$B \to D^* \ell \nu$ at nonzero recoil from Fermilab-MILC

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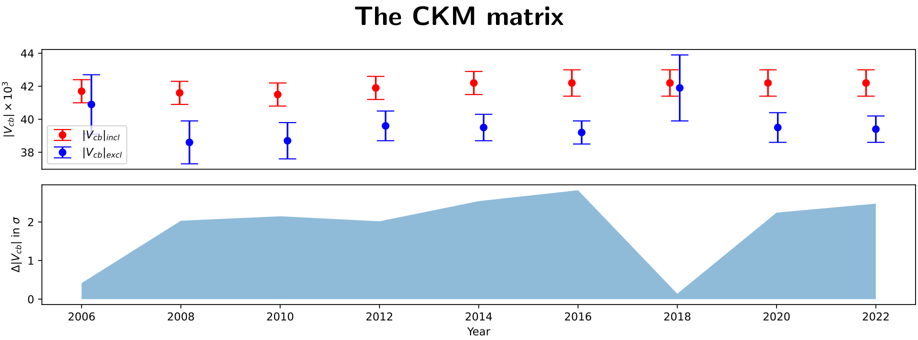
The motivation

Motivation: CKM matrix elements



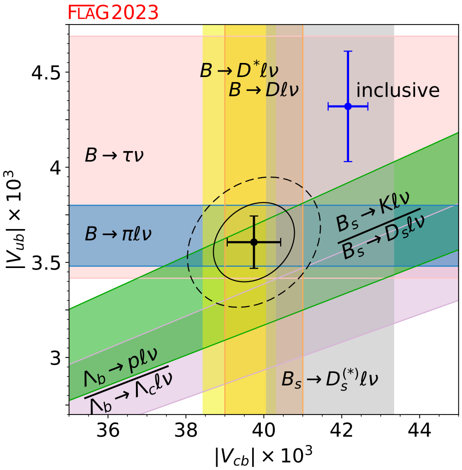
- Current values (PDG 2024):

$$|V_{cb}|_{\text{excl}} \times 10^{-3} = 39.8(6)$$

$$|V_{cb}|_{\text{incl}} \times 10^{-3} = 42.2(5)$$

- The 3σ difference between these two values shows that we have not improved much

Motivation: CKM matrix elements

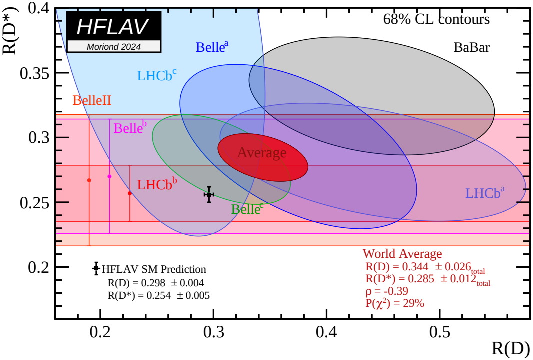


Strong arguments disfavoring new physics

Phys. Rev. Lett. 114, 011802 (2015)

Motivation: Tensions in LFU ratios

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3.3\sigma$ combined tension with the SM (HFLAV)
  - Tension in $R(D) \approx 1.6\sigma$ Tension in $R(D^*) \approx 2.5\sigma$

The theory

Semileptonic B decays on the lattice: Exclusive $|V_{cb}|$

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^* \ell \bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{K_1(w, m_\ell \approx 0)}_{\text{Known factors}} \underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} |V_{cb}|^2 , \quad w = v_{D^*} \cdot v_B$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q \to \infty} \mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2 - 1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $|V_{cb}|^2 |\eta_{ew} \mathcal{F}(w)|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

Semileptonic B decays on the lattice: Universality ratios

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^* \ell \bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w, m_\ell)}_{\text{Known factors}} \underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w, m_\ell)}_{\text{Known factors}} \underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right] \times |V_{cb}|^2$$

- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
- Since $K_2(w, 0) = 0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
  - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw \left[K_1(w, m_\tau)|\mathcal{F}(w)|^2 + K_2(w, m_\tau)|\mathcal{F}_2(w)|^2\right] \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw \left[K_1(w, 0)|\mathcal{F}(w)|^2\right] \times \cancel{|V_{cb}|^2}}$$

- $|V_{cb}|$ cancels out

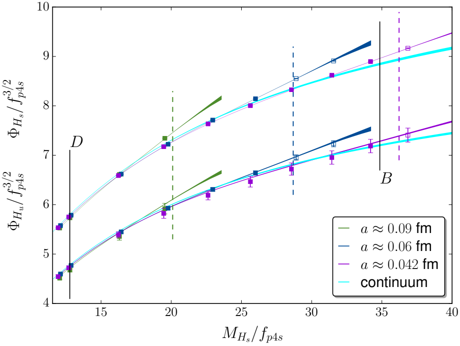
Semileptonic B decays on the lattice: Heavy quarks

