

QED CORRECTIONS TO EXCLUSIVE LEPTONIC B DECAYS

Lattice meets continuum

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Siegen, Germany

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Based on arXiv:2212.14430 [hep-ph] and work in progress
In collab. w/ C. Cornella, M. König and M. Neubert (JGU Mainz)



Motivations

► Why leptonic B decays ?

$$\mathcal{B}(B^- \rightarrow \ell^- \bar{\nu}_\ell) = \tau_B G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

- Direct determination of the CKM element $|V_{ub}|$
- **Chirality-suppressed** in the SM \rightarrow powerful probe of (pseudo) scalar new physics
- Testing flavor universality in charged current : Belle II will measure the $\ell = \tau, \mu$ channels at 5 – 7 % [Belle II Physics Book]. FCC-ee prospects are promising.

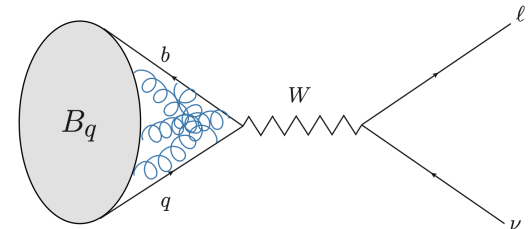
► Why QED corrections are needed?

- Pure hadronic effects are simple and well-understood:

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B_q(p) \rangle = i f_{B_q} p^\mu$$

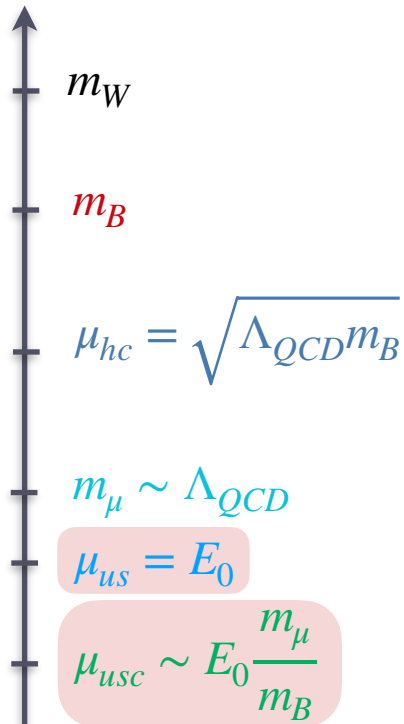
and f_{B_q} is known with $\mathcal{O}(1\%)$ precision : $f_{B_q} = 189.4 \pm 1.4 \text{ MeV}$ [FNAL/MILC 1712.09262]

- In the exclusive channel, with strong cuts on additional soft radiations, QED corrections can be **sizable** and compete with QCD uncertainties \rightarrow need a **precise** estimation



A multi-scale process

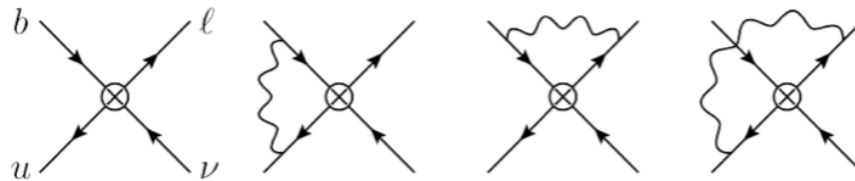
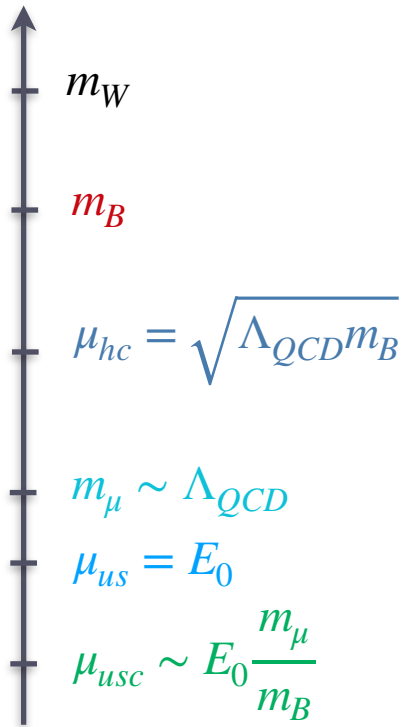
- ▶ Focusing on $\ell = \mu$ case, including QED effects introduce new scales (both static and dynamic) to which $B^- \rightarrow \mu^- \bar{\nu}_\mu$ is sensitive :



Static scales dictated by the energy cut on the additional final radiation

A multi-scale process

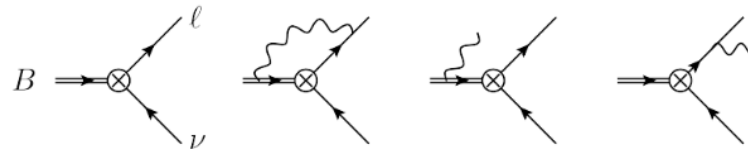
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B-meson described as a superposition of Fock-states: $|\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \dots$

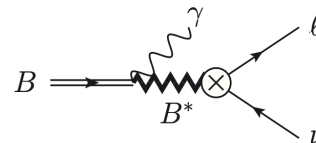
[Beneke et al(2019),JHEP10232]

B-meson described as a point-like pseudo-scalar boson...



[Dai et al (2022), Phys.Rev.D 105 3]

...including its first vector excited state as off-shell intermediate propagator !



[Becirevic et al (0907.1845)]

The plan : running down with the scale !

Turning a multi-scale problem to a product of single scale objects by :

- ▶ Identifying the **appropriate effective description** at each scale.
- ▶ Performing a step-by-step **matching** between each EFT.
- ▶ Deriving a **factorization theorem** to break this multi-scale problem into a convolution of single-scale objects.
- ▶ Using the **renormalisation group** to evaluate each object at its natural scale and run it to a common scale to **resum logarithms**.

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In this talk we are going to embark on an EFT journey :

- ▶ Discussing the **(re)factorization** of the **virtual** amplitude in the partonic picture.
- ▶ Constructing the proper EFT valid at **low energy** to include the **real emissions**.
- ▶ Deriving the **factorization formula for the decay rate**.

