Semileptonic Charm Decays in the Weak Effective Theory

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Study of $c \to s\ell\nu$ transitions

Motivation

- Extraction of V_{cs}

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Current value by PDG

Leptonic $D_s \to \{\mu, \tau\}\nu$

Semileptonic $D \to K\{e, \mu\}\nu$

$$|V_{cs}| = 0.975 \pm 0.006$$

Study of $c \to s\ell\nu$ transitions

Motivation

- Extraction of V_{cs}

Current value by PDG

Leptonic $D_s \to \{\mu, \tau\}\nu$

HFLAV averages of $\mathscr{B}$ combined with PDG averages for mass, lifetime, decay constant

Semileptonic $D \to K\{e, \mu\}\nu$

HFLAV averages of $|V_{cs}|f_+^K(0)$ combined with FLAG average for form factor

$$|V_{cs}| = 0.975 \pm 0.006$$

Study of $c \to s\ell\nu$ transitions

Motivation

- Extraction of V_{cs} using Bayesian analysis with additional decay channels, with dispersive bounds applied to the full set of theoretical inputs simultaneously

Study of $c \to s\ell\nu$ transitions

Motivation

- Extraction of V_{cs} using Bayesian analysis with additional decay channels, with dispersive bounds applied to the full set of theoretical inputs simultaneously

- How compatible is the current data with what we predict theoretically?
- Is there preference for Standard Model or treatment in full Weak Effective Theory?

Experimental data

- Branching ratio:

$$D_s \to \tau^+\nu_\tau$$

$$D_s \to \mu^+\nu_\mu$$

$$D^0 \to K^-\mu^+\nu_\mu$$

$$D^0 \to K^-e^+\nu_e$$

$$D^+ \to K_S^0 e^+\nu_e$$

$$D^+ \to K_S^0\mu^+\nu_\mu$$

Experimental data

- Branching ratio:

$D_s \to \tau^+\nu_\tau$

$D_s \to \mu^+\nu_\mu$

$D_s^* \to e^+\nu_e$

$D^0 \to K^-\mu^+\nu_\mu$

$D^0 \to K^-e^+\nu_e$

$D^+ \to K_S^0 e^+\nu_e$

$D^+ \to K_S^0 \mu^+\nu_\mu$

$\Lambda_c^+ \to \Lambda^0\mu^+\nu_\mu$

$\Lambda_c^+ \to \Lambda^0 e^+\nu_e$

- Shape distribution:

$D^0 \to K^-\mu^+\nu_\mu$

$D^0 \to K^-e^+\nu_e$

$D^+ \to K_S^0 e^+\nu_e$

Updated measurements here

New!
not included in PDG value

Theory inputs

- Decay constants f_{D_s} , $f_{D_s^*}$ and $f^T_{D_s^*}$ → $\left.\frac{f_{D_s^*,T}}{f_{D_s}}\right|_{\text{QCDSR}}$
  - ETM, FNAL/MILC
  - CLQCD

FLAG Review 2021
ETM, Phys.Rev.D 91 (2015)
FNAL/MILC, Phys.Rev.D 98 (2018)
CLQCD, Phys.Rev.D 109 (2024)
Pullin, Zwicky, JHEP 09 (2021) 023

- $D \to K$ form factors → HPQCD, FNAL/MILC, ~~ETM~~

HPQCD, Phys.Rev.D 107 (2023)
FNAL/MILC, Phys.Rev.D 107 (2023)
ETM, Phys.Rev.D 96 (2017)
ETM, Phys.Rev.D 98 (2018)

- $\Lambda_c \to \Lambda$ form factors
  - → (Axial)vector and (pseudo)scalar: LQCD
  - → Tensor: HQET + SCET relations to (axial)vector FF's

Meinel, Phys.Rev.Lett. 118 (2017)

Our work

Combining theory inputs

Dispersive bounds

- Ensures unitarity, correlates most hadronic parameters

  Caprini, *Functional Analysis and Optimization Methods in Hadron Physics*

- Dispersion relations ⇒ perturbatively calculated quantities χ

- Hadronic representation of correlators ⇒ $\chi_A^{(J=0)}\big|_{1\text{pt}} = \dfrac{M_{D_s}^2 f_{D_s}^2}{(M_{D_s}^2 - Q^2)^2}$

- BGL-like parametrisation of FF: $f(q^2) = \dfrac{1}{\phi_f(z)B(z)} \sum_{k=0}^{K} a_k^{(f)} p_k^{(f)}(z)\Big|_{z=z(q^2)}$ $\quad \sum_f \sum_{k=0}^{K} |a_k^{(f)}|^2 < 1$

Further discussion on form factor approach: Gubernari, (Reboud), van Dyk, Virto 2021 & 2022; Blake et al. 2022; Flynn et al. 2023

