Semileptonic $B \to D^*$ decays: The long path to 1%

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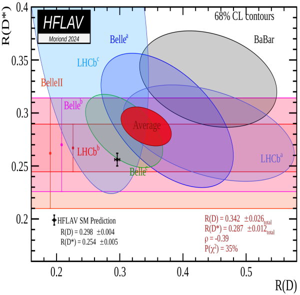
Tensions as a motivation for semileptonic decays?

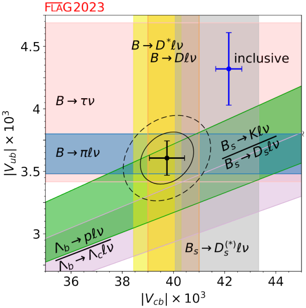
Present tensions in $B \to D^* l \nu$ decays:

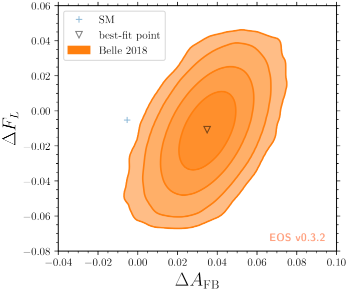
$$R(X) \equiv \frac{\mathrm{Br}(B \to X\tau\nu)}{\mathrm{Br}(B \to X\ell\nu)}$$

$$V_{cb}^{\mathrm{excl}} \neq V_{cb}^{\mathrm{incl}}$$

$$A_{\mathrm{FB}}^{\mu} - A_{\mathrm{FB}}^{e} \neq 0$$







- These are not the main motivations to study this mode
- Whatever your interpretation: necessary to understand!
  - potentially triggering progress
- Potential explanations: Exp. vs. QCD vs. BSM?
- Partly discussed in the following

What could go wrong?

Standard workflow:

1. Experimental measurement of (partial) rates [Raynette's talk]
2. Theoretical expressions for measured observables: SM plus potentially BSM
3. Theoretical/Phenomenological parametrization: Form factors

➥ Extract FF parameters, $|V_{cb}|$ (and Wilson bilinears)

- Some tensions within points 1+3, no issues (afaik) in point 2
- In the following: scrutinize/detail all three points, essentially no BSM discussion [$\rightarrow$ Uli's talk]
- One elephant in the room not discussed: e/m effects
  ➥ What I'm discussing *should* be larger effects

Substructure of a measurement from a pheno perspective

Experiment makes contact with phenomenology via background-subtracted, unfolded spectra. Structure:

$$\overbrace{N(B^{0/\pm} \to D^*(\to D(\to X_D)\pi)\ell\nu)}^{\text{Measured}} =$$

$$\underbrace{2N_{\Upsilon(4S)} f_{00/\pm}}_{\text{B production}} \underbrace{BR(D^* \to D\pi) BR(D \to X_D)\tau_B}_{\text{universal external inputs}} \underbrace{\epsilon}_{\text{MC}} \quad \underbrace{\Gamma(B \to D^*\ell\nu)}_{\text{Observable of interest}}$$

- Counting rate: Main experimental result
- Experiment-dependent B production: # initial B mesons
- Universal ext. inputs: connecting to specific final state
- Channel- + experiment-dependent efficiency: Monte Carlo
- Observable: (Partial) rate of interest for phenomenology

➥ All of these problematic when aiming at 1%!

Going into even more detail

Universal external inputs:

- Measured by the the same and/or other experiments (LHC, Belle(-II),BaBar, BES-III, Tevatron, CLEO, LEP, ...)
- No issue in principle, but for instance $\sigma_{\rm rel}(BR(D^+ \to K^- \pi^+ \pi^+)) \approx 2\%$, PDG-scaling 1.6

Measured number of events, efficiency:

- Background subtraction + efficiency typically include (outdated?) models + depend on SM vs BSM
  - ➥ Can reweighting correct correct for this?

B production:

- LHCb: f_u/f_d relative production fractions, absolute normalization unfeasible. $f_u/f_d = 1$??
- B factories: $N_{\Upsilon(4S)}$ measured, requires sub-threshold runs Theoretical assumptions entering? $f_{0,\pm}$: $\Upsilon(4S)$ BRs, $\sigma_{\rm rel}(f_{0,\pm}) \approx 1.5\%$, depends on assumptions

➥ This is something I want to discuss in more detail

Production fractions at the B factories

To get an absolute BR, number of decaying B's has to be known

➥ From $N_{\Upsilon(4S)}$ typically, double-tagging possible

$\Upsilon(4S) \to B\bar{B}$ decays:

- Naively: $R^{\pm 0} \equiv \frac{BR(\Upsilon(4S)\to B^+B^-)}{BR(\Upsilon(4S)\to B^0\bar{B}^0)} \overset{\text{Isospin}}{=} 1 \overset{f_{\not{B}}=0}{=} \frac{1/2}{1/2}$
- However: close to threshold $\to$ sizable isospin breaking!
  Phase space: $R^{\pm 0}_{\rm PS} = 1.048$
  Naive Coulomb enhancement: $R^{\pm 0}_{\rm CE} = 1.20$!?
  [Atwood/Marciano,Lepage'90]
  ➥ More detailed calculations: still (too) large
  [Byers+'90,Kaiser+'03,Voloshin+'03'04,Dubynski+'07,Milstein'21]
- Υ decays in to non-$B\bar{B}$ states: observed ($f_{\not{B}} > 0.264\%$)
  ➥ Uncertainty? CLEO: $f_{\not{B}} = (-0.11 \pm 1.43 \pm 1.07)\%$
  With $f_{\not{B}} \neq 0$, $R^{\pm 0}$ not sufficient for $f_{00,\pm}$!
- $R^{\pm 0}_{\rm HFLAV} = 1.058 \pm 0.024$: sizable, not huge
  Note: PDG averages ignore this largely!

Stops you from knowing any B BR to better than $1 - 2\%$!

