Nonperturbative QCD in $D^0\overline{D}^0$ mixing

Lovro Dulibić (*Ruđer Bošković Institute,* Croatia)

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HRZZ Croatian Science Foundation
Alexander von HUMBOLDT STIFTUNG

in collaboration with:
Blaženka Melić (*Ruđer Bošković Institute*, Croatia), Alexey Petrov (*University of South Carolina*, USA)

Quirks in Quark Flavor Physics, Zadar 2024

Outline

1. Introducing the formalism
2. Presenting the problem
3. Our approach - **nonlocal condensates**
4. (Preliminary) results
5. Future research

Introduction

The formalism

$$i\frac{d}{dt}\begin{pmatrix} |D^0(t)\rangle \\ |\overline{D}^0(t)\rangle \end{pmatrix} = \hat{\mathcal{H}} \begin{pmatrix} |D^0\rangle \\ |\overline{D}^0\rangle \end{pmatrix}$$

$$\hat{\mathcal{H}} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)$$

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<u>*lifetime difference*</u>

$$x = \frac{\Delta M}{\Gamma_D} = 2\frac{M_{12}}{\Gamma_D} \qquad y = \frac{\Delta\Gamma}{2\Gamma_D} = \frac{\Gamma_{12}}{\Gamma_D}$$

<u>*mass difference*</u>

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mass difference

Introduction

Experiment

- $D^0\overline{D}^0$ mixing was discovered in **2007** by Belle and BaBar
  - Later confirmed by many others (to such an extent that the **no-mixing hypothesis is excluded** at $> 10\sigma$)

- The current **experimental world average** for the mass difference mixing parameter

$$x_{EXP} = -\,0.44^{+0.13}_{-0.15} \times 10^{-2}$$

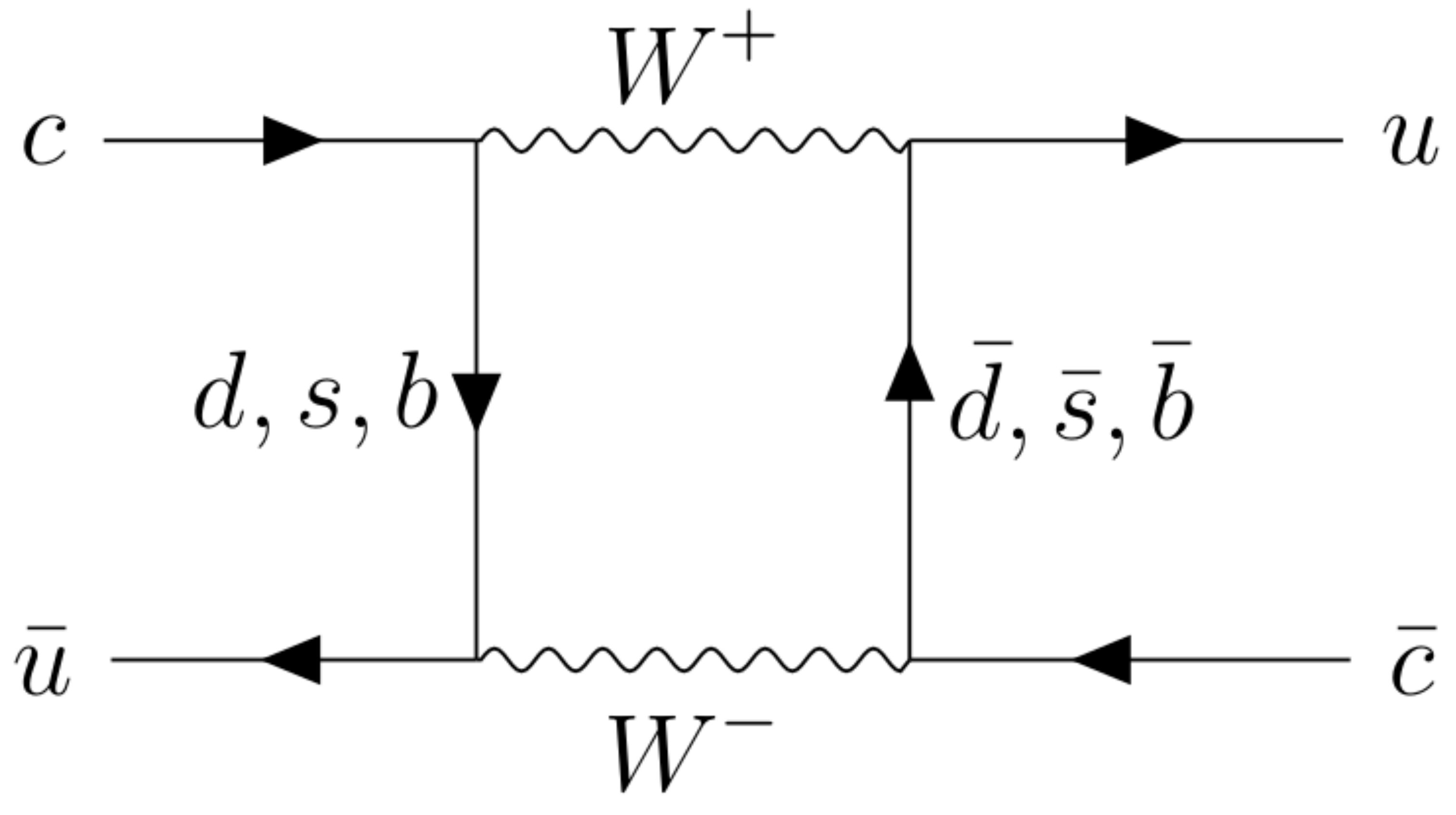
BELLE collaboration, *Evidence for D – D Mixing, Phys. Rev. Lett. 98 (2007) 211803,* [hep-ex/0703036]
BaBar collaboration, *Evidence for D – D Mixing, Phys. Rev. Lett. 98 (2007) 211802,* [hep-ex/0703020]
HFLAV collaboration, *Averages of b-hadron, c-hadron, and τ -lepton properties as of 2021, Phys. Rev. D 107 (2023) 052008,* [2206.07501]

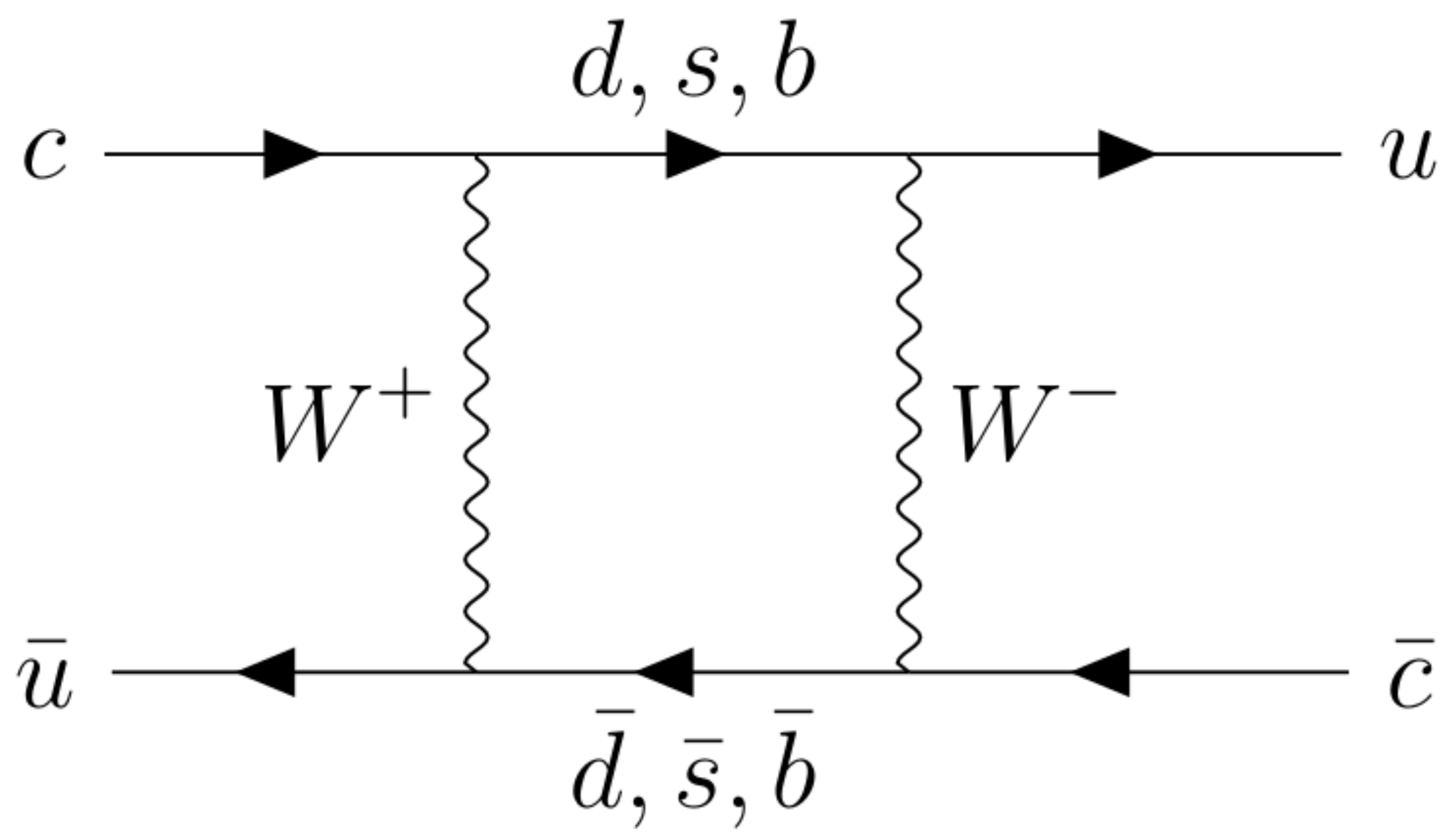
Presenting the problem

Theory - box diagram

$x_{EXP} \approx 10^{-3}$

$x_{TH} \approx 10^{-6}$





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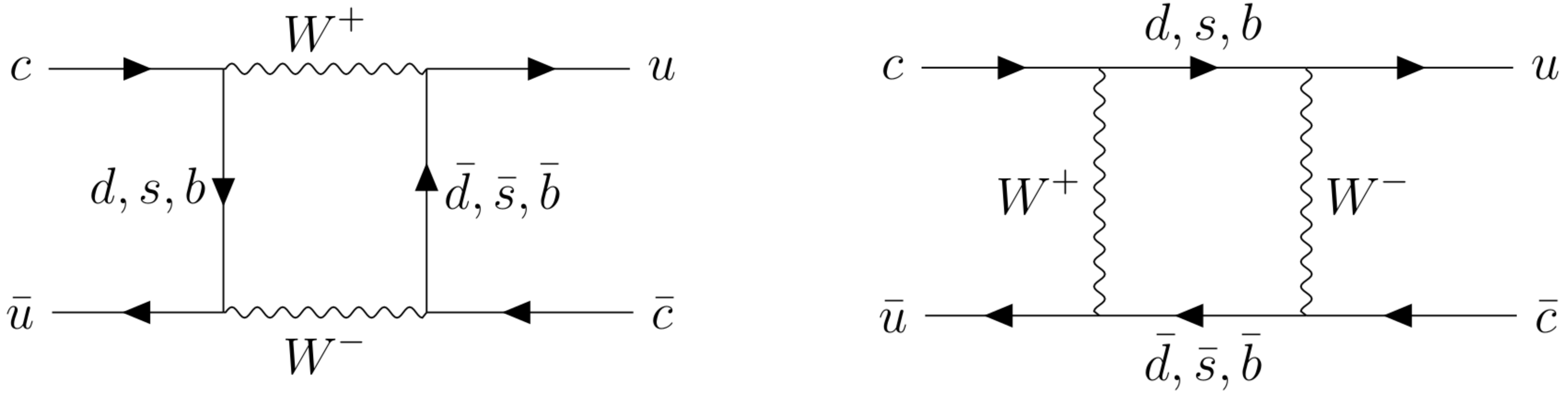

$$\mathscr{A} = \xi_s^2(A_{ss} - 2A_{ds} + A_{dd}) + 2\xi_s\xi_b(A_{dd} - A_{ds}) + \xi_b^2 A_{dd}$$

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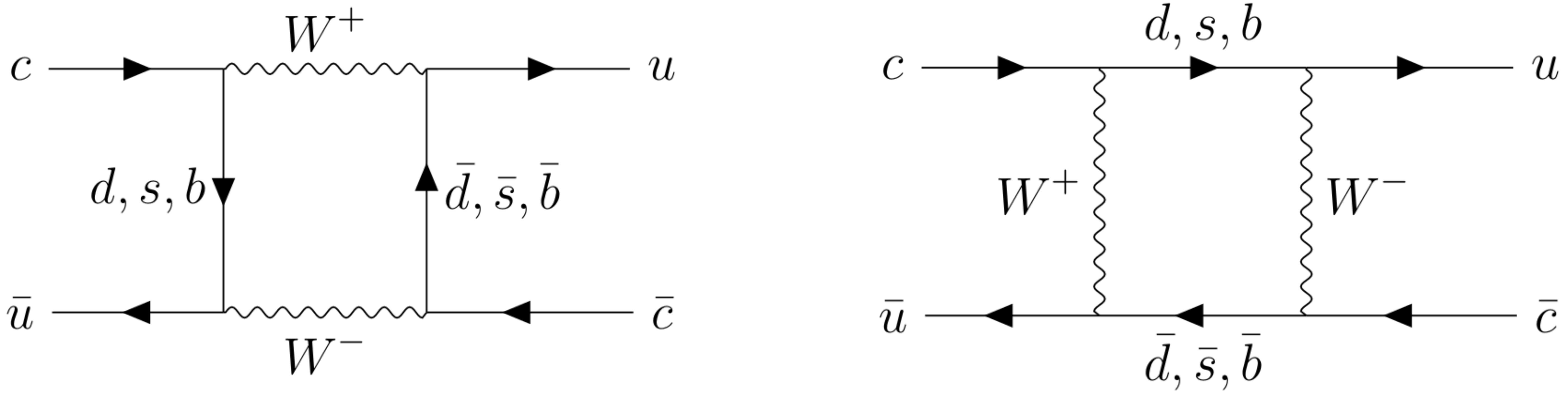

