Short Flow-Time Expansion of the LEFT basis: Background Field Method and Chiral Symmetry

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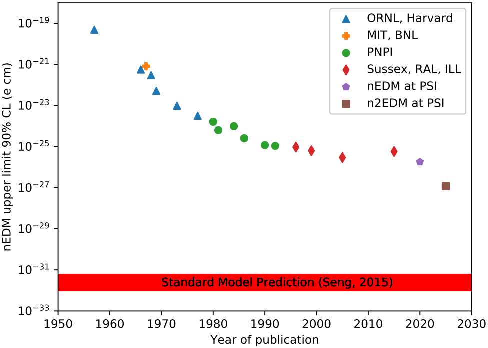
Motivation: Neutron Electric Dipole Moment

- Baryon asymmetry of the universe $\implies$ BSM CP violation.
- EDMs (e.g. nEDM) are interesting probes of BSM CP violation.
- $$d_n \sim \sum_i L_i \langle N | \mathcal{O}_i^{\mathrm{MS}} | N\gamma \rangle$$
- These matrix elements are non-perturbative quantities $\implies$ Lattice QCD.
- Lattice scheme has to be converted to MS!
- Ultimate goal: Translate the bounds on low-energy observables (nEDM) to constraints on heavy new physics.

Experimental status of the nEDM



Two possibilities:

- Detecting a signal in the unexplored region $\Longrightarrow$ CP violating New Physics.
- Not detecting a signal, lowering the bound. Still very interesting! Constrains the shape of New Physics Models.

nEDM and the GF: Status

At one-loop, the Short Flow Time expansions for operators contributing to the nEDM are now known

- Quark dipoles $\bar{q}qX$ (Mereghetti, Monahan, Rizik, Shindler, Stoffer, 2021)
- Four quark operators $\bar{q}q\bar{q}q$ (Bühler, Stoffer, 2023)
- Weinberg operator $GG\tilde{G}$ (ÒLC, Monahan, Rizik, Shindler, Stoffer, 2023)

$\mathcal{O}\left(10-25\%\right)$ uncertainties are required for meaningful nEDM studies.

(Alarcon et al., 2022)

- $\mathcal{O}\left(40\%\right)$ uncertainties at one loop $\implies$ we are not precise enough!
- Progress on the 2-loop dipole SFTE.

(Borgulat, Harlander, Rizik, Shindler, 2022)

Nuisance operators for the LEFT SFTE

In a general off-shell matching

$$\mathcal{O}_i(t,x) = C_{ij}\underbrace{\mathcal{O}_j}_{\text{physical}} + C_{ik}^{\text{EOM}}\underbrace{\mathcal{O}_k^{\text{EOM}}}_{\text{EOM}} + C_{il}^{\mathcal{E}}\underbrace{\mathcal{E}_l}_{\text{evanescent}} + C_{is}^{\mathcal{N}}\underbrace{\mathcal{N}_s}_{\text{gauge variant}}.$$

The basis of nuisance operators $\mathcal{N}$ relevant for the nEDM is known.

(Cirigliano, Mereghetti, Stoffer, 2020)

However:

- Basis of nuisance operators is large, computationally expensive to compute at higher orders!
- Basis of nuisance operators not known for the whole Low Energy Effective Field Theory (LEFT) basis.

Background Field Method applied to the Gradient Flow

- We decompose flowed gauge fields into background and quantum parts

$$B_\mu(t,x) = \underbrace{\hat{B}_\mu(t,x)}_{\text{background}} + \underbrace{b_\mu(t,x)}_{\text{quantum}} .$$

- Let us define the notion of background gauge transformation

$$\hat{B}_\mu(t,x) \to \hat{B}_\mu(t,x) + \hat{D}_\mu \omega(x) , \, b_\mu(t,x) \to b_\mu(t,x) + \left[b_\mu(t,x), \omega(x)\right]$$

which reproduces the full gauge transformation

$$B_\mu(t,x) \to B_\mu(t,x) + D_\mu \omega(x) .$$

Background Field Method applied to the Gradient Flow

We split the conventional flow equation

$$\partial_t B_\mu (t,x) = D_\nu G_{\nu\mu} (t,x) + \alpha_0 D_\mu \partial_\nu B_\nu (t,x)$$

into two flow equations (Suzuki, PTEP 2015 (2015) 10, 103B03)

$$\partial_t B_\mu (t,x) = D_\nu G_{\nu\mu} (t,x) + \alpha_0 D_\mu \hat{D}_\nu B_\nu (t,x) \ ,$$
$$\partial_t \hat{B}_\mu (t,x) = \hat{D}_\nu \hat{G}_{\nu\mu} (t,x)$$

where both flow equations transform covariantly under background gauge transformations!

