Continuum approaches to semileptonic $s \to d$ and $b \to s(d)$ transitions

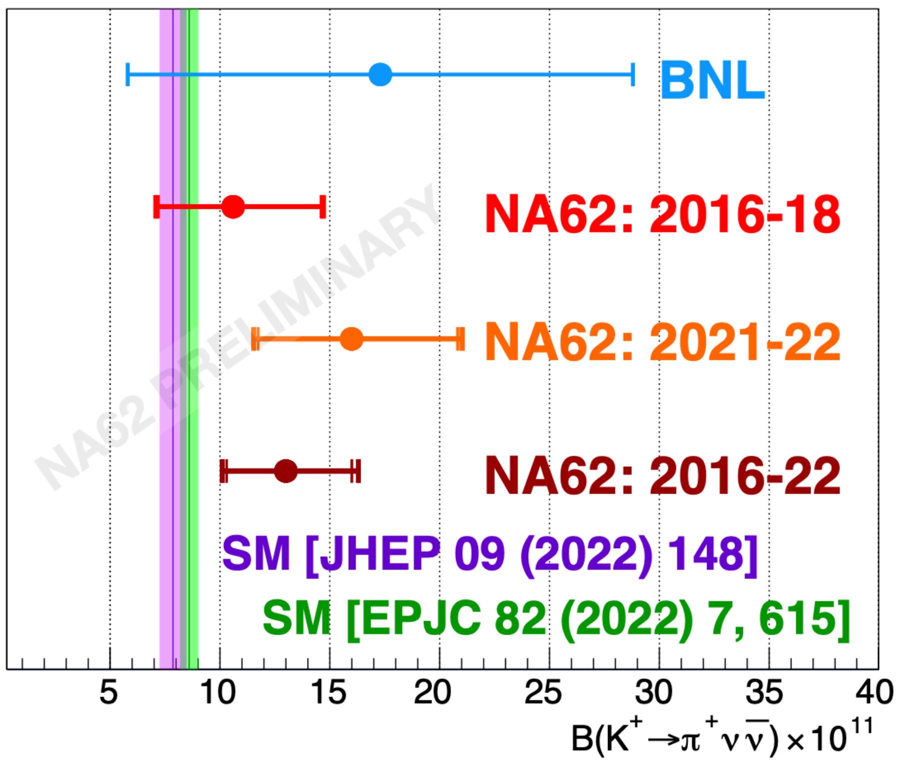
Lattice meets Continuum
Siegen, Oct 2 2024

Bansal, JJ, Winney [preliminary]
Huber, Hurth, JJ, Lunghi, Qin, Vos [2404.03517]

TP1 Theoretical Particle Physics

Jack Jenkins

Some motivation: (preliminary) results from NA62 last week



$$10^{11} \times BR|_{exp} = 13.0(^{+3.0}_{-2.7})_{stat}(^{+1.3}_{-1.2})_{sys}$$

$$10^{11} \times BR|_{SM} = 7.73 \pm 0.16_{pert} \pm 0.25_{non-pert} \pm 0.54_{param}$$

Brod, Gorbahn, Stamou [2105.02868]

Four frontiers for precision in $K^+ \to \pi^+ \nu \bar{\nu}$:

- Experiment! .. still statistically limited..
- Progress on the $|V_{cb}|$ puzzle in B sector: $K^+ \to \pi^+ \nu \bar{\nu}$ rate is proportional to $|V_{cb}|^4$
- $V_{ts}^* V_{td} X_t(m_t)$ at higher order in perturbative QCD
- Intrinsic hadronic uncertainties (local and nonlocal FFs)

This talk

Scale separation in $K^+ \to \pi^+ \nu\bar{\nu}$

$$O_1^q = (\bar{d}_L \gamma_\mu s_L)(\bar{q}_L \gamma^\mu q_L)\,, \;\; q = u, c$$
$$O_2^q = (\bar{d}_L \gamma_\mu T^a s_L)(\bar{q}_L \gamma^\mu q_L)$$
$$O_3 = (\bar{d}_L \gamma_\mu s_L)\Sigma_{q'}(\bar{q}' \gamma^\mu q')\,, \;\; q' = u, d, s, c$$
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$$O_{10} = (\bar{d}_L \gamma_\mu s_L)\Sigma_{\ell'}(\bar{\ell}' \gamma^\mu \gamma_5 \ell')\,,$$
$$O_\nu = (\bar{d}_L \gamma_\mu s_L)\Sigma_\nu(\bar{\nu}_L \gamma^\mu \nu_L)$$

Dominant contribution from O_ν sensitive to large top quark mass (GIM), known at NLO QCD and NLO EW

Brod, Gorbahn, Stamou [1009.0947]

RGE invariant below the weak scale (CVC)

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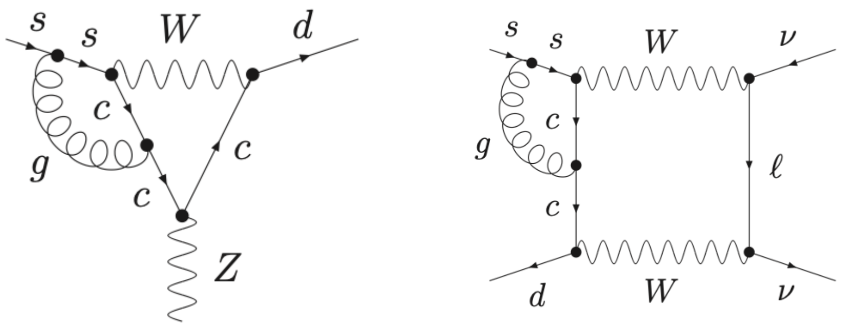

Buras, Gorbahn, Haisch, Nierste [0603079]

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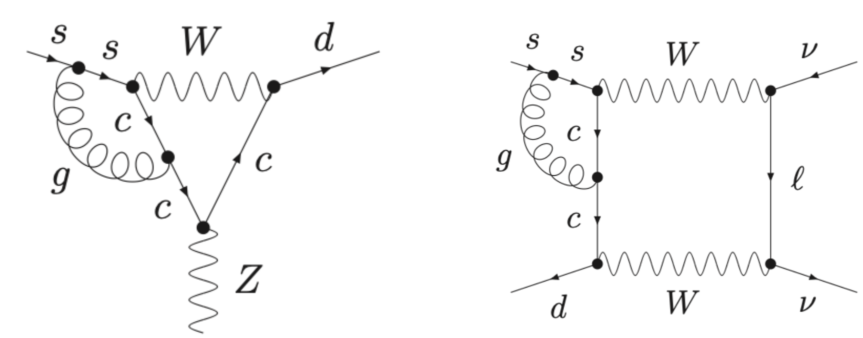

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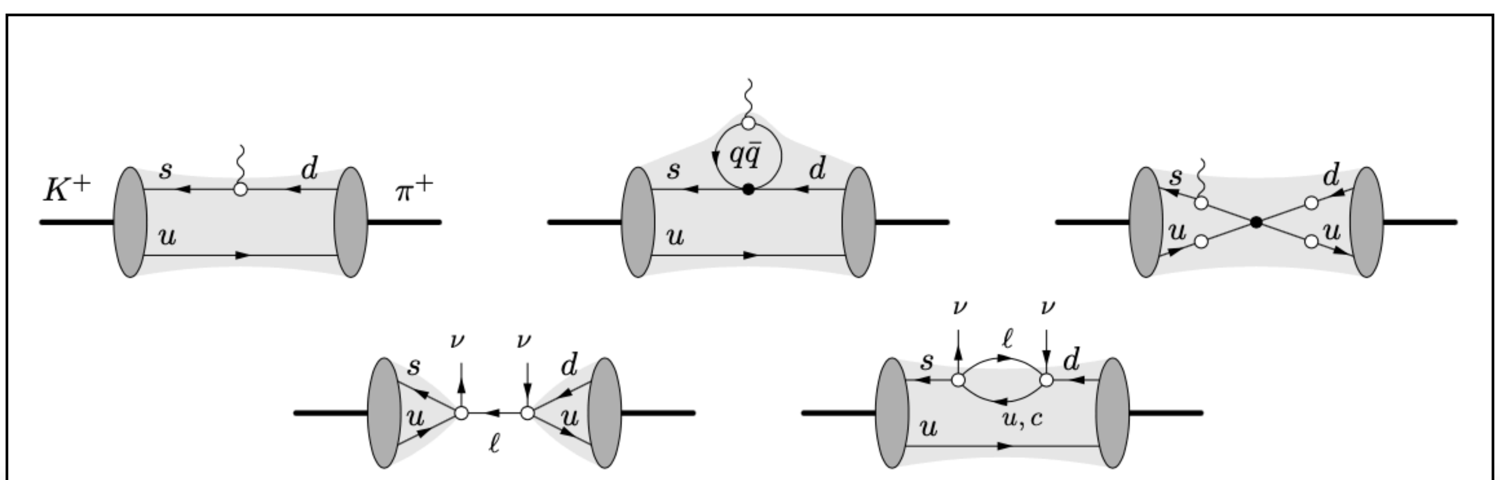

Nonlocal operators / matrix elements from factorisation ✔

Actual values of these FFs (??)

