Constraining CP violation in charm meson two-body decays *

Eleftheria Solomonidi, IFIC (U. Valencia/CSIC)

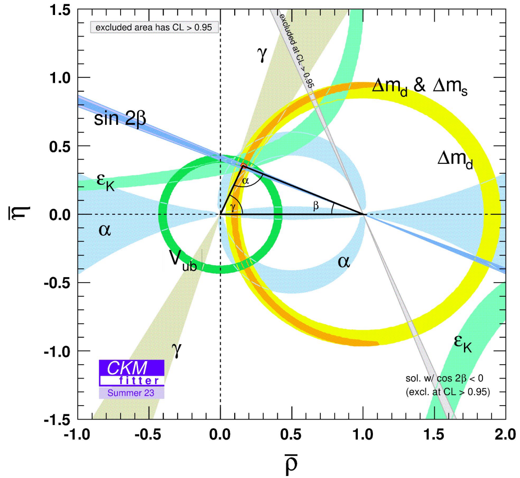
Quirks in quark flavour physics, Zadar, 19 June 2024

*not Zadar

Introduction

Based on *Phys.Rev.* D 108 (2023) 3, 036026
with Antonio Pich and Luiz Vale Silva

Why look into charm?



The CKM matrix is (generally) well probed from various exp. processes: lots of processes,

only 4 independent parameters

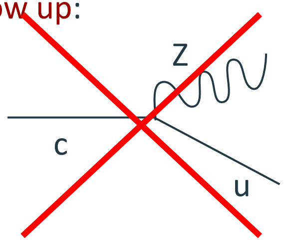
Charm is the only weakly decaying up-type quark bound in hadrons

→ Can still perform complementary CKM tests from the charm sector

Otherwise, assuming good control over CKM matrix:

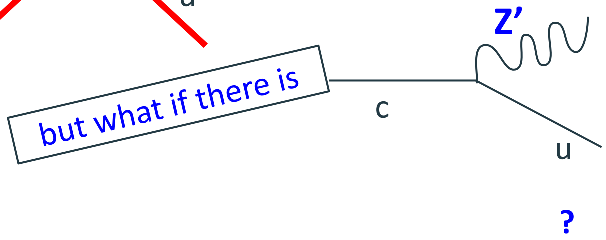
→ Can look for rare processes where there is more room for NP to show up:

- b⟶sμμ, b⟶sνν, s⟶dνν, … [lots of work there!]



In this search,

different NP scenarios can be explored by starting off from the charm quark



“No stone left unturned” approach

Rich experimental programme (LHCb, Belle II, BESIII, future facilities,...)

The measurements

$$A_{\rm CP}(f) \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D^0} \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D^0} \to \overline{f})}$$

Up to date, the only observation of CP violation in charm systems: [LHCb 2019]

$$\Delta A_{\rm CP} = A_{\rm CP}(K^-K^+) - A_{\rm CP}(\pi^-\pi^+) = (-15.4 \pm 2.9) \times 10^{-4}$$

$\longrightarrow$ at least one of them is non-zero and large

$\longrightarrow$ CPV from D-anti-D mixing largely cancels

Followed up by the measurement of an individual CP asymmetry: [LHCb 2022]

$$A_{\rm CP}(K^-K^+) = [6.8 \pm 5.4 \ (\text{stat}) \ \pm 1.6 \ (\text{syst}) \ ] \times 10^{-4}$$

(systematics would be the same if π-π+ was measured instead)

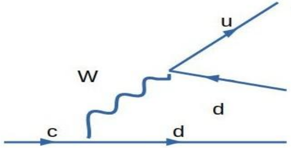
- Mixing-induced CPV also measured to be small

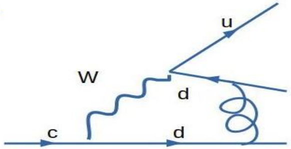
$\longrightarrow$ **large *direct* CPV at least in the *decay* of D^0 to π-π+**

$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

Can this be explained within the SM?

How CP violation arises





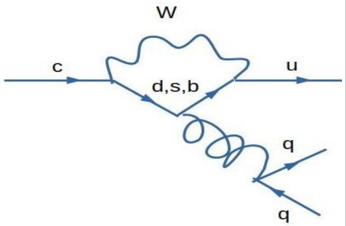
Generally: at least **2 interfering amplitudes**

Can be parameterised as

$$\mathcal{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$
$$\mathcal{A}(\overline{D^0} \to f) = A(f) - i r_{CKM} B(f)$$

where $r_{CKM} = \mathrm{Im}\frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}} \approx 6.5 \times 10^{-4}$

and consequently $$A_{\mathrm{CP}}^{\mathrm{direct}} \approx 2 \underbrace{r_{CKM}}_{\text{weak phases}} \frac{|B(f)|}{|A(f)|} \cdot \underbrace{\sin\arg\frac{A(f)}{B(f)}}_{\text{strong phases}}$$



At the scale of the charm quark mass:

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} C_i(\mu) \left(\underbrace{\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu)}_{\text{current-current operators}}\right) - \lambda_b(\Sigma_{i=3}^{6} C_i(\mu) \underbrace{Q_i(\mu)}_{\text{penguin operators}} + C_{8g}(\mu) Q_{8g}(\mu))\right]$$

$$\left.\begin{aligned} \lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b. \\ |\lambda_d| \approx |\lambda_s| = \mathcal{O}(\lambda) \\ \lambda_d + \lambda_s + \lambda_b = 0 \\ |C_{3-6}| < 0.1 C_2,\ 0.03 C_1 \end{aligned}\right\}$$

affect branching ratios & aCP's

affect only aCP's

$\boxed{\text{Challenge: to calculate} \quad \langle P^+ P^- | Q_i | D^0 \rangle,\ P = \pi, K}$

How to incorporate strong phases: isospin & unitarity

Isospin is a good symmetry

of strong interactions - Use Wigner-Eckart theorem

$$A(D^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} - \frac{1}{2\sqrt{3}}\boxed{|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}}$$

$$A(D^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} + \frac{1}{\sqrt{3}}\boxed{|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}}$$

$$A(D^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2\sqrt{2}}\boxed{|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}}$$

$$A(D^0 \to K^-K^+) = \frac{1}{2}\left(\boxed{|A_{KK}^{I=1}|e^{i\delta_{KK,1}}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^0 \to \overline{K}^0K^0) = \frac{1}{2}\left(-\boxed{|A_{KK}^{I=1}|e^{i\delta_{KK,1}}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^+ \to \overline{K}^0K^+) = \boxed{|A_{KK}^{I=1}|e^{i\delta_{KK,1}}}$$

