Extracting excited-state contributions to the B_s to D_s semi-leptonic decays from inclusive lattice simulations

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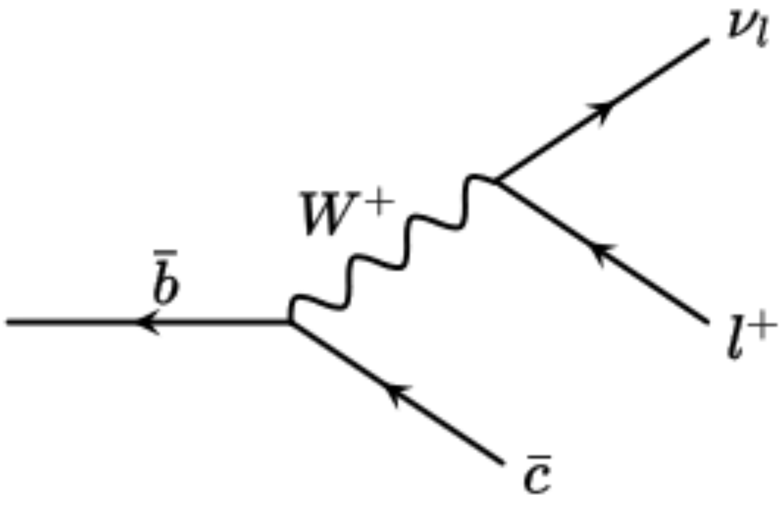
Lattice Meets Continuum, 2024/10/1

Introductions

Semi-leptonic decays

- in the quark level it corresponds to $\boldsymbol{b \to c l \nu_l}$
- it is a CKM-favoured decay corresponding to $\boldsymbol{V_{cb}}$, a fundamental parameter in SM
- we concentrate on the **exclusive processes** with X_{cs} being $\boldsymbol{D_s, D_s^*, D_{s0}^*, D_{s1}, D_{s1}', D_{s2}^*}$
- for l, we only consider **light leptons**
- two problems
  - tension between **inclusive and exclusive** V_{cb}
  - **1/2 vs 3/2** puzzle



$$B_s \to X_{cs} + l\,\nu_l$$

$$v = \frac{p_{B_s}}{M_{B_s}} \qquad v' = \frac{p_{X_{cs}}}{M_{X_{cs}}} \qquad q$$

$$\omega = v \cdot v' \qquad p_{B_s} = p_X + q$$

Inclusive and exclusive determinations of $|V_{cb}|^2$

inclusive

$42 \pm 1 \times 10^{-3}$

exclusive

$38 \pm 1 \times 10^{-3}$

experimental data

Inclusive and exclusive determinations of $|V_{cb}|^2$

OPE expansion of moments for inclusive decays

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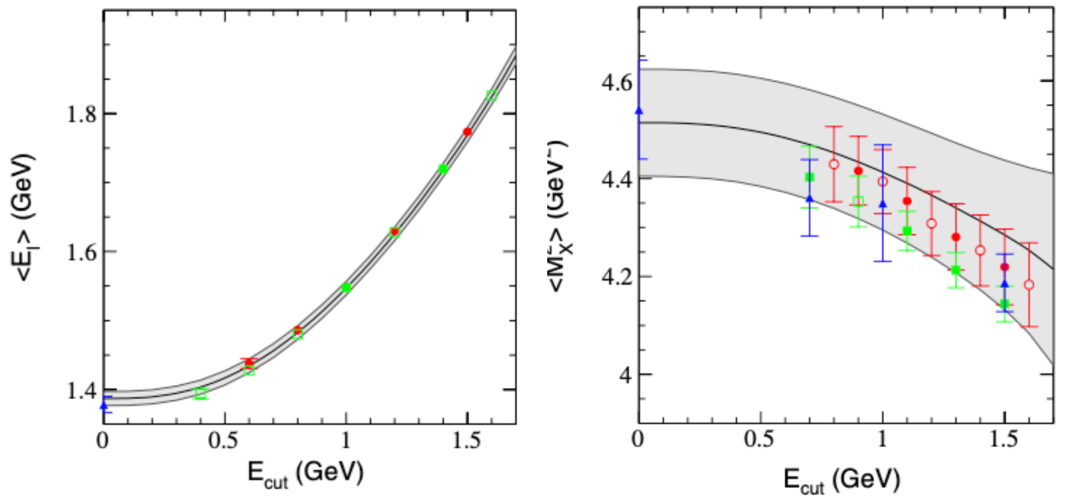

$$X = X^{(0)} + \frac{\alpha_s}{\pi} X^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 X^{(2)} + \left(\frac{\mu_\pi}{m_b}\right)^2 X^{(\pi)}$$
$$+ \left(\frac{\mu_G}{m_b}\right)^2 X^{(G)} + \left(\frac{\mu_D}{m_b}\right)^3 X^{(D)} + \left(\frac{\mu_{LS}}{m_b}\right)^2 X^{(LS)} + \ldots$$

[Gambino et al., *PRD* 89.1 (2014)]

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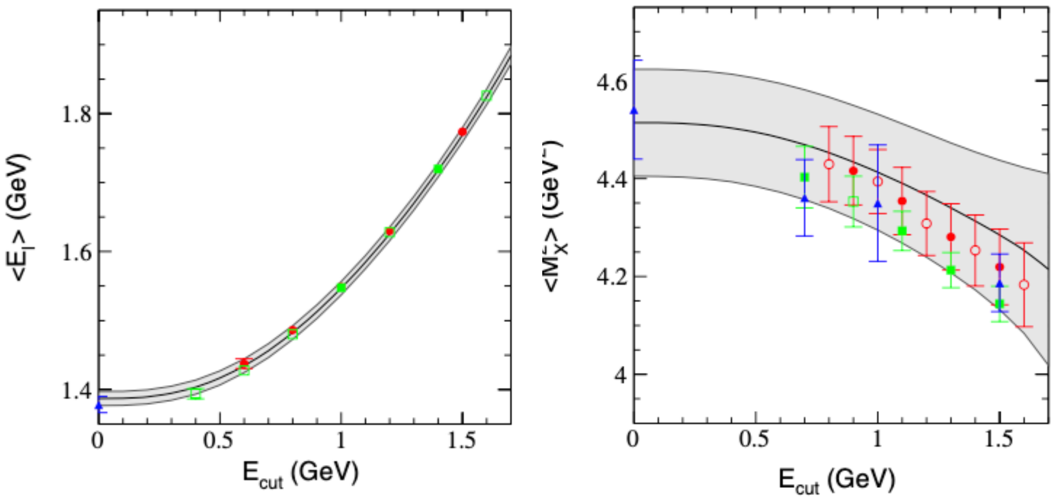
lattice data + CLN/BGL parameterizations of exclusive form factors



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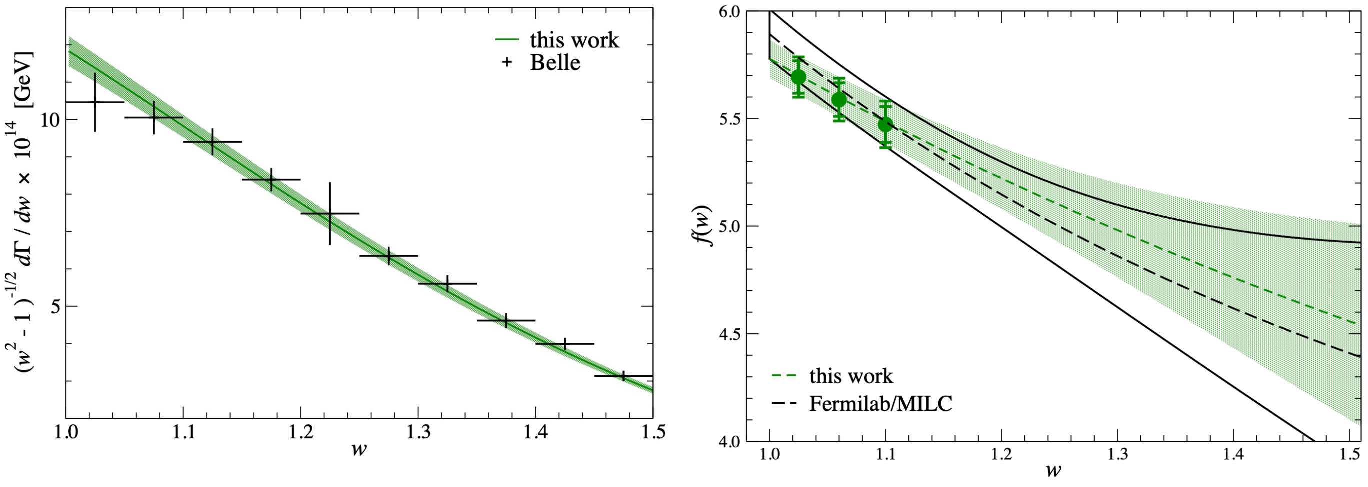

$$\frac{d\Gamma}{d\omega} = G(f, \mathcal{F}_1, \mathcal{F}_2)$$

$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{k=0}^{N_F} a_{F,k} z^k, \; z(\omega) = \frac{\sqrt{\omega+1} - \sqrt{2}}{\sqrt{\omega+1} + \sqrt{2}}$$

[Aoki et al., *PRD* **109**.7 (2024)]

Inclusive and exclusive determinations of $|V_{cb}|^2$

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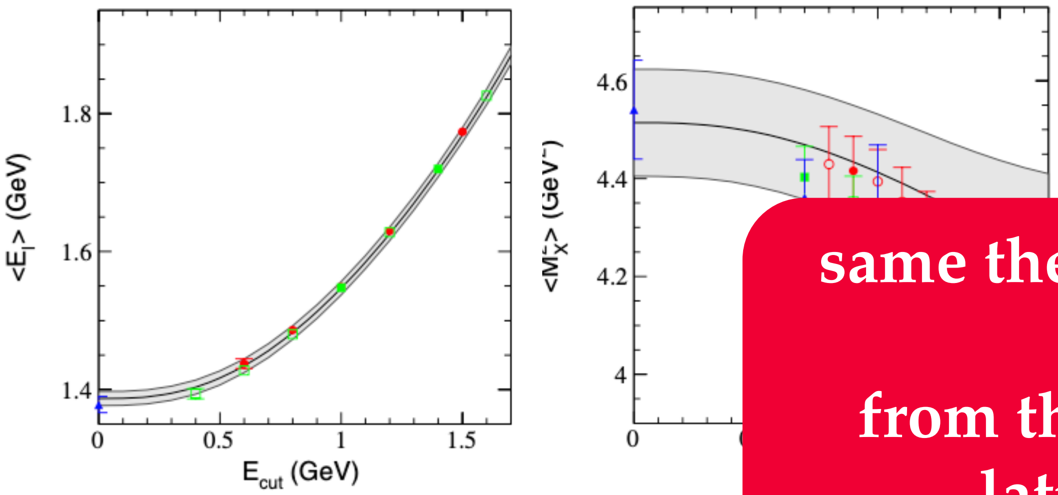

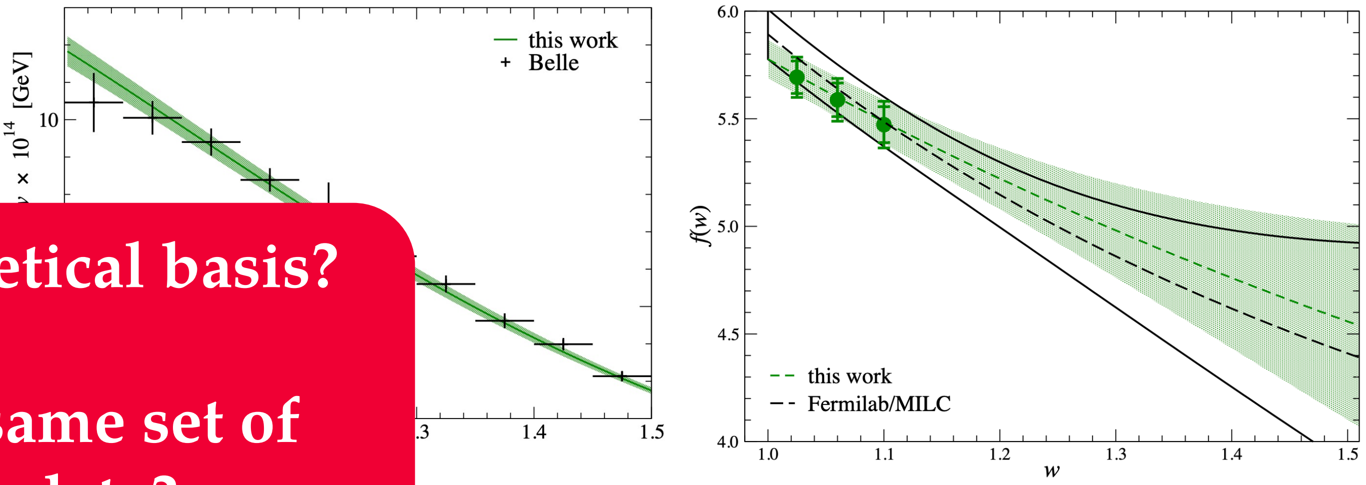

same theoretical basis?

from the same set of lattice data?

$$X = X^{(0)} + \frac{\alpha_s}{\pi} X^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 X^{(2)} + \left(\frac{\mu_\pi}{m_b}\right)^2 X^{(\pi)}$$
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[Gambino et al.,PRD89.1(2014)]

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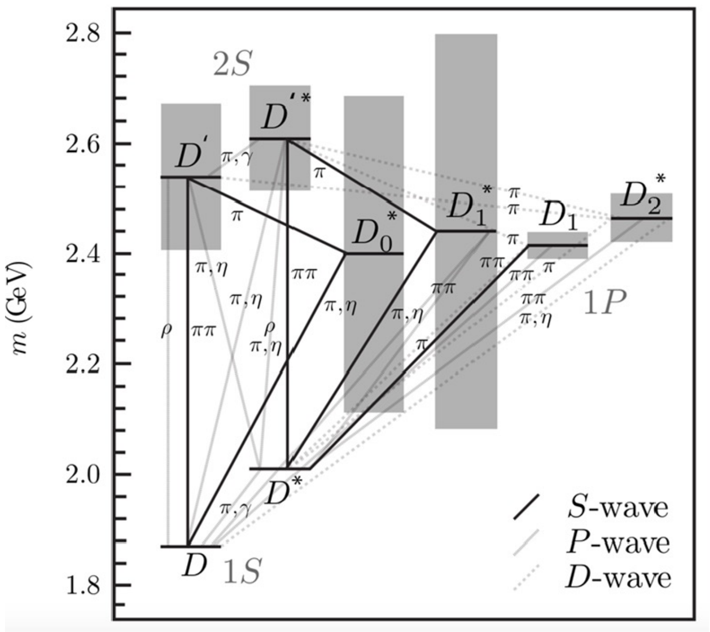
1/2 and 3/2 puzzle



j^P	J^P	
$(1/2)^- \equiv S$	0^-	D
	1^-	D^*
$(1/2)^+ \equiv P_{1/2}$	0^+	D_0^*
	1^+	D_1'
$(3/2)^+ \equiv P_{3/2}$	1^+	D_1
	2^+	D_2^*

$$j = L + s_{\text{light quark}}$$

$$J = j + s_{\text{heavy quark}}$$

[Bigi et al.,*EPJC***52**.4(2007)]