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CKM leading
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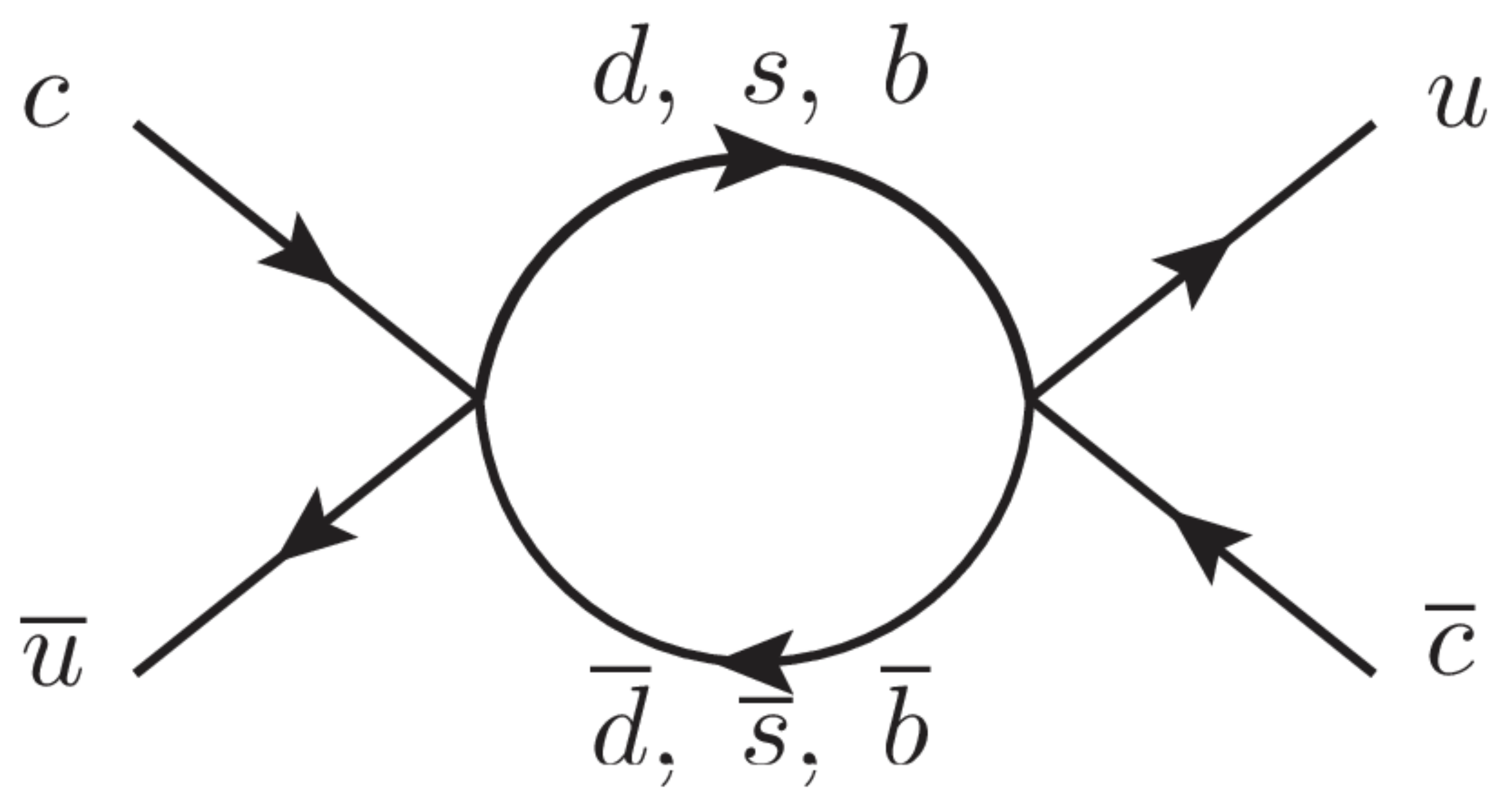

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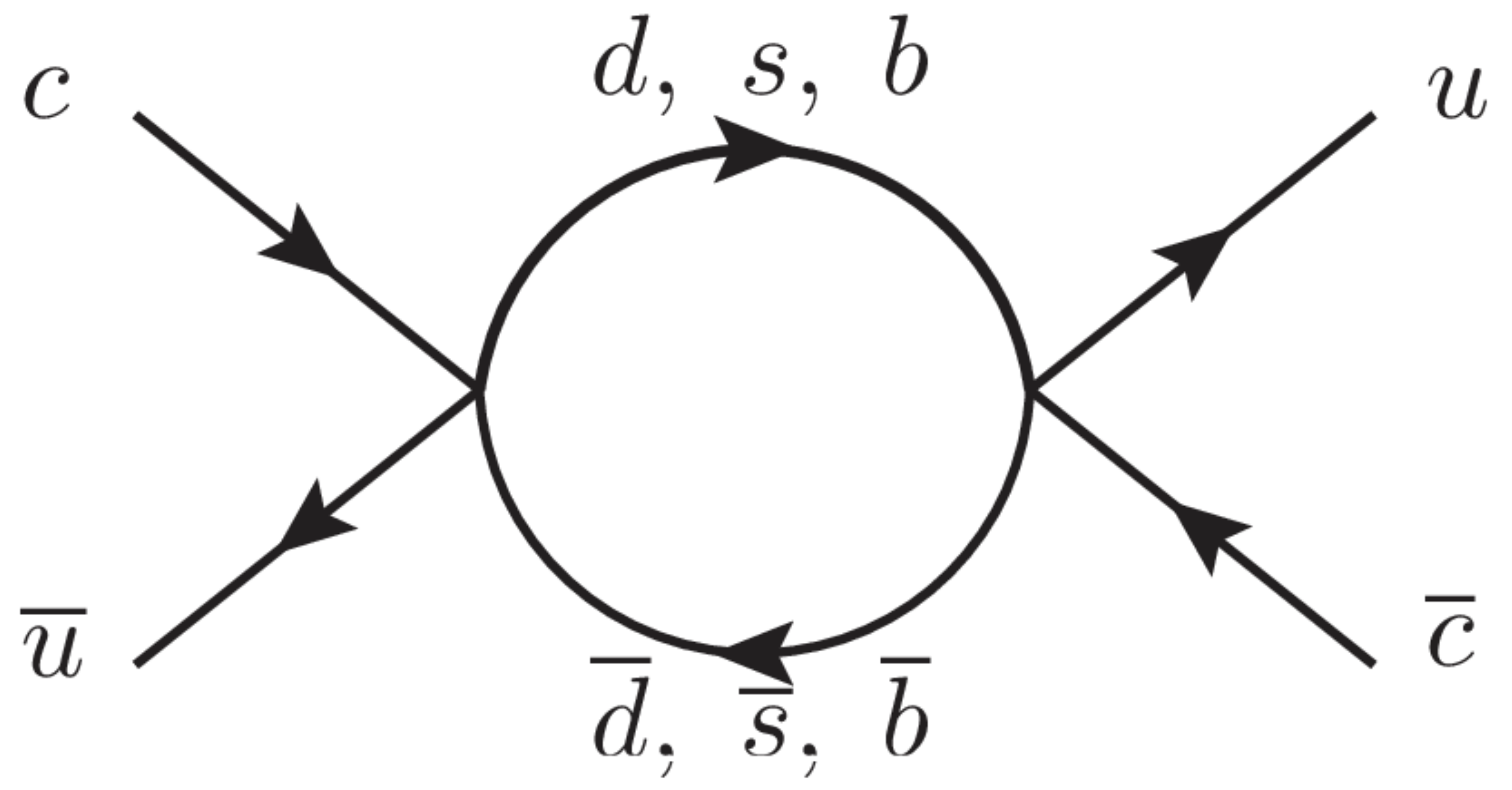

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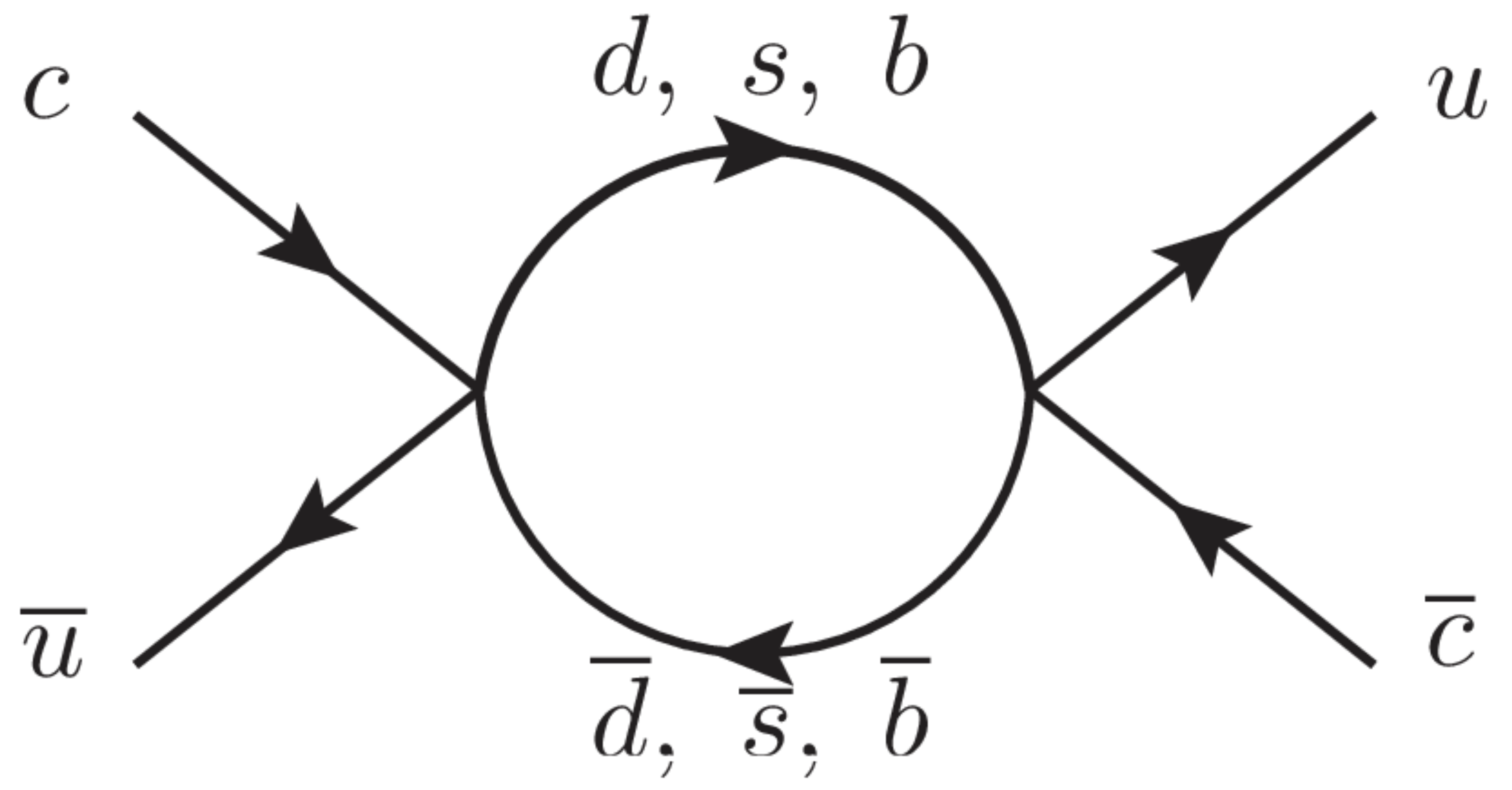

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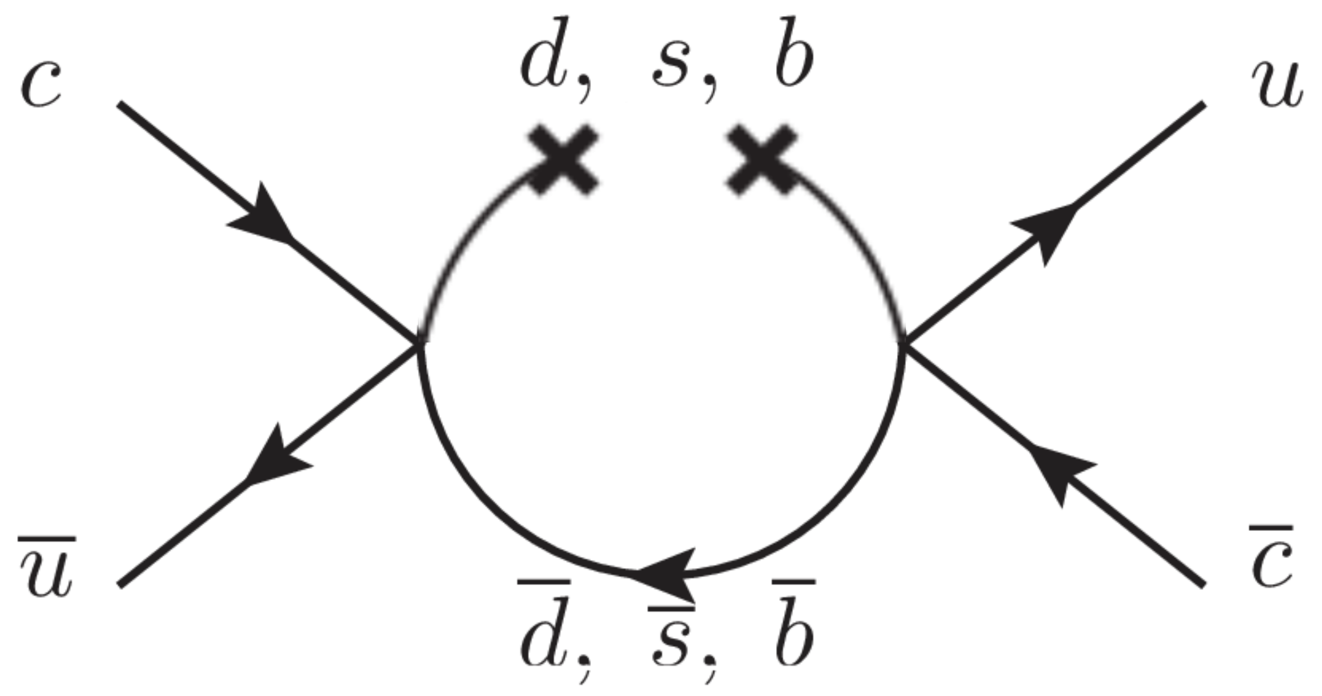
- It seems ***other*** contributions are actually leading. *NLO, nonperturbative effects, ...*

- Sticking to the Standard Model, we look for the missing contribution from ***nonperturbative physics***.

Different approaches

Inclusive

- SU(3) breaking contributions from new **higher-dimension operators**



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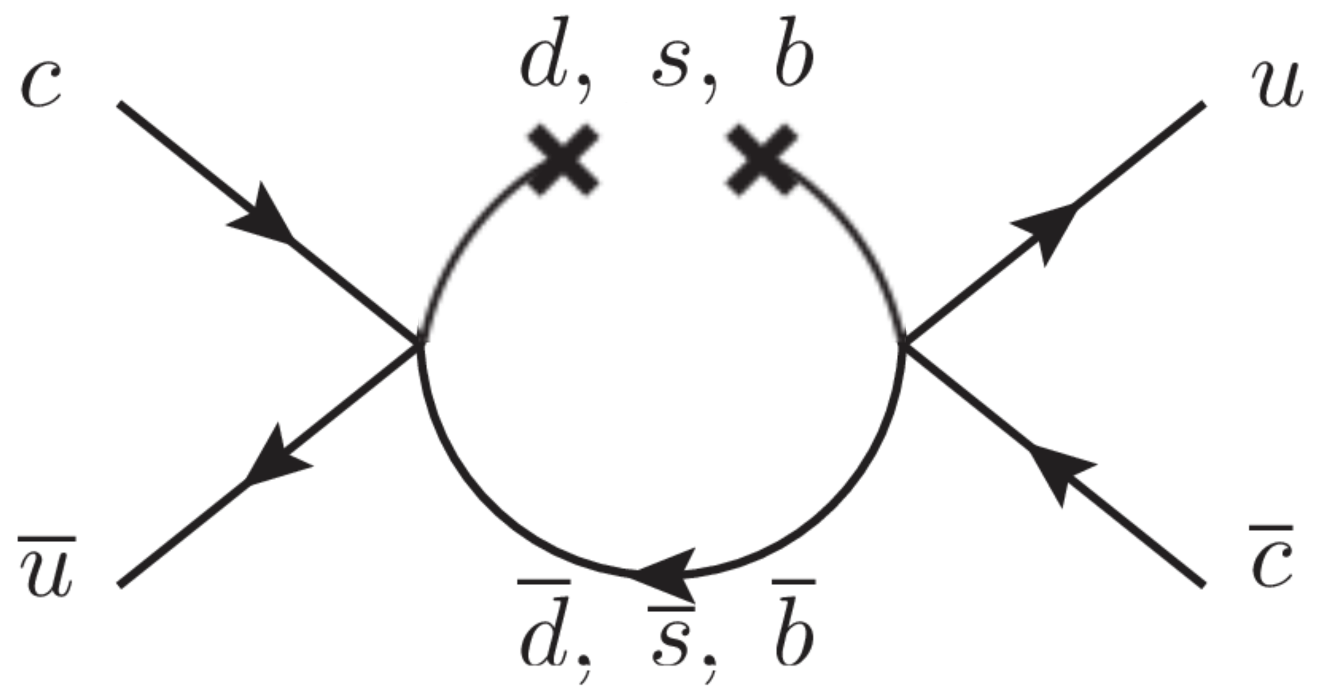

