“$B \to D^* \ell \nu$ semileptonic decays in lattice QCD”

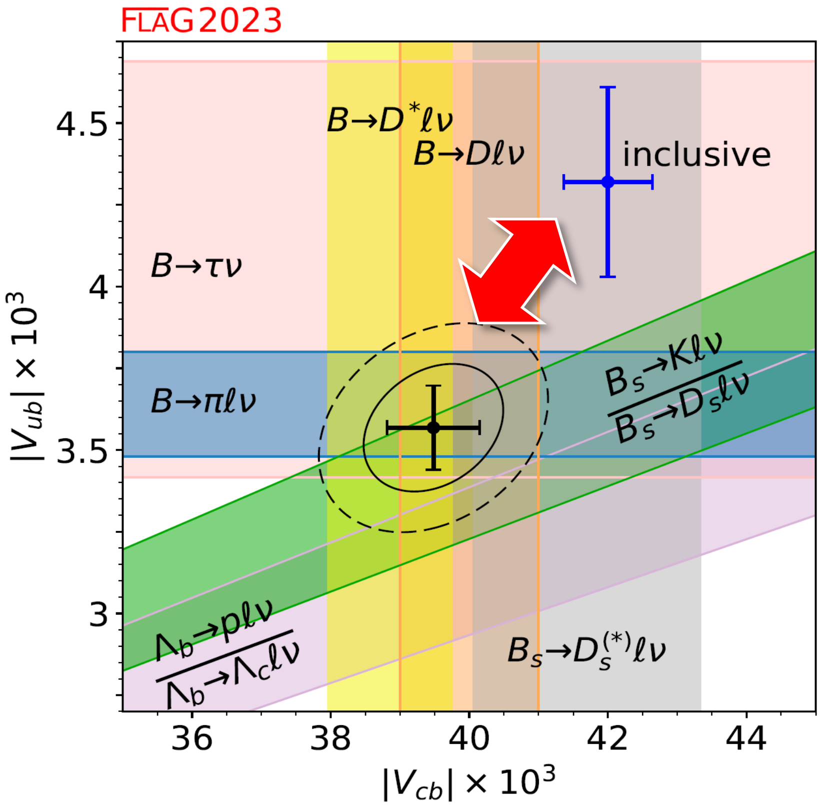
KEK, SOKENDAI Takashi Kaneko

Lattice meets Continuum @ University of Siegen, Sep 30 – Oct 3, 2024

$B \to D^* \ell \nu$ & lattice QCD

$\ell = e, \mu$ modes

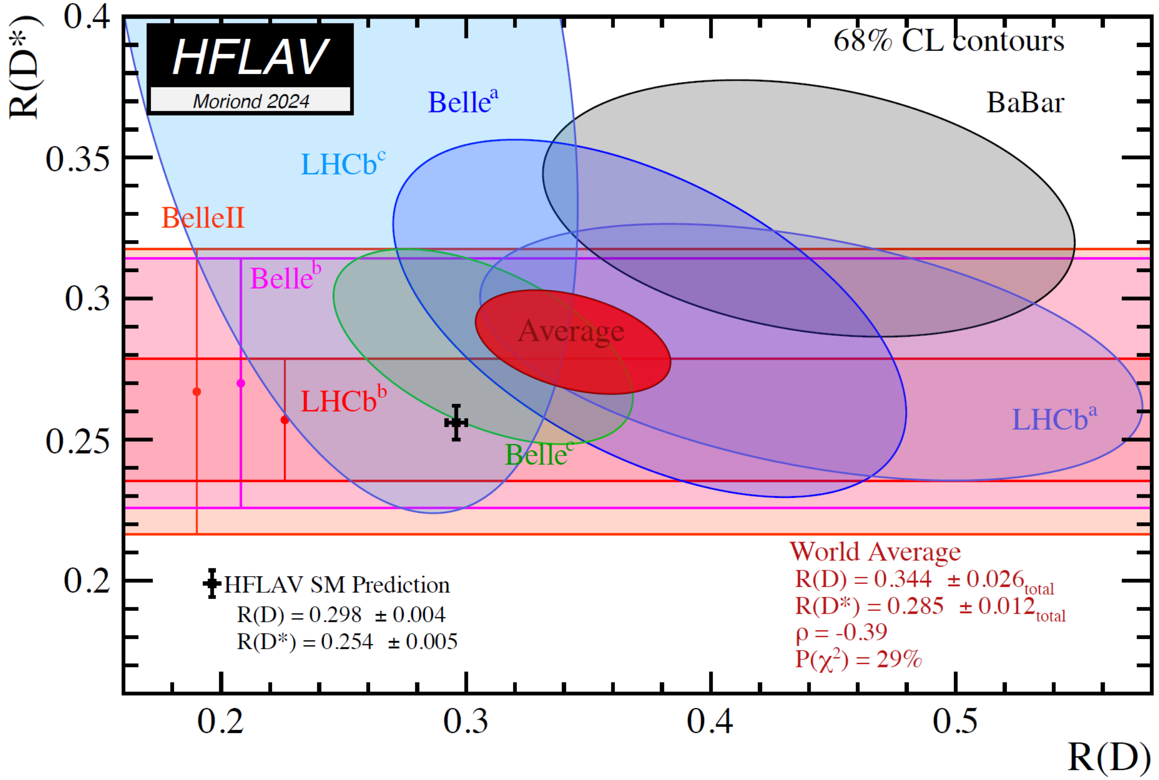
– determination of $|V_{cb}|$



– ≥ 8%, 3σ tension w/ inclusive decay

semitauonic mode $\ell = \tau$

– hint of NP thru LF universality violation



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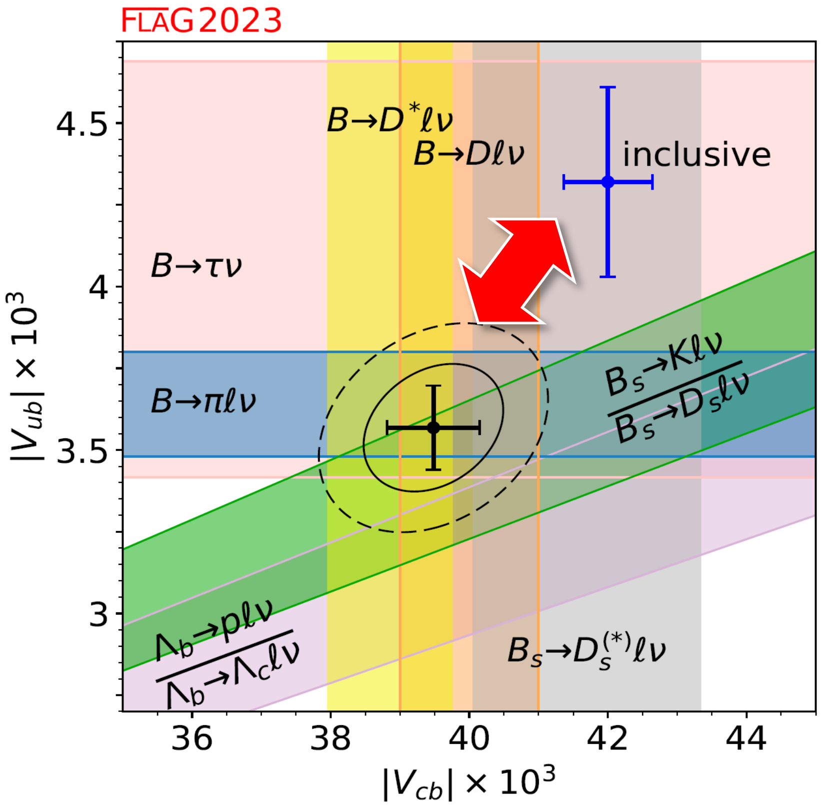

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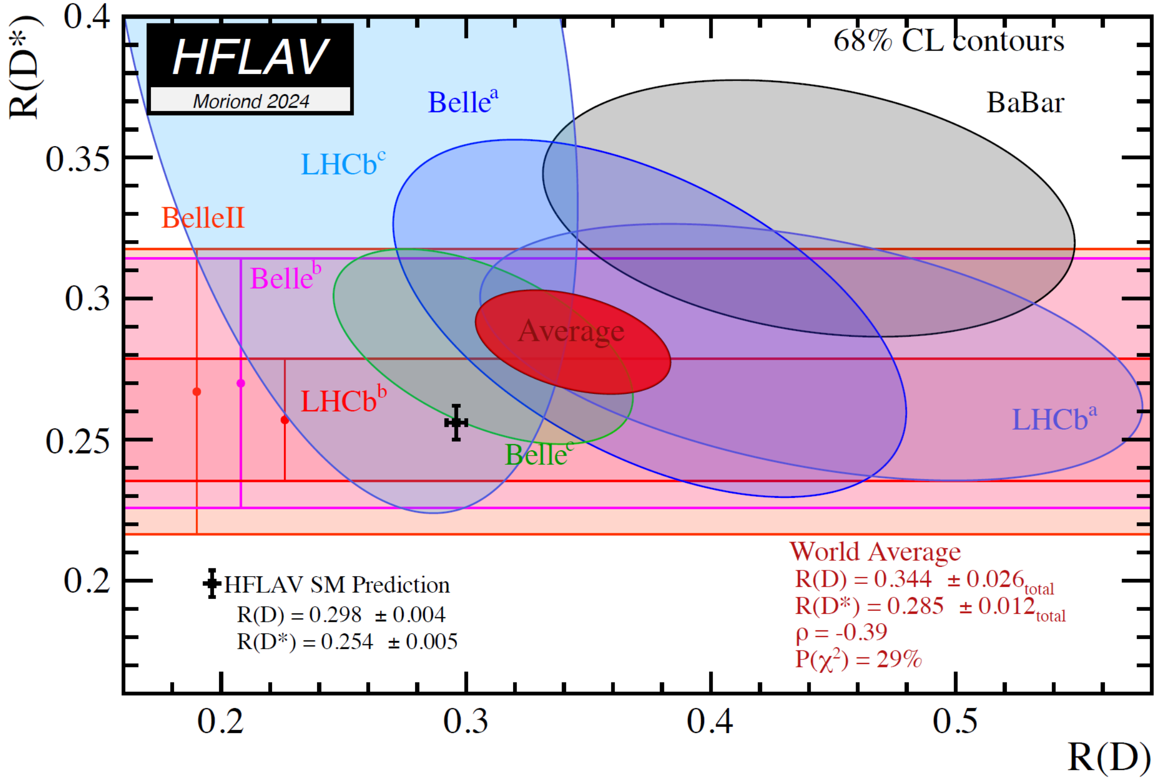


– for pure SM estimates of $R(D_*)$

lattice QCD – all relevant form factors (FFs)

this talk

"cover $B \to D^* \ell \nu$ from the perspectives of your own collaboration"

JLQCD data in some technical/physics details + some comparisons

- calculation of FFs @ simulation points
- extrapolation to the real world
- parameterization of FF data
- [$|V_{cb}|$ and $R(D^*)$]

see talks by Alex Vaquero, Martin Jung and Raynette van Tonder

for lattice, phenomenology, experiment status

calculation of FFs @ simulation points

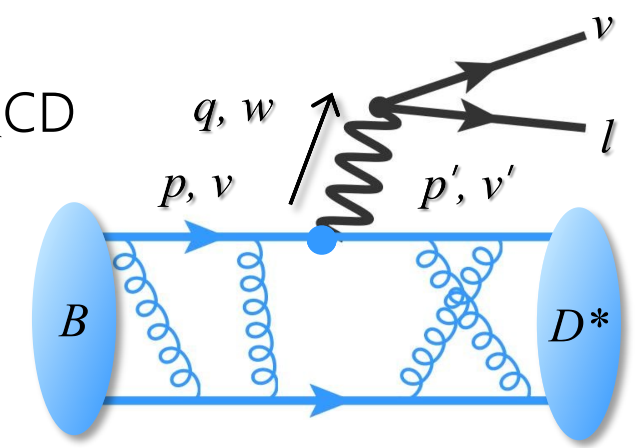
$B \to D^* \ell \nu$ form factors

in "heavy quark" convention

w/ NR normalization, v's instead of p's $\Rightarrow$ less sensitive to m_Q $\Rightarrow$ lattice QCD

$$\langle D^*(p',\varepsilon')|V_\mu|B(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\varepsilon'^{\nu} v'^{\rho} v^{\sigma} h_V(w)$$

$$\langle D^*(p',\varepsilon')|A_\mu|B(p)\rangle = \varepsilon'_\mu (w+1) h_{A_1}(w) - \varepsilon' v \{ v_\mu h_{A_2}(w) + v'_\mu h_{A_3}(w) \}$$



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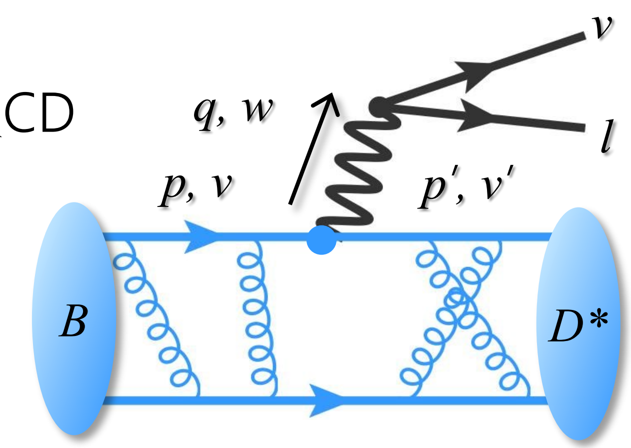

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before ‘21

- minimal input to determine $|V_{cb}|$ had been calculated : only h_{A1} at w=1
- other FFs [$h_{A1}(w)$ @ $w \neq 1$, $h_{A2}(w)$, $h_{A3}(w)$, $h_V(w)$] from exp't $\Rightarrow$ previous $|V_{cb}|$, $R(D^*)$

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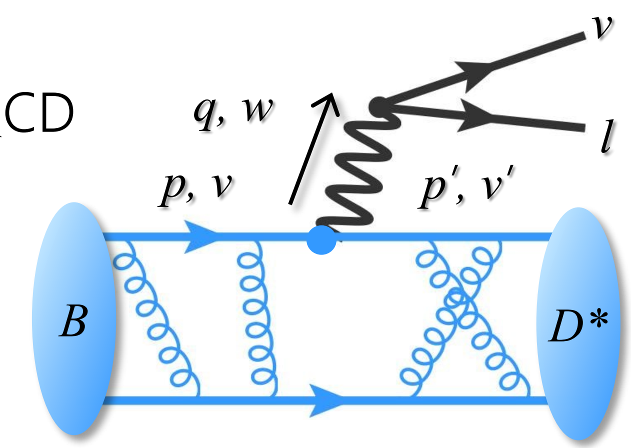

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3 collaborations recently calculated all FFs also at $w \neq 1$

Fermilab/MILC '21 : 1st calculation w/ EFT-based b quarks at physical $m_{b,\text{phys}}$

HPQCD '23 : computationally-inexpensive & relativistic b at unphysically small m_b

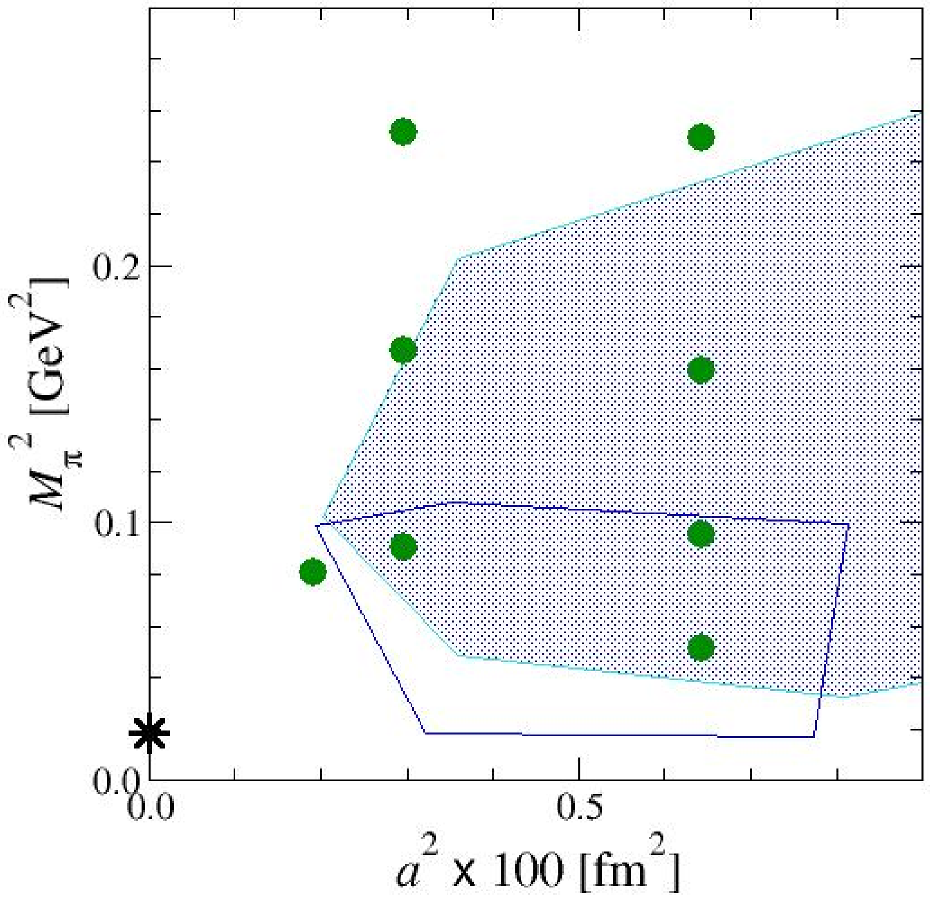
JLQCD '23 : theoretically clean & relativistic b at unphysically small m_b

gauge ensembles

JLQCD '23

- a chiral fermion-formulation to preserve chiral symmetry
- N_f = 2+1 ($m_u = m_d = m_{ud}$)
- 3 cutoffs $a^{-1} \lesssim$ 4.5 GeV ~ $m_{b,\mathrm{phys}}$ ⇔ ~~$O(a)$ error~~
- $M_\pi \gtrsim$ 230 MeV ⇔ ChPT at a=0

M_π^2 vs a^2



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- much higher statistics than JLQCD

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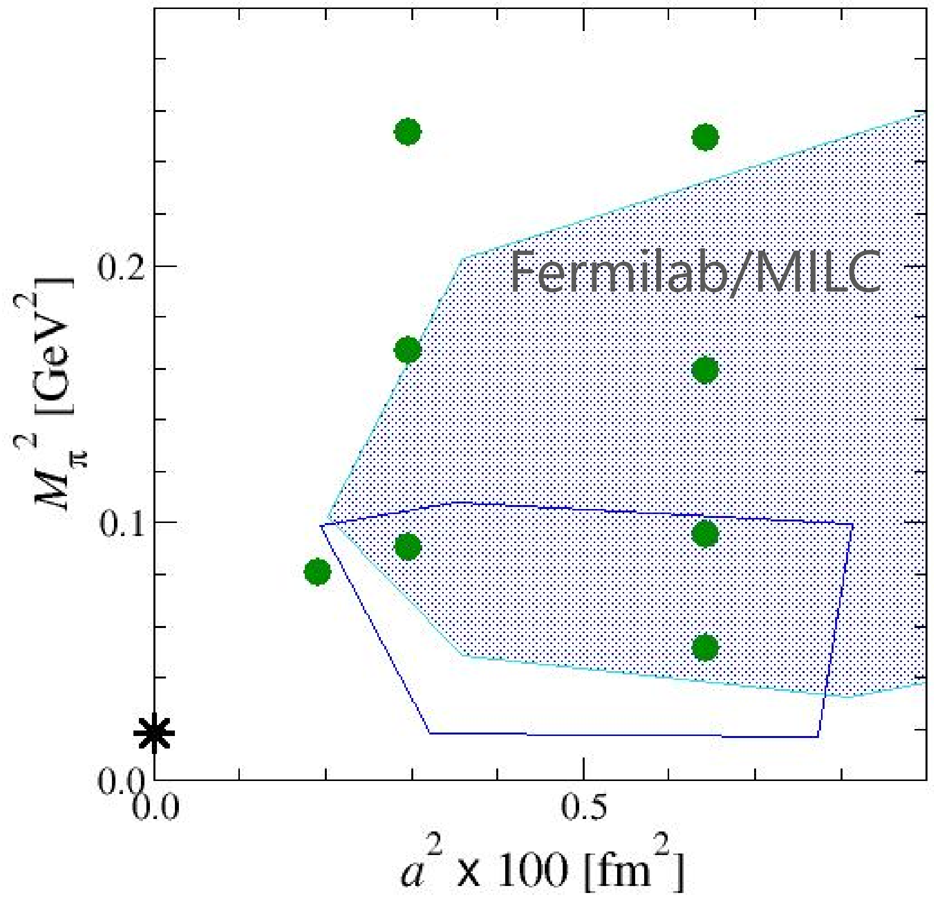

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- similar cutoffs ($B \to D^* \ell \nu$) w/ $O(\alpha_s \nu a^2)$ errors
- physical $M_{\pi,\text{phys}}$ ⇔ mild M_π dependence (later) ⇔ $B \to \pi \ell \nu$