Both ππ and KK have an isospin-zero component

Isospin=1, 2: only KK, ππ channels respectively

The S-matrix is unitary

In *isospin-zero*, *spin-zero*, the strong S-submatrix

is also unitary

(assumption: no other channels leak to ππ and KK)

$$\boxed{\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK)\end{pmatrix} = \underbrace{\begin{pmatrix} S_0(\pi\pi\to\pi\pi) & S_0(\pi\pi\to KK) \\ S_0(KK\to\pi\pi) & S_0(KK\to KK)\end{pmatrix}}_{\text{strong-interaction-driven}} \cdot \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK)\end{pmatrix}}$$

$$S_0 = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2}\end{pmatrix}$$

***If** isospin-zero ππ and KK channels didn't communicate*:

Watson's theorem

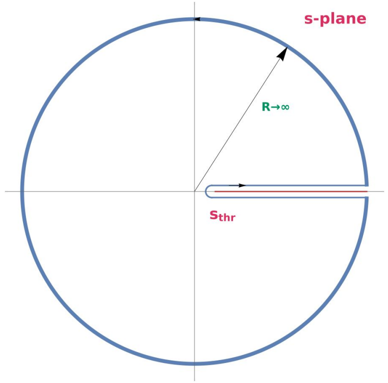
$$\arg A(D\to\pi\pi) = \arg(\pi\pi\to\pi\pi,\ \text{S-wave}) \mod \pi$$

Instead, now the phases of D⟶PP depend on the *magnitudes* of D⟶PP + the strong S-submatrix S_0

How the phases affect the amplitudes

Through analyticity by applying Cauchy's theorem

$$\mathrm{Re}A(s) = \frac{1}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\mathrm{Im}A(s')}{s'-s}$$



and ***if rescattering is elastic,***

through unitarity we get the

dispersion relation

$$\mathrm{Re}A(s) = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{\tan\delta_1(s')}{s'-s}\mathrm{Re}A(s')$$

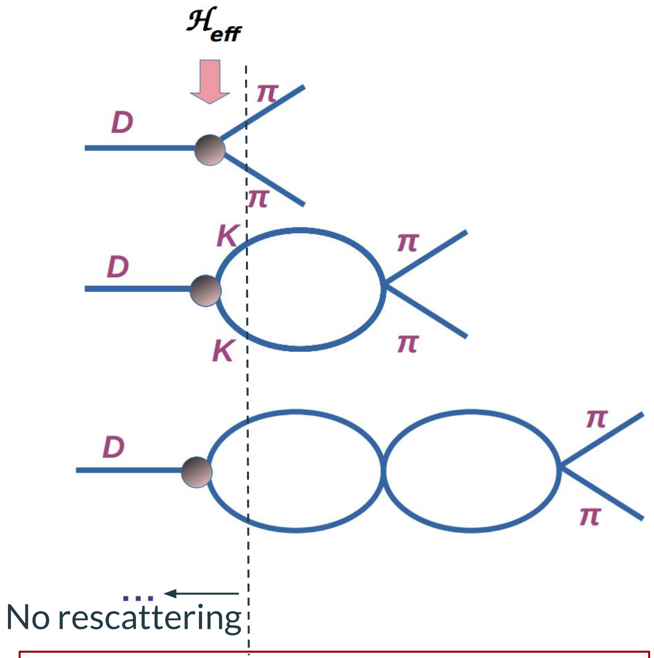
which has the solution (Omnes)

$$|A(s)| = A(s_0)\exp\{\underbrace{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1(s')}{(s'-s_0)(s'-s)}}_{}\}$$

limit of no rescattering → large N_C

Omnes factor Ω;

Large phases modify amplitude



We correct large N_C /factorization by incorporating s-channel rescattering of the final states

Two-channel case

In the isospin-zero block there are both ππ and KK : the elastic

$$|A(D\to\pi\pi)(s)| = A(s_0)\cdot exp\{\frac{s-s_0}{\pi}PV\int_{4M_\pi^2}^{\infty} dz\frac{\delta_1(z)}{(z-s_0)(z-s)}\}$$

now becomes

$$\begin{pmatrix}\mathcal{A}(D\to\pi\pi)\\ \mathcal{A}(D\to KK)\end{pmatrix} = \Omega\cdot\begin{pmatrix}\mathcal{A}_{(\text{large }N_C)}(D\to\pi\pi)\\ \mathcal{A}_{(\text{large }N_C)}(D\to KK)\end{pmatrix}$$

rescattering

no rescattering

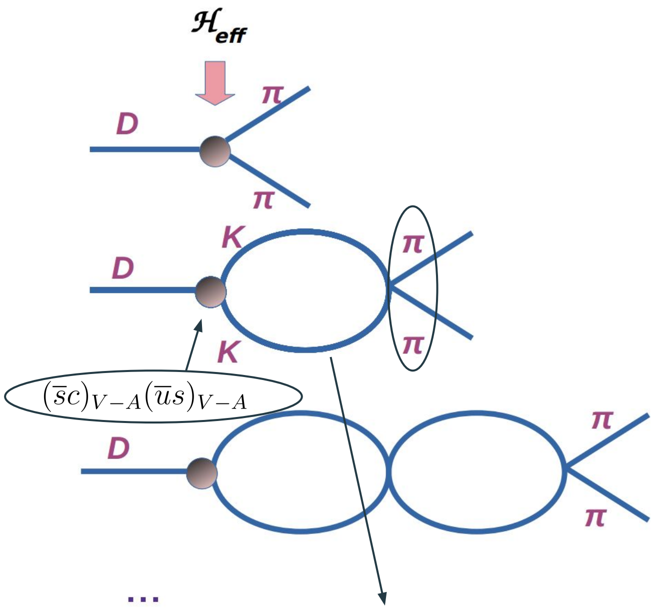
where Ω is a 2-by-2 matrix that has to be found **numerically**

by solving the two-channel dispersion relation

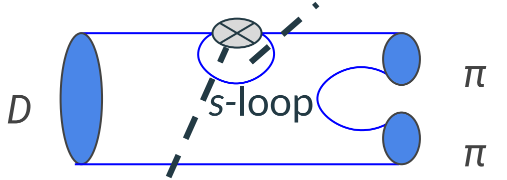
- In the language of hadronic matrix elements:

Non-diagonal Ω creates

$$\langle\pi\pi(I=0)|Q_i^s|D\rangle \neq 0$$
$$\langle KK(I=0)|Q_i^d|D\rangle \neq 0$$

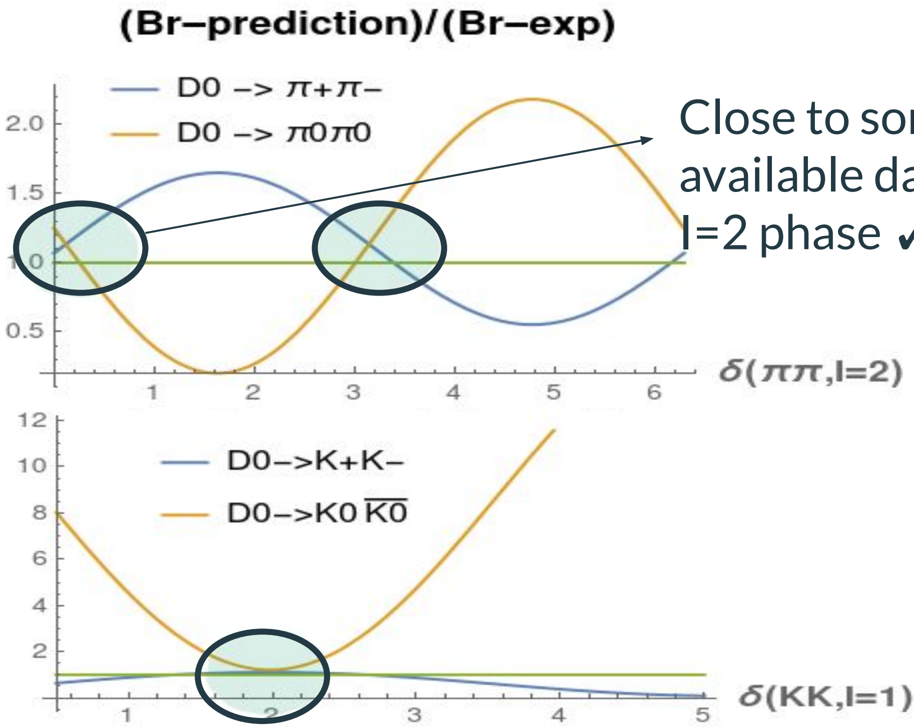


“Long-distance penguin”

