Are there quirks in the $D \to \pi\ell^+\ell^-$ decays ?

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(work in progress with Anshika Bansal and Thomas Mannel)

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Introduction

- $D^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$ - the simplest decay with $c \to u\ell^{+}\ell^{-}$ transition
- FCNC in the charmed sector, enhanced in various BSM scenarios
- in SM the smallness of

$$\left|\frac{\lambda_b}{\lambda_d}\right| \equiv \left|\frac{V_{ub}V_{cb}^*}{V_{ud}V_{cd}^*}\right| \sim 0.7 \times 10^{-3}$$

and the CKM unitarity $\lambda_d + \lambda_s + \lambda_b = 0$ yield $\lambda_s \simeq -\lambda_d$

- two different approximations: GIM limit, $\lambda_b = 0$, $\lambda_d = -\lambda_s$

  $SU(3)_{fl}$ limit, $m_s = m_{u,d}$

$c \to u$ transitions in Standard Model

- effective Hamiltonian

$$H_{eff}^{(\Delta S=0)} = \frac{4G_F}{\sqrt{2}} \sum_{\mathcal{D}=d,s} \lambda_{\mathcal{D}} \left[C_1(\mu) O_1^{\mathcal{D}} + C_2(\mu) O_2^{\mathcal{D}} \right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i \,,$$

$$O_1^{\mathcal{D}} = (\bar{u}_L \gamma_\mu \mathcal{D}_L)\,(\bar{\mathcal{D}}_L \gamma^\mu c_L)\ , \quad O_2^{\mathcal{D}} = (\bar{u}_L \gamma_\mu t^a \mathcal{D}_L)\,(\bar{\mathcal{D}}_L \gamma^\mu t^a c_L)\ , \quad (\mathcal{D} = d, s)\,,$$

- the Wilson coefficients
  at $\mu = 1.3$ GeV at NNLO:

$C_1 = 1.034,\ C_2 = -0.633,$

$|C_{3,\ldots,8}| < 0.09,\ C_9 = -0.488,\ C_{10} = 0$


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Higher-order Wilson coefficients for $c \to u$ transitions in the standard model

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- Hamiltonian in the GIM limit:

$$H_{eff}^{(\Delta S=0,\, \lambda_b=0)} = \frac{4G_F}{\sqrt{2}} \lambda_d \left[C_1 \;\left(O_1^d - O_1^s\right) + C_2 \;\left(O_2^d - O_2^s\right) \right]$$

- the largest effect beyond GIM limit $\sim \lambda_b C_9$

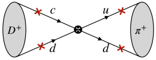
Decay mechanism

- Singly-Cabibbo-suppressed (SCS) weak transition

  $\oplus$ e.m. lepton pair emission

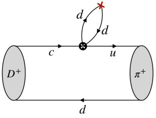
- the two topologies:

  annihilation $\Rightarrow$



(a)

d, s loops $\Rightarrow$



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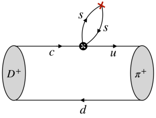


(c)

- the decay amplitude:

$$A(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em}G_F}{\sqrt{2}}\right) \lambda_d \frac{\bar{u}_\ell(q_-)\gamma^\mu v_\ell(q_+)}{q^2} \mathcal{A}_\mu^{(D^+ \to \pi^+ \gamma^*)}(p,q) \,,$$

- hadronic matrix element – a “nonlocal form factor”

  cf. $B \to K\ell\ell$ with a different hierarchy of topologies

$$\mathcal{A}_\mu^{(D^+ \to \pi^+ \gamma^*)}(p,q) = i \int d^4x e^{iq\cdot x} \langle \pi^+(p) | T\left\{ j_\mu^{em}(x), H_{eff}^{(\Delta S=0,\, \lambda_b=0)}(0) \right\} | D^+(p+q) \rangle$$

$$= \left[(p\cdot q) q_\mu - q^2 p_\mu \right] A^{(D^+ \to \pi^+ \gamma^*)}(q^2) \,,$$

- we need this amplitude at $0 < q^2 < (m_D - m_\pi)^2$

Employing the U-spin symmetry

- adopting, in addition to GIM, also the $SU(3)_{fl}$ limit.
- the three Hamiltonians of CF, SCS, DCS form a U-triplet:

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L\gamma_\mu s_L)\,(\bar{d}_L\gamma^\mu c_L) \\ 1/\sqrt{2}\,[(\bar{u}_L\gamma_\mu d_L)\,(\bar{d}_L\gamma^\mu c_L) - (\bar{u}_L\gamma_\mu s_L)\,(\bar{s}_L\gamma^\mu c_L)] \\ (\bar{u}_L\gamma_\mu d_L)\,(\bar{s}_L\gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} & |1,+1\rangle \\ - & |1,0\rangle \\ - & |1,-1\rangle \end{pmatrix},$$