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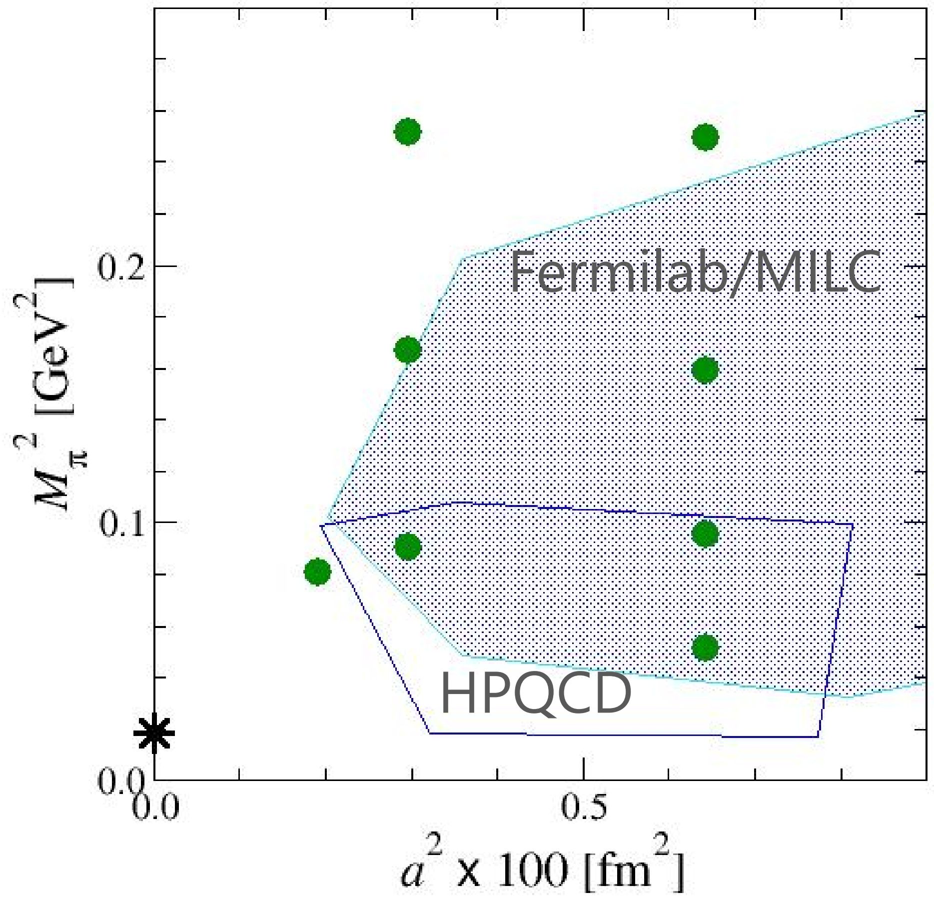

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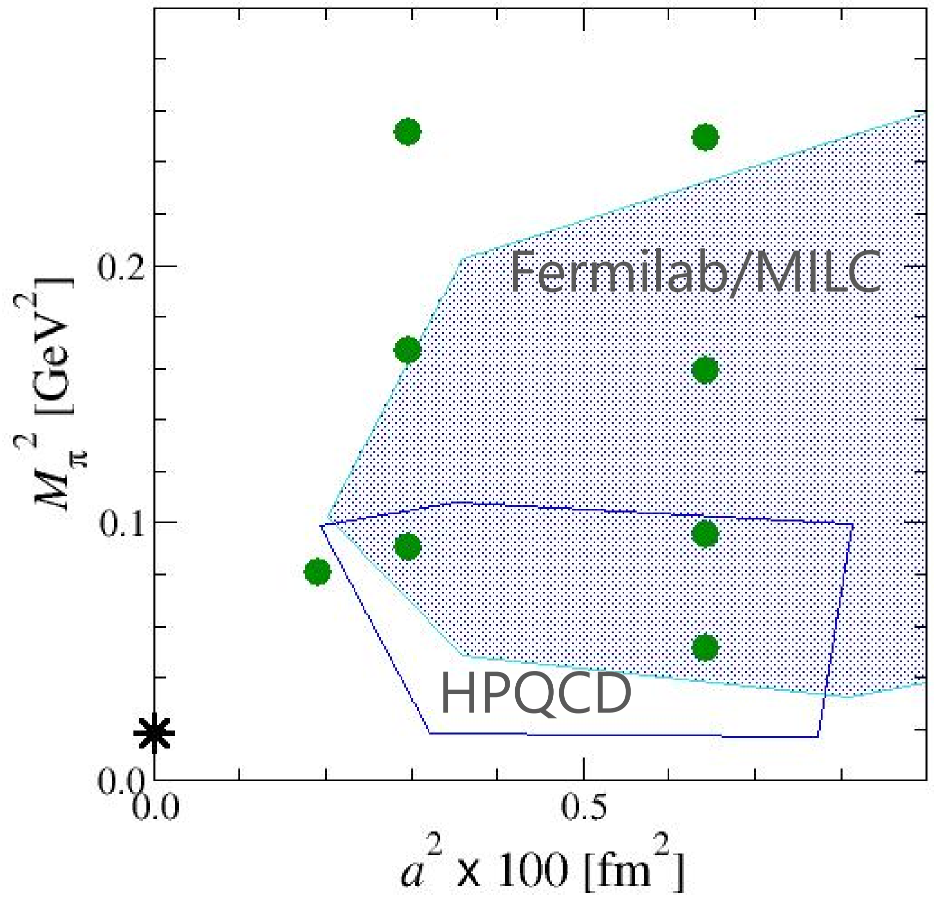


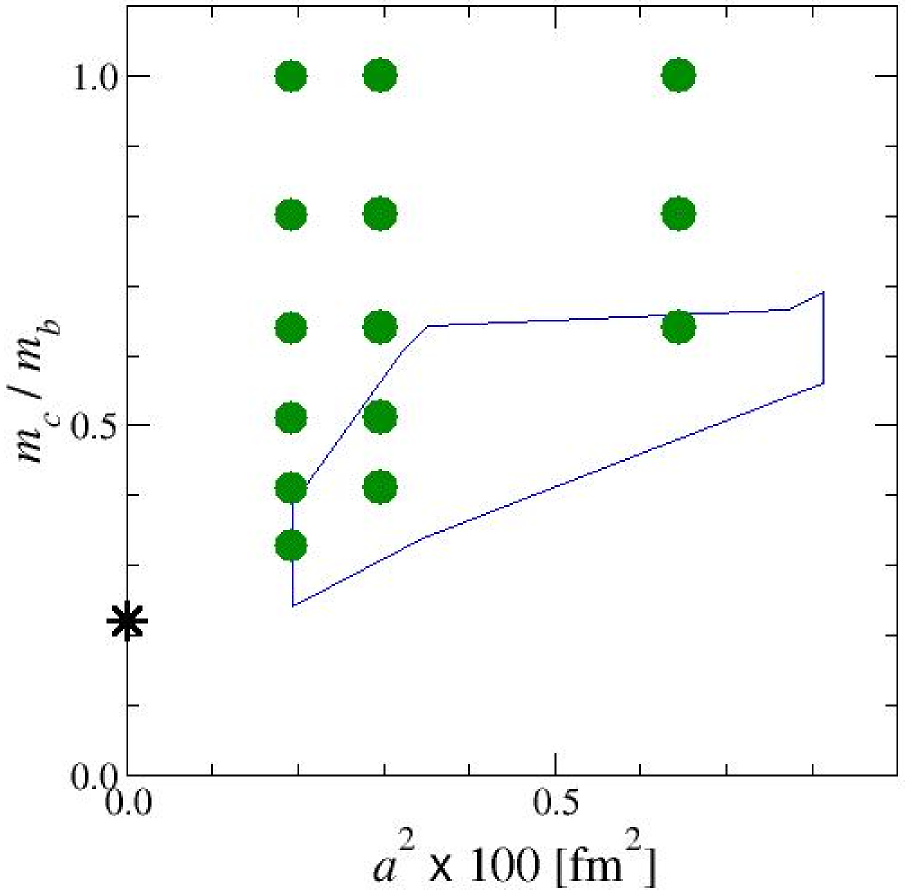
⇒ *e.g.* Fermilab/MILC, HPQCD : w/ more extended set of N_f=2+1+1 ensembles (*e.g.* Lattice '24)

$B \to D^*$ correlation functions

JLQCD '23

- $am_b < 0.7$ $\Leftrightarrow$ $a \neq 0$ errors << order counting $O((am_b)^2)$
- $m_b \lesssim 3.2$ GeV $\Rightarrow$ extrap to $m_{b,\text{phys}}$ $\Leftrightarrow$ mild m_b dependence
- $w \lesssim 1.1$ w/ $|a\mathbf{p}| \leq 0.40$
- simple renormalization of lattice operators [see below]

m_c/m_b vs a^2



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- relativistic approach w/ $am_b < 0.8$
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- v2 : more conservative error for renormalization of lattice op.s

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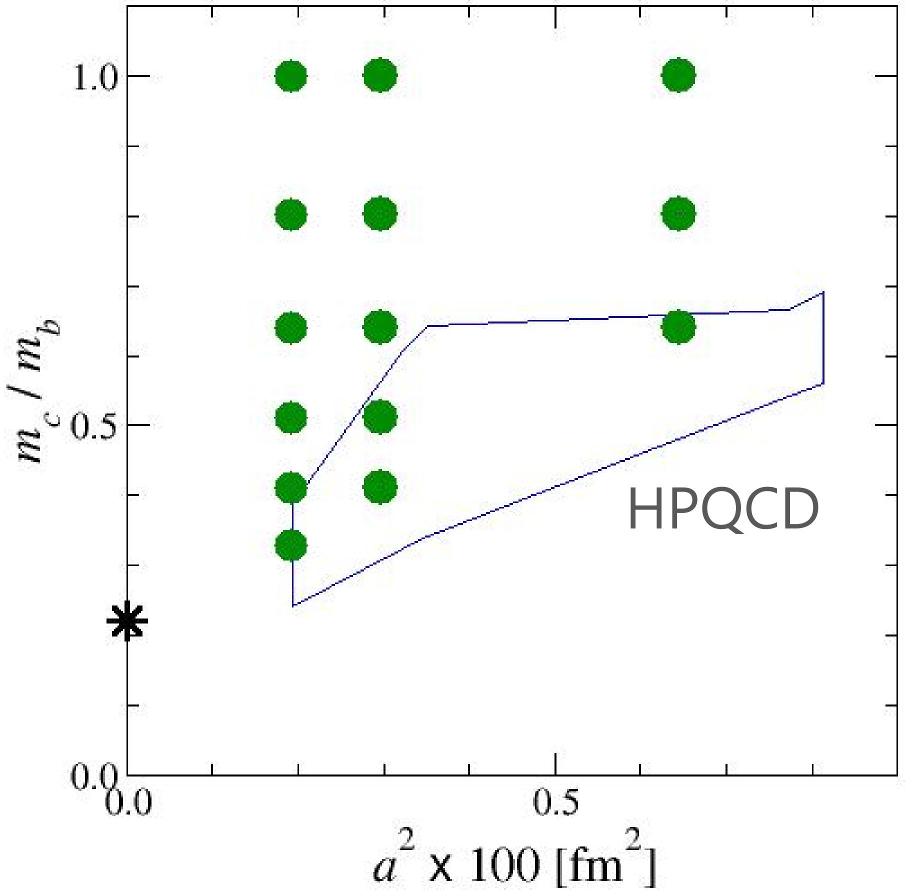

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- Fermilab approach (HQET re-interpretation of Wilson-type)
- $f(am_b)O(|a\mathbf{p}_b|^n)$ $a \neq 0$ errors $\Rightarrow$ directly @ $m_{b,\text{phys}}$
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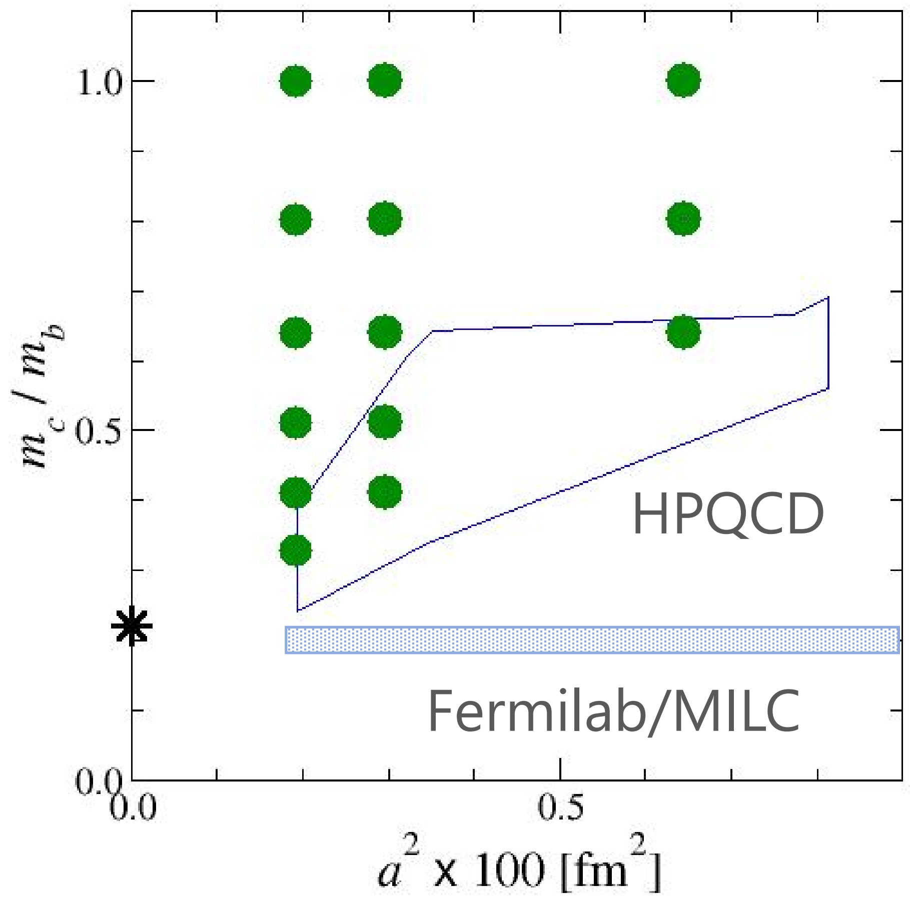

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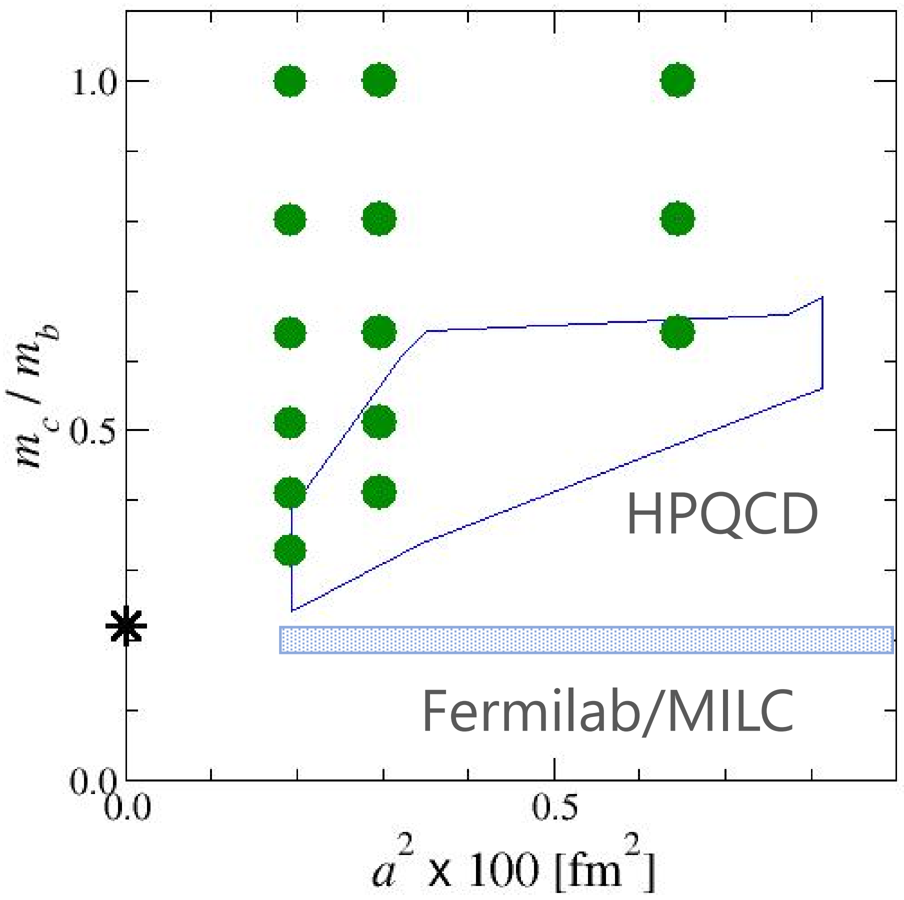
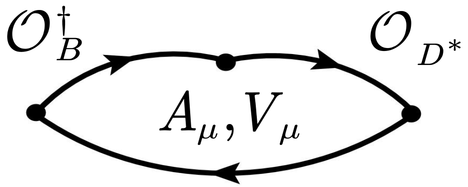


independent studies w/ different systematics

extraction of FFs

main observables – B and D^* correlation functions on the lattice → MEs



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simultaneous fit to all correlators : HPQCD

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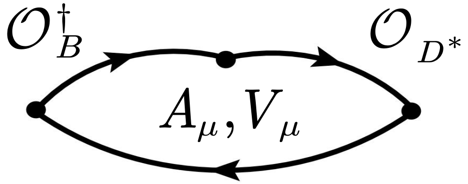

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ratio method [Hashimoto et al. '99] : Fermilab/MILC, JLQCD

– designed to cancel unnecessary factors / reduce excited state contribution, ...

e.g. to determine h_{A1} @ $|\mathbf{p}| = |\mathbf{p}'| = 0$

$$R(\Delta t,\Delta t') = \frac{C_{A_4}^{BD^*}(\Delta t,\Delta t')\, C_{A_4}^{D^*B}(\Delta t,\Delta t')}{C_{V_4}^{BB}(\Delta t,\Delta t')\, C_{V_4}^{D^*D^*}(\Delta t,\Delta t')} = \frac{\langle D^*|A_4|B\rangle \langle B|A_4|D^*\rangle}{\langle B|V_4|B\rangle \langle D^*|V_4|D^*\rangle} = h_{A_1}(w=1)^2$$

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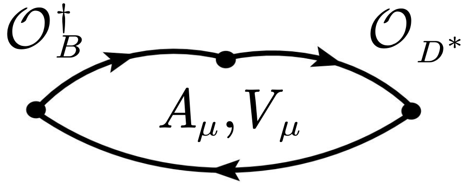

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– JLQCD w/ chiral symmetry : do not need explicit renormalization for SM FFs ⇔ HPQCD's update v2

ground state saturation : JLQCD

could be serious issue for $B\to\pi\ell\nu$ (talk by Oliver Bär) : process / lattice op's dependent issue – $B\to D^*\ell\nu$?

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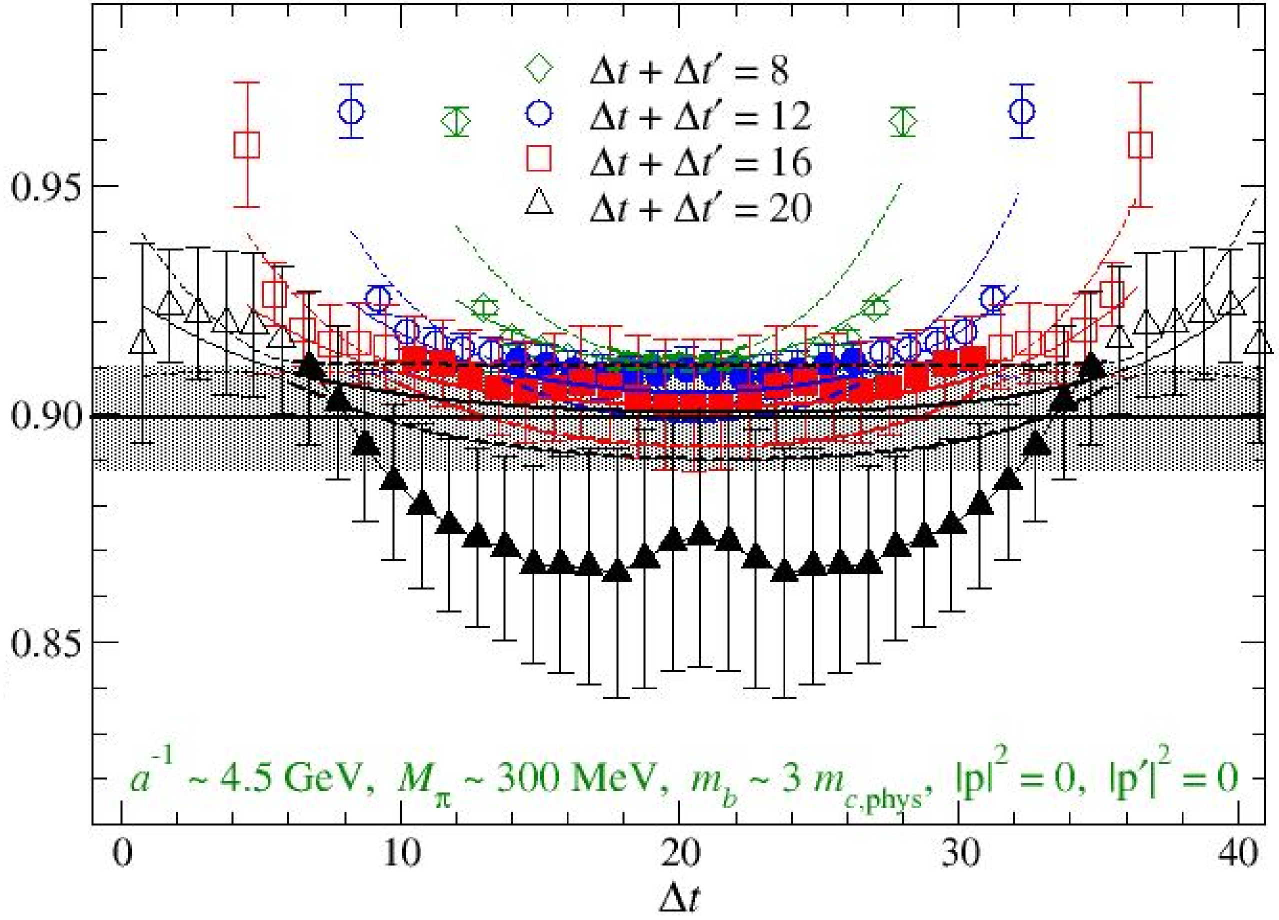