LEFT SFTE

- Goal: SFTE of all the operators in the LEFT containing quark and gluons $\rightarrow$ connection to lattice and extraction of RGEs (talk by R. Harlander).
- Basis of physical, EOM and evanescent operators is known.

(Naterop, Stoffer, 2023)

- The background field method lets us avoid nuisance operators.
- Flowed calculations at one-loop are accessible: master integrals + method of projectors (talk by R. Harlander).
- $\Longrightarrow$ all ingredients ready for the LEFT SFTE at one-loop!

(ÒLC, Stoffer, to appear)

Chiral Symmetry Breaking in the 't Hooft Veltman scheme

- Chiral symmetry broken in the LEFT by mass terms and higher dimensional operators

$$\mathcal{L}_{\text{mass}} = -\overline{\psi}_R M \psi_L - \overline{\psi}_L M^\dagger \psi_R \,.$$

- We can preserve chiral symmetry by promoting masses and Wilson coeffs. to be spurions

$$M \to U_R M U_L^\dagger \,, \quad M^\dagger \to U_L M^\dagger U_R^\dagger \,.$$

- HV breaks chiral symmetry

$$\mathcal{L}_{\text{kin}} = \overline{\psi} i \not{D} \psi = \overline{\psi}_L i \overline{\not{D}} \psi_L + \overline{\psi}_R i \overline{\not{D}} \psi_R + \overline{\psi}_L i \hat{\not{D}} \psi_R + \overline{\psi}_R i \hat{\not{D}} \psi_L \,.$$

- Breaking of spurious chiral symmetry due to HV can be cured by finite counterterms. (Naterop, Stoffer, 2023)

Summary

- One-loop SFTEs relevant for the neutron electric dipole moment are done.
- Background Field Method applied to the gradient flow lets us avoid mixing into gauge variant operators.
- One-loop SFTE of the QCD sector of the Low Energy Effective Field Theory ready soon.

Future work

- SFTEs relevant for the nEDM at two loops.
- Other SFTEs at two loops that are relevant to make a connection to lattice (e.g. talks by F. Lange and R. Harlander).
- LEFT SFTEs at two loops give RGEs that are not known yet and are relevant for phenomenology studies!

Thank you! 😊

Back up slides 💪

nEDM in terms of LEFT Wilson Coefficients

$$d_n \sim \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$$

$$\begin{aligned} d_n &= -(1.5 \pm 0.7) \cdot 10^{-3}\ \bar{\theta}\, e\, \mathrm{fm} - (0.20 \pm 0.01) d_u \\ &\quad + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e\tilde{d}_u \\ &\quad - (1.1 \pm 0.55) e\tilde{d}_d + (50 \pm 40)\, \mathrm{MeV} e\, \tilde{d}_G\ . \end{aligned}$$

where d_q denotes the EDM of a quark q, $\tilde{d}_q$ denotes its chromo EDM, and $\tilde{d}_G$ denotes the gluon-chromo EDM. (Alarcon et al., 2022)

Lessons learned:

- We need to measure multiple EDMs.
- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- Lattice is key!

Operator basis relevant for the nEDM: Physical (I)

Basis from (Cirigliano, Mereghetti, Stoffer, 2020):

$$\mathcal{O}_1^{(4)} = \frac{1}{g^2} Tr[G_{\mu\nu}\hat{G}_{\mu\nu}]\,,$$

$$\mathcal{O}_2^{(4)} = \partial_\mu \left(\bar{q}\gamma_\mu\gamma_5 q\right)\,,$$

$$\mathcal{N}_1^{(4)} = (\bar{q}_E\gamma_5 q + \bar{q}\gamma_5 q_E)\,,$$

$$\mathcal{O}_1^{(5)} = \epsilon_{ijk}\epsilon_{lmn}\mathcal{M}_{mj}\mathcal{M}_{nk}\bar{q}^i i\gamma_5 q^l$$

$$\mathcal{O}_1^{(6)} = \frac{1}{g^2} Tr[G_{\mu\nu}G_{\mu_\lambda}\hat{G}_{\nu\lambda}] = \mathcal{O}_{GG\hat{G}}\,,$$

$$\mathcal{O}_2^{(6)} = (\bar{q}\tilde{\sigma}_{\mu\nu}\mathcal{M}t^a q)G^a_{\mu\nu}\,,$$

$$\mathcal{O}_3^{(6)} = (\bar{q}\tilde{\sigma}_{\mu\nu}\mathcal{M}Qq)F_{\mu\nu}\,.$$