Theory inputs

$D \to K$ form factors

- HPQCD + FNAL/MILC are incompatible with ETM determination

HPQCD + FNAL/MILC: p-value = 4%

HPQCD + FNAL/MILC + ETM: p-value < 0.1%

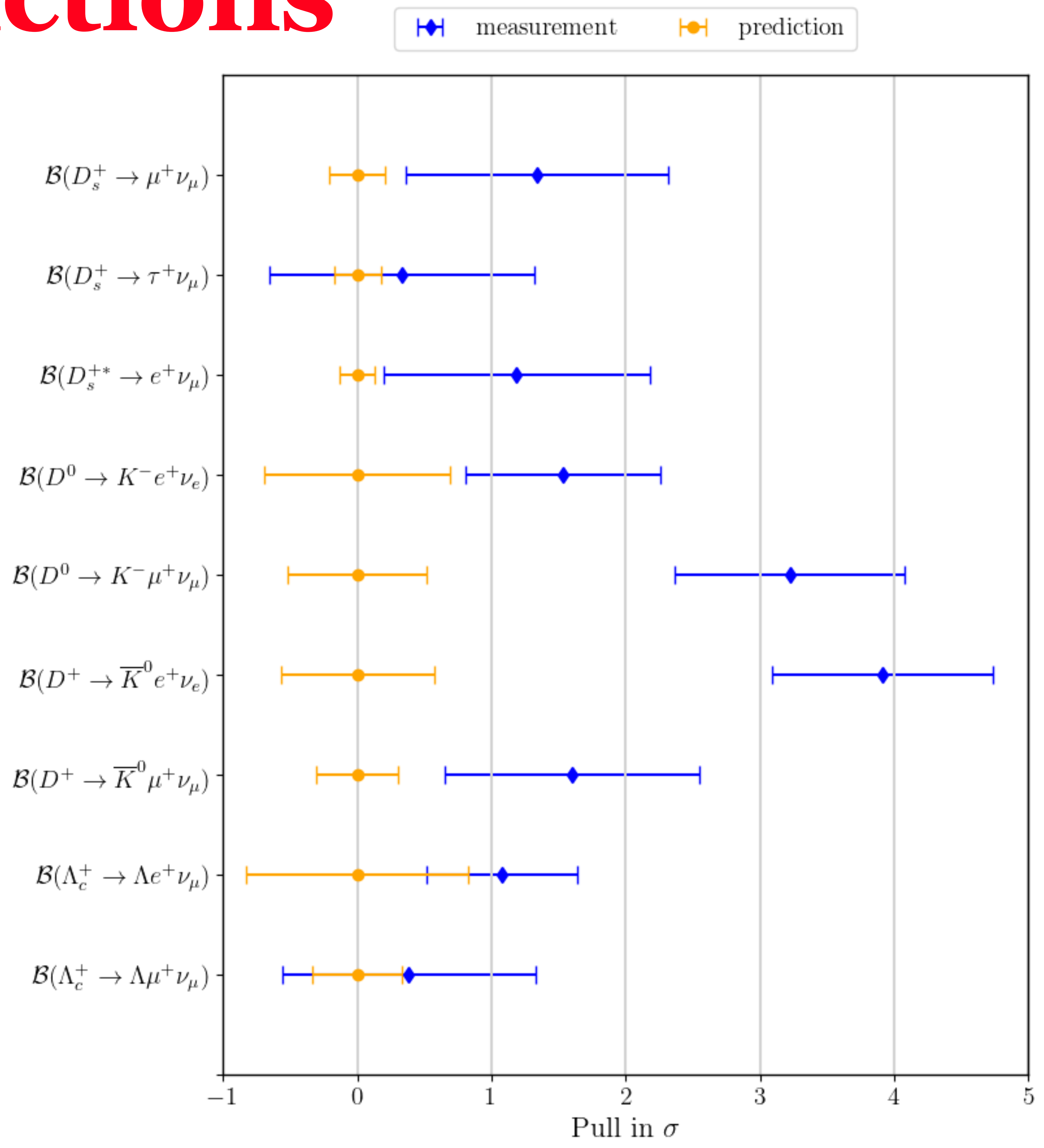
Theory inputs

$D \to K$ form factors

- HPQCD + FNAL/MILC are more compatible with data

Theory predictions

- Identified local inconsistencies between measured data and theoretical predictions
- Outliers in our fits



- Shape distributions are well fitted

Analyses

- SM
- CKM
- WET

Bayesian model comparison between all three models

Same set of hadronic nuisance parameters

Same experimental likelihood

Analyses

- SM
- CKM
- WET

Checks compatibility of data and SM

No parameter of interest in fit

Fixed value of $|V_{cs}| = 0.975$

Fixed the only Wilson coefficient to SM value

Analyses

- SM
- CKM
- WET

Joint fit to all contributions as well as
for the different decay modes individually

One parameter of interest in fit
$|V_{cs}|$ in [0.88 , 1.03]