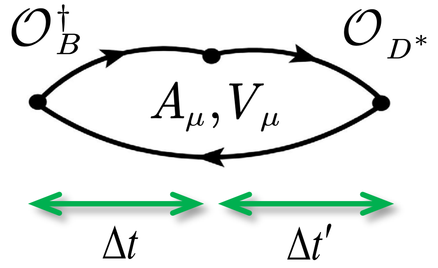

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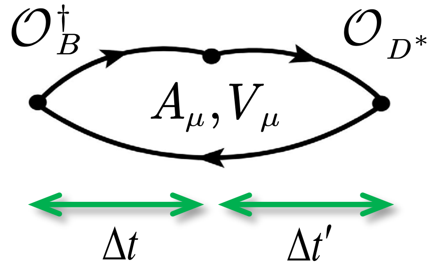

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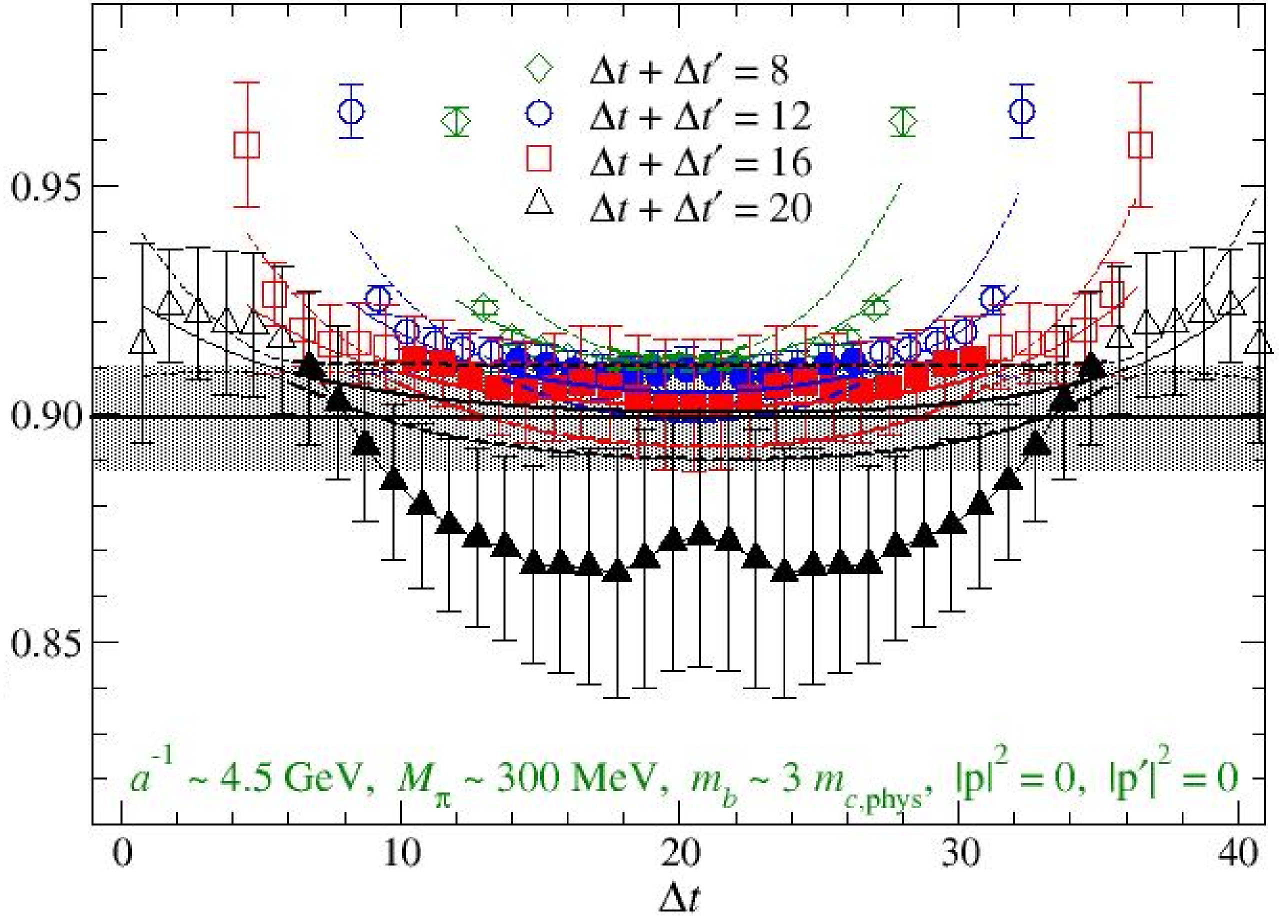

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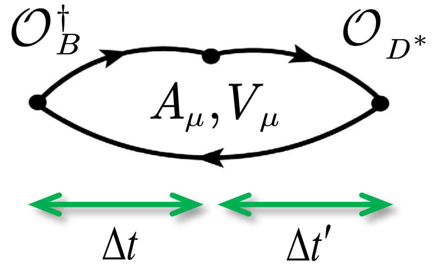

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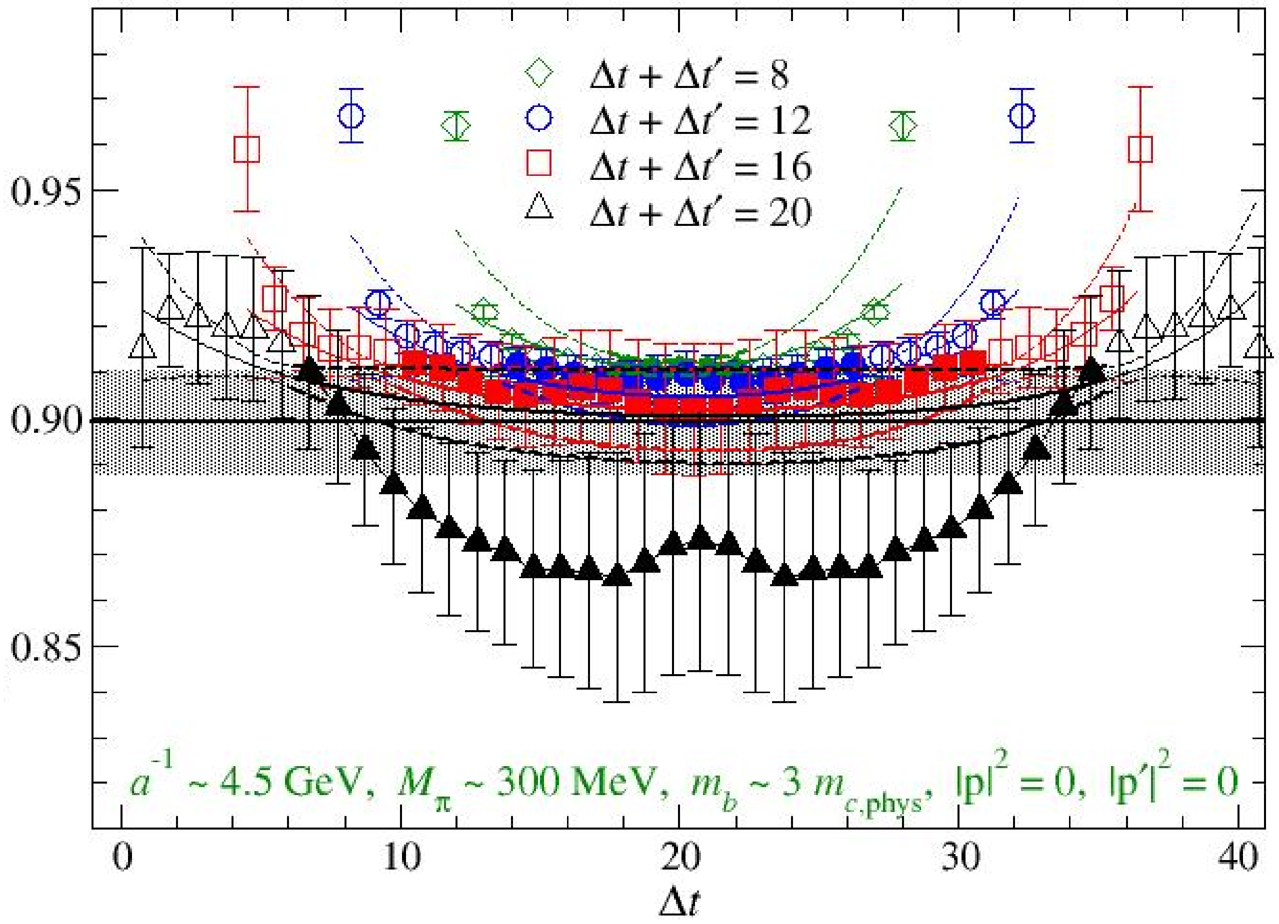

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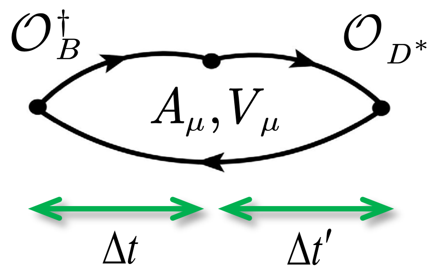

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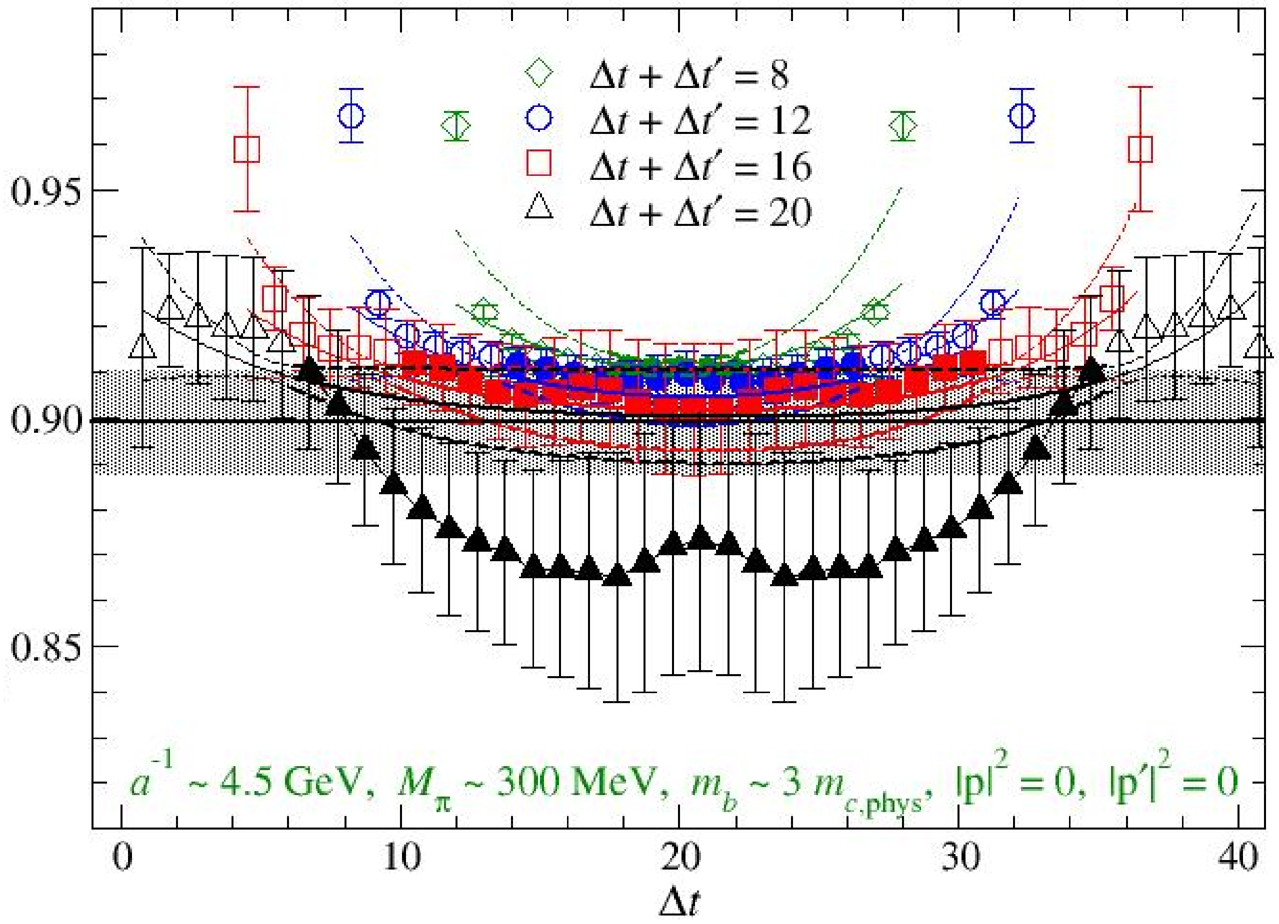

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excited state contributions are reasonably controlled in JLQCD's study

extrapolation to the real world

2 step analysis of FF data

1. "continuum + chiral extrapolation"
   - extrapolate to $a=0$, $m_{q,\mathrm{phys}}$ based on HMChPT, HQET, …
   - AND, polynomial-interpolate to reference values of w

   ⇒ "synthetic data" of FFs at $a=0$, $m_{q,\mathrm{phys}}$ but at reference values of w

2. model-independent (BGL) parameterization (Boyd-Grinstein-Lebed '97)

   ⇒ FFs at $a=0$, $m_{q,\mathrm{phys}}$ as a function of w

continuum + chiral extrapolation : JLQCD

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

$$\frac{h_{A_1}(w)}{\eta_{A_1}} = c + \frac{3g_{D^*D\pi}^2}{32\pi^2 f_\pi^2}\Delta_c^2 \bar{F}_{\log}(M_\pi, \Delta_c, \Lambda_\chi)$$

$$+ c_w(w-1) + d_w(w-1)^2 + c_b(w-1)\varepsilon_b + c_\pi \xi_\pi + c_{\eta_s}\xi_{\eta_s} + a_a \xi_a + a_{m_b a}\xi_{am_b}$$

$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b}, \quad \xi_\pi = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \quad \xi_{\eta_s} = \frac{M_{\eta_s}^2}{(4\pi f_\pi)^2}, \quad \xi_a = (a\Lambda_{\mathrm{QCD}})^2, \quad \xi_{am_b} = (am_b)^2$$

η_X : one-loop radiative correction (Neubert '92); Luke's theorem ⇒ $O((w\text{-}1)/m_b)$ for h_{A1}

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$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b}, \quad \xi_\pi = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \quad \xi_{\eta_s} = \frac{M_{\eta_s}^2}{(4\pi f_\pi)^2}, \quad \xi_a = (a\Lambda_{\rm QCD})^2, \quad \xi_{am_b} = (am_b)^2$$

η_X : one-loop radiative correction (Neubert '92); Luke's theorem ⇒ $O((w\text{-}1)/m_b)$ for h_{A1}

– appropriately normalized $\xi_X \Rightarrow c_X = O(1)$ or zero-consistent

continuum + chiral extrapolation : JLQCD

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

$$\frac{h_{A_1}(w)}{\eta_{A_1}} = c + \frac{3g^2_{D^*D\pi}}{32\pi^2 f_\pi^2}\Delta_c^2 \bar{F}_{\log}(M_\pi, \Delta_c, \Lambda_\chi)$$

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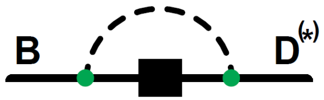
- appropriately normalized ξ_X ⇒ c_X = $O(1)$ or zero-consistent
- Fermilab/MILC and HPQCD : similar form of NLO chiral log + polynomial corrections
- large correlation matrix w.r.t. a, m_{ud}, m_s, w and m_b for relativistic approach

  e.g. JLQCD w/ time-consuming chiral fermions ⇒ poorly determined low-lying eigenpairs

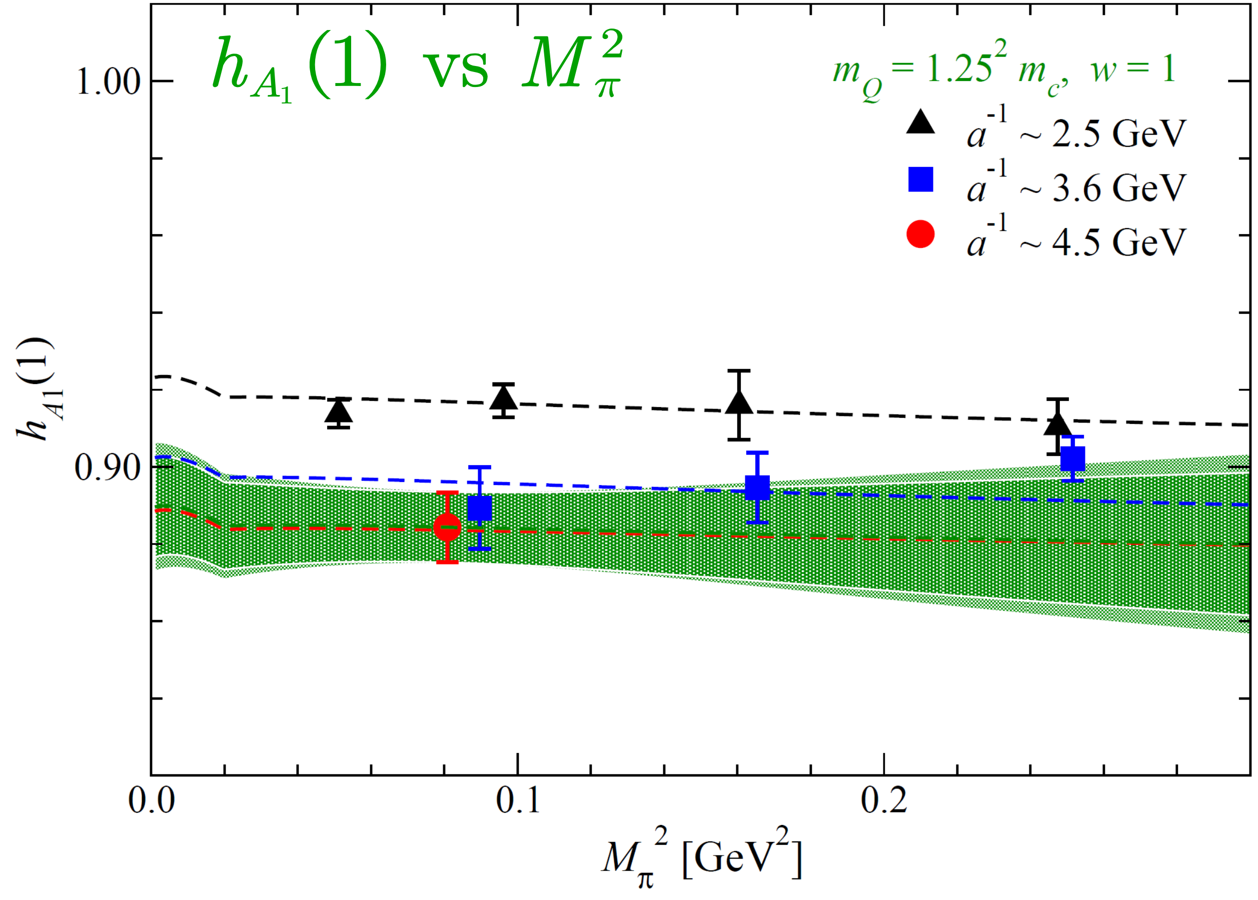
  ⇒ SVD cut, shrinkage ⇒ additional systematic uncertainty ⇔ Fermilab/MILC

M_π dependence

NLO chiral log in HMChPT (Randall-Wise '92, Savage '01)



$$\Delta_c^2 \bar{F}_{\log} = \Delta_c^2 \ln\left[\frac{M_\pi^2}{\Lambda_{\rm QCD}^2}\right] - 2(2) - \Delta_c \sqrt{\Delta_c^2 - M_\pi^2} \ln\left[\frac{\Delta_c - \sqrt{\Delta_c^2 - M_\pi^2}}{\Delta_c + \sqrt{\Delta_c^2 - M_\pi^2}}\right] + \ldots = \Delta_c^2 \ln\left[\frac{M_\pi^2}{\Lambda_{\rm QCD}^2}\right] + O(\Delta_c^3)$$



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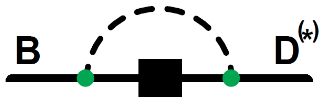

