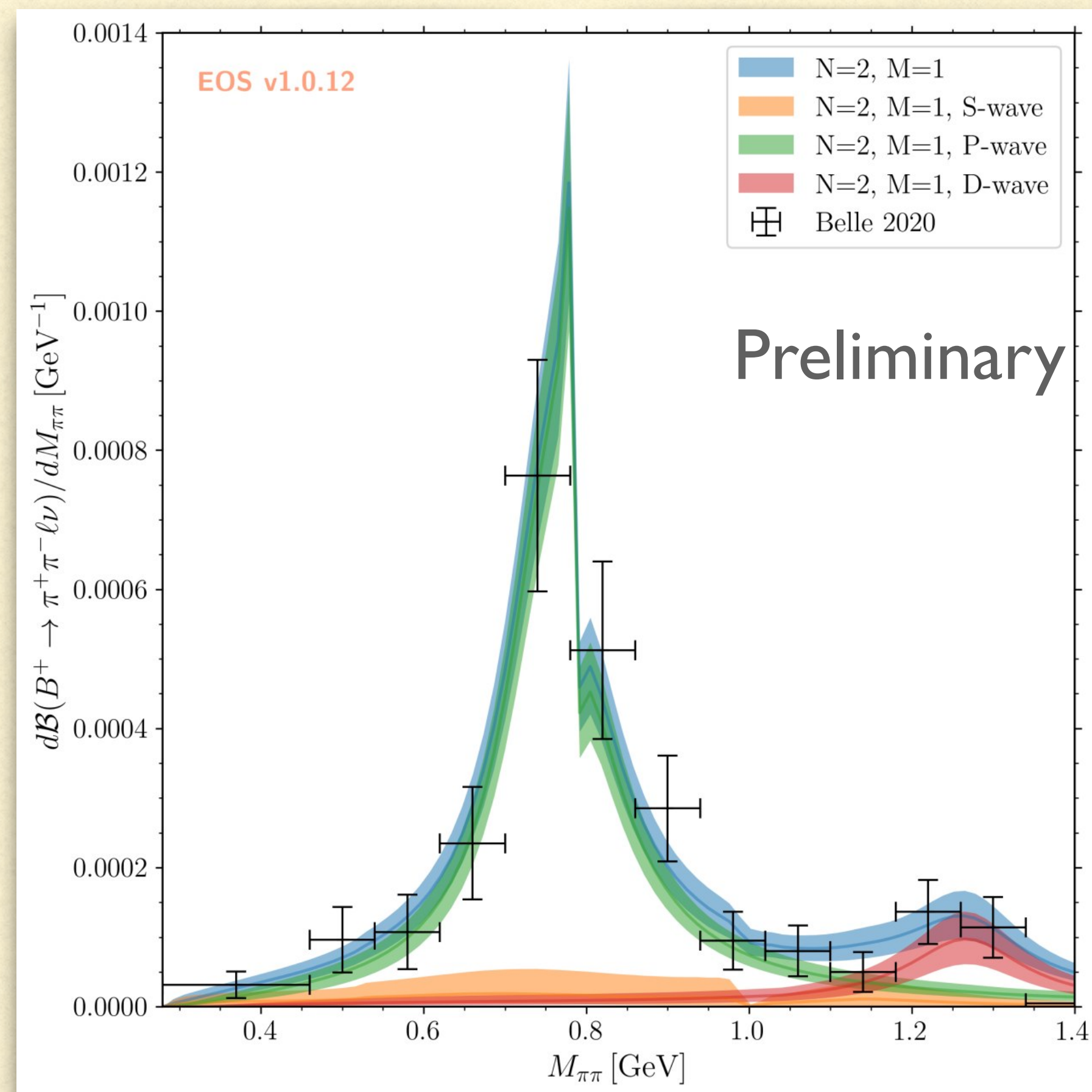


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