Fixed the only Wilson coefficient to SM value

Analyses

- SM
- CKM
- WET

Weak Effective Theory allows for BSM physics

Fixed scale $|V_{cs}| = 0.975$

Fit for the parameters of 5 different Wilson Coefficients

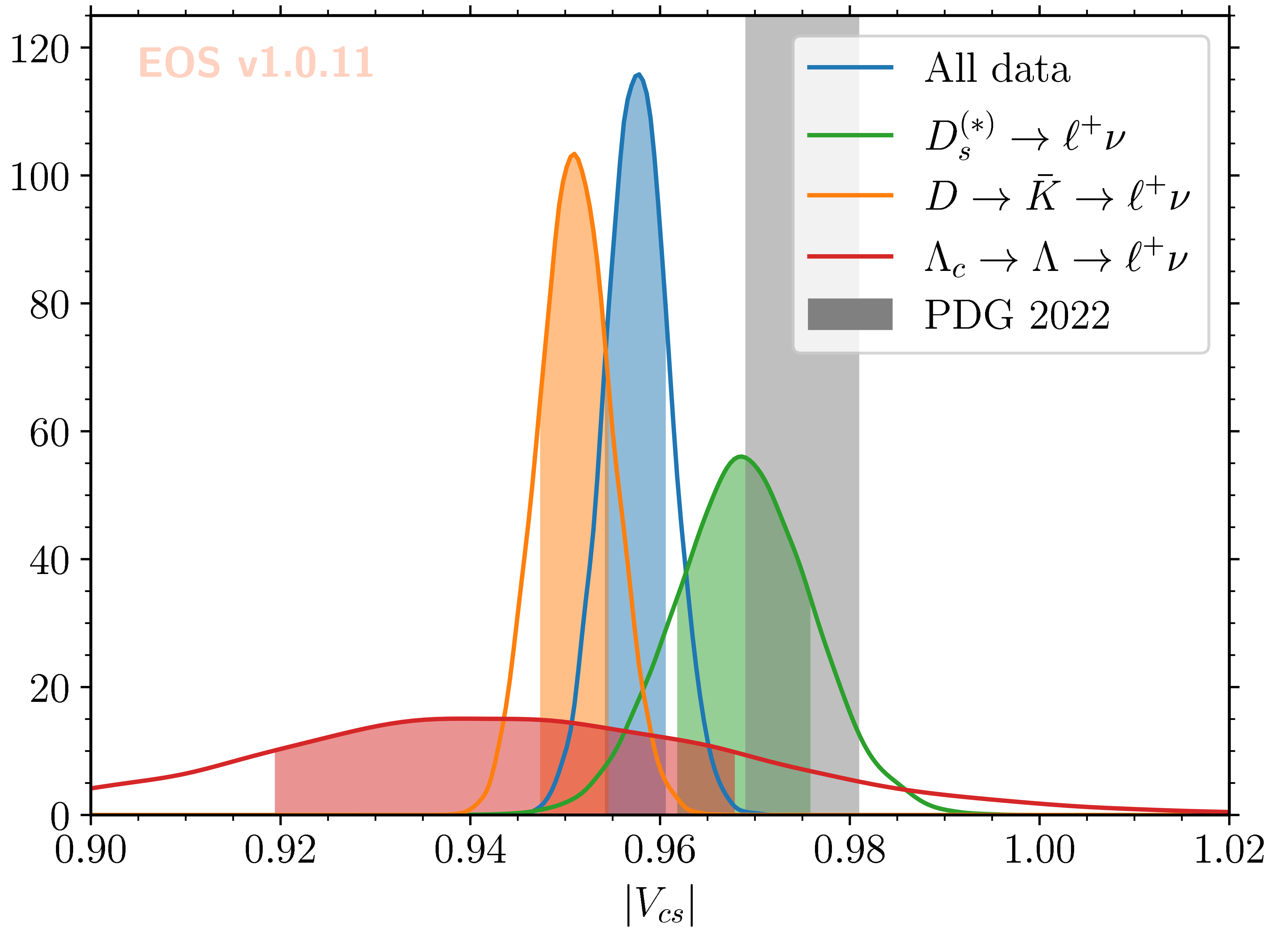
Preliminary

CKM fit

Extraction of V_{cs}

Data set	χ^2	d.o.f.	p value [%]	$\|V_{cs}\|$
	Goodness of fit			
$D_s^{(*)+} \to \ell^+\nu$	2.5	2	28.1	$0.969^{+0.007}_{-0.007}$
$D \to \bar{K}\ell\nu$	44.2	45	50.6	$0.953^{+0.004}_{-0.004}$
$\Lambda_c \to \Lambda\ell\nu$	0.3	1	58.4	$0.947^{+0.027}_{-0.026}$
joint fit	51.0	50	43.4	$0.958^{+0.003}_{-0.003}$

- Compatible with PDG at 2.5σ



CKM fit

Unitarity

- Checking unitarity in the second column of the CKM matrix

$$\left|V_{us}\right|^{\text{world avg.}} = 0.2243 \pm 0.0008 \qquad \left|V_{ts}\right|^{\text{world avg.}} = (41.5 \pm 0.9) \times 10^{-3}$$

$$\left|V_{cs}\right|^{\text{our result}} = 0.958 \pm 0.003$$

$$\sum_{U=u,c,t} \left|V_{Us}\right|^2 \simeq 0.9698$$

Assuming perfect positive correlation between determinations $\Rightarrow 4.8\sigma$ deviation from unitarity!