Local form factors

Charged-currents:

$$\langle \pi^+(k) | \bar{u}\gamma_\mu s | K^0(p)\rangle = f_+^{K^0\pi^+}(q^2)(p+k)_\mu + f_-^{K^0\pi^+}(q^2)q_\mu$$

Neutral-currents:

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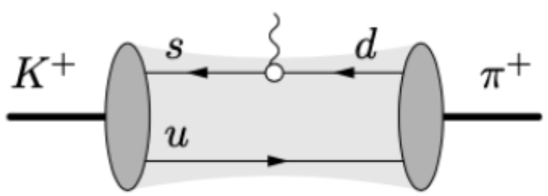

Local vector form factors from V-A currents in SM (also V+A for FCNCs, hadronic current is the same)

Universal to charged-current and neutral-current $K \to \pi^+$ transitions up to isospin corrections ($K^+ \to \pi^0$ complicated by $\pi^0 - \eta$ mixing LECs)

$$\frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = 1.0015 \pm 0.0007 \qquad \frac{\lambda_+^{K^+\pi^+}}{\lambda_+^{K^0\pi^+}} = 0.9986 \pm 0.0002$$

Mescia, Smith [0705.2025]

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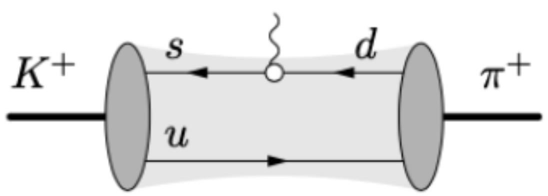

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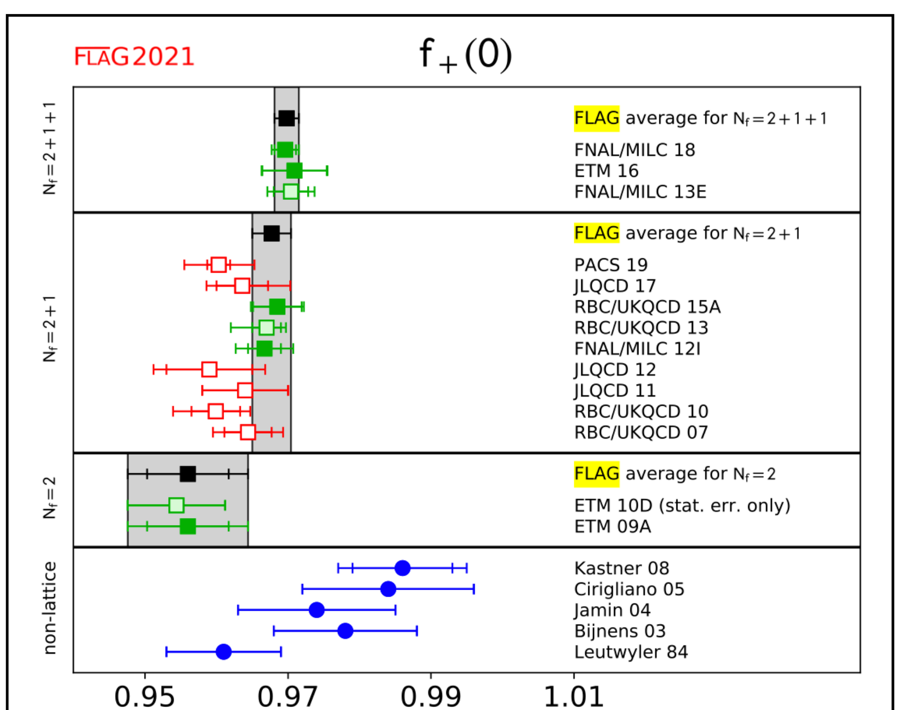
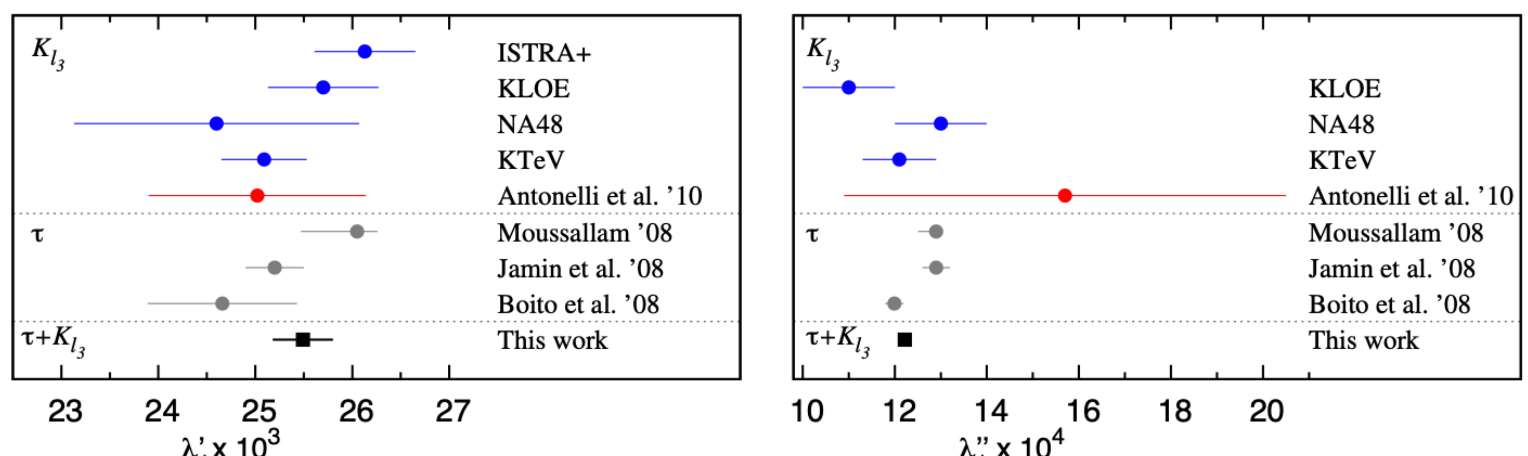
Normalisation: LQCD



FLAG [2111.09849]

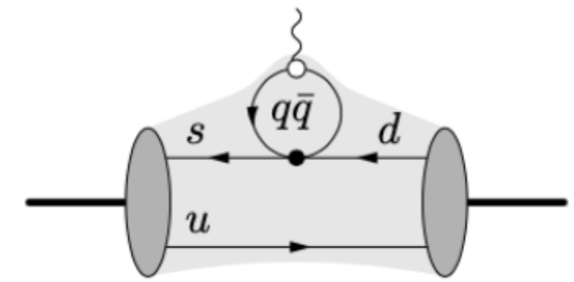
Slope parameters: $K \to \pi\ell\nu$ and $\tau \to K\pi\bar{\nu}_\tau$ (analyticity)



Boito, Escribano, Jamin [1007.1858]

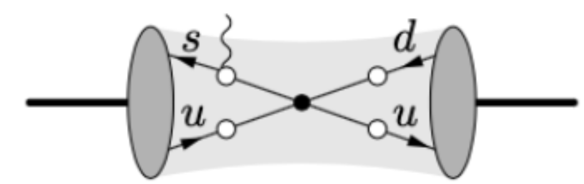
$$f_+(q^2) \simeq f_+(0)\left[1 + \lambda'_+\frac{q^2}{M_{\pi^+}^2} + \lambda''\frac{q^4}{2M_{\pi^+}^4}\right]$$

Nonlocal form factors (Z, γ)



Electromagnetic form factor dominates $K^+ \to \pi^+ \ell^+ \ell^-$ and can be extracted directly from the spectrum up to phase

$$\int d^4x\, e^{-iqx} \langle \pi^+ | T\mathscr{L}_{\Delta S}(0)\, J^\mu_\gamma(x) | K^+ \rangle = \left(q^\mu p \cdot q - p^\mu q^2\right) F_\gamma^{K^+\pi^+}(q^2)$$



Weak neutral-current form factor in $K^+ \to \pi^+ \nu \bar{\nu}$

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No $\perp$ contribution ($m_\nu = 0$)

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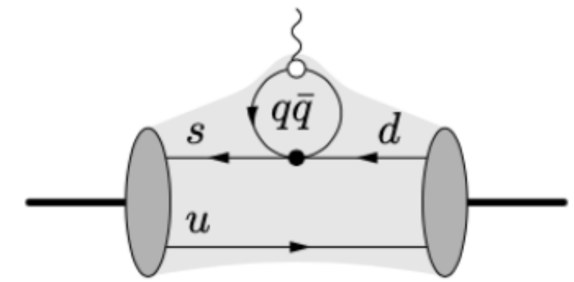

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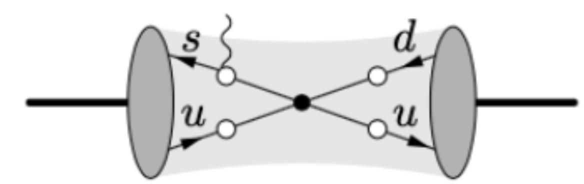

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Weak and electric charges are not aligned

$$\frac{Q_u}{Q_d} = -2$$

$$J^\mu_\gamma = \frac{2}{3}(\bar{u}\gamma_\mu u + \bar{c}\gamma_\mu c) - \frac{1}{3}(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)$$

$$J^\mu_Z = c^u_v(\bar{u}\gamma_\mu u + \bar{c}\gamma_\mu c) + c^d_v(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)$$

$$\frac{c^u_V}{c^d_V} = \frac{1 - 8/3 \sin^2\theta_w}{-1 + 4/3 \sin^2\theta_w} = -0.58$$

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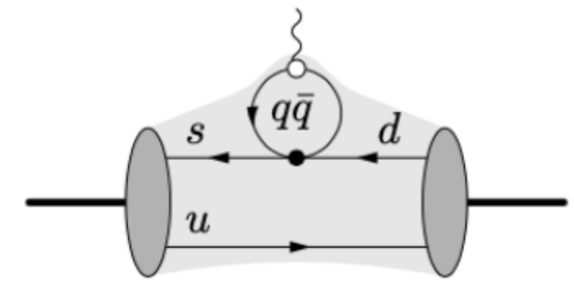

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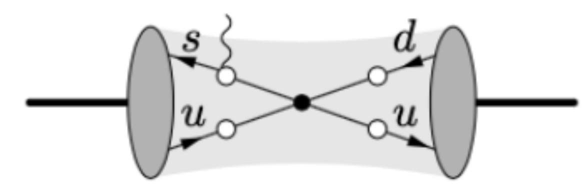

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$$F_{Z\parallel}(q^2) = \frac{3c^u_V}{2} F_\gamma(q^2) + \left(c^d_V + \frac{c^u_V}{2}\right) \int d^4x e^{-iqx} \langle \pi^+ \,|\, T\mathscr{L}_{\Delta S}(0) J^\mu_{d+s}(x) \,|\, K^+ \rangle$$

$$J^\mu_{d+s} = \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s$$

Absorbs u,c contributions (UV)

Residual (light) d,s contributions (IR)

More information needed to isolate the isospin contribution unique to the weak current..