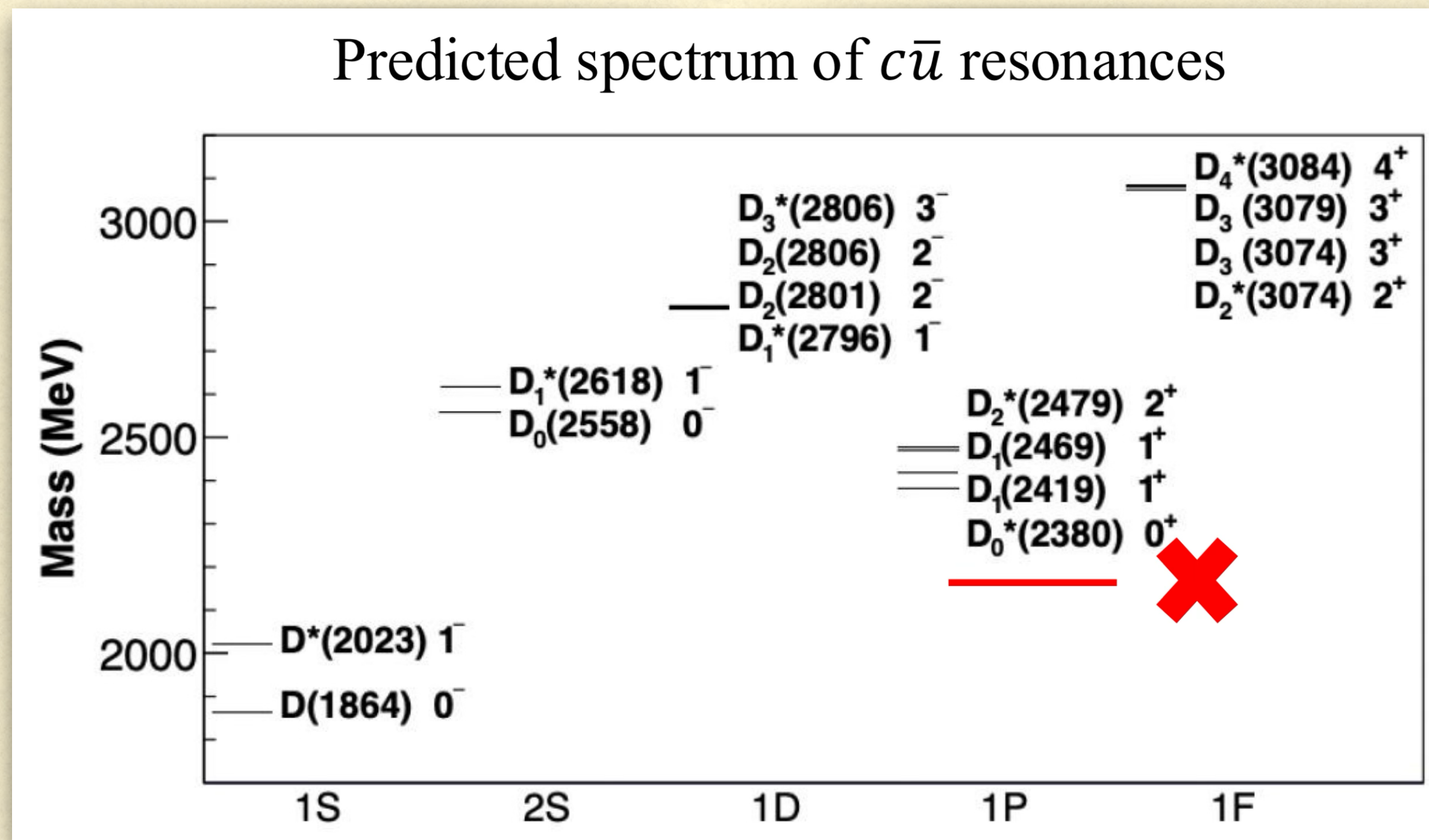