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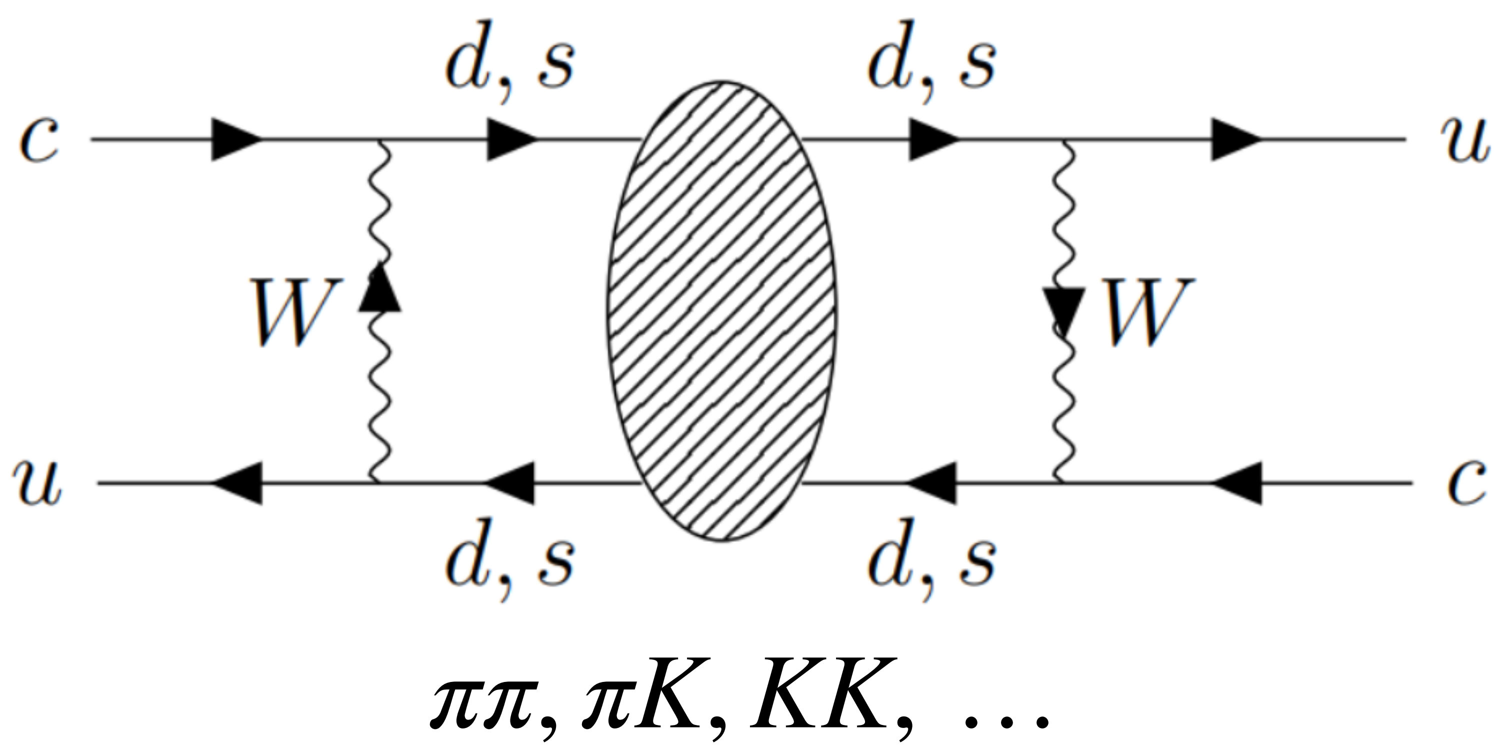

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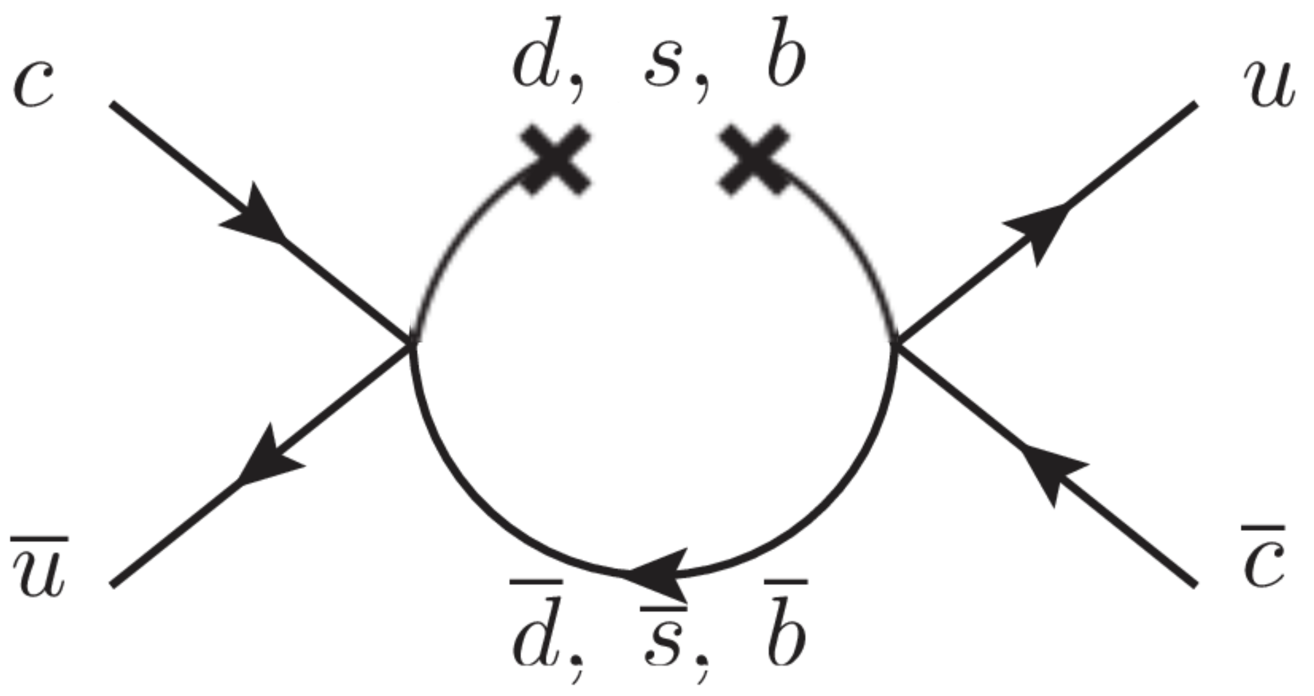

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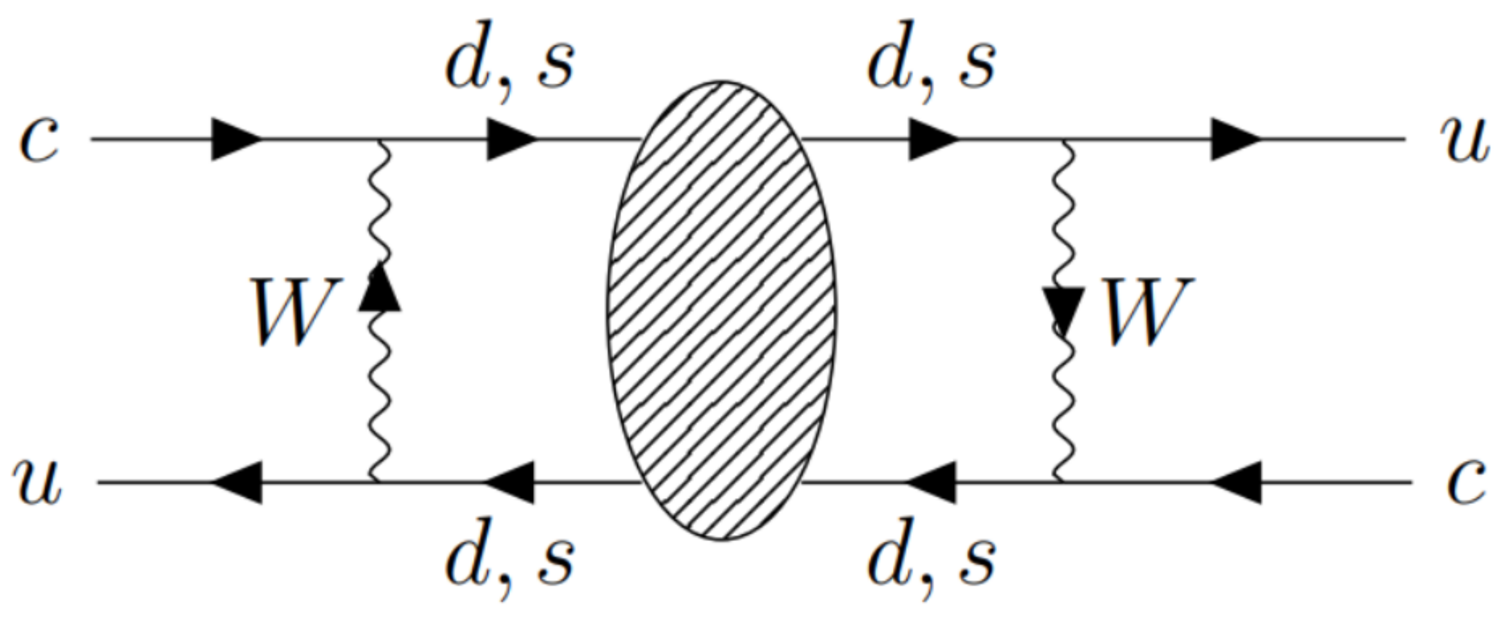

$$\pi\pi, \pi K, KK, \; \ldots$$

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Dispersive

- SU(3) breaking contributions from **'threshold' effects** from different D meson decay channels

A. F. Falk, Y: Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, *$D0$–$D0$ mass difference from a dispersion relation, Phys. Rev. D 69, 114021,* [hep-ph/0402204]

H.-N. Li, H. Umeeda, F. Xu and F.-S. Yu, *D meson mixing as an inverse problem, Phys. Lett. B 810 (2020) 135802,* [2001.04079]

H.-n. Li, *Dispersive analysis of neutral meson mixing, Phys. Rev. D 107 (2023) 054023,* [2208.14798]

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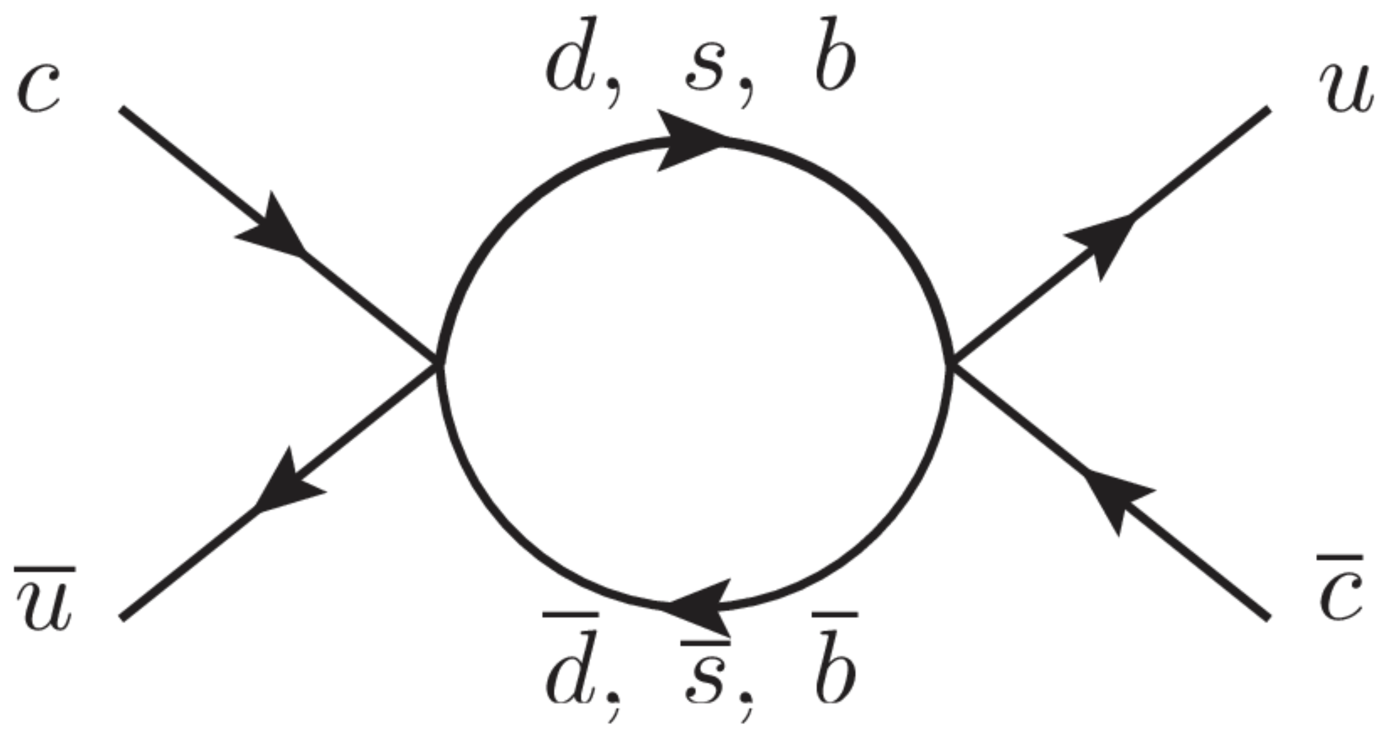
Our approach - nonlocal QCD condensates

Motivation



- **Box diagram** is $\propto \left(m_s/m_c\right)^4$

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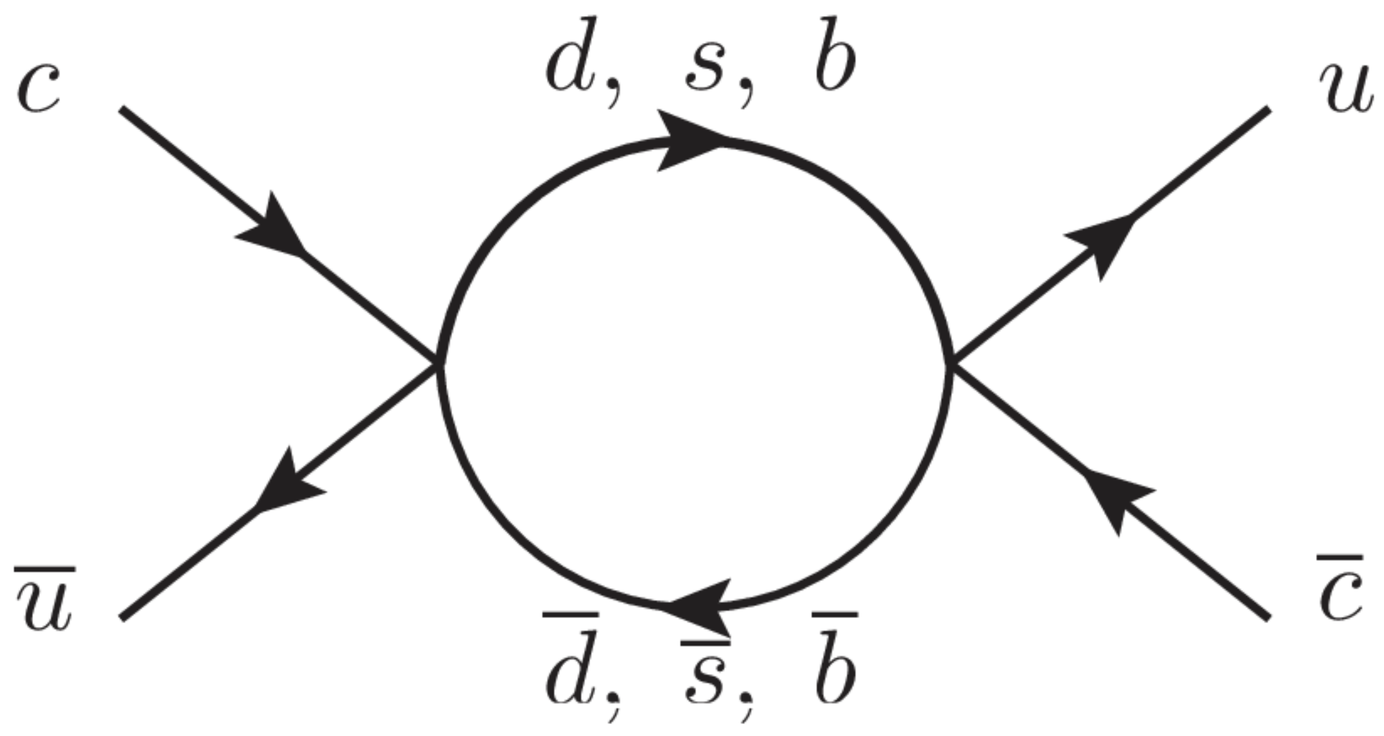