Fermi theory \rightarrow HQET \otimes SCET_I $\mu \sim m_B$

Power counting: $\lambda = \frac{m_\ell}{m_B} \sim \frac{\Lambda_{QCD}}{m_B}$

Relevant scalings: $p^\mu = (\bar{n} \cdot p, n \cdot p, p_\perp)$

$p \sim (1, \lambda^2, \lambda)$ « collinear »,

$p^2 \sim m_\ell^2 \sim \mathcal{O}(\lambda^2)$

\rightarrow given by the lepton virtuality

$p \sim (1, \lambda, \sqrt{\lambda})$ « hard-collinear »,

$p^2 \sim m_\ell m_b \sim \mathcal{O}(\lambda)$

\rightarrow soft and collinear quark X-talk

$p \sim (\lambda, \lambda, \lambda)$ « soft »,

$p^2 \sim \Lambda_{QCD}^2 \sim \mathcal{O}(\lambda^2)$

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Fermi theory \rightarrow HQET \otimes SCET_I $\mu \sim m_B$

- The b quark is described by a **soft** HQET field :

$$b(x) \rightarrow e^{-im_b(v \cdot x)} \left(1 + \mathcal{O}(\sqrt{\lambda}) \right) h_v(x)$$

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- ▶ QED corrections \rightarrow hard-collinear momentum exchange between partons & lepton.

\rightarrow need a SCET_I subleading power description for the different modes of the spectator and the lepton :

$$q(x) \rightarrow \left(1 + \frac{1}{i\bar{n} \cdot D_s} (i \mathcal{D}_\perp + m_\ell) \frac{\not{n}}{2} \right) \xi_C^{(q)}(x) + \left(1 + \frac{1}{i\bar{n} \cdot D_s} e Q_q A_{C\perp} \frac{\not{n}}{2} \right) q_s(x)$$

$$\ell(x) \rightarrow \left(1 + \frac{1}{i\bar{n} \cdot D_s} (i \mathcal{D}_\perp + m_\ell) \frac{\not{n}}{2} \right) \xi_C^{(\ell)}(x) + \left(1 + \frac{1}{i\bar{n} \cdot D_s} e Q_q A_{C\perp} \frac{\not{n}}{2} \right) \ell_s(x)$$

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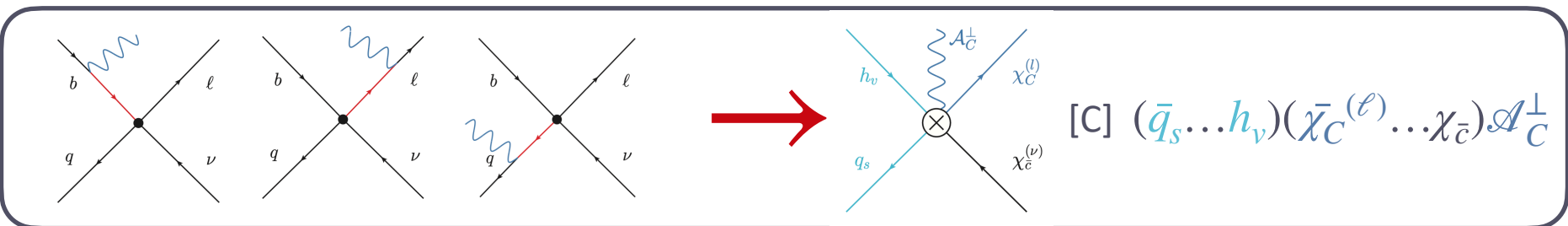
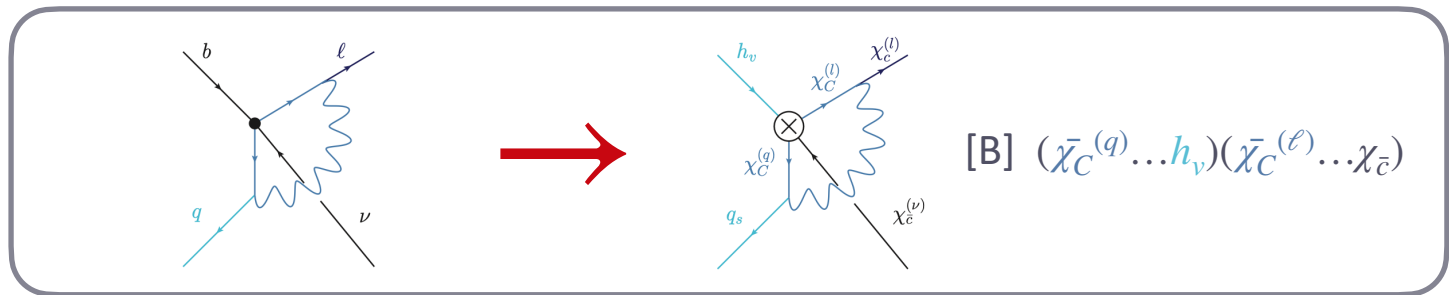
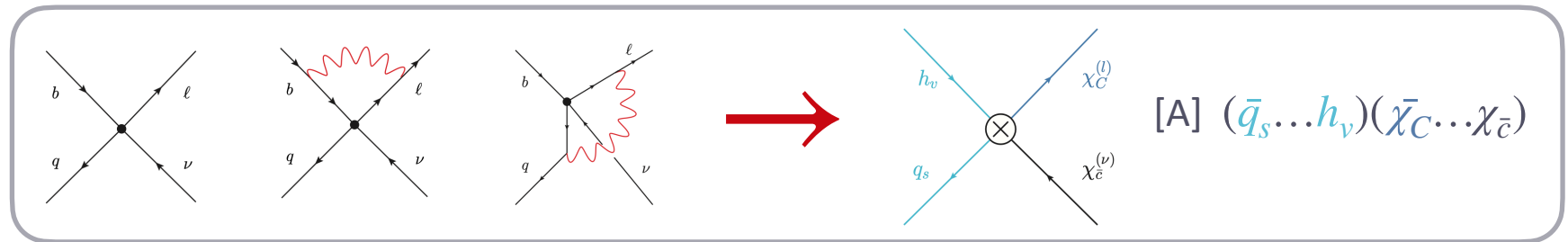
\rightarrow given by the spectator quark virtuality