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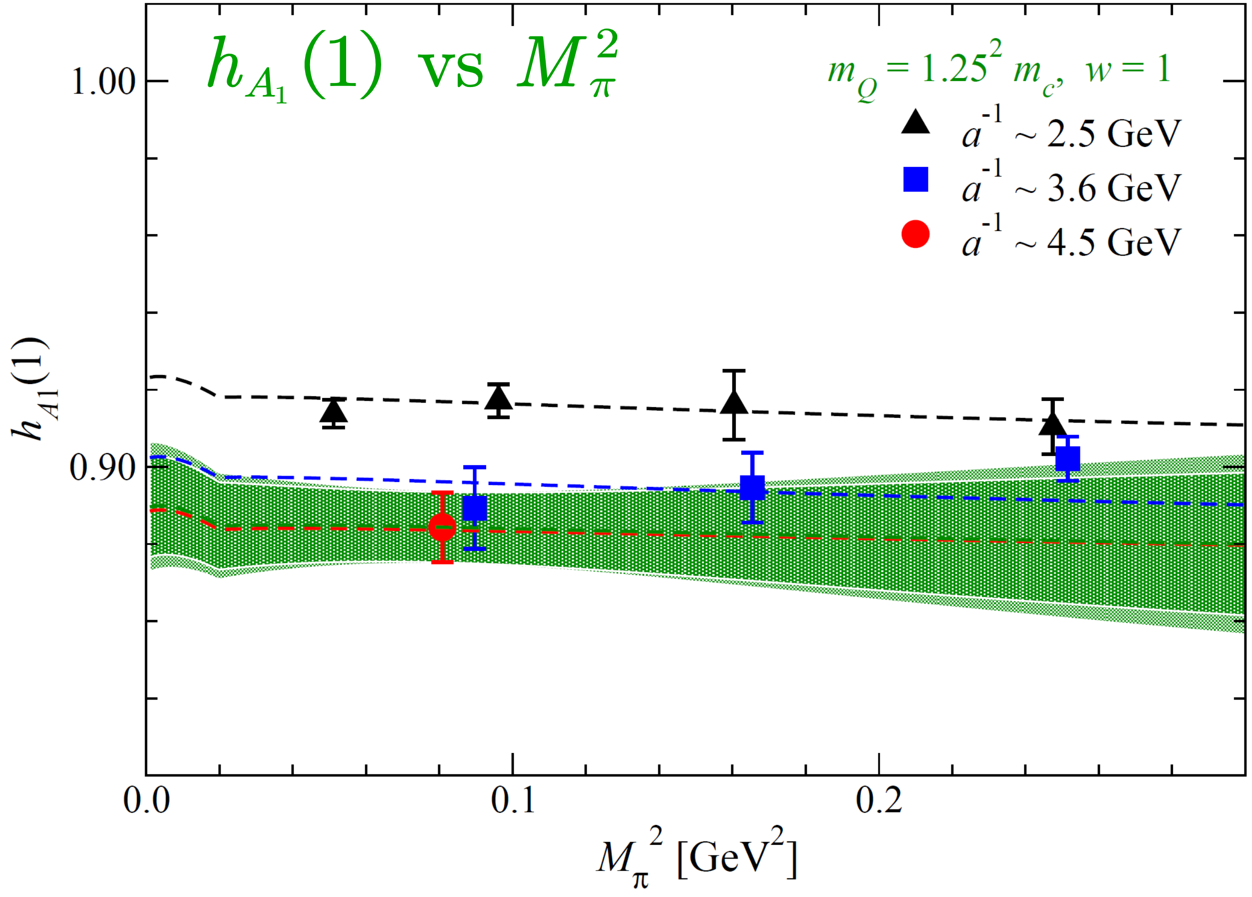

– chiral log suppressed by $\Delta_c^2 = (M_{D*} - M_D)^2$

$$\frac{3 g_{D^*D\pi}^2}{32\pi^2 f_\pi^2}\Delta_c^2 \bar{F}_{\log}$$

+ "ξ" scheme f(LEC) $\to f_\pi$

+ $g_{D^*D\pi}$=0.53(8) (Fermilab/MILC '14)

$\Rightarrow$ small systematic uncertainty

M_π dependence

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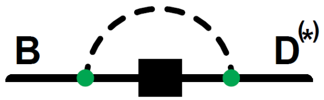

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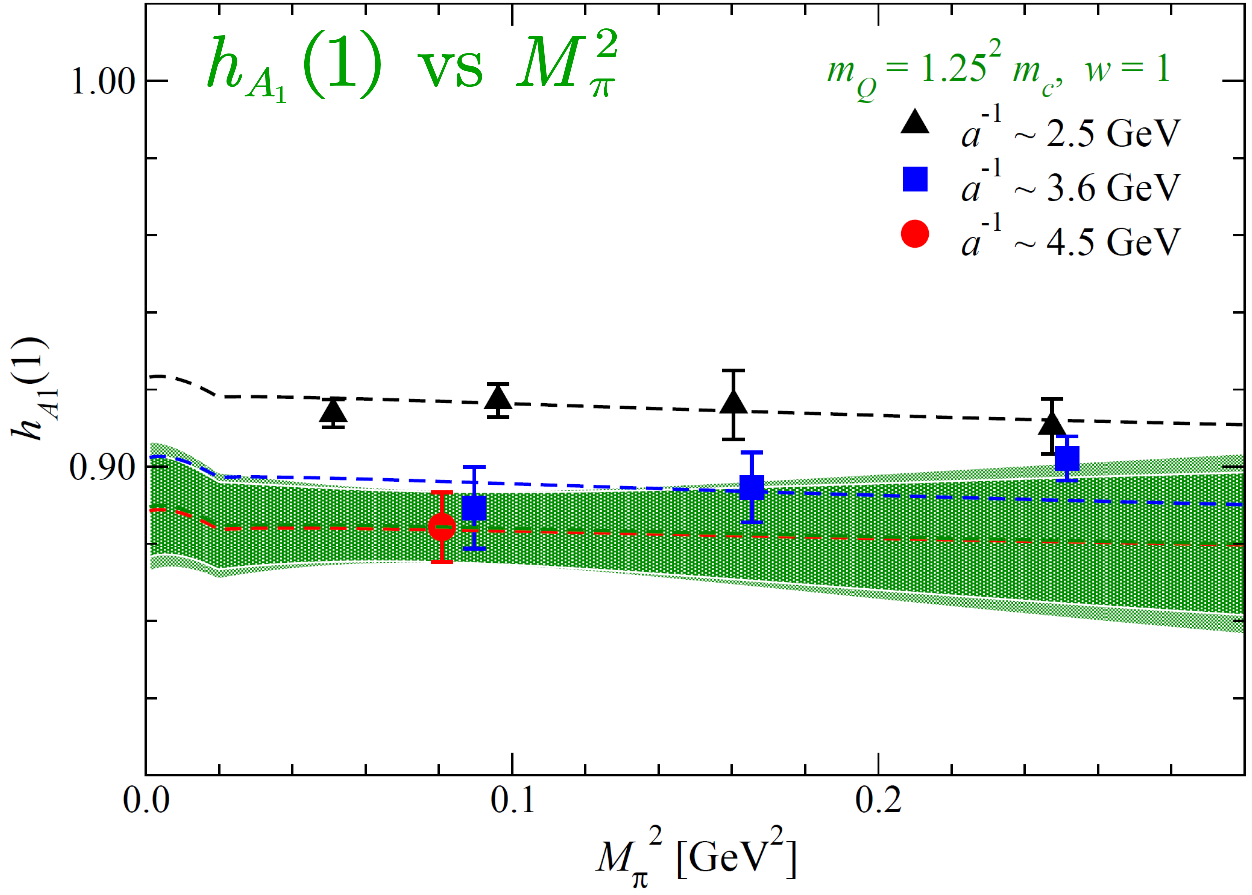

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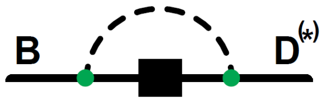

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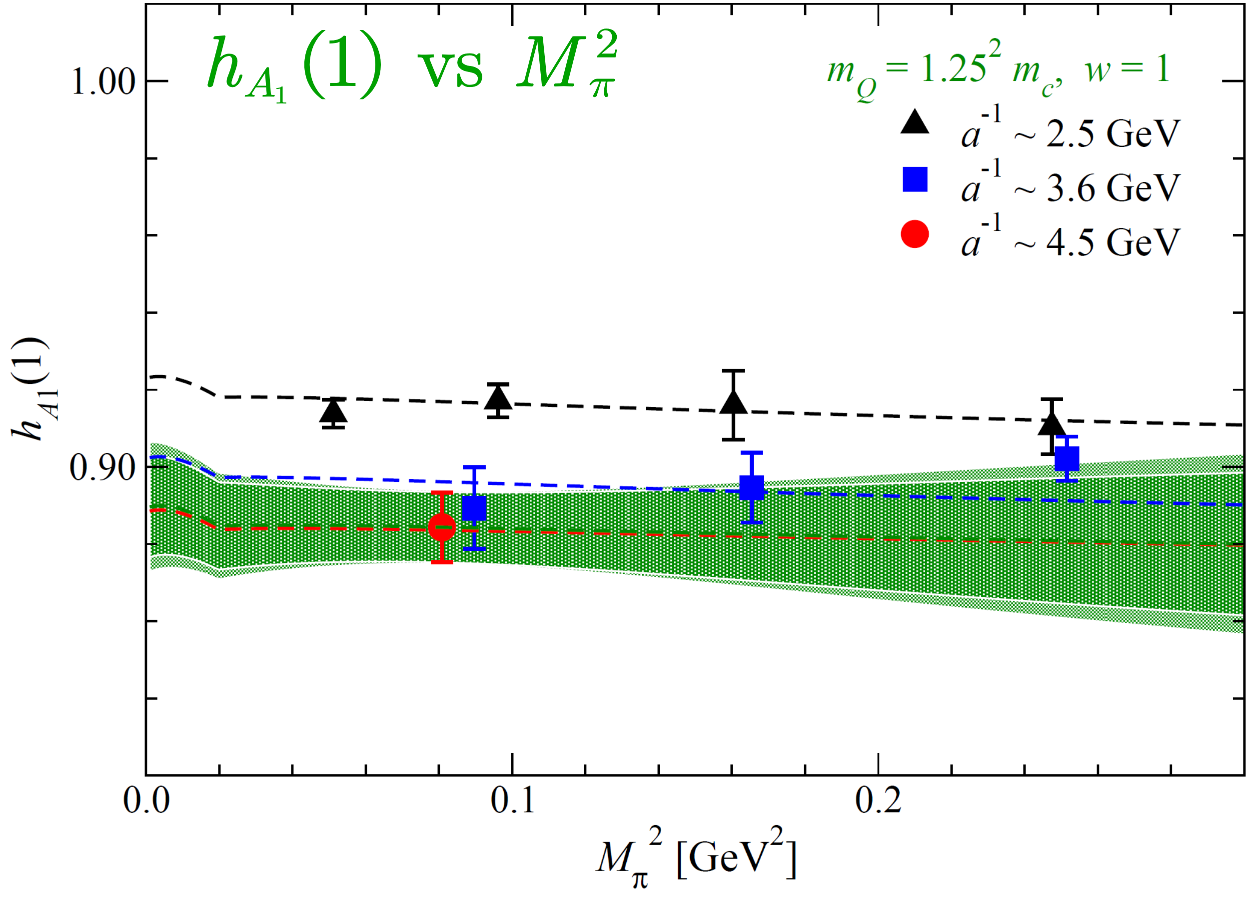

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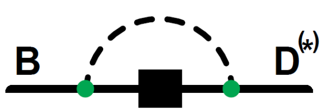
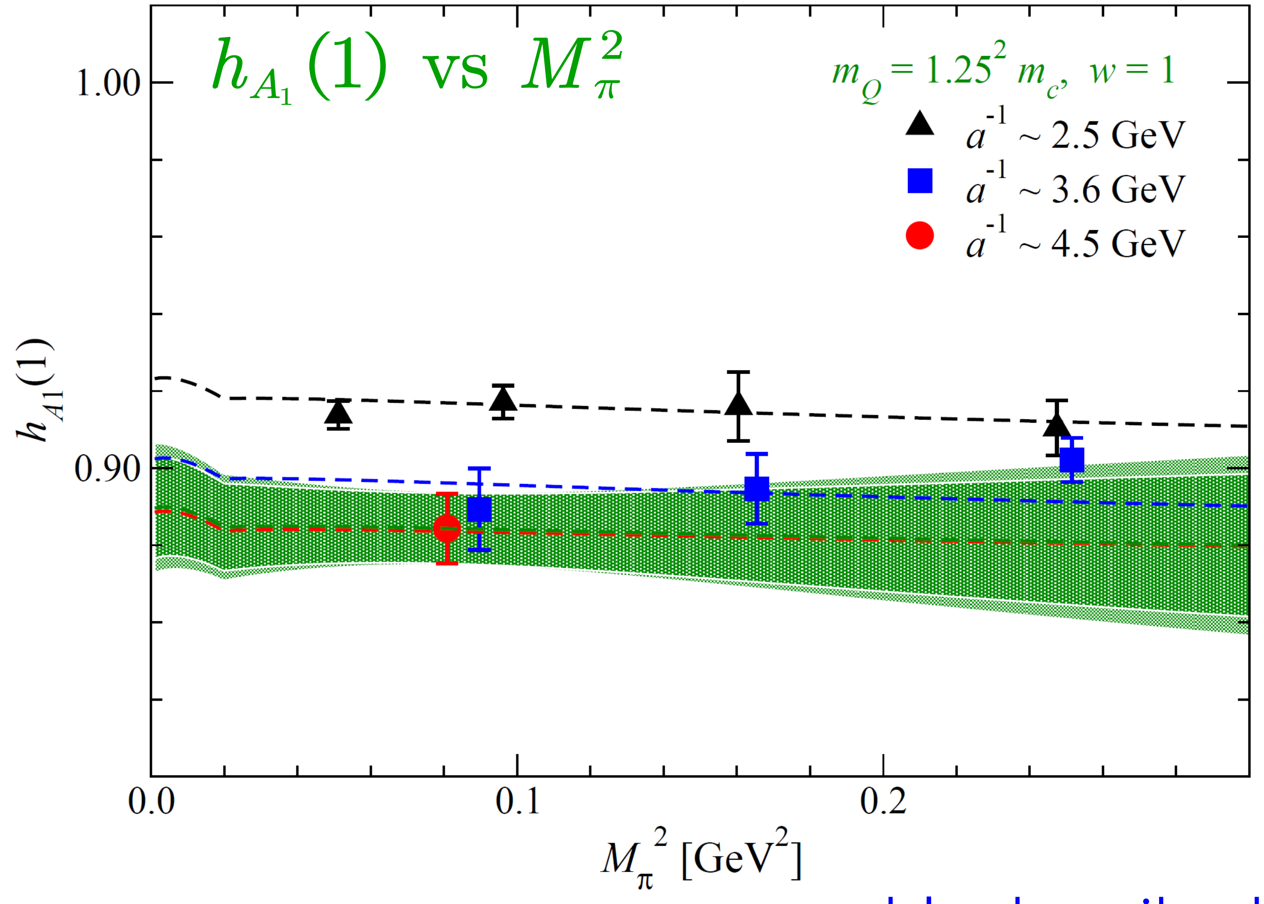


$$\Delta_c^2 \bar{F}_{\log} = \Delta_c^2 \ln\left[\frac{M_\pi^2}{\Lambda_{QCD}^2}\right] - 2(2) - \Delta_c\sqrt{\Delta_c^2 - M_\pi^2}\,\ln\left[\frac{\Delta_c - \sqrt{\Delta_c^2-M_\pi^2}}{\Delta_c+\sqrt{\Delta_c^2-M_\pi^2}}\right] + \dots = \Delta_c^2\ln\left[\frac{M_\pi^2}{\Lambda_{QCD}^2}\right] + O(\Delta_c^3)$$



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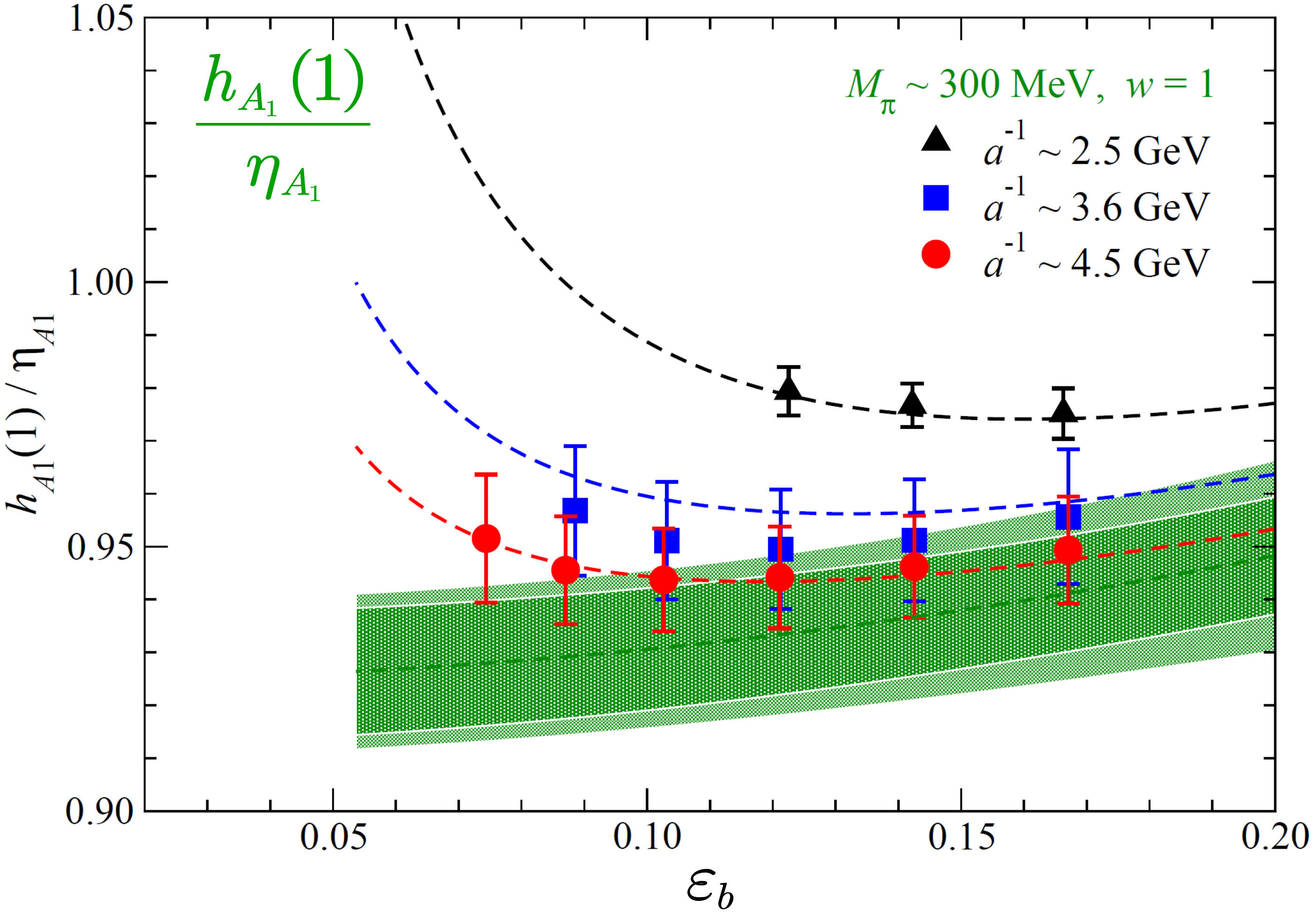
reasonably described by NLO log + analytic

m_b dependence

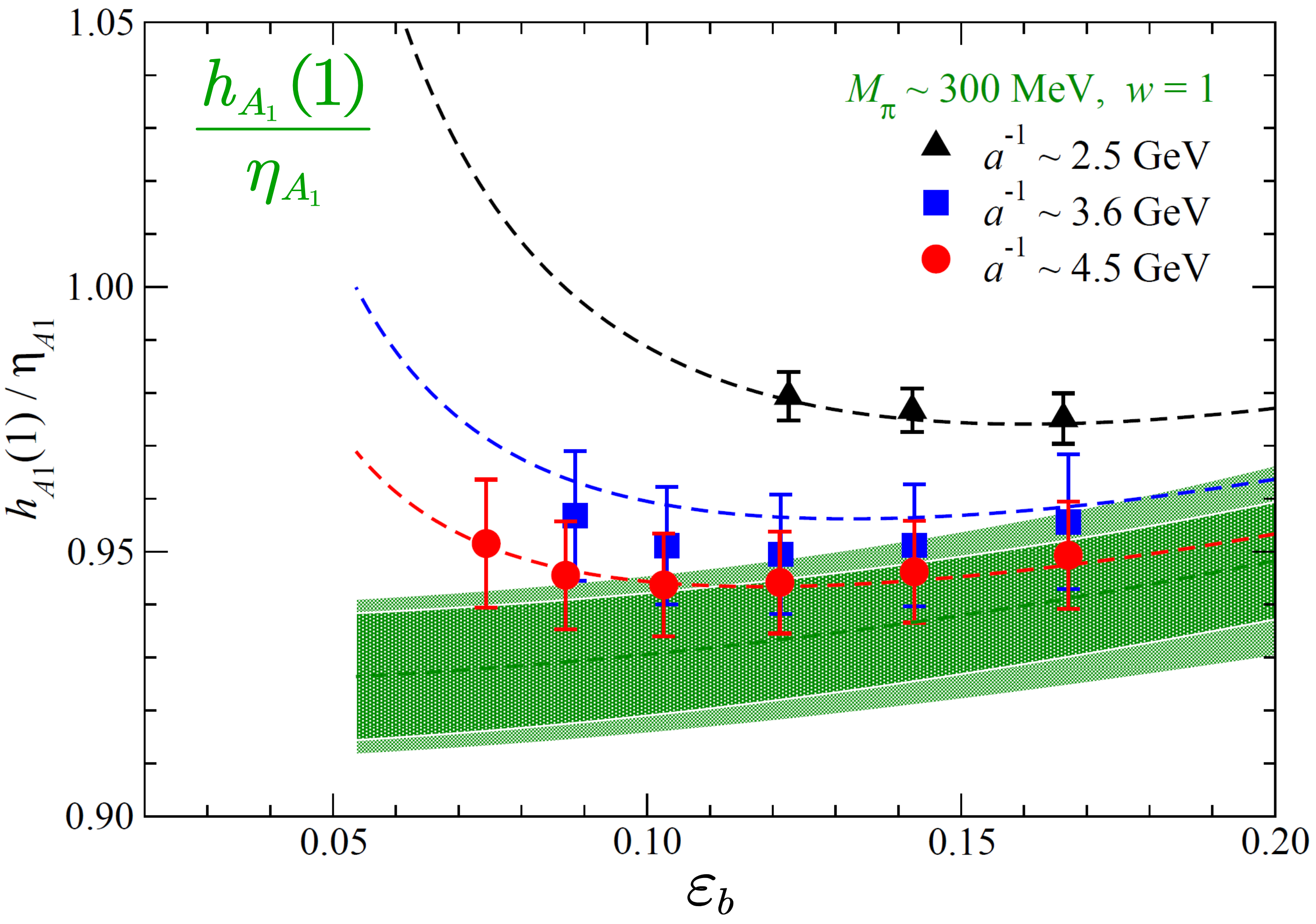


- physical m_b dependence

$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b} \to \frac{\bar{\Lambda}}{M_{\eta_b}} \quad \bar{\Lambda} = 0.5\,\mathrm{GeV}$$