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$$S_q = \frac{\not{p} + m_q}{p^2 - m_q^2} = \frac{\not{p} + m_q}{p^2}\left(1 + \frac{m_q^2}{p^2} + \ldots\right)$$

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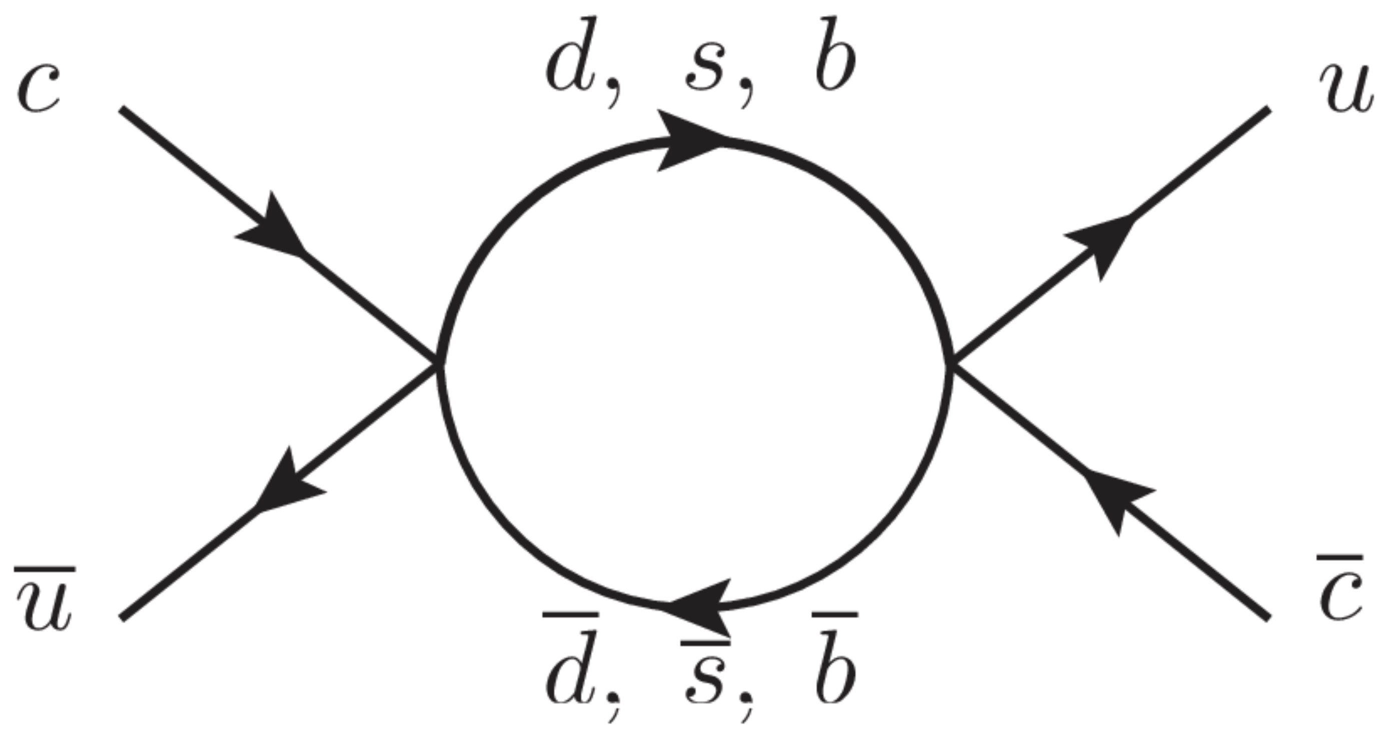

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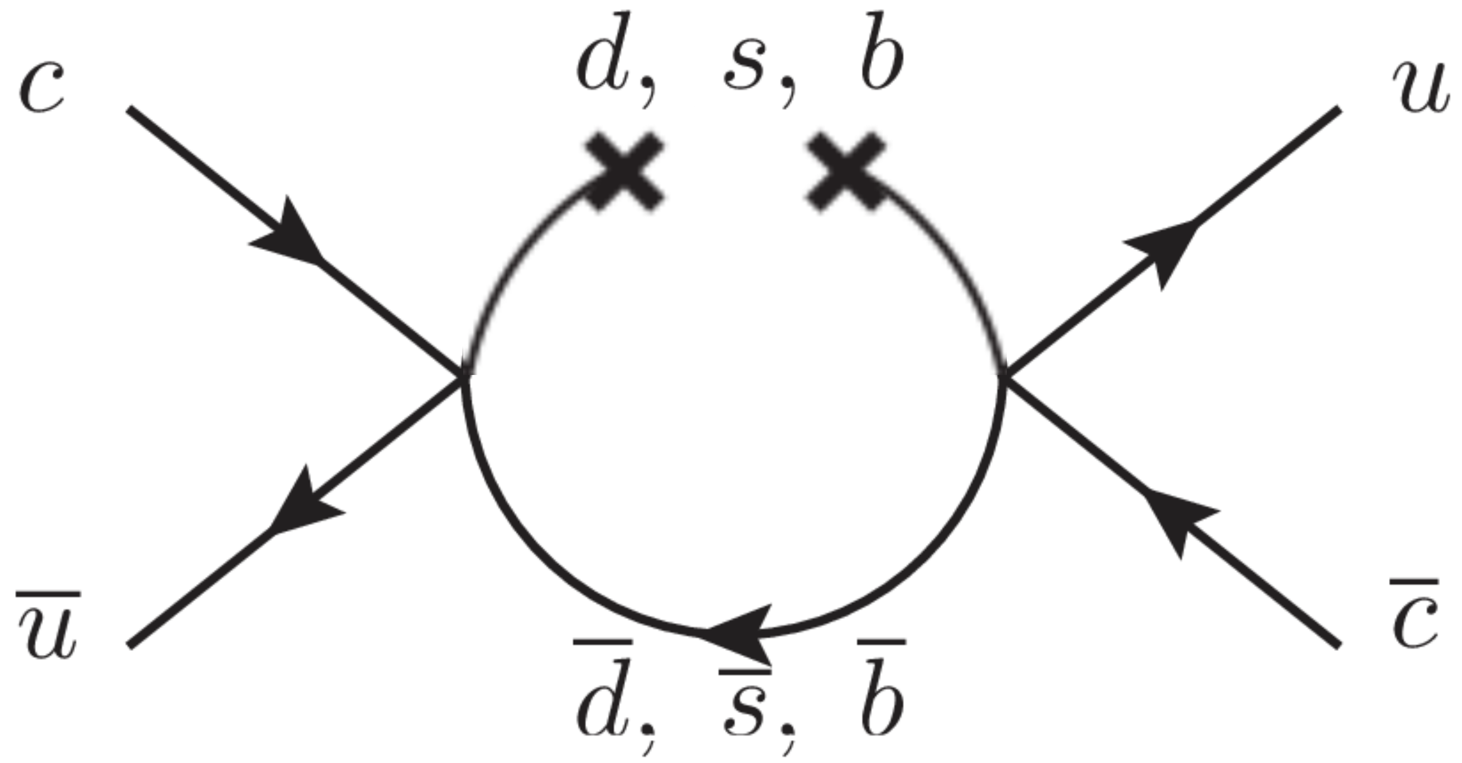

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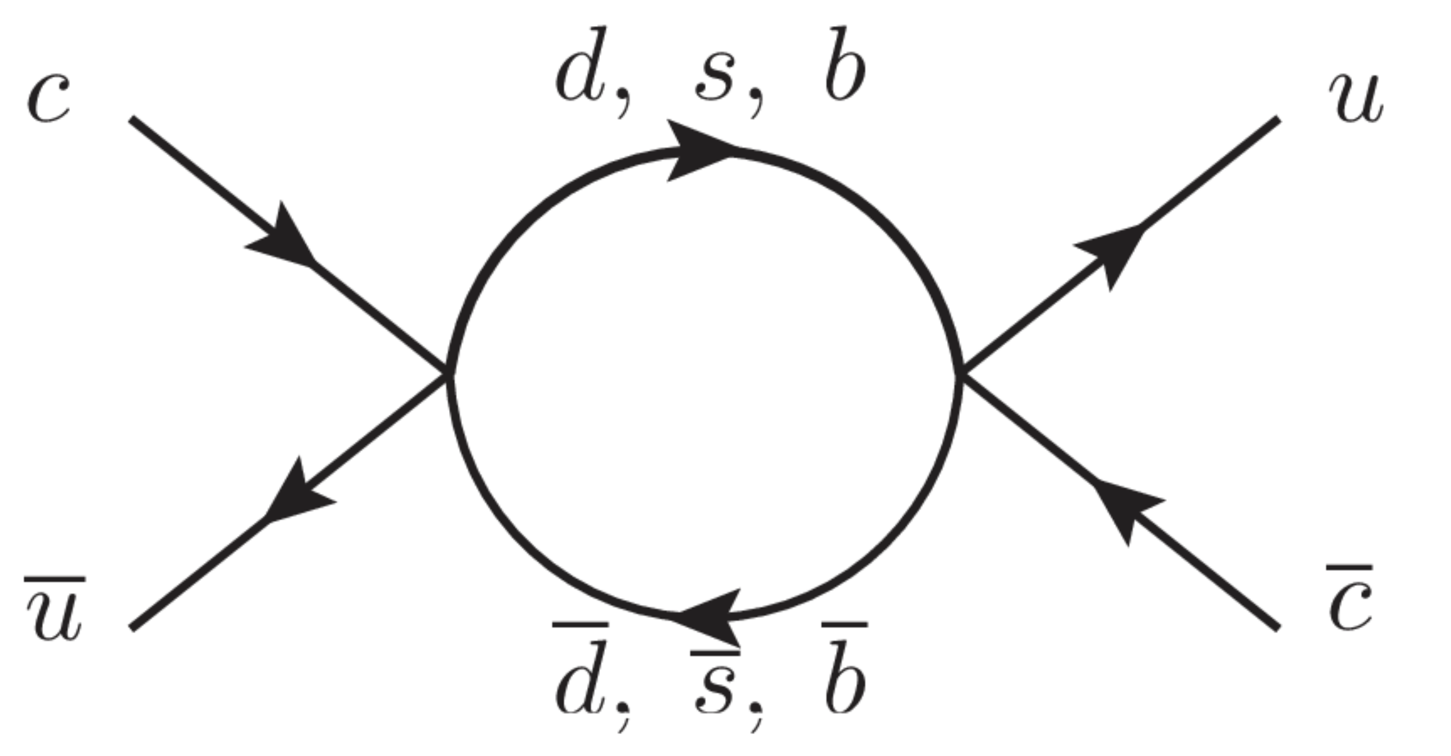

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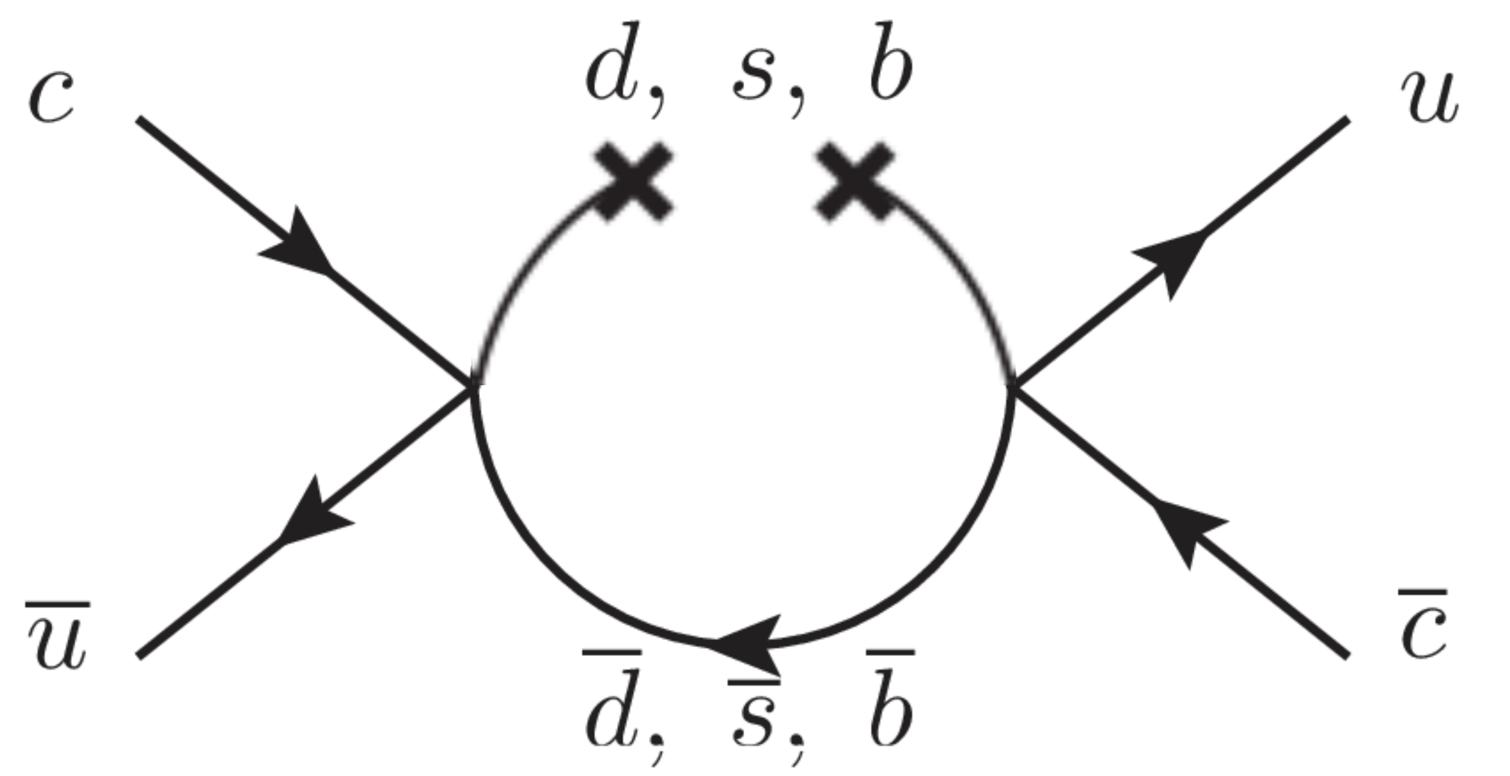

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$$\langle \overline{q}(x)q(0)\rangle \propto 1 + i\not{x}\frac{m_q}{4} + \ldots$$

