Gradient Flow Renormalisation for Meson Mixing and Lifetimes

Lattice meets Continuum – Siegen

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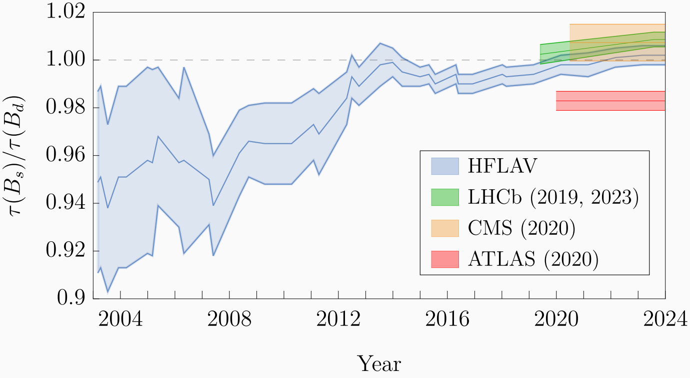
B meson mixing and lifetimes

- B meson mixing and lifetimes important tests of our understanding of QCD and as probes of physics beyond the SM
- High and further increasing experimental precision

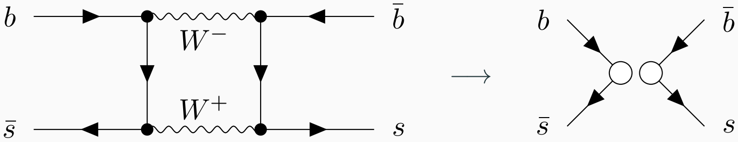
⇒ Theory effort needed to catch up



[Albrecht, Bernlochner, Lenz, Rusov 2024]

Theory in a nutshell $\Rightarrow$ see Maria Laura Piscopo's talk for more details

- Tool of choice: effective theories



- $+$ Heavy Quark Expansion (HQE) for lifetimes:

$$\Gamma(H_b) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right)$$

- Observables factorize into perturbative Wilson coefficients and non-perturbative matrix elements
- This talk: dimension-six four-quark matrix elements $\langle \tilde{\mathcal{O}}_6 \rangle$ for B meson mixing and lifetimes on the lattice
- Mixing well established $\Rightarrow$ use as validation, then move to lifetimes

$\Delta Q = 2$ meson mixing

- Consider mixing of neutral heavy mesons Q_q and $\bar{Q}_q$
- Only

$$\tilde{Q}^q_{6,1} = (\bar{q}\gamma_\mu(1-\gamma_5)Q)(\bar{q}\gamma^\mu(1-\gamma_5)Q)$$

  contributes to mass difference in the SM
- No mixing with other operators
- Bag parameter

$$B_1^{Q_q}(\mu) = \frac{\langle \bar{Q}_q | \tilde{Q}^q_{6,1} | Q_q \rangle}{\frac{8}{3} M_{Q_q}^2 f_{Q_q}^2}$$

  for charm and bottom mesons determined by sum rule computations and several lattice groups $\Rightarrow$ see Felix Erben's talk
- Nonetheless, $B_1^{Q_q}$ dominate uncertainty
- We may not be competitive here, but test bed for more difficult $\Delta Q = 0$ lifetimes

The gradient flow and the short-flow-time expansion $\Rightarrow$ more details in Robert Harlander's talk

- Extend fields along new parameter flow time τ [Narayanan, Neuberger 2006; Lüscher 2010; Lüscher 2013]

$$\partial_\tau B_\mu^a = \mathcal{D}_\nu^{ab} G_{\nu\mu}^b \quad \text{with} \quad B_\mu^a(\tau,x)\big|_{\tau=0} = A_\mu^a(x)$$