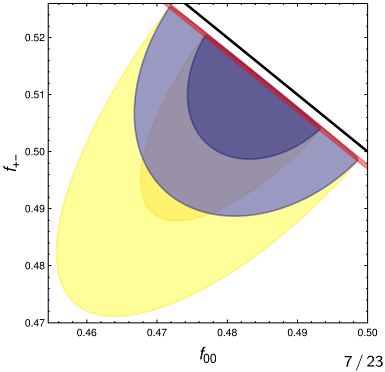
How is this measured? [MJ'12,Bernlochner/MJ+'23,HFLAV]

$R^{\pm 0}$	Method	Comment	Reference
1.047(44)(36)	Single vs. double-tag	Uses $f_{\mathcal{B}}$, see text	[10, 16, 17]
1.039(31)(50)	$B \to X_c \ell\nu$	Assumes negligible isospin violation	[18, 19]
1.068(32)(20)(21)	$B \to X_s \gamma$	Third uncertainty due to resolved photon contributions	[20]
1.055(30)		Average categories I and II	
1.065(12)(19)(32)	$B \to J/\psi K$	Third uncertainty due to isospin violation in $B \to J/\psi K$	[21, 22]
1.013(36)(27)(30)	$B \to J/\psi K$	Third uncertainty due to isospin violation in $B \to J/\psi K$	[23]
1.100(35)(35)(33)	$B \to J/\psi(ee)K$	Third uncertainty due to isospin violation in $B \to J/\psi K$	[24]
1.066(32)(34)(32)	$B \to J/\psi(\mu\mu)K$	Systematic uncertainties $\sim 100\%$ correlated with ee mode	[24]
1.060(18)(32)		Average for $B \to J/\psi K$	
1.057(23)		**Average of all categories I–III**	

Problem: separate production and decay

Three main methods:

- I Single vs. double-tag [MARK-II]
  - Independent of decay mode
- II "Known" ratios
  - Suppression beyond isospin
- III (Quasi-)Isospin assumptions
  - Uncertainty?
- Desirable: precision, FS-independent



Can we do better? [Bernlochner/Jung/Khan/Landsberg/Ligeti'23]

Observation: $R^{\pm 0}$ compatible with phase-space enhancement, only!

➥ Additional enhancement at most few %

Idea: use B production @$\Upsilon(5S)$

➥ $R^{\pm 0}_{\rm PS} \simeq 1$, $R^{\pm 0}_{\rm CE}(\Upsilon(5S)) \approx \frac{1}{4} R^{\pm 0}_{\rm CE}(\Upsilon(4S)) \longrightarrow R^{\pm 0}(\Upsilon(5S)) \approx 1$

Proposal: measure double-ratios for final states f, f':

$$r(f,f') \equiv \left[\frac{N(B^+ \to f)}{N(B^0 \to f')}\right]_{\Upsilon(4S)} \Big/ \left[\frac{N(B^+ \to f)}{N(B^0 \to f')}\right]_{\Upsilon(5S)} \approx R^{\pm 0}(\Upsilon(4S))$$

- Independent of isospin violation in the final state!

➥ Can choose most convenient states f, f', even completely unrelated states, no isospin necessary

	Belle	Belle II partial	Belle II full
$\mathcal{L}_{\Upsilon(5S)}$ / $\mathcal{L}_{\Upsilon(4S)}$ [ab^{-1}/ab^{-1}]	0.12 / 0.71	0.5 / 5	5 / 50
$N^{\Upsilon(5S)}_{B^{(*)}B^{(*)}}$ / $N^{\Upsilon(4S)}_{BB}$	2.74×10^7 / 7.72×10^8	1.13×10^8 / 5.55×10^9	1.13×10^9 / 5.55×10^{10}
$f,\ f'$		$\Delta r(f,f')/r(f,f')$	
$J/\psi K^+,\ J/\psi K^0$	7.1%	3.5%	1.1%
$\bar{D}^0\pi^+,\ D^-\pi^+$	2.4%	1.2%	0.4%
$\bar{D}^{*0}\ell^+\nu,\ D^{*-}\ell^+\nu$	4.5%	2.2%	0.7%
$\bar{D}^0\pi^+,\ D^{*-}\ell^+\nu$	1.8%	0.9%	0.3%

Theoretical expression for the differential decay rate

Four-fold differential rate for $B \to D^*(\to D\pi)\ell\nu$ (P-wave) given as [Duraisamy+'14, also Ivanov+'16]

$$\begin{aligned}\frac{8\pi}{3}\frac{d^4\Gamma^{(l)}}{dq^2 d\cos\theta_l d\cos\theta_D d\chi} &= \left(J_{1s}^{(l)} + J_{2s}^{(l)}\cos 2\theta_l + J_{6s}^{(l)}\cos\theta_l\right)\sin^2\theta_D \\ &+ \left(J_{1c}^{(l)} + J_{2c}^{(l)}\cos 2\theta_l + J_{6c}^{(l)}\cos\theta_l\right)\cos^2\theta_D \\ &+ \left(J_3^{(l)}\cos 2\chi + J_9^{(l)}\sin 2\chi\right)\sin^2\theta_D\sin^2\theta_l \\ &+ \left(J_4^{(l)}\cos\chi + J_8^{(l)}\sin\chi\right)\sin 2\theta_D\sin 2\theta_l \\ &+ \left(J_5^{(l)}\cos\chi + J_7^{(l)}\sin\chi\right)\sin 2\theta_D\sin\theta_l\end{aligned}$$

- This expression is valid including any heavy BSM physics
- $J_i^{(l)}$ are q^2-dependent functions $\to$ numbers after integration
- $J_{7,8,9}^{(l)}$ change sign under CP

Only CP-averaged measurements available $\to$ use $S_i^{(l)} = \frac{J_i^{(l)} + \bar{J}_i^{(l)}}{\Gamma^{(l)} + \bar{\Gamma}^{(l)}}$

➡ $S_{7,8,9}^{(l)} = 0$, even beyond the SM [BBGJvD'21]

➡ Only 4 observables in single-differential distributions!

Sensitivity to BSM physics [Bobeth/Bordone/Gubernari/MJ/vanDyk'21]

4 effective operators in $B \to D^* \ell \nu \overset{?}{\to} 4 \times 2 = 8$ parameters?

- Clearly not, at least 1 phase always unobservable
- Sensitivity only to bilinears: $\mathrm{Re}(C_i C_j^*)$, $\mathrm{Im}(C_i C_j^*)$, $|C_i|^2$
- $m_\ell \to 0$: P-T and V-A sectors decouple
- relations among $J_i^{(l)}$ [Algueró+'20]