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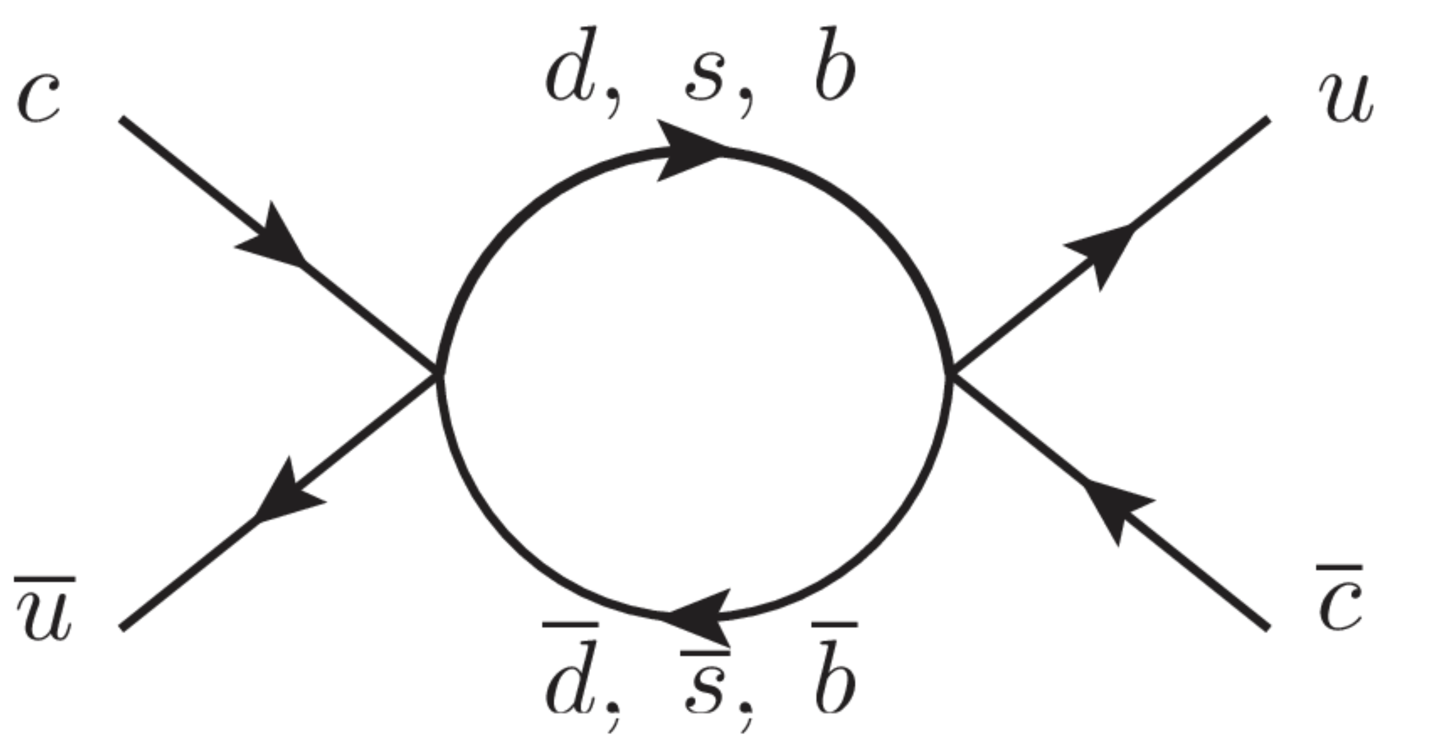

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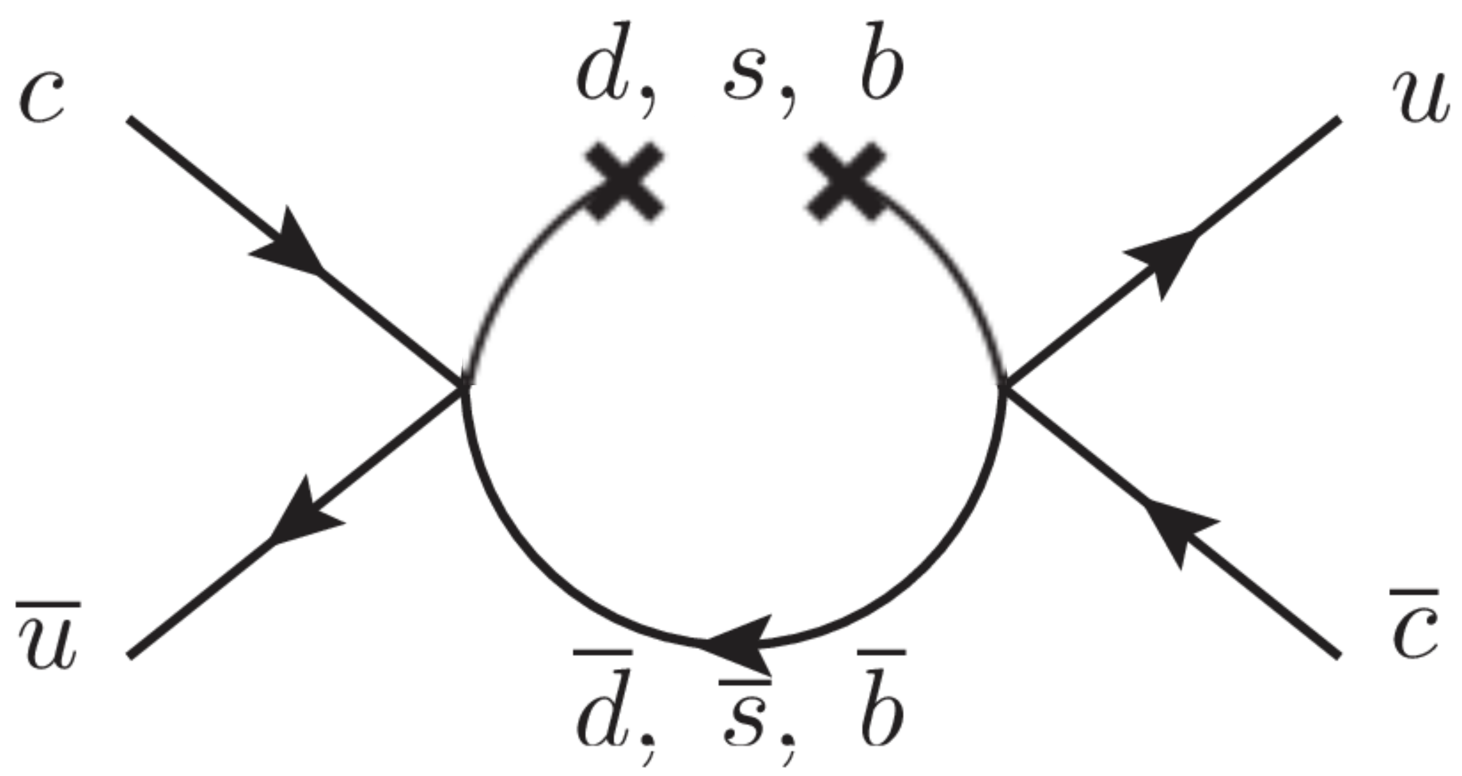

- Using **QCD condensates** we expect $\propto (m_s/m_c)^3$
  - Essentially, we trade a power of m_s/m_c suppression for a suppression of the higher dimensional operator.
  - Don't forget - there is also $16\pi^2$ relative enhancement since this is not a loop calculation

QCD condensates

- Condensates are well-known and have been widely used in sum rules calculations for decades

$$\langle \bar{q}(x)^a_\alpha q(0)^b_\beta \rangle = \frac{\langle \bar{q}q\rangle_0}{4N_C}\delta^{ab}\left[\delta_{\alpha\beta}\left(1 - \frac{x^2}{4}\left(\frac{m^2}{2} - \frac{\langle \bar{q}i\sigma G q\rangle_0}{2\langle \bar{q}q\rangle_0}\right)\dots\right) + \right.$$

$$\left. +i(\not{x})_{\beta\alpha}\left(\frac{m}{4} - \frac{x^2}{4}\left(\frac{m^3}{12} - \frac{m}{12}\frac{\langle \bar{q}i\sigma G q\rangle_0}{\langle \bar{q}q\rangle_0} + \frac{2}{81}\pi\alpha_s^{NP}\frac{\langle \bar{q}q\rangle_0^2}{\langle \bar{q}q\rangle_0}\right)\dots\right)\right]$$

M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *QCD and resonance physics. theoretical foundations, Nucl. Phys. B147, Issue 5, 1979, 385-447*

QCD condensates

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$$\langle \bar{q}q\rangle_0 = (-243\,\mathrm{MeV})^3$$

$$\frac{\langle \bar{q}i\sigma Gq\rangle_0}{2\langle \bar{q}q\rangle_0} = 0.4 \pm 0.1\,\mathrm{GeV}^2$$

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$$\frac{\langle \bar{s}s\rangle}{\langle \bar{q}q\rangle} = 0.8 \pm 0.3$$

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quark virtuality

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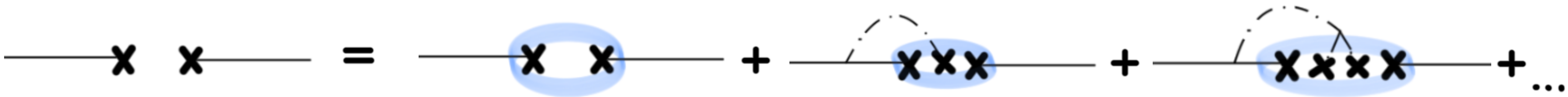

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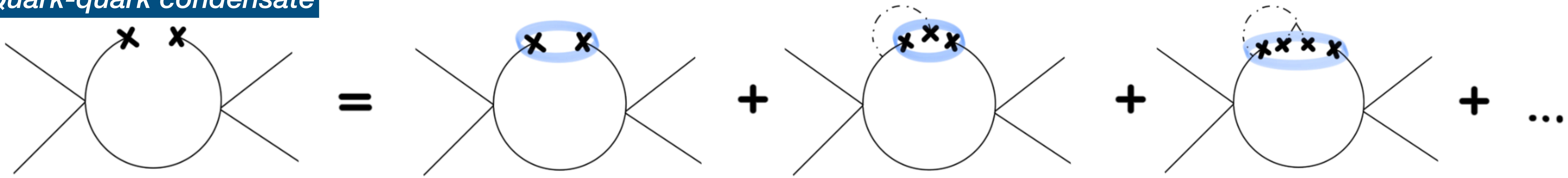
QCD condensates

The relevant contributions in the local expansion

Quark-quark condensate



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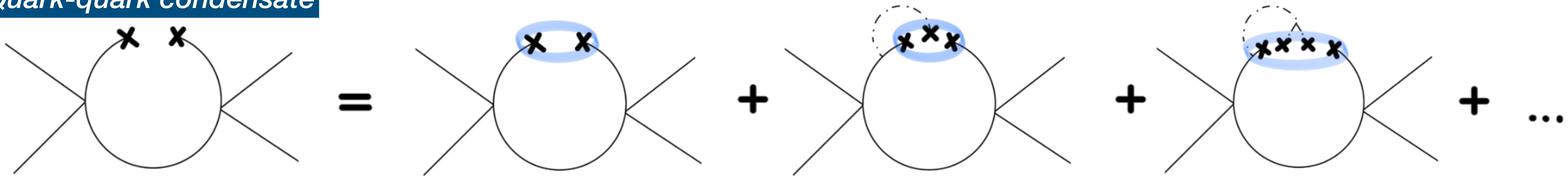
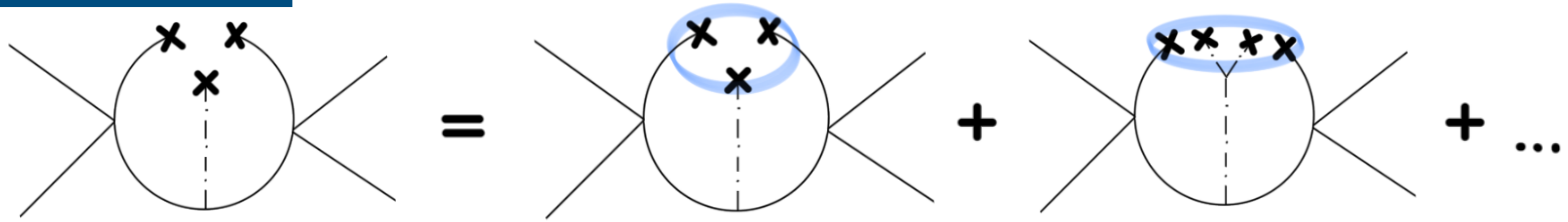


Mixed condensate



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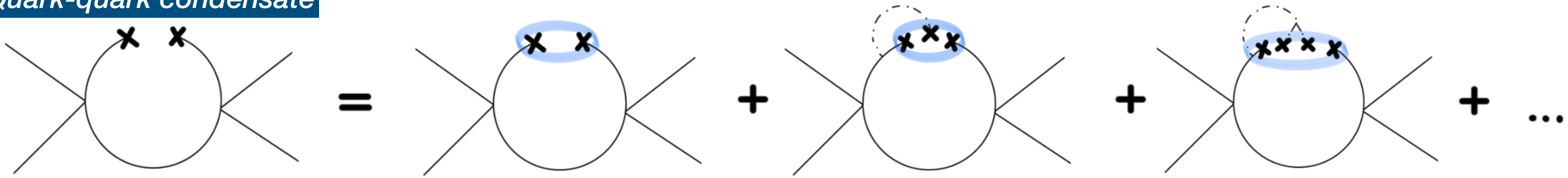
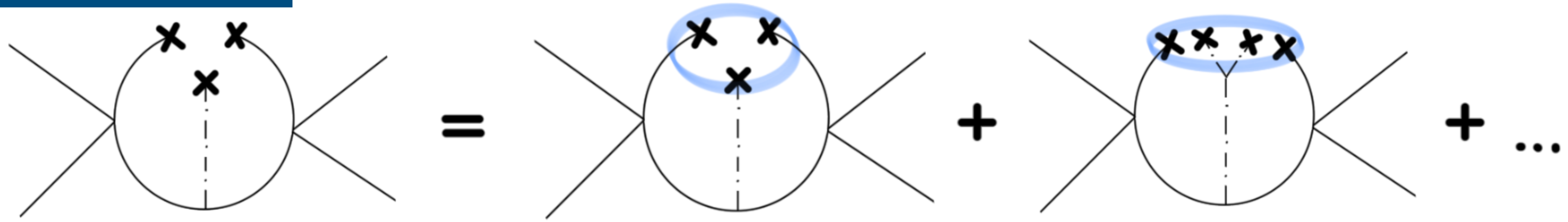
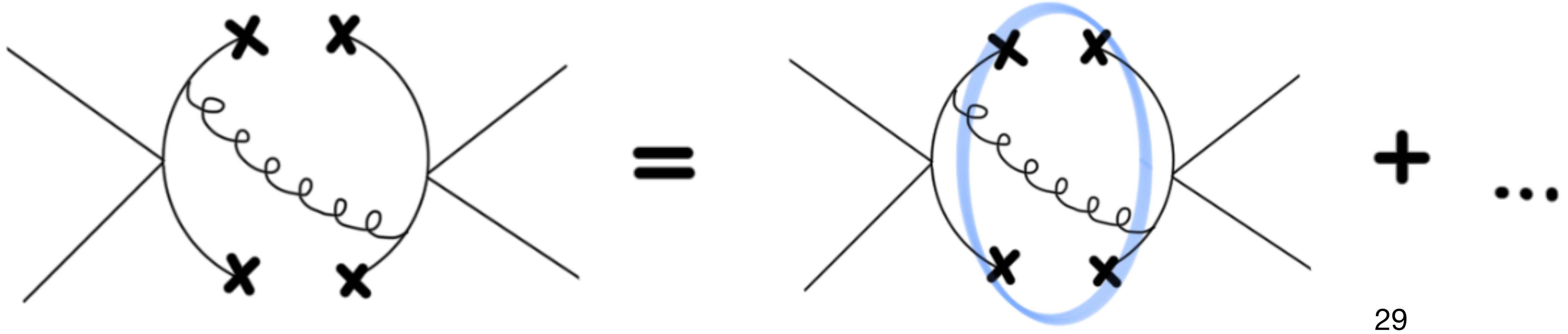