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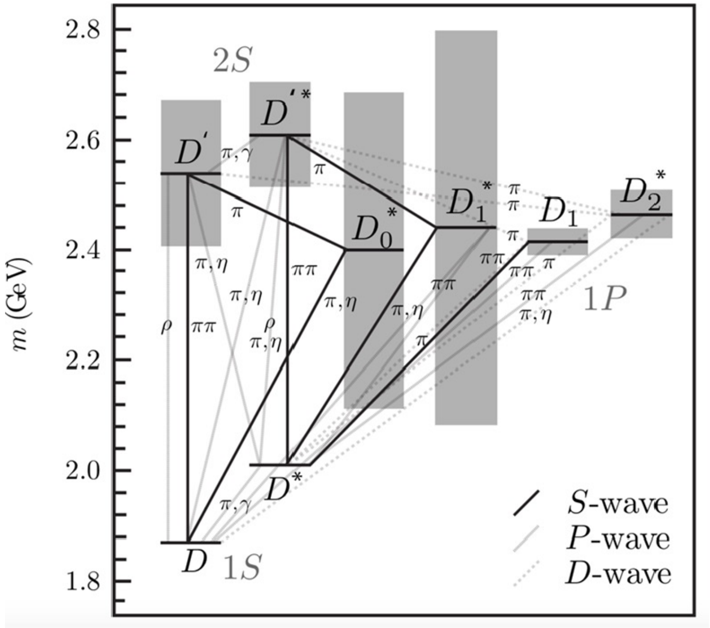

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$D \approx 20\%$

$D^* \approx 50\%$

$D^{**}_{3/2} \approx 15\%$

$? \approx 15\%$

- heavy quark limit $m_Q \to \infty$
- OPE+HQET

- **$\tau_{1/2}(\omega = 1) \ll \tau_{3/2}(\omega = 1)$**
- transition form factors from B to $D^{**}_{3/2}$ and $D^{**}_{1/2}$, and thus their decay ratios, can be expressed by **Isgur-Wise form factor $\tau_{1/2}$ and $\tau_{3/2}$**

$\Gamma_{1/2} \ll \Gamma_{3/2}$

[Uraltsev,*PLB***501**.1-2(2001)]

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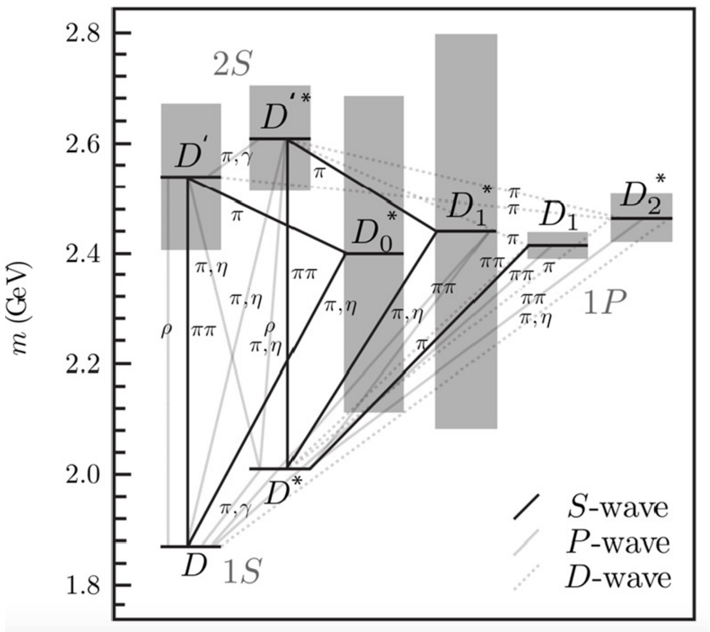

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$$\Gamma_{1/2} \ll \Gamma_{3/2}$$

what is the nature of the remaining 15%?
can we understand it from lattice?

[Uraltsev,*PLB* **501**.1-2(2001)]

Methods

Lattice setup

more details from Alessandro's talk on Thursday

$$C_{J_\mu J_\nu}(\boldsymbol{q}, t) \equiv \int \mathrm{d}^3 \boldsymbol{x} \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2M_{B_s}} \left\langle B_s \left| J_\mu^\dagger(\boldsymbol{x}, 0) e^{-\hat{H}t} J_\nu(0) \right| B_s \right\rangle$$

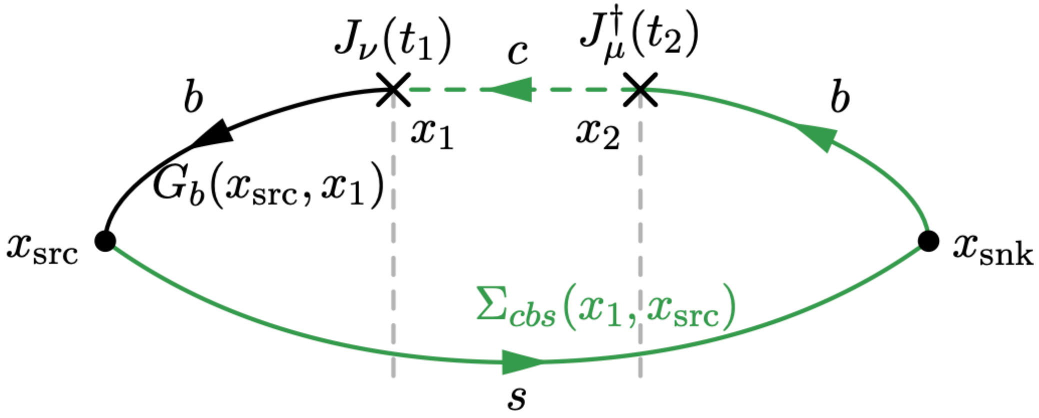
Gauge ensemble

- data from RBC/UKQCD
- lattice size: $24^3 \times 64$
- lattice spacing: $a \approx 0.11$ fm, energy unit: $\frac{1}{a} \approx 1.785$ GeV
- 2+1-flavour DWF actions with approximately physical masses are utilized for light quarks
- relativistic-heavy quark action for b and c quarks

$M_\pi \approx 330$ MeV, $M_{B_s} \approx 5370$ MeV, $\boldsymbol{M_{D_s} \approx 1680}$ **MeV**

Simulation of 4-pt correlators

- $t_{snk} - t_{src} = 20$, $t_2 - t_{src} = 14$, fixed
- t_1 range from 0 to 14 $\Longrightarrow$ $\boldsymbol{t = t_2 - t_1}$ **range from 14 to 0**
- we work in the rest frame of B_s, $v \equiv \frac{p_{B_s}}{M_{B_s}} = (1, 0, 0, 0)$
- $-\boldsymbol{q} = (q_k, q_k, q_k)$
- almost non-perturbative renormalization for the current [El-Khadra et. al., PRD.64(2001):014502]



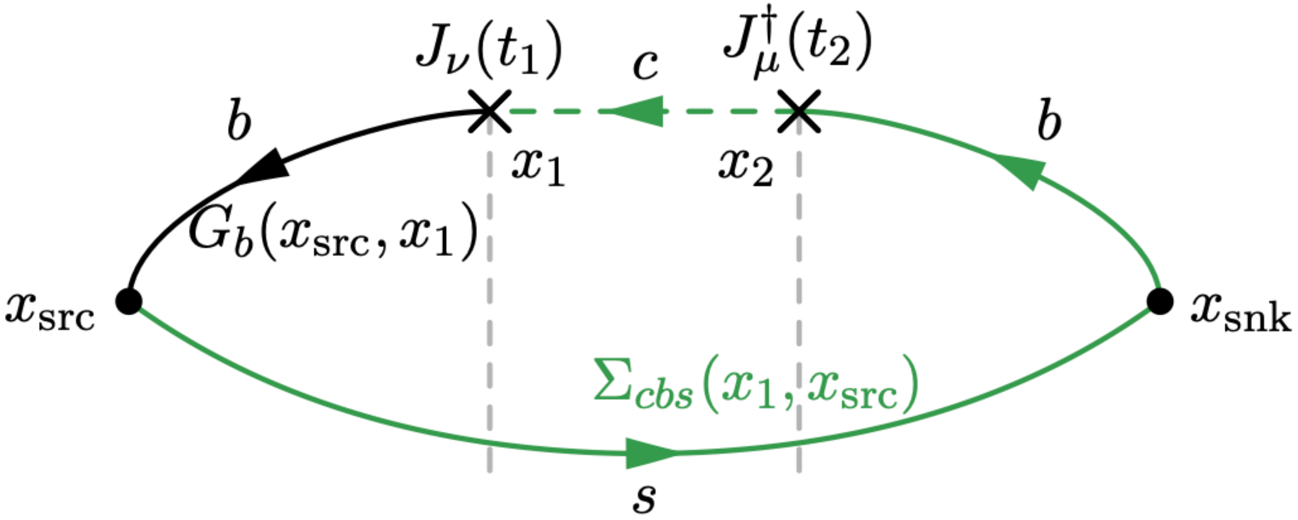
Four-point correlators

$$C_{J_\mu J_\nu}(\boldsymbol{q}, t) \equiv \int d^3\boldsymbol{x} \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2M_{B_s}} \left\langle B_s \middle| J_\mu^\dagger(\boldsymbol{x}, 0) e^{-\hat{H}t} J_\nu(0) \middle| B_s \right\rangle$$

inclusive process with B_s as initial state

see Alessandro's talk on Thursday

$J_\mu \equiv V_\mu - A_\mu, V_\mu = \bar{b}\gamma^\mu c, A_\mu = \bar{b}\gamma^\mu\gamma^5 c$



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$$\equiv \int d^3\boldsymbol{x}\, \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2M_{B_s}} \langle B_s | J_\mu^\dagger(\boldsymbol{x}, 0) e^{-\hat{H}t} J_\nu(0) | B_s \rangle$$
$$= \sum_{X_{cs}} \frac{1}{2M_{B_s} 2E_{X_{cs}}} \langle B_s | J_\mu^\dagger(0) | X_{cs}, -\boldsymbol{q} \rangle \langle X_{cs}, -\boldsymbol{q} | J_\nu(0) | B_s \rangle e^{-E_{X_{cs}} t}$$

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contributions from all possible final states

exclusive information could be extracted from 4-pt correlators

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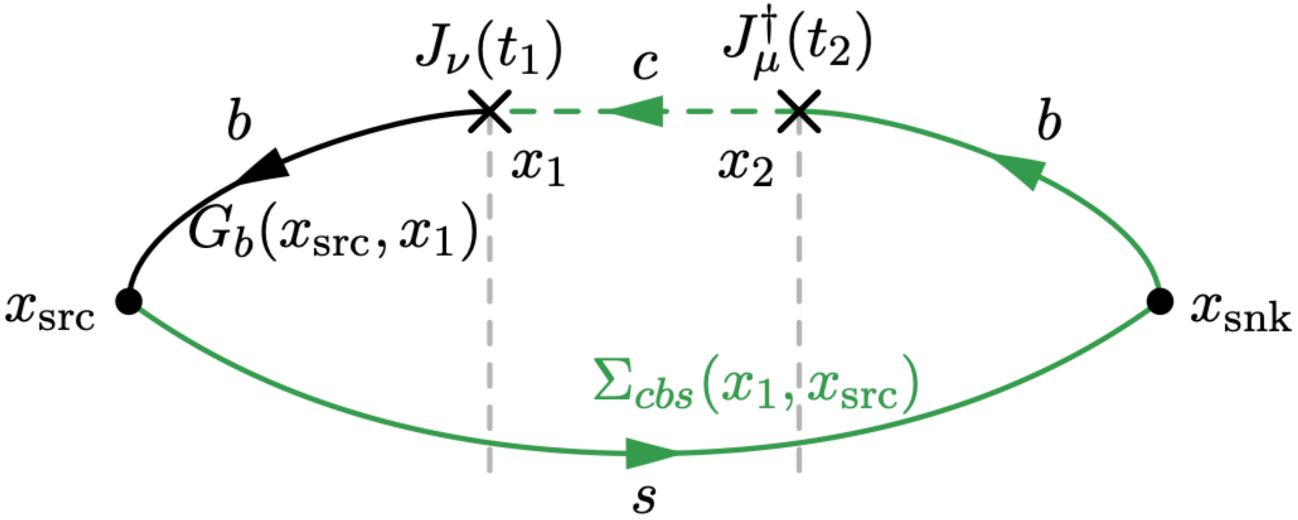

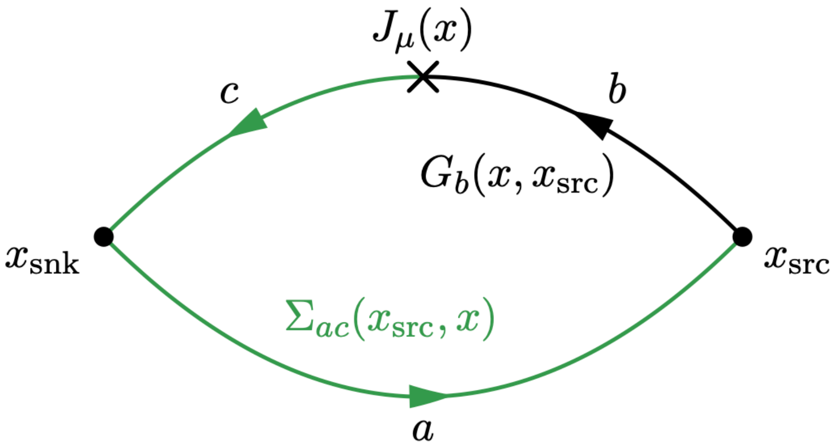

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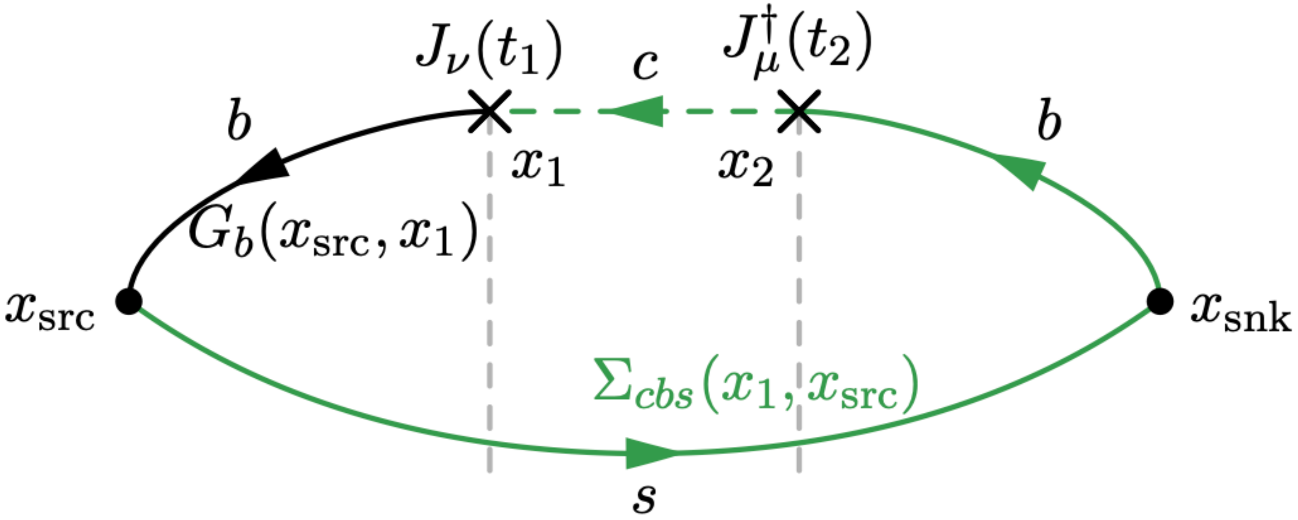
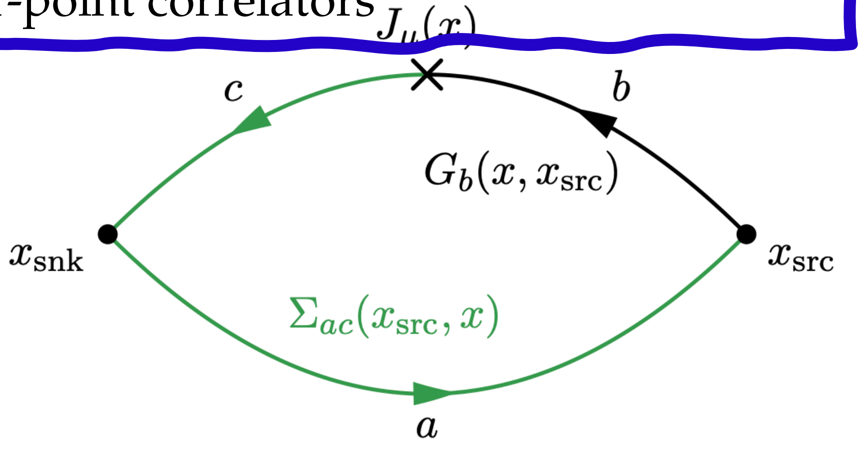
$$J_\mu \equiv V_\mu - A_\mu, V_\mu = \bar{b}\gamma^\mu c, A_\mu = \bar{b}\gamma^\mu\gamma^5 c$$