- Heavy quark treatment in Lattice QCD
  - For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
  - For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
    - Typically $O(a^2 m_Q^2)$
- Need special actions and ETs to describe the bottom quark
  - Feasible to reach physical bottom quark masses
  - Require matching between EFT and lattice, complex renormalization, etc
- If the action is improved enough, one can treat the bottom as a light quark
  - Highly improved action AND small lattice spacing, so $O(a^2 m_Q^2)$ is small
  - Most cases use unphysical values for m_b and extrapolate
    - Assuming $am_Q \approx 0.65$, we need at least $a \approx 0.03$ fm to reach the physical value of m_b (unrenormalized)

Semileptonic B decays on the lattice: Heavy quarks

- HISQ fermions from Fermilab/MILC — Phys.Rev.D 98 (2018) 7, 074512; Phys.Rev.D 107 (2023) 9, 094516
- From HPQCD — Phys.Rev.D 75 (2007) 054502; Phys.Rev.D 87 (2013) 3, 034017
- Errors **start** at $O(\alpha_s v a^2 m_Q^2)$, one order of magnitude smaller than $O(a^2 m_Q^2)$
- Reasonable correction, even at large am_Q, without ap issues
- HISQ corrects at all orders, theoretical limit with fine tuning $am_Q = \pi/2$

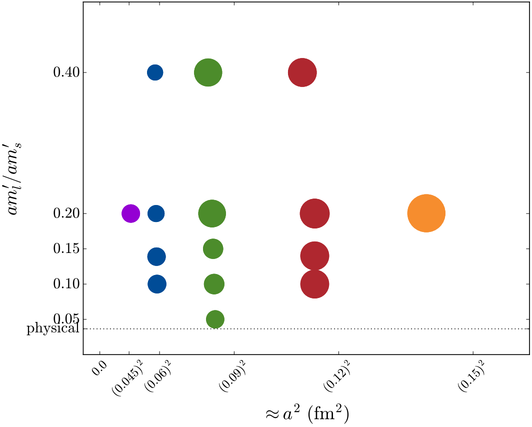
$$a \approx 0.042 - 0.088 \text{ fm}, \ M_\pi \approx 135 \text{ MeV}, \ am_Q \lesssim 1.3, \ m_Q \sim m_b$$



The calculation

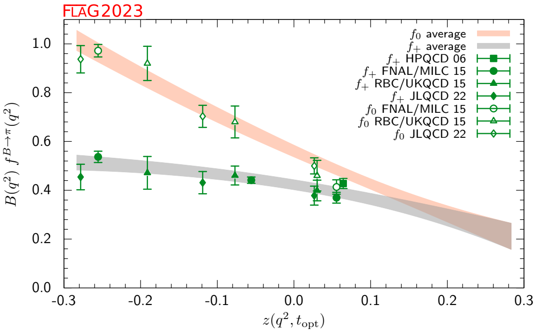
$B \to D^* \ell \nu$: Setup

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action

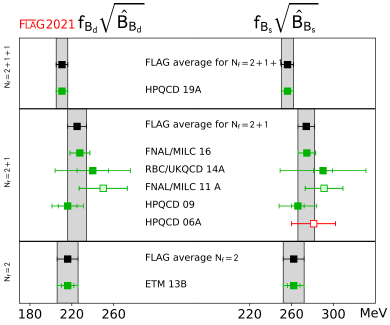


$B \to D^* \ell \nu$: Asqtad ensembles

- The asqtad data is being superseded by newer data with improved actions
  - 2^{nd} generation $N_f = 2+1+1$ HISQ and Fermilab charm/bottom quarks
  - 3^{rd} generation $N_f = 2+1+1$ HISQ and a HISQ bottom quark
- Some results from the asqtad ensembles are still competitive today



PRD**92**, (2015) 014024, arXiv:1503.07839



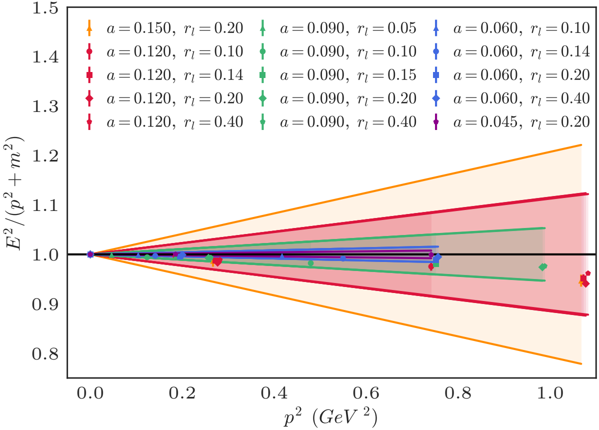
PRD**93**, (2016) 113016, arXiv:1602.03560