Observable	$\|C_A\|^2$	$\|C_V\|^2$	$\|C_P\|^2$	$\|C_T\|^2$	$\mathrm{Re}(C_A C_V^*)$	$\mathrm{Re}(C_A C_P^*)$	$\mathrm{Re}(C_A C_T^*)$	$\mathrm{Re}(C_V C_P^*)$	$\mathrm{Re}(C_V C_T^*)$	$\mathrm{Re}(C_P C_T^*)$
$J_{1c} = V_1^0$	✓	–	✓	✓	–	(m)	(m)	–	–	–
$J_{1s} = V_1^T$	✓	✓	–	✓	–	–	(m)	–	(m)	–
$J_{2c} = V_2^0$	✓	–	–	✓	–	–	–	–	–	–
$J_{2s} = V_2^T$	✓	✓	–	✓	–	–	–	–	–	–
$J_3 = V_4^T$	✓	✓	–	✓	–	–	–	–	–	–
$J_4 = V_1^{0T}$	✓	–	–	✓	–	–	–	–	–	–
$J_5 = V_2^{0T}$	(m^2)	–	–	(m^2)	✓	(m)	(m)	–	(m)	✓
$J_{6c} = V_3^0$	(m^2)	–	–	–	–	(m)	(m)	–	–	✓
$J_{6s} = V_3^T$	–	–	–	(m^2)	✓	–	(m)	–	(m)	–
$d\Gamma/dq^2$	✓	✓	✓	✓	–	(m)	(m)	–	(m)	–
$\mathrm{num}(A_{\mathrm{FB}})$	(m^2)	–	–	(m^2)	✓	(m)	(m)	–	(m)	✓
$\mathrm{num}(F_L)$	✓	–	✓	✓	–	(m)	(m)	–	–	–
$\mathrm{num}(F_L\text{-}1/3)$	✓	✓	✓	✓	–	(m)	(m)	–	(m)	–
$\mathrm{num}(\widetilde{F}_L)$	✓	(m^2)	✓	✓	–	(m)	(m)	–	(m)	–
$\mathrm{num}(\widetilde{F}_L\text{-}1/3)$	✓	✓	–	✓	–	–	–	–	–	–
$\mathrm{num}(S_3)$	✓	✓	–	✓	–	–	–	–	–	–
Observable	–	–	–	–	$\mathrm{Im}(C_A C_V^*)$	$\mathrm{Im}(C_A C_P^*)$	$\mathrm{Im}(C_A C_T^*)$	$\mathrm{Im}(C_V C_P^*)$	$\mathrm{Im}(C_V C_T^*)$	$\mathrm{Im}(C_P C_T^*)$
$J_7 = V_3^{0T}$					(m^2)	–	(m)	(m)	–	✓
$J_8 = V_4^{0T}$					✓	–	–	–	–	–
$J_9 = V_5^T$					✓	–	–	–	–	–

Consistency of experimental data [Gambino/MJ/Schacht, in prep.]

This allows to compare measurements without FF input:

$$\Sigma X = \frac{X^e + X^\mu}{2}\,, \quad \Delta X = X^\mu - X^e\,, \quad \delta X = X_{\rm hi} - X_{\rm lo}\,.$$

Measurement	Belle-II 23a [3]	Belle-II 23b [4]	Belle 23a [1]	Belle 23b [2]	Belle 18 [5]
χ^2/dof	7.5/16	53/42	113/118	118/118	48/52
Observable					
$\Sigma A_{\rm FB,tot}$	0.171(23)	0.189(19)	0.238(11)	0.244(14)	0.212(5)
$\Sigma S_{3,\rm tot}$	-0.130(29)	-0.141(8)	-0.126(22)	-0.126(24)	-0.139(6)
$\Sigma S_{5,\rm tot}$	0.173(25)	—	—	0.177(20)	—
$\Sigma F_{\rm L,tot}$	—	0.524(8)	0.500(13)	0.530(18)	0.5302(35)
$\Sigma \tilde{F}_{\rm L,tot}$	—	0.515(20)	0.523(20)	0.514(23)	0.543(7)
$\Delta A_{\rm FB,tot}$	-0.03(5)	-0.020(22)	-0.002(22)	0.020(27)	0.035(9)
$\Delta S_{3,\rm tot}$	-0.08(6)	-0.023(17)	-0.04(4)	-0.04(5)	-0.013(11)
$\Delta S_{5,\rm tot}$	-0.03(5)	—	—	0.04(4)	—
$\Delta F_{\rm L,tot}$	—	0.007(9)	0.027(25)	0.02(4)	-0.006(6)
$\Delta \tilde{F}_{\rm L,tot}$	—	-0.015(28)	0.001(38)	-0.02(5)	-0.011(14)
$\Delta\delta A_{\rm FB}$	-0.14(6)	—	—	0.04(6)§	—
$\Delta\delta \tilde{F}_{\rm L}$	—	—	—	0.28(10)§	—

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q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points
- ➥ Knowledge of functional dependence on q^2 crucial
- This is where discussions start...
- ➥ Most $B \to D^*$ data not usable due to model dependence!

Give as much information as possible independently of this choice!

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Even with FF-model-dependent data:

Consistent HFLAV $B \to D^*$ fit in CLN
➥ Experimental w-dependence well established!

In the following: mostly BGL and HQE ($\to$ CLN) parametrizations

Generalized Unitairty constraints [Gambino/MJ/Schacht preliminary]

Problem in BGL for $B \to M$ transition: cuts below $t_+ = (M_B + M_M)^2$

➥ In $B \to D^*$: $(M_{B_c} + 2M_\pi)^2 < t_+^{B\to D^*}$

Already discussed by BGL: model yields small effect

➥ Still true by today's standards?

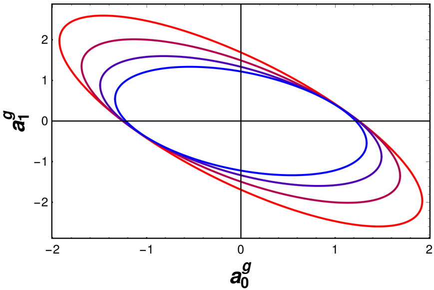
GUCs model-independent approach to address this issue [Gubernari+'20]

[also Blake+'22,Flynn+'23, Bordone+'24 talks by Florian and Tobias]

Lower threshold $\to$ integration only over part of the unit circle

➥ Monomials in z not orthogonal anymore!

Treatment [Flynn+'23] : non-diagonal unitarity constraints. Convergence?



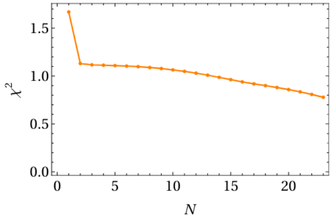
Unitarity only:
(blue→red N=1...4)

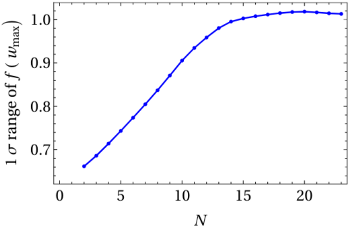
- Adding higher orders in z affects low orders
- Convergence should be guaranteed, but where?