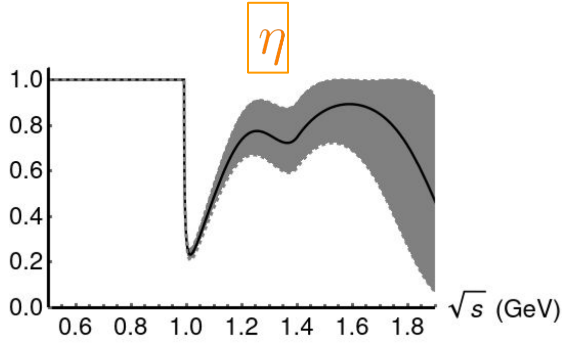


Sorting through the uncertainties

Solving the Omnes equations provides a full description of the decay amplitudes
→ *Select* among the strong rescattering input
the one that yields values close to exp. Br's <u>for all decay channels simultaneously</u>



Close to some available data for I=2 phase ✓



Only this I=0 inelasticity survives, giving an Omnes matrix like

$$\Omega_{I=0} = \begin{pmatrix} 0.58e^{1.8i} & 0.64e^{-1.7i} \\ 0.58e^{-1.4i} & 0.61e^{2.3i} \end{pmatrix}$$

→ large rescattering between ππ and KK in the I=0 channel

Results: CP asymmetry predictions

We find $$\Delta A_{\rm CP}^{\rm direct} \approx -5 \cdot 10^{-4}$$

$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) \approx 3 \cdot 10^{-4} \qquad A_{\rm CP}^{\rm direct}(K^-K^+) \approx -2 \cdot 10^{-4}$$

and similar levels predicted for $\pi^0\pi^0,\ K^0\overline{K^0}$

- SU(3) not considered; its breaking turns out comparable to known levels
- Multiple amplitudes interfere: I=2 vs I=0, I=1 vs I=0, **I=0 vs I=0** (present because of ππ->KK rescattering)

LHCb:

$$\Delta A_{\rm CP} = (-15.4 \pm 2.9) \times 10^{-4}$$
$$A_{\rm CP}^{\rm direct}(\pi^-\pi^+) = (23.2 \pm 6.1) \times 10^{-4}$$

Theoretical values much smaller than experimental !!
The discrepancy between theory and exp persists in $D^0 \to \pi^+\pi^-$

Bounds on the CP asymmetries

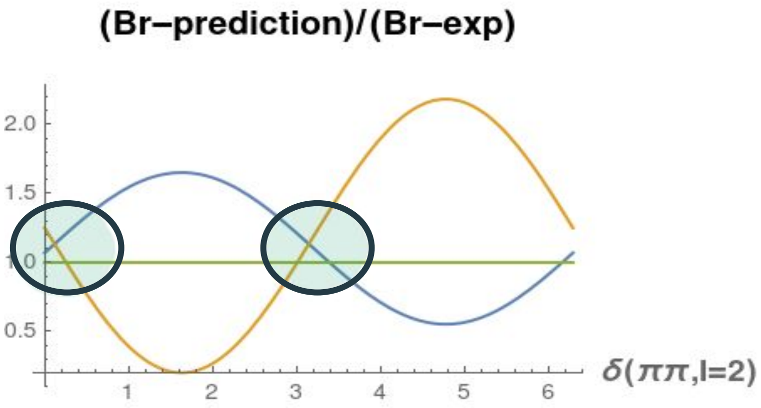
Based on *work in progress*
with Antonio Pich and Luiz Vale Silva

Stretching the theory predictions

We have not provided uncertainties

⟶ <u>Parametric uncertainties:</u> check for Br's *close enough* to the exp. value

	solution I	solution II	solution III
$\eta_0^0,\ m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.80\,e^{+1.60\,i} & 1.01\,e^{-1.69\,i} \\ 0.56\,e^{-1.50\,i} & 0.59\,e^{+2.07\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.39\,e^{+1.64\,i} & 0.59\,e^{-2.13\,i} \\ 0.51\,e^{-1.31\,i} & 0.56\,e^{+2.43\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.71\,e^{+0.53\,i} & 1.35\,e^{-2.67\,i} \\ 0.38\,e^{-0.98\,i} & 0.42\,e^{+2.65\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.56\,e^{+1.84\,i} & 0.61\,e^{-1.73\,i} \\ 0.57\,e^{-1.41\,i} & 0.58\,e^{+2.25\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.42\,e^{+1.75\,i} & 0.54\,e^{-2.05\,i} \\ 0.51\,e^{-1.33\,i} & 0.55\,e^{+2.43\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.35\,e^{+1.13\,i} & 0.74\,e^{-2.47\,i} \\ 0.50\,e^{-1.18\,i} & 0.55\,e^{+2.48\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 2$	$\Omega^{(0)} = \begin{pmatrix} 0.58\,e^{+1.80\,i} & 0.64\,e^{-1.74\,i} \\ 0.58\,e^{-1.37\,i} & 0.61\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.43\,e^{+1.64\,i} & 0.58\,e^{-2.10\,i} \\ 0.52\,e^{-1.25\,i} & 0.57\,e^{+2.48\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.40\,e^{+1.01\,i} & 0.80\,e^{-2.50\,i} \\ 0.50\,e^{-1.11\,i} & 0.56\,e^{+2.53\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 3$	$\Omega^{(0)} = \begin{pmatrix} 0.60\,e^{+1.76\,i} & 0.66\,e^{-1.74\,i} \\ 0.60\,e^{-1.33\,i} & 0.63\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.44\,e^{+1.53\,i} & 0.61\,e^{-2.16\,i} \\ 0.52\,e^{-1.17\,i} & 0.59\,e^{+2.53\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.45\,e^{+0.91\,i} & 0.86\,e^{-2.53\,i} \\ 0.50\,e^{-1.04\,i} & 0.57\,e^{+2.58\,i} \end{pmatrix}$
sol. B': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 2.01\,e^{+1.39\,i} & 2.47\,e^{-1.76\,i} \\ 0.37\,e^{-0.33\,i} & 0.54\,e^{+3.05\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.91\,e^{+0.60\,i} & 2.78\,e^{-2.55\,i} \\ 0.31\,e^{-0.23\,i} & 0.45\,e^{+3.30\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.20\,e^{+0.43\,i} & 3.55\,e^{-2.72\,i} \\ 0.35\,e^{+0.03\,i} & 0.57\,e^{+3.40\,i} \end{pmatrix}$
sol. C': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 1.83\,e^{+1.38\,i} & 2.65\,e^{-1.76\,i} \\ 0.34\,e^{-0.40\,i} & 0.57\,e^{+3.00\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.80\,e^{+0.59\,i} & 3.11\,e^{-2.56\,i} \\ 0.29\,e^{-0.24\,i} & 0.49\,e^{+3.24\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.09\,e^{+0.43\,i} & 3.94\,e^{-2.72\,i} \\ 0.32\,e^{-0.03\,i} & 0.61\,e^{+3.34\,i} \end{pmatrix}$



Within the uncertainties the CP asymmetries are **still very far from the experimental values**

⟶ <u>Systematic uncertainties:</u> The main one comes from the **two-channel** isospin-zero hypothesis.