.. but still may be interesting since F_γ absorbs UV divergence in $F_{Z\parallel}$ (including charm penguin) and can be fixed to data in a phenomenological approach

Hadronic amplitudes

$s \to dq\bar{q}$ operators decompose into isospin $\Delta I = 1/2, 3/2$

$$\langle \pi_b \pi_c \,|\, T_{\Delta I} \,|\, K_i \pi_a \rangle = C^{ia;bc}_{\Delta I} \langle I_{\pi\pi} \| T_{\Delta I} \| I_{K\pi} \rangle$$ [Wigner-Eckart]

$$i = \pm 1/2 : (K^+, K^0)$$
$$a, b, c = 0, \pm 1 : (\pi^0, \pi^\pm)$$

Reduced amplitudes functions of s, t and can be expanded in partial waves

$$p_K = p_a + p_b + p_c$$

$$s = (p_K - p_a)^2 = (p_b + p_c)^2$$
$$t = (p_K - p_b)^2 = (p_a + p_c)^2$$

$$T^{I_{K\pi} I_{\pi\pi}}_{\ell, \Delta I}(s) = \int_{-1}^{1} dz \, P_\ell(z) \, T^{I_{K\pi}, I_{\pi\pi}}_{\Delta I}(s, t(z))$$

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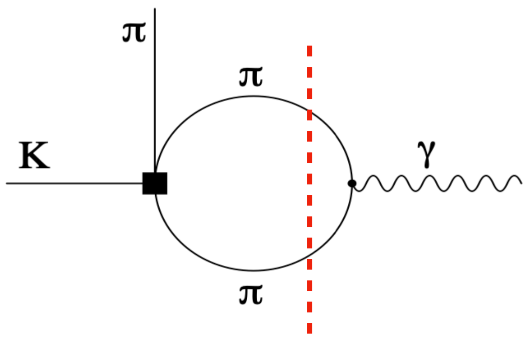
Discontinuity of $K^+ \to \pi^+ \ell^+ \ell^- (\nu\bar{\nu})$ from the hadronic amplitude and pion vector form factor

Disc $F_{\gamma,Z}(q^2) \sim \rho_\pi(s) T_1(q^2) F^*_\pi(q^2)$



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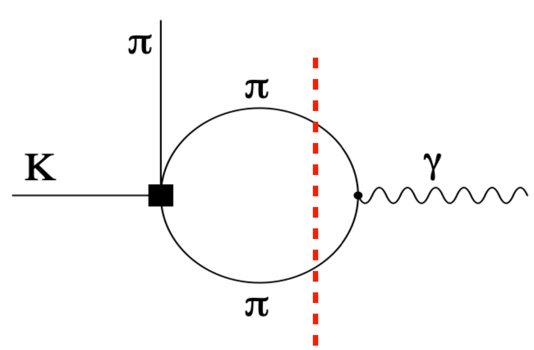


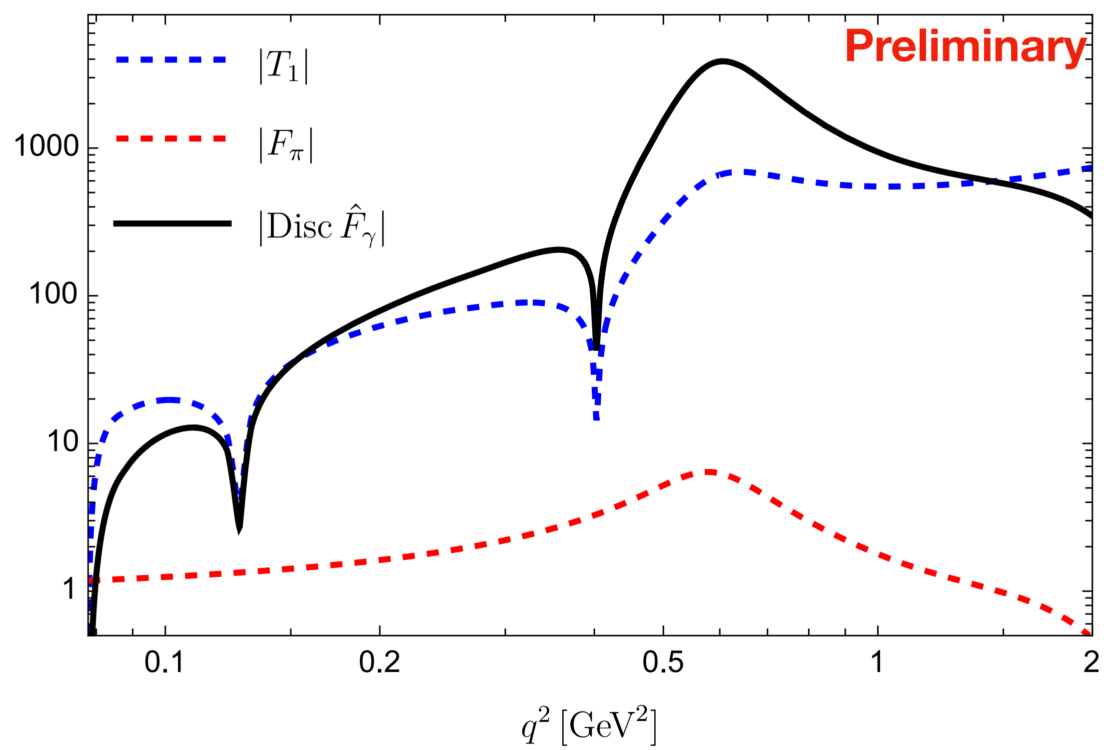
Hadronic amplitudes recently available in Khuri-Treimann formalism (pion rescattering in s,t,u-channel)

Bernard, Descotes-Genon, Knecht, Moussallam [2403.17570]

With one subtraction:

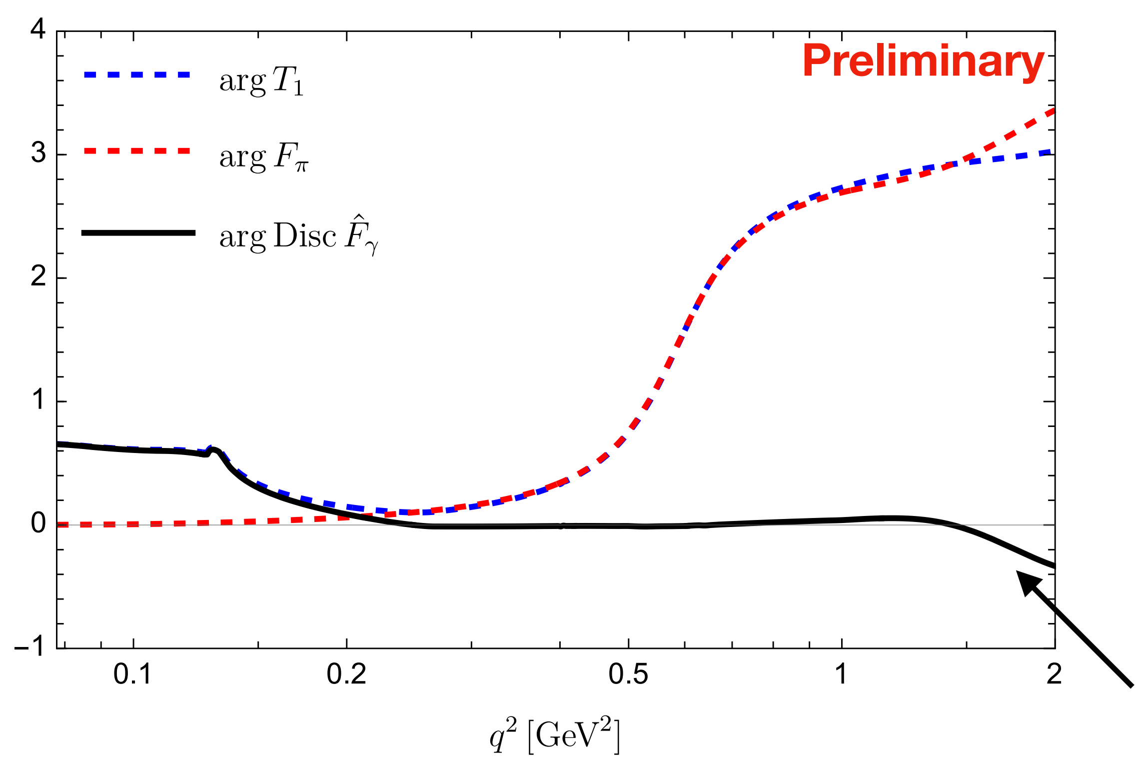
$$F_{\gamma,Z}(s) = F_{\gamma,Z}(-Q_0^2) + \frac{s + Q_0^2}{\pi} \int_{4M_\pi^2}^{\infty} dt \frac{\text{Disc}[F_{\gamma,Z}(t)]}{(t + Q_0^2)(t - s)} + \text{LH cuts}$$

Discontinuity of FFs (Resonance Region)



- P-wave amplitude vanishes at various (pseudo)-thresholds $q^2 = 4M_\pi^2$, $(M_K - M_\pi)^2$, $(M_K + M_\pi)^2$

- Error from $K \to 3\pi$ Dalitz parameters negligible, theory errors from KT should be scrutinised (PW truncation)..