- pro: don't need to construct the **complicated operators** representing the final state
- con: need to perform **multi-state fit** of the four-point correlators





Decomposition of four-point correlators

four-point correlators as the summations of a series of three-point correlators

$$
\begin{aligned}
&C_{J_\mu J_\nu}(\boldsymbol{q},t)\\
&=\sum_{X_c}\frac{1}{2E_{X_c}2M_B}\langle B|J_\mu^\dagger(0)|X_c\rangle\langle X_c|J_\nu(0)|B\rangle e^{-E_{X_c}t}\\
&=\frac{1}{2E_D2M_B}\langle B|J_\mu^\dagger(0)|D\rangle\langle D|J_\nu(0)|B\rangle e^{-E_Dt}\\
&+\frac{1}{2E_{D^*}2M_B}\sum_\sigma\langle B|J_\mu^\dagger(0)|D^*,\sigma\rangle\langle D^*,\sigma|J_\nu(0)|B\rangle e^{-E_{D^*}t}\\
&+\frac{1}{2E_{D_0^*}2M_B}\langle B|J_\mu^\dagger(0)|D_0^*\rangle\langle D_0^*|J_\nu(0)|B\rangle e^{-E_{D_0^*}t}\\
&+\frac{1}{2E_{D_1'}2M_B}\sum_\sigma\langle B|J_\mu^\dagger(0)|D_1',\sigma\rangle\langle D_1',\sigma|J_\nu(0)|B\rangle e^{-E_{D_1'}t}\\
&+\frac{1}{2E_{D_1}2M_B}\sum_\sigma\langle B|J_\mu^\dagger(0)|D_1,\sigma\rangle\langle D_1,\sigma|J_\nu(0)|B\rangle e^{-E_{D_1}t}\\
&+\frac{1}{2E_{D_2^*}2M_B}\sum_\sigma\langle B|J_\mu^\dagger(0)|D_2^*,\sigma\rangle\langle D_2^*,\sigma|J_\nu(0)|B\rangle e^{-E_{D_2^*}t}
\end{aligned}
$$

Decomposition of four-point correlators

four-point correlators as the summations of a series of three-point correlators

three-point correlators parameterized by form factors and final-state masses/energies

- **S wave**
  - $D\,(0^-)$

$$\langle B,v|V_\mu|D,v'\rangle = \left[h_+(v_\mu + v'_\mu) + \boxed{h_-(v_\mu - v'_\mu)}\right]\sqrt{M_B M_D} \quad (3.97)$$

$$\langle B,v|A_\mu|D,v'\rangle = 0 \quad (3.98)$$

  - $D^*\,(1^-)$

$$\langle B,v|V_\mu|D^*,v',\sigma\rangle = \left[\boxed{h_V \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\alpha v'^\beta v^\gamma}\right]\sqrt{M_B M_{D^*}} \quad (3.99)$$

$$\langle B,v|A_\mu|D^*,v',\sigma\rangle = i\left[(\omega+1)h_{A1}\epsilon_\mu - \boxed{(\epsilon\cdot v)\left(h_{A2}v_\mu + h_{A3}v'_\mu\right)}\right]\sqrt{M_B M_{D^*}} \quad (3.100)$$

- **P wave, $j = \frac{1}{2}$**
  - $D_0^*\,(0^+)$

$$\langle B,v|V_\mu|D_0^*,v'\rangle = 0 \quad (3.101)$$

$$\langle B,v|A_\mu|D_0^*,v'\rangle = \left[g_+(v_\mu + v'_\mu) + \boxed{g_-(v_\mu - v'_\mu)}\right]\sqrt{M_B M_{D_0^*}} \quad (3.102)$$

  - $D'_1\,(1^+)$

$$\langle B,v|V_\mu|D'_1,v',\sigma\rangle = \left[g_{V1}\epsilon_\mu + \boxed{(\epsilon\cdot v)\left(g_{V2}v_\mu + g_{V3}v'_\mu\right)}\right]\sqrt{M_B M_{D'_1}}, \quad (3.103)$$

$$\langle B,v|A_\mu|D'_1,v',\sigma\rangle = -i\left[\boxed{g_A \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\alpha v'^\beta v^\gamma}\right]\sqrt{M_B M_{D'_1}}. \quad (3.104)$$

- **P wave, $j = \frac{3}{2}$**
  - $D_1\,(1^+)$

$$\langle B,v|V_\mu|D'_1,v',\sigma\rangle = \left[f_{V1}\epsilon_\mu + \boxed{(\epsilon\cdot v)\left(f_{V2}v_\mu + f_{V3}v'_\mu\right)}\right]\sqrt{M_B M_{D_1}}, \quad (3.105)$$

$$\langle B,v|A_\mu|D'_1,v',\sigma\rangle = -i\left[\boxed{f_A \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\alpha v'^\beta v^\gamma}\right]\sqrt{M_B M_{D_1}}. \quad (3.106)$$

  - $D_2^*\,(2^+)$

$$\langle B,v|V_\mu|D_2^*,v',\sigma\rangle = -i\left[\boxed{k_V \varepsilon_{\mu\alpha\beta\gamma}\epsilon^{\alpha\rho}v_\rho v'^\beta v^\gamma}\right]\sqrt{M_B M_{D_2^*}}, \quad (3.107)$$

$$\langle B,v|A_\mu|D_2^*,v',\sigma\rangle = \left[\boxed{k_{A1}\epsilon_{\mu\rho}v^\rho + (\epsilon_{\alpha\beta}v^\alpha v^\beta)\left(k_{A2}v_\mu + k_{A3}v'_\mu\right)}\right]\sqrt{M_B M_{D_2^*}}. \quad (3.108)$$

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three-point correlators parameterized by form factors and final-state masses/energies

four-point correlators parameterized by a series of exponentials with form factors and masses/energies of different final states as prefactors

$$
\begin{aligned}
&C_{A_0A_0}\\
&=\frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}}3q_k^2\left[\left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}}\right)h_{A1}-h_{A2}-\frac{E_{D^*}}{M_{D^*}}h_{A3}\right]^2\\
&+\frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}}\left[g_+\left(E_{D_0^*}+M_{D_0^*}\right)-g_-\left(E_{D_0^*}-M_{D_0^*}\right)\right]^2\\
&+\frac{e^{-E_{D_2^*}t}}{4E_{D_2^*}M_{D_2^*}}\cdots
\end{aligned}
$$

	S		$P_{1/2}$		$P_{1/2}$	
	D	D^*	D_0^*	D_1'	D_1	D_2^*
V_0V_0	h_+,h_-			g_{V1},g_{V2},g_{V3}	f_{V1},f_{V2},f_{V3}	
$V_\parallel V_\parallel$	h_+,h_-			g_{V1},g_{V3}	f_{V1},f_{V3}	
$V_\perp V_\perp$		h_V		g_{V1}	f_{V1}	k_V
$V_0V_\parallel$	h_+,h_-			g_{V1},g_{V2},g_{V3}	f_{V1},f_{V2},f_{V3}	
A_0A_0		h_{A1},h_{A2},h_{A3}	g_+,g_-			...
$A_\parallel A_\parallel$		h_{A1},h_{A3}	g_+,g_-			...
$A_\perp A_\perp$		h_{A1}		g_A	f_A	...
$A_0A_\parallel$		h_{A1},h_{A2},h_{A3}	g_+,g_-			...

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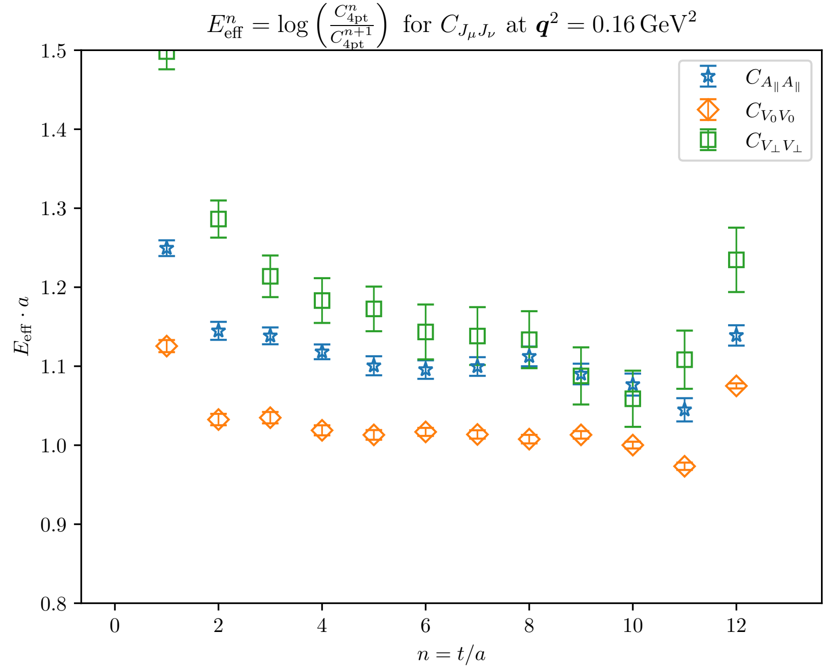
lattice simulation of four-point correlators

values of the masses/energies of the final states and the corresponding form factors

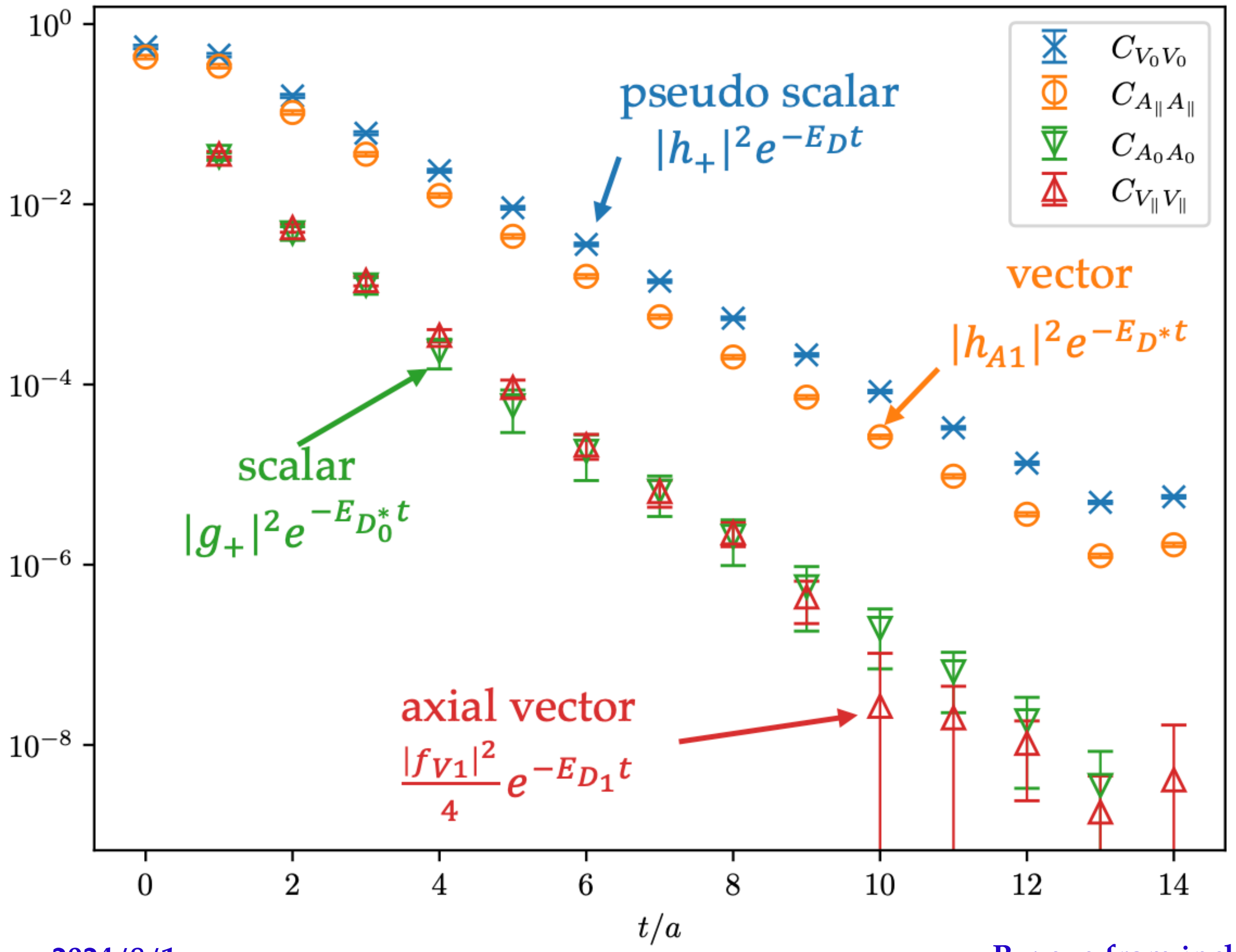
	S		$P_{1/2}$		$P_{1/2}$	
	D	D^*	D_0^*	D_1'	D_1	D_2^*
V_0V_0	h_+, h_-			g_{V1}, g_{V2}, g_{V3}	f_{V1}, f_{V2}, f_{V3}	
$V_\parallel V_\parallel$	h_+, h_-			g_{V1}, g_{V3}	f_{V1}, f_{V3}	
$V_\perp V_\perp$		h_V		g_{V1}	f_{V1}	k_V
$V_0V_\parallel$	h_+, h_-			g_{V1}, g_{V2}, g_{V3}	f_{V1}, f_{V2}, f_{V3}	
A_0A_0		h_{A1}, h_{A2}, h_{A3}	g_+, g_-			$\cdots$
$A_\parallel A_\parallel$		h_{A1}, h_{A3}	g_+, g_-			$\cdots$
$A_\perp A_\perp$		h_{A1}		g_A	f_A	$\cdots$
$A_0A_\parallel$		h_{A1}, h_{A2}, h_{A3}	g_+, g_-			$\cdots$