How sizable is the effect of a potential third channel?

Goal: scrutinise the two-channel hypothesis, see how far the CP asymmetries can reach

Lose on predictivity - provide an <u>upper bound</u> for the asymmetries *as an alternative for uncertainties*

Limiting the sources of uncertainties

Within a data-driven approach: can we *bypass some of the input data*?

- Biggest source of uncertainties in the input: inelasticity *ππ→ KK*

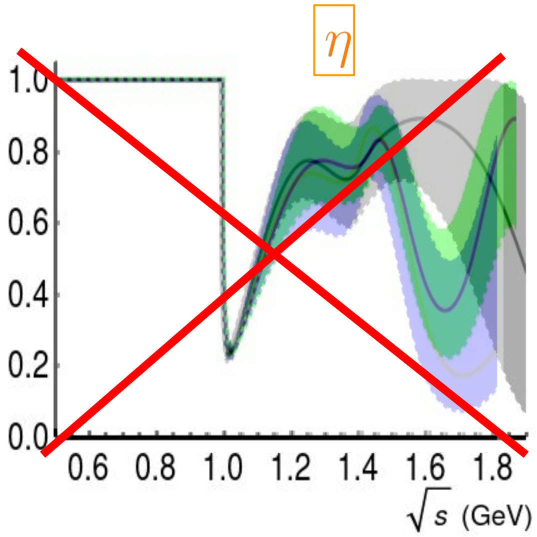
$$S_0 = \begin{pmatrix} \boxed{\eta} e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}$$
$$= \begin{pmatrix} \boxed{S_0(\pi\pi \to \pi\pi)} & S_0(\pi\pi \to KK) \\ S_0(KK \to \pi\pi) & S_0(KK \to KK) \end{pmatrix}$$

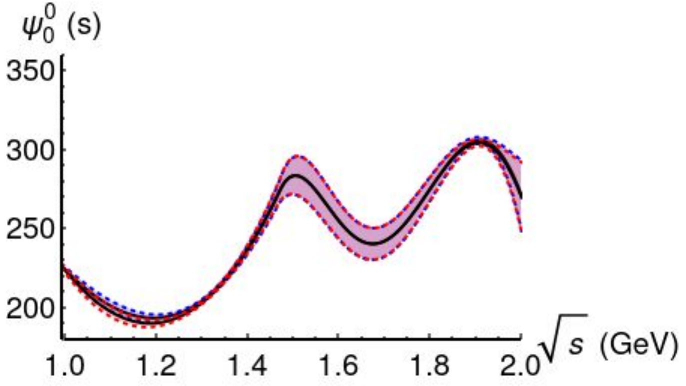
- Phase $\boxed{\delta_1}$ also relatively uncertain
- <u>Less uncertain input</u>: phase of *ππ+KK*

→ Parameterisation in three energy regions:

1. Below the inelastic threshold (= phase of ππ): very well known
2. Below ~1.5 GeV: dispersion relations respected
3. Below ~1.9 GeV: analytical parameterisation fitting data

→ Extrapolation for higher energies





Consequences of the two-channel hypothesis

Assumption: no further rescattering to 4π or any other channels of isospin zero

1. $$\boxed{\begin{pmatrix} \mathcal{A}(D\to\pi\pi) \\ \mathcal{A}(D\to KK) \end{pmatrix} = \Omega \cdot \begin{pmatrix} \mathcal{A}_{(\text{large } N_C)}(D\to\pi\pi) \\ \mathcal{A}_{(\text{large } N_C)}(D\to KK) \end{pmatrix}}$$

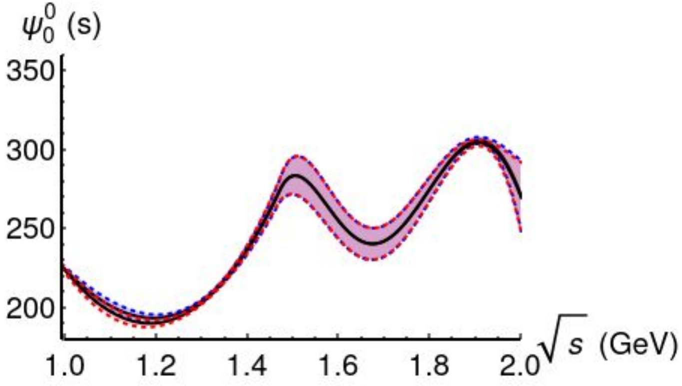
We **do not solve** the DRs for the Omnes matrix Ω.
Instead, just **solve for the determinant of Ω**: obeys an elastic dispersion relation
⟶ has an analytical solution:

$$\det\Omega(s) = \exp\{i\psi_0^0(s)\}\exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\psi_0^0(z)}{(z-s_0)(z-s)}\}$$

2. $\psi_0^0(s)$ at infinity has to go to **2π or a higher multiple of π**
Reasonable assumption: $\psi_0^0(s)\to 2\pi$
(no further resonances at higher energies)

3. **CPT constraint**: unitarity + CPT symmetry

$$\frac{A_{\rm CP}(\pi\pi(0-0))}{A_{\rm CP}(KK(0-0))} = -\frac{2|A(K^+K^-)|^2}{3|A(\pi^+\pi^-)|^2}\frac{\sigma_K}{\sigma_\pi}$$



Experimental information to be used

We make use of all the available experimental Br's for all the decay channels related through isospin

$$A(D^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} - \frac{1}{2\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} + \frac{1}{\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2\sqrt{2}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to K^-K^+) = \frac{1}{2}\left(|A_{KK}^{I=1}|e^{i\delta_{KK,1}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^0 \to \overline{K}^0 K^0) = \frac{1}{2}\left(-|A_{KK}^{I=1}|e^{i\delta_{KK,1}} - |A_{KK}^{I=0}|e^{i\delta_{KK,0}}\right)$$

$$A(D^+ \to \overline{K}^0 K^+) = |A_{KK}^{I=1}|e^{i\delta_{KK,1}}$$

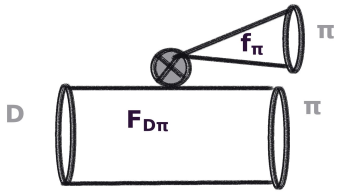
$\to$ can fit to $|A_{\pi\pi}^{I=2}|,\ |A_{\pi\pi}^{I=0}|, \cos(\delta_{\pi\pi,2} - \delta_{\pi\pi,0})$

and to $|A_{KK}^{I=1}|,\ |A_{KK}^{I=0}|, \cos(\delta_{KK,1} - \delta_{KK,0})$

$$\boxed{\begin{pmatrix} \mathcal{A}(D \to \pi\pi) \\ \mathcal{A}(D \to KK) \end{pmatrix} = \Omega \cdot \begin{pmatrix} \mathcal{A}_{(\text{large } N_C)}(D \to \pi\pi) \\ \mathcal{A}_{(\text{large } N_C)}(D \to KK) \end{pmatrix}}$$