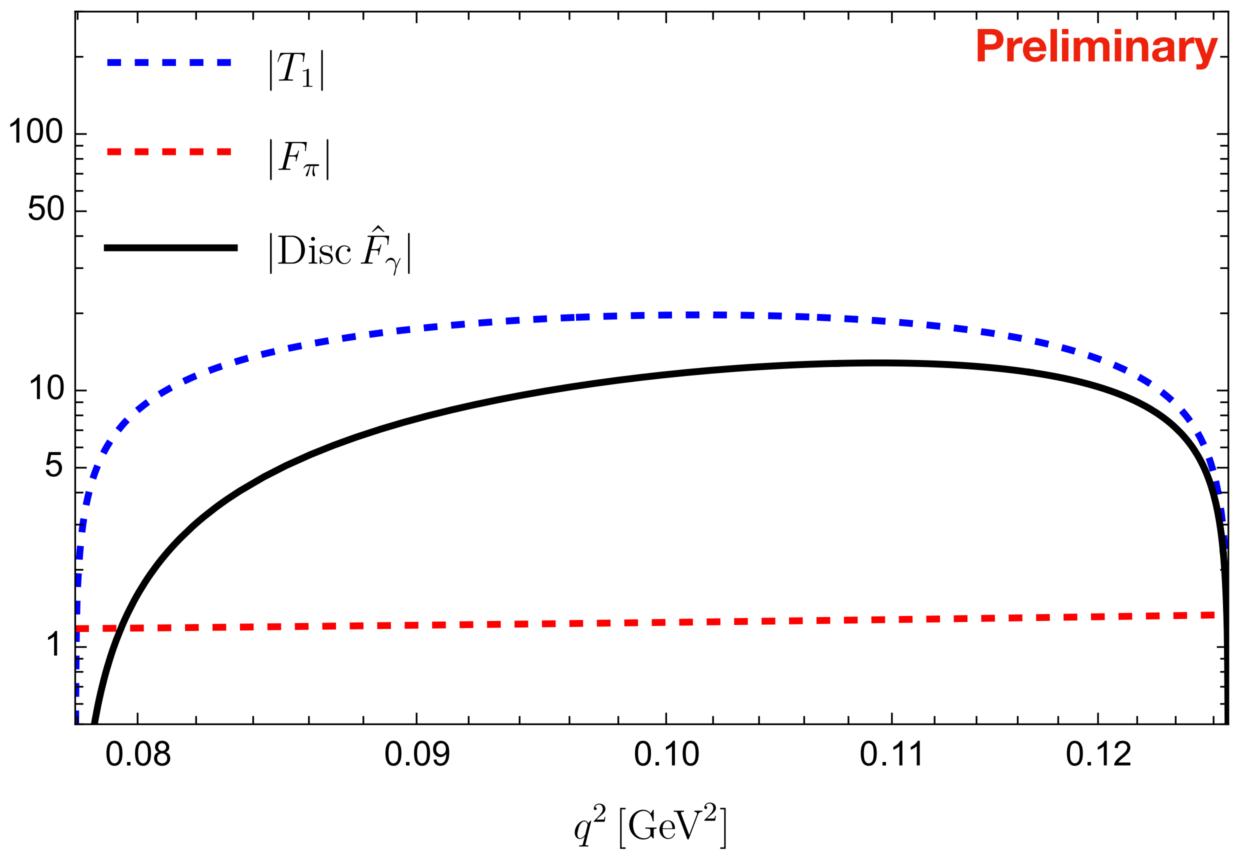
- Phase of the $K \to 3\pi$ amplitude and pion VFF are dominated by the $\rho(770)$ above $K\pi$ threshold

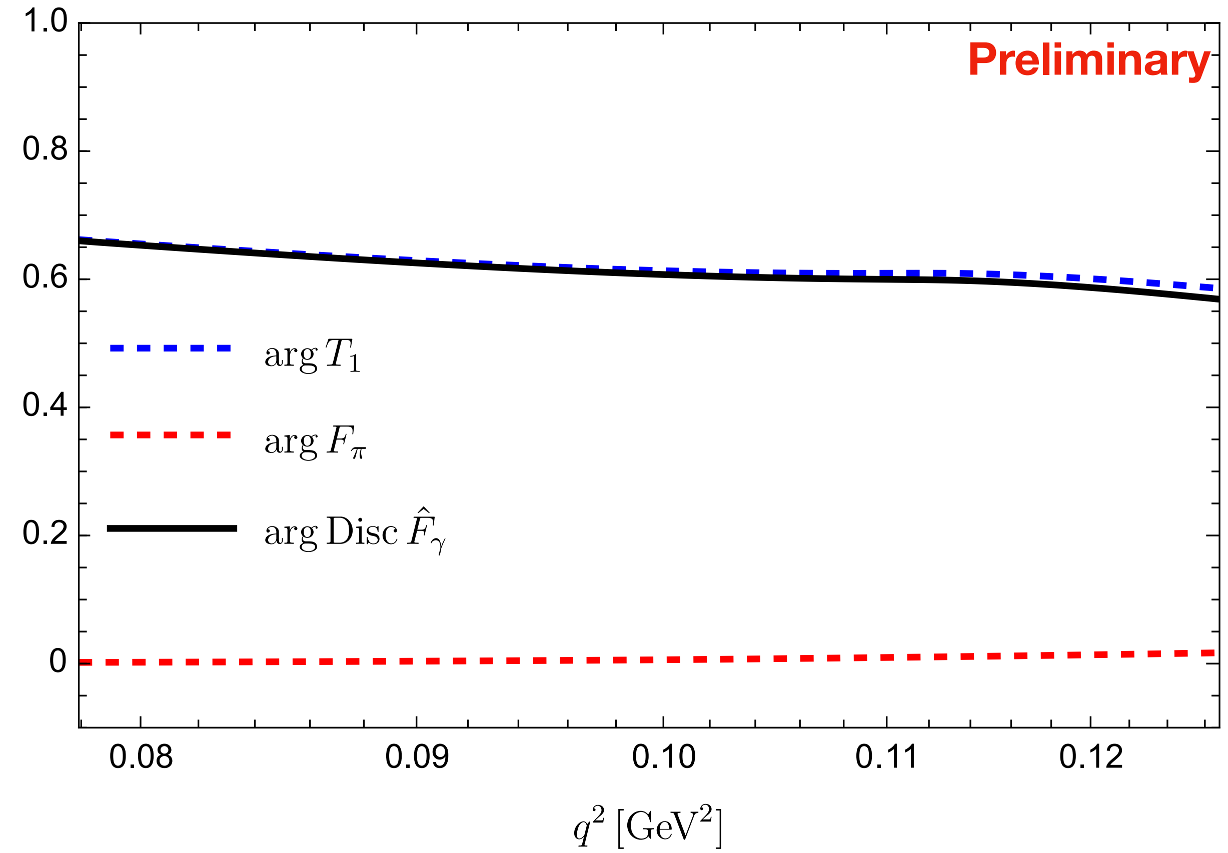
- In the decay region the $K \to 3\pi$ amplitude determines the phase of the discontinuity (VFF phase is small)

πK and KK phase shifts included in VFF but not in KT (yet)

Gonzalez-Solis, Roig [1902.02273]

Discontinuity of FFs (Decay Region)





Electromagnetic Form Factor

"Standard" parameterisation NLO SU(3) ChiPT

$\sim F_{\gamma}(q^2)$ including Fermi constant and Cabibbo angle normalisation

$$\frac{d\Gamma}{dq^2} = \frac{\alpha^2 M_K}{12\pi(4\pi)^4}\lambda^{3/2}\left(1,\frac{q^2}{M_K^2},\frac{M_\pi^2}{M_K^2}\right)\sqrt{1-\frac{4m_\ell^2}{q^2}}\left(1+\frac{2m_\ell^2}{q^2}\right)|W(q^2)|^2$$

$$W(q^2) = G_F^2 M_K^2\left(a_+ + b_+\frac{q^2}{M_K^2}\right) + W_{\pi\pi}(q^2)$$

Subtractions (fit)

Explicit pion loop function (chiral logs)

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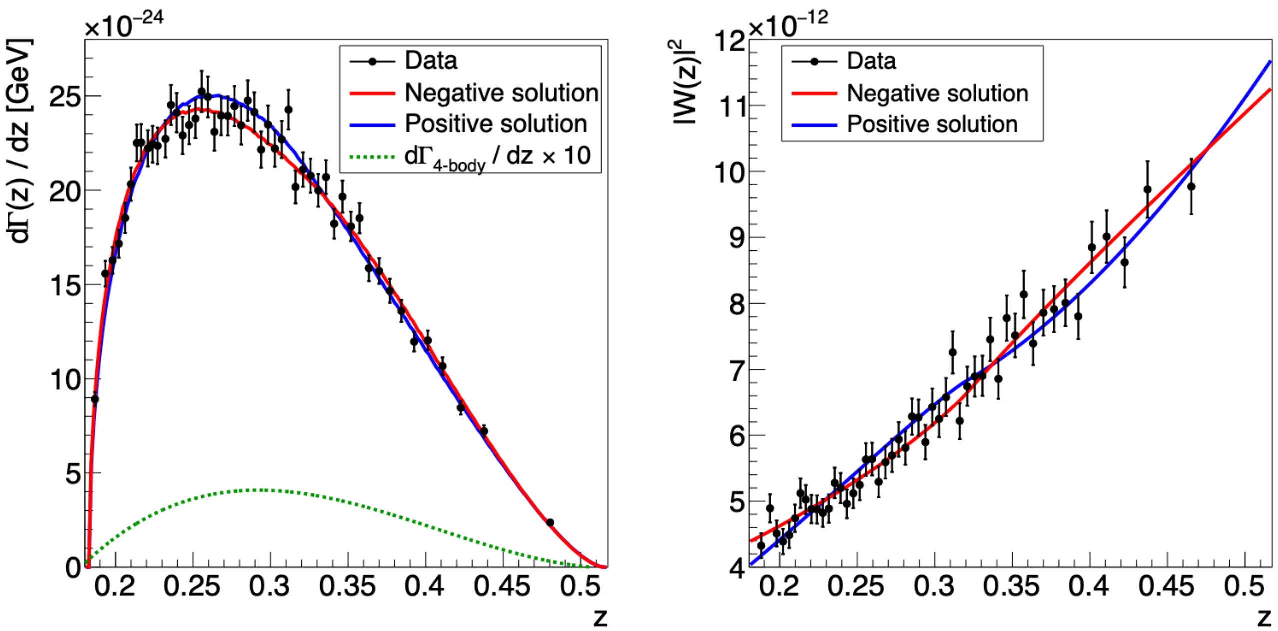
Explicit pion loop function (chiral logs)