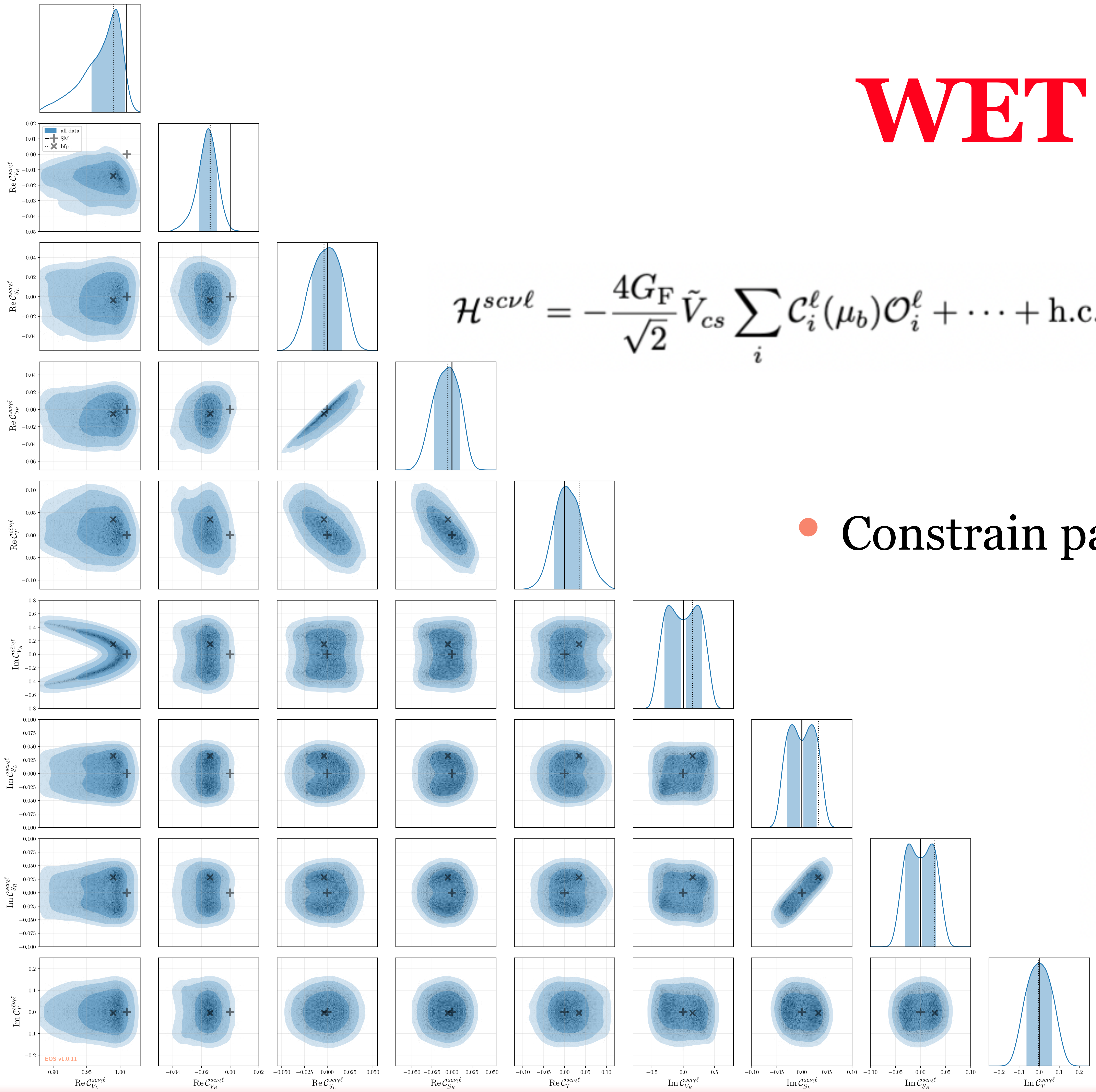
WET fit

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_{\rm F}}{\sqrt{2}}\tilde{V}_{cs}\sum_i \mathcal{C}_i^\ell(\mu_b)\mathcal{O}_i^\ell + \cdots + \text{h.c.}\,.$$

$$\begin{aligned}
\mathcal{O}^\ell_{V,L} &= [\bar{s}\gamma^\mu P_L c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,, & \mathcal{O}^\ell_{V,R} &= [\bar{s}\gamma^\mu P_R c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,,\\
\mathcal{O}^\ell_{S,L} &= [\bar{s}P_L c]\,[\bar{\nu}P_L \ell]\,, & \mathcal{O}^\ell_{S,R} &= [\bar{s}P_R c]\,[\bar{\nu}P_L \ell]\,,\\
\mathcal{O}^\ell_{T} &= [\bar{s}\sigma^{\mu\nu} b]\,[\bar{\nu}\sigma_{\mu\nu} P_L \ell]\,.
\end{aligned}$$

- Constrain parameter space for Wilson Coefficients

WET fit



$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_F}{\sqrt{2}}\tilde{V}_{cs}\sum_i \mathcal{C}_i^\ell(\mu_b)\mathcal{O}_i^\ell + \cdots + \text{h.c.}\,.$$

$$\mathcal{O}_{V,L}^\ell = [\bar{s}\gamma^\mu P_L c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,,\quad \mathcal{O}_{V,R}^\ell = [\bar{s}\gamma^\mu P_R c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,,$$
$$\mathcal{O}_{S,L}^\ell = [\bar{s}P_L c]\,[\bar{\nu}P_L \ell]\,,\quad \mathcal{O}_{S,R}^\ell = [\bar{s}P_R c]\,[\bar{\nu}P_L \ell]\,,$$
$$\mathcal{O}_T^\ell = [\bar{s}\sigma^{\mu\nu}b]\,[\bar{\nu}\sigma_{\mu\nu}P_L \ell]\,.$$

- Constrain parameter space for Wilson Coefficients

$$\text{Re}\,\mathcal{C}_{V,L}^\ell = [\;0.941,\;0.998]\,,$$
$$\text{Re}\,\mathcal{C}_{V,R}^\ell = [-0.023,-0.009]\,,\quad \text{Im}\,\mathcal{C}_{V,R}^\ell = [-0.277,0.277]\,,$$
$$\text{Re}\,\mathcal{C}_{S,L}^\ell = [-0.018,\;0.015]\,,\quad \text{Im}\,\mathcal{C}_{S,L}^\ell = [-0.028,0.028]\,,$$
$$\text{Re}\,\mathcal{C}_{S,R}^\ell = [-0.024,\;0.009]\,,\quad \text{Im}\,\mathcal{C}_{S,R}^\ell = [-0.029,0.029]\,,$$
$$\text{Re}\,\mathcal{C}_{T}^\ell = [-0.023,\;0.045]\,,\quad \text{Im}\,\mathcal{C}_{T}^\ell = [-0.065,0.065]\,.$$