Isospin-zero component of e.g.

$$\mathcal{A}(D^0 \to \pi^+\pi^-) \propto C_1 \lambda_d \langle \pi^+\pi^- | Q_1^d | D^0 \rangle_{fac}$$



and for the CP asymmetries: include

$$-\lambda_b \left(C_4 \langle \pi^+\pi^- | Q_4 | D^0 \rangle_{fac} + C_6 \langle \pi^+\pi^- | Q_6 | D^0 \rangle_{fac}\right)$$

CP asymmetry from I=0/I=0 interference

Jarlskog, 3*10^-5

We find:

$$A_{\rm CP}(\pi\pi(0-0)) = \frac{\mathcal{J}\,\omega_\pi^{(\rm Im)}}{3|A(D^0\to\pi^+\pi^-)|^2}\underbrace{\left(T^{CC}_{\pi\pi}T^{CC}_{KK} + T^{P}_{\pi\pi}T^{CC}_{KK} + T^{CC}_{\pi\pi}T^{P}_{KK}\right)}_{\text{large } N_C \text{ limit}} \approx 1.7\cdot 10^{-3}\,\omega_\pi^{(\rm Im)}$$

experiment

large N_C limit (of current-current and penguin operators)
No cancellations between different terms

rescattering dynamics:

$$\omega_\pi^{(\rm Im)} \equiv \mathrm{Im}\{\Omega_{11}^*\Omega_{12}\}$$

and similarly for KK

$$\omega_K^{(\rm Im)} \equiv \mathrm{Im}\{\Omega_{21}^*\Omega_{22}\}$$

The constraints from $\det\Omega(s)$, the exp. isospin-zero invariant amplitudes *and* CPT result in

$$|\omega_\pi^{(\rm Im)}| \leq \frac{|\det\Omega(m_D^2)|}{1+\sigma_\pi/\sigma_K}\ ,\quad |\omega_K^{(\rm Im)}| \leq \frac{|\det\Omega(m_D^2)|}{1+\sigma_K/\sigma_\pi}$$

with an **opposite sign**; both up to ~0.2

therefore

$$\boxed{|A_{\rm CP}(\pi\pi(0-0))| \leq 3.1\cdot 10^{-4}}\qquad \boxed{|A_{\rm CP}(KK(0-0))| \leq 1.7\cdot 10^{-4}}$$

very small fraction of the experimental π+π- ACP

CP asymmetry from I=2/I=0 interference

The other source of CP violation for the pion channels:

common penguin-tree interference present in K→ππ

$$A_{\rm CP}(\pi^+\pi^-(2-0)) = \frac{|A_{\pi\pi}^{I=2}|\,\mathcal{J}}{3\sqrt{2}|\lambda_d||A(D^0\to\pi^+\pi^-)|^2}[(T_{KK}^{CC}+T_{KK}^{P})|\Omega_{12}|\sin(\delta_{\pi\pi,2}-\arg\Omega_{12}) + \underline{T_{\pi\pi}^{P}|\Omega_{11}|\sin(\delta_{\pi\pi,2}-\arg\Omega_{11})}]$$

$$\approx (2.1|\Omega_{11}|\sin(\arg\Omega_{11}-\delta_{\pi\pi,2}) + 3.2|\Omega_{12}|\sin(\arg\Omega_{12}-\delta_{\pi\pi,2}))\cdot 10^{-4}$$

In order to reach the observed asymmetry we would need at least $|\Omega_{11}| \approx |\Omega_{12}| \approx 4$

<u>There exist</u> Omnes matrices that satisfy **all the imposed constraints** of this work & *do not spoil the smallness* of ACP in D→K+K-

but some "fine tuning" needed to keep $\det\Omega = \Omega_{11}\Omega_{22} - \Omega_{12}\Omega_{21}$ to the determined value of ~0.4

& to reproduce the I=0 amplitudes extracted from the exp. fit

$$\Omega = \begin{pmatrix} 4e^{1.5i} & 4.13e^{1.5i} \\ 3.79e^{-0.92i} & 3.84e^{-0.93i} \end{pmatrix}$$

If the 2/0 interference is the only significant CPV source,

CPV should be equally sizable in the neutral pion mode (i.e. no cancellations)

[<u>From the full dispersive calculation</u> [2305.11951], with all the rescattering input implemented:

<u>none of the Omnes matrices</u> comes close to the required values

while reproducing the experimental Br's for the decay channels]

Outlook

Independent theoretical determinations **agree on small CPV**: LCSRs [Khodjamirian et al ‘17 + Lenz et al ‘23] ✔, U-spin breaking arguments [Schacht ‘23] ✔

Could something be missing from the theory prediction?

- 3rd channel in isospin zero? e.g. *ρρ*, a_1 *π* (⟶ *4π*)

  ⟶ no data available as required for dispersion relations - would need model dependence [Kubis et al.]

Theoretical cross-checks:

- Could try to *understand better* I=2 (we do not calculate Omnes but use exp. Br’s)

  ⟶ no known resonances that would lead to inelastic *ππ*⟶*ππ*

- More theoretical determinations of related channels: D⟶3π (could highlight the enhancement of CPV from some resonance), D⟶ππμμ
- Address indirect CPV theoretically? (could shed light into underlying long-distance dynamics)

Experimental cross-checks: $\pi^0\pi^0,\ K^0\overline{K^0}$ already theoretically calculated

⟶ **if large CPV observed in charged pion mode, equally sizable in the neutral pion mode** [see also Nierste, Schacht ‘15]

⟶ **if two-channel hypothesis not valid, CPV should manifest in other channels (4π)**

NP? Z’ model breaking U-spin, see [Hiller et al. ‘23] also [Lenz, Rusov et al. ‘19] etc.

Outlook

The ACP remains an open question! An exciting flavour anomaly

Independent theoretical determinations **agree on small CPV**: LCSRs [Khodjamirian et al '17 + Lenz et al '23] ✔, U-spin breaking arguments [Schacht '23] ✔

Could something be missing from the theory prediction?

- 3rd channel in isospin zero? e.g. $\rho\rho$, $a_1\,\pi$ ($\rightarrow 4\pi$)

  → no data available as required for dispersion relations - would need model dependence [Kubis et al.]

Theoretical cross-checks:

- Could try to *understand better* I=2 (we do not calculate Omnes but use exp. Br's)

  → no known resonances that would lead to inelastic $\pi\pi \rightarrow \pi\pi$

- More theoretical determinations of related channels: D→3π (could highlight the enhancement of CPV from some resonance), D→ππμμ
- Address indirect CPV theoretically? (could shed light into underlying long-distance dynamics)

More predictions, more measurements needed

A lot of work to be done!