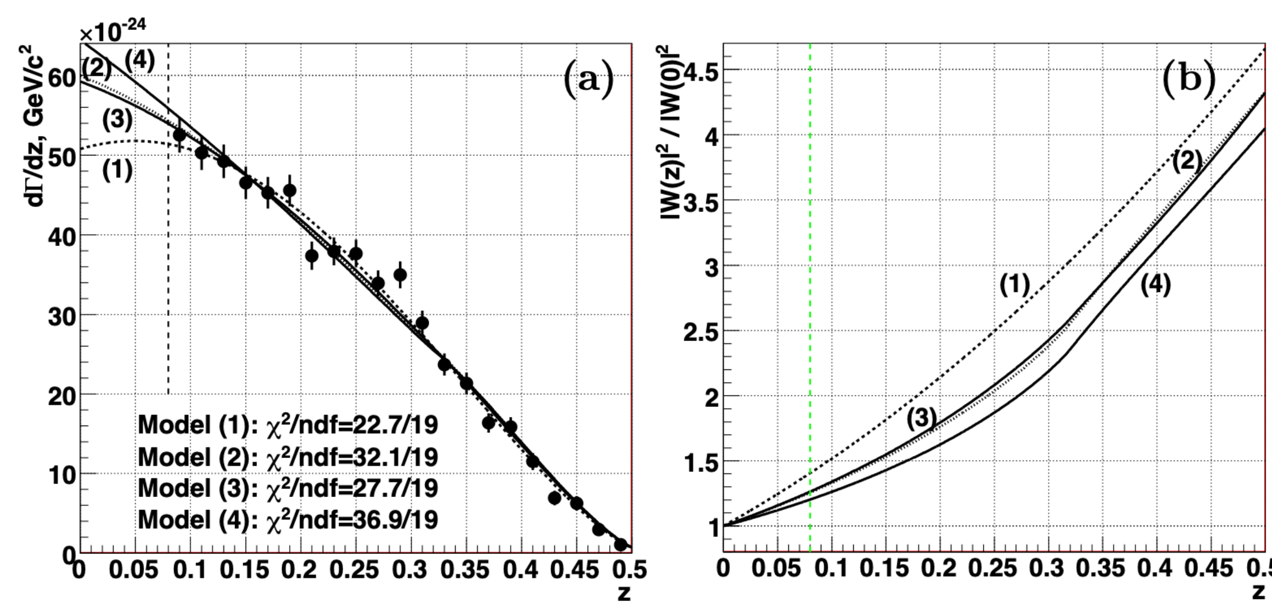
(Roughly): BSM+non-pert (a_+), non-pert (b_+)

Measurement	Signal candidates	a_+	b_+	$\rho(a_+, b_+)$
E865, $K_{\pi ee}$ [14]	10300	-0.587 ± 0.010	-0.655 ± 0.044	—
NA48/2, $K_{\pi ee}$ [15]	7253	-0.578 ± 0.016	-0.779 ± 0.066	-0.913
NA48/2, $K_{\pi\mu\mu}$ [11]	3120	-0.575 ± 0.039	-0.813 ± 0.145	-0.976
NA62, $K_{\pi\mu\mu}$, this result	27679	-0.575 ± 0.013	-0.722 ± 0.043	-0.972

$K^+ \to \pi^+\mu^+\mu^-$ NA62 [2209.05076]



$K^+ \to \pi^+e^+e^-$ NA48/2 [0903.3130]



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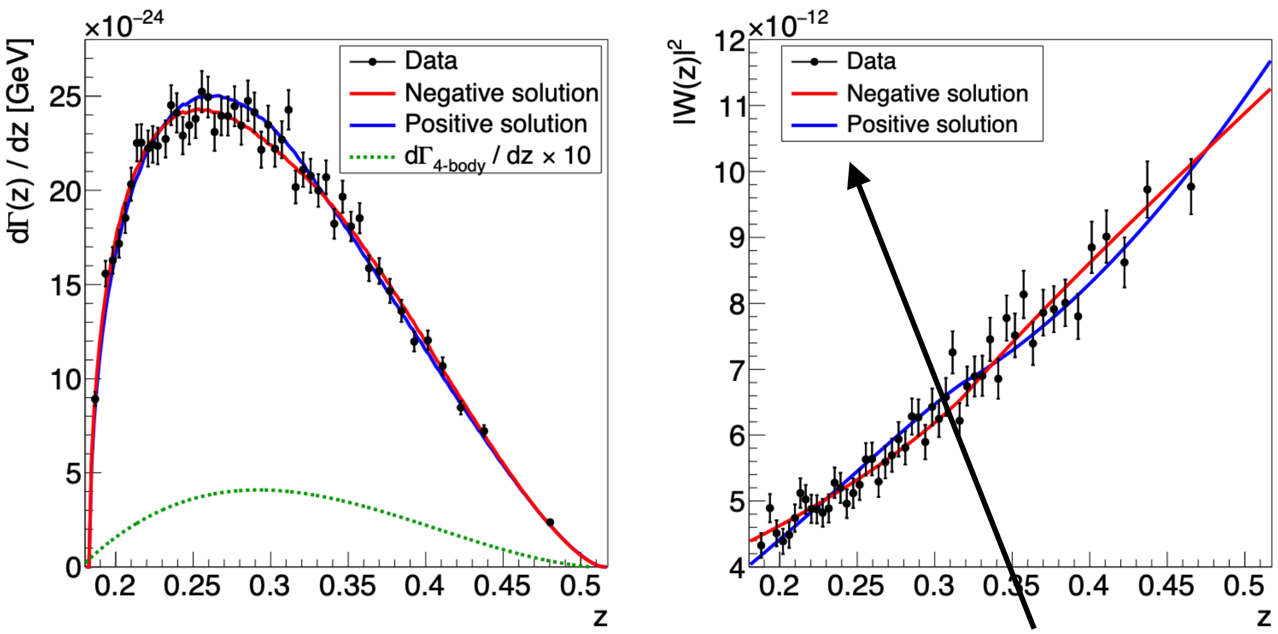

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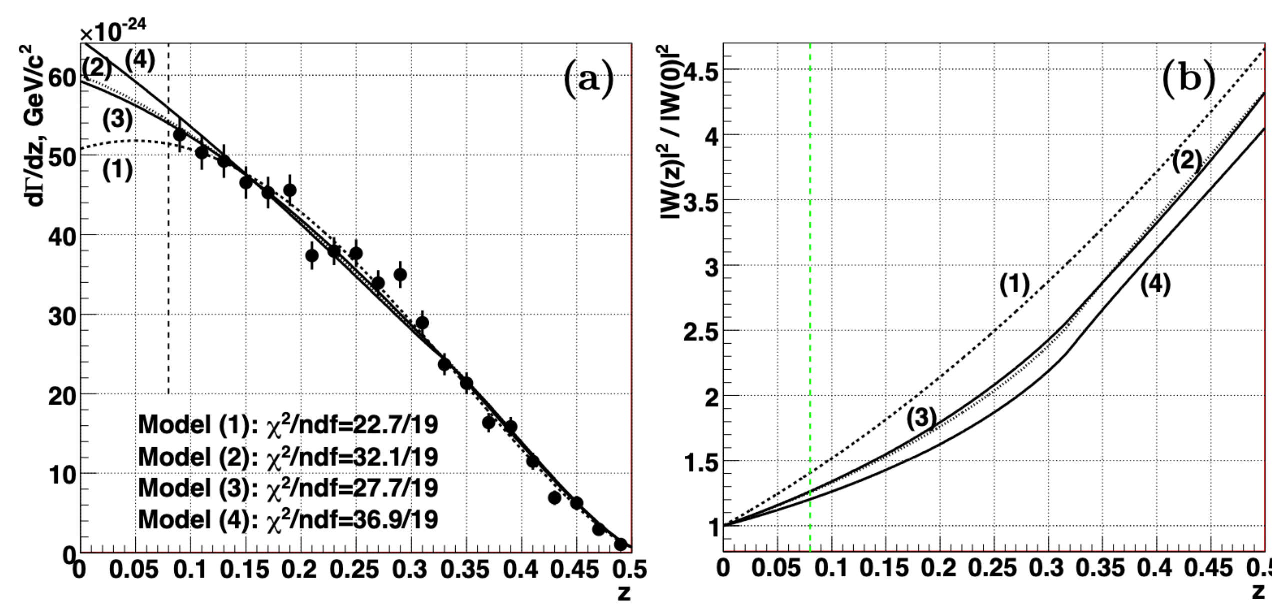
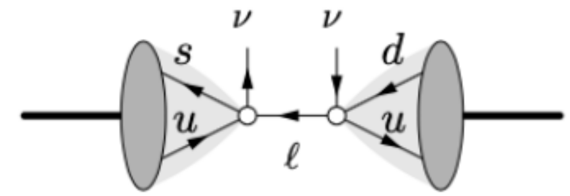
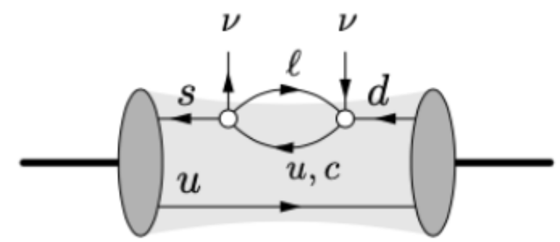