- Regulates UV divergencies and composite operators do not require renormalisation [Lüscher, Weisz 2011]
- Expand composite operators in small τ [Lüscher, Weisz 2011]

$$\tilde{\mathcal{O}}_i(\tau,x) = \sum_j \zeta_{ij}(\tau)\mathcal{O}_j(x) + O(\tau)$$

and invert [Suzuki 2013; Lüscher 2013]

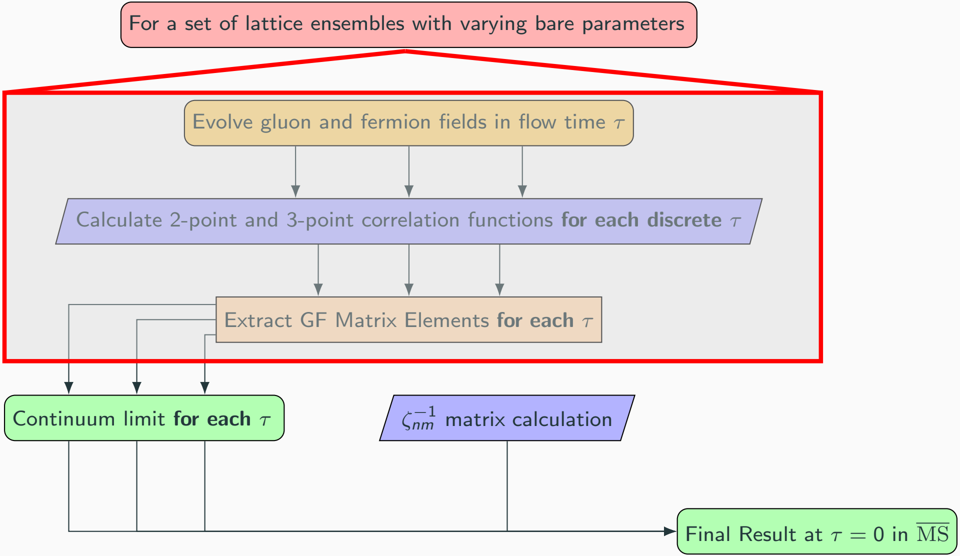
$$T = \sum_i C_i \mathcal{O}_i = \sum_{i,j} C_i \zeta_{ij}^{-1}(\tau)\tilde{\mathcal{O}}_j(\tau)$$

$\Rightarrow$ Expressed physical observable through better behaved flowed operators

- Three ingredients for physical prediction:
  Wilson coefficients C_i, matching matrix $\zeta_{ij}^{-1}(\tau)$, flowed matrix elements $\langle\tilde{\mathcal{O}}_j(\tau)\rangle$

$\Rightarrow$ see Òscar Lara Crosas' talk for another application

Matrix elements with gradient flow (schematic)



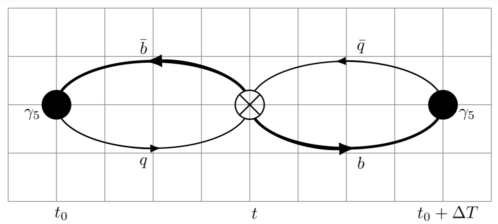
Lattice setup

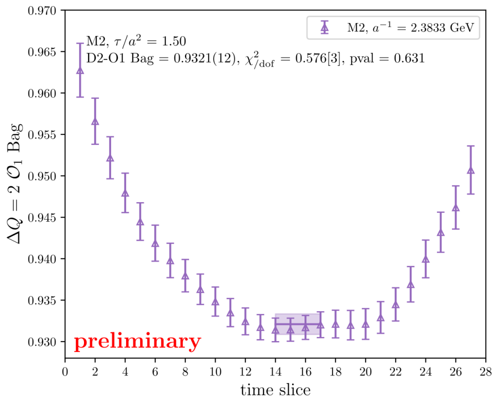
- Use 6 RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles
- For pilot study, simplified setup without additional extrapolations
  $\Rightarrow$ physical charm and strange quarks $\Rightarrow$ simulating a charm-strange meson
- Stout-smeared Möbius DWF for charm [Cho, Hashimoto, Jüttner, Kaneko, Marinkovic, Noaki, Tsang 2015]
- Neutral charm-strange meson mixing $\Rightarrow$ proxy to short-distance D^0 mixing up to spectator effects
- Charm-strange meson $\Delta Q = 0$ operators $\Rightarrow$ D_s meson lifetimes