Zero-recoil results

Four-point correlators at zero recoil

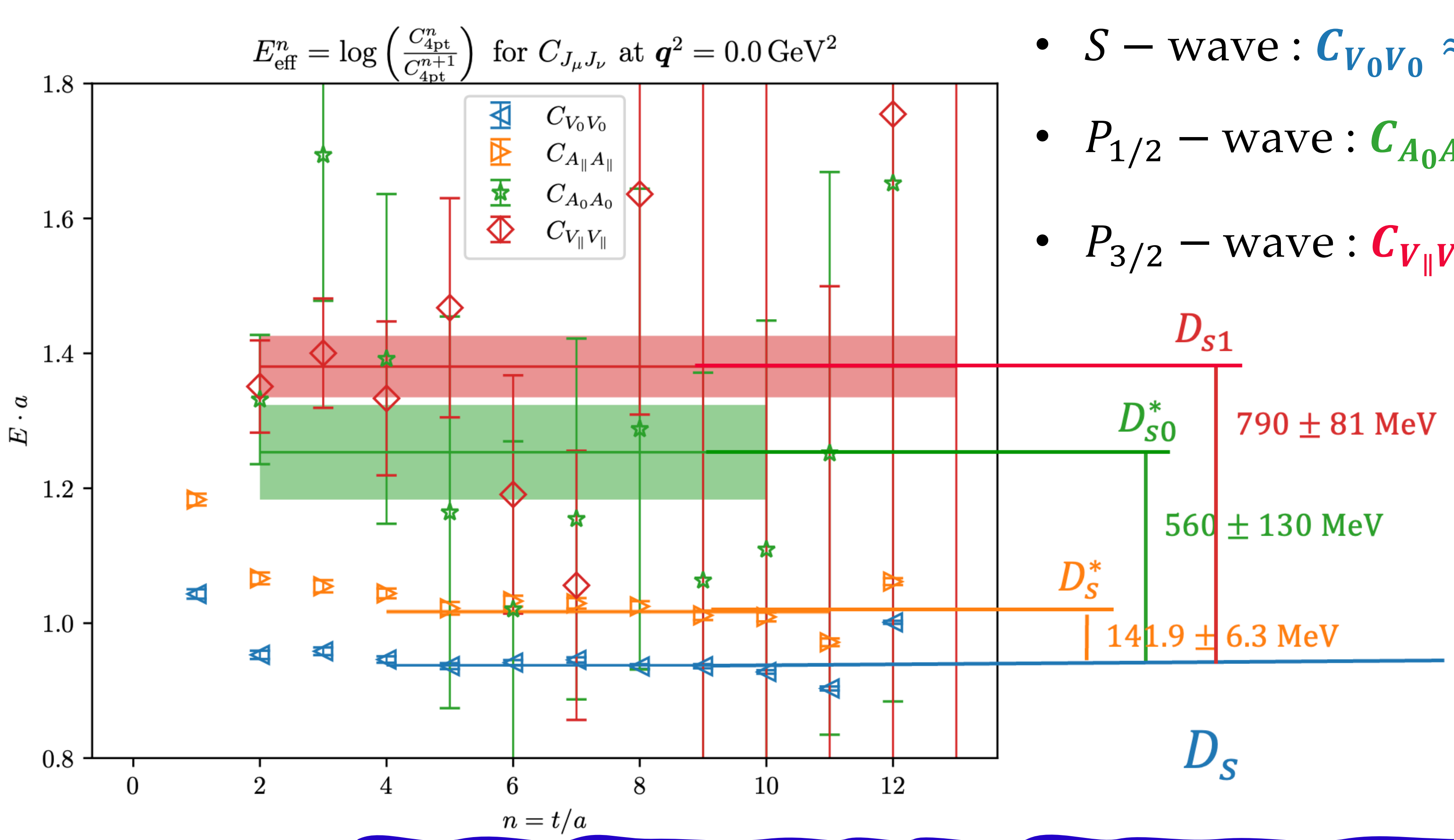
at zero recoil limit, $\vec{q}^2 = 0$, $\omega = 1$,
parity is well defined, thus parity symmetry dedicates the isolation of final states



- S – wave : $C_{V_0V_0} \approx h_+^2\, e^{-M_{D_s} t}$, $C_{A_\parallel A_\parallel} \approx h_{A1}^2\, e^{-M_{D_s^*} t}$
- $P_{1/2}$ – wave : $C_{A_0A_0} \approx g_+^2\, e^{-M_{D_{s0}^*} t}$
- $P_{3/2}$ – wave : $C_{V_\parallel V_\parallel} \approx \frac{f_{V1}^2}{4}\, e^{-M_{D_1} t}$

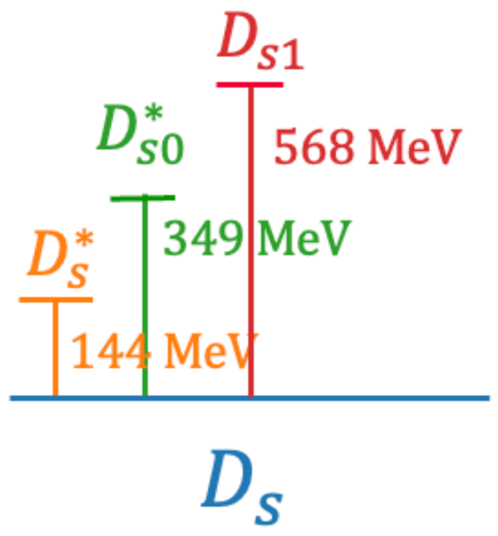
the magnitudes of P-wave contributions are around 1/10 smaller than those of the S-wave contributions, but, information can still be extracted from the lattice simulation

Comparison of the effective mass and the fitted mass



- S − wave : $C_{V_0V_0} \approx h_+^2\, e^{-M_{D_s}t}$, $C_{A_\parallel A_\parallel} \approx h_{A1}^2\, e^{-M_{D_s^*}t}$
- $P_{1/2}$ − wave : $C_{A_0A_0} \approx g_+^2\, e^{-M_{D^*_{s0}}t}$
- $P_{3/2}$ − wave : $C_{V_\parallel V_\parallel} \approx \frac{f_{V1}^2}{4}\, e^{-M_{D_1}t}$

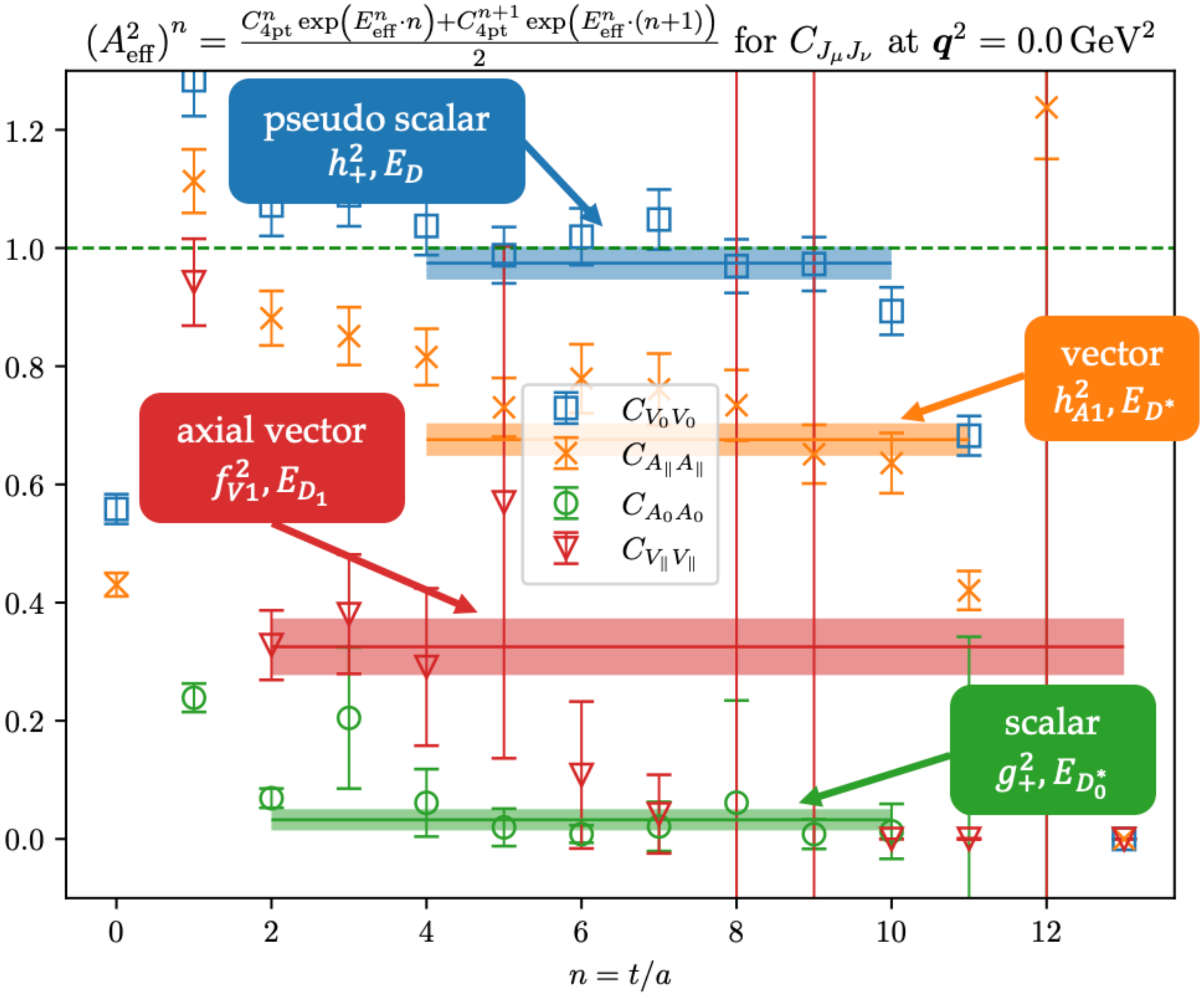
PDG values of D_s mass differences



- $M_\pi \approx 330$ MeV
- D_{s1}/D^*_{s0} and $D^{(*)}K$

} we obtain reasonable results for the masses

Comparision of the fitted and effective form factor



- S-wave form factors at zero recoil (ω=1)

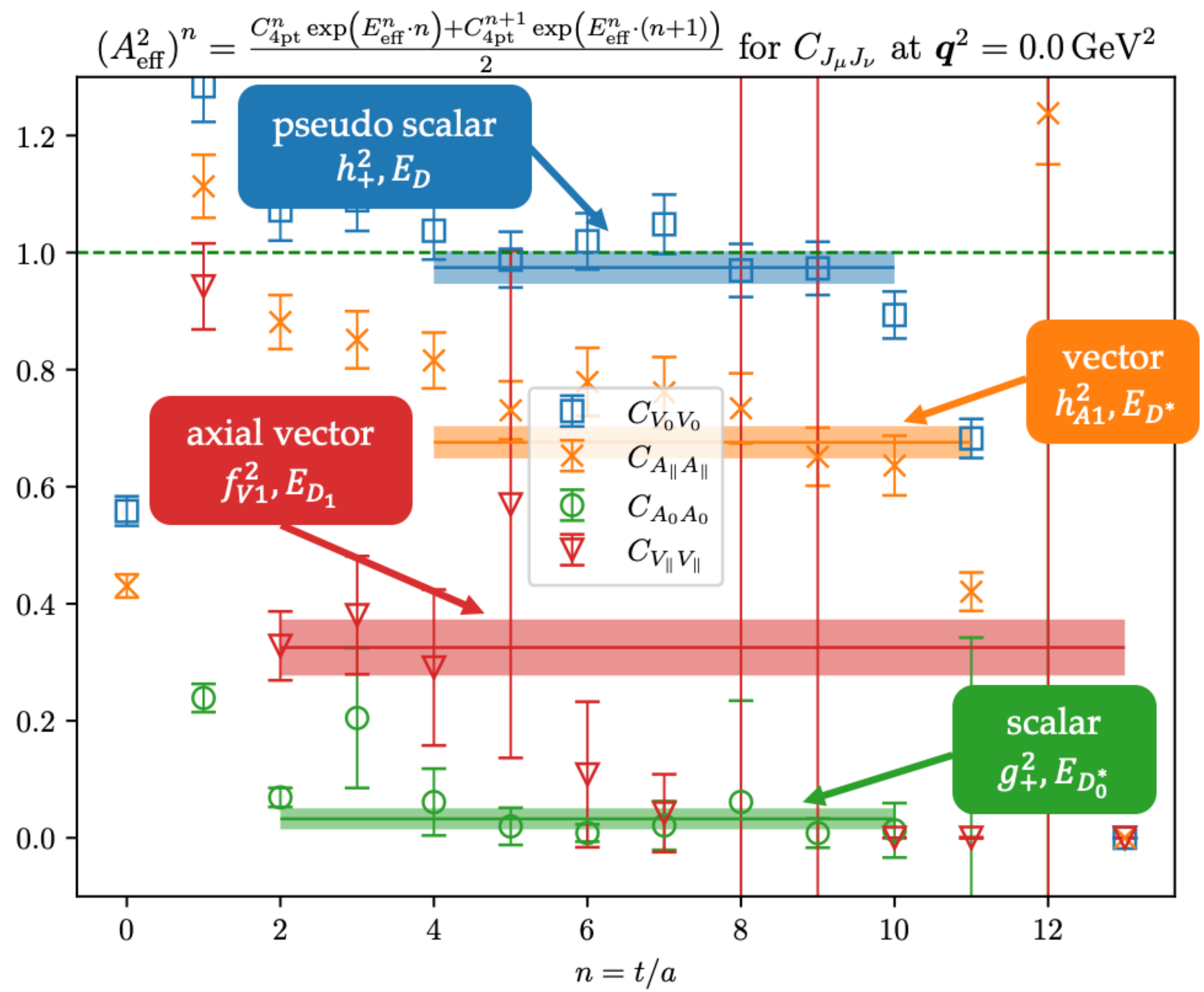
$$\langle B_s|V_\mu|D_s\rangle \propto h_+(\omega)(v_\mu + v'_\mu)$$
$$\langle B_s|A_\mu|D_s^*, \epsilon_\mu\rangle \propto (\omega+1)h_{A1}(\omega)\epsilon_\mu$$

$$h_+(\omega=1) = 0.987(14)$$
$$h_{A1}(\omega=1) = 0.822(17)$$

compatible with the zeroth order calculations from the heavy quark effective theory (HQET)

$$h_+(\omega=1) \approx h_{A1}(\omega=1) \approx 1$$

Comparision of the fitted and effective form factor



- P-wave form factors at zero recoil

$$\langle B_s | A_\mu | D^*_{s0} \rangle \propto g_+(\omega)(v_\mu + v'_\mu)$$
$$\langle B_s | V_\mu | D_{s1}, \epsilon_\mu \rangle \propto f_{V1}(\omega)\epsilon_\mu$$