- $O((am_b)^2)$ $a \neq 0$ effects
- non-trivial mixture $x^{-2} + x^2$
- $am_b < 0.7 \Rightarrow$ both @ a few %

m_b dependence



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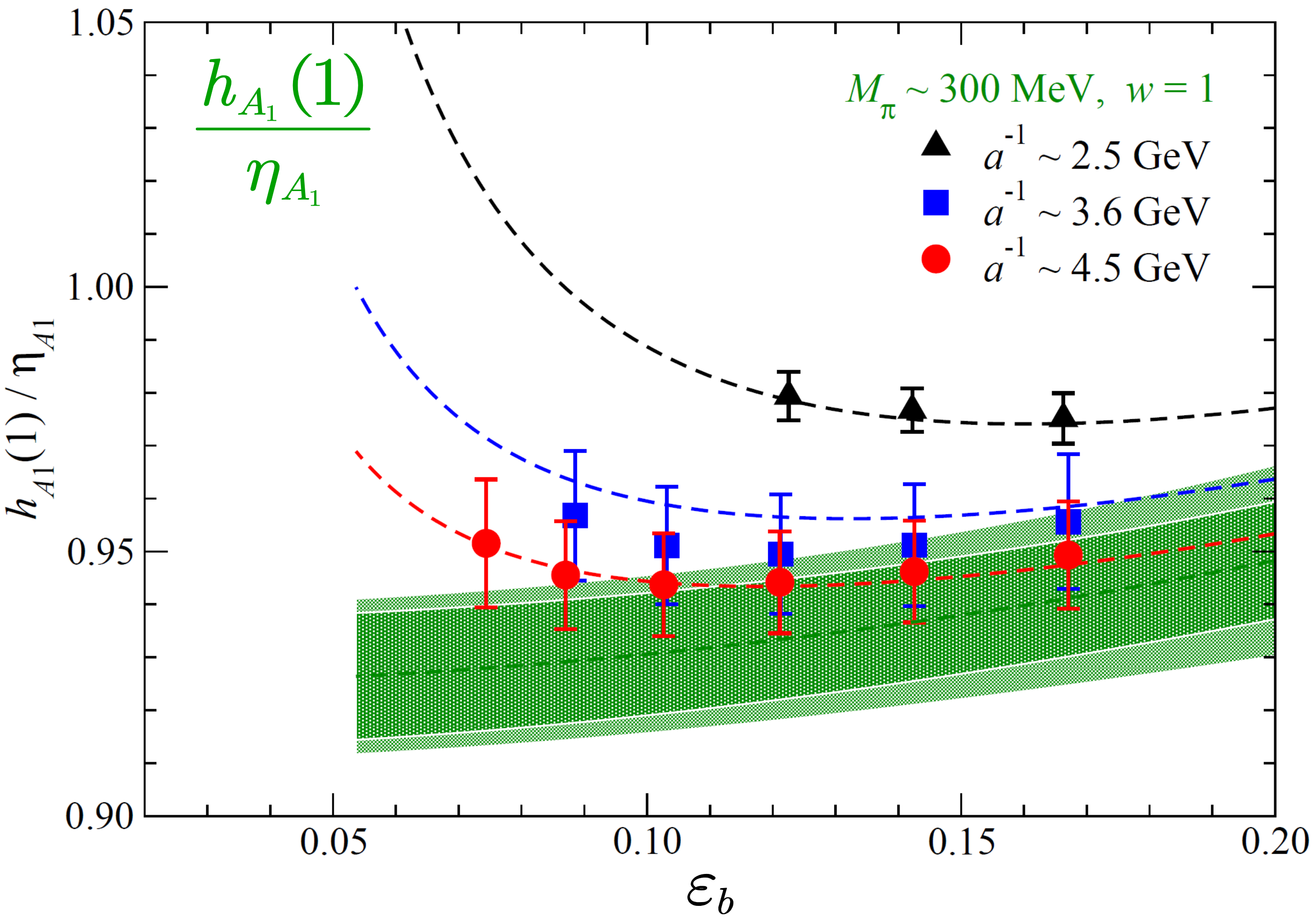
fitted to expected functional form of $1/m_b^{(2)}$ plus $(am_b)^2$ $\Rightarrow$ ~~higher order terms~~

m_b dependence



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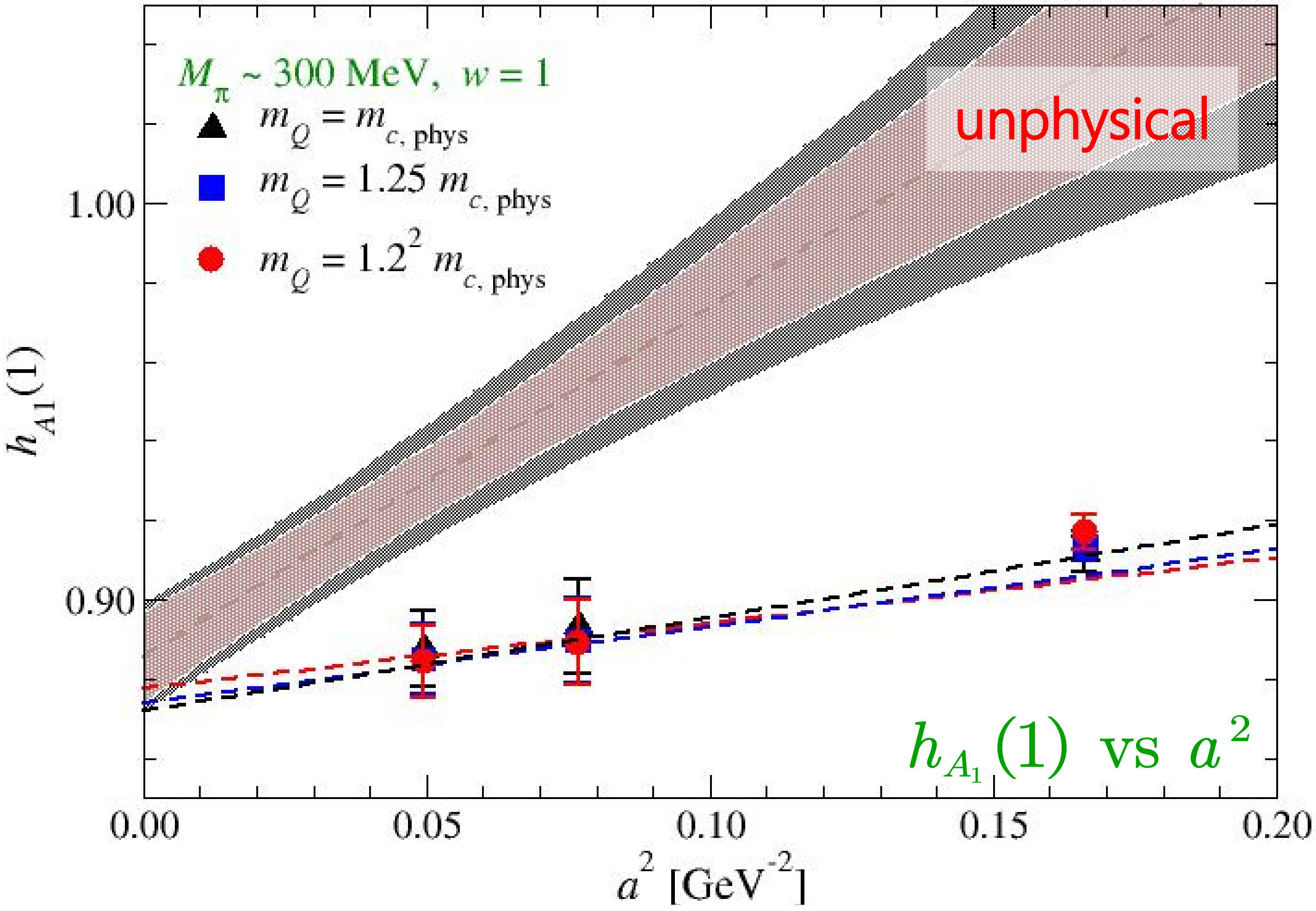
- physical m_b dependence on lattice vs HQET analysis of exp data

fitted to expected functional form of $1/m_b^{(2)}$ plus $(am_b)^2$ ⇒ ~~higher order terms~~

a dependence



– if naively simulate physical m_b

⇒ 3 – 12% $a \neq 0$ errors

– w/ our condition $am_b < 0.7$

⇒ a few % $a \neq 0$ errors

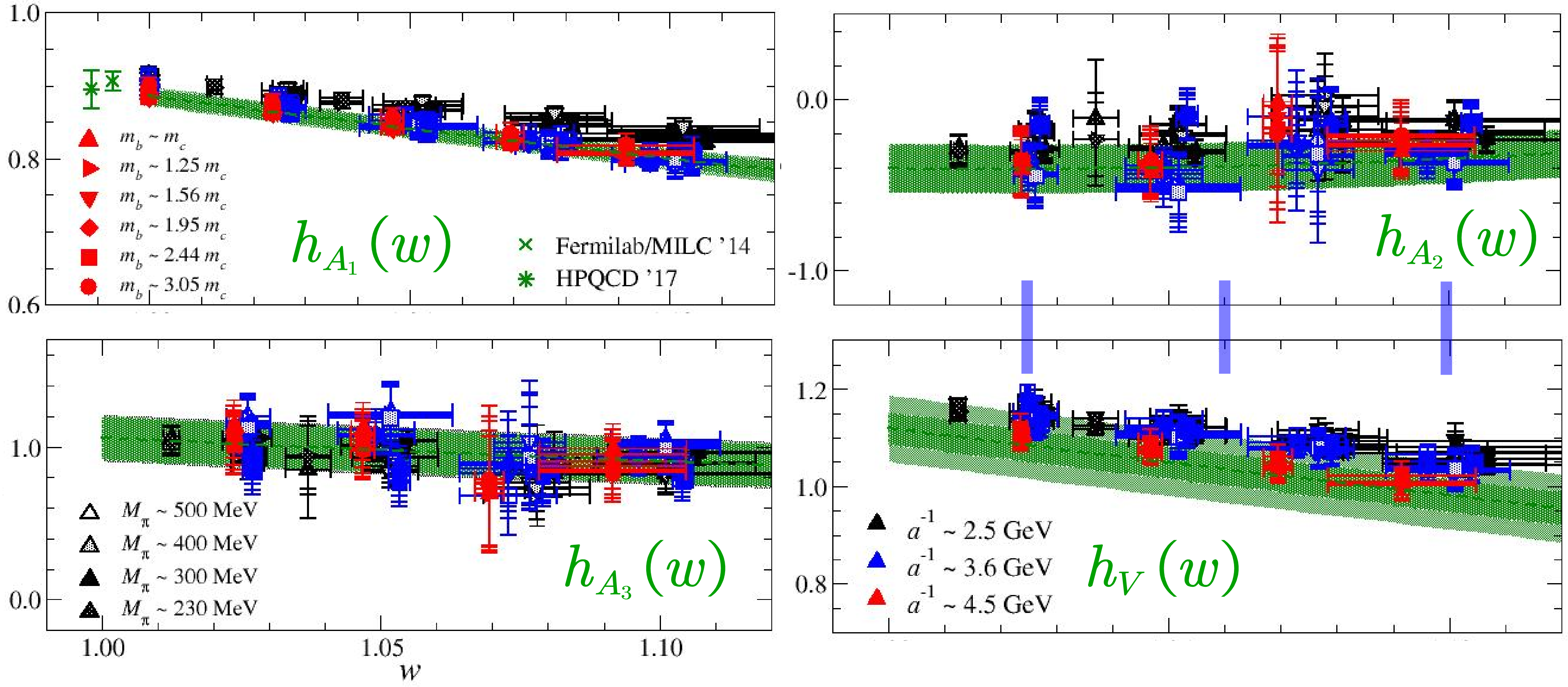
⇒ controlled continuum extrap

– *e.g.* similar $a \neq 0$ error @ physical m_b

⇒ $a^{-1} \geq 4.5$ GeV

well described by $O(a^2)$ extrapolation

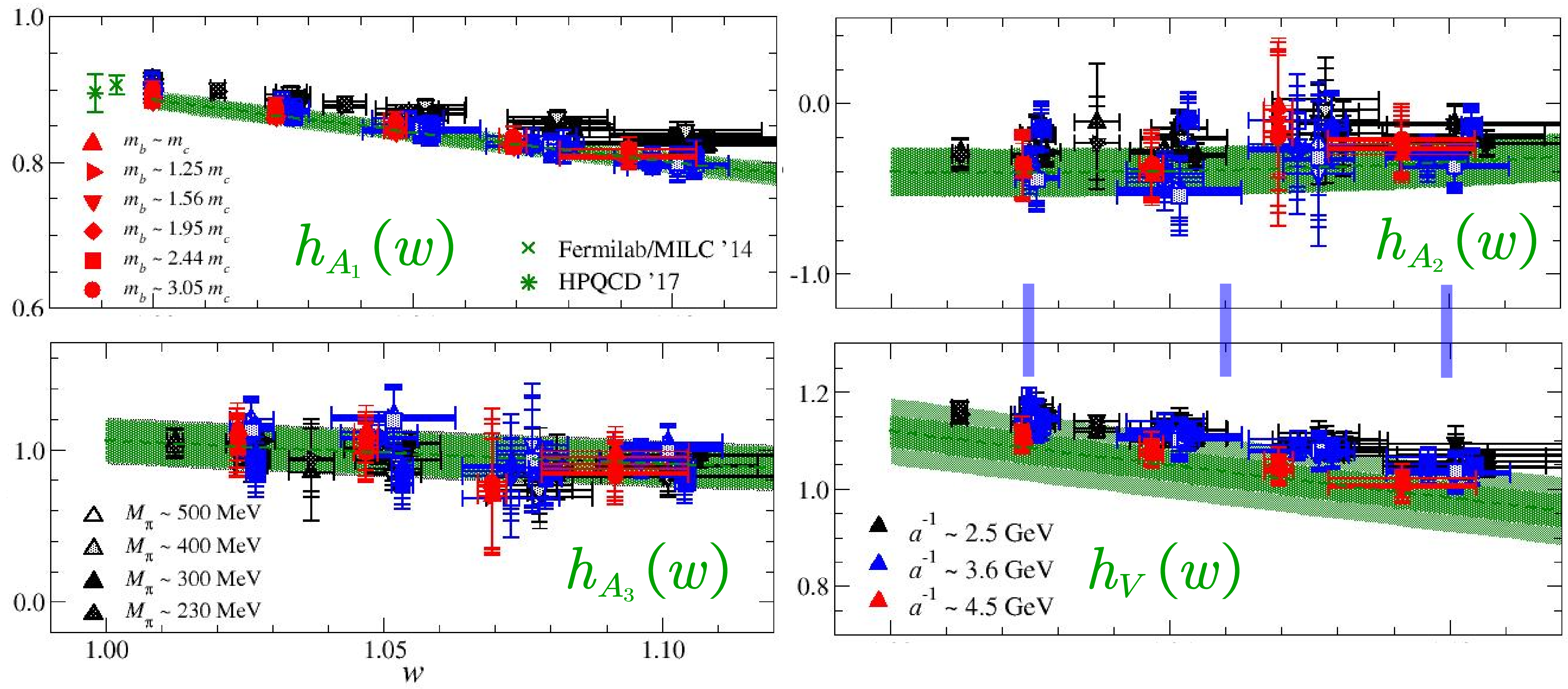
w dependence



– no strong curvature in w $\Rightarrow$ quadratic interpolation in $(w\text{-}1)$ to reference values of w

w/ quadratic coefficients : 3.0σ for h_{A1}; consistent w/ zero for others

w dependence



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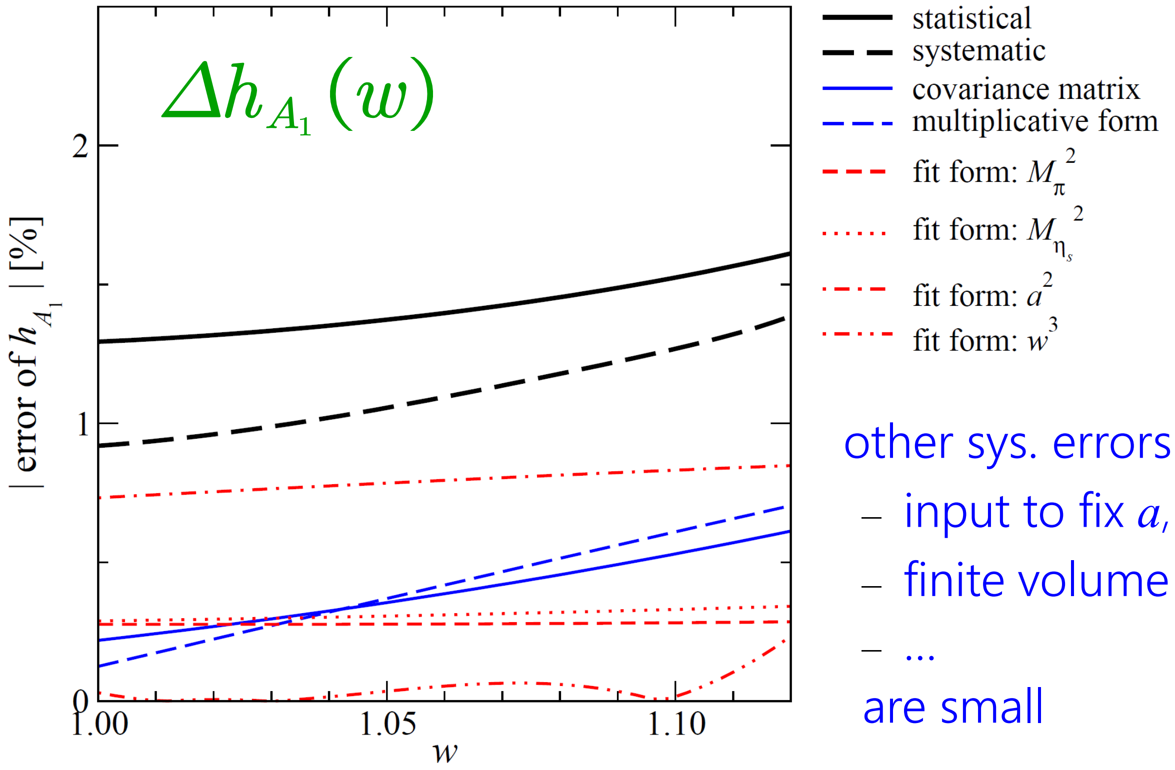
w/ quadratic coefficients : 3.0σ for h_{A1}; consistent w/ zero for others

– much noisier for $h_{A\{2,3\}}$ (later)

– mild dependence on a, M_{π}, m_b $\Rightarrow$ reasonably controlled extrapolation w/ χ^2/dof ~ 0.5

uncertainties

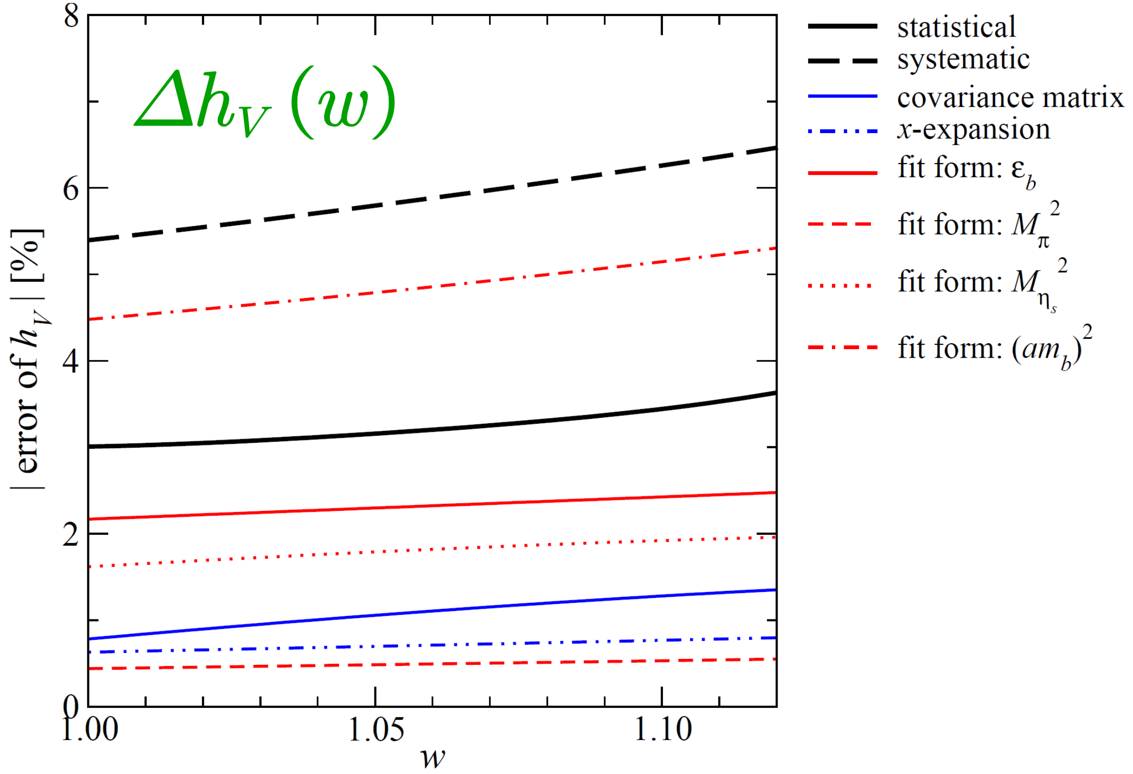
h_{A1}, h_V



other sys. errors due to

- input to fix a, m_q's
- finite volume effects
- ...