$\Delta Q = 2$ bag parameter computation

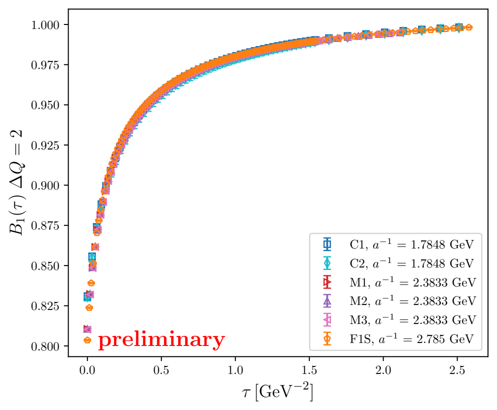
- Measure three-point correlation function
- Normalize by two-point correlation functions:

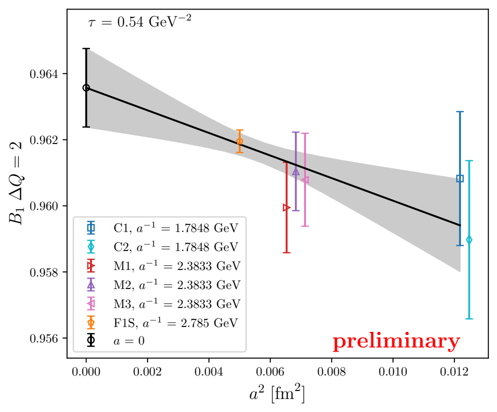
$$\frac{C^{\rm 3pt}_{\mathcal{Q}_i}(t, \Delta T, \tau)}{\frac{8}{3} C^{\rm 2pt}_{\rm AP}(t,\tau) C^{\rm 2pt}_{\rm PA}(\Delta T - t, \tau)} \to B_1^{\rm GF}(\tau)$$





$\Delta Q = 2$ continuum limit





- Different lattice spacings overlap at finite physical flow time

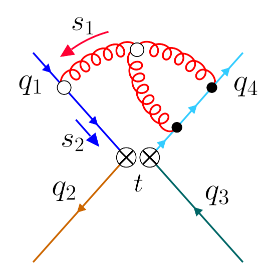
$\Rightarrow$ Mild continuum limit

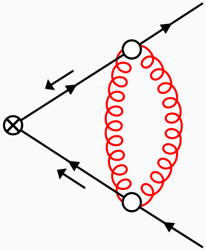
Matching to $\overline{\text{MS}}$

- In the continuum

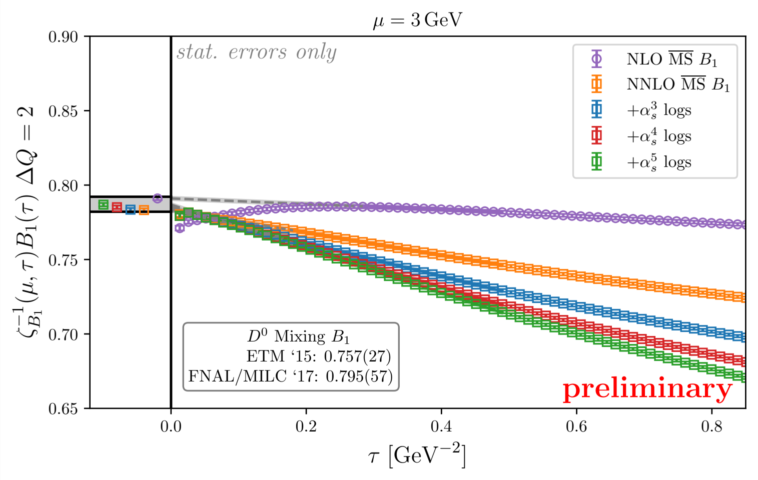
$$\frac{C^{\rm 3pt}_{\mathcal{Q}_i}(t,\Delta T,\tau)}{\frac{8}{3}C^{\rm 2pt}_{\rm AP}(t,\tau)C^{\rm 2pt}_{\rm PA}(\Delta T-t,\tau)} \to B_1^{\rm GF}(\tau) \propto \frac{\langle \tilde{Q}^q_{6,1}\rangle(\tau)}{\left(\langle\bar{\chi}\gamma_\mu\gamma_5\chi\rangle(\tau)\right)^2}$$