Pion loop function depends on $K \to 3\pi$ Dalitz parameters. KT can resolve their phase

Nonlocal form factors (WW)

Also the double insertion of charged-current semileptonic operators (nonlocal WW box)





$$W^{\mu\nu} = \int d^4x\, e^{-iqx}\, \langle \pi^+ |\, T\left[J_u^\mu(0)\, J_u^\nu(x) - J_c^\mu(0)\, J_c^\nu(x)\right]\, | K^+ \rangle = W_i(q^2) T_i^{\mu\nu}$$

Nice property of CKM hierarchy:
CKM phase factors out

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$

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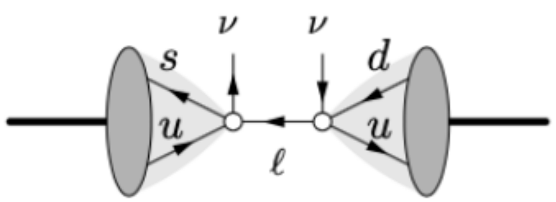

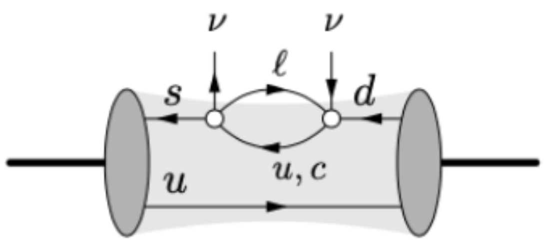

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- Discontinuity of $W_i(q^2)$ is given in terms of decay constants $f_{K,\pi}$ (lepton pole), form factors $f_\pm^{K^+\pi^0}(q^2)$ and the muiltihadron continuum.
- The issue is the subtractions (in this case not constrained from phenomenology, e.g $K^+ \to \pi^+ \ell^+ \ell^-$), but fortunately the nonlocal WW is numerically smaller than the nonlocal Z-penguin (from SU(3) ChiPT or VMD)

$$\delta P_{c,u} = (4.0 \pm 2.0_{\chi p^4}) \times 10^{-2}$$

Isidori, Mescia, Smith [0503107]

Constructive ~5% contribution to the rate

$(0.3 \pm 0.3)_W$

$$\delta P_{c,u} = (3.1 \pm 0.3_\rho \pm 0.3_W) \times 10^{-2}$$

$$\delta P_{c,u} = (3.0 \pm 1.5_{\chi p^4}) \times 10^{-2}$$

Lunghi, Soni [2408.11190]

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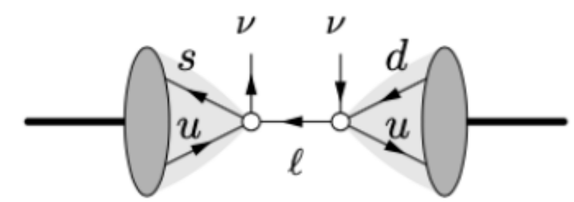

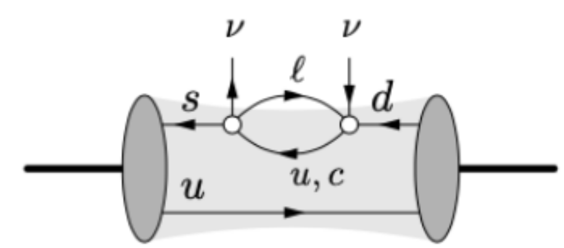

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Lunghi, Soni [2408.11190]

UV divergences of the nonlocal operators are absorbed by the local current, the transition vector form factor $f_+(q^2)$

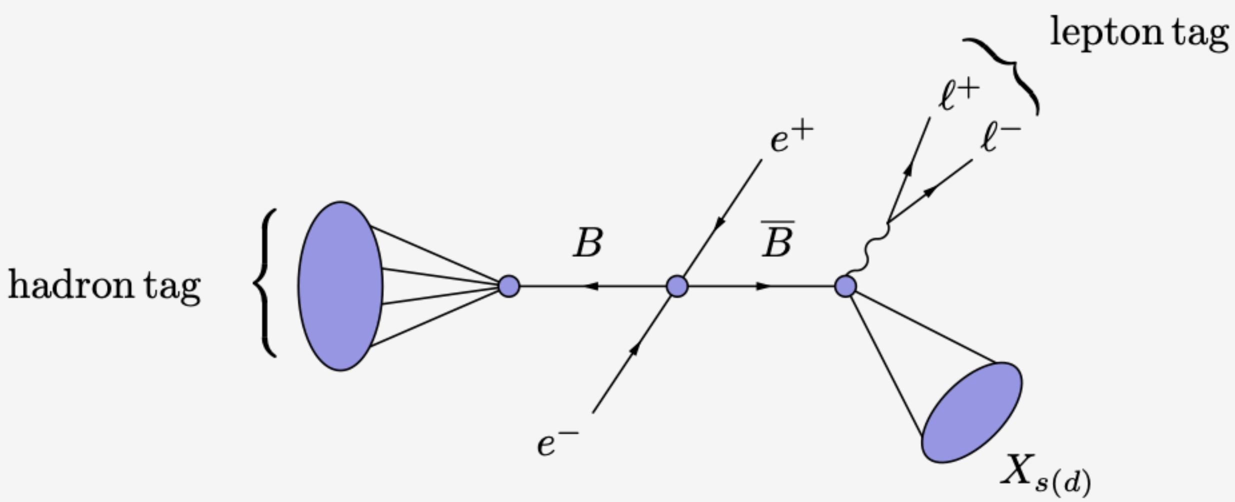
$$\int e^{-iqx} \langle \pi^+ | \, T[O_i^\dagger(0) O_j(x)] \, | K^+ \rangle \sim \langle \pi^+ | \bar{d}\gamma_\mu s \, | K^+ \rangle T_{ij}^\mu \times C_{ij}(-Q^2, \mu) + O(\Lambda^2/Q^2)$$

Take $Q_0 \sim \mu \gg \Lambda$ for the subtractions, minimise logs in the WC and match to PT at $\mu \sim m_c \gg \Lambda$.

Try to avoid SU(3) ChiPT and Λ/m_c corrections. Stay $N_f = 4$

$B \to X_{s(d)} \ell^+ \ell^-$

Ideal environment for inclusive modes (recoil tagging or sum-over-exclusive)



Three angular observables with q^2-dependent sensitivity to $C_{9,10}$
[Lee, Ligeti, Stewart, Tackmann 0612156]

$$\frac{d^3\Gamma}{dq^2 dM_X dz} = \frac{3}{8}\left[(1+z^2)H_T + 2z\,H_A + 2(1-z^2)H_L\right]$$

$$H_T \sim 2(1-\hat{q}^2)^2\hat{q}^2\left[\left(C_9 + 2C_7/\hat{q}^2\right)^2 + C_{10}^2\right],$$
$$H_A \sim -4(1-\hat{q}^2)^2\hat{q}^2\,C_{10}(C_9 + 2C_7/\hat{q}^2),$$
$$H_L \sim (1-\hat{q}^2)^2\left[(C_9+2C_7)^2 + C_{10}^2\right]$$

$B \to X_{s(d)}\ell^+\ell^-$

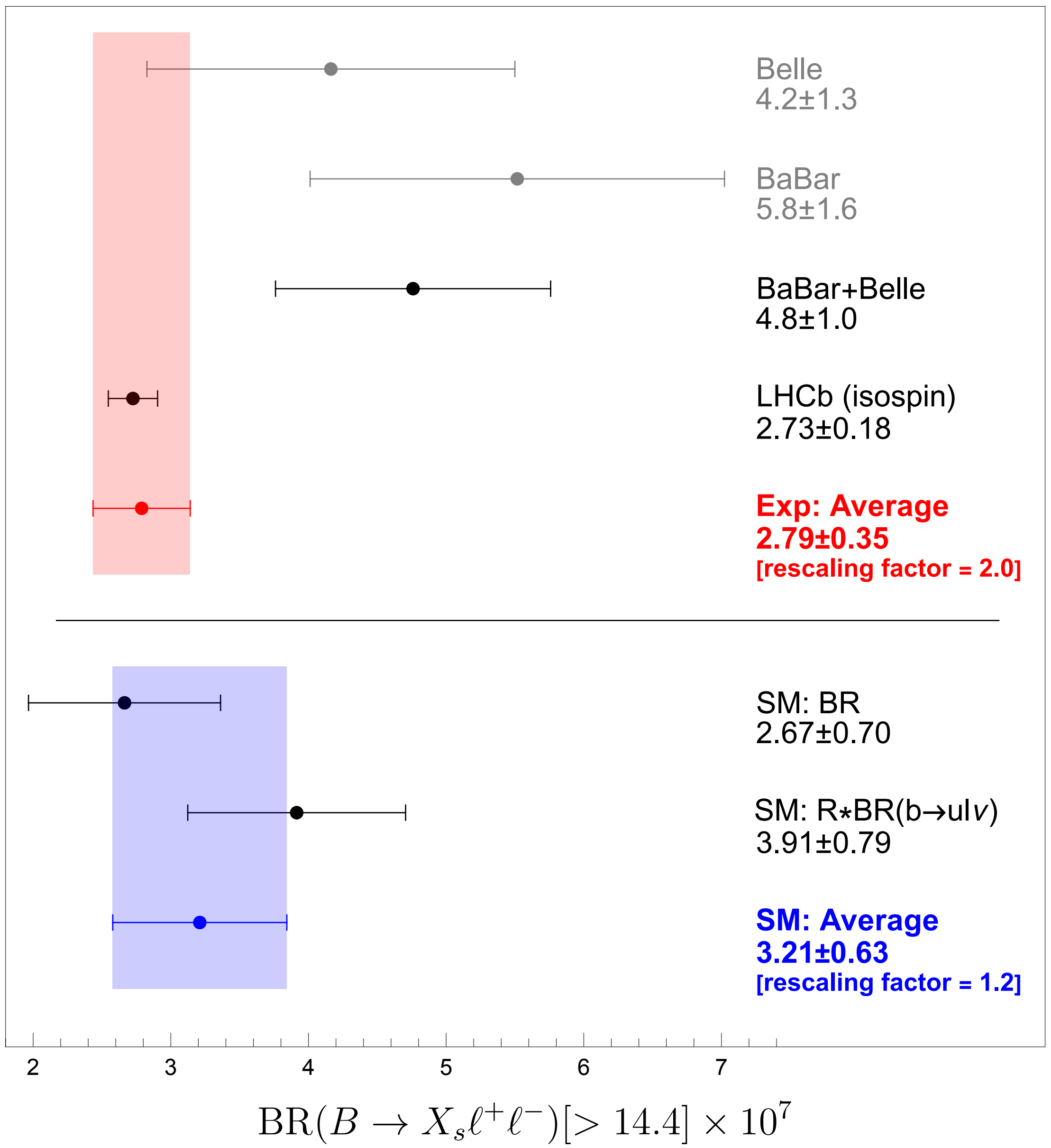
Power corrections dominate the error at high-q^2, in particular four-quark operators which are suppressed in the ratio [Ligeti, Tackmann 0707.1694]