Generalized Unitairty constraints II [Gambino/MJ/Schacht preliminary]

Lattice-only fit example: Fitting JLQCD FFs at varying order N

➥ With "standard" BGL saturation at $N = 3$





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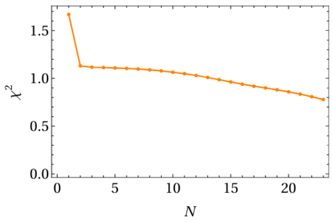
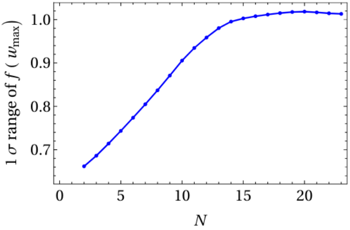
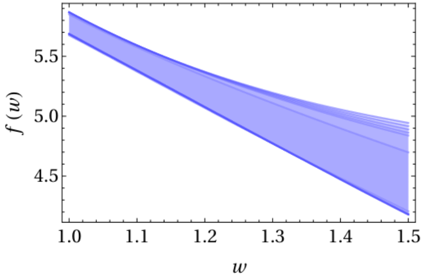




N=1...6:



- Very late convergence
- May change w/ data relevant for V_{cb}?
  ➥ under investigation
- This case: convergence to "standard" BGL
  ➥ Not always true

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  - ➥ Parameter reduction, necessary for NP analyses!

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  - ➥ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
➥ Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
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Solution: Include systematically $1/m_c^2$ corrections
[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] , using [Falk/Neubert'92]
[Bernlochner+'22] : model for $1/m_c^2$ corrections $\to$ fewer parameters

Theory determination of $b \to c$ Form Factors

[Bordone/MJ/vanDyk'19,Bordone/Gubernari/MJ/vanDyk'20]

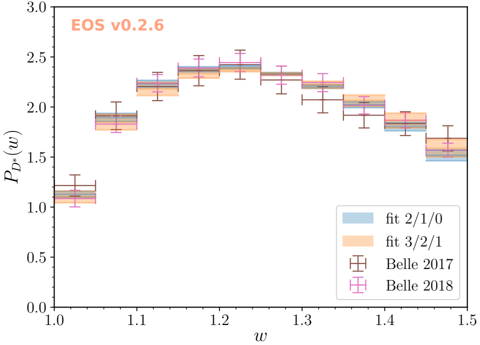
For general NP analysis, FF shapes needed from theory!

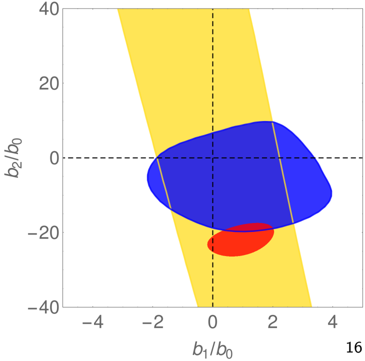
Fit to all $B \to D^{(*)}$ FFs, using lattice, LCSR, QCDSR and unitarity

[CLN,BGL,HPQCD'15'17,FNAL/MILC'14'15,Gubernari+'18,Ligeti+'92'93]

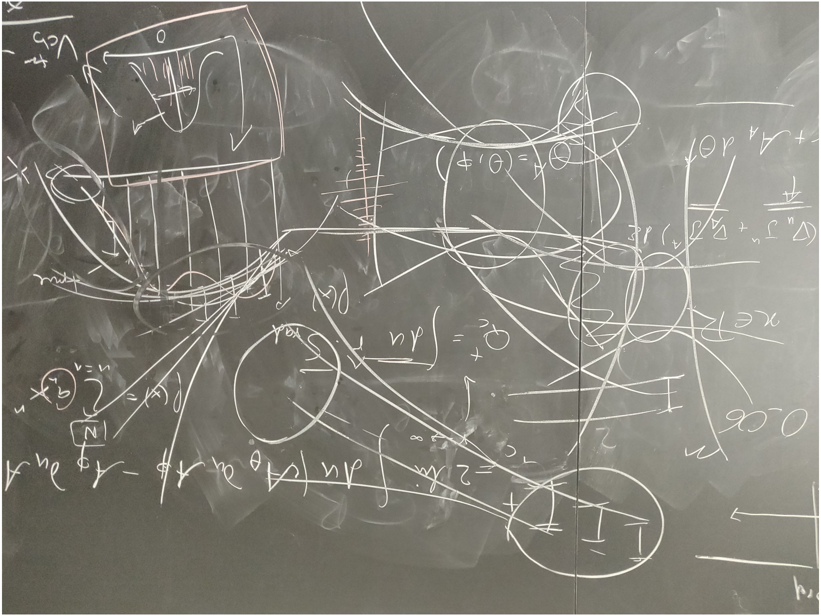
k/l/m: order in z for leading/subleading/subsubleading IW functions

- 2/1/0 works, but only 3/2/1 captures uncertainties
- Consistent V_{cb} value from Belle'17+'18
- Predictions for diff. rates, perfectly confirmed by data





Form-factor truncation



Form-factor truncation

Key question: Where do we truncate our expansions?

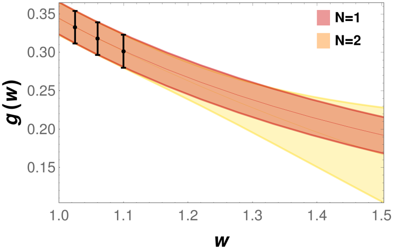
- A [Bernlochner+'19] : include parameter only if χ^2 decreases significantly
- B (GJS, BGJvD): include one "unnecessary" order

Comments:

- Large difference, $\sim 50\%$ difference in uncertainty
- Motivation for A: convergence, avoid overfitting
- Motivation for B: avoid underestimating uncertainties
- Different perspectives: only describing data, A is ok.
  However: we extrapolate to regions where we lack sensitivity

Example: $g(w)$ from FNAL/MILC

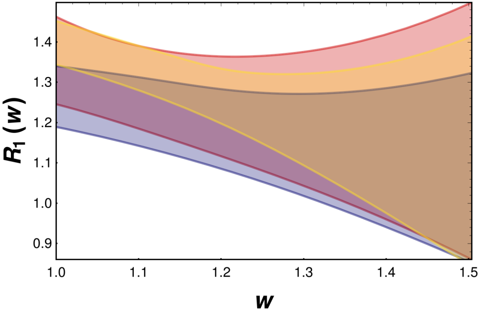
- perfect description at $\mathcal{O}(z)$
- large impact from $\mathcal{O}(z^2)$
- Nevertheless: $\mathcal{O}(z^2) \leq 6\% \times \mathcal{O}(z)$
  - overfitting limited



Just because you're not sensitive, doesn't mean it's not there!

Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$!



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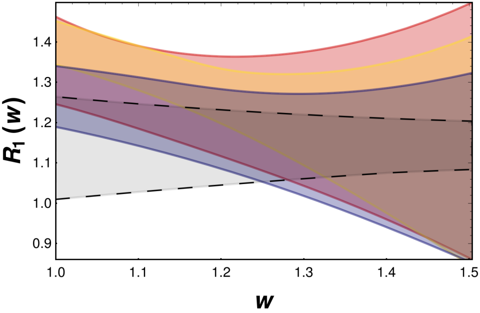

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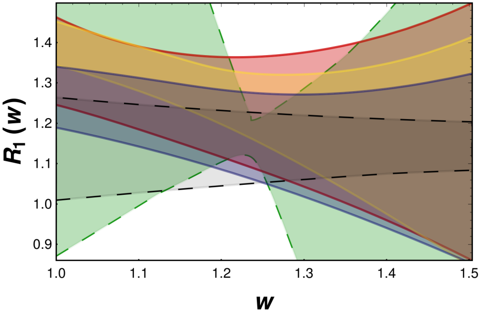

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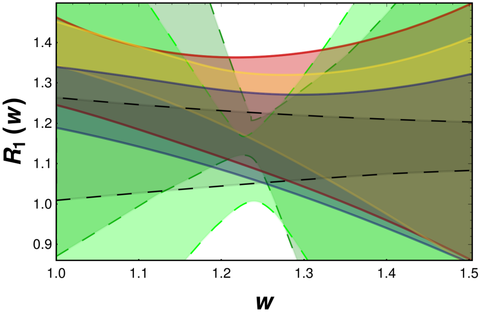

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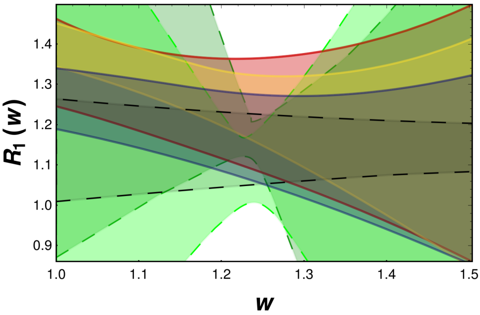

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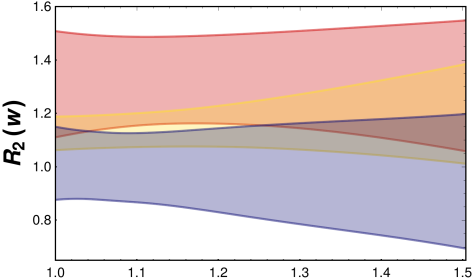

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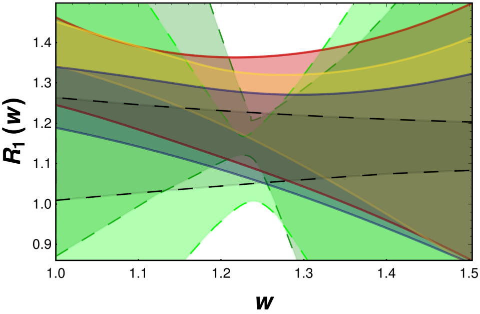

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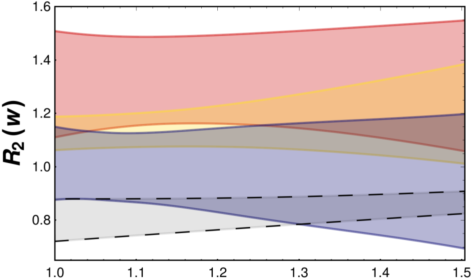

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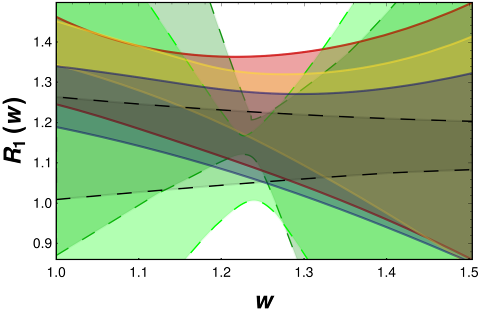

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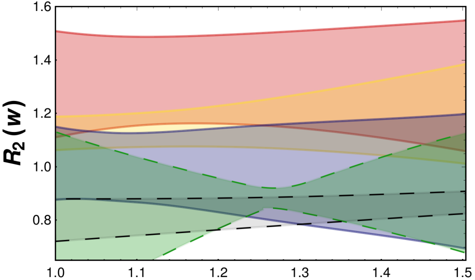

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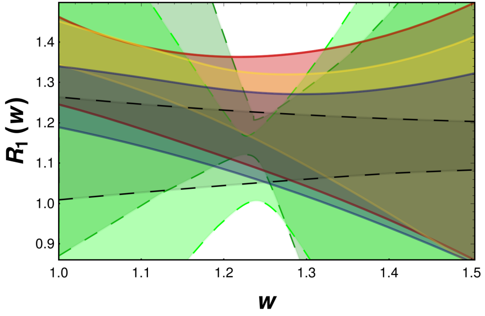

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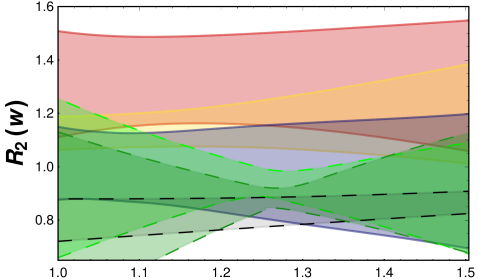

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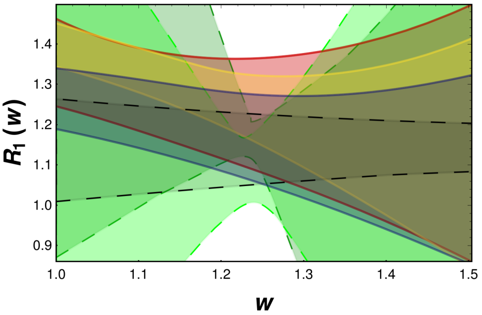

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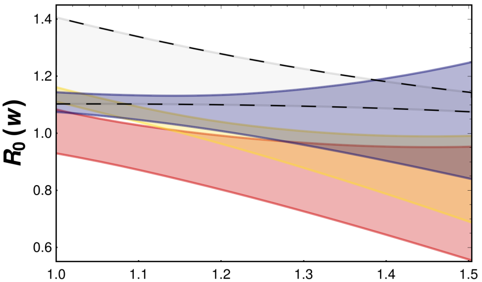