- Matching of $\tilde{Q}^q_{6,1}(\tau)$ available through NNLO from [Suzuki, Taniguchi, Suzuki, Kanaya 2020; Harlander, **FL** 2022]
- Matching of axial current available through NNLO from [Endo, Hieda, Miura, Suzuki 2015; Borgulat, Harlander, Kohnen, **FL** 2023]





Preliminary results for $\Delta Q = 2$ mixing



- $\tau \to 0$ extrapolation smooth, different perturbative orders close
- $B_1^{\overline{\mathrm{MS}}} = 0.787(5)$
- Statistical uncertainties and perturbative spread only!
- In the ballpark of:
  - [ETM 2015] : $B_1^{\overline{\mathrm{MS}}} = 0.757(27)$
  - [FNAL/MILC 2015] : $B_1^{\overline{\mathrm{MS}}} = 0.795(57)$
  - HQET sum rules [Kirk, Lenz, Rauh 2017] : $B_1^{\overline{\mathrm{MS}}} = 0.654^{+0.060}_{-0.052}$

$\Delta Q = 0$ lifetimes

- Lifetime differences described by four operators:

$$\mathcal{O}_1 = (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)Q) \qquad \langle \mathcal{O}_1(\mu)\rangle = f_{B_q}^2\, M_{B_q}^2\, B_1^{\Delta Q=0}(\mu)$$

$$\mathcal{O}_2 = (\bar{Q}(1-\gamma_5)q)(\bar{q}(1+\gamma_5)Q) \qquad \langle \mathcal{O}_2(\mu)\rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2\, M_{B_q}^2\, B_2^{\Delta Q=0}$$

$$T_1 = (\bar{Q}\gamma_\mu(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1-\gamma_5)T^A Q) \qquad \langle T_1(\mu)\rangle = f_{B_q}^2\, M_{B_q}^2\, \epsilon_1^{\Delta Q=0}(\mu)$$

$$T_2 = (\bar{Q}(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1+\gamma_5)T^A Q) \qquad \langle T_2(\mu)\rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2\, M_{B_q}^2\, \epsilon_2^{\Delta Q=0}(\mu)$$

- Some ancient, preliminary lattice results available [Di Pierro, Sachrajda 1998; Di Pierro, Sachrajda, Michael 1999; Becirevic 2001] , new calculation ongoing [Lin, Detmold, Meinel 2022]
- More operators and mixing with lower dimensional operators for absolute lifetimes

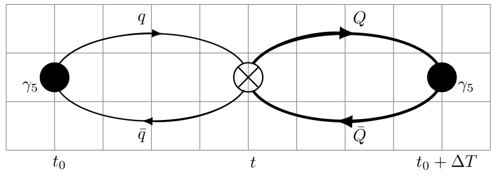
$\Rightarrow$ Power divergencies!