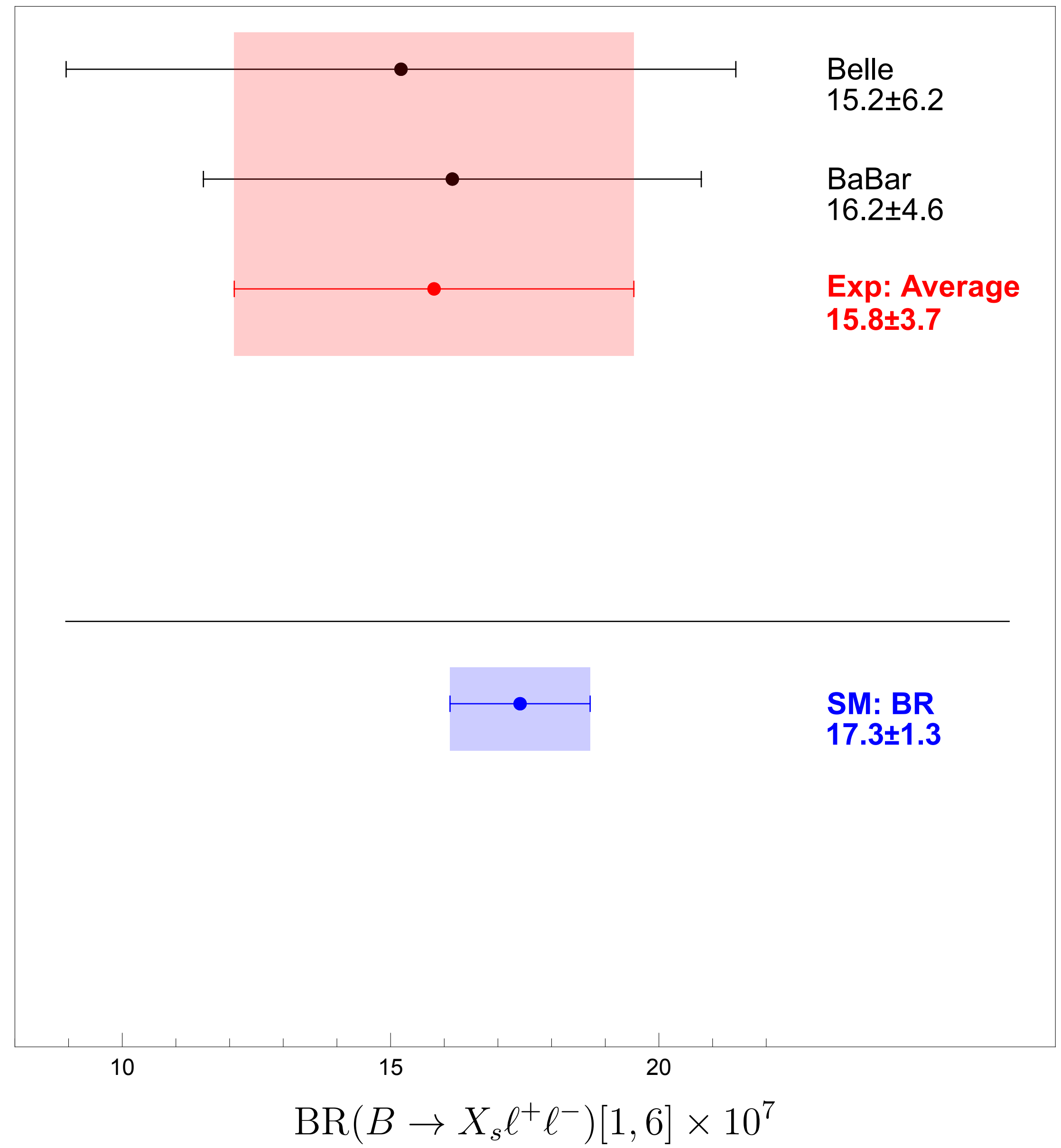
$$\mathcal{R}(q_0^2) = \int_{q_0^2}^{M_B^2} dq^2 \frac{d\mathcal{B}(B \to X_s \ell\ell)}{dq^2} \Bigg/ \int_{q_0^2}^{M_B^2} dq^2 \frac{d\mathcal{B}(B \to X_u \ell\nu)}{dq^2}$$

This normalization provides an indirect determination of the $B \to X_s \ell\ell$ rate [Huber, Hurth, Jenkins, Lunghi, Qin, Vos 2404.03517]

$$\begin{aligned}\mathcal{B}[>15] = (&2.59 \pm 0.21_{\rm scale} \pm 0.03_{m_t} \pm 0.05_{C,m_c} \pm 0.19_{m_b} \pm 0.004_{\alpha_s} \pm 0.002_{\rm CKM} \\ &\pm 0.04_{\rm BR_{sl}} \pm 0.26_{\rho_1} \pm 0.10_{\lambda_2} \pm 0.54_{f_{u,s}}) \times 10^{-7} \\ = (&2.59 \pm 0.68) \times 10^{-7} \\ \mathcal{R}(15) = (&27.00 \pm 0.25_{\rm scale} \pm 0.30_{m_t} \pm 0.11_{C,m_c} \pm 0.17_{m_b} \pm 0.15_{\alpha_s} \pm 1.16_{\rm CKM} \\ &\pm 0.37_{\rho_1} \pm 0.07_{\lambda_2} \pm 1.43_{f_{u,s}}) \times 10^{-4} \\ = (&27.00 \pm 1.94) \times 10^{-4}\,.\end{aligned}$$

Pheno Overview (Exp+Th)





Summary

- $K \to 3\pi$ amplitudes from KT approach are on the market, which opens the door to determinations of the nonlocal Z-penguin form factor in $K^+ \to \pi^+ \nu \bar{\nu}$

- $K^+ \to \pi^+ \ell^+ \ell^-$ as a first/complementary step. The idea is to fix the normalisation of the electromagnetic FF to data for the charged lepton mode to constrain the long distance effect in the $\nu \bar{\nu}$ mode using the universality of the pion vector form factor

- External determinations for at least a single value of q^2 needed (subtractions are unconstrained in dispersive approach), e.g. $q^2 = -Q^2$ with $Q \sim m_c$ to match to $N_f = 4$ perturbation theory

- Long distance effects in $B \to X_{s(d)} \mu^+ \mu^-$ are more difficult to control especially at high-q^2

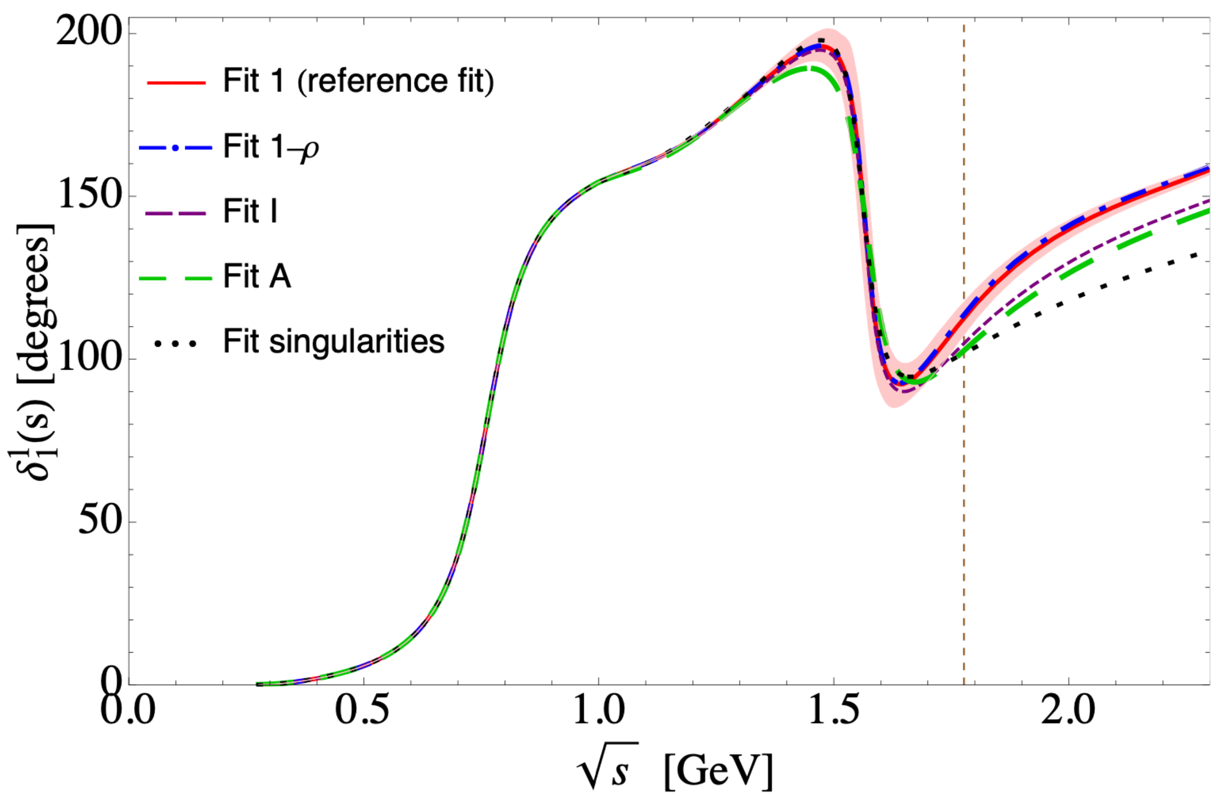
- Needs HQE parameters (including weak annihilation) at higher precision: $|V_{cb}|$ fits and four-quark operators).