Application: $B \rightarrow \pi\pi\ell\nu$

WIP with Bastian Kubis, Ruth Van der Water, Raynette van Tonder



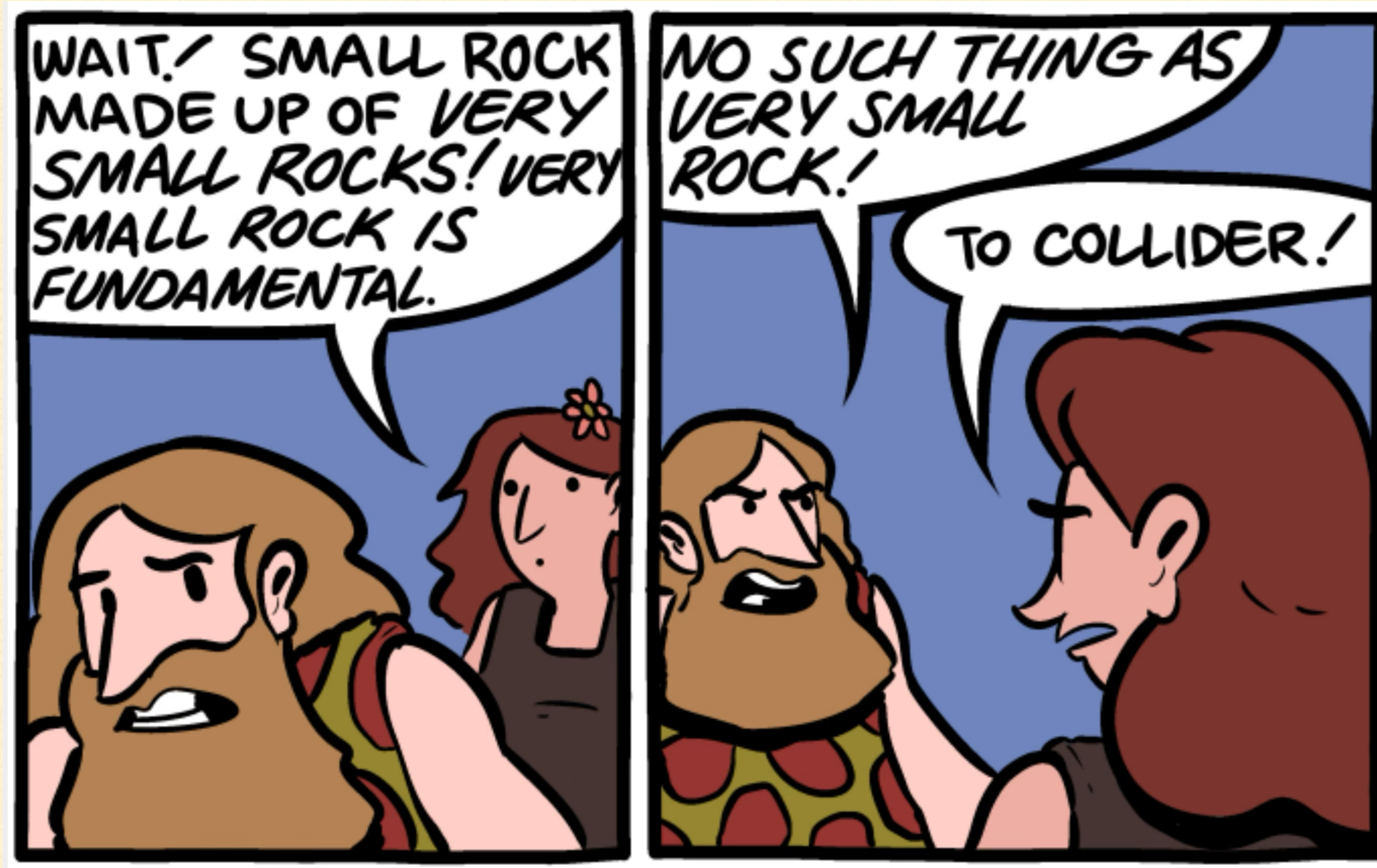
Can we find the $D_0^*(2105)$ in semileptonic decays?



Taken from Abhijit Mathad's [talk last week](#)