SCET_I operators

We build our SCET_I basis with the following power counting :

$$h_\nu, q_s \sim \mathcal{O}(\lambda^{3/2}) \quad \chi_C \sim \mathcal{O}(\lambda^{1/2}) \quad \mathcal{A}_C^\perp \sim \mathcal{O}(\lambda^{1/2}) \quad \chi_c \sim \mathcal{O}(\lambda) \quad \mathcal{A}_c^\perp \sim \mathcal{O}(\lambda) \quad \chi_{\bar{c}} \sim \mathcal{O}(\lambda)$$

At $\mathcal{O}(\alpha)$, three classes of 4-fermions operators are relevant:



SCET_I → SCET_{II}

$$\mu \sim \mu_{hc} \sim \sqrt{\Lambda_{QCD} m_B}$$

- ▶ At $\mu \sim \mu_{hc}$, we lower the virtuality removing **hard-collinear** modes → pure SCET_{II} construction where **collinear** and **soft** carry the same virtuality :

$$p_c \sim (1, \lambda^2, \lambda), \quad p_s \sim (\lambda, \lambda, \lambda), \quad p_c^2 \sim p_s^2 \sim \mathcal{O}(\lambda^2)$$

- ▶ Integrating out **hard-collinear** propagators introduces **non-localities** even in **soft** product :

$$\frac{1}{n \cdot \partial} q_s, \left(\frac{1}{n \cdot \partial} \mathcal{G}_s^\perp \right) \left(\frac{1}{n \cdot \partial} q_s \right), \dots$$

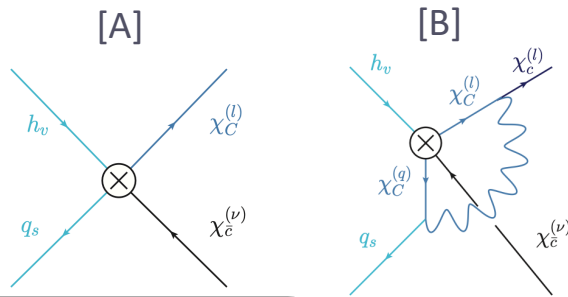
→ Contains more fields but are of the same order !

- ▶ Contrary to the $B_s \rightarrow \mu^+ \mu^-$ case, those structure-dependent contributions to $B^- \rightarrow \mu^- \bar{\nu}_\mu$ carry the same suppression as the tree level result and do not overcome the chiral suppression.

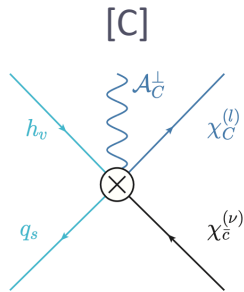
[Beneke, Bobeth, Szafron 2017, 2019]

SCET_{II} basis

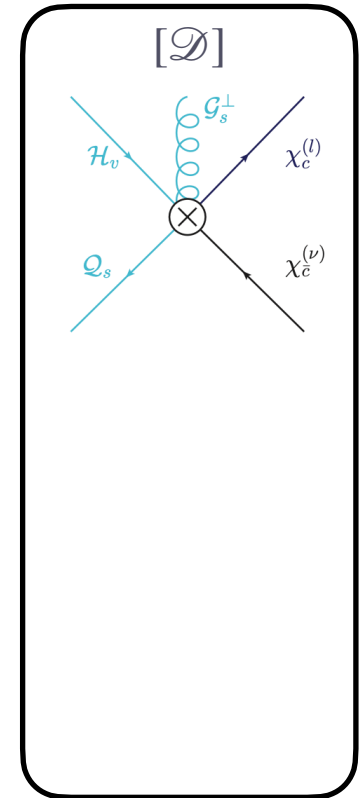
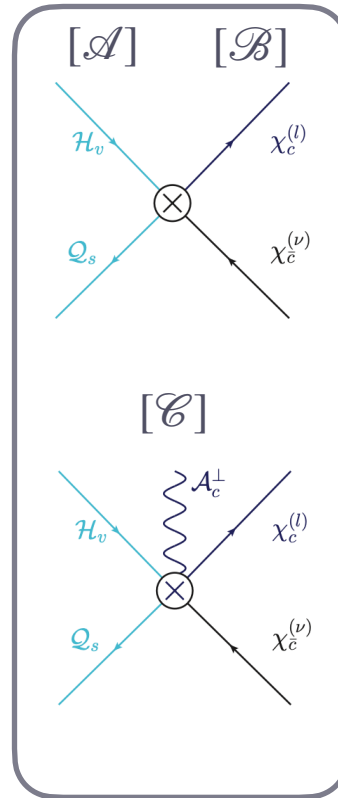
► Starting from our SCET_I operators, we can construct our SCET_{II} basis :



No tree level matching



No tree level matching



$$|B\rangle = |\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \dots$$

Refactorization

- ▶ The matrix element of B-type SCET II operators involve an **endpoint-divergent** integral → the theory fails to properly separate **hard-collinear modes** with low energy (hard-collinear fraction $v \rightarrow 0$) from **soft modes**.
- ▶ This region can be identified with the energetic limit of the soft region → we rearrange terms between the two (**RBS scheme**) [Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang 2020; Beneke et al. 2022]

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Jet function

$$\llbracket \mathcal{J}_{B1} \rrbracket(v, \omega, \Lambda) = \mathcal{J}_{B1}(v, \omega) - \theta(r_\Lambda - v) \delta \mathcal{J}_{B1}(v, \omega)$$

$$\llbracket \mathcal{J}_{B1}^{3part.}(v, \omega, \omega_g, \Lambda) \rrbracket = \mathcal{J}_{B1}^{3part.}(v, \omega, \omega_g) - \theta(r_\Lambda - v) \delta \mathcal{J}_{B1}(v, \omega, \omega_g)$$

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$$\langle O_{\mathcal{A}}^{(1)} \rangle = \left\langle \frac{m_\ell}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \not{n} P_L S_n^{(l)\dagger} h_\nu \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_c^{(\nu)} \right) \right\rangle$$

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Add it back

Soft function

$$\langle O_{\mathcal{A}}^{\Lambda(1)} \rangle = \left\langle \frac{m_\ell}{(\vec{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \theta(\Lambda + i\vec{n} \cdot \vec{D}_s) \not{n} P_L S_n^{(l)\dagger} h_\nu \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_c^{(\nu)} \right) \right\rangle$$

[Cornella, König, Neubert 2023]

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New hadronic parameter !