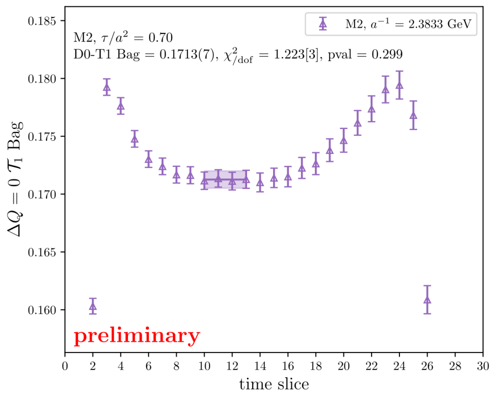
- Only HQET sum rules result available [Kirk, Lenz, Rauh 2017; King, Lenz, Rauh 2021]
- This talk: focus on $\mathcal{O}_1$ and T_1, ignore eye diagrams and lower-dimensional operators

Computing ϵ_1

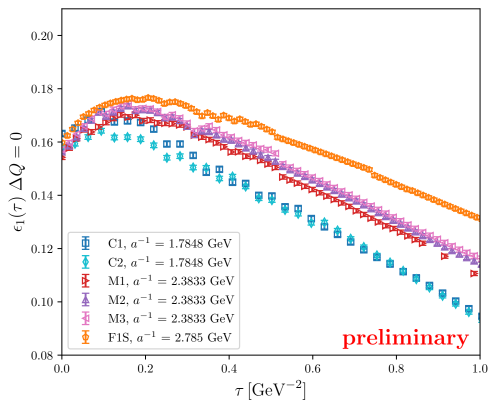
- $B_1^{\Delta Q=0}$ very similar to $B_1^{\Delta Q=2}$
- $\epsilon_1^{\Delta Q=0}$ has different functional form due to asymmetric signal $Q\bar{Q} \to q\bar{q}$

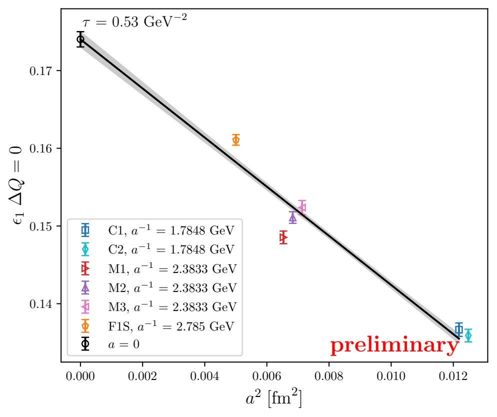
$\Rightarrow$ Extraction more difficult





$\epsilon_1^{\Delta Q=0}$ continuum limit





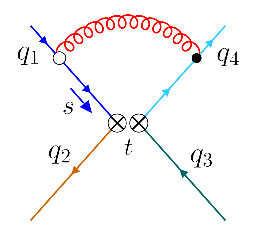
- Some systematic effects in the correlator fits to be understood
- Continuum limit steeper

Matching to $\overline{\text{MS}}$

- Again compute

$$B^{\rm GF}(\tau) = \frac{\langle \tilde{Q} \rangle(\tau)}{\left(\langle \bar{\chi}\gamma_\mu\gamma_5\chi\rangle(\tau)\right)^2}$$

- Matching of axial current available through NNLO from [Endo, Hieda, Miura, Suzuki 2015; Borgulat, Harlander, Kohnen, **FL** 2023]
- Matching of $\tilde{Q}(\tau)$ computed through NLO in this work, ignoring lower-dimensional operators so far ($\equiv$ lifetime differences)