Results

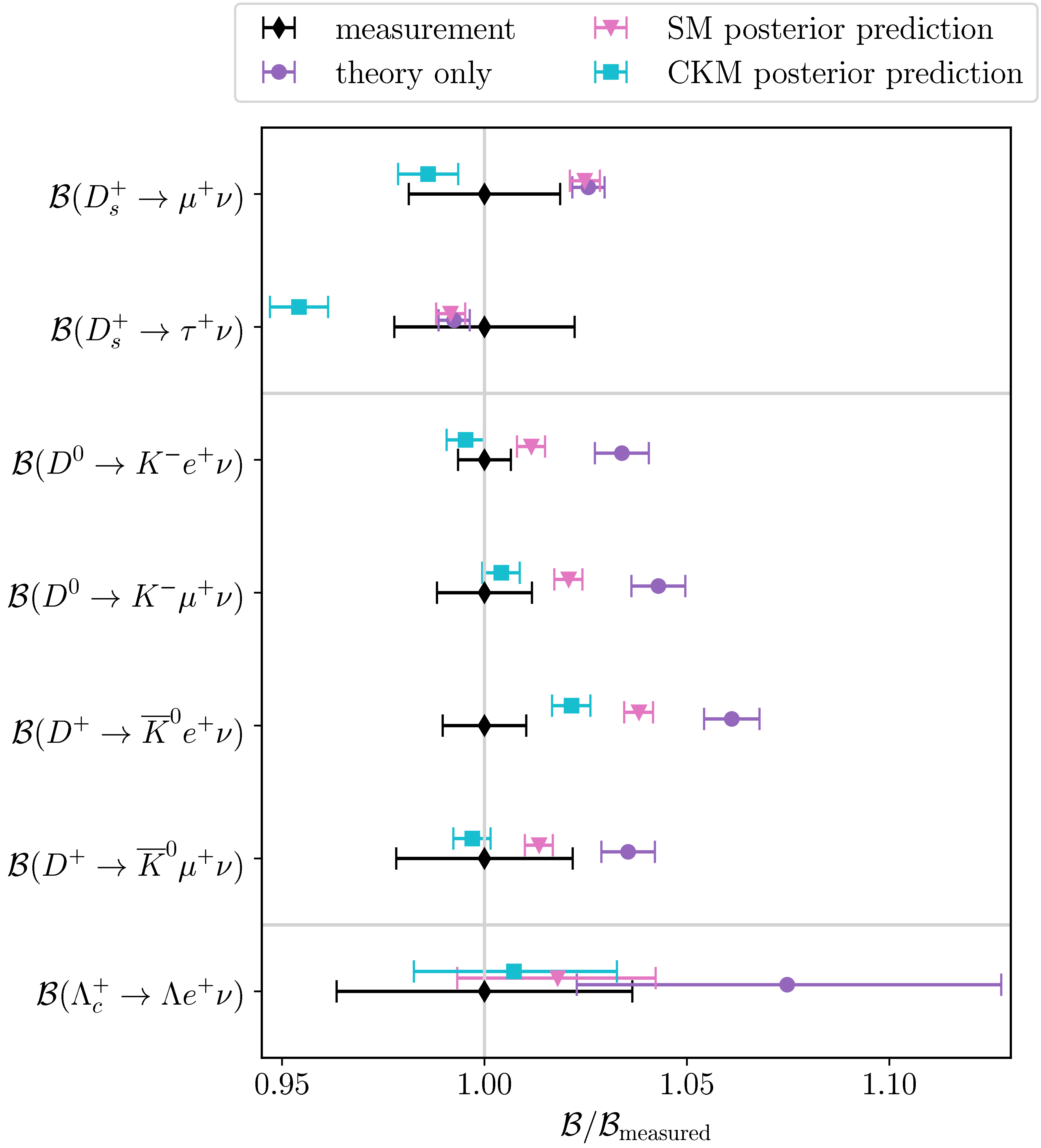
fit model M	χ^2	d.o.f.	p value [%]	$\ln P(D, M)$
	goodness of fit			
SM	61.2	51	15.5	239.1 ± 0.4
CKM	52.1	50	39.2	251.4 ± 0.4
WET	47.2	42	26.8	251.0 ± 0.4

$$\frac{P(\text{all data}\,|\,\text{WET})}{P(\text{all data}\,|\,\text{SM})} = 147267 \qquad \frac{P(\text{all data}\,|\,\text{WET})}{P(\text{all data}\,|\,\text{CKM})} = 0.7$$

CKM corresponds to barely worth mentioning improvement wrt. WET

Integral over much larger parameter space for WET provides basically same efficiency in describing the data as for CKM

Cannot distinguish between the two



Conclusions

- Analysed compatibility between current $c \to s\ell\nu$ data and theoretical predictions

- Extracted new determination of the CKM element

$$|V_{cs}| = 0.958 \pm 0.003$$

- Investigated preference for WET treatment $\Rightarrow$ cannot distinguish from CKM fit
  - ★ Placed new constraints on parameter space for Wilson coefficients
  - ★ Data on angular distribution on $\Lambda_c \to \Lambda\ell\nu$ decays may resolve model preference

BACK-UP

Hadronic matrix elements

$$\langle 0|\, \bar{s}\gamma^\mu\gamma_5 c\,|D_s^+(p)\rangle = i f_{D_s} p^\mu\,, \quad \langle 0|\, \bar{s}\gamma_5 c\,|D_s^+(p)\rangle = -i\frac{M_{D_s}^2}{m_c(\mu_c)+m_s(\mu_c)} f_{D_s}\,.$$

$$\langle 0|\, \bar{s}\gamma^\mu c\,|D_s^+(p,\varepsilon)\rangle = f_{D_s^*} M_{D_s}\varepsilon^\mu\,, \quad \langle 0|\, \bar{s}\sigma^{\mu\nu} c\,|D_s^{*+}(p,\varepsilon)\rangle = i f_{D_s^*}^T\,(\varepsilon^\mu p^\nu - p^\mu\varepsilon^\nu)\,.$$

$$\begin{aligned}
\langle K(k)|\, \bar{s}\gamma^\mu c\,|D(p)\rangle &= f_+^{D\to K}(q^2)\left[(p+k)^\mu - q^\mu\frac{M_D^2-M_K^2}{q^2}\right] + f_0^{D\to K}(q^2) q^\mu\frac{M_D^2-M_K^2}{q^2}\,,\\
\langle K(k)|\, \bar{s}c\,|D(p)\rangle &= f_0^{D\to K}(q^2)\frac{M_D^2-M_K^2}{m_c(\mu_c)-m_s(\mu_c)}\,,\\
\langle K(k)|\, \bar{s}\sigma^{\mu\nu}q_\nu c\,|D(p)\rangle &= \frac{i f_T^{D\to K}(q^2)}{M_D+M_K}\left[q^2(p+k)^\mu - (M_D^2-M_K^2)q^\mu\right]\,.
\end{aligned}$$

Hadronic matrix elements

$$\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,\gamma^\mu\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle = \bar{u}_\Lambda(k,s_\Lambda)\bigg[f_{V,t}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}-m_\Lambda)\frac{q^\mu}{q^2}$$
$$+ f_{V,0}^{\Lambda_c\to\Lambda}(q^2)\frac{m_{\Lambda_c}+m_\Lambda}{s_+}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)$$
$$+ f_{V,\perp}^{\Lambda_c\to\Lambda}(q^2)\left(\gamma^\mu-\frac{2m_\Lambda}{s_+}p^\mu-\frac{2m_{\Lambda_c}}{s_+}k^\mu\right)\bigg]u_{\Lambda_c}(p,s_{\Lambda_c})\,,$$