- adding e.m. interaction does not alter the U-spin properties

$$\tilde{O}_\mu^{(U=1)}(q) = i\int d^4x e^{iq\cdot x}\, T\{j_\mu^{em}(x), [C_1(\mu)O_1^{U=1}(0) + C_2(\mu)O_2^{U=1}(0)]\}\,.$$

- new relations (two independent U-invariant amplitudes)

$$A^{(D^+\to\pi^+\gamma^*)}(q^2) = -A^{(D_s^+\to K^+\gamma^*)}(q^2) = A^{(D_s^+\to\pi^+\gamma^*)}(q^2) = A^{(D^+\to K^+\gamma^*)}(q^2)$$

$$A^{(D^0\to\bar{K}^0\gamma^*)}(q^2) = A^{(D^0\to K^0\gamma^*)}(q^2) = -1/2A^{(D^0\to\pi^0\gamma^*)}(q^2) + \sqrt{3}/2A^{(D^0\to\eta_8\gamma^*)}(q^2)$$

$$A^{(D^0\to\eta_8\gamma^*)}(q^2) = -\sqrt{3}A^{(D^0\to\pi^0\gamma^*)}(q^2)\,,$$

$$A^{(D^0\to\eta'\gamma^*)}(q^2) = 0\,,$$

- in this limit only the annihilation topology contributes
- measuring the CF modes, e.g. $D_s \to \pi^+\ell^+\ell^-$ will allow to disentangle this topology

Use of QCD factorization

- the method suggested originally for $B \to K^* \ell^+ \ell^-$ decays

M. Beneke, T. Feldmann and D. Seidel, [arXiv:hep-ph/0106067 [hep-ph]].

- first used for $D \to \rho \ell^+ \ell^-$ in

T. Feldmann, B. Müller and D. Seidel, [arXiv:1705.05891 [hep-ph]].

  - the loop topology diagram modified to include resonances

(Shifman model of loop-resonance duality)

- a similar method applied to $D \to \pi \ell^+ \ell^-$:

A. Bharucha, D. Boito and C. Meaux, [arXiv:2011.12856 [hep-ph]].

- annihilation at LO,
included is only one of the four diagrams
(emission from the initiial d-quark)

- some NLO and nonfactorizable
corrections to the loop topology

- missing $1/m_c^2$ corrections
e.g., use of D-meson distribution
amplitude

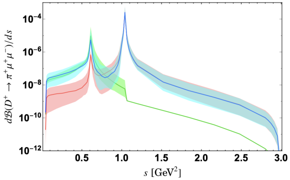


Figure 4. Distribution of the $\mathcal{B}(D^+ \to \pi^+\mu^+\mu^-)$ in the SM. In blue the total branching ratio, in red the contribution from $Y^{(d)}$, and in green that of $C_9^{(d)}|_{\text{Ann}}$ including resonance effects as per eq. (4.18). The error bands on the SM prediction, determined as explained in section 4.3, are shaded.

What does experiment tell us?

- Upper bounds from PDG :

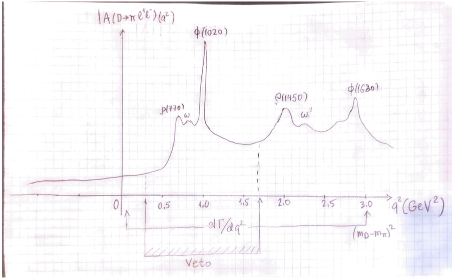
Decay mode	Cabibbo hierarchy	BR, exp. upper limit [10]
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1 \times 10^{-6}(\ell = e)$ $6.7 \times 10^{-8}(\ell = \mu)$
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5 \times 10^{-7}(\ell = e)$ $5.4 \times 10^{-8}(\ell = \mu)$
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4 \times 10^{-5}(\ell = e)$ $2.6 \times 10^{-4}(\ell = \mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3 \times 10^{-6}(\ell = e)$ $5.3 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-7}(\ell = \mu)$
$D_s^+ \to K^+\ell^+\ell^-$	SCS	$3.7 \times 10^{-6}(\ell = e)$ $1.4 \times 10^{-7}(\ell = \mu)$

Table 1. The $D_{(s)} \to P\ell^+\ell^-$ decay modes and their currently measured upper limits of the branching fractions.

- Most recent LHCb measurements: [2011.00217]

vetoing the region [525 MeV, 1250 MeV]

Can we isolate resonances ?