This is the last analysis done with asqtad data

$B \to D^* \ell \nu$: Dispersion relation for heavy mesons

$$a^2 E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- Heavy quark discretization effects break the dispersion relation
- Deviations from the continuum expression measure the size of the discretization errors



$B \to D^*\ell\nu$: Ratios and excited states

Ratios

$$\frac{\langle D^*(p)|\mathbf{V}|D^*(0)\rangle}{\langle D^*(p)|V_4|D^*(0)\rangle} \to x_f \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\langle D^*(p_\perp,\varepsilon_\parallel)|\mathbf{A}|\bar{B}(0)\rangle\langle \bar{B}(0)|\mathbf{A}|D^*(p_\perp,\varepsilon_\parallel)\rangle}{\langle D^*(0)|V_4|D^*(0)\rangle\langle \bar{B}(0)|V_4|\bar{B}(0)\rangle}^* \to R_{A_1}^2 \qquad h_{A_1} = \left(1-x_f^2\right) R_{A_1}$$

$$\frac{\langle D^*(p_\perp,\varepsilon_\perp)|\mathbf{V}|\bar{B}(0)\rangle}{\langle D^*(p_\perp,\varepsilon_\parallel)|\mathbf{A}|\bar{B}(0)\rangle} \to X_V \qquad h_V = \frac{w+1}{w-1} h_{A_1} X_V$$

$$\frac{\langle D^*(p_\parallel,\varepsilon_\parallel)|\mathbf{A}|\bar{B}(0)\rangle}{\langle D^*(p_\perp,\varepsilon_\parallel)|\mathbf{A}|\bar{B}(0)\rangle} \to R_1 \qquad h_{A_3} = h_{A_1}\frac{w-R_1}{w-1}$$

$$\frac{\langle D^*(p_\perp,\varepsilon_\parallel)|A_4|\bar{B}(0)\rangle}{\langle D^*(p_\perp,\varepsilon_\parallel)|\mathbf{A}|\bar{B}(0)\rangle} \to R_0 \qquad h_{A_2} = h_{A_1}\frac{wR_1-\sqrt{w^2-1}R_0-1}{w-1}$$

* Phys.Rev. **D**66, 01503 (2002)

- B meson always smeared $\to$ suppresses excited states
- Excited states explicitly fitted

$B \to D^*\ell\nu$: Chiral-continuum extrapolation

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

$$h_{A_1}(w) = \underbrace{\left[1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2}\text{logs}_{\text{SU3}}(a, m_l, m_s, \Lambda_{QCD})\right.}_{\text{NLO }\chi\text{PT} + \text{HQET}}$$

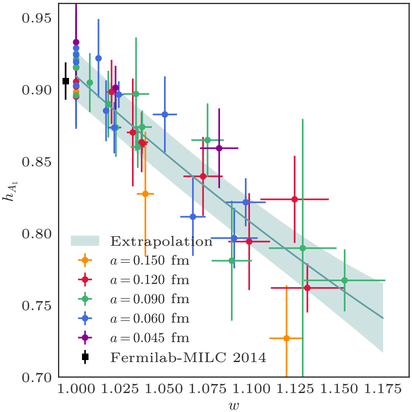
$$\left.\underbrace{+c_1 x_l + c_{a1} x_{a^2}}_{\text{NLO }\chi\text{PT}} \underbrace{-\rho^2_{A_1}(w-1) + k_{A_1}(w-1)^2}_{w\text{ dependence}} \underbrace{+ c_2 x_l^2 + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\text{NNLO }\chi\text{PT}}\right] \times$$

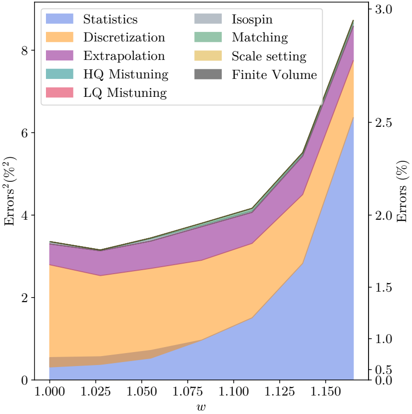
$$\underbrace{\left(1 + \beta_{11}^{A_1}\alpha_s a \Lambda_{\text{QCD}} + \cancel{\beta_{02}^{A_1} a^2 \Lambda^2_{\text{QCD}}} + \beta_{03}^{A_1} a^3 \Lambda^3_{\text{QCD}}\right)}_{\text{HQ discretization errors}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