- according to HQET [*Leibovich et. al.*,PRD.57.1], at lowest non-zero order
  - $|g_+(1)| = 3(\epsilon_c + \epsilon_b)(\overline{\Lambda^*} - \overline{\Lambda})\tau_{1/2}(1)$
  - $|f_{V1}(1)| = \frac{8}{\sqrt{2}}(\epsilon_c)(\overline{\Lambda'} - \overline{\Lambda})\tau_{3/2}(1)$

heavy quark mass

$$\epsilon_c = \frac{1}{2m_c} \approx 0.801$$
$$\epsilon_b = \frac{1}{2m_b} \approx 0.12$$

difference of spin-averaged hadron mass

$P_{1/2} - S : \overline{\Lambda^*} - \overline{\Lambda}$

$P_{3/2} - S : \overline{\Lambda'} - \overline{\Lambda}$

[Bernlochner et al.,PRD95.1(2017)]

Comments about $\tau_{1/2} \ll \tau_{3/2}$

$$\tau_{1/2} = 0.181(50)$$
$$\tau_{3/2} = 0.315(42)$$
$$\tau_{1/2} \lesssim \tau_{3/2}$$

- it is valid at **infinite heavy quark mass**, or zeroth order of HQET
- we perform calculations at finite heavy quark mass, thus **consistent but not exactly the same** results
- it is a conclusion valid in the **zero-recoil limit**

→ **what about non-zero $\vec{q}^2$?**

- heavy quark limit $m_Q \to \infty$
- OPE+HQET

↓

- $\tau_{1/2}(\omega = 1) \ll \tau_{3/2}(\omega = 1)$
- transition form factors from B to $D^{**}_{3/2}$ and $D^{**}_{1/2}$, and thus their decay ratios, can be expressed by **Isgur-Wise form factor $\tau_{1/2}$ and $\tau_{3/2}$**

↓

$$\Gamma_{1/2} \ll \Gamma_{3/2}$$

[Uraltsev, *PLB* **501**.1-2(2001)]

Non-zero recoil

Multi-state (?)

$$E = \cosh^{-1}\left(\cosh M + 3 - 3\cos q_k\right)$$

	S		$P_{1/2}$		$P_{1/2}$	
	D	D^*	D_0^*	D_1'	D_1	D_2^*
V_0V_0	h_+, h_-			g_{V1}, g_{V2}, g_{V3}	f_{V1}, f_{V2}, f_{V3}	
$V_\parallel V_\parallel$	h_+, h_-			g_{V1}, g_{V3}	f_{V1}, f_{V3}	
$V_\perp V_\perp$		h_V		g_{V1}	f_{V1}	k_V
$V_0V_\parallel$	h_+, h_-			g_{V1}, g_{V2}, g_{V3}	f_{V1}, f_{V2}, f_{V3}	
A_0A_0		h_{A1}, h_{A2}, h_{A3}	g_+, g_-			...
$A_\parallel A_\parallel$		h_{A1}, h_{A3}	g_+, g_-			...
$A_\perp A_\perp$		h_{A1}		g_A	f_A	...
$A_0A_\parallel$		h_{A1}, h_{A2}, h_{A3}	g_+, g_-			...

excited-state contribution estimations

- ignoring the $\omega/\vec{q}^2$ dependence of form factors
  $$f(\omega) \approx f(1)$$
- use the lowest order relation from HQET
  $$h_{A1} \approx h_{A3}\,, h_{A2} \approx 0$$
  $$g_+ \approx -\frac{3}{2}(\epsilon_c + \epsilon_b)(\overline{\Lambda^*} - \overline{\Lambda})g_-$$
- we only need the values of $h_{A1}(1)$ and $g_+(1)$ to estimate the prefactors and thus the contributions from different states

$$C_{A_0A_0} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} 3q_k^2 \xi(1)^2 + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} \left(3E_{D_0^*} - M_{D_0^*}\right)^2 g_+^2(1)$$

$$C_{A_\parallel A_\parallel} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} (E_{D^*} + M_{D^*})^2 \xi^2(1) + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} 27 q_k^2 g_+^2(1)$$

$$C_{A_0A_\parallel} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} \sqrt{3} q_k (E_{D^*} + M_{D^*}) \xi^2(1) + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} \sqrt{3} q_k (9E_{D_0^*} - 3M_{D_0^*}) g_+^2(1)$$

$$\vec{q}^2 = 0.08, 0.16, 0.25\ \mathrm{GeV}^2$$

contributions from excited state

$$\boldsymbol{A_0A_0(\sim 50\%) > A_0A_\parallel(\sim 10\%) > A_\parallel A_\parallel(< 10\%)}$$

KEK

Let the data talk

- we fit A_0A_0, $A_0A_\parallel$ and $A_\parallel A_\parallel$ separately with multiple exponentials **without assuming the functional forms of the prefactors**

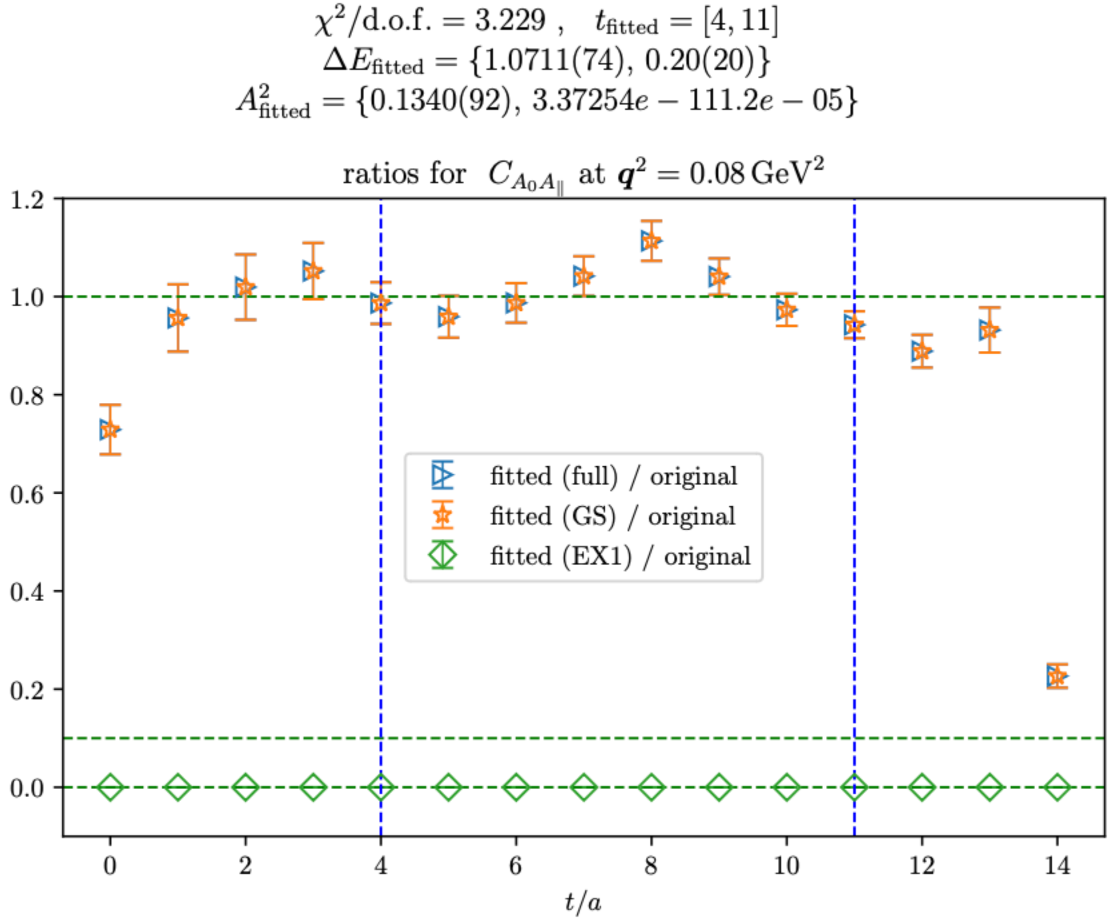
$$C_{J_\mu J_\mu} = \sum_{i=1}^{N_{\text{fit}}} A^2_{i \text{ in } J_\mu J_\nu} \exp\left(-E_{i \text{ in } J_\mu J_\nu} \times t\right)$$

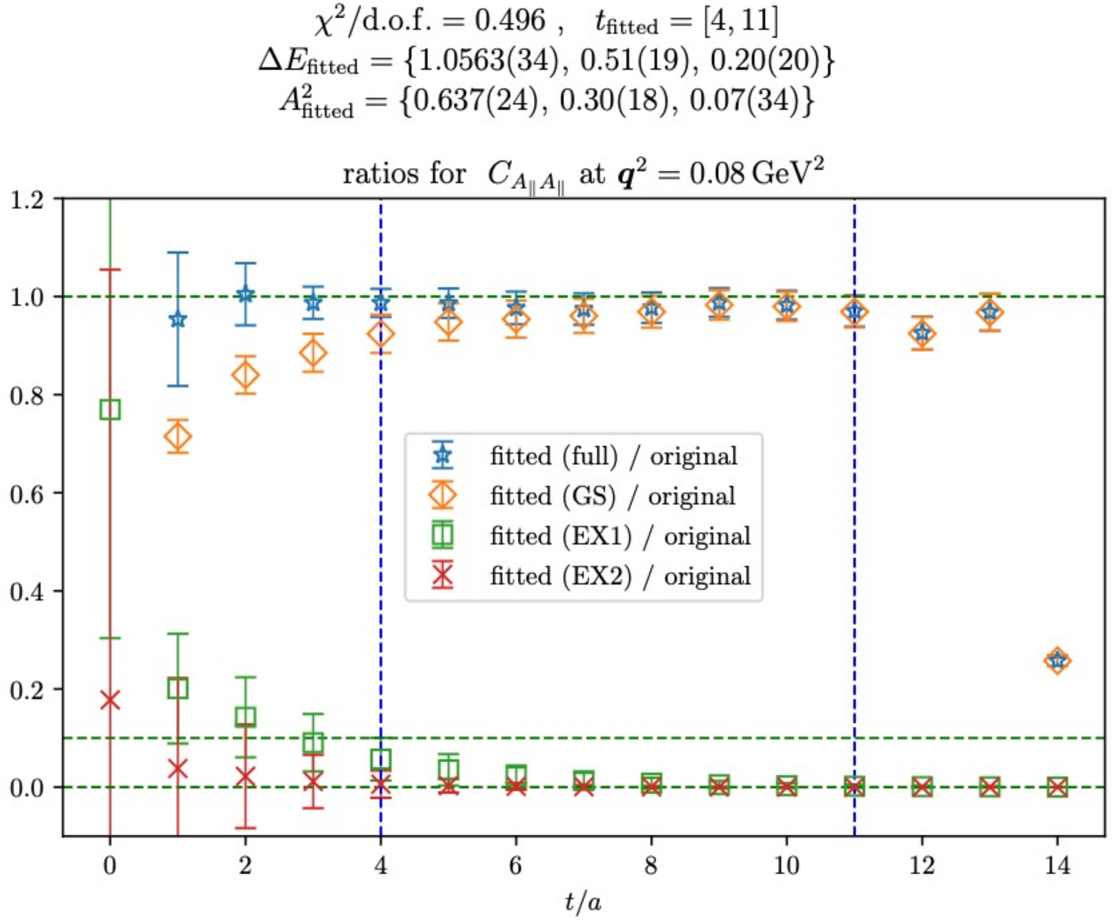
- $N_{\text{fit}} = 1, 2, 3, \ldots\ldots$ step-by-step fitting, with prior from the outcomes of the previous fit
- **full loop** over t_{min} and t_{max}, the **fitting range**
- chose the best fit to be the fit with **smallest** $\left|\frac{\chi^2}{\text{d.o.f.}} - 1\right|$ and
  - t_{max} to be **fairly large** for all fits
  - t_{min} to be **fairly large for $N_{fit} = 1$ fits**
  - t_{min} to be **fairly small for $N_{fit} > 1$ fits**
- look at the fitting results to manually the physics of the fitting results are correct

KEK

Clear sign of non-existence of excited states

- The fitted parameters are occupied by error and/or the central values are extremely small