Mixed condensate



Four quark condensate



Nonlocal QCD condensates

Nonlocal generalization

- Assumption - long distances play a crucial role in $D^0\overline{D}^0$ mixing

$$\langle \overline{q}(x)^a_\alpha q(0)^b_\beta \rangle = \frac{\langle \overline{q}q\rangle_0}{4N_C}\delta^{ab}\left[\delta_{\alpha\beta}\left(1-\frac{x^2}{4}\left(\frac{m^2}{2}-\frac{\langle \overline{q}\sigma G q\rangle_0}{2\langle \overline{q}q\rangle_0}\right)\dots\right)+i(\not{x})_{\beta\alpha}\left(\frac{m}{4}-\frac{x^2}{4}\left(\frac{m^3}{12}-\frac{m}{12}\frac{\langle \overline{q}\sigma G q\rangle_0}{\langle \overline{q}q\rangle_0}+\frac{2}{81}\pi\alpha_s^{NP}\frac{\langle \overline{q}q\rangle_0^2}{\langle \overline{q}q\rangle_0}\right)\dots\right)\right]$$

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S. V. Mikhailov and A. V. Radyushkin, *Nonlocal condensates and QCD sum rules for the pion wave function, Phys. Rev. D 45 (Mar, 1992) 1754–1759*

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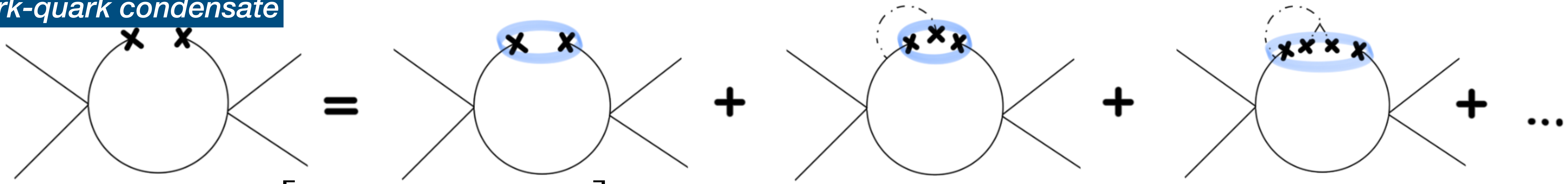
Nonlocal QCD condensates

Nonlocal generalization

Quark-quark condensate



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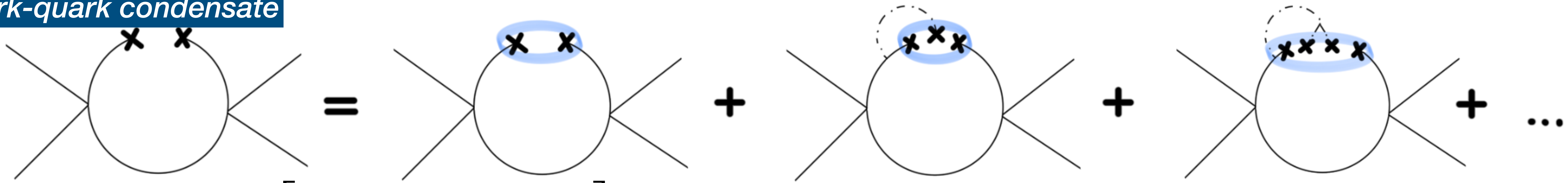

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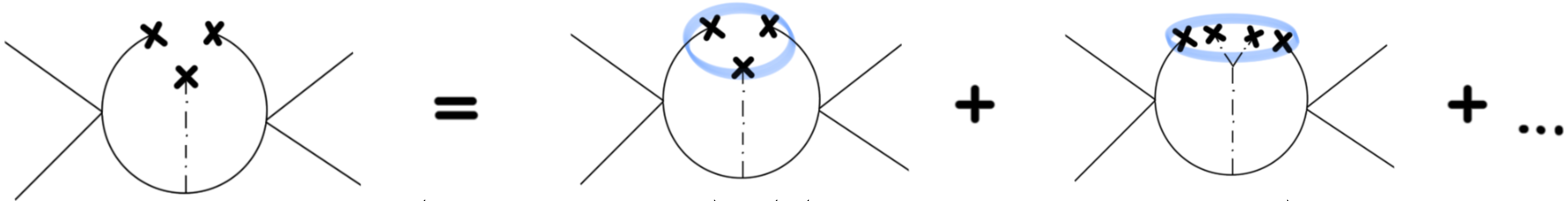

$$\langle \bar{q}^a_\alpha(x) G^{cd}_{\mu\nu}(0) q^b_\beta(0) \rangle = \frac{4}{1536}\left(\delta^{ac}\delta^{bd} - \frac{1}{3}\delta^{ab}\delta^{cd}\right)\left(\left(\sigma_{\mu\nu} + \frac{m}{2}(i\sigma_{\mu\nu}\not{x} + \gamma_\mu x_\nu - \gamma_\nu x_\mu)\right)\langle \bar{q} i\sigma G q\rangle + \frac{i}{2}\not{x}\sigma_{\mu\nu}\frac{16}{9}\pi\alpha_s^{NP}\langle \bar{q}q\rangle^2\right)_{\beta\alpha} F_G(x)$$

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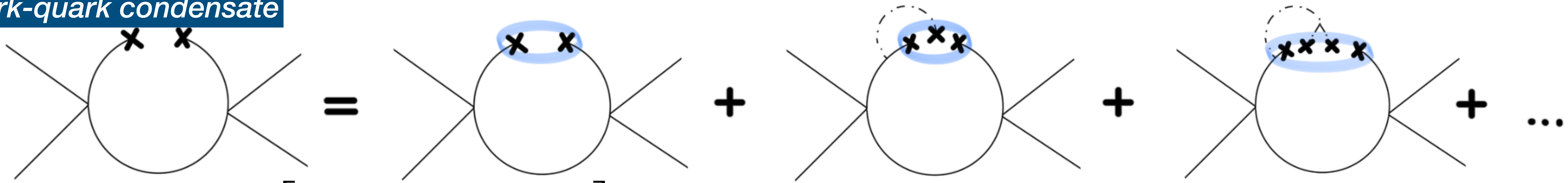

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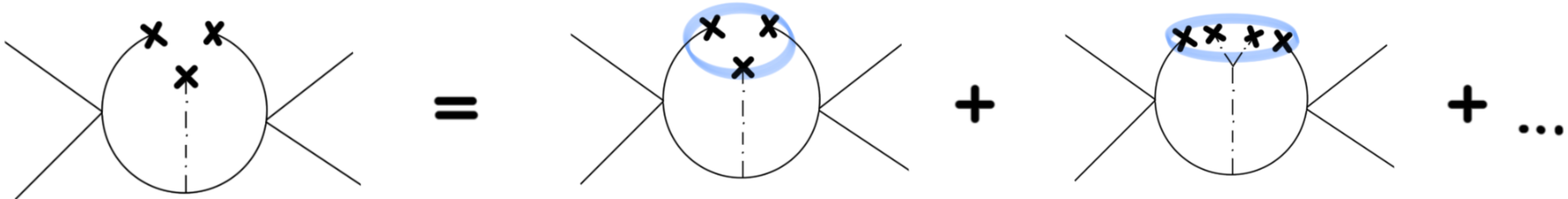

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How is the nonlocality modeled?

- *Condition 1:* must reproduce the expansion in small-x limit
- *Condition 2:* must decay in the large-x limit

$$F_{S,V,G}(x) = \int_0^\infty d\alpha \left(B_{S,V,G} f(\alpha) + A_{S,V,G} f'(\alpha)\right) e^{-\alpha \frac{x^2}{4}}$$

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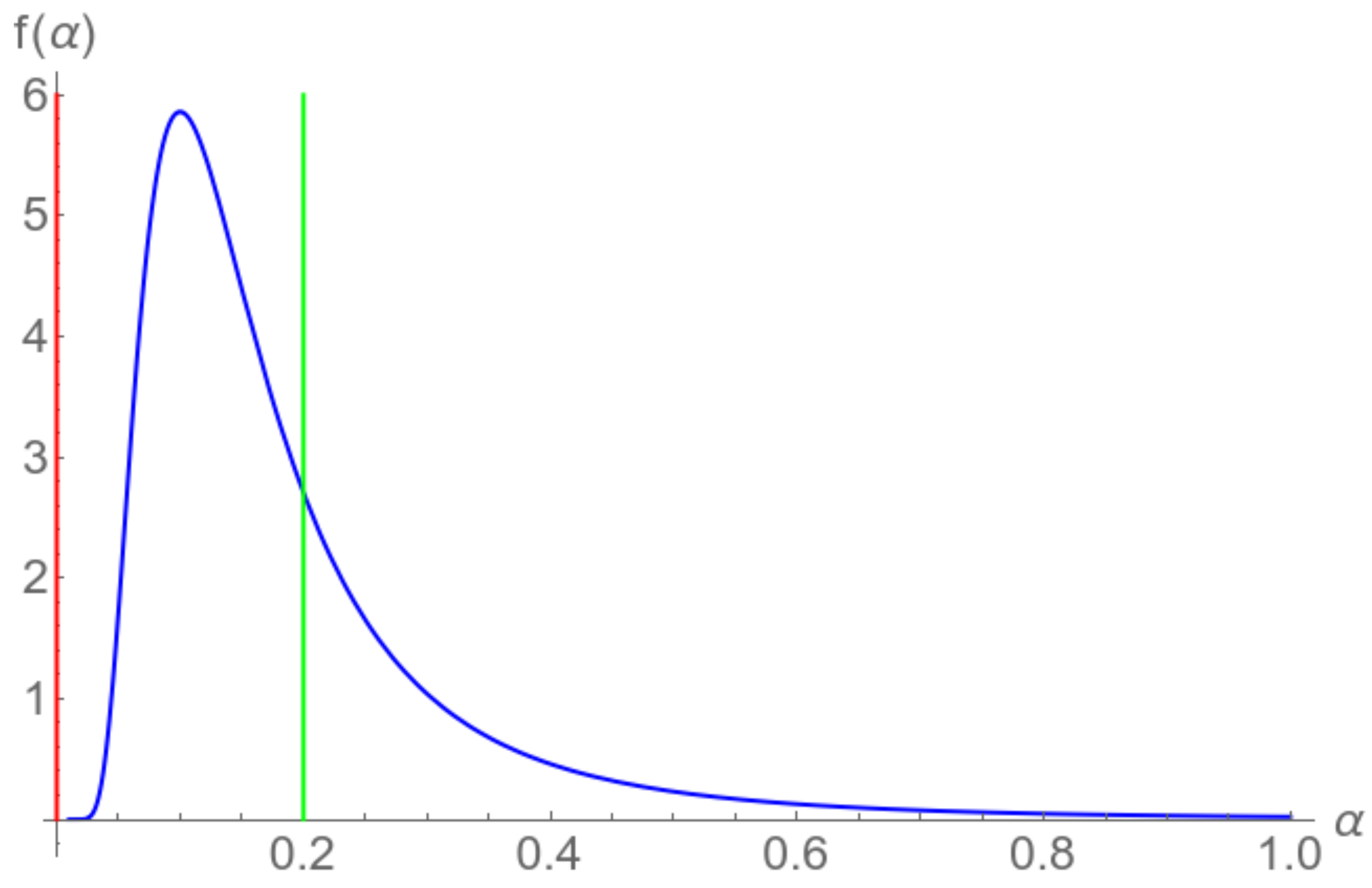

V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, *The B meson distribution amplitude in QCD, Phys. Rev. D 69 (2004) 034014*, [hep-ph/0309330]

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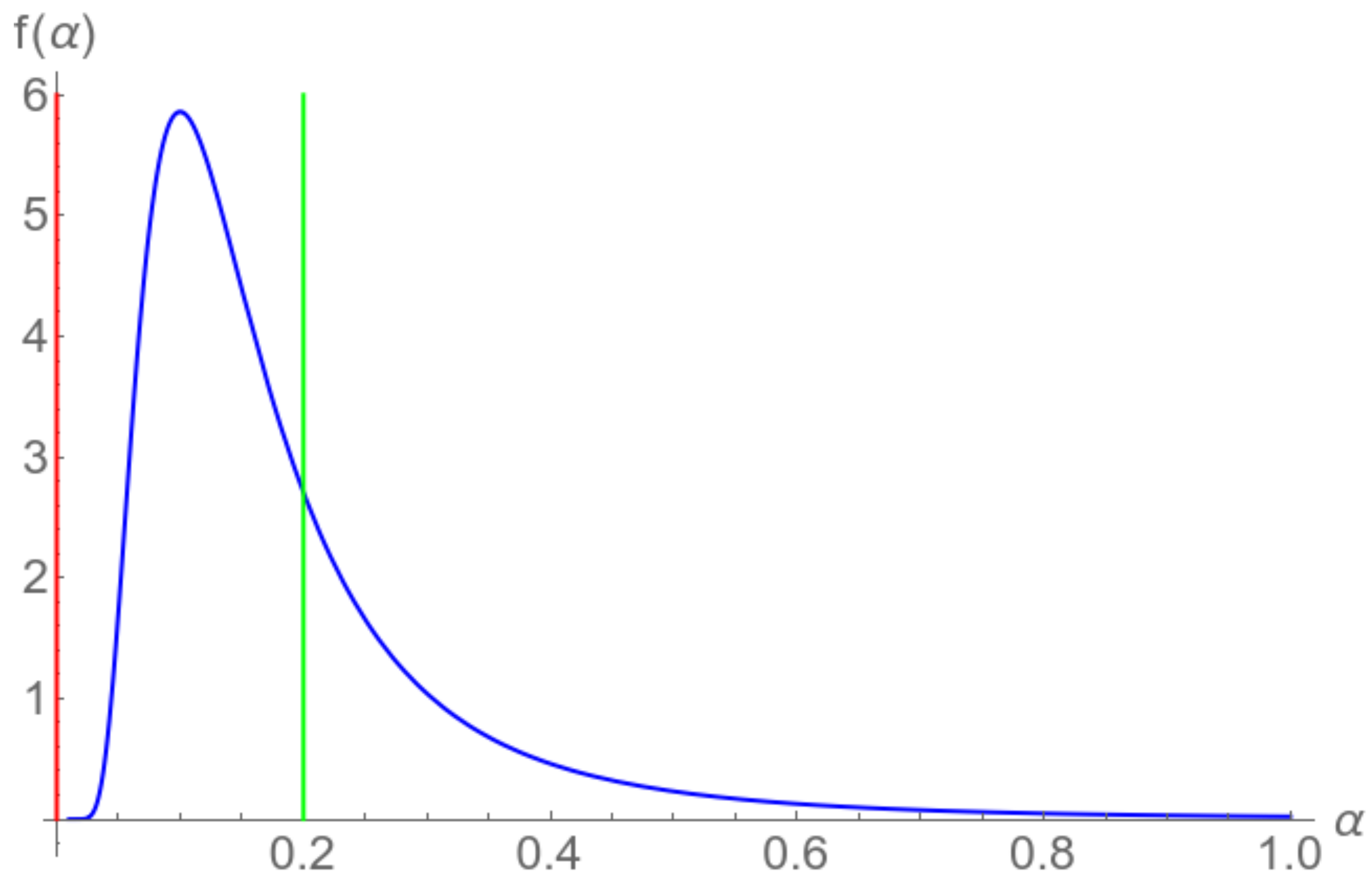