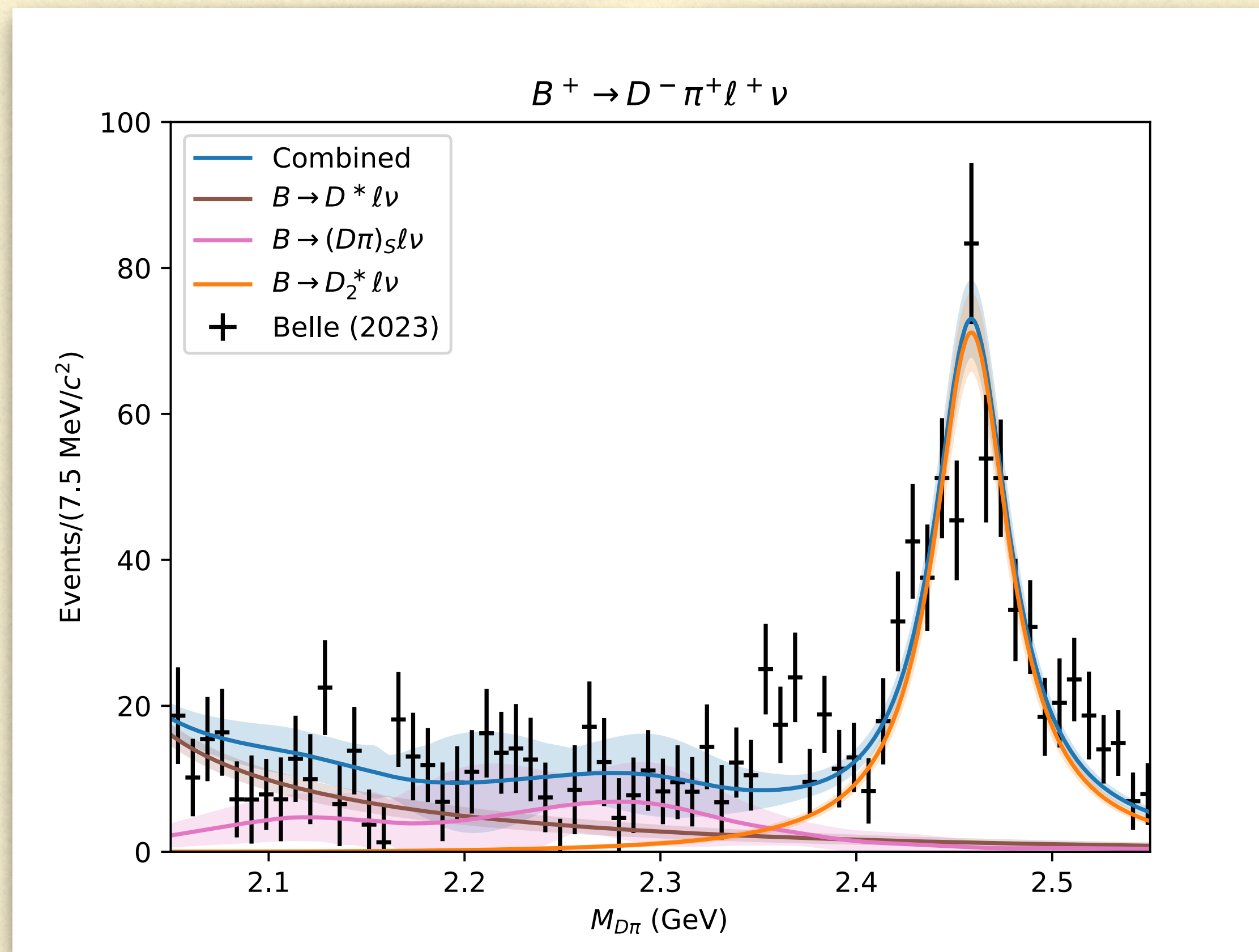
- Convincing experimentalists and theorists working on semileptonic decays to change the way they model broad resonances can be hard
- Some are more than happy to throw away the old picture, because “it doesn’t describe data anyways”
- Others will claim that “partial wave expansions are ad hoc”, “HQET disagrees with what you say” or “you don’t have any experimental evidence”
- So, can we directly find the $D_0^*(2105)$ in semileptonic decays?

Can we find the $D_0^*(2105)$ in semileptonic decays?