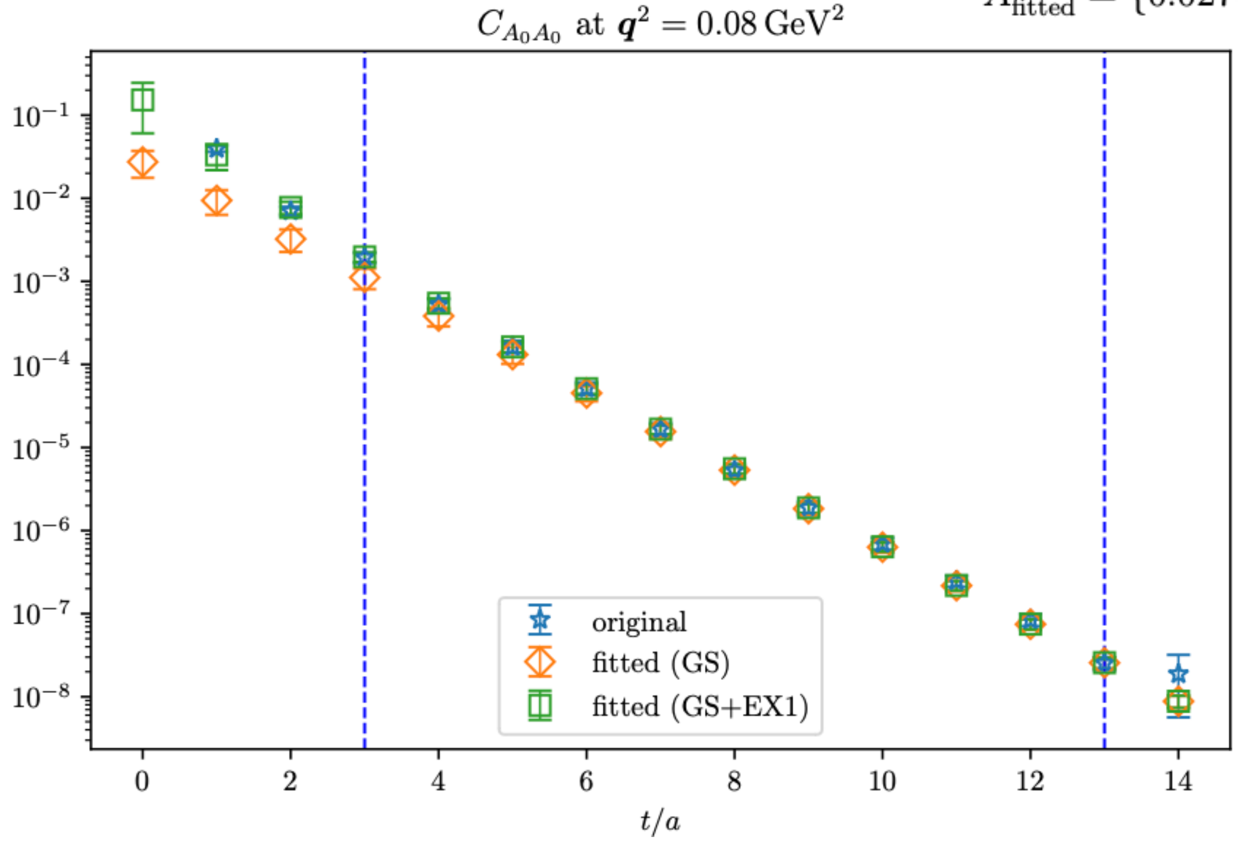


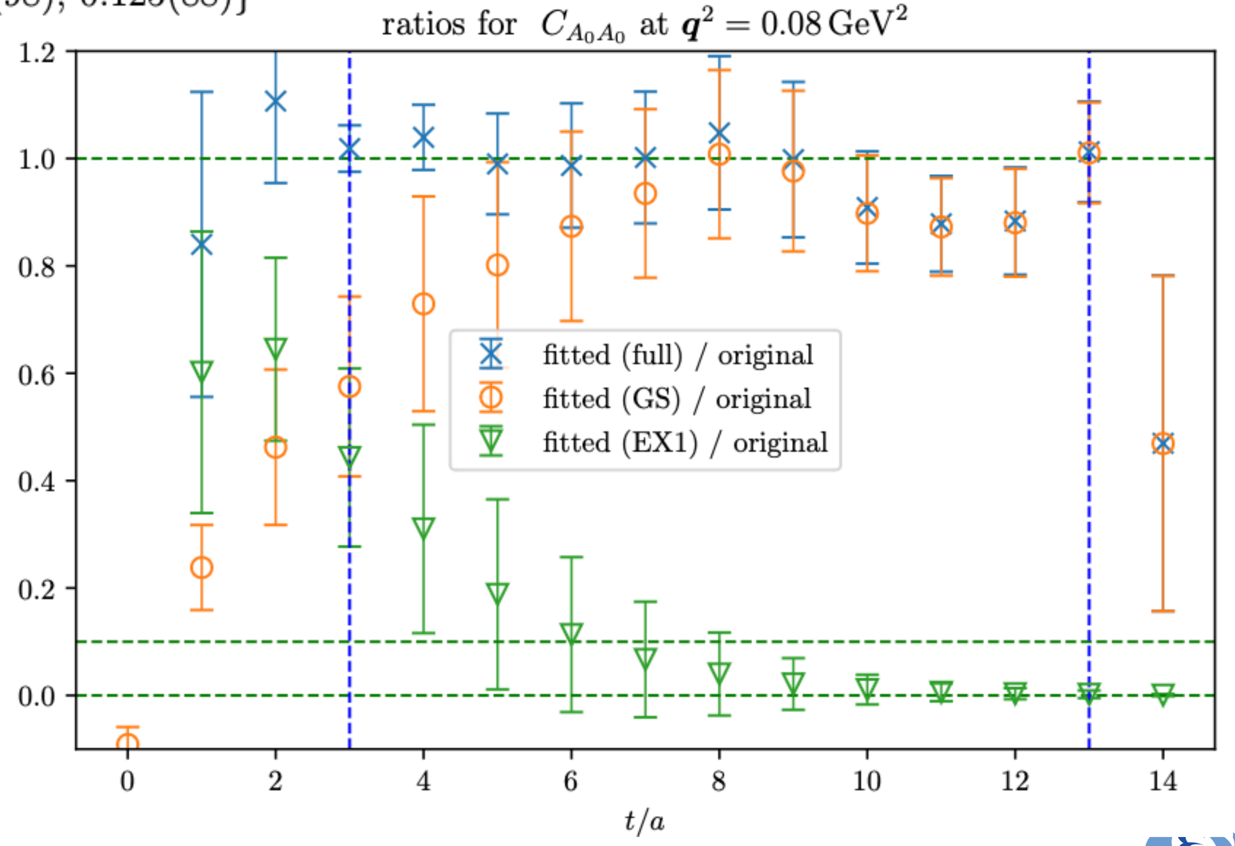
Positive sign for the existence of excited states

- the turn in the correlators at small time slice and clear portion of contributions from the excited states

$C_{A_0A_0}$ at $\vec{q}^2 = 0.08$ GeV^2

$$\chi^2/\text{d.o.f.} = 0.472\ , \quad t_{\text{fitted}} = [3, 13]$$
$$\Delta E_{\text{fitted}} = \{1.068(30),\ 0.59(30)\}$$
$$A^2_{\text{fitted}} = \{0.0274(98),\ 0.125(88)\}$$



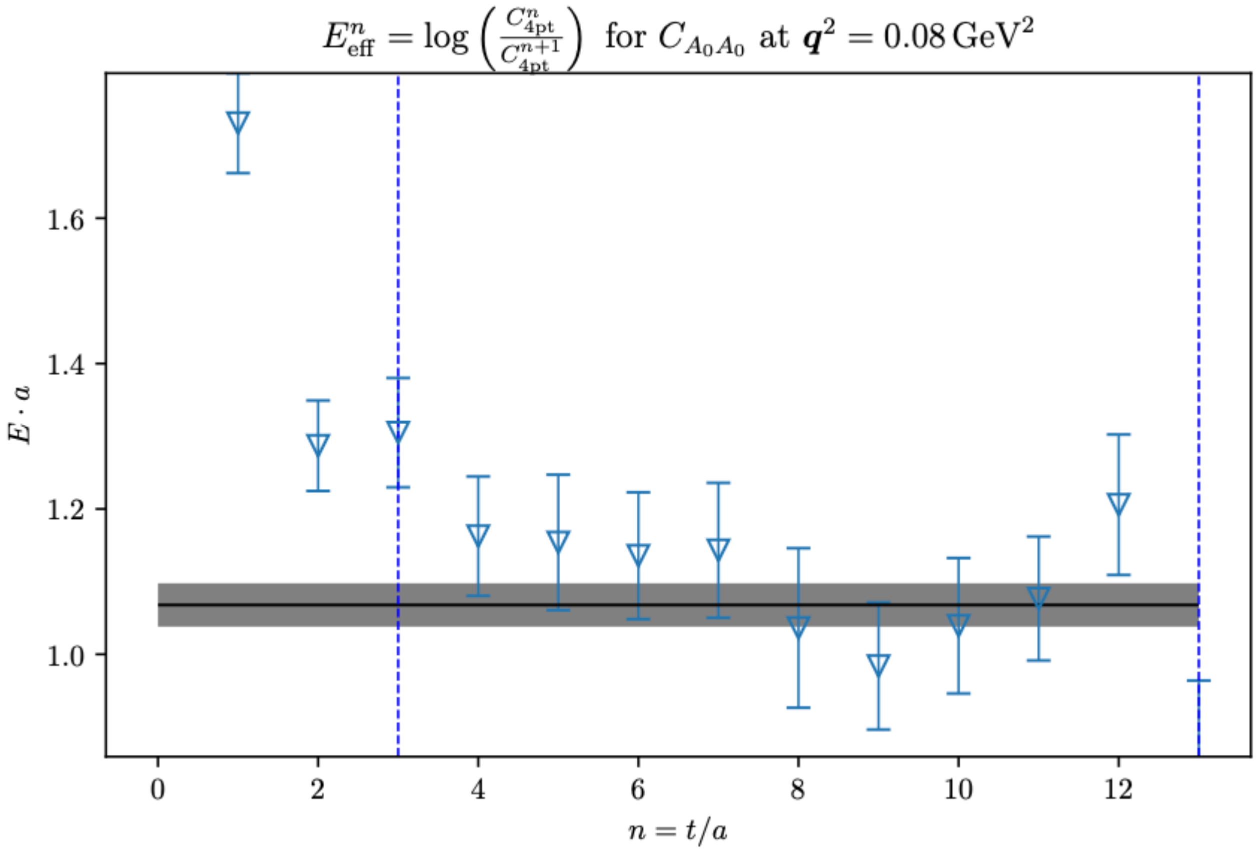


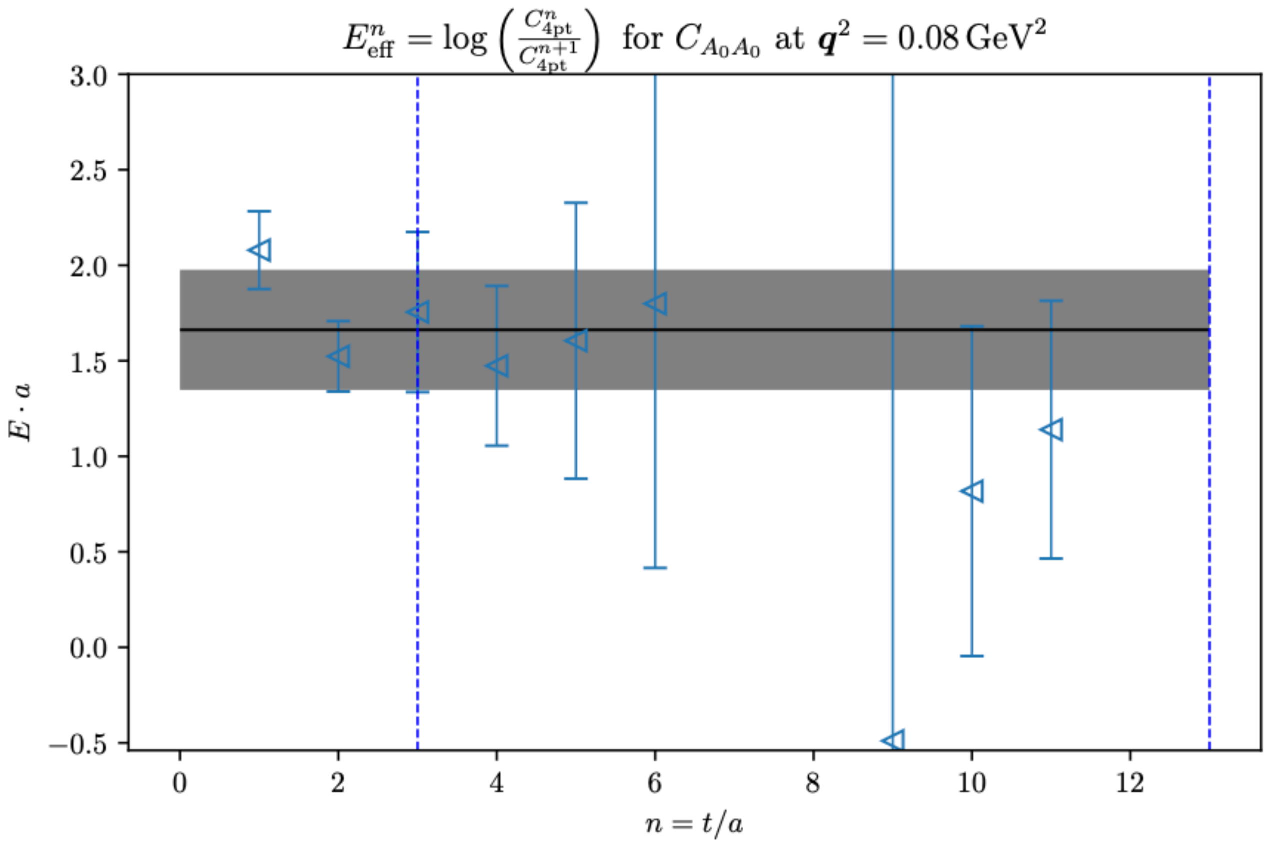
Positive sign for the existence of excited states