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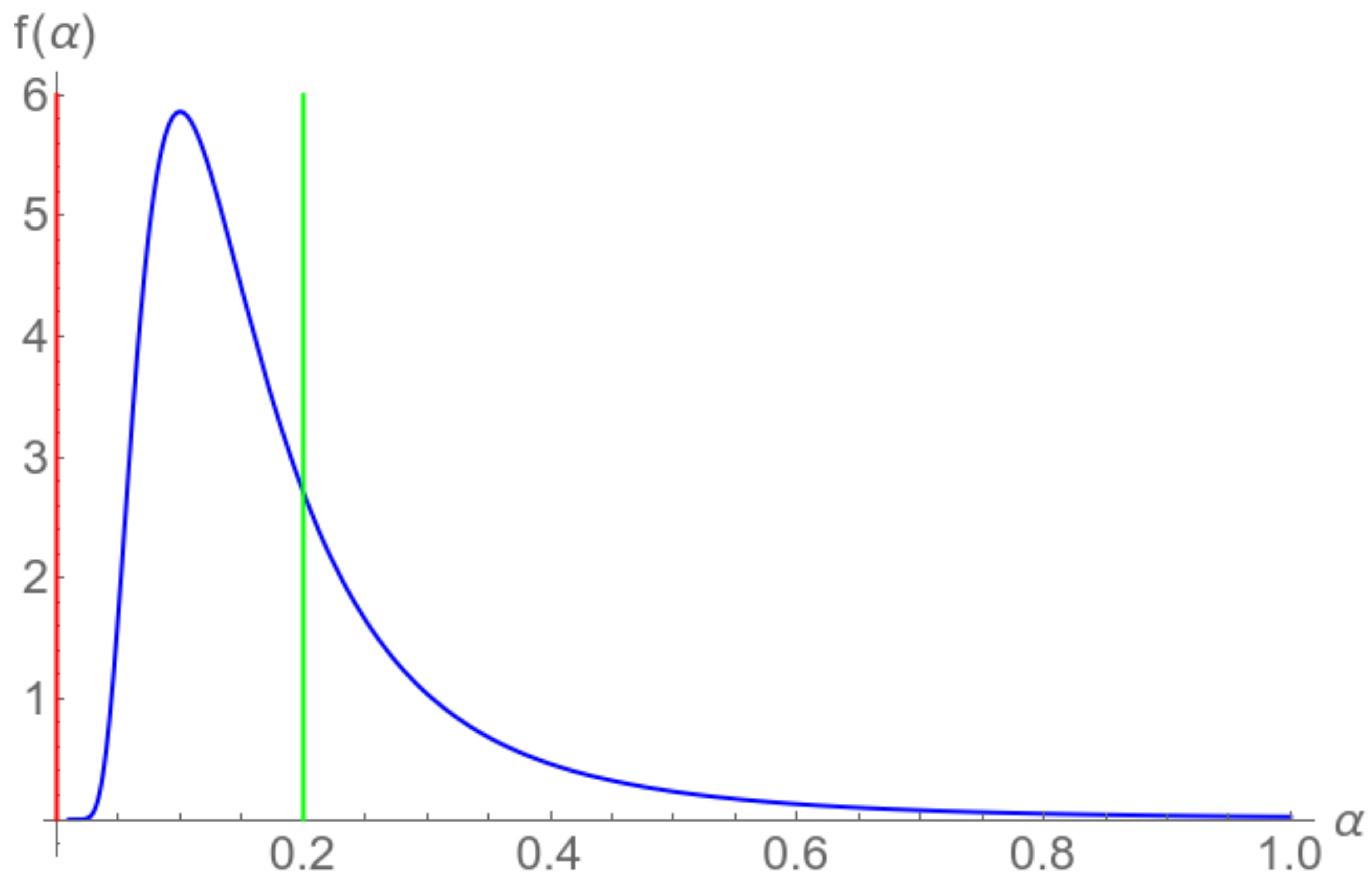
average quark virtuality



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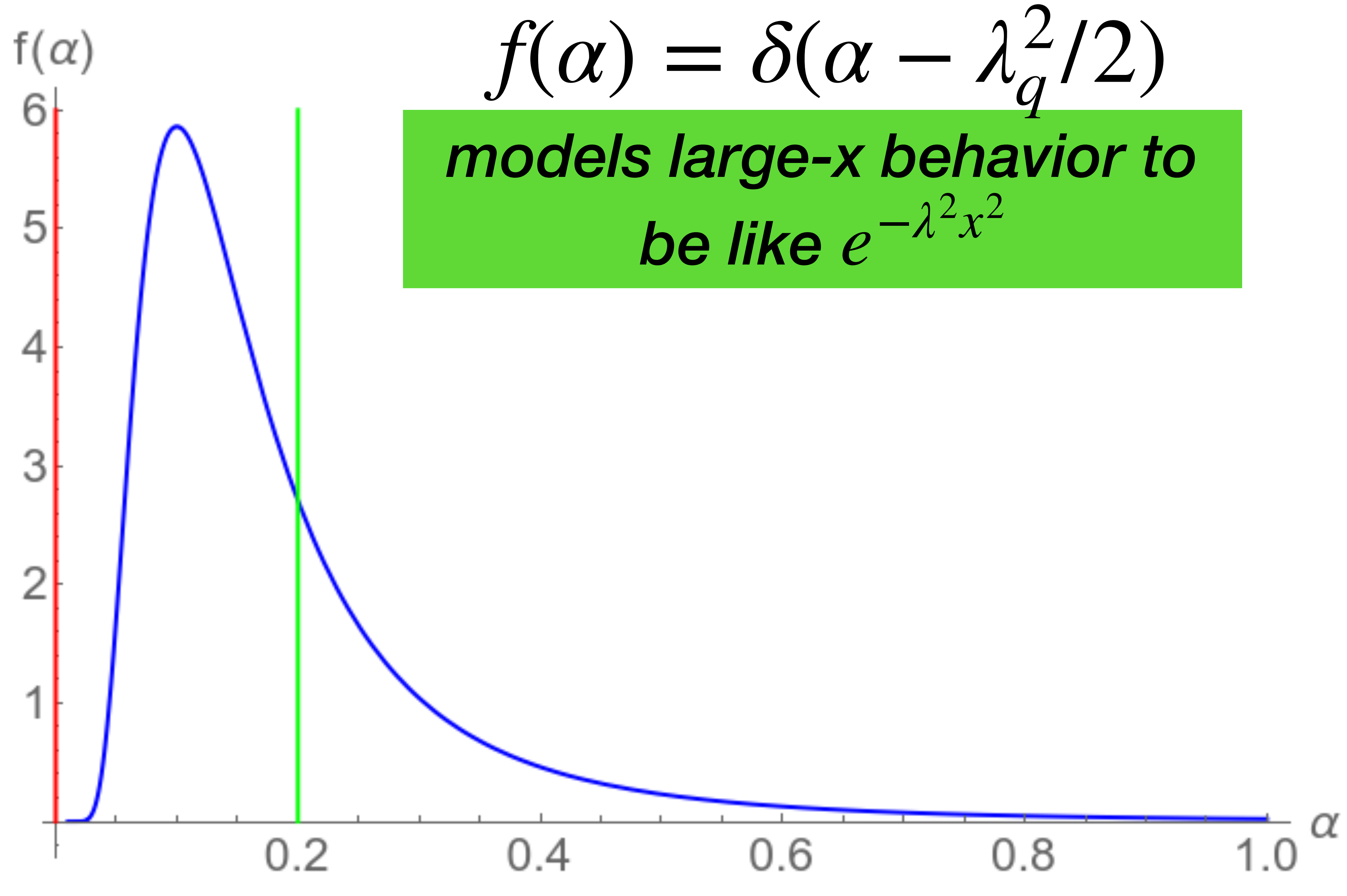

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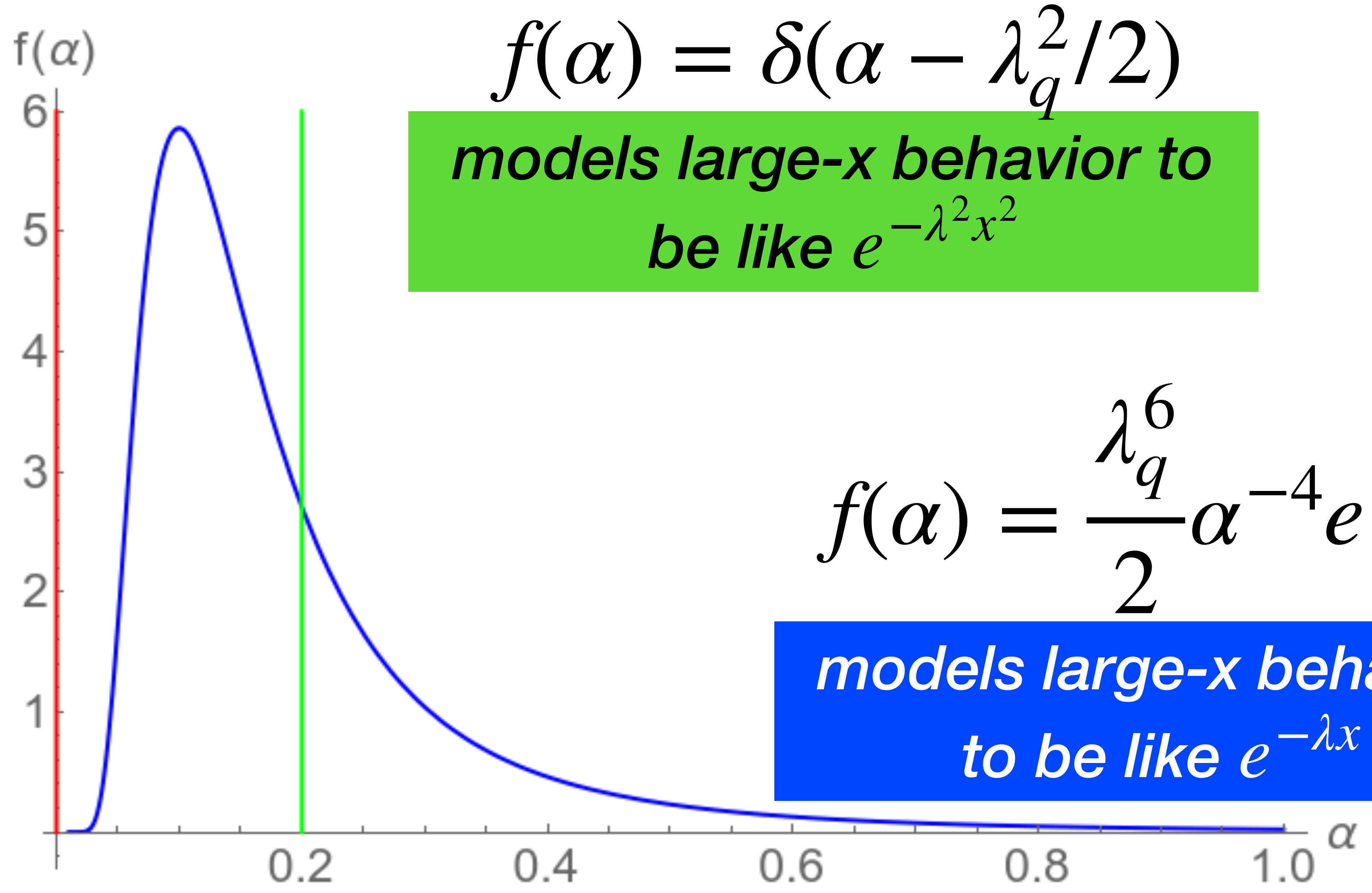

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$$f(\alpha) = \frac{\lambda_q^6}{2}\alpha^{-4} e^{-\lambda_q^2/\alpha}$$

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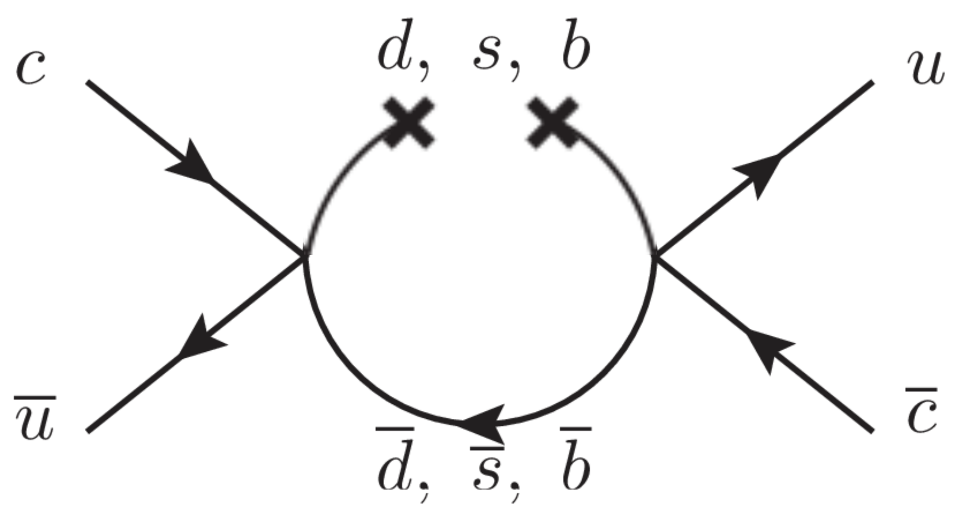
(Preliminary) **Results**

For the mass difference parameter x

expanding in terms of local condensates



$$-4.6 \times 10^{-6}$$

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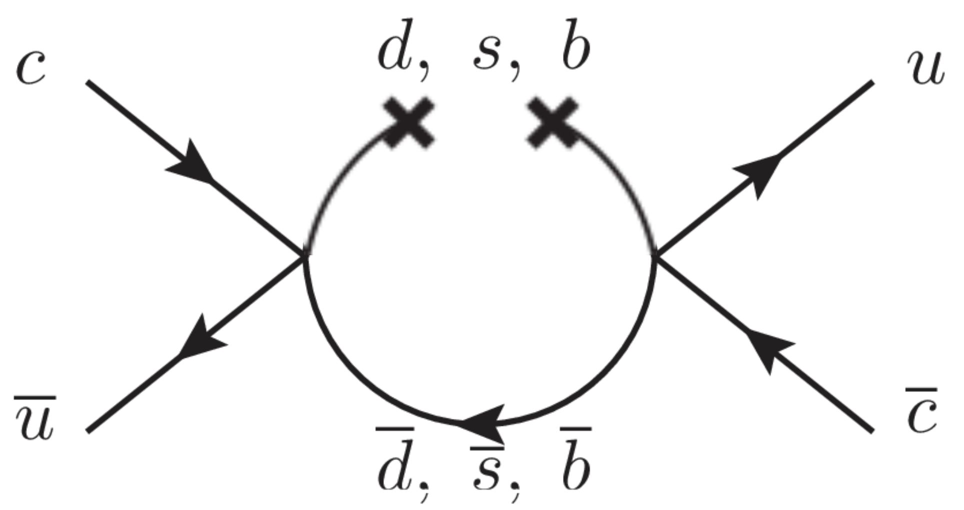	*expanding in terms of local condensates*	*using the simplest nonlocal model*
	-4.6×10^{-6}	-5.8×10^{-6}