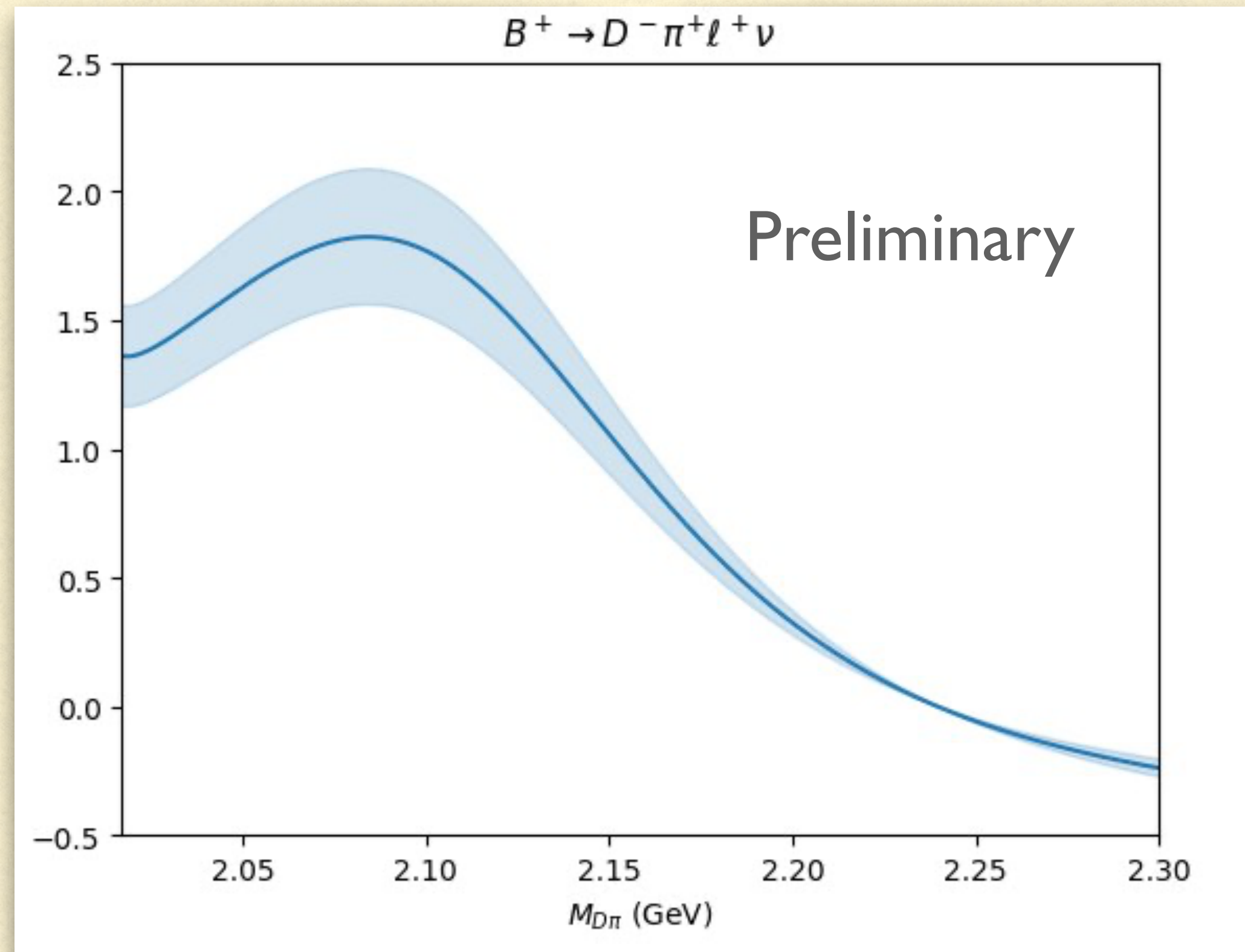
SMBC comics: To the collider!

Can we find the $D_0^*(2105)$ in semileptonic decays?