- steps in the effective energies

effective energies for the original $C_{A_0A_0}$

effective energies for $C_{A_0A_0}$ – the fitted ground-state contribution



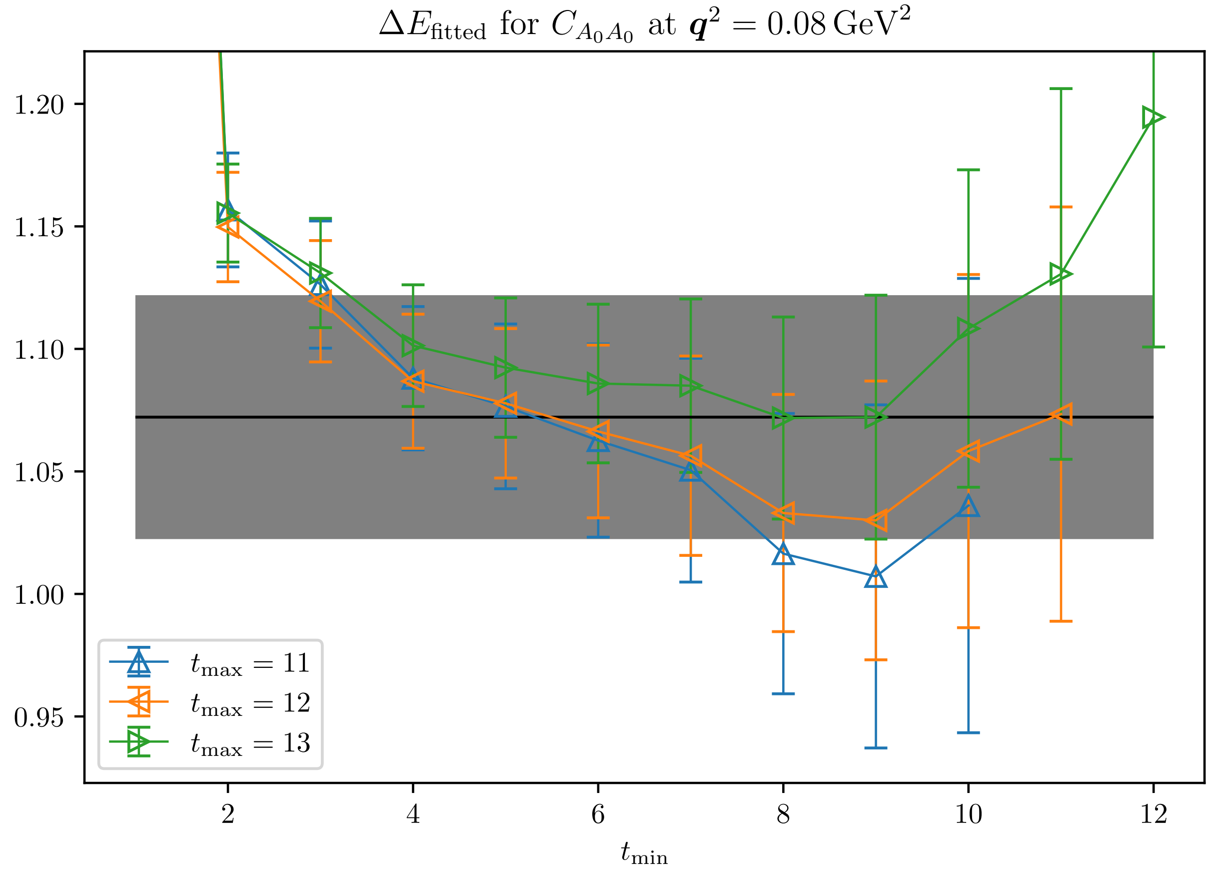


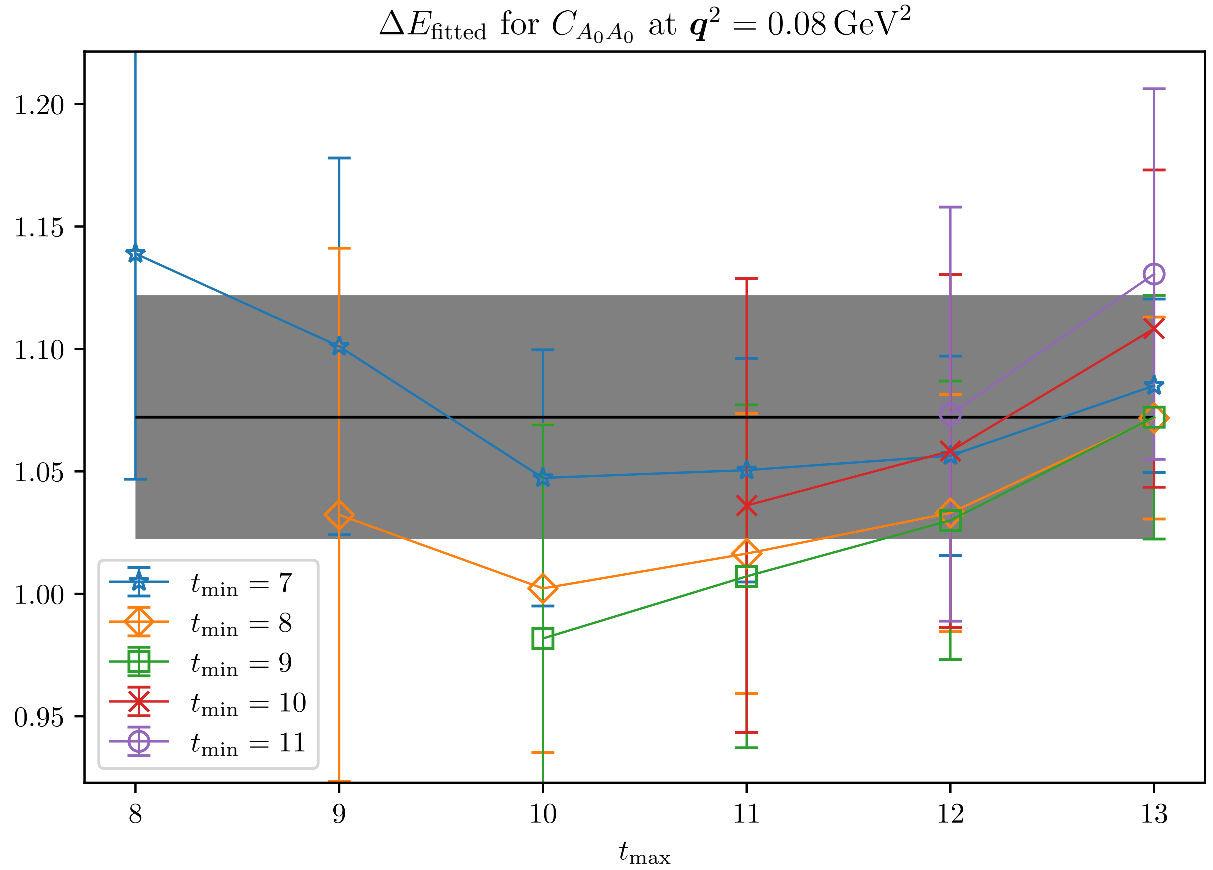
Positive sign for the existence of excited states

- changes of the parameters vs $t_{\min}$ and $t_{\max}$

fitted value of the ground-state energy in 1-state fit for $C_{A_0A_0}$ at $\vec{q}^2 = 0.08\ \mathrm{GeV}^2$

$\chi^2/\mathrm{d.o.f.} = 0.537\ ,\quad t_{\mathrm{fitted}} = [9, 13]$
$\Delta E_{\mathrm{fitted}} = \{1.072(50)\}\ ,\quad A^2_{\mathrm{fitted}} = \{0.029(17)\}$



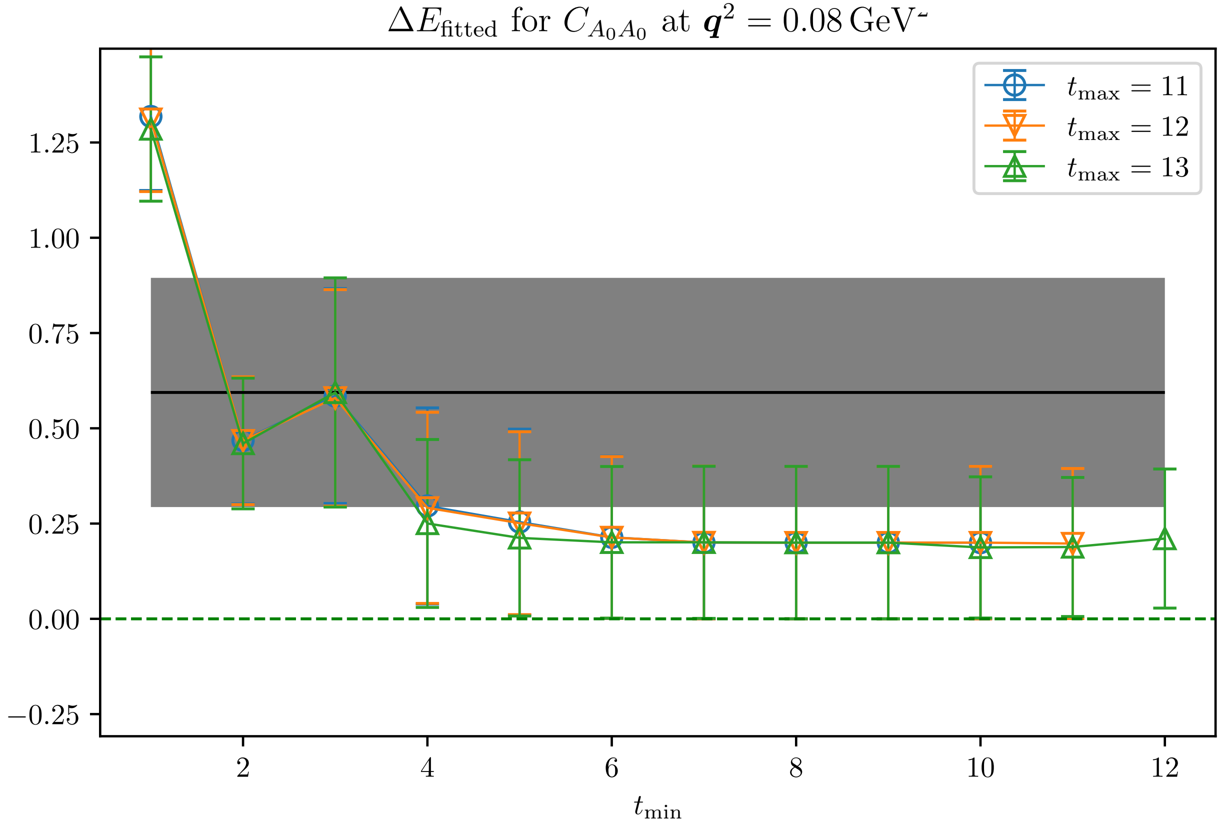


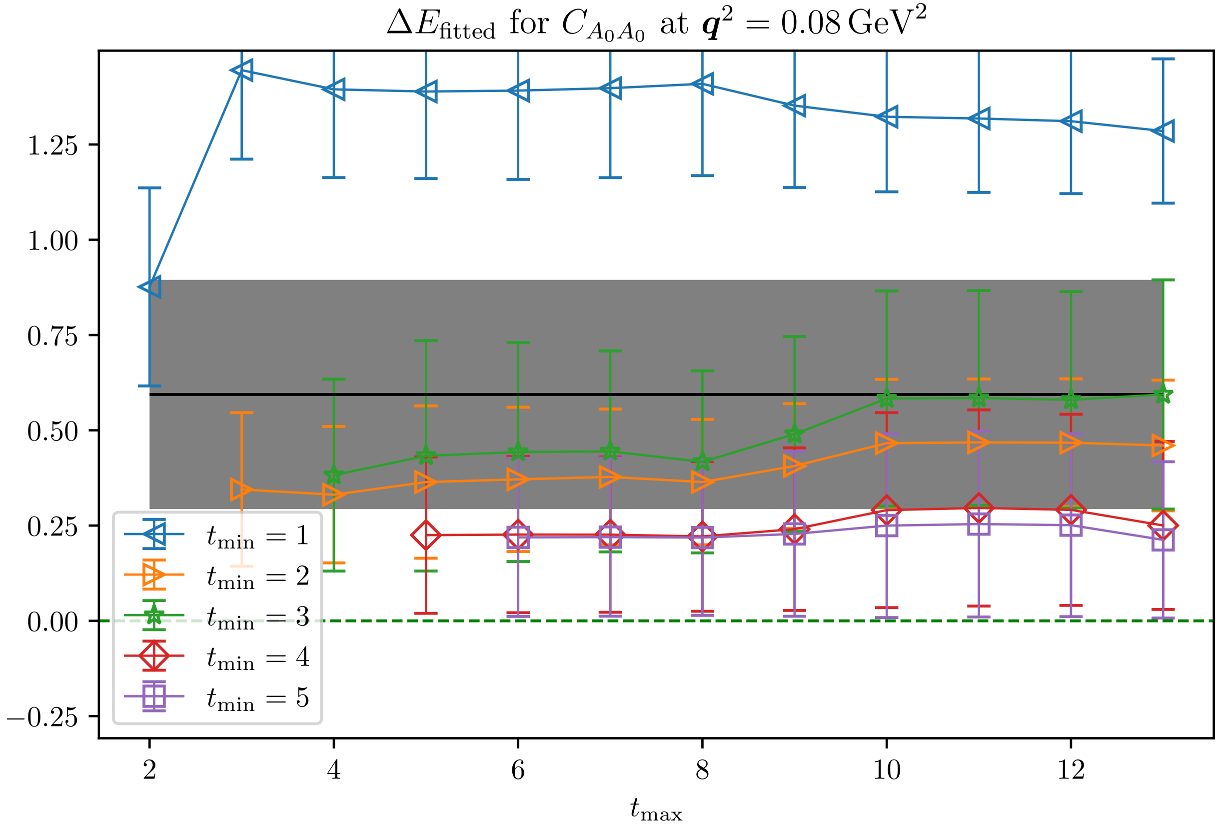
Positive sign for the existence of excited states

- changes of the parameters vs $t_{\min}$ and $t_{\max}$

fitted value of the excited-state energy in 2-state fit for $C_{A_0A_0}$ at $\vec{q}^2 = 0.08\ \mathrm{GeV}^2$

$$\chi^2/\mathrm{d.o.f.} = 0.472\ , \quad t_{\mathrm{fitted}} = [3, 13]$$
$$\Delta E_{\mathrm{fitted}} = \{1.068(30),\ 0.59(30)\}$$
$$A^2_{\mathrm{fitted}} = \{0.0274(98),\ 0.125(88)\}$$





Consistency check

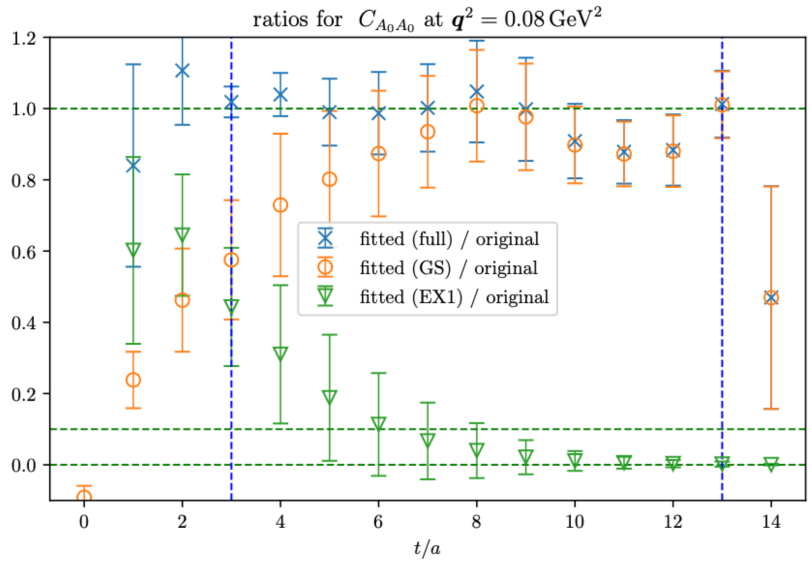
- Do we recover the estimation from the previous estimations of the contribution ratio?

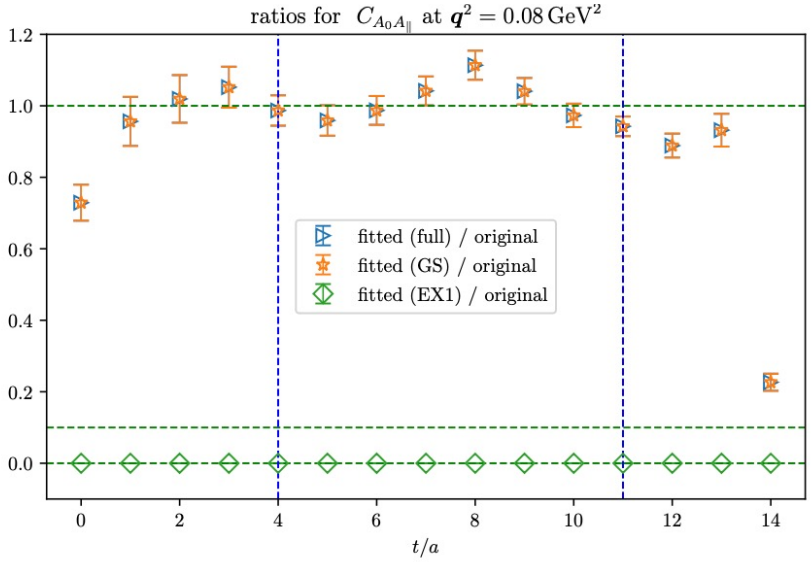
estimations of the contributions from excited state

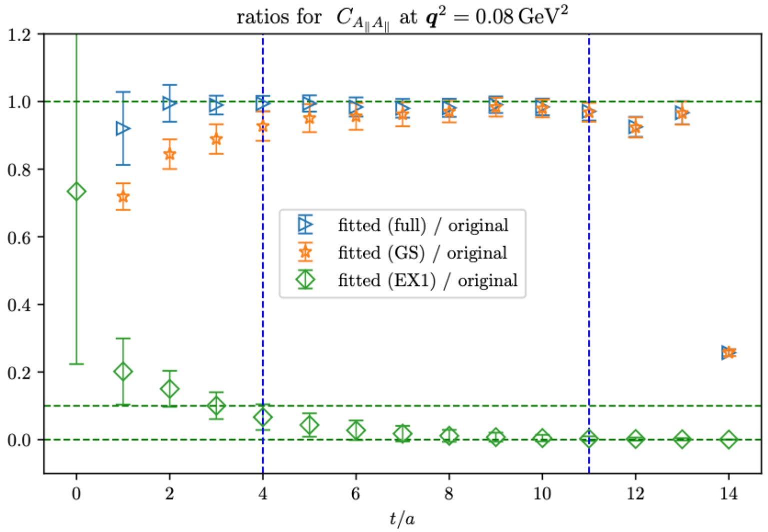
$$\boldsymbol{A_0 A_0 (\sim 50\%) > A_0 A_\parallel (\sim 10\%) > A_\parallel A_\parallel (< 10\%)}$$

Consistency check

- Do we recover the estimation from the previous estimations of the contribution ratio?