Experimental cross-checks: $\pi^0\pi^0,\ K^0\overline{K^0}$ already theoretically calculated

→ **if large CPV observed in charged pion mode, equally sizable in the neutral pion mode** [see also Nierste, Schacht '15]

→ **if two-channel hypothesis not valid, CPV should manifest in other channels (4π)**

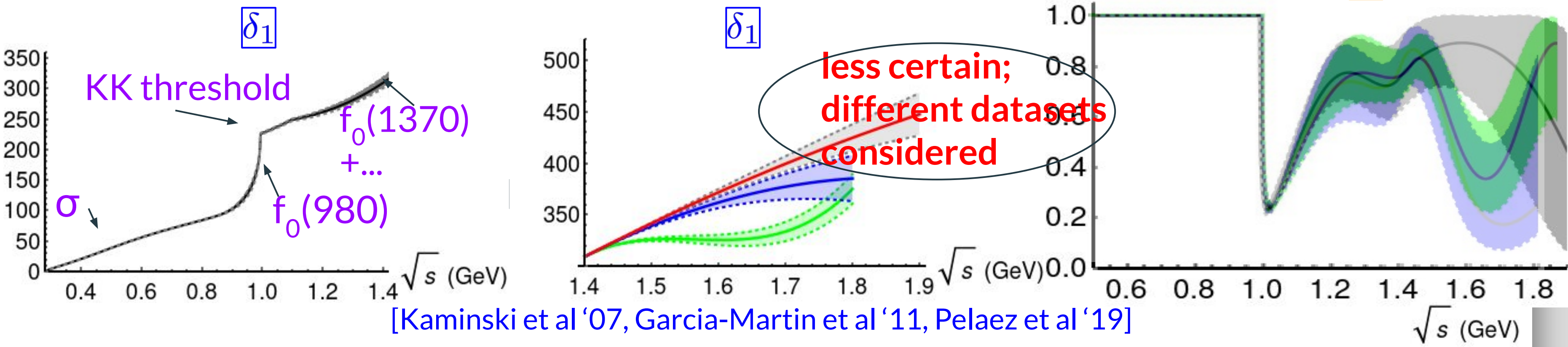
NP? Z' model breaking U-spin, see [Hiller et al. '23] also [Lenz, Rusov et al. '19] etc.

BACKUP

Full implementation of the strong rescattering

Isospin zero:

$$S_0 = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix} = \begin{pmatrix} S_0(\pi\pi\to\pi\pi) & S_0(\pi\pi\to KK) \\ S_0(KK\to\pi\pi) & S_0(KK\to KK) \end{pmatrix}$$



[Kaminski et al ‘07, Garcia-Martin et al ‘11, Pelaez et al ‘19]

- Data-driven parameterizations, incorporating the effect of known resonances & other features
- Extrapolations for energies higher than 1.9 GeV

Isospins 1 and 2:

- Elastic ππ, KK rescattering
- No (adequate) data available → use measured Br’s of D+ decays

$$\begin{cases} A(D^+\to\pi^+\pi^0) = \dfrac{\sqrt{3}}{2\sqrt{2}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}} \\ A(D^+\to\overline{K}^0K^+) = |A_{KK}^{I=1}|e^{i\delta_{KK,1}} \end{cases}$$

free

ACP, direct and indirect

$$A_{\rm CP}(f;t) \equiv \frac{\frac{d\Gamma(D^0_{phys}(t)\to\overline{f})}{dt} - \frac{d\Gamma(\overline{D^0}_{phys}(t)\to f)}{dt}}{\frac{d\Gamma(D^0_{phys}(t)\to\overline{f})}{dt} + \frac{d\Gamma(\overline{D^0}_{phys}(t)\to f)}{dt}}$$

$$\begin{aligned}\frac{d\Gamma(D^0_{phys}(t)\to\overline{f})}{dt} - \frac{d\Gamma(\overline{D^0}_{phys}(t)\to f)}{dt} \propto & \left(|A_f|^2 - |\overline{A}_{\overline{f}}|^2\right)\left(\cosh(\Delta\Gamma/2t) + \cos(\Delta m t)\right) \\ & + \left(|\tfrac{q}{p}|^2|\overline{A}_f|^2 - |\tfrac{p}{q}|^2|A_{\overline{f}}|^2\right)\left(\cosh(\Delta\Gamma/2t) - \cos(\Delta m t)\right) \\ & + 2\mathrm{Re}\left(\frac{q}{p}A_f^*\overline{A}_f - \frac{p}{q}A_{\overline{f}}\overline{A}_{\overline{f}}^*\right)\sinh(\Delta\Gamma/2t) \\ & - 2\mathrm{Im}\left(\frac{q}{p}A_f^*\overline{A}_f - \frac{p}{q}A_{\overline{f}}\overline{A}_{\overline{f}}^*\right)\sin(\Delta m t).\end{aligned}$$

In charm: $\left\{x \equiv \frac{\Delta m}{M},\ y \equiv \frac{\Delta\Gamma}{2\Gamma}\right\} \ll 1$ which results in

$$A_{\rm CP}(f;t) \simeq A_{\rm CP}^{\rm direct}(f) + \frac{t}{\tau_D} a_{\rm CP}^{\rm ind}$$

and time-integrated

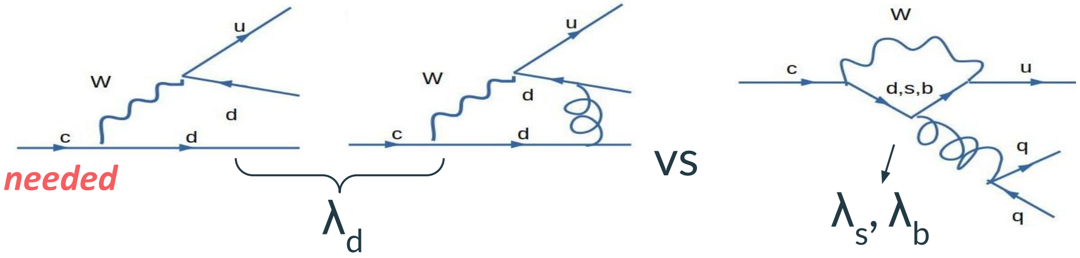
$$A_{\rm CP}(f) \simeq A_{\rm CP}^{\rm direct}(f) + \frac{\langle t_f\rangle}{\tau_D} a_{\rm CP}^{\rm ind}$$

$$a_{\rm CP}^{\rm ind} = \frac{\eta_{CP}}{2}\left[\underbrace{\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)}_{\text{mixing CPV}} y\ \cos\arg\lambda_f - \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\ \sin\arg\lambda_f\right]$$

interference CPV + final-state dependence

Sources of CP violation

At the quark level (full theory):



At the level of amplitudes:
Recall: ***different weak phases & strong phases needed***
For D→ ππ (similarly for D→KK):
One I=2 amplitude

$$\lambda_d \cdot \langle \pi\pi_{I=2} | (\bar{d}c)(\bar{u}d) | D \rangle$$

(current-current operators implied)

and several I=0 amplitudes

$$\lambda_d \cdot \langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle + \lambda_s \cdot \underbrace{\langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle}_{\text{“Long-distance penguin”}} - \lambda_b \cdot \underbrace{\langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle}_{\text{Short-distance penguin}}$$

Short-distance penguin
(**significant for Q_6 operator-annihilation topology**)