$$\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,\gamma^\mu\gamma_5\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle = -\bar{u}_\Lambda(k,s_\Lambda)\,\gamma_5\bigg[f_{A,t}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}+m_\Lambda)\frac{q^\mu}{q^2}$$
$$+ f_{A,0}^{\Lambda_c\to\Lambda}(q^2)\frac{m_{\Lambda_c}-m_\Lambda}{s_-}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)$$
$$+ f_{A,\perp}^{\Lambda_c\to\Lambda}(q^2)\left(\gamma^\mu+\frac{2m_\Lambda}{s_-}p^\mu-\frac{2m_{\Lambda_c}}{s_-}k^\mu\right)\bigg]u_{\Lambda_c}(p_{\Lambda_c},s_{\Lambda_c}),$$

$$\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\,b\,|\Lambda_c(p,s_{\Lambda_c})\rangle = -\bar{u}_\Lambda(k,s_\Lambda)\bigg[f_{T,0}^{\Lambda_c\to\Lambda}(q^2)\frac{q^2}{s_+}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)$$
$$+ f_{T,\perp}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}+m_\Lambda)\left(\gamma^\mu-\frac{2m_\Lambda}{s_+}\,p^\mu-\frac{2m_{\Lambda_c}}{s_+}\,k^\mu\right)\bigg]u_{\Lambda_c}(p,s_{\Lambda_c})\,,$$

$$\langle\Lambda(k,s_\Lambda)|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu\gamma_5\,c\,|\Lambda_c(p,s_{\Lambda_c})\rangle = -\bar{u}_\Lambda(k,s_\Lambda)\,\gamma_5\bigg[f_{T5,0}^{\Lambda_c\to\Lambda}(q^2)\,\frac{q^2}{s_-}\left(p^\mu+k^\mu-(m_{\Lambda_c}^2-m_\Lambda^2)\frac{q^\mu}{q^2}\right)$$

$$+ f_{T5,\perp}^{\Lambda_c\to\Lambda}(q^2)\,(m_{\Lambda_c}-m_\Lambda)\left(\gamma^\mu+\frac{2m_\Lambda}{s_-}\,p^\mu-\frac{2m_{\Lambda_c}}{s_-}\,k^\mu\right)\bigg]u_{\Lambda_c}(p,s_{\Lambda_c})\,,$$

Theory inputs

Dispersive bounds

$$J_V^\mu(x) = \bar{s}(x)\,\gamma^\mu c(x)\,, \qquad J_A^\mu(x) = \bar{s}(x)\,\gamma^\mu\gamma_5 c(x)\,,$$
$$J_T^\mu(x) = \bar{s}(x)\,\sigma^{\mu\alpha} q_\alpha c(x)\,, \qquad J_{AT}^\mu(x) = \bar{s}(x)\,\sigma^{\mu\alpha} q_\alpha \gamma_5 c(x)\,.$$

- Two-point correlation functions ⇒ minimally subtracted correlators

$$\chi_\Gamma^{(\lambda)}(Q^2) = \frac{1}{n!}\left[\frac{\partial}{\partial q^2}\right]^n \Pi_\Gamma^{(\lambda)}(q^2)\Big|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty ds\, \frac{\operatorname{Im}\Pi_\Gamma^{(\lambda)}(s)}{(s-Q^2)^{n+1}}$$

calculated numerically: NLO in α_s, up to order $1/m_c^5$

- Hadronic representation of correlation functions ⇒ $\chi_A^{(J=0)}\big|_{1\text{pt}} = \dfrac{M_{D_s}^2 f_{D_s}^2}{(M_{D_s}^2 - Q^2)^2}$

- BGL-like parametrisation of FF: $f(q^2) = \dfrac{1}{\phi_f(z)B(z)} \displaystyle\sum_{k=0}^{K} a_k^{(f)} p_k^{(f)}(z)\Big|_{z=z(q^2)}$ $\qquad \displaystyle\sum_f \sum_{k=0}^{K} |a_k^{(f)}|^2 < 1$

Further discussion on form factor approach: Gubernari, (Reboud), van Dyk, Virto 2021 & 2022; Blake et al. 2022; Flynn et al. 2023

$\Lambda_c \to \Lambda \ell \nu$

- Heavy Quark limit ⇒ expansion in α_s/π , Λ_{QCD}/m_c
- Large Energy limit ⇒ expansion in α_s/π , Λ_{had}/m_c , $\Lambda_{\text{had}}/E_\Lambda$

$$\begin{aligned}
\frac{\xi}{m_{\Lambda_c}} &= f_{V,t}(0) = f_{V,\perp}(0) = f_{V,0}(0) = f_{A,t}(0) = f_{A,\perp}(0) = f_{A,0}(0) \\
&= f_{T,\perp}(0) = f_{T,0}(0) = f_{T5,\perp}(0) = f_{T5,0}(0)\,, \\
\frac{\xi_1 - \xi_2}{m_{\Lambda_c}} &= f_{V,\perp}(q^2_{\text{max}}) = f_{V,0}(q^2_{\text{max}}) = f_{A,t}(q^2_{\text{max}}) = f_{T,\perp}(q^2_{\text{max}}) = f_{T,0}(q^2_{\text{max}})\,, \\
\frac{\xi_1 + \xi_2}{m_{\Lambda_c}} &= f_{A,\perp}(q^2_{\text{max}}) = f_{A,0}(q^2_{\text{max}}) = f_{V,t}(q^2_{\text{max}}) = f_{T5,\perp}(q^2_{\text{max}}) = f_{T5,0}(q^2_{\text{max}})\,.
\end{aligned}$$

- Constraint through relations

$$\begin{aligned}
f_{T,\perp}/f_{V,\perp} &= 1 \pm 0.35\,, \quad & f_{T,0}/f_{V,0} &= 1 \pm 0.35\,, \\
f_{T5,\perp}/f_{A,\perp} &= 1 \pm 0.35\,, \quad & f_{T5,0}/f_{A,0} &= 1 \pm 0.35\,,
\end{aligned}$$