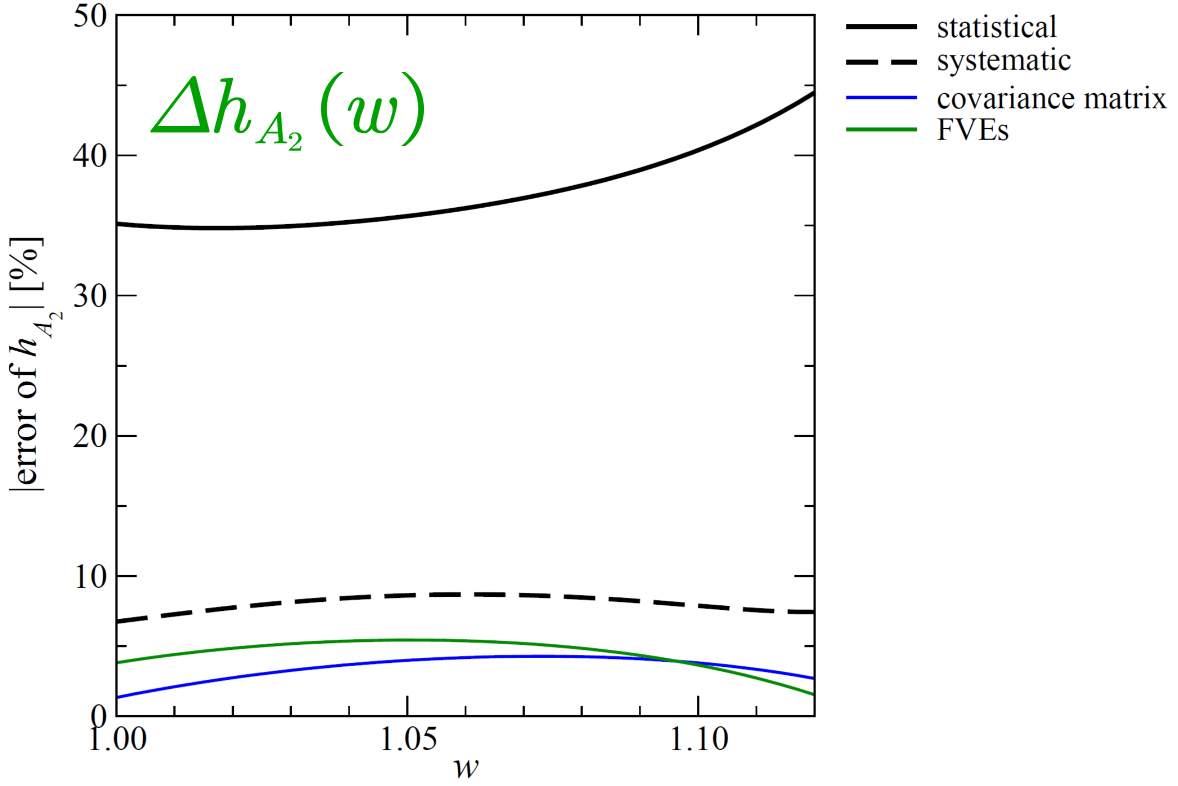
are small

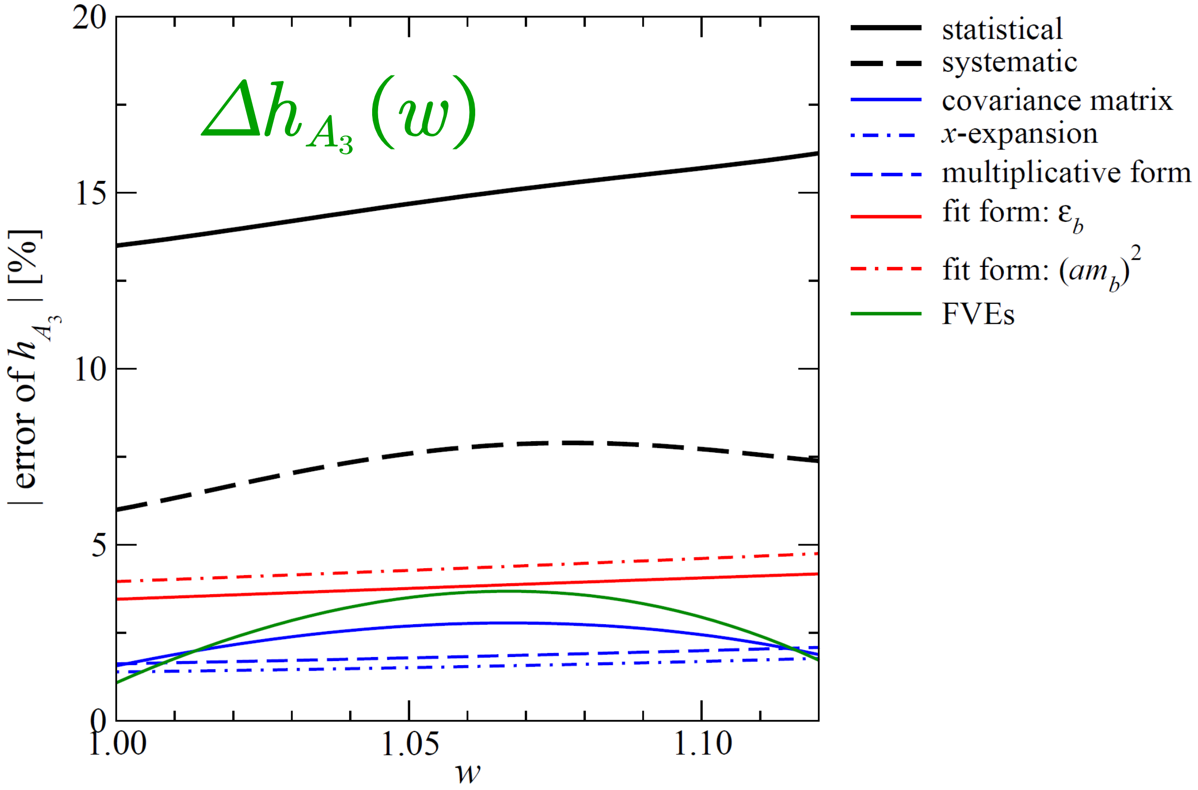


- $C^{BD*}{}_{A1}$ ($\varepsilon_{D*} \perp v_B$), $C^{BD*}{}_V$ sensitive only to h_{A1}, h_V $\Rightarrow$ statistically more accurate than $h_{A\{2,3\}}$
- largest errors from statistics and discretization : 1-2% for h_{A1}, 3-5% for h_V
- systematic error as |"analysis A" – "analysis B"| ~ 1σ?

uncertainties

$h_{A2,}\ h_{A3}$

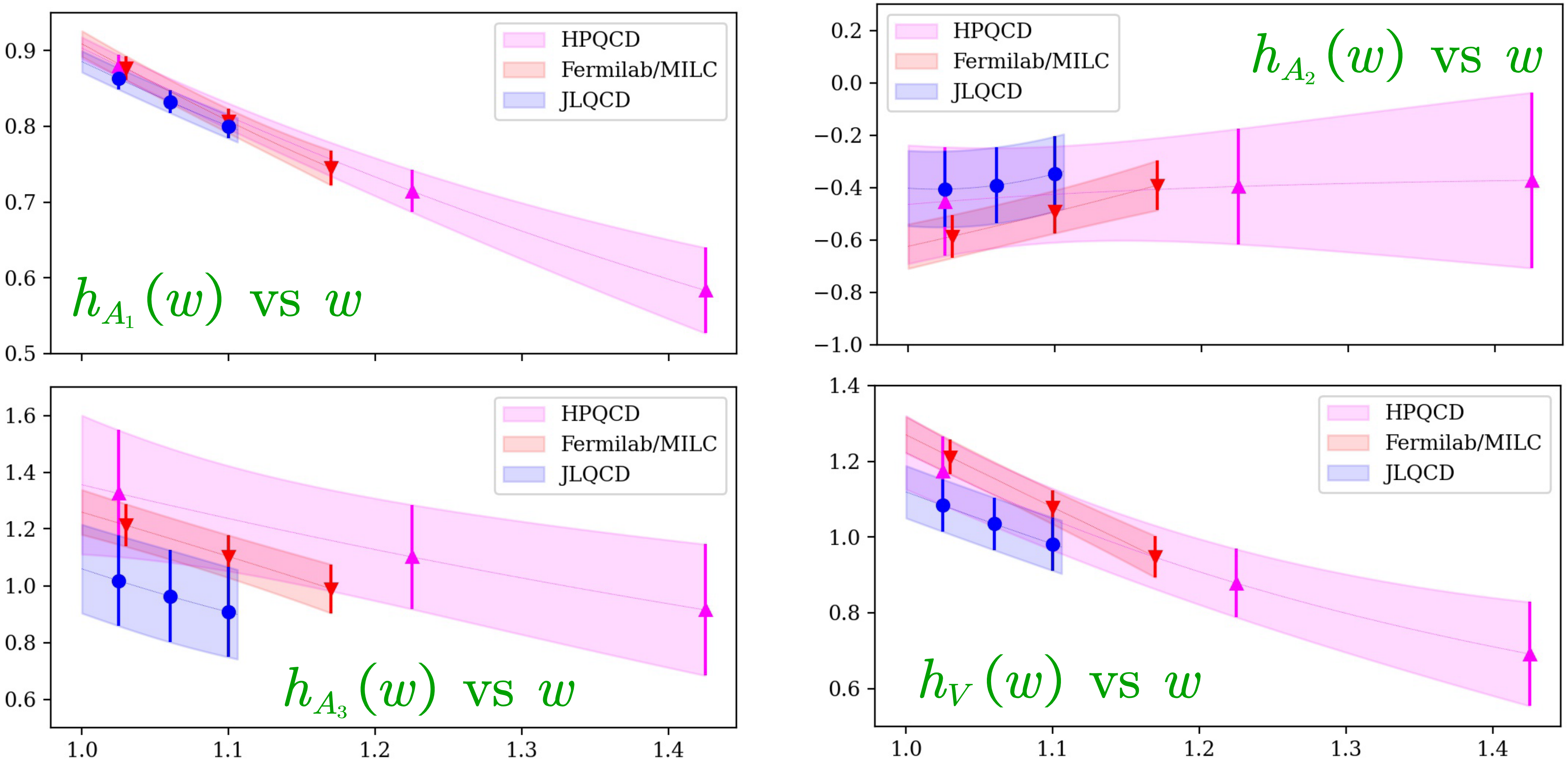




– no correlators exclusively sensitive to these ⇒ statistics limited

– much room to improve (later)

comparison w/ Fermilab/MILC and HPQCD



- HPQCD v2 $\Rightarrow$ (1 σ level) $h_{A2}\downarrow$, $h_{A3}\uparrow$ w/ slightly larger uncertainties
- reasonable consistency

parametrization of “synthetic” FF data

FFs in “relativistic” convention

“relativistic” convention $|H\rangle_{\mathrm{rel}} = |H\rangle_{\mathrm{heavy}} / \sqrt{M_H}$

$$\langle D^*(p',\varepsilon')|V_\mu|B(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\varepsilon'^{*\nu} p'^{\rho} p^{\sigma} g(q^2)$$

$$\langle D^*(p',\varepsilon')|A_\mu|B(p)\rangle = \varepsilon'^*_\mu f(q^2) - \varepsilon'^* p\{(p+p')_\mu a_+(q^2) + (p-p')_\mu a_-(q^2)\}$$

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$a_\pm \;\Rightarrow\; \mathcal{F}_{1,2}$ = linear combinations of f, $a_\pm$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}|V_{cb}|^2\frac{k}{q^5}(q^2 - m_\ell^2)\{(2q^2 + m_\ell^2)(2q^2 f^2 + \mathcal{F}_1^2 + 2k^2q^4g^2) + 3k^2q^2m_\ell^2\mathcal{F}_2^2\}$$

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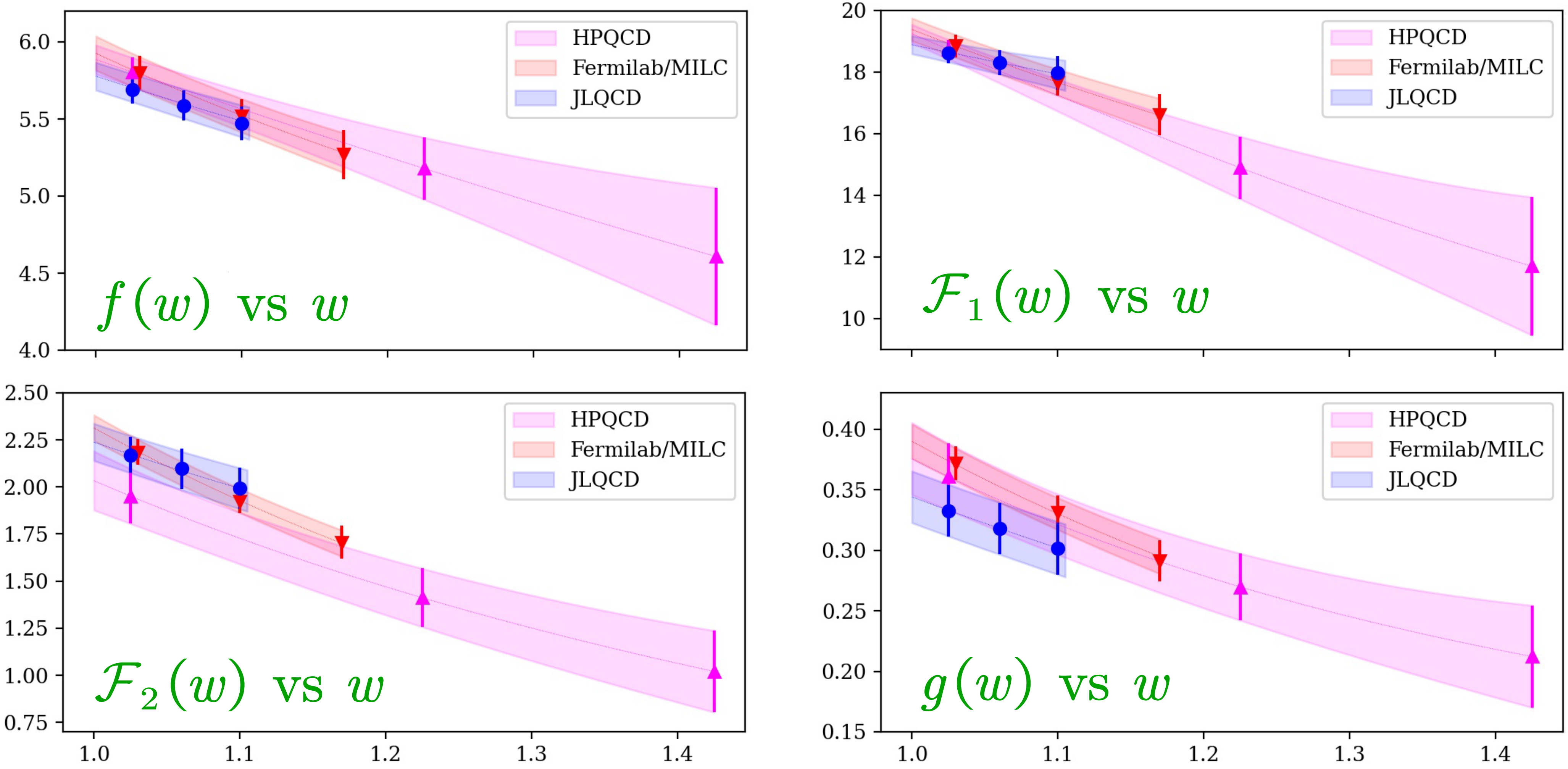
- $f \propto h_{A1}$: only this @ w=1
- $g \propto h_V$: only this for vector ME
- $\mathcal{F}_1 \ni f, h_{A\{2,3\}}$: contributions of h_{A2}, h_{A3} @ $w\neq1$
- $\mathcal{F}_2 \ni f, h_{A\{2,3\}}$: m_ℓ^2 suppressed contributions → $R(D^*)$

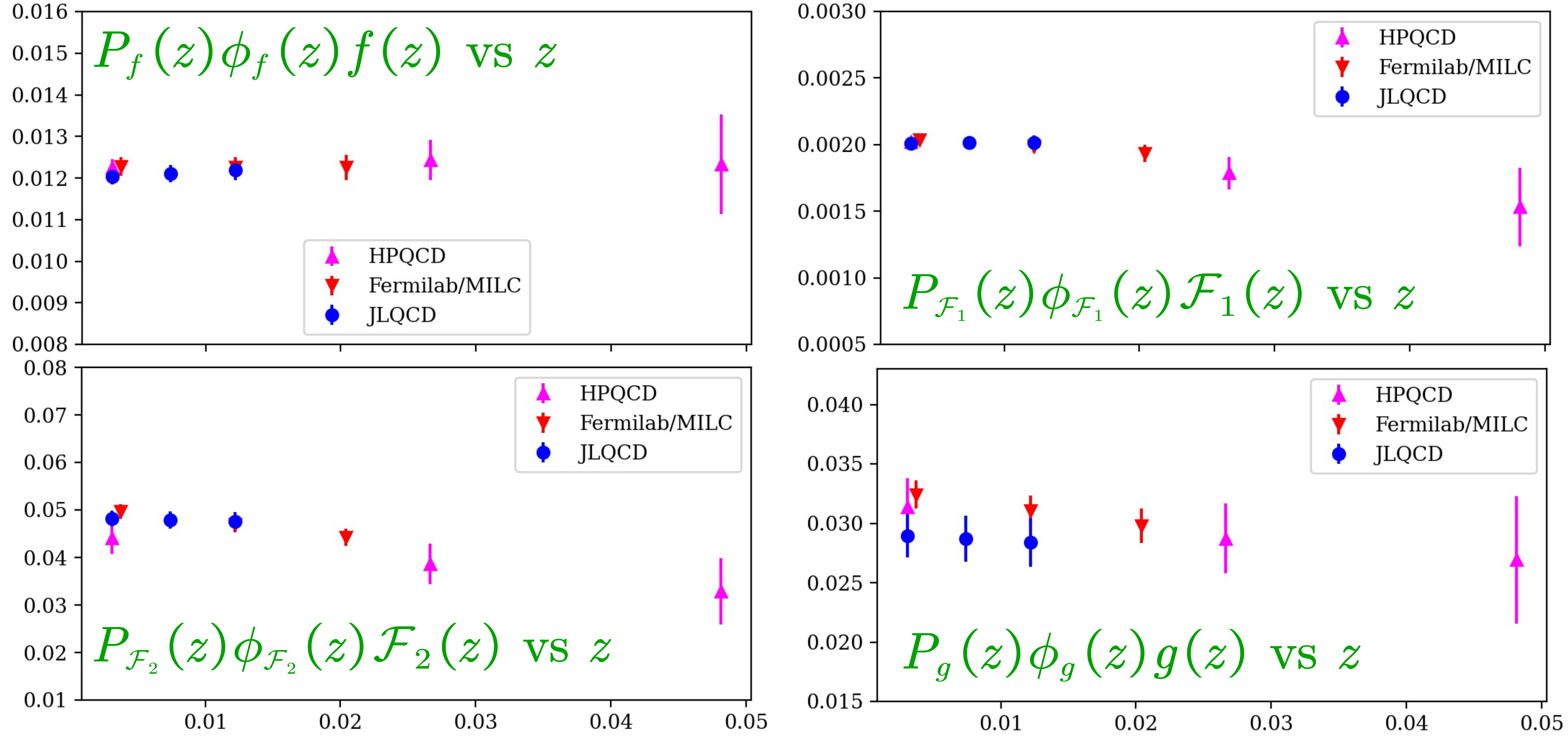
synthetic data of FFs in relativistic convention



- "tensions"? : normalization of $\mathcal{F}_2$ and g … but $\lesssim 2\sigma$ level
- limited regions w for JLQCD = [1.025,1.100] ⇔ Fermilab/MILC [1.03,1.17], HPQCD [1.025,1.425]

done :^)

data for BGL fit



- factoring out pole contributions $\Rightarrow$ milder dependence on z (w)
- "tensions" remain: normalization of $\mathcal{F}_2$ and g

fit to BGL parametrization (Boyd+ '97)

$$f(q^2) = \frac{1}{P_f(z)\phi_f(z)} \sum_n^{N_f} a_n^f z^n \qquad z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}$$

- small z parameter $\lesssim$ 0.015 JLQCD ; $\lesssim$ 0.02 Fermilab/MILC ; $\lesssim$ 0.05 HPQCD

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$\Rightarrow$ quadratic fits in recent lattice studies : good χ^2/dof $\lesssim$ 1 & zero-consistent quadratic coefficients

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- resonance masses & susceptibilities $\Rightarrow$ those in Bigi-Gambino-Schacht '17 $\Rightarrow$ comparison of $a^f{}_n$

  $\Rightarrow$ small systematic uncertainties on physical quantities $|V_{cb}|$, $R(D^*)$ (*e.g.* JLQCD)

fit to BGL parametrization (Boyd+ '97)

$$f(q^2) = \frac{1}{P_f(z)\phi_f(z)} \sum_n^{N_f} a_n^f z^n \qquad z = \frac{\sqrt{1+w}-\sqrt{2}}{\sqrt{1+w}+\sqrt{2}}$$