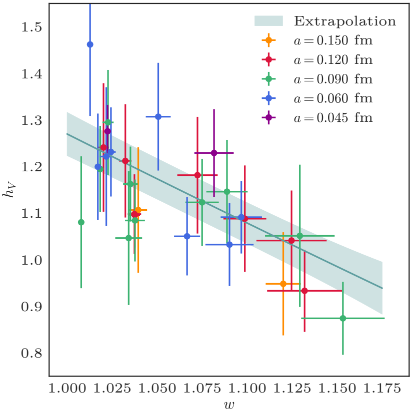
$B \to D^*\ell\nu$: Chiral-continuum extrapolation

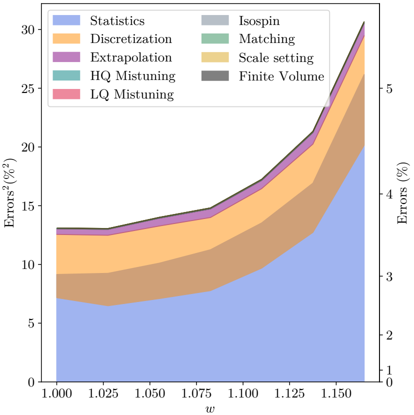




- Combined fit $\chi^2/\mathrm{dof} = 85.2/92$
- $h_{A_1}(1) = 0.909(17)$

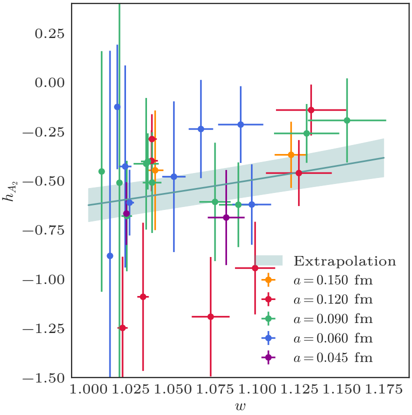
$B \to D^* \ell \nu$: Chiral-continuum extrapolation

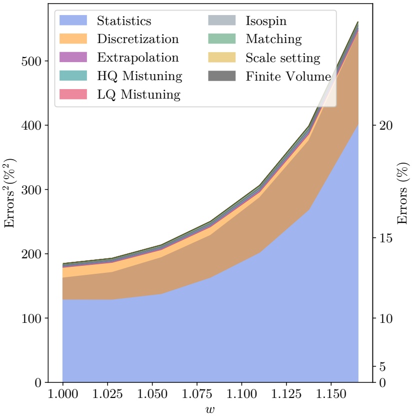




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_V(1) = 1.270(48)$

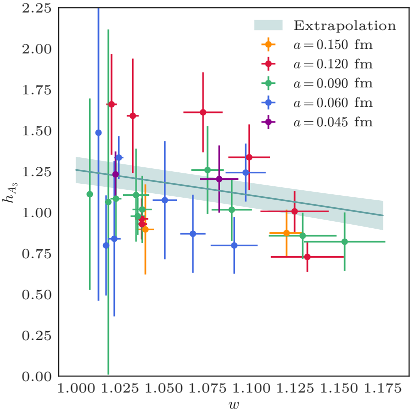
$B \to D^*\ell\nu$: Chiral-continuum extrapolation

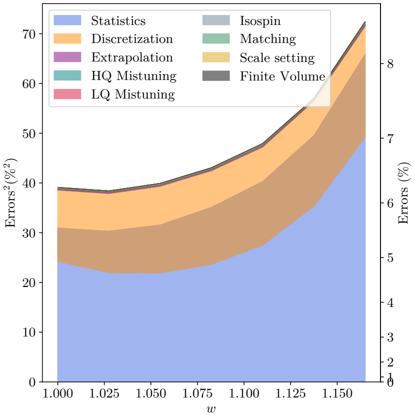




- Combined fit $\chi^2/\mathrm{dof} = 85.2/92$
- $h_{A_2}(1) = -0.624(85)$

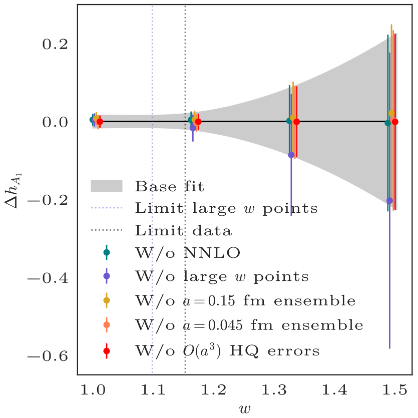
$B \to D^* \ell \nu$: Chiral-continuum extrapolation