Form factors in the full q^2 range

$z\left(q^2 = t_0\right) = 0$

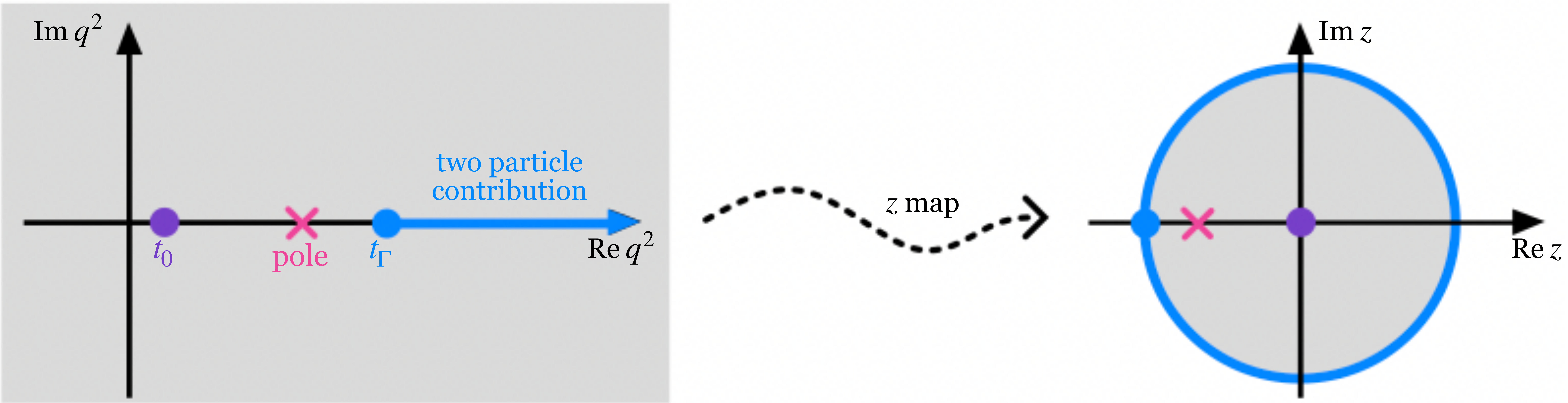
Parametrisation

- Modified BGL: analyticity + unitarity

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma - t_0}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma - t_0}}$$

$$f(q^2) = \frac{1}{\sqrt{\chi}\,\phi(q^2)\,B(q^2)} \sum_k^K a_k\, p_k\left(z(q^2)\right)$$

$$f_+(q^2 = 0) = f_0(q^2 = 0)$$



Used $K = 4$

Form factors in the full q^2 range

$z\left(q^2 = t_0\right) = 0$
$t_\Gamma = (m_B + m_\pi)^2$
$t_+ = (m_{B_s} + m_K)^2$

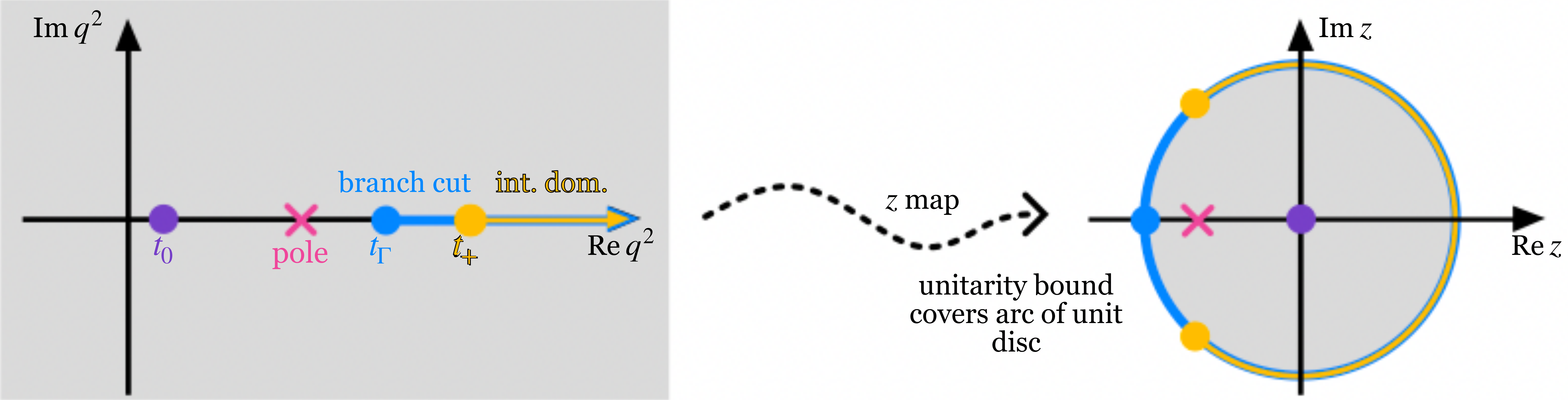
Parametrisation

- Modified BGL: analyticity + unitarity + (pair production ≠ first branch point)

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma - t_0}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma - t_0}}$$

$$f(q^2) = \frac{1}{\sqrt{\chi}\,\phi(q^2)\,B(q^2)} \sum_k^K a_k\, p_k\left(z(q^2)\right)$$