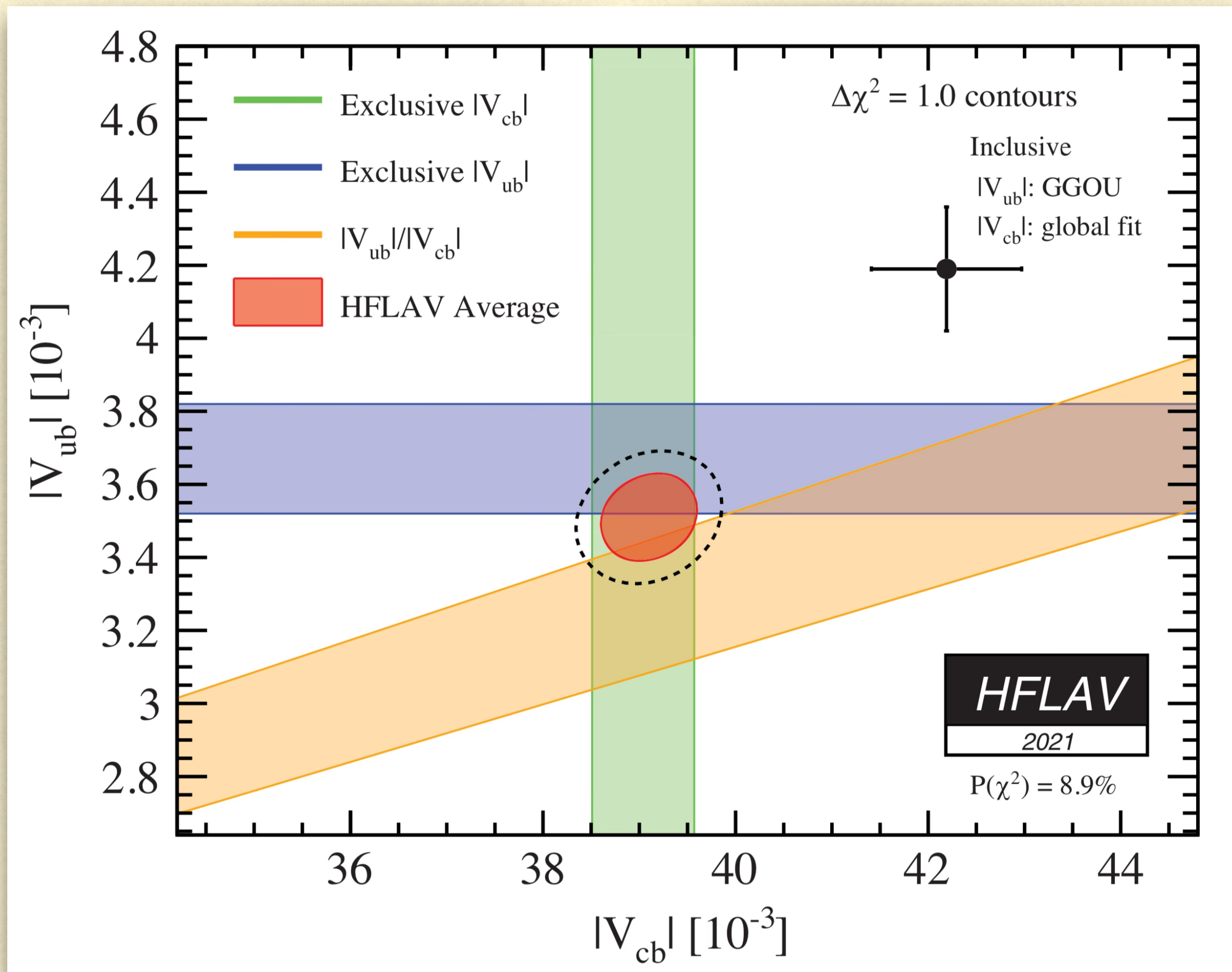
- Just the invariant mass spectrum is insufficient
- Interplay between 3 partial waves
- Below ≈ 2.3 GeV the D-wave can be neglected
- As Christoph mentioned in the case of $B^+ \rightarrow D^- \pi^+ \pi^+$: we understand the P-wave well \rightarrow Reference phase
- $B^+ \rightarrow D^- \pi^+ \ell \nu$ ideal since D^* subthreshold
- $K \rightarrow \pi \pi \ell \nu$ decays serve as inspiration

Can we find the $D_0^*(2105)$ in semileptonic decays?