[Cornella, König, Neubert 2023]

Factorization formula (virtual corrections)

- ▶ Taking the matrix elements of our SCET_{II} operators, the virtual corrections can be expressed as :

$$\begin{aligned}
 i\mathcal{A}_{\text{virt}} = & H_{A_0}^{(1)}(\mu) \langle O_{\mathcal{A}}^{\Lambda(1)} \rangle + H_{A_0}^{(2)}(\mu) \langle O_{\mathcal{A}}^{(2)} \rangle \\
 & + \int_0^\infty d\omega \int_0^1 dv \left[\sum_{i=1,2} H_{B_0}^{(i)}(v, \mu) \mathcal{F}_{B_0}^{(i)}(v, \omega) + H_{B_1}(v, \mu) \llbracket \mathcal{F}_{B_1}(v, \omega, \Lambda) \rrbracket \right] \langle O_{\mathcal{B}}^{(1)}(\omega) \rangle_s \\
 & + \sum_{i=1,2} \int_0^1 du H_{C_0}^{(i)}(u, \mu) \langle O_{\mathcal{C}}^{(i)} \rangle \\
 & + \int_0^\infty d\omega \int_0^\infty d\omega_g \int_0^1 dv H_{B_1}(v, \mu) \llbracket \mathcal{F}_{B_1}^{3part.}(v, \omega, \omega_g) \rrbracket \langle O_{\mathcal{D}}(\omega, \omega_g) \rangle
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 & \propto F_{\Lambda}(\mu) \\
 & + \int_0^{\infty} d\omega \int_0^1 dv \left[\sum_{i=1,2} H_{B_0}^{(i)}(v, \mu) \mathcal{F}_{B_0}^{(i)}(v, \omega) + H_{B_1}(v, \mu) \llbracket \mathcal{F}_{B_1}(v, \omega, \Lambda) \rrbracket \right] K_{\mathcal{B}}^{(1)}(\mu) \langle O_{\mathcal{B}}^{(1)}(\omega) \rangle_s \\
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Generalized decay constant

- ▶ The QED corrected $O_A^{\Lambda(1)}$ operator defines a **generalized decay constant** :

$$\begin{aligned} \langle O_A^{\Lambda(1)} \rangle_s &\equiv \left\langle 0 \left| \bar{q}_s \theta(\Lambda + i\vec{n} \cdot \vec{D}_s) \not{n} P_L S_n^{(l)\dagger} h_\nu \right| B^- \right\rangle \\ &= -\frac{i\sqrt{m_B}}{2} F_\Lambda(\mu) \langle 0 | S_{\nu_B}^{(B)} S_n^{(l)\dagger} | 0 \rangle \end{aligned}$$

$$S_r^{(i)}(x) = \mathcal{P} \exp \left\{ ieQ_i \int_{-\infty}^0 ds r \cdot A_s(x + sr) \right\}$$

- ▶ For $\alpha \rightarrow 0$, F_Λ reduces to the standard HQET decay constant :

$$\sqrt{m_B} f_B \left[1 + C_F \frac{\alpha_s(m_b)}{2\pi} + \mathcal{O}(\alpha_s^2) \right] = F_\Lambda(m_b) \Big|_{\alpha \rightarrow 0} = F_{\text{QCD}}(m_b),$$

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$$S_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_s(x + sr) \right\}$$

- ▶ F_Λ is an **unknown non-perturbative parameter** following evolution equations for Λ and μ :

$$F_\Lambda(\mu) = F_{m_B}(\mu) \left[1 - \frac{\alpha}{4\pi} Q_\ell Q_u \log \frac{\Lambda^2}{m_B^2} \int d\omega \left(\phi_-(\omega) \log \frac{\mu^2}{\omega \sqrt{m_B} \Lambda} + 2 \int d\omega_g \frac{\phi_3(\omega, \omega_g)}{\omega_g^2} \log \frac{\omega}{\omega + \omega_g} \right) + \dots \right]$$

two-particle LCDA three-particle LCDA

$$F_{m_B}(\mu) = F_{m_B}(m_b) \left[1 + \frac{\alpha}{8\pi} Q_\ell Q_u \log^2 \frac{\mu^2}{m_b^2} + \dots \right]$$

- ▶ Need to be computed at a **single value** of Λ and μ . Lattice determination ? QCD SR estimate ?

Low-energy theory : bHLET \otimes HSET $\mu < \Lambda_{QCD} \sim m_\mu$

- ▶ Below $\mu \sim \Lambda_{QCD}$, quarks **hadronize** and the meson can be described by a charged scalar of mass m_B . We can describe it with a **heavy scalar (HS)** :

$$\Phi_B(x) \rightarrow e^{-im_B(v_B \cdot x)} \varphi_B(x)$$

- ▶ Since $\Lambda_{QCD} \sim m_\mu$, also the muon becomes infinitely heavy. We describe it as a **boosted heavy lepton field (bHL)** :

$$\ell(x) \rightarrow e^{-im_\ell(v_\ell \cdot x)} \chi_{v_\ell}(x)$$

[Fleming et al (hep-ph/0703207);
Beneke et al (2305.06401)]

- ▶ We match the resulting low energy EFT by taking the hadronic matrix element :

$$\langle \ell \nu | \mathcal{L}_{SCET_H \otimes HQET} | B \rangle = \langle \ell \nu | \mathcal{L}_{bHLET \otimes HSET} | B \rangle$$



A theory of Wilson lines ?