- Experimental situation is a bit scattered, updates from Belle (II) and LHCb would be nice to clarify the saturation of the inclusive mode by K and K^*

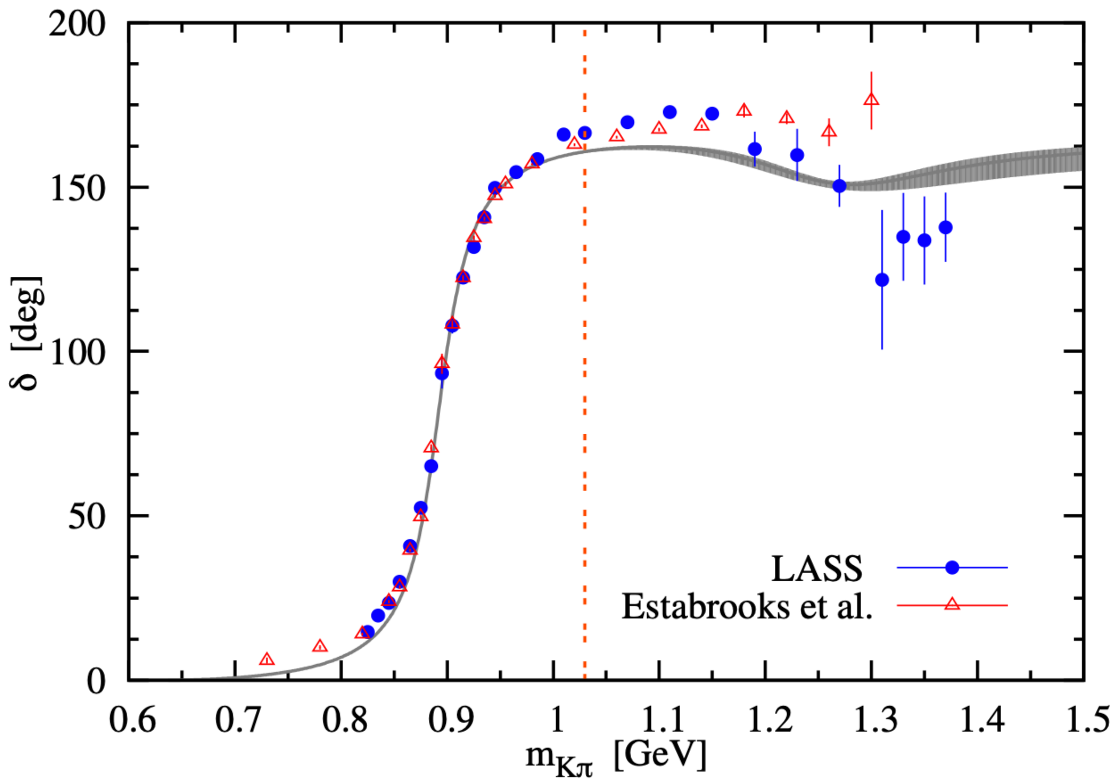
Backup

Phase shifts

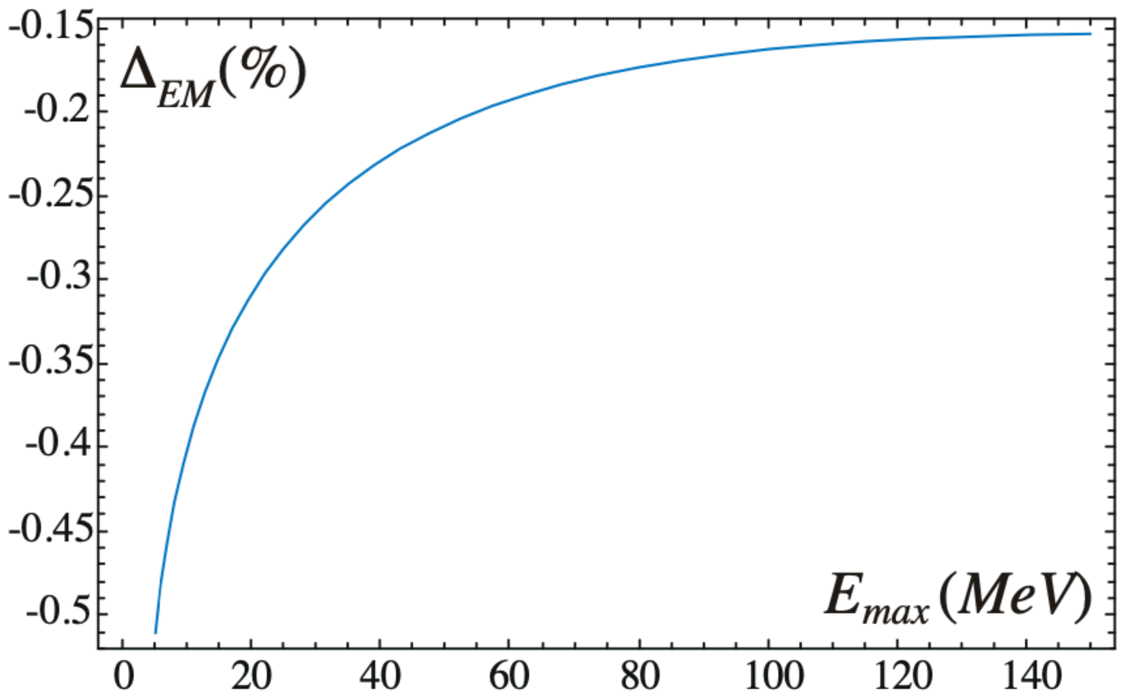
$\pi\pi$



$K\pi$



QED corrections



$K^+ \to \pi^+ \nu \bar{\nu}(\gamma)$

Mescia, Smith [0705.2025]

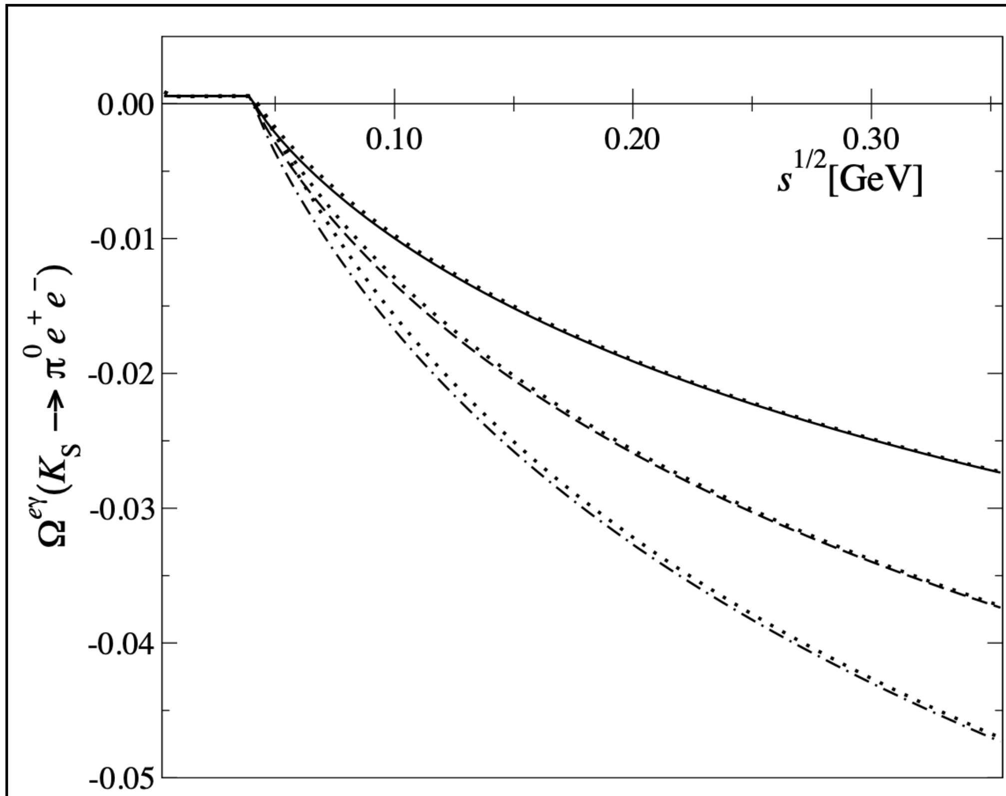


Fig. 5. Correction factor for $d\Gamma/ds(K_S \to \pi^0 e^+ e^-)$ in the massless limit, with cuts on $\theta_{e^\pm\gamma} = 20^\circ$ (full line), 10° (dashed), and 5° (dash-dotted), all with $E_{\rm cut} = 20$ MeV. For comparison, the dotted lines nearby show the same curves for the physical electron mass.

$K_s \to \pi^0 \ell^+ \ell^- (\gamma)$ (similar for $K^+ \to \pi^+$)

Kubis, Schmidt [1007.1887]