- The full amplitude represented via hadronic dispersion relation (modulo subtraction)

$$A^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{f_V|A_{D^+\pi^+V}|e^{i\varphi_V}}{(m_V^2 - q^2 - im_V\Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{s - q^2 - i\epsilon} \,.$$

$A_{D^+\pi^+V}$ - amplitude of nonleptonic $D\to\pi V$ decay , f_V - decay constant

- Dispersion relation tells us:

vetoing a certain q^2-region does not remove resonances from the amplitude

- the radial excitations of ρ,ω,ϕ and the "tail" at $s>(m_D-m_\pi)^2$ are indispensable (cf. pion e.m. formfactor)

The method: LCSR-supported dispersion relation

- The idea: to calculate $A^{(D^+\to\pi^+\gamma^*)}(q^2)$ at $q^2<0$ from LCSRs and match the result to dispersion relation
- resembling partly the analysis of nonlocal effects in $B\to K^*\ell^+\ell^-$

  AK, T. Mannel and Y. M. Wang [1211.0234], AK and A. V. Rusov [1703.04765], N. Gubernari, M. Reboud, D. van Dyk and J. Virto, [2011.09813].
- The correlation function depending on $P^2=(p+q-k)^2$, $(p+q)^2$, q^2

$$\mathcal{F}_\mu(p,q,k)=-\int d^4x\, e^{iq\cdot x}\int d^4y\, e^{-i(p+q)\cdot y}$$
$$\times\langle\pi^+(p-k)|T\Big\{j_\mu^{em}(x)\, H_{eff}^{(\Delta S=0,\,\lambda_b=0)}(0) j_5^D(y)\Big\}|0\rangle\,,$$

- finite m_c, OPE with pion distribution amplitudes
- introducing artifitial four-momentum in the weak vertex

  used before in LCSR analysis of $B\to 2\pi$ and $D\to 2\pi, K\bar{K}$

  AK, [arXiv:hep-ph/0012271]

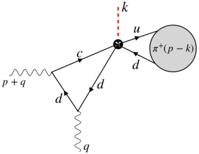
  AK, T. Mannel, M. Melcher and B. Melic, [arXiv: hep-ph/0304179, hep-ph/0509049].

  AK and A. A. Petrov, [arXiv:1706.07780].
- common input with LCSRs for $D\to\pi$ form factors

  AK, C. Klein, T. Mannel and N. Offen,[arXiv:0907.2842 [hep-ph]].

the LO diagrams

- annihilation diagram



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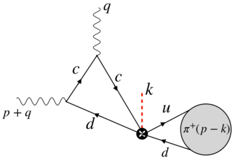

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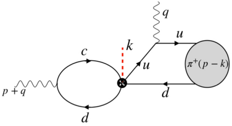

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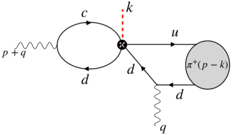
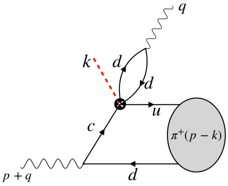


(d)

- loop diagrams (factorizable into loop function and LCSR for $D \to \pi$ FF)



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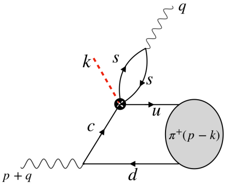

(b)

getting the LCSR

- dispersion relation at $q^2 < 0$ in D-meson channel

$$\frac{1}{\pi}\int\limits_{m_c^2}^{\infty} ds \frac{\mathrm{Im}F^{(OPE)}(s,q^2,P^2=m_D^2)}{s-(p+q)^2} = \frac{m_D^2 f_D A^{(D^+\to\pi^+\gamma^*)}(q^2)}{m_D^2-(p+q)^2} + \int_{s_{h_d}}^{\infty} ds \frac{\rho_{h_D}(s,q^2,P^2=m_D^2)}{s-(p+q)^2},$$

- quark-hadron duality in D meson channel and Borel transform

$$A^{(D^+\to\pi^+\gamma^*)}(q^2) = \frac{e^{-m_D^2/M^2}}{m_D^2 f_D}\frac{1}{\pi}\int\limits_{m_c^2}^{s_0^D} ds e^{-s/M^2}\,\mathrm{Im}F^{(OPE)}(s,q^2,m_D^2)\,, \qquad (2)$$

- using this result valid at $q^2 < 0$ we will fit the unknown parameters of the hadronic dispersion relation,

using data on nonleptonic $D \to \pi V$ amplitudes and f_V

- going beyond GIM limit is straightforward:

$$A^{(O_9)}(D^+ \to \pi^+\ell^+\ell^-) = -\frac{\alpha_{em} G_F}{\sqrt{2}\pi}\lambda_b C_9\,[\bar{u}_\ell(q_+)\gamma_\mu u_\ell(q_-)]\,p^\mu\,[f^+_{D\pi}(q^2)]_{LCSR}\,,$$

Outlook

- Numerical analysis at LO in progress ,
- what else can be done:
  - soft-gluon corrections to annihilation
  - CF modes, $m_s \neq 0$ in the correlator, pion DA$\rightarrow$ kaon DA
  - varying resonance ansatz in the dispersion relation,
    including η as well as ρ', ω', ϕ', *employingShifmanmodel*
  - beyond GIM limit: CP-asymmetry, inflating C_9 (BSM) etc

- to test SM in these decays: need measurements of the differential decay rates $d\Gamma/dq^2(D_{(s)} \rightarrow P\ell^+\ell^-)$ in the whole q^2 region