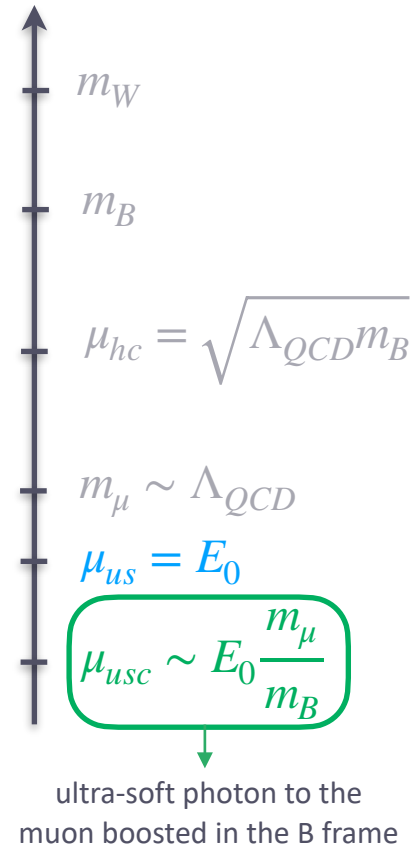
- ▶ We can decouple the interactions of the B and the muon with **ultrasoft** and **ultrasoft-collinear** photons :

$$\varphi_B(x) \rightarrow Y_{\nu_B}^{(B)}(x) C_{\bar{n}}^{(B)}(x_+) h_{\nu_B} \quad \chi_{\nu_\ell}(x) \rightarrow Y_n^{(\ell)}(x_-) C_{\nu_\ell}^{(\ell)}(x) \frac{\not{n} \not{\bar{n}}}{4} h_{\nu_\ell}$$

$$Y_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_{us}(x + sr) \right\} \quad C_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_{usc}(x + sr) \right\}$$

- ▶ Real corrections can be expressed as matrix elements of these Wilson lines, convoluted with the **measurement function** implementing the experimental radiation veto

$$S(E_0, \mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc} \theta \left(\frac{E_0}{2} - \omega_{us} - \omega_{usc} \right) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$



Factorization formula...

- ▶ Using this framework, we can write the **factorization formula** for $B^- \rightarrow \mu^- \bar{\nu}_\mu$ at the level of the decay rate.
- ▶ **Real emissions** are factorized at the level of the **decay rate**. **Ultrasoft/ultrasoft-collinear** logarithms can be **resummed** in Laplace space ($\sim \mathcal{O}(10\%)$ corrections).
- ▶ The **virtual corrections** already factorize at the **amplitude level** and appear as an effective **Yukawa coupling** containing the **hard/hard-collinear** logarithms (which could be resummed but lead to a $\sim \mathcal{O}(1\%)$ correction).

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$$y_B(\mu) = C_L(\mu) \times H(\mu) \times \mathcal{J}(\mu)$$

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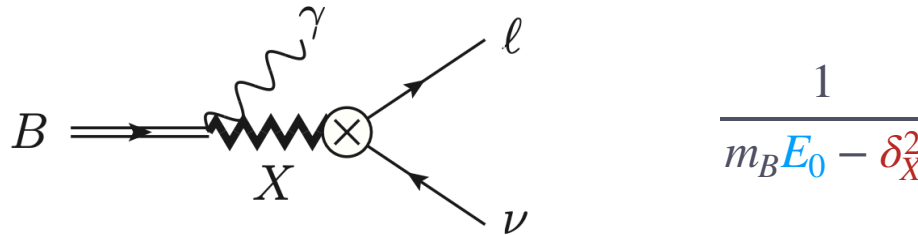
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- ▶ Below Λ_{QCD} , the initial B meson can emit a photon and transition into an excited state X :

[Becirevic et al (0907.1845)]



- ▶ For $E_0 \lesssim \lambda$, only the B^* needs to be included (**heavy-spin symmetry**), contributions from higher states being power-suppressed \rightarrow **Pole-dominance approximation**

$$\delta_X^2 = \frac{m_X^2 - m_B^2}{2m_B} \lesssim E_0$$

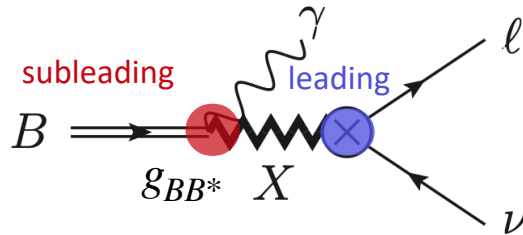
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$$\frac{1}{m_B E_0 - \delta_X^2}$$

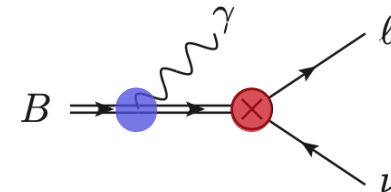
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- ▶ The $B \rightarrow B^* \gamma$ interaction carries the **same power suppression** as the effective **Yukawa**.



- ▶ g_{BB^*} is **unobserved**, has to be estimated by a mixture of **QCD sum rules**, **quark models** and **lattice QCD**.

Factorization formula... now complete !

$$\Gamma = \Gamma_0 \left(\underbrace{|y_B(\mu)|^2 |K(\mu)|^2}_{\text{Non-radiative}} \underbrace{S(E_0, \mu)}_{\text{Radiative}} + \underbrace{|y_B^*|^2 S_{B^*}(E_0)}_{\text{Structure-dependent radiative corrections}} \right)$$

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- Virtual QED correction breaks the **heavy-spin symmetry** for the effective **Yukawa couplings** :

$$y_B(m_\ell) = 6.30 \cdot 10^{-8} (1 \pm 0.05_{V_{ub}} \pm 0.02_{F_{m_B}} \pm 0.01_{f_B}),$$

$$y_{B^*}(m_\ell) = 6.69 \cdot 10^{-8} (1 \pm 0.05_{V_{ub}} \pm 0.06_{f_{B^*}}).$$