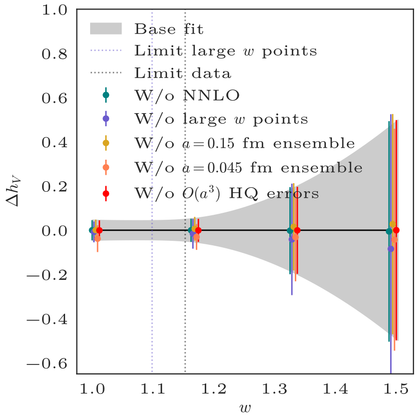




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_{A_3}(1) = 1.259(79)$

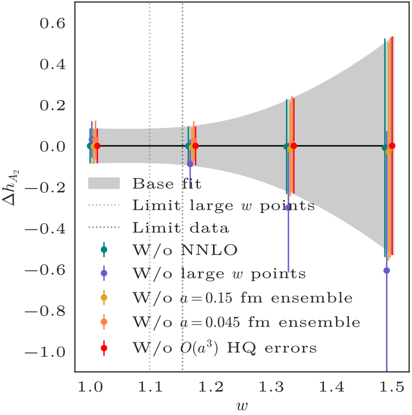
$B \to D^* \ell \nu$: Chiral-continuum extrapolation

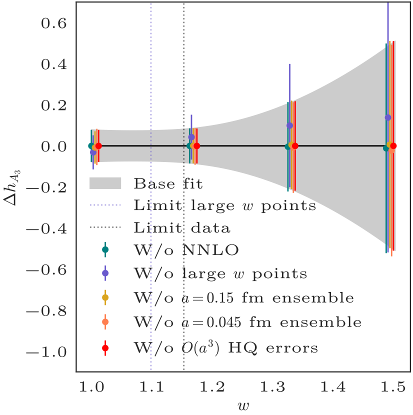




	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.2/92**	86.0/107	71.1/75	79.4/86
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.6/86	85.3/99	

$B \to D^* \ell \nu$: Chiral-continuum extrapolation





	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	$\mathbf{85.2/92}$	86.0/107	71.1/75	79.4/86
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.6/86	85.3/99	

$B \to D^*\ell\nu$: z expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z)B_g(z)}\sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z)B_f(z)}\sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2}H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z)B_{\mathcal{F}_1}(z)}\sum_j c_j z^j$$

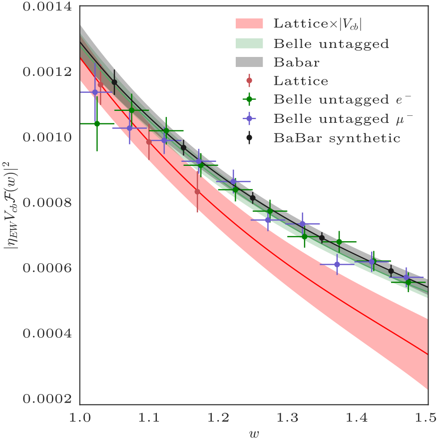
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}}H_S = \frac{1}{\phi_{\mathcal{F}_2}(z)B_{\mathcal{F}_2}(z)}\sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*})f(z=0)$
- Constraint $(1+w)m_B^2(1-r)\mathcal{F}_1(z=z_{\rm Max}) = (1+r)\mathcal{F}_2(z=z_{\rm Max})$
- BGL (weak) unitarity constraints

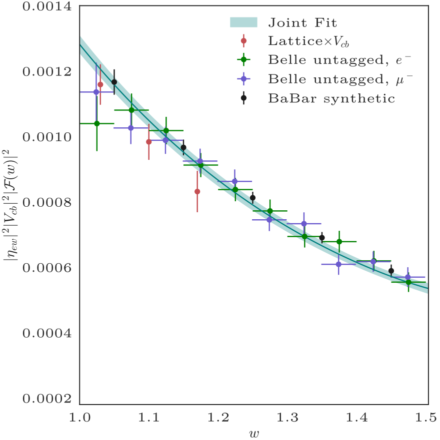
$$\sum_j a_j^2 \le 1, \qquad \sum_j b_j^2 + c_j^2 \le 1, \qquad \sum_j d_j^2 \le 1$$

$B \to D^*\ell\nu$: BGL fits

Separate fits



Joint fit

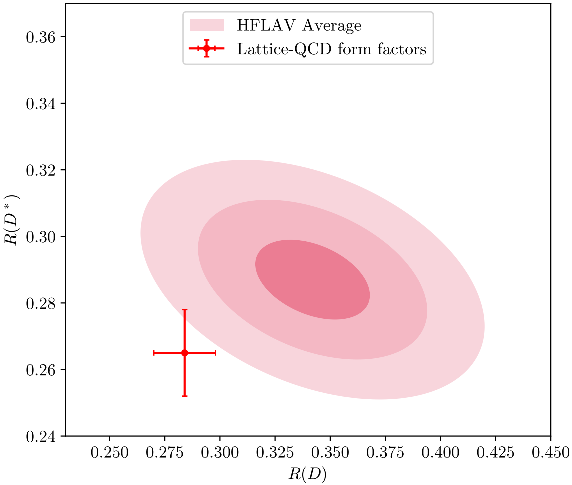


Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
χ^2/dof	$0.63/1$	$104/76$	$111/79$	$8.50/4$	$126/84$