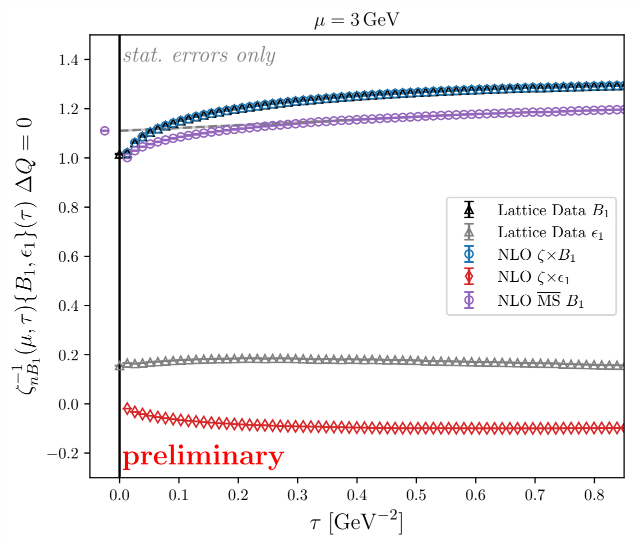


$$\zeta^{-1}(\tau) = \begin{pmatrix} 1 & \alpha_s(-\frac{11}{4} - \frac{3}{2}L_{\mu t}) \\ \alpha_s(-\frac{11}{18} - \frac{1}{3}L_{\mu t}) & 1 + \alpha_s(\frac{11}{12} + \frac{1}{2}L_{\mu t}) \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

$$L_{\mu t} = \ln 2\mu^2 t + \gamma_E$$

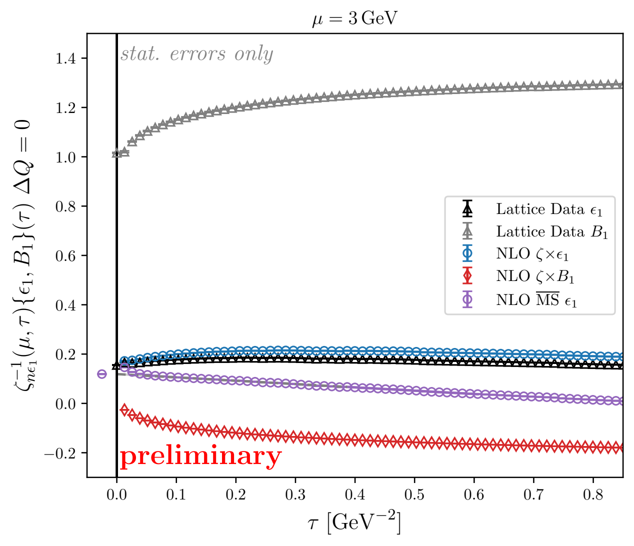
- Matching matrix mixes $B_1^{\Delta Q=0,\rm GF}$ and $\epsilon_1^{\Delta Q=0,\rm GF}$

Preliminary results for $B_1^{\Delta Q=0}$



- $B_1^{\Delta Q=0,\overline{\text{MS}}} = 1.110(2)$
- Only statistical uncertainties, missing eye diagrams, missing lower-dimensional operators
- HQET sum rules for lifetime differences [Kirk, Lenz, Rauh 2017] : $B_1^{\overline{\text{MS}}} = 0.902^{+0.077}_{-0.051}$
- Same order of magnitude

Preliminary results for $\epsilon_1^{\Delta Q=0}$



- $\epsilon_1^{\Delta Q=0,\overline{\mathrm{MS}}} = 0.119(1)$
- Only statistical uncertainties, missing eye diagrams, missing lower-dimensional operators
- HQET sum rules for lifetime differences [Kirk, Lenz, Rauh 2017] : $\epsilon_1^{\overline{\mathrm{MS}}} = -0.132^{+0.041}_{-0.046}$
- Magnitude agrees, sign differs

Summary

- $\Delta B = 0$ bag parameters important for flavour phenomenology
- Only HQET sum rules results available
- We aim to compute them in full QCD on the lattice using gradient flow renormalisation
- Preliminary result for test bed $\Delta C = 2$ mixing consistent with literature
- Performed first analysis for $\Delta C = 0$ lifetimes
- Missing eye diagrams, mixing with lower dimensional operators, and controlling the arising power divergencies, e.g. following the strategy of [Kim, Luu, Rizik, Shindler 2021]