We choose $\Lambda = m_B$

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$$\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) \Big|_{E_\gamma^{max}=25 \text{ MeV}} = 4.00 \cdot 10^{-7} \left(1 \pm 0.09_{V_{ub}} \pm 0.04_{F_{m_B}} \pm 0.01_{f_B} \right)$$

$$\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) \Big|_{E_\gamma^{max}=100 \text{ MeV}} = 4.16 \cdot 10^{-7} \left(1 \pm 0.09_{V_{ub}} \pm 0.04_{F_{m_B}} \pm 0.01_{f_B} \pm 0.01_{g_{BB^*}} \right)$$

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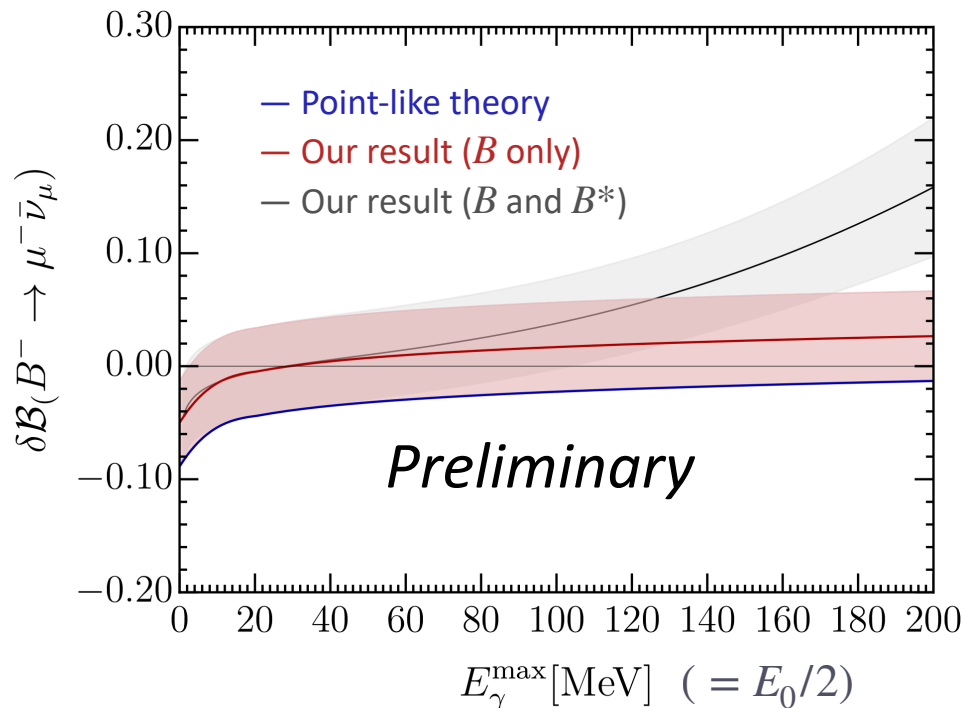
Precision for the extraction of V_{ub}

$$\mathcal{B} = \mathcal{B}_0 \left(|y_B(\mu)|^2 |K(\mu)|^2 S(E_S, \mu) + |y_B^*|^2 S_{B^*}(E_S) + |y_{B'_1}|^2 S_{B'_1}(E_S) \right)$$

$$\delta\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) \equiv \frac{\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu)}{\mathcal{B}_0(B^- \rightarrow \mu^- \bar{\nu}_\mu)} - 1$$

$$F_{m_B}(m_b) = (1.00 \pm 0.02) \cdot F_{\text{QCD}}(m_b).$$

Conservative assumption !



$$S(E_0, \mu) \sim \log \frac{E_0}{m_B} \log \frac{m_\ell}{m_B}$$

$$S_{B^*}(E_0) \sim E_0^2$$

Conclusions

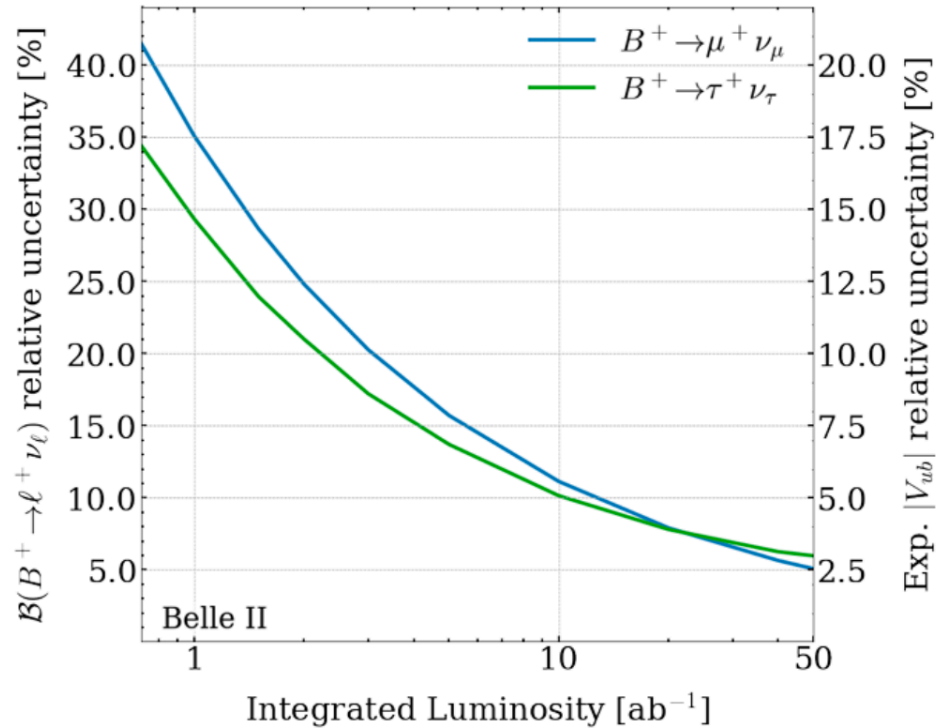
- ▶ The **exclusive leptonic decay** $B^- \rightarrow \mu^- \bar{\nu}_\mu$, is an important channel for test of physics in **SM and beyond**. Since future experimental accuracy calls for percent precision in the theory prediction, QED corrections are needed.
- ▶ On top of **large logarithms** of lepton mass and photon cuts, we find (single logarithmic) **structure-dependent** effects in the virtual & real corrections:
 - virtual: **hard-collinear** photons between the **lepton** and light **spectator quark**
 - real: B^* contribution; gets **more important** for a looser cut.
- ▶ These corrections introduce **new uncertainties**: they probe the inner structure of the B meson & introduce **new hadronic parameters**. Could F_Λ be computed on the lattice ?