- FNAL/MILC'21
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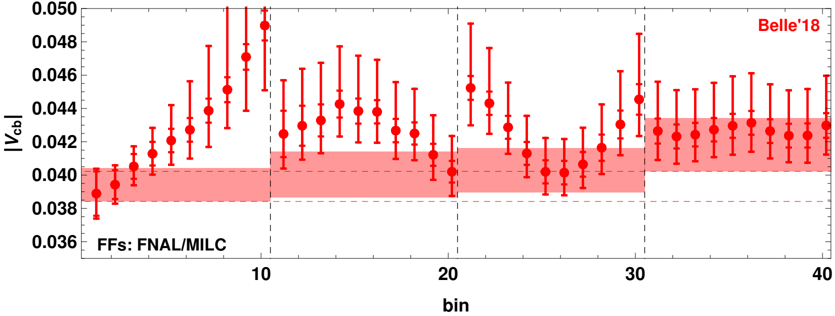
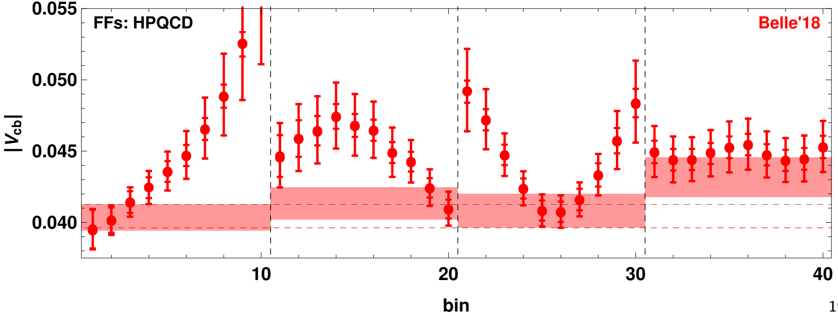
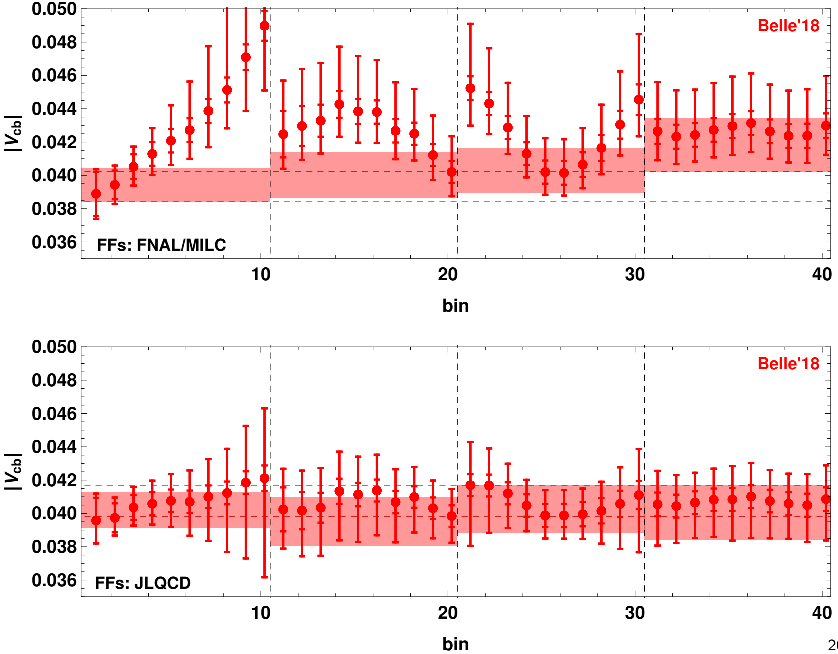
- HQE@$1/m_c^2$

Binned V_{cb} from Belle'18 data: FNAL/MILC vs HPQCD

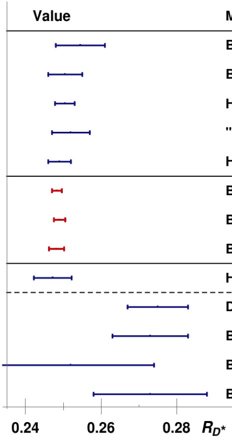




Binned V_{cb} from Belle'18 data: FNAL/MILC vs JLQCD



Overview over predictions for $R(D^*)$



Method	Input Theo	Input Exp	Reference
BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
"Average"			HFLAV'21
HQET_{RC}@$1/m^2,\alpha_s^{(2)}$	Belle'17'18	Lattice	Bernlochner et al.'22
BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
BGL	Lattice	Belle'18	JLQCD'23 (MJ)
BGL	Lattice	Belle'18	Davies, Harrison'23
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
DM	Lattice	---	Martinelli et al.
BGL	Lattice	---	Vaquero et al.'21v2
BGL	Lattice	---	JLQCD'23
BGL	Lattice	---	Davies, Harrison'23

Lattice $B \to D^*$: $h_{A_1}(w=1)$ [FNAL/MILC'14,HPQCD'17] , [FNAL/MILC'21]
Other lattice: $f_{+,0}^{B\to D}(q^2)$ [FNAL/MILC,HPQCD'15]
QCDSR: [Ligeti/Neubert/Nir'93,'94] , LCSR: [Gubernari/Kokulu/vDyk'18]

Overall consistent SM predictions!
"Explaining" $R(D^*)$ by FM/HPQCD $\to$ NP in $B \to D^*(e,\mu)\nu$!

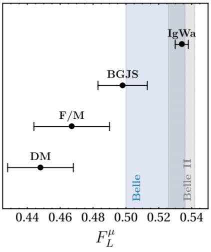
Can we resolve the $R(D^*)$ puzzle with different FFs?