Operator basis relevant for the nEDM: Physical (II)

$$\mathcal{O}_4^{(6)} = \frac{1}{g^2} Tr[\mathcal{M}^2] Tr[G_{\mu\nu}\hat{G}_{\mu\nu}]\,,$$

$$\mathcal{O}_5^{(6)} = \frac{1}{g^2}\partial_\nu Tr[(D_\mu G_{\mu\lambda})\hat{G}_{\nu\lambda}]$$

$$\mathcal{O}_6^{(6)} = \partial_\mu\left(\bar{q}\gamma_\mu\gamma_5\mathcal{M}^2 q\right) - \frac{1}{N_f}\mathcal{O}_7^{(6)}\,,$$

$$\mathcal{O}_7^{(6)} = Tr[\mathcal{M}^2]\partial_\mu\left(\bar{q}\gamma_\mu\gamma_5 q\right)\,,$$

$$\mathcal{O}_8^{(6)} = \partial_\mu\left(\bar{q}\gamma_\nu Q q\hat{F}_{\mu\nu}\right)\,,$$

$$\mathcal{O}_9^{(6)} = \frac{1}{g^2}\Box\, Tr[G_{\mu\nu}\hat{G}_{\mu\nu}]\,,$$

$$\mathcal{O}_{10}^{(6)} = \Box\partial_\mu\left(\bar{q}\gamma_\mu\gamma_5 q\right).$$

Operator basis relevant for the nEDM: EOM

$$\mathcal{N}_1^{(6)} = (\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) G^a_{\mu\nu}\,,$$

$$\mathcal{N}_2^{(6)} = (\bar{q}_E \tilde{\sigma}_{\mu\nu} Q q + \bar{q} \tilde{\sigma}_{\mu\nu} Q q_E) F_{\mu\nu}\,,$$

$$\mathcal{N}_3^{(6)} = (\bar{q}_E \mathcal{M} \gamma_\mu \gamma_5 D_\mu q + \bar{q} \overleftarrow{D}_\mu \gamma_\mu \gamma_5 \mathcal{M} q_E)\,,$$

$$\mathcal{N}_4^{(6)} = \left(\bar{q}_E \mathcal{M}^2 \gamma_5 q + \bar{q} \mathcal{M}^2 \gamma_5 q_E\right) - \frac{1}{N_f} \mathcal{N}_5^{(6)}\,,$$

$$\mathcal{N}_5^{(6)} = Tr[\mathcal{M}^2](\bar{q}_E \gamma_5 q + \bar{q} \gamma_5 q_E)\,,$$

$$\mathcal{N}_6^{(6)} = \partial_\mu \left(\bar{q}_E \gamma_5 D_\mu q - \bar{q} \overleftarrow{D}_\mu \gamma_5 q_E\right)\,,$$

$$\mathcal{N}_7^{(6)} = \partial_\mu (\bar{q}_E \tilde{\sigma}_{\mu\nu} D_\nu q - \bar{q} \overleftarrow{D}_\nu \tilde{\sigma}_{\mu\nu} q_E)\,,$$

$$\mathcal{N}_8^{(6)} = \frac{1}{g^2} \partial_\lambda \left(G^a_{\mu\nu} \left(D_\rho G^a_{\rho\sigma} - g^2 \bar{q} t^a \gamma_\sigma q \right) \right) \epsilon_{\mu\nu\lambda\sigma}\,,$$

$$\mathcal{N}_9^{(6)} = \partial_\mu \left(\bar{q}_E \mathcal{M} \gamma_\mu \gamma_5 q + \bar{q} \mathcal{M} \gamma_\mu \gamma_5 q_E \right)\,,$$

Operator basis relevant for the nEDM: Gauge variant (I)

$$\mathcal{N}_{11}^{(6)} = \frac{1}{g^2} G^a_{\mu\nu} \left(\partial_\lambda \left(D_\rho G^a_{\rho\sigma} - g^2 \bar{q} t^a \gamma_\sigma q - g^2 f^{abc} (\partial_\sigma \bar{c}^b) c^c \right) \right) \epsilon_{\mu\nu\lambda\sigma} \,,$$

$$\mathcal{N}_{12}^{(6)} = (\bar{q}_E \gamma_5 q + \bar{q} \gamma_5 q_E) G^a_\mu G^a_\mu \,,$$

$$\mathcal{N}_{13}^{(6)} = (\bar{q}_E \gamma_5 t^a q + \bar{q} \gamma_5 t^a q_E) G^b_\mu G^c_\mu d^{abc} \,,$$

$$\mathcal{N}_{14}^{(6)} = (\bar{q}_E \gamma_5 t^a q - \bar{q} \gamma_5 t^a q_E) \partial_\mu G^a_\mu \,,$$

$$\mathcal{N}_{15}^{(6)} = (\bar{q}_E \gamma_5 t^a D_\mu q - \bar{q} \overleftarrow{D}_\mu \gamma_5 t^a q_E) G^a_\mu \,.$$