Thanks for listening !



Backup-slides

Belle II projections



[Belle II Physics Book]

Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \rightarrow \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \rightarrow \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

SCET_{II} basis

$$\mathcal{H}_v = Y_n^{(q)\dagger} h_v, \quad \mathcal{Q}_s = Y_n^{(q)\dagger} q_s, \quad \mathcal{G}_s^\mu = Y_n^\dagger (iD_s^{(G)\mu} Y_n)$$

$$\omega = n \cdot p_q, \quad \omega_g = n \cdot p_g$$

► Starting from our SCET_I operators, we can construct our SCET_{II} basis :

[\mathcal{A}]

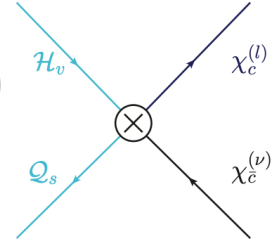
$$O_{\mathcal{A}}^{(1)} = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_s \not{n} P_L \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

$$O_{\mathcal{A}}^{(2)} = m_\ell \left(\bar{\mathcal{Q}}_s P_R \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

[\mathcal{B}]

$$O_{\mathcal{B}}^{(1)}(\omega) = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_s^{[\omega]} \not{n} P_L \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

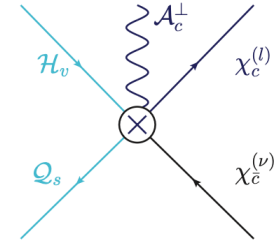
$$O_{\mathcal{B}}^{(2)}(\omega) = m_\ell \left(\bar{\mathcal{Q}}_s^{[\omega]} P_R \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$



[\mathcal{C}]

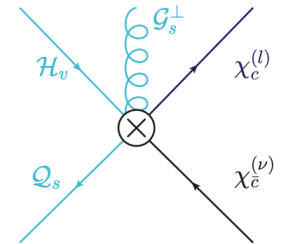
$$O_{\mathcal{C}}^{(1)}(u, \omega) = \frac{1}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_s^{[\omega]} \not{n} P_L \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[u]} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

$$O_{\mathcal{C}}^{(2)}(u, \omega) = \left(\bar{\mathcal{Q}}_s^{[\omega]} P_R \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[u]} P_L \chi_{\bar{c}}^{(\nu)} \right)$$



[\mathcal{D}]

$$O_{\mathcal{D}}^{(1)}(\omega, \omega_g) = m_\ell \left(\bar{\mathcal{Q}}_s^{[\omega]} \left(\frac{\not{n}}{in \cdot \partial} \left(\mathcal{G}_{s\perp}^{[\omega_g]} \right)^\mu \right) P_R \mathcal{H}_v \right) \left(S_n^{\dagger(\ell)} \bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$



Virtual amplitude at NLO QED

$$\mathcal{A}_{\text{virtual}} = i\sqrt{2} G_F K_{EW}(\mu) V_{ub} \frac{m_\ell}{m_B} \sqrt{m_B} F_\Lambda(\mu) \bar{u}(p_\ell) P_L v(p_\nu) \sum_j \mathcal{M}_j(\mu),$$

$$\begin{aligned} \mathcal{M}_{2p}(\mu) = & 1 + \frac{\alpha}{4\pi} \left\{ Q_b^2 \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right] \right. \\ & - Q_\ell^2 \left[\frac{1}{\epsilon_{\text{IR}}} \left(2 + \log \frac{m_\ell^2}{m_B^2} \right) - \frac{1}{2} \log^2 \frac{\mu^2}{m_\ell^2} + \frac{3}{2} \log \frac{\mu^2}{m_\ell^2} - \frac{\pi^2}{12} + 2 \right] \\ & + Q_b Q_\ell \left[\frac{1}{R} \left(\log \frac{\mu^2}{m_B^2} + 1 + \log R \right) - f(R) + \frac{1}{2} R f'(R) - \log \frac{\mu^2}{m_\ell^2} \right] \\ & \left. + Q_\ell Q_u \left[-5 - 2 \log \frac{\mu^2}{m_\ell^2} + \int_0^\infty d\omega \phi_-(\omega) \left(\log \frac{\mu^2}{\omega \sqrt{m_B} \Lambda} \log \frac{\Lambda^2}{m_B^2} - 2 \log \frac{\mu^2}{\omega \sqrt{m_B} \Lambda} + \log \frac{\Lambda}{m_B} - 6 - \frac{\pi^2}{3} \right) \right] \right\} \\ & + \frac{C_F \alpha_s}{4\pi} \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right], \end{aligned}$$

$$R = \frac{m_b}{q_-} = \frac{m_b}{m_B}$$

$$f(R) = \frac{1}{2} \log^2 \frac{\mu^2 R^2}{m_b^2} + \frac{\pi^2}{12} - 2 \text{Li}_2(1-R) - \log^2 R + 1$$

$$\mathcal{M}_{3p}(\mu) = -\frac{\alpha}{\pi} Q_\ell Q_u \left(1 + \log \frac{\Lambda}{m_B} \right) \times \int_0^\infty d\omega \int_0^\infty d\omega' \frac{\phi_3(\omega, \omega')}{\omega'} \left[\frac{1}{\omega'} \ln \left(1 + \frac{\omega'}{\omega} \right) - \frac{1}{\omega + \omega'} \right].$$

Coft Wilson-lines cancellation

- ▶ One can also define F_Λ using **time-like Wilson-lines** :

$$-\frac{i\sqrt{m_B}}{2}F_\Lambda(\mu) \equiv \frac{\langle 0 | \bar{q}_s \theta(\Lambda + i\vec{n} \cdot \overleftarrow{D}_s) \not{n} P_L \mathcal{S}_{v_\ell}^{(l)\dagger} h_v | B^- \rangle}{\langle 0 | \mathcal{S}_{v_B}^{(B)} \mathcal{S}_{v_\ell}^{(l)\dagger} | 0 \rangle}$$