If ππ did not rescatter to KK:

$$\langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle = 0 \quad \text{AND}$$

$$\arg\langle \pi\pi_{I=0} | \text{penguin operators} | D \rangle = \arg\langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle$$

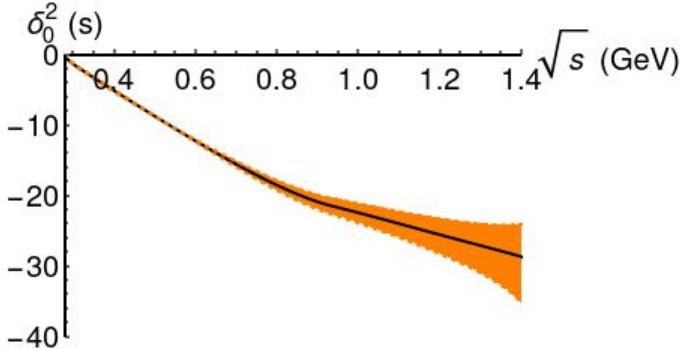
(Watson's theorem)

→ Only source ***would be*** interference of I=2 vs I=0 short-distance penguin

Instead: **more sources of CP violation now** ; no significant cancellations between different CPV sources

Isospin-two rescattering

S-wave isospin-two ππ phase



Elastic - admits Omnes solution

$$|A_{I=2}(D \to \pi\pi)(s)| = A_{I=2}(s_0) \times \underbrace{exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_0^2(z)}{(z-s_0)(z-s)}\}}_{\text{Omnès factor } \Omega}$$

which at infinity behaves as

$$\Omega(s) \sim \frac{1}{s^n}, \quad n = \frac{\delta_0^2(\infty) - \delta_0^2(4m_\pi^2)}{\pi}$$

and has to go to zero

$\longrightarrow$ phase has to go to positive multiples of π

KK in I=1: not available

Naive estimate of final-state-interaction effects

We can write [Bauer, Stech, Wirbel '86]

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S^{1/2} \cdot \begin{pmatrix} A^{I=0}_{\pi\pi,\mathrm{bare}} \\ A^{I=0}_{KK,\mathrm{bare}} \end{pmatrix}$$

$$\boxed{S_S = \begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}}$$

where the bare amplitudes come from factorization (no strong phases)

This reproduces correctly Watson's theorem in the limit of elastic rescattering

What S-matrix unitarity gives:

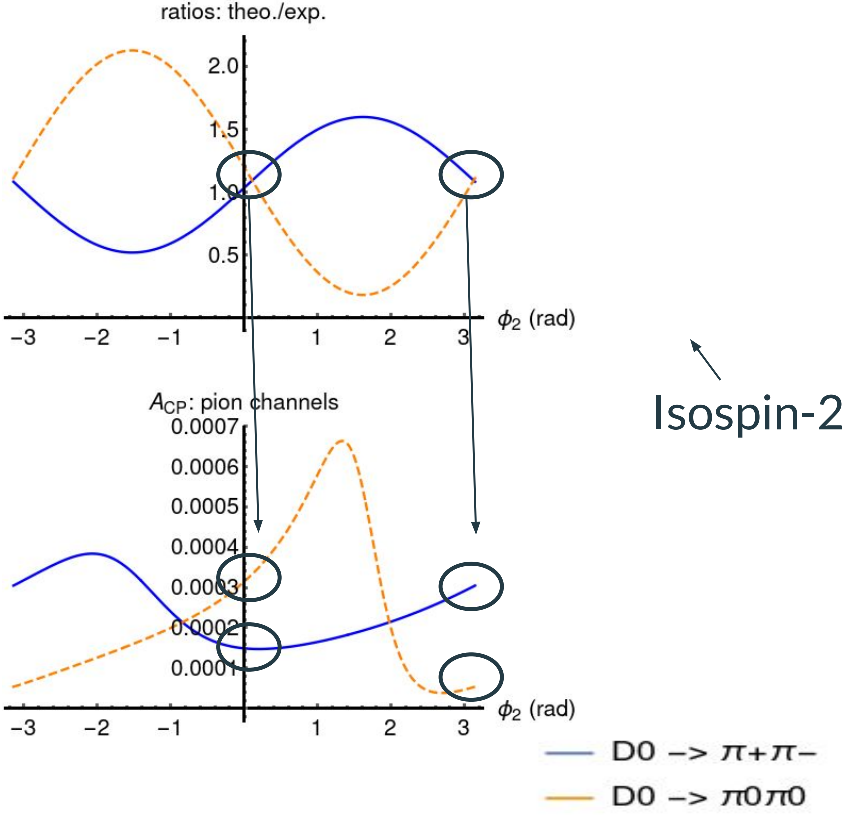
$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S \cdot \begin{pmatrix} (A^{I=0}_{\pi\pi})^* \\ (A^{I=0}_{KK})^* \end{pmatrix}$$

$\longrightarrow$ No direct solution for the amplitudes; can relate them to the rescattering phases

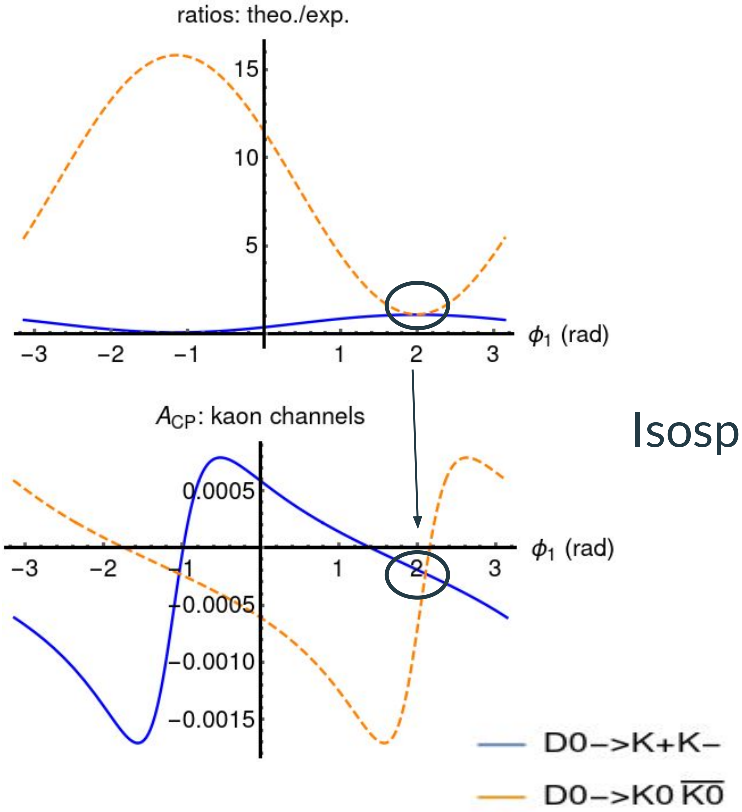
$$arg A^{I=0}_{\pi\pi} = \delta_1 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{KK}|}{|A^{I=0}_{\pi\pi}|}\right)^2(1-\eta^2)}{4\eta}}$$

$$arg A^{I=0}_{KK} = \delta_2 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{\pi\pi}|}{|A^{I=0}_{KK}|}\right)^2(1-\eta^2)}{4\eta}}$$