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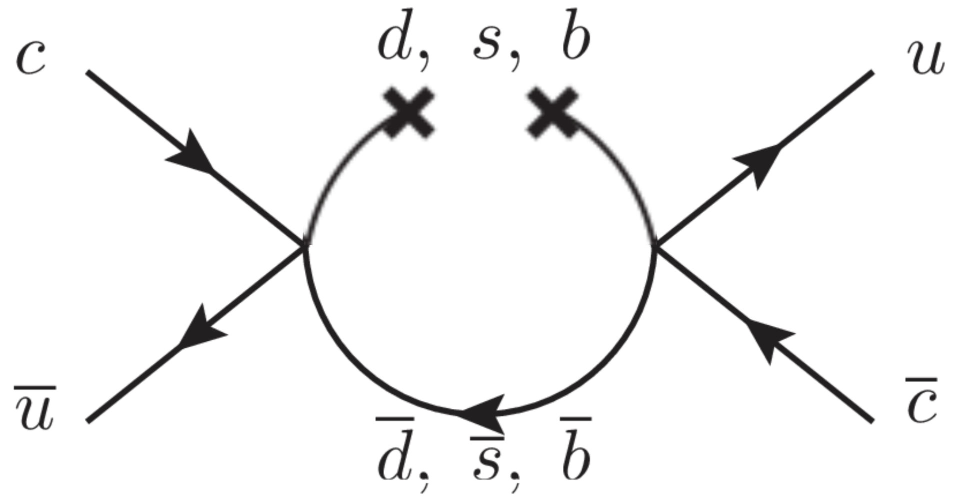
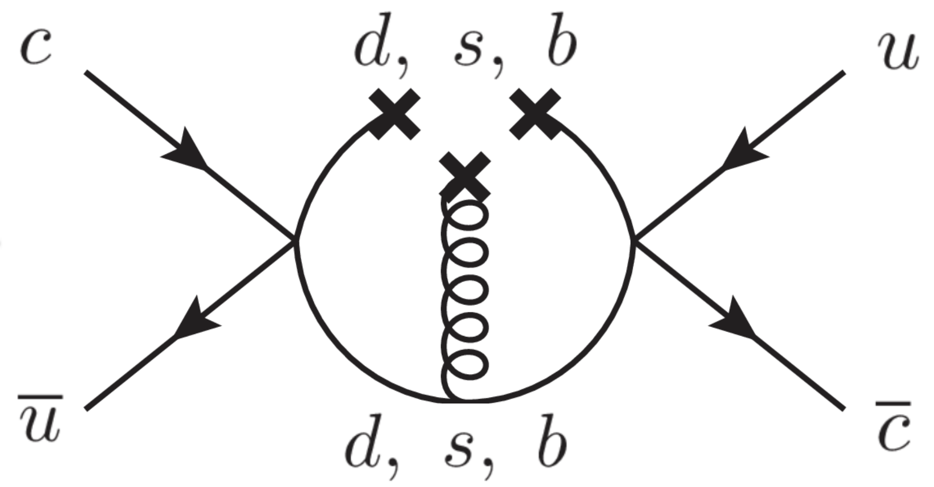
	-4.6×10^{-6}	-5.8×10^{-6}
		-1.9×10^{-6}

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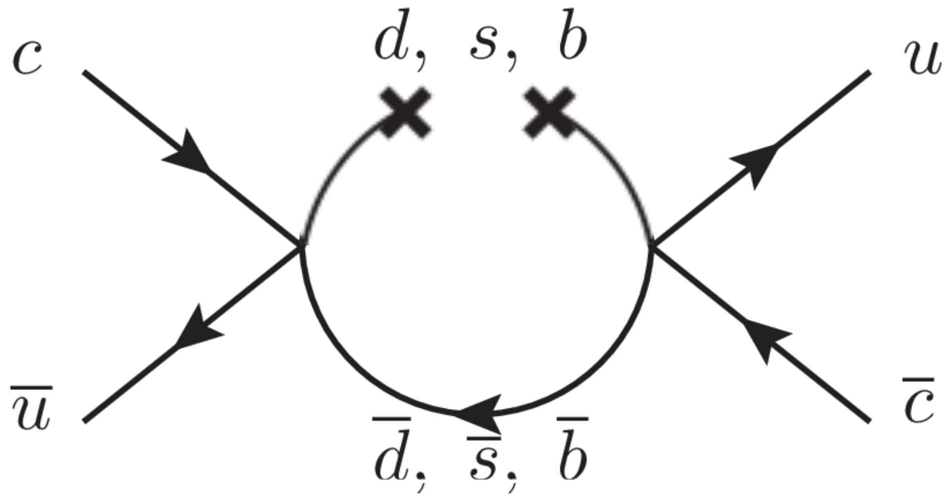
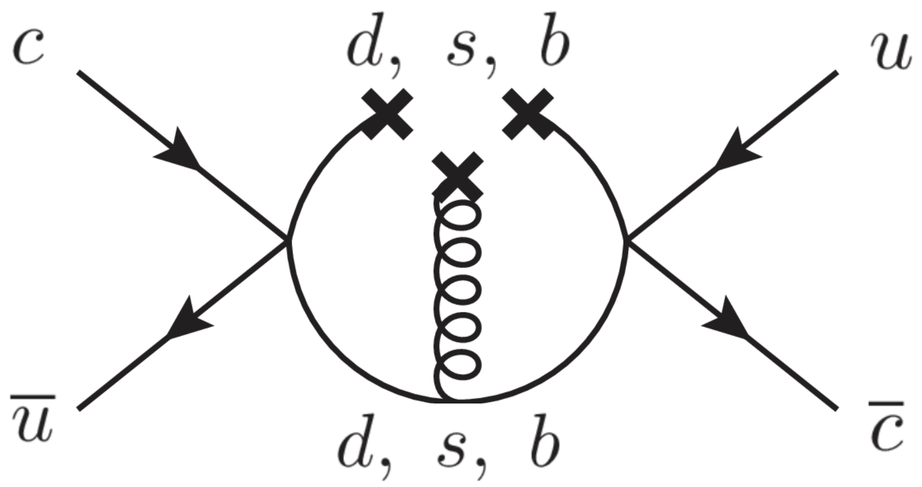
	-4.6×10^{-6}	-5.8×10^{-6}
		-1.9×10^{-6}
		total from nonlocal condensates -7.7×10^{-6}

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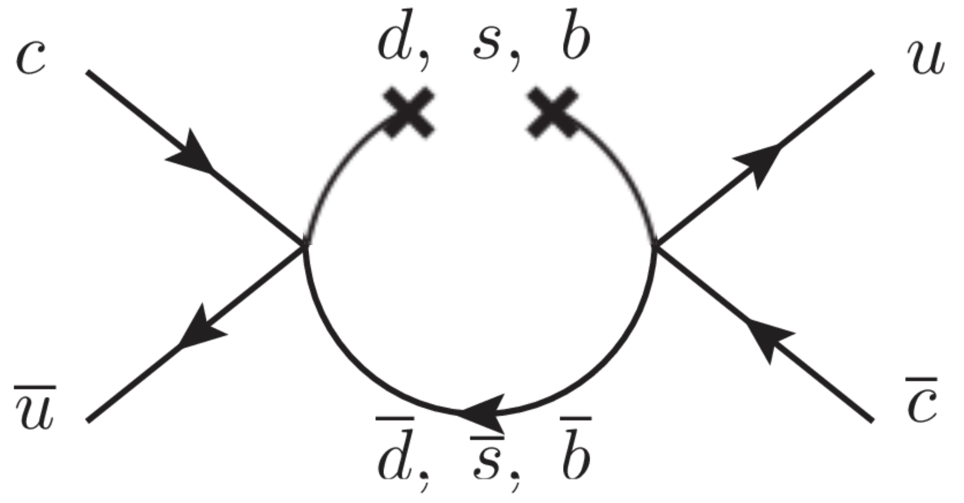

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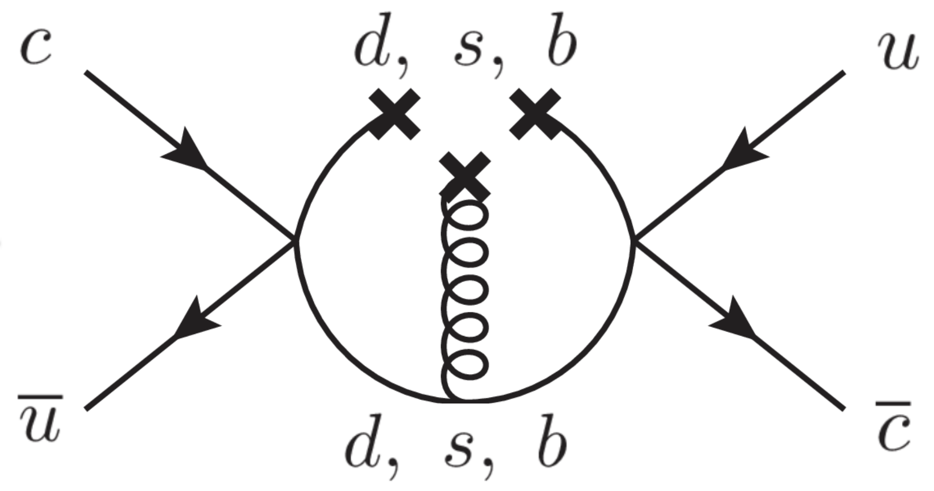

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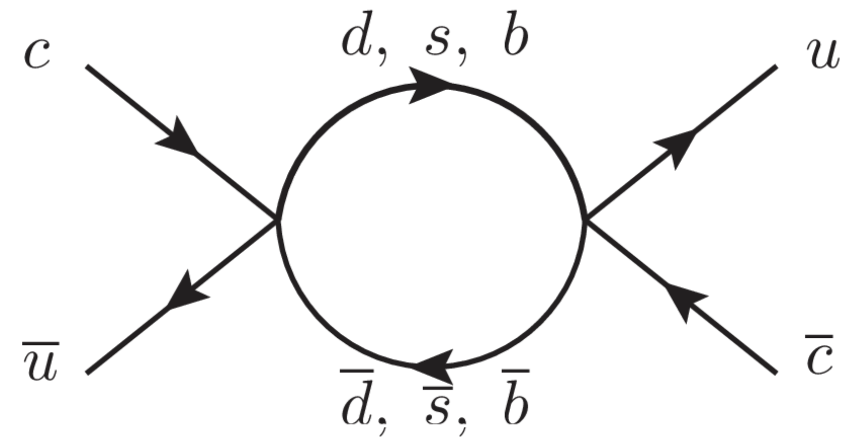

the perturbative contribution

-3.6×10^{-6}

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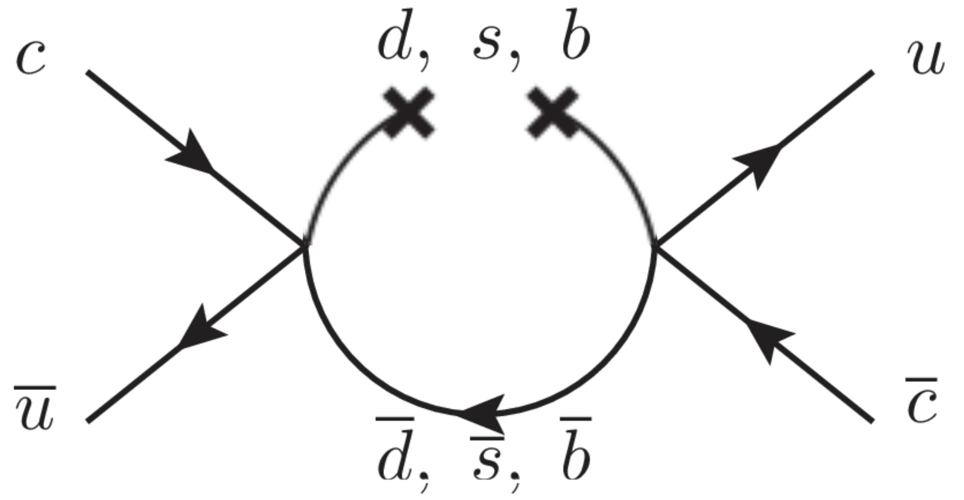
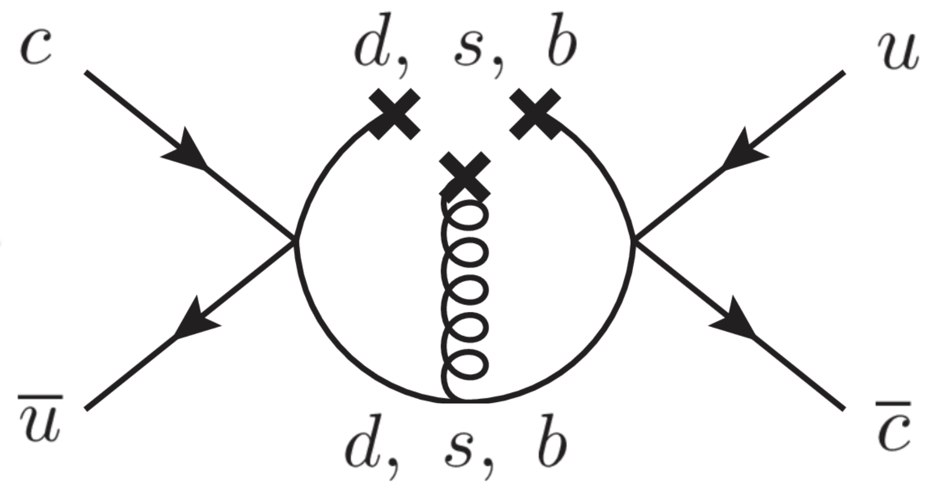
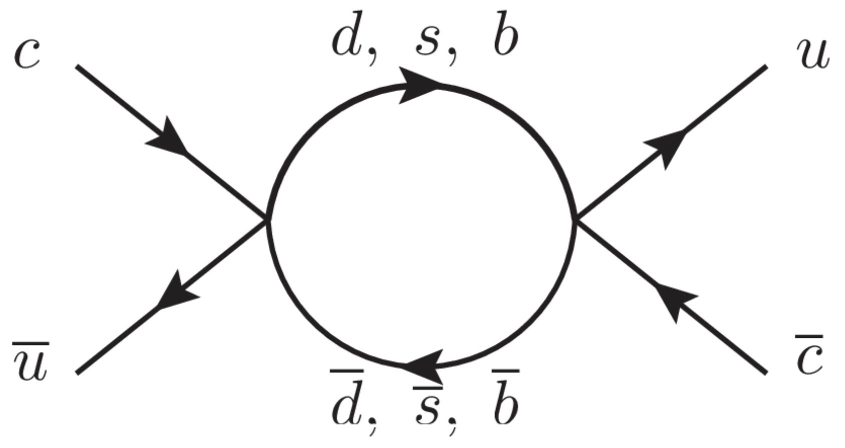
	-4.6×10^{-6}	-5.8×10^{-6}
		-1.9×10^{-6}
	*the perturbative contribution* -3.6×10^{-6}	*total from nonlocal condensates* -7.7×10^{-6}

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FINAL RESULT

Concluding remarks

Future research & possible improvements

- The result is sensitive to
  - **Condensate values**, as well as the ratio $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$.
  - **Quark virtuality** for which the often quoted value is $\sim 0.4\,\mathrm{GeV}^2$ which we used, but some report values as high as $\sim 2.5\,\mathrm{GeV}^2$

P. Gubler and D. Satow, *Recent Progress in QCD Condensate Evaluations and Sum Rules, Prog. Part. Nucl. Phys. 106 (2019) 1–67*, [1812.00385]

A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, *SU(3) breaking and $D0-\bar{D}0$ mixing, Phys. Rev. D 65, 054034,* [hep-ph/0110317]

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- Future research - calculation of the **four-quark condensate** contribution
  - Both propagators in the box diagram are replaced by condensates - expected dependence on strange mass $\propto (m_s/m_c)^2$
  - This is supposed to be the (parametrically) **leading contribution!**

$$x = -1.13 \times 10^{-5}$$

FINAL RESULT

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Thank you!

Lovro Dulibić (*Ruđer Bošković Institute,* Croatia)

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Hrzz Croatian Science Foundation

Alexander von HUMBOLDT STIFTUNG

in collaboration with:
Blaženka Melić (*Ruđer Bošković Institute*, Croatia), Alexey Petrov (*University of South Carolina*, USA)