- ▶ Those time-like WL can be splitted between **soft** and **soft-collinear (coft)** contributions :

$$\mathcal{S}_{v_B}^{(l)}(x) = S_{v_B}^{(B)}(x) C_{\vec{n}}^{(B)}(x_+) = S_{v_B}^{(B)}(x) C_{\vec{n}}^{(b)}(x_+) C_{\vec{n}}^{(q)\dagger}(x_+) \quad \mathcal{S}_{v_\ell}^{(l)}(x) = S_n^{(\ell)}(x_-) C_{v_\ell}^{(\ell)}(x)$$

- ▶ After **soft-collinear** decoupling, $h_v \rightarrow C_{\vec{n}}^{(b)} h_v$, $q_s \rightarrow C_{\vec{n}}^{(q)} q_s$

contributions from the **coft Wilson-lines** cancel in the ratio :

$$-\frac{i\sqrt{m_B}}{2}F_\Lambda(\mu) = \frac{\langle 0 | \bar{q}_s \theta(\Lambda + i\vec{n} \cdot \overleftarrow{D}_s) \not{n} P_L S_n^{(\ell)\dagger} h_v | B^- \rangle}{\langle 0 | S_{v_B}^{(B)} S_n^{(\ell)\dagger} | 0 \rangle}$$

A theory of Wilson lines ?

$$Y_r^{(i)}(x) = \mathcal{P} \exp \left\{ ieQ_i \int_{-\infty}^0 ds r \cdot A_{us}(x+sr) \right\} \quad C_r^{(i)}(x) = \mathcal{P} \exp \left\{ ieQ_i \int_{-\infty}^0 ds r \cdot A_{usc}(x+sr) \right\}$$

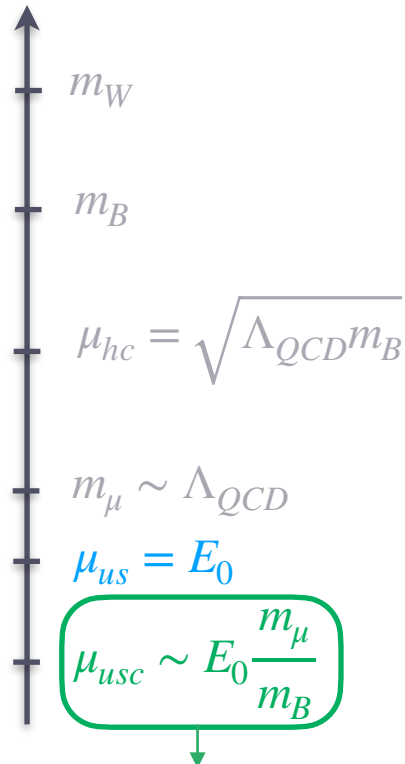
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$$\varphi_B(x) \rightarrow Y_{v_B}^{(B)}(x) C_{\bar{n}}^{(B)}(x_+) h_{v_B} \quad \chi_{v_\ell}(x) \rightarrow Y_n^{(\ell)}(x_-) C_{v_\ell}^{(\ell)}(x) \frac{\not{x} \not{\bar{x}}}{4} h_{v_\ell}$$

- ▶ The real corrections can be expressed as matrix elements of these Wilson lines :

$$W_s(\omega_{us}, \mu) = \left[\sum_{n_{us}=0}^{\infty} \prod_{i=1}^{n_{us}} \int d\Pi_i(q_i) \right] \left| \langle n_{us} \gamma_{us}(q_i) | Y_{v_B}^{(B)} Y_n^{\dagger(\ell)} | 0 \rangle \right|^2 \delta(\omega_{us} - q_0^{us}) \quad , \quad q_0^{us} = \sum_i q_{0i}$$

$$W_{usc}(\omega_{usc}, \mu) = \left[\sum_{n_{usc}=0}^{\infty} \prod_{j=1}^{n_{usc}} \int d\Pi_j(q_j) \right] \left| \langle n_{usc} \gamma_{usc}(q_j) | C_{\bar{n}}^{(B)} C_{v_\ell}^{\dagger(\ell)} | 0 \rangle \right|^2 \delta(\omega_{usc} - q_0^{usc}) \quad , \quad q_0^{usc} = \sum_j q_{0j}$$



Soft photon to the muon boosted in the B frame

- ▶ These radiative functions are convoluted with the **measurement function** implementing the experimental radiation veto

$$S(E_0, \mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc} \theta\left(\frac{E_0}{2} - \omega_{us} - \omega_{usc}\right) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$

First pole dominance

- ▶ This structure-dependent correction can be implemented at the level of the radiative hadronic matrix element $\langle \gamma | J_{A/V}^\mu | B \rangle$, where in the quark picture:

$$J_A^\mu = \bar{q} \gamma^\mu \gamma_5 b \quad J_V^\mu = \bar{q} \gamma^\mu b$$

- ▶ In the **ultrasoft/ultrasoft-collinear** region for the photon, it is reasonable to assume **first pole dominance** for both matrix elements [Becirevic et al (0907.1845)]

$$\langle \gamma | J_V^\mu | B \rangle \simeq \frac{\langle 0 | J_V^\mu | B^* \rangle \langle B^* | B \gamma \rangle}{q^2 - m_{B^*}^2} + \dots \quad \langle \gamma | J_A^\mu | B \rangle \simeq \frac{\langle 0 | J_A^\mu | B'_1 \rangle \langle B'_1 | B \gamma \rangle}{q^2 - m_{B^*}^2} + \dots$$

$\sim g_{BB^*}$ $\sim g_{BB'_1}$

- ▶ Below Λ_{QCD} , quark currents can be replaced by **effective hadronic currents** involving meson fields:

$$J_A^\mu \rightarrow J_{Had,A}^\mu = f_B D^\mu \Phi_B + i m_{B'_1} f_{B'_1} V_1^\mu \quad J_V^\mu \rightarrow J_{Had,V}^\mu = m_{B^*} f_{B^*} V_*^\mu$$