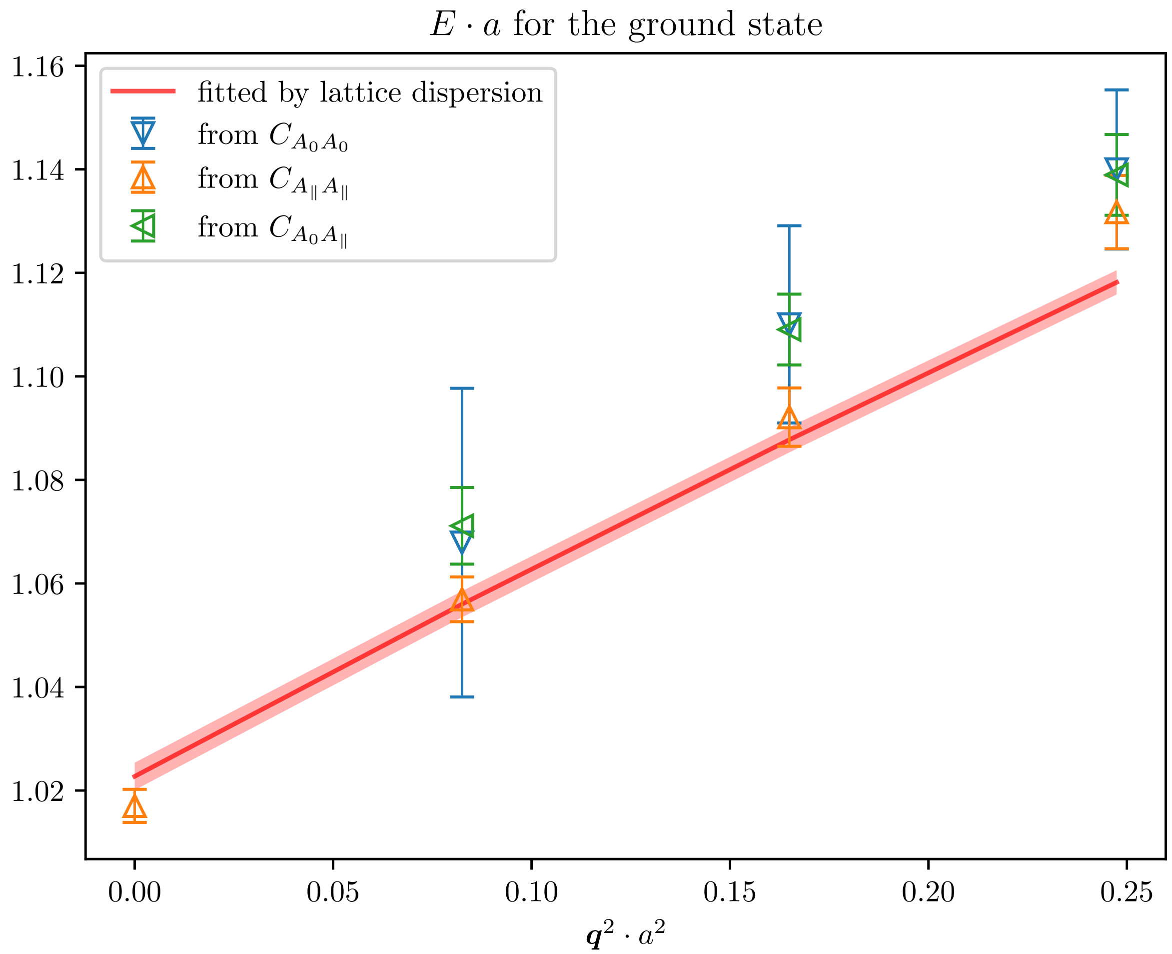
fitted contributions from excited state

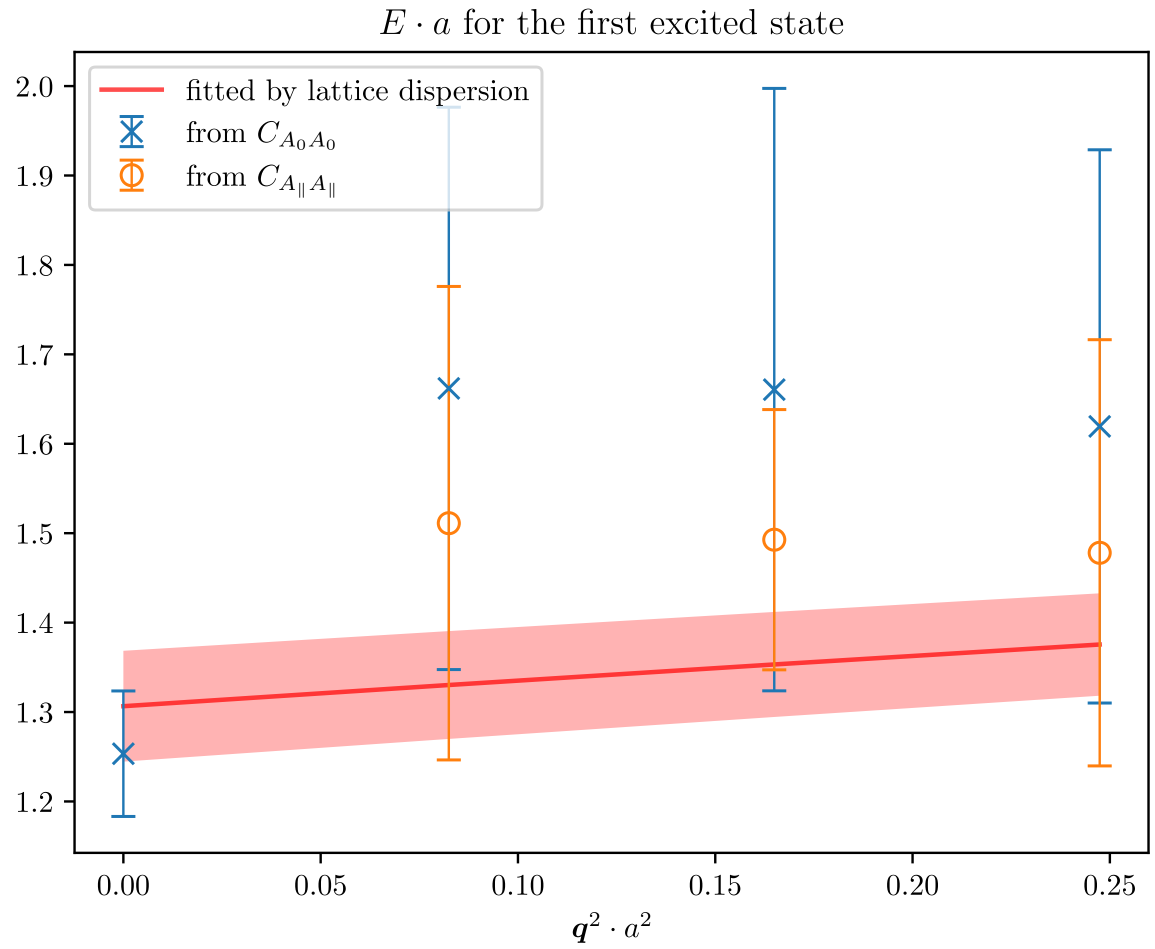
$$A_0A_0(\sim 50\%) > A_\parallel A_\parallel(< 10\%) > A_0A_\parallel(\sim 0\%)$$

Consistency check

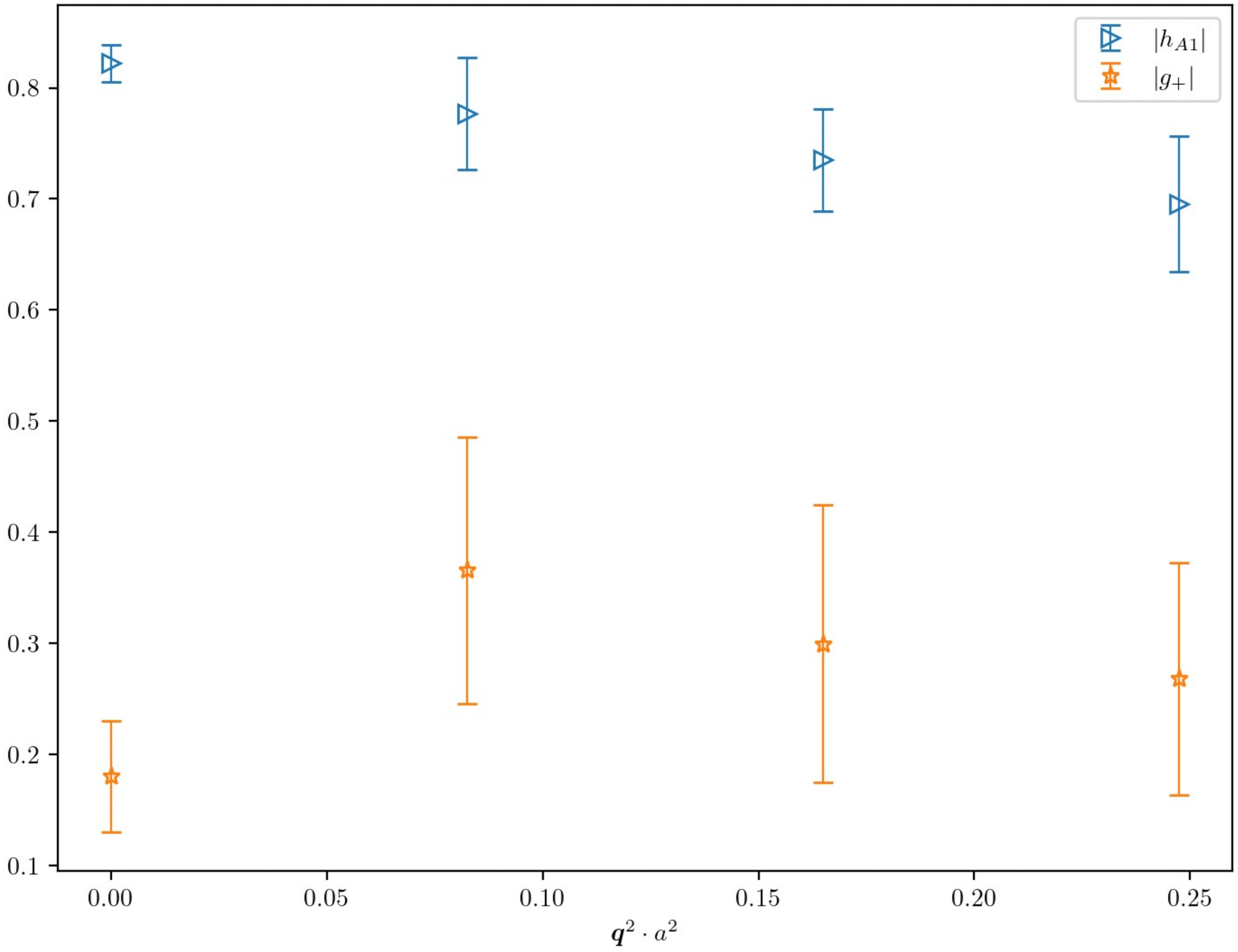
- are there the same set of states?

$$E = \cosh^{-1}\left(\cosh M + 3 - 3\cos q_k\right)$$





Resulting form factors



$$C_{A_0A_0} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} 3q_k^2 \left[\left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) h_{A1} - h_{A2} - \frac{E_{D^*}}{M_{D^*}} h_{A3} \right]^2 + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} \left[g_+ \left(E_{D_0^*} + M_{D_0^*} \right) - g_- \left(E_{D_0^*} - M_{D_0^*} \right) \right]^2 + \frac{e^{-E_{D_2^*}t}}{4E_{D_2^*}M_{D_2^*}} \cdots$$

$$C_{A_\parallel A_\parallel} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} \left[h_{A1} \frac{(E_{D^*}+M_{D^*})\, E_{D^*}}{M_{D^*}} - h_{A3} \frac{3q_k^2}{M_{D^*}} \right]^2 + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} 3q_k^2 \left[g_+ - g_- \right]^2 + \frac{e^{-E_{D_2^*}t}}{4E_{D_2^*}M_{D_2^*}} \cdots$$

$$C_{A_0A_\parallel} = C_{A_\parallel A_0} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} \sqrt{3} q_k \Bigg[\left(h_{A1} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) \right)^2 E_{D^*} + (h_{A3})^2 \frac{3q_k^2 E_{D^*}}{M_{D^*}^2} - h_{A1}h_{A2} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) E_{D^*} - h_{A1}h_{A3} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) \left(\frac{2E_{D^*}^2}{M_{D^*}} - M_{D^*} \right) + h_{A3}h_{A2} \frac{3q_k^2}{M_{D^*}} \Bigg]$$

Summary and prospect

Summary

- as a preliminary study, we show the feasibility to extract exclusive informations from inclusive correlator

Prospect

- can we do the fit just blindly using the functional form?
- we should investigate the continuum and infinite volume limit

$$C_{A_0A_0} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} 3q_k^2 \left[\left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) h_{A1} - h_{A2} - \frac{E_{D^*}}{M_{D^*}} h_{A3} \right]^2 + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} \left[g_+ \left(E_{D_0^*} + M_{D_0^*} \right) - g_- \left(E_{D_0^*} - M_{D_0^*} \right) \right]^2 + \frac{e^{-E_{D_2^*}t}}{4E_{D_2^*}M_{D_2^*}} \cdots$$

$$C_{A_\parallel A_\parallel} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} \left[h_{A1} \frac{(E_{D^*}+M_{D^*})E_{D^*}}{M_{D^*}} - h_{A3} \frac{3q_k^2}{M_{D^*}} \right]^2 + \frac{e^{-E_{D_0^*}t}}{4E_{D_0^*}M_{D_0^*}} 3q_k^2 \left[g_+ - g_- \right]^2 + \frac{e^{-E_{D_2^*}t}}{4E_{D_2^*}M_{D_2^*}} \cdots$$

$$C_{A_0A_\parallel} = C_{A_\parallel A_0} = \frac{e^{-E_{D^*}t}}{4E_{D^*}M_{D^*}} \sqrt{3} q_k \left[\left(h_{A1} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) \right)^2 E_{D^*} + (h_{A3})^2 \frac{3q_k^2 E_{D^*}}{M_{D^*}^2} - h_{A1}h_{A2} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) E_{D^*} - h_{A1}h_{A3} \left(\frac{E_{D^*}+M_{D^*}}{M_{D^*}} \right) \left(\frac{2E_{D^*}^2}{M_{D^*}} - M_{D^*} \right) + h_{A3}h_{A2} \frac{3q_k^2}{M_{D^*}} \right]$$

KEK