Br’s and ACP’s as functions of the free phases



Isospin-2



Isospin-1

Quantified sources of CP asymmetry

$A_{CP}(\pi^-\pi^+)$; $A_{CP}(\pi^0\pi^0)$	interference	expression	final numerics
numerator	I=0/I=0	$0.0019 \times \omega_\pi^{(\mathrm{Im})}$	0.00027
	I=0/I=2	$0.00041 \times \tilde{\omega}_{\pi 2}^{(\mathrm{Im})} + 0.00026 \times \tilde{\omega}_{\pi 1}^{(\mathrm{Im})}$; $-0.00081 \times \tilde{\omega}_{\pi 2}^{(\mathrm{Im})} - 0.00052 \times \tilde{\omega}_{\pi 1}^{(\mathrm{Im})}$	-0.00009; 0.00018
denominator	I=0/I=0	$\lvert\Omega_{11}^{(0)}\rvert^2 + 0.57 \times \lvert\Omega_{12}^{(0)}\rvert^2 - 1.51 \times \omega_\pi^{(\mathrm{Re})}$	1.11
	I=0/I=2	$0.64 \times \tilde{\omega}_{\pi 1}^{(\mathrm{Re})} - 0.49 \times \tilde{\omega}_{\pi 2}^{(\mathrm{Re})}$; $-1.28 \times \tilde{\omega}_{\pi 1}^{(\mathrm{Re})} + 0.97 \times \tilde{\omega}_{\pi 2}^{(\mathrm{Re})}$	0.03; -0.07
	I=2/I=2	$\lvert\Omega^{(2)}\rvert^2 \times 0.10$; $\lvert\Omega^{(2)}\rvert^2 \times 0.41$	0.08; 0.33

$A_{CP}(K^-K^+)$; $A_{CP}(K_S K_S)$	interference	expression	final numerics
numerator	I=0/I=0	$0.0019 \times \omega_K^{(\mathrm{Im})}$	-0.00032
	I=0/I=1	$0.0019 \times \tilde{\omega}_K^{(\mathrm{Im})}$; $-0.0019 \times \tilde{\omega}_K^{(\mathrm{Im})}$	-0.00019; 0.00019
denominator	I=0/I=0	$\lvert\Omega_{21}^{(0)}\rvert^2 + 0.57 \times \lvert\Omega_{22}^{(0)}\rvert^2 - 1.51 \times \omega_K^{(\mathrm{Re})}$	1.05
	I=0/I=1	$1.15 \times \tilde{\omega}_{K2}^{(\mathrm{Re})} - 1.51 \times \tilde{\omega}_{K1}^{(\mathrm{Re})}$; $-1.15 \times \tilde{\omega}_{K2}^{(\mathrm{Re})} + 1.51 \times \tilde{\omega}_{K1}^{(\mathrm{Re})}$	1.23; -1.23
	I=1/I=1	$\lvert\Omega^{(1)}\rvert^2 \times 0.57$	0.36

If I=2 was inelastic

$$A(D^0 \to \pi^+\pi^-) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} - \frac{1}{2\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{6}}|A_{\pi\pi}^{I=0}|e^{i\delta_{\pi\pi,0}} + \frac{1}{\sqrt{3}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

$$A(D^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2\sqrt{2}}|A_{\pi\pi}^{I=2}|e^{i\delta_{\pi\pi,2}}$$

Still true; extraction from D+ Br is valid, just no longer representing the Omnes function
If inelasticities are to ππKK: In large Nc corresponds to initial weak decay like D⟶?φ (?=an isospin-2 resonance), CKM factor associated: λ_s
⟶ additional I=2/2 interference would contribute to CPV; different interferences between I=2 and I=0

Extrapolation to high energies

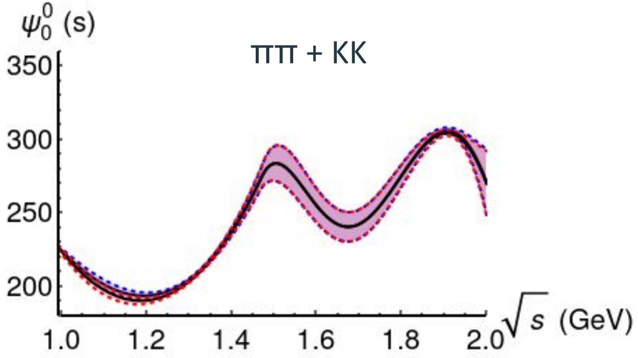
$$\delta_0^0(E) = n^*\pi + (\delta_0^0(E_0) - n^*\pi) f_\delta\left(\cancelto{0}{\frac{E}{E_0}}\right),$$

$$\delta_K(E) = \ell^*\pi + (\delta_K(E_0) - \ell^*\pi) f_\delta\left(\cancelto{0}{\frac{E}{E_0}}\right)$$

$$\eta_0^0(E) = \eta_\infty + (\eta_0^0(E_0) - \eta_\infty) f_\eta\left(\cancelto{0}{\frac{E}{E_0}}\right)$$

$\eta_\infty \rightarrow 1$ (strong coupling $\rightarrow$ 0)

No data available; if only resonances, sum of phases of all communicating channels → multiple of π



If there was a third channel: we would still need the sum of all phases to go to 3π. Then ππ + KK could go to e.g. π

More “natural” to send to 2π than ≥3π ; no resonances past that energy

CP asymmetries in the rare decays

see also [2312.07501]

The unnormalised CP-asymmetric observables e.g. from the P-wave go as

$$\mathrm{Im}(\lambda_s \lambda_d^*)\mathrm{Im}(C_{9d}^P C_{9s}^{P*}) \tag{1}$$

where roughly

$$C_{9d}^P = \frac{e^{i\delta_{\rho\rho}}}{P_\rho(q^2)} \tag{2}$$

$$C_{9s}^P = \frac{e^{i\delta_{\rho\phi}}}{P_\phi(q^2)} \tag{3}$$

This on top of the resonance gives $3 \cdot 10^{-5}$ (from the CKM) x (up to 500).

On the other hand, the observables are normalised to the decay rates, which go as

$$|\lambda_d|^2 |C_{9d}^P + C_{9d}^S|^2 \tag{4}$$

which gives $5 \cdot 10^{-2}$ (from the CKM) x (up to $5 \cdot 10^4$).

Thus the effect on top of the resonances is very small. On the contrary, away from the resonances there are some comparative enhancement patterns. Still because of the typical CKM suppression factor $6.4 \cdot 10^{-4}$ of charm decays the overall, normalised CP-asymmetrical observables are expected to be very small, less than per mille.

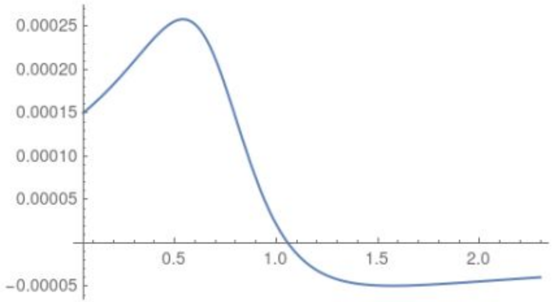


Figure 1: Generic CP-asymmetric observable A over generic CP-symmetric observable S/differential decay rate, as a function of the invariant mass of the dimuon. CKM factors included.