Operator basis relevant for the nEDM: Gauge variant (II)

$$
\begin{aligned}
\mathcal{N}_{16}^{(6)} &= (\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) \partial_\mu G^a_\nu \,, \\
\mathcal{N}_{17}^{(6)} &= (\bar{q}_E \mathcal{M} \gamma_\mu \gamma_5 t^a q - \bar{q} \mathcal{M} \gamma_\mu \gamma_5 t^a q_E) G^a_\mu \,, \\
\mathcal{N}_{18}^{(6)} &= \frac{1}{g^2} \partial_\lambda \left((\partial_\mu G^a_\nu) \Big(D_\rho G^a_{\rho\sigma} - g^2 \bar{q} t^a \gamma_\sigma q - g^2 f^{abc} (\partial_\sigma \bar{c}^b) c^c \Big) \right) \epsilon_{\mu\nu\lambda\sigma} \,, \\
\mathcal{N}_{19}^{(6)} &= \partial_\mu \Big((\bar{q}_E \gamma_5 t^a q - \bar{q} \gamma_5 t^a q_E) G^a_\mu \Big) \,, \\
\mathcal{N}_{20}^{(6)} &= \partial_\mu \Big((\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) G^a_\nu \Big) \,.
\end{aligned}
$$

Background field method

Background Field Method in QCD (QFT and the SM, Schwartz, 2014): Split gauge field in background and quantum parts

$$A^a_\mu \to \underbrace{\hat{A}^a_\mu}_{\text{bckg.}} + \underbrace{A^a_\mu}_{\text{quantum}} \, .$$

QCD Lagrangian is invariant under background gauge transformations:

$$\hat{A}^a_\mu \to \hat{A}^a_\mu + \frac{1}{g}\partial_\mu \alpha^a - f^{abc}\alpha^b \hat{A}^c_\mu \, , \quad A^a_\mu \to A^a_\mu - f^{abc}\alpha^b A^c_\mu \, .$$

We can now gauge fix A_μ through Faddeev-Popov and choose the gauge condition $\hat{D}_\mu A_\mu = 0$ to obtain

$$\mathcal{L}_{\text{QCD,BFM}} = \mathcal{L}_{\text{QCD},A^a_\mu \to \hat{A}^a_\mu + A^a_\mu} - \frac{1}{2\tilde{\xi}} \left(\hat{D}_\mu A^a_\mu \right)^2 + \text{ghost terms} \, .$$

Background field method applied to the GF: Fermions

We split the fermion fields in a quantum and background part

$$\chi\left(t,x\right)=\hat{\chi}\left(t,x\right)+k\left(t,x\right)$$

and change the flow equation from

$$\partial_t\chi=D_\mu D_\mu\chi-\alpha_0\left(\partial_\mu B_\mu\right)\chi$$

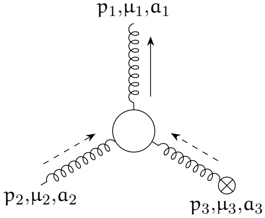
to

(Suzuki, PTEP 2015 (2015) 10, 103B03)

$$\partial_t\chi=D_\mu D_\mu\chi-\alpha_0\left(\hat{D}_\mu B_\mu\right)\chi\,,$$
$$\partial_t\hat{\chi}=\hat{D}_\mu\hat{D}_\mu\chi\,.$$

Background field method applied to the GF: Feynman Rules

- Flow vertices involving only quantum gauge fields are equal to the ones of R_ξ gauge.
- Flow vertices involving only background gauge fields are equal to the ones of R_ξ gauge setting $\alpha_0 = 0$.
- Flow vertices involving both quantum and background fields involve new formulae:



$$= -if^{a_1,a_2,a_3}\int_0^\infty dt\Big(-g^{\mu_1\mu_3}\left(2p_3^{\mu_2}+(1-\alpha_0)\,p_2^{\mu_2}\right) - g^{\mu_1\mu_2}\left(2p_2^{\mu_3}+p_3^{\mu_3}\right) - g^{\mu_2\mu_3}\left(2(\alpha_0-1)p_2^{\mu_1}+(\alpha_0+1)p_3^{\mu_1}\right)\Big)$$