- small z parameter $\lesssim$ 0.015 JLQCD ; $\lesssim$ 0.02 Fermilab/MILC ; $\lesssim$ 0.05 HPQCD
- Blaschke factor $P_f \sim q^2\text{-}M_{\rm pole}^2 \Rightarrow$ ~~pole singularity~~ $\Rightarrow$ mild dependence of regular parts

  $\Rightarrow$ quadratic fits in recent lattice studies : good χ^2/dof $\lesssim$ 1 & zero-consistent quadratic coefficients
- outer function $\phi_f \Rightarrow$ (weak) unitarity bound $\Sigma_n |a^f{}_n|^2 \leq 1 \Rightarrow$ 3 studies just check in their fit results ...
- resonance masses & susceptibilities $\Rightarrow$ those in Bigi-Gambino-Schacht '17 $\Rightarrow$ comparison of $a^f{}_n$

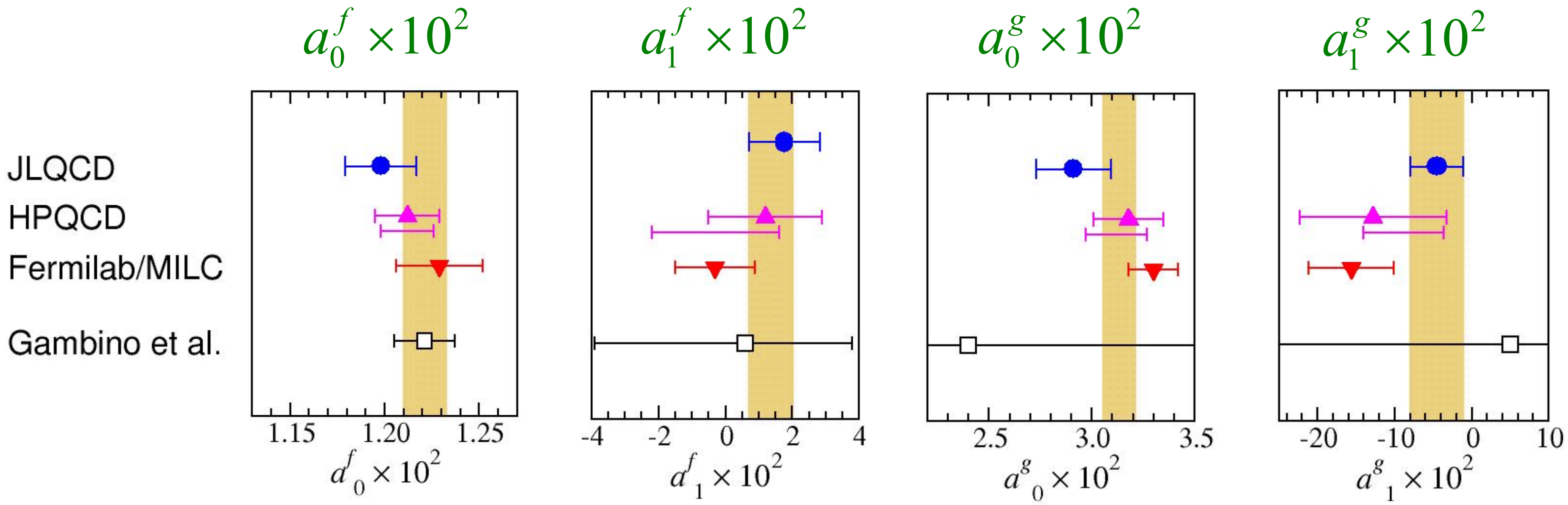
  $\Rightarrow$ small systematic uncertainties on physical quantities $|V_{cb}|$, $R(D^*)$ (*e.g.* JLQCD)
- kinematical constraints

  @ w=1: $\mathcal{F}_1(1) = (M_B\text{-}M_{D^*})\, f(1) \Rightarrow$ 3 studies : fix $a^{\mathcal{F}1}{}_0$

  @ $w_{\rm max}(m_\ell=0)$: $\mathcal{F}_1(w_{\rm max}) \Leftrightarrow \mathcal{F}_2(w_{\rm max}) \Rightarrow$ JLQCD: fix $a^{\mathcal{F}2}{}_0$; other two: confirm in their fit results

comparison of coefficients : *f* and *g*

constant terms $a^X{}_{0\ \mathrm{a}}$, linear coefficients $a^X{}_1$



Gambino et al. '19: BGL analysis (quadratic) of Belle data '18 + $h_{A1}(w\!=\!1)$ from lattice

Bordone+ '24: frequentist BGL fit to all lattice data $\Rightarrow$ FLAG '24 (?)

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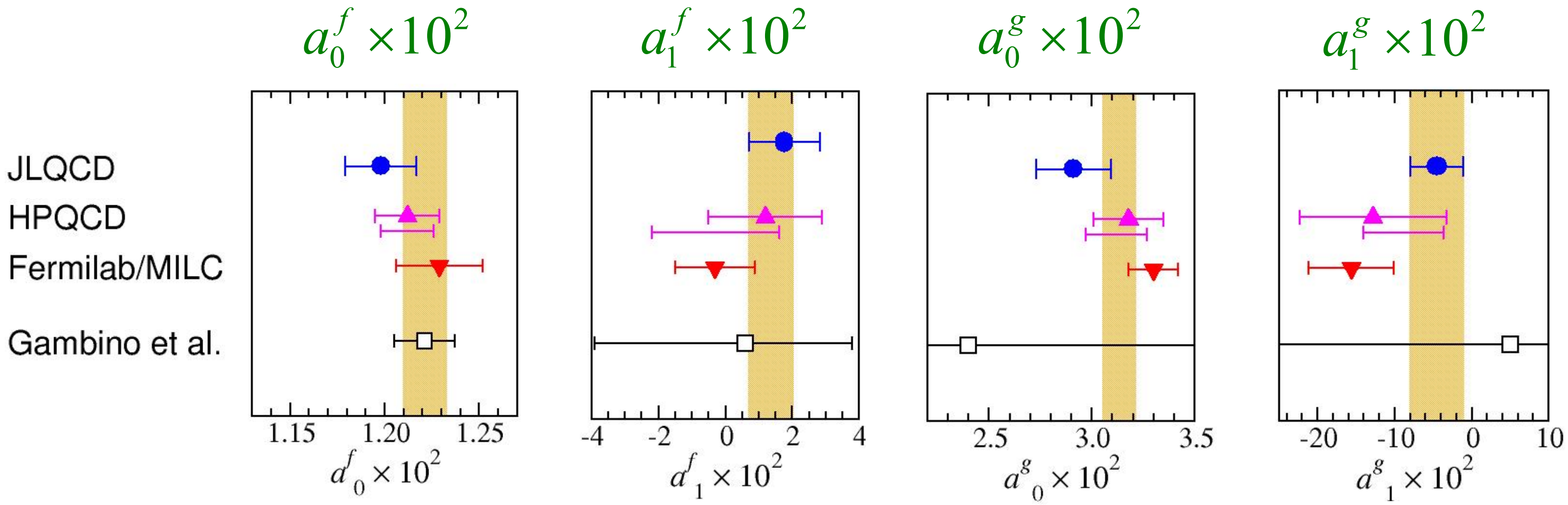

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Bordone+ '24: frequentist BGL fit to all lattice data $\Rightarrow$ FLAG '24 (?)

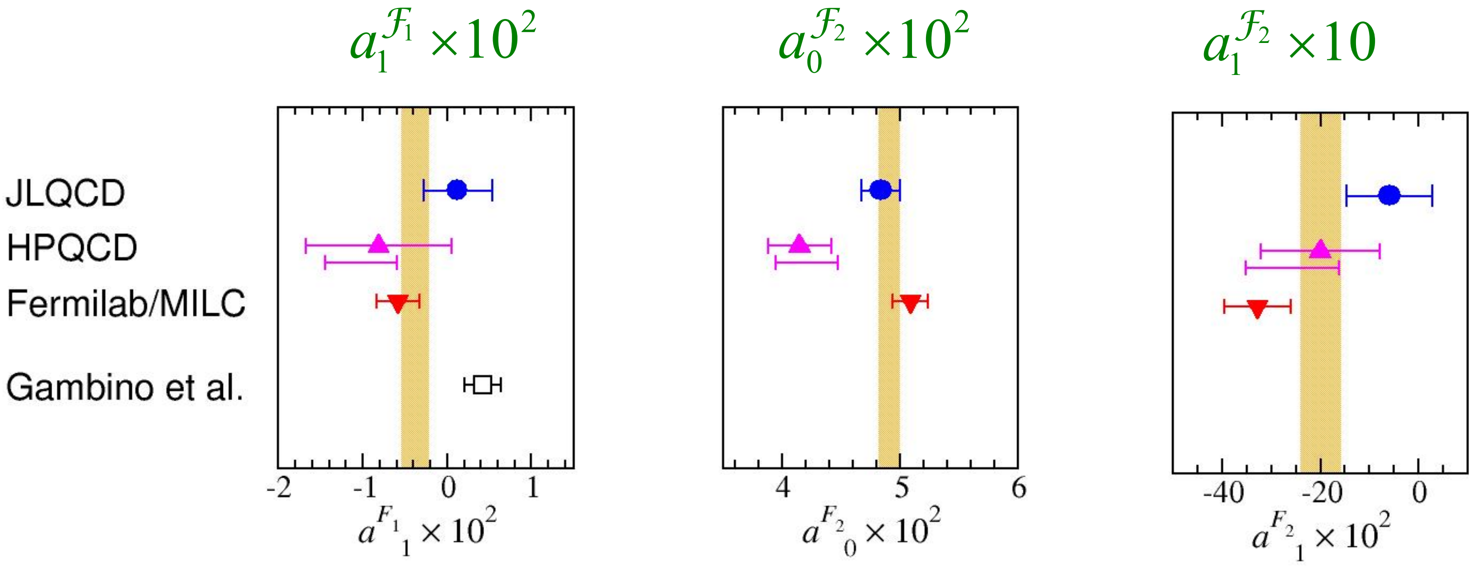
– $f \sim d\Gamma/dw$ @ $w \sim 1$, $g \sim \langle D^* | V_\mu | B \rangle$

+ lattice data @ $w \neq 1$ helpful in constraining BGL $\Leftrightarrow$ $a^f{}_0$: constrained by $h_{A1}(w=1)$ from lattice

+ reasonable agreement among independent lattice studies $\Leftrightarrow$ very different systematics

JLQCD $\Leftrightarrow$ Fermilab/MILC : 1.8σ for $a^g{}_0$; 1.7σ for $a^g{}_1$

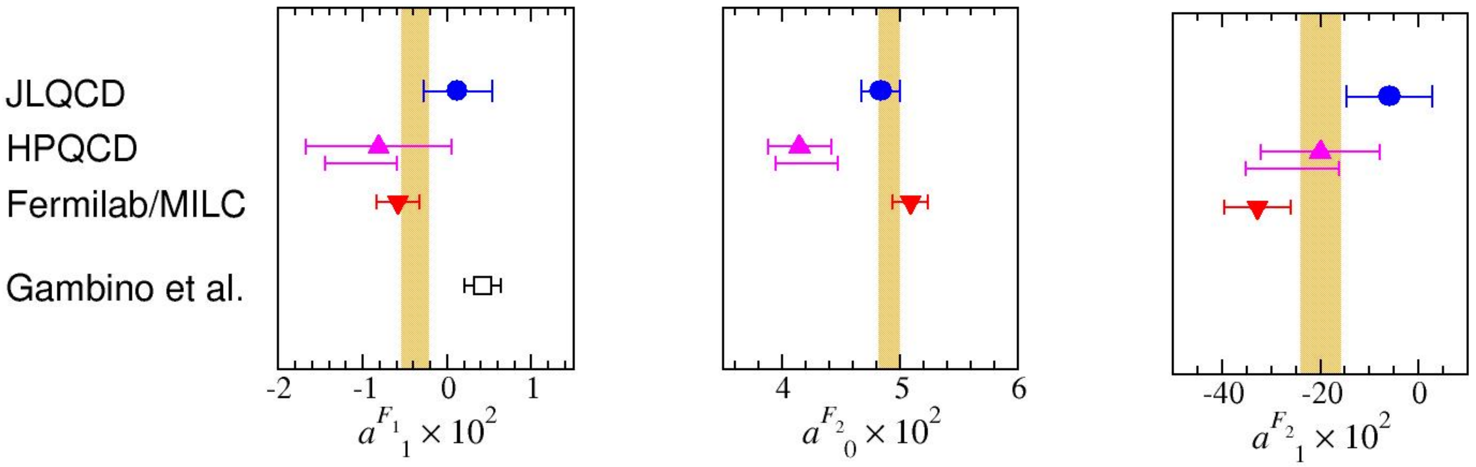
comparison of coefficients : $\mathcal{F}_1$, $\mathcal{F}_2$



- kinematical constraints @ w=1 $\Rightarrow a^{\mathcal{F}1}{}_0 \propto a^f{}_0$

comparison of coefficients : $\mathcal{F}_1$, $\mathcal{F}_2$

$a_1^{\mathcal{F}_1} \times 10^2$ $a_0^{\mathcal{F}_2} \times 10^2$ $a_1^{\mathcal{F}_2} \times 10$



- kinematical constraints @ w=1 $\Rightarrow a^{\mathcal{F}1}{}_0 \propto a^f{}_0$

– $\mathcal{F}_1$ ~ h_{A2}, h_{A3} @ $w \neq 1$

  + well constrained by exp'tal data, consistent w/ JLQCD; Fermillab/MILC ???

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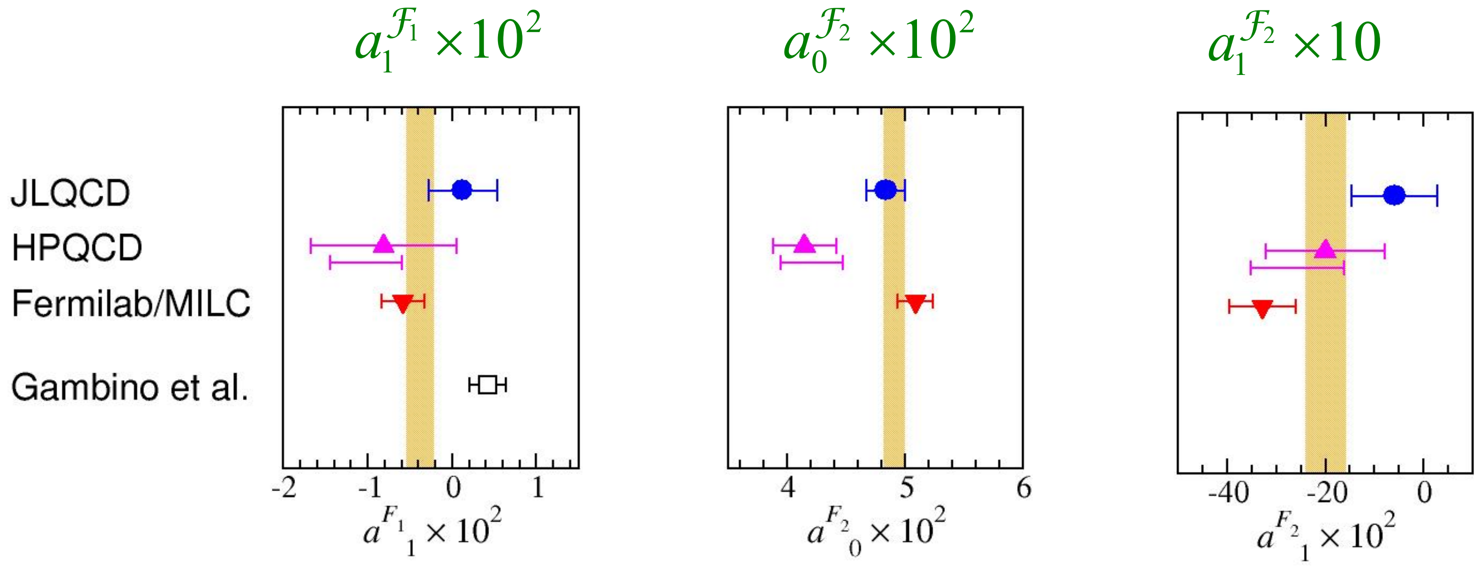


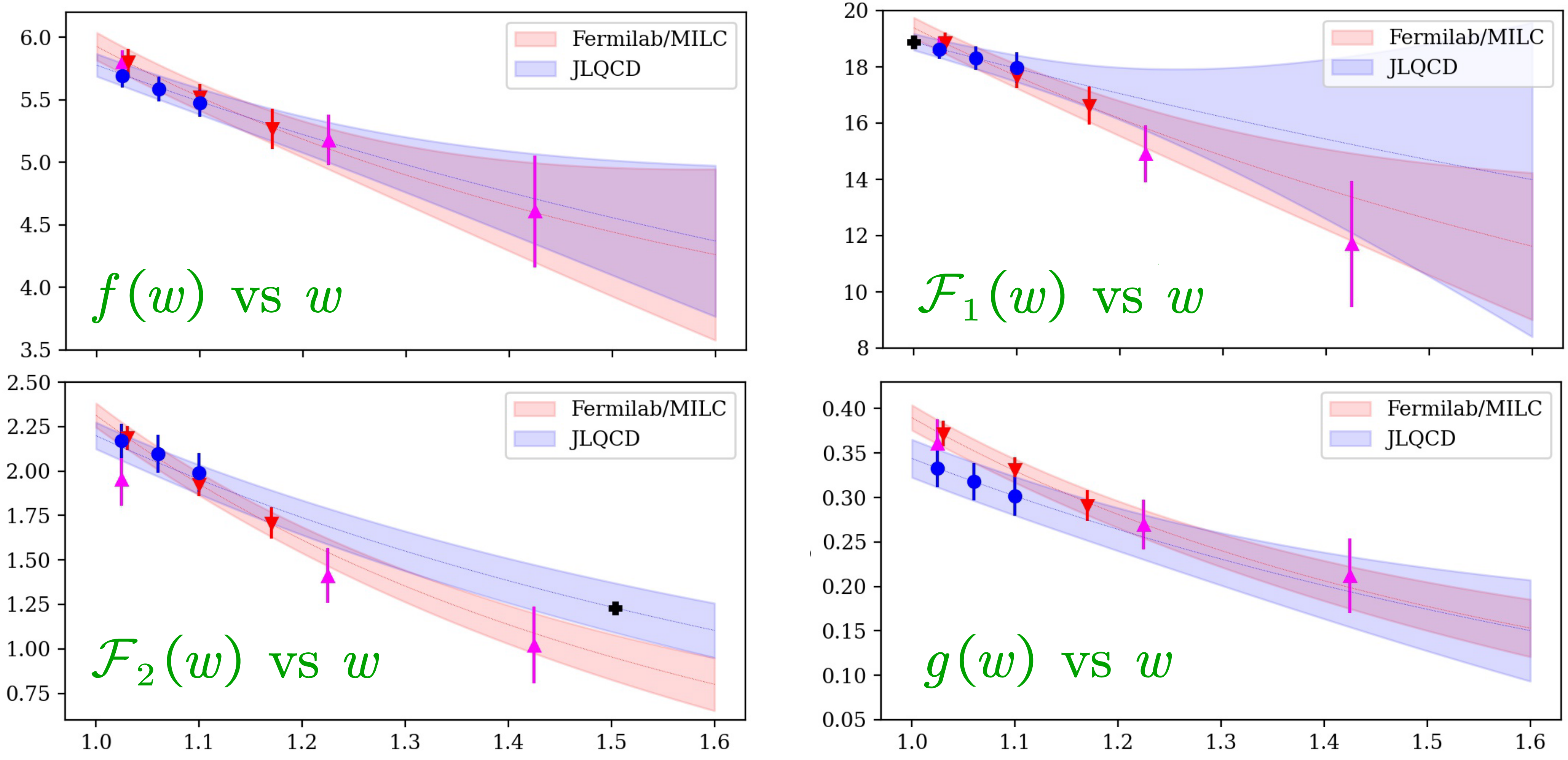
- kinematical constraints @ w=1 $\Rightarrow a^{\mathcal{F}1}{}_0 \propto a^f{}_0$

– $\mathcal{F}_1 \sim h_{A2},\ h_{A3}$ @ $w \neq 1$
  - well constrained by exp'tal data, consistent w/ JLQCD; Fermillab/MILC ???