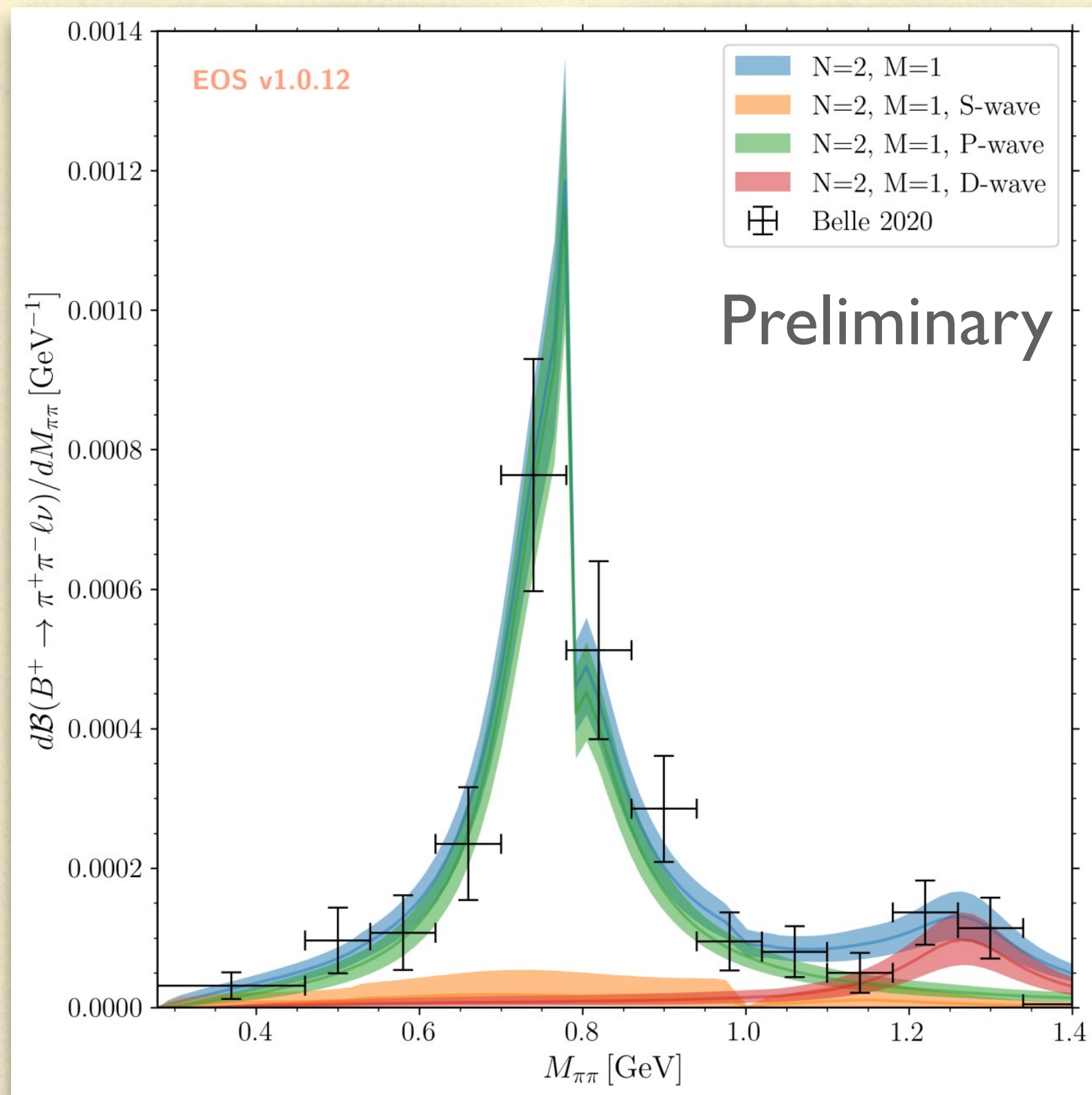
- Similar to $\langle P_{13} \rangle$ the forward-backward asymmetry of the D is directly related to $\cos(\delta_0 - \delta_1)$
- Slight complication w.r.t. non-leptonic: FFs depend on q^2 ; but for this analysis, they are known well enough (partially cancel)
- Sensitivity study for Belle II currently in progress; $\mathcal{O}(300)$ events in Belle analysis
- Belle + current Belle II data set might already give us some sensitivity!

Conclusion & Outlook



- Semileptonic decays are phenomenologically crucial for precise tests of the SM (see Talks by Raynette, Martin, Uli, Jack, Jaime and Keri later this week)
- The analytic structure of $1 \rightarrow 1$ form factors is well understood
- However, for many of these interesting processes, decays to higher states need to be taken into account as backgrounds

Conclusion & Outlook



- Model-independent parameterizations of $1 \rightarrow 2$ decays are possible and directly make connection to scattering phases
- Parameterization + Lineshapes already enough for some experiments to improve their backgrounds modelling
- Combined with LCSR calculations (see works by Alex & collaborators) and/or Lattice calculations (see talk by Fernando) systematic uncertainties can be significantly reduced
- Even better: semileptonic measurements can feed back to our spectroscopic understanding