$$f_+(q^2 = 0) = f_0(q^2 = 0)$$



Used $K = 4$

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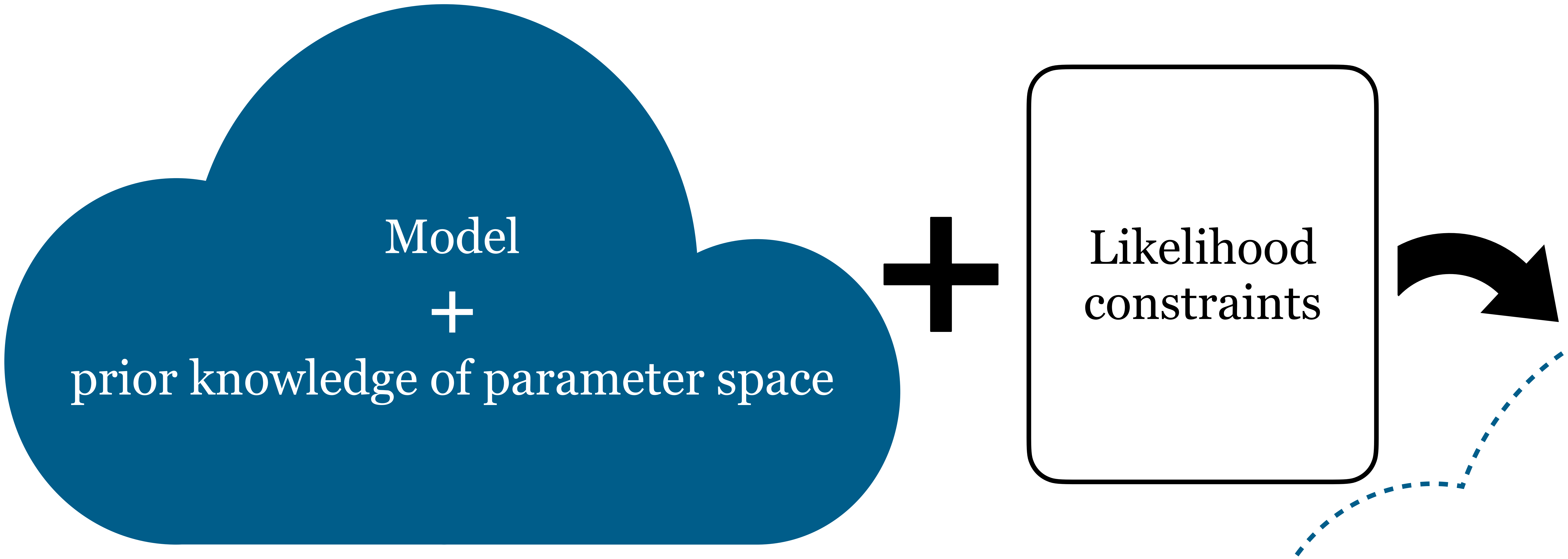
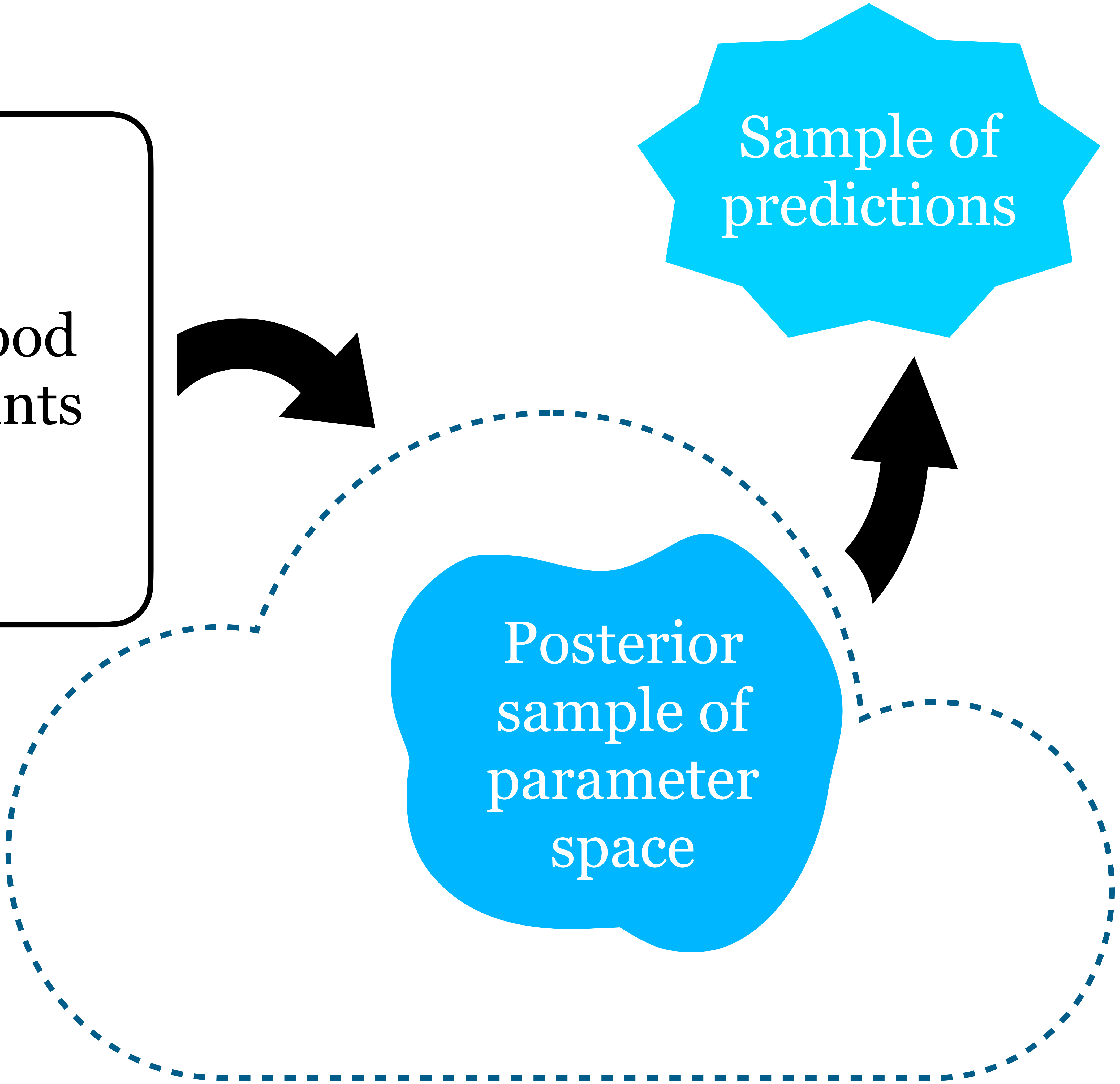
Statistical treatment in EOS

Bayesian inference





- (Dynamic) nested sampling:
  Access probabilities of the predicted samples