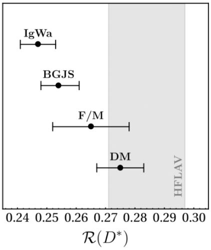
Rewriting $R(D^*)$: [Bigi/Gambino/Schacht'17]

$$R(D^*) = \underbrace{R_{\tau,1}}_{\text{determined by } d\Gamma/dw|_\ell} + \underbrace{R_{\tau,2}}_{\sim m_\tau^2 F_2^2, \sim R_{\tau,1}/10}$$

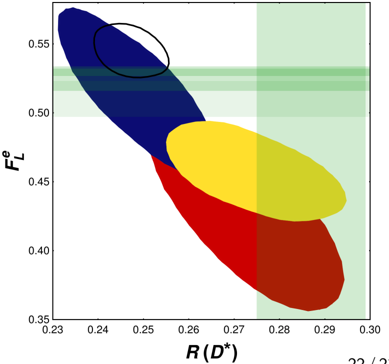
- 0.25 $\to\sim$ 0.27 (FNAL/MILC,HPQCD) $\Leftrightarrow$ 100% correction to $R_{\tau,2}$!
- $R(D^*)$ prediction to 90% "measurable"

  More specifically: strong correlation between F_L^e and $R(D^*)$:





[Fedele+'23]



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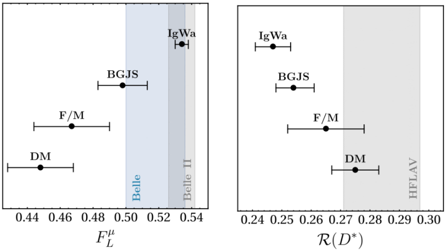

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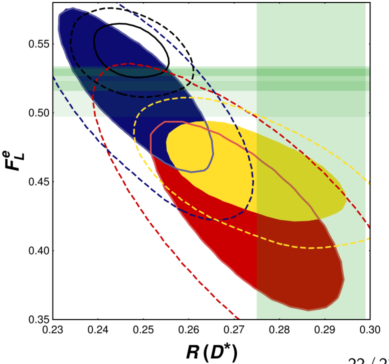

Conclusions

We have work ahead of us!

1. Need to control all inputs to very good precision
   - ➥ Proposed new method(s) to determine B production
2. q^2 dependence of FFs critical
   - ➥ Need parametrization-independent data
3. Inclusion of higher-order (theory) uncertainties essential
   - ➥ Affects a lot of subfits
4. HQE: systematic expansion in $1/m, \alpha_s$, relates FFs
   - ➥ $\mathcal{O}(1/m_c)$ ($\to$ CLN) not sufficient anymore
5. Important LQCD analyses in $B_{(s)} \to D^*_{(s)}$ @ finite recoil

➥ Agreement for f, g tensions in ratios ($F_{1,2}$) – correlations?

6. Despite complications: $R(D^{(*)})$ SM prediction robust!

Central lesson:
Experiment and theory (lattice + pheno) need to work closely together!

Exclusive decays: Form factors

In exclusive decays, hadronic information encoded in Form Factors
They parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks)
vs.
Experiment with hadrons

$$\left\langle D_q(p')|\bar{c}\gamma^\mu b|\bar{B}_q(p)\right\rangle = (p+p')^\mu f_+^q(q^2) + (p-p')^\mu f_-^q(q^2)\,,\; q^2 = (p-p')^2$$

Most general matrix element parametrization, given symmetries:
Lorentz symmetry plus P- and T-symmetry of QCD
$f_\pm(q^2)$: real, scalar functions of one kinematic variable

How to obtain these functions?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...)
  Precision?
- Measurable e.g. in semileptonic transitions
  Normalization? Suppressed FFs? NP?

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD symmetries (unitarity, crossing)
   ➥ dispersion relation
4. Calculate partonic part (mostly) perturbatively

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Result: Model-independent parametrization

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t, t_0)]^n \,.$$

- a_n: real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_+
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$

Series in z with bounded coefficients (each $|a_n| \leq 1$)!
➥ Uncertainty related to truncation is calculable!

$B \to D\ell\nu$

$B \to D\ell\nu$, aka "The teacher's pet":

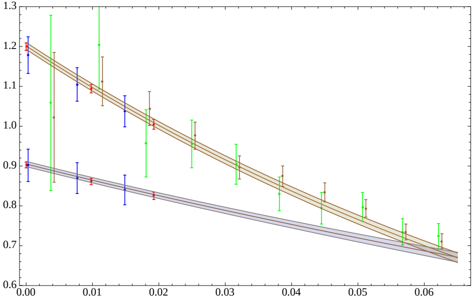
- Excellent agreement between experiments [BaBar'09,Belle'16]
- Excellent agreement between two lattice determinations [FNAL/MILC'15,HPQCD'16]

➥ Lattice data inconsistent with CLN parametrization! (but consistent w/ HQE@1/m, discussed later)

- BGL fit [Bigi/Gambino'16] :

$$R(D) = 0.299(3)\,.$$

See also [Jaiswal+,Berlochner+'17,MJ/Straub'18,Bordone/MJ/vanDyk'19]



$f_{+,0}(z)$, inputs:

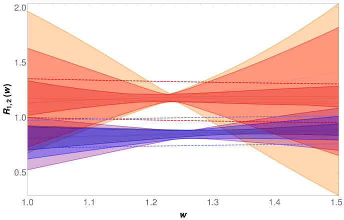
- FNAL/MILC'15
- HPQCD'16
- BaBar'09
- Belle'16

$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'18(+'17) provide FF-independent data for 4 single-differential rates

BGL analysis:

- Datasets compatible
- d'Agostini bias + syst. important
- Expand FFs to z^2
  ➥ 50% increased uncertainties



- Belle'18: no parametrization dependence
- Belle'17 never published → replace w/ Belle'23, not available yet
- Tension w/ inclusive reduced, but not removed

$$R(D^*) = 0.253^{+0.007}_{-0.006} \quad \text{(including LCSR point)}$$

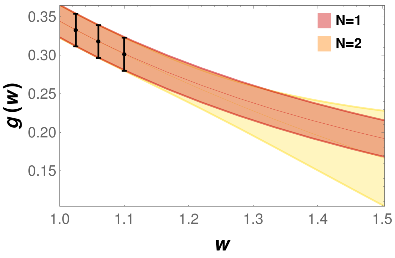
Comparison to Bernlochner+'22

Bernlochner et al. also perform HQE analysis @$1/m_c^2$. Differences:

- Postulate different counting within HQET
  - ➥ Highly constraining model for higher-order corrections
- Avoid use of LCSR (and mostly QCDSR) results
- Include partial α_s^2 corrections
- Include FNAL/MILC results partially
- Expansion in z: 2/1/0 (justified in [Bernlochner+'19])

Observations:

- $1/m_c^2$ corrections necessary
- Overall small uncertainties
- $V_{cb} = (38.7 \pm 0.6) \times 10^{-3}$
  - ➥ smaller due to larger $\mathcal{F}(1)$
- $R(D^*)$: agreement w/ BGJvD
- $R(D) \sim 3\sigma$ from GJS + BGJvD
  - ➥ In my opinion due to model