Unblinded, final result $|V_{cb}| = 38.40(78) \times 10^{-3}$

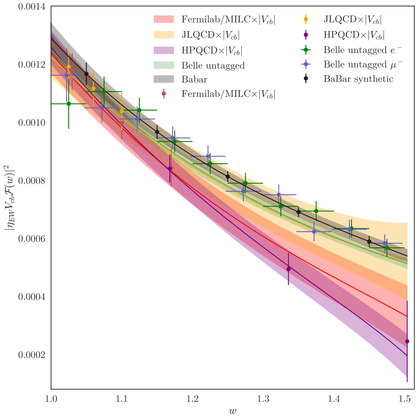
$B \to D^*\ell\nu$: $R(D^*)$

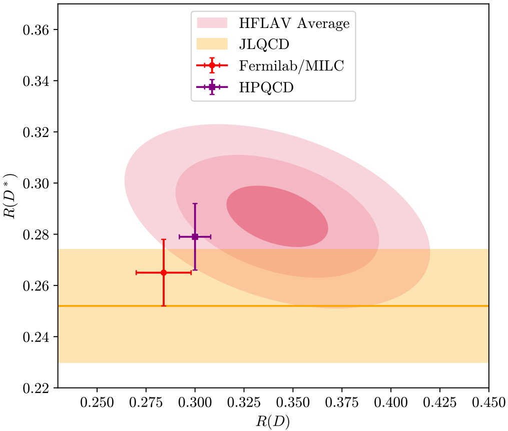
$$R(D^*)_{\mathbf{Lat}} = 0.265(13)$$



The mess

The mess: Lattice results





$$|V_{cb}|^{\mathrm{FM}} = 38.40(78)\times 10^{-3}$$

$$|V_{cb}|^{\mathrm{JLQCD}} = 39.19(90)\times 10^{-3}$$

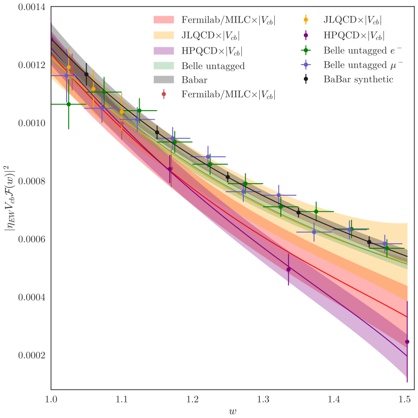
$$|V_{cb}|^{\mathrm{HPQCD}} = 39.31(74)\times 10^{-3}$$

$$R(D^*)^{\mathrm{FM}} = 0.265(13)$$

$$R(D^*)^{\mathrm{JLQCD}} = 0.252(22)$$

$$R(D^*)^{\mathrm{HPQCD}} = 0.279(13)$$

The mess: Lattice results



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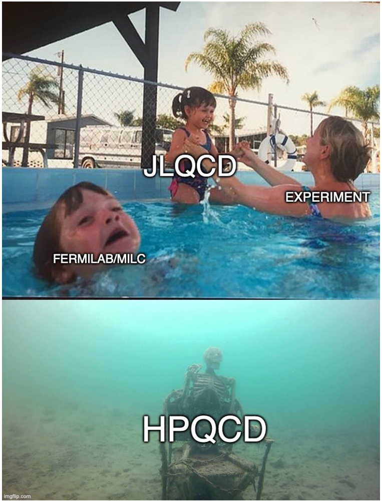
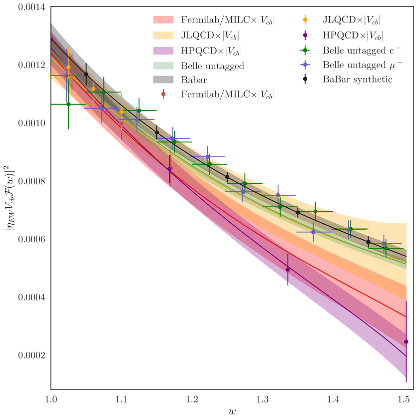


The mess: Lattice results



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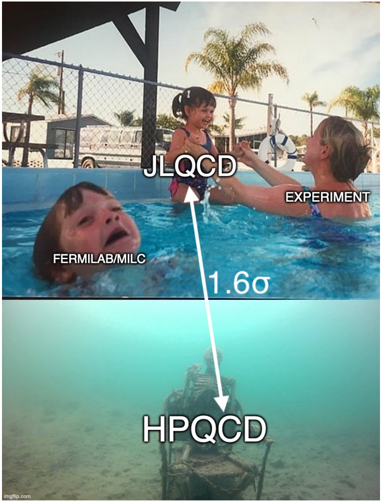