– $\mathcal{F}_2 \sim O(m_l^{\,2})$ to $d\Gamma/dw$
  - poorly constrained by exp't (e,μ) / input to new physics search by τ channel
  - well constrained by lattice studies but tension among them

    $a^{\mathcal{F}2}{}_0$: 3.0σ b/w Fermilab/MILC ⇔ HPQCD; $a^{\mathcal{F}2}{}_1$: 2.4σ b/w Fermilab/MILC ⇔ JLQCD

BGL fit curves



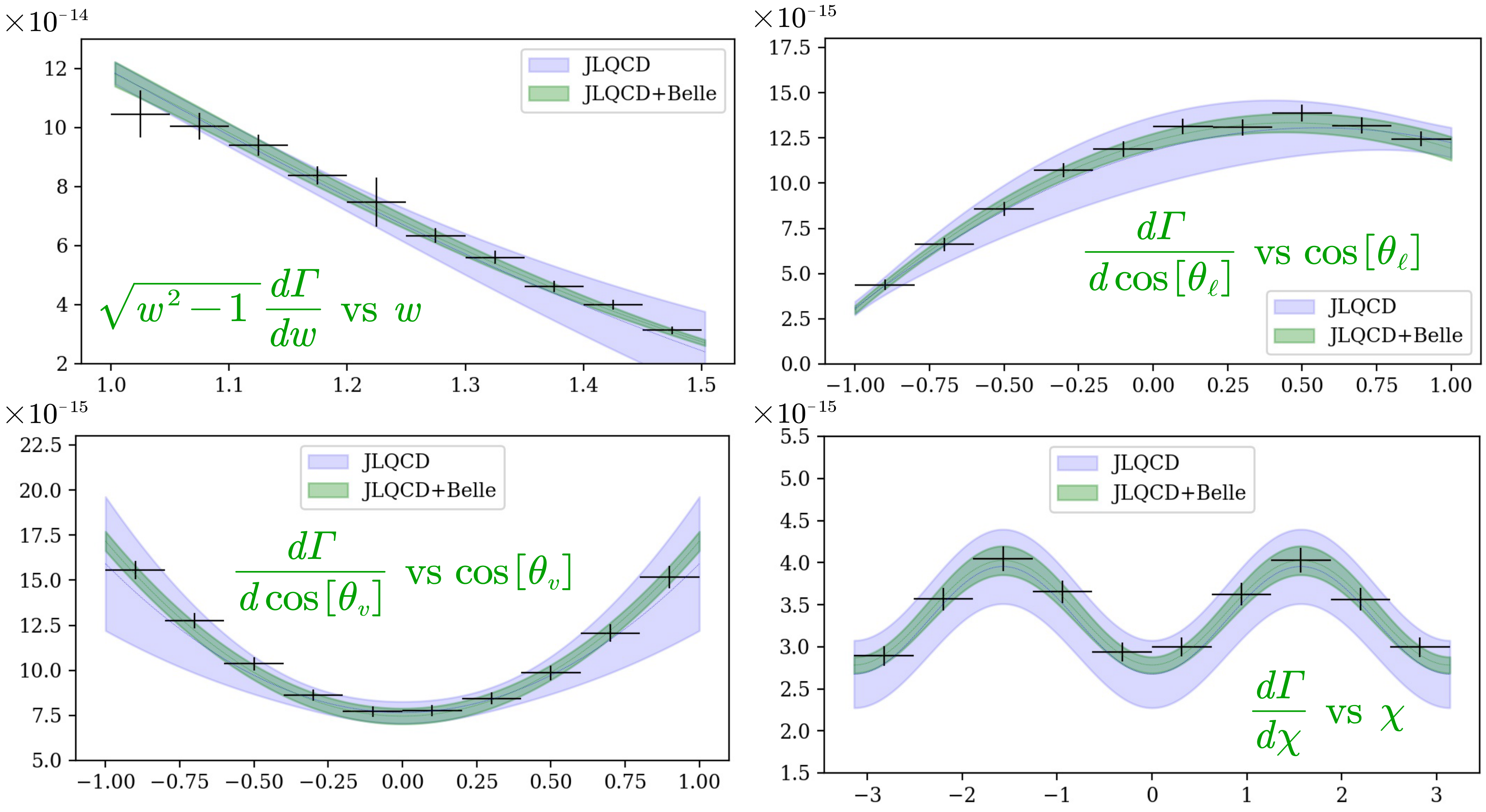
only synthetic data for HPQCD

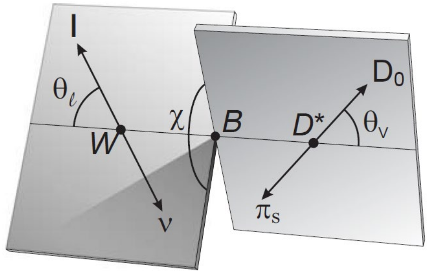
- 2σ consistency among lattice studies
- constraint at $w_{\max}$: helpful to constrain JLQCD's extrapolation of $\mathcal{F}_2$ to large recoils

observables : $d\Gamma/dX$, $|\mathrm{V}_{cb}|$, $R(D^{*})$

comparison w/ Belle

differential decay rate w.r.t. w and 3 decay angles





blue : fit to JLQCD data

⊗ an input value of $|V_{cb}|$

good consistency w/ Belle

green: fit to JLQCD+Belle

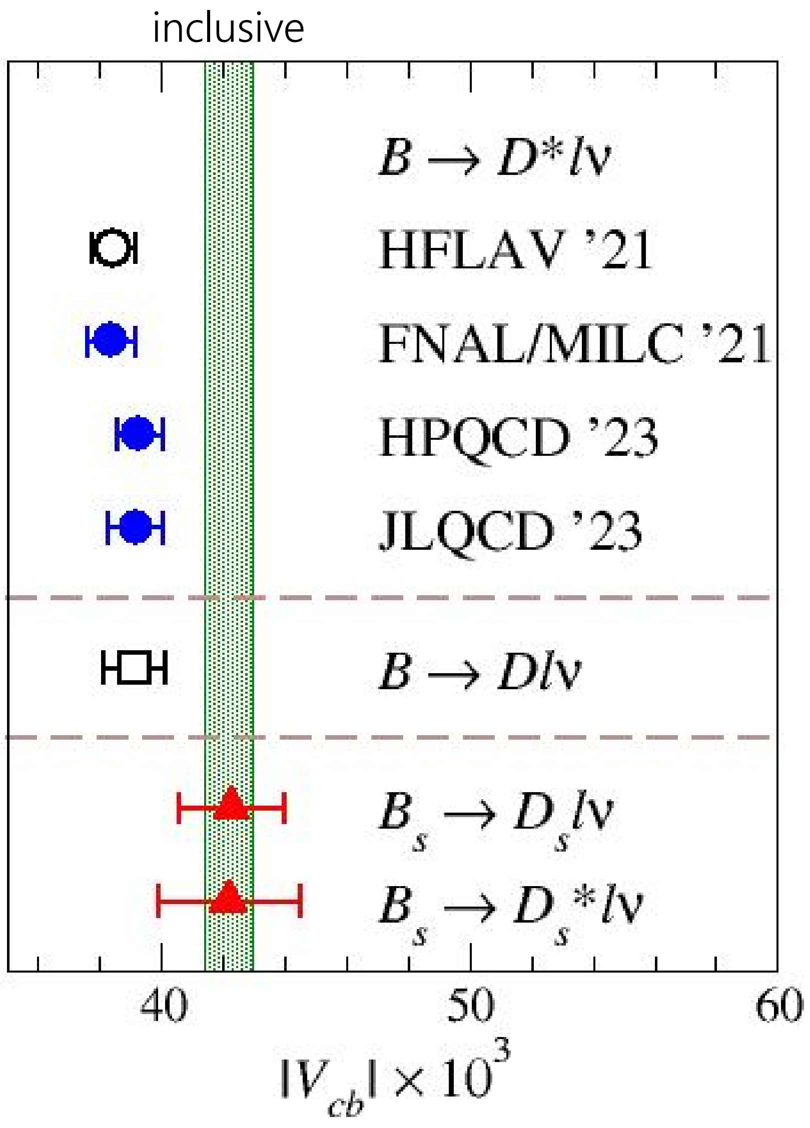
JLQCD and Belle data

determination of $|V_{cb}|$

good consistency w/ Belle; extension of JLQCD to larger w ⇒ precise th. predictions

$|V_{cb}|$ and $R(D^*)$

$|V_{cb}|$ from $B_{(s)}$ decays



– consistency among 3 studies and previous determination w/ $h_{A1}(1)$

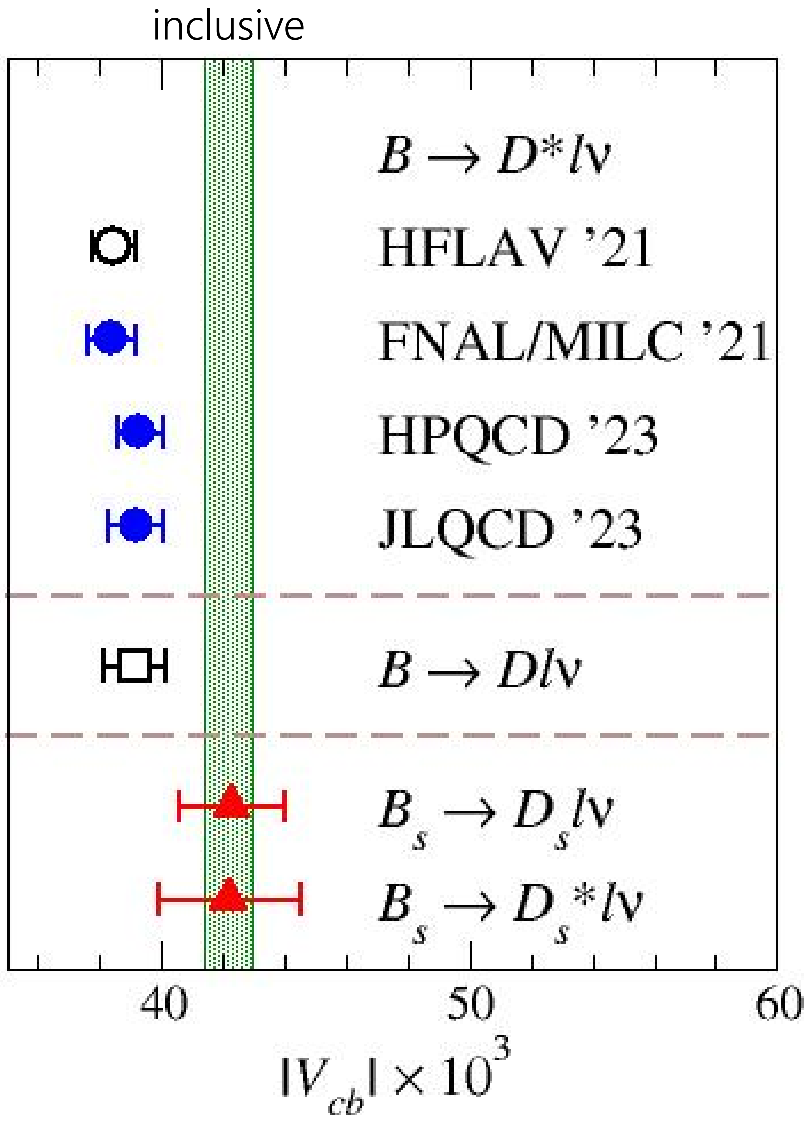
$\Rightarrow$ $|V_{cb}|$ tension remains

+ truncation of BGL expansion
+ D'Agostini bias

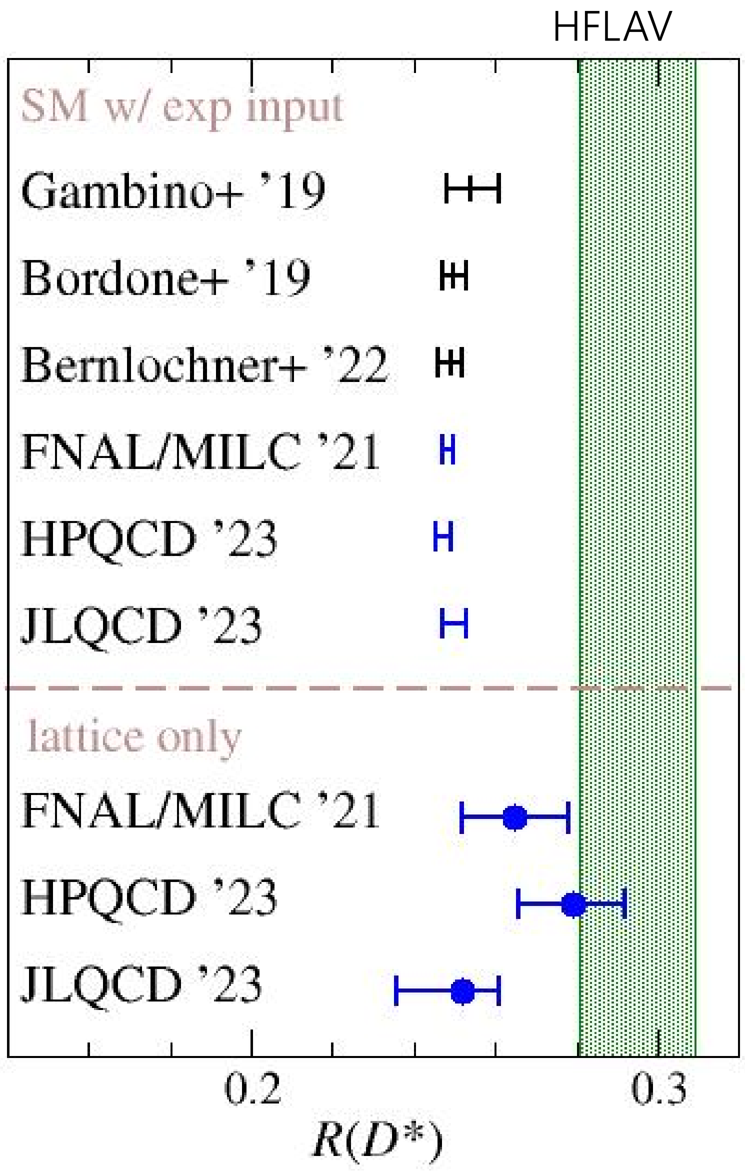
$|V_{cb}|$ and $R(D^*)$

$|V_{cb}|$ from $B_{(s)}$ decays

$R(D^*) = \Gamma(B \to D^* \tau\nu)/\Gamma(B \to D^* \ell\nu) \;\; (\ell = e, \mu)$



- consistency among 3 studies and previous determination w/ $h_{A1}(1)$
  - $\Rightarrow$ $|V_{cb}|$ tension remains
    - + truncation of BGL expansion
    - + D'Agostini bias
- $R(D^*)$ w/ exp input : 3σ from exp
- pure SM value of $R(D^*)$
  - $\Rightarrow$ average 0.267(8) 3%, 1.7σ from exp
    - + more precise $\mathcal{F}_2$ up to w_{max}^{τ}
    - + extention to larger recoils $w_{max}^{\{e,\mu\}}$



future possibilities of JLQCD

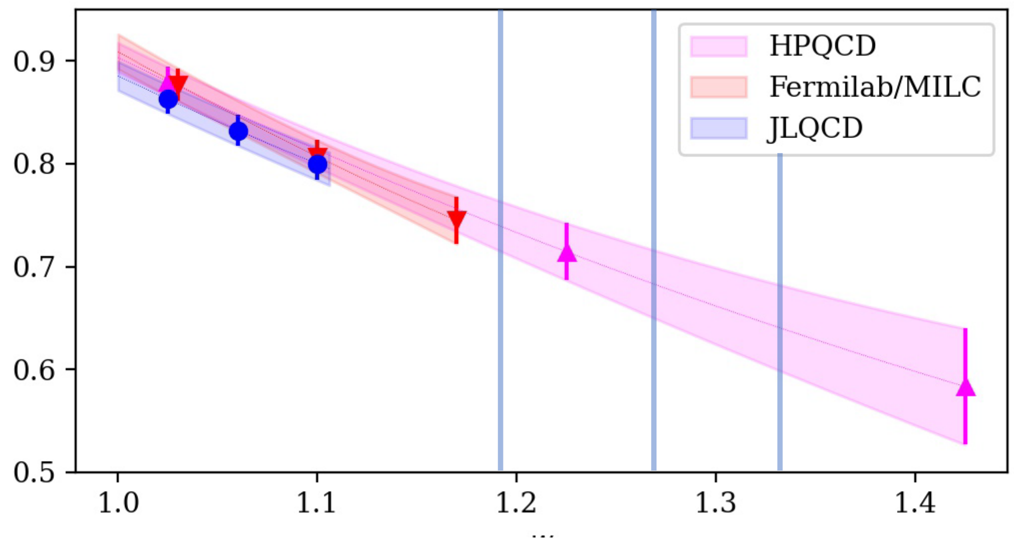
short term

– extension to large recoils

+ JLQCD: limit n for $|a\mathbf{p}| = (2\pi/N_s)n$ as $a \to 0$

HPQCD like: keep $|a\mathbf{p}| \propto n/N_s$ as $a \to 0 \;\Rightarrow\; w \le 1.33$

+ need much more statistics: #tsrc = 2 → 16



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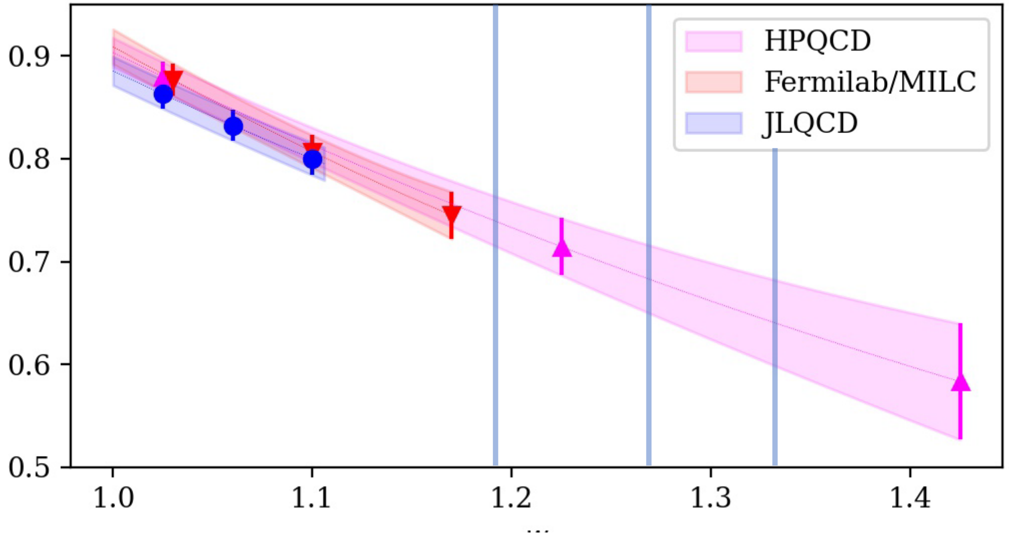

- correlator ratios for h_{A2} and h_{A3} $\Rightarrow$ more precise $\mathcal{F}_1$, $\mathcal{F}_2$

$$\langle D^*(p',\varepsilon')|A_\mu|B(p)\rangle = \varepsilon'_\mu(w+1)h_{A_1}(w) - \varepsilon' v\{v_\mu h_{A_2}(w) + v'_\mu h_{A_3}(w)\}$$

w/ $\mathbf{p}=\mathbf{0}$ $\Rightarrow$ limit (p′,ε′)'s

$\Rightarrow$ need study efficiency on the existing ensembles

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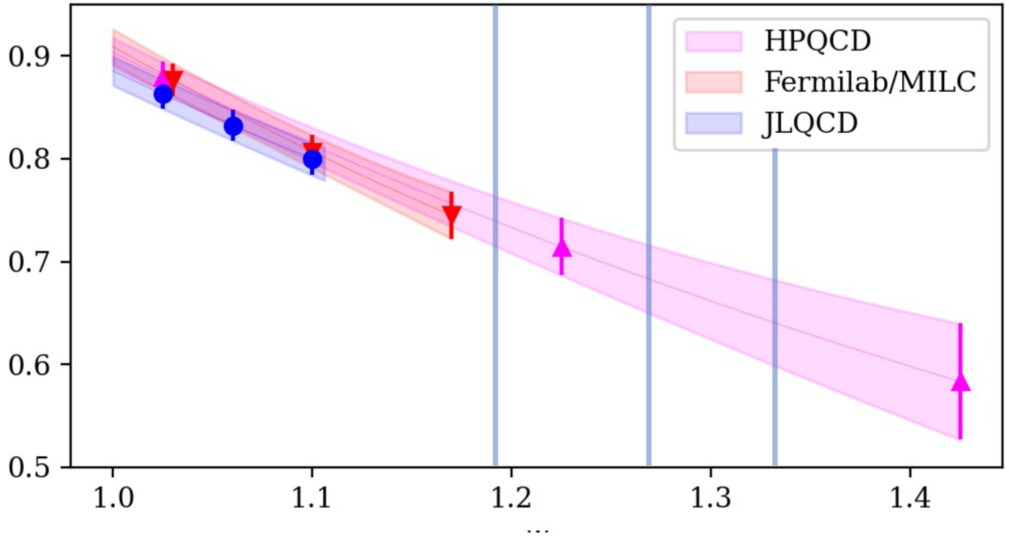

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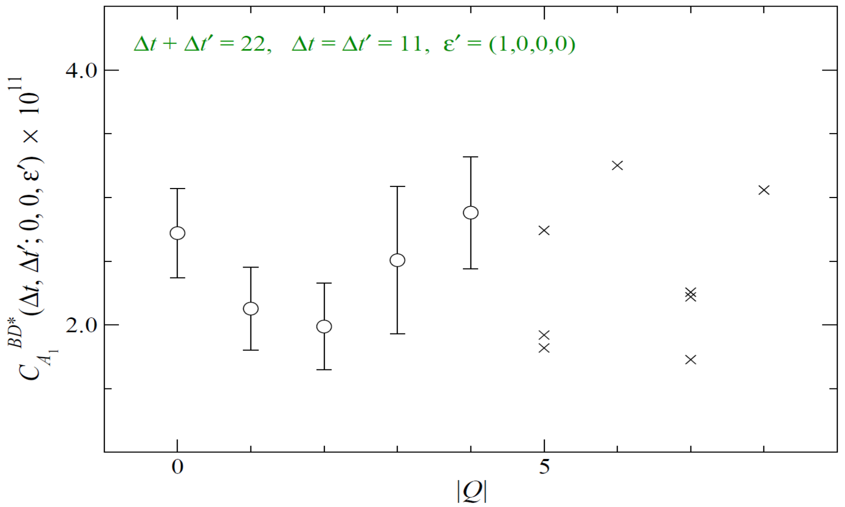
⇒ need study efficiency on the existing ensembles

long term

– simulating a finer lattice

+topology freezing? ⇒ Q dependence not large



summary

$B \to D^* \ell \nu$ from the perspectives of JLQCD

JLQCD's approach

– independent study w/ a theoretically-clean relativistic formulation to control systematics

comparison w/ Fermilab/MILC and HPQCD

– 2-3σ tension in BGL coefficients, but 2σ consistency for FFs

– systematic error as |"fit A" – "fit B"| = "1σ"?

future possibilities

– larger recoils; more precise calculation of $\mathcal{F}_1$, $\mathcal{F}_2$