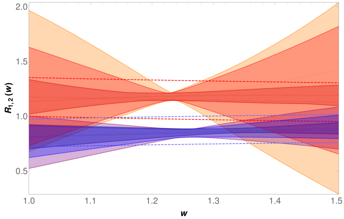


$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'17+'18 provide FF-independent data for 4 single-differential rates

Analysis of these data with BGL form factors:

- Datasets roughly compatible
- d'Agostini bias + syst. important
- All FFs to z^2 to include uncertainties
  ➥ 50% increased uncertainties
- 2018: no parametrization dependence



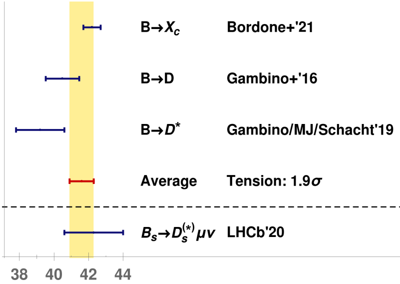
$$|V_{cb}^{D^*}| = 39.6^{+1.1}_{-1.0}\,[39.2^{+1.4}_{-1.2}] \times 10^{-3}$$

$$R(D^*) = 0.254^{+0.007}_{-0.006}\,[0.253^{+0.007}_{-0.006}]$$

In brackets: 2018 only ($\Delta V_{cb}^{\mathrm{Belle}} = 0.9$)

Updating the $|V_{cb}|$ puzzle:

- Tension 1.9σ (larger $\delta V_{cb}^{B\to D^*}$)
- $B_s \to D_s^{(*)}$ reduces tension further
- $V_{cb}^{B\to D^*}$ vs. V_{cb}^{incl} still problematic



See also [Bigi+,Bernlocher+,Grinstein+'17,Jaiswal+'17'19,MJ/Straub'18,Bordone+'19/'20]

Theory determination of $b \to c$ Form Factors

SM: BGL fit to data + FF normalization $\to |V_{cb}|$

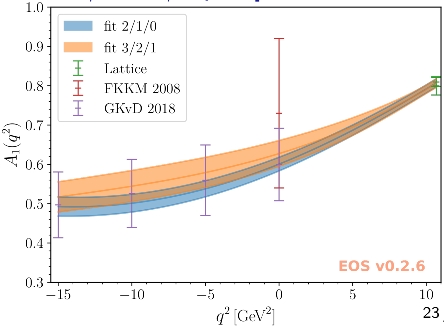
NP: can affect the q^2-dependence, introduces additional FFs

➥ To determine general NP, FF shapes needed from theory

[MJ/Straub'18,Bordone/MJ/vDyk'19] used all available theory input:

- Unitarity bounds (using results from [CLN, BGL])
  - ➥ non-trivial $1/m$ vs. z expansions
- LQCD for $f_{+,0}(q^2)$ $(B \to D)$, $h_{A_1}(q^2_{\max})$ $(B \to D^*)$
  [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod f_T) [Gubernari/Kokulu/vDyk'18]
- QCDSR results for $1/m$ IW functions [Ligeti+'92'93]
- HQET expansion to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$

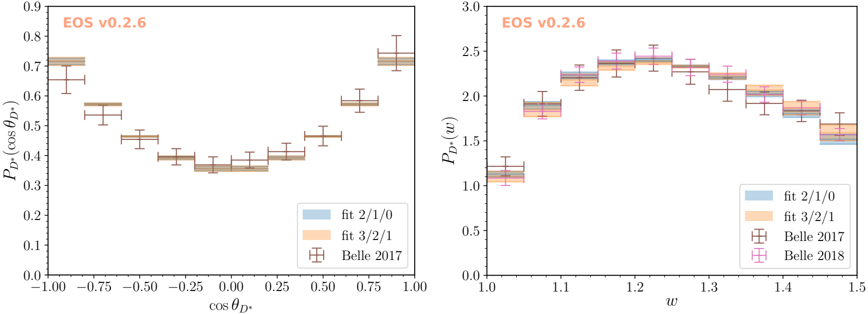
FFs under control;
$R(D^*) = 0.247(6)$
[Bordone/MJ/vDyk'19]



Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- Fits 3/2/1 and 2/1/0 are theory-only fits(!)
- $k/l/m$ denotes orders in z at $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- w-distribution yields information on FF shape $\to V_{cb}$
- Angular distributions more strongly constrained by theory, only

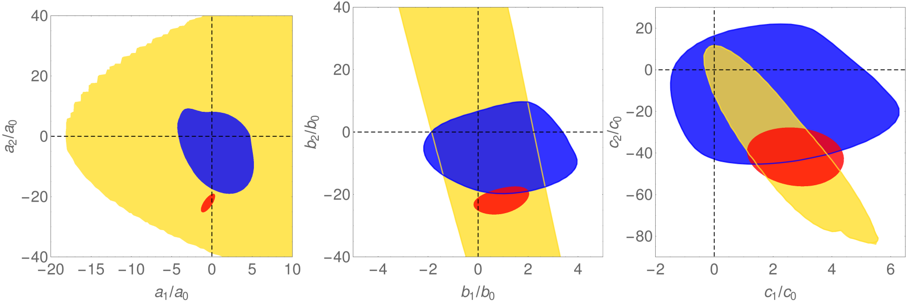
➥ Predicted shapes perfectly confirmed by $B \to D^{(*)}\ell\nu$ data

➥ V_{cb} from Belle'17 compatible between HQE and BGL!

Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- $B \to D^*$ BGL coefficient ratios from:
  1. Data (Belle'17+'18) + weak unitarity (yellow)
  2. HQE theory fit 2/1/0 (red)
  3. HQE theory fit 3/2/1 (blue)

➥ Again compatibility of theory with data

➥ 2/1/0 underestimates the uncertainties massively

➥ For b_i, c_i ($\to f, \mathcal{F}_1$) data and theory complementary

Application: Flavour universality in $B \to D^*(e, \mu)\nu$

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

So far: Belle'18 data used in SM fits, flavour-averaged

However: Bins 40 × 40 covariances given separately for $\ell = e, \mu$

- Belle'18: $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$
- What can we learn about flavour-non-universality? → 2 issues:

  1. $e - \mu$ correlations not given, but constructible from Belle'18
  2. 3 bins linearly dependent, but covariances not singular

Two-step analysis:

1. 2 × 4 angular observables suffice for 2 × 30 angular bins
   - Model-independent description including NP!
2. Compare with SM predictions, using FFs@$1/m_c^2$ [Bordone+'19]
   - $\sim 4\sigma$ discrepancy in $\Delta A_{\rm FB} = A_{\rm FB}^\mu - A_{\rm FB}^e$