The mess: Combined lattice fits

- Combined fits with priors 0(1)
- Kinematic constraint imposed with priors
- BGL fit 2222

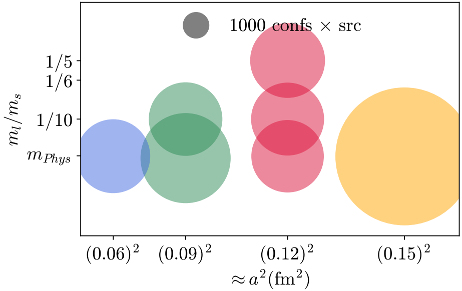
	w Constraint		w/o Constraint	
	p	$R_2(1)$	p	$R_2(1)$
MILC	0.51	1.20(12)	0.43	1.27(13)
JLQCD	0.52	0.98(19)	0.25	0.97(19)
HPQCD	0.77	1.39(16)	0.65	1.39(16)
MILC+JLQCD	0.40	1.118(97)	0.36	1.16(11)
MILC+HPQCD	0.44	1.262(93)	0.37	1.262(93)
JLQCD+HPQCD	0.73	1.18(12)	0.67	1.18(12)
All	0.56	1.193(83)	0.50	1.193(83)

- p-value of Belle untagged + BaBar BGL fit 223 is ≈ 0.04
- Combined $R(D^*) = 0.2667(57)$

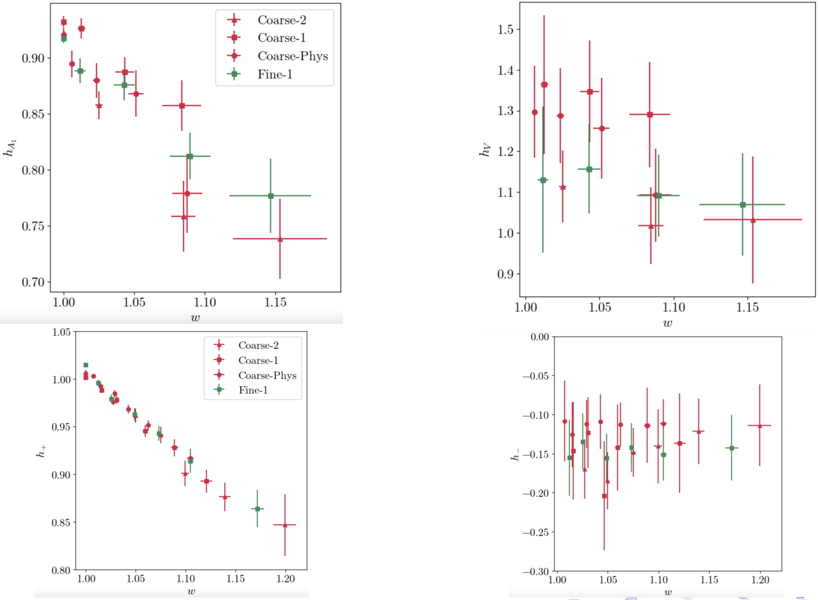
The future

Future projects: HISQ + Fermilab

- Fermilab/MILC calculation
- Using 7 $N_f = 2+1+1$ ensembles of sea HISQ quarks
- The heavy quarks use the Fermilab effective action
  - Correlated with a $B \to L\ell\nu$ analysis using the same data
  - Four channels in a single correlated analysis

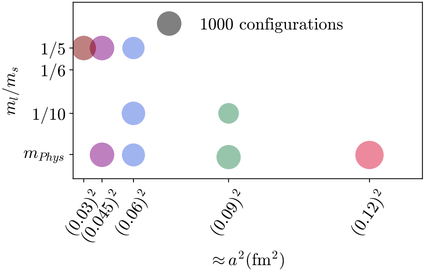


Future projects: HISQ + Fermilab

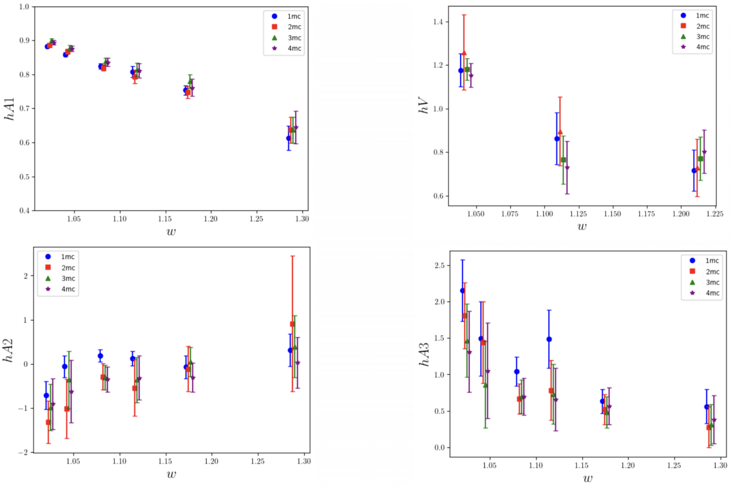


Future projects: HISQ^2

- Fermilab/MILC calculation
- Planning to use 9 $N_f = 2 + 1 + 1$ ensembles of sea HISQ quarks
- The heavy quarks use the HISQ action
  - Physical bottom mass reachable with the finest ensembles
- m_π physical in several ensembles



Future projects: HISQ^2



Preliminary results $B_s \to D_s^* \ell \nu$, statistics 24×426

Single ensemble $a = 0.06$ fm and $m_l/m_s = \frac{1}{5}$ at different values of am_b

Conclusions

- Great progress in LQCD calculations of $B \to D^* \ell \nu$ form factors
- Good agreement between different LQCD results
  - Not so good between LQCD and experiment
- New calculations are needed to clarify the situation

- Fermilab/MILC working on the next two calculations of $B \to D^*$

Thank you for your attention

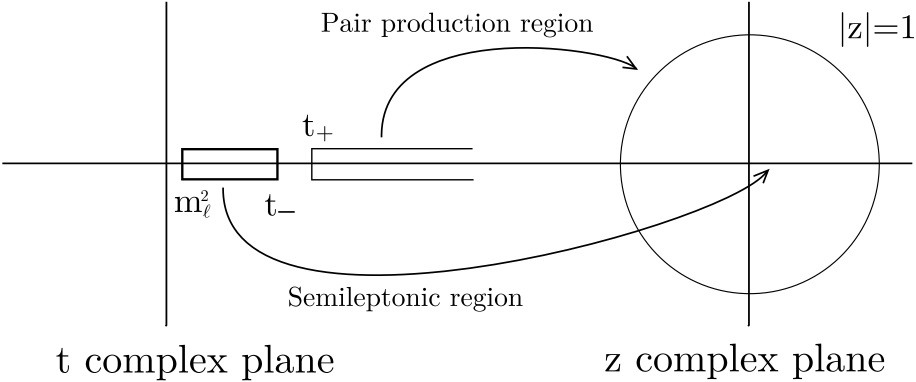
BACKUP SLIDES

Semileptonic B decays on the lattice: Parametrizations

Most parametrizations perform an expansion in the z parameter

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}, \qquad z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

with $t_\pm = (m_B \pm m_{D^*})^2\,, \quad t = (p_B - p_{D^*})^2\,, \quad w = v_B \cdot v_{D^*}$



Semileptonic B decays on the lattice: Parametrizations

- Boyd-Grinstein-Lebed (BGL) *Phys. Rev. Lett. 74 (1995) 4603-4606*

Phys.Rev. D56 (1997) 6895-6911

Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

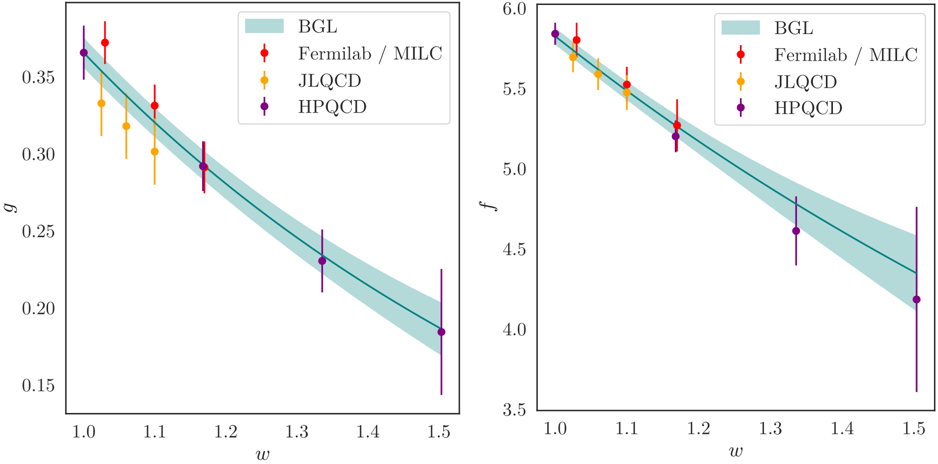
- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys. B530 (1998) 153-181*

$$F(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $F(w)$: four independent parameters, one relevant at $w = 1$
- Current consensus: abandon CLN
  - Spiritual sucessors of CLN Bernlochner *et al. Phys.Rev.D 95 (2017) 115008, Phys.Rev.D 97 (2018) 059902*

Bordone, Gubernari, Jung, Straub, Van Dyk... *Eur.Phys.J.C 80 (2020) 74, Eur.Phys.J.C 80 (2020) 347, JHEP 01 (2019) 009*

The mess: Combined lattice fits



The mess: Combined lattice fits