

# Exploring Semileptonic $B_s \rightarrow D_s^* l \bar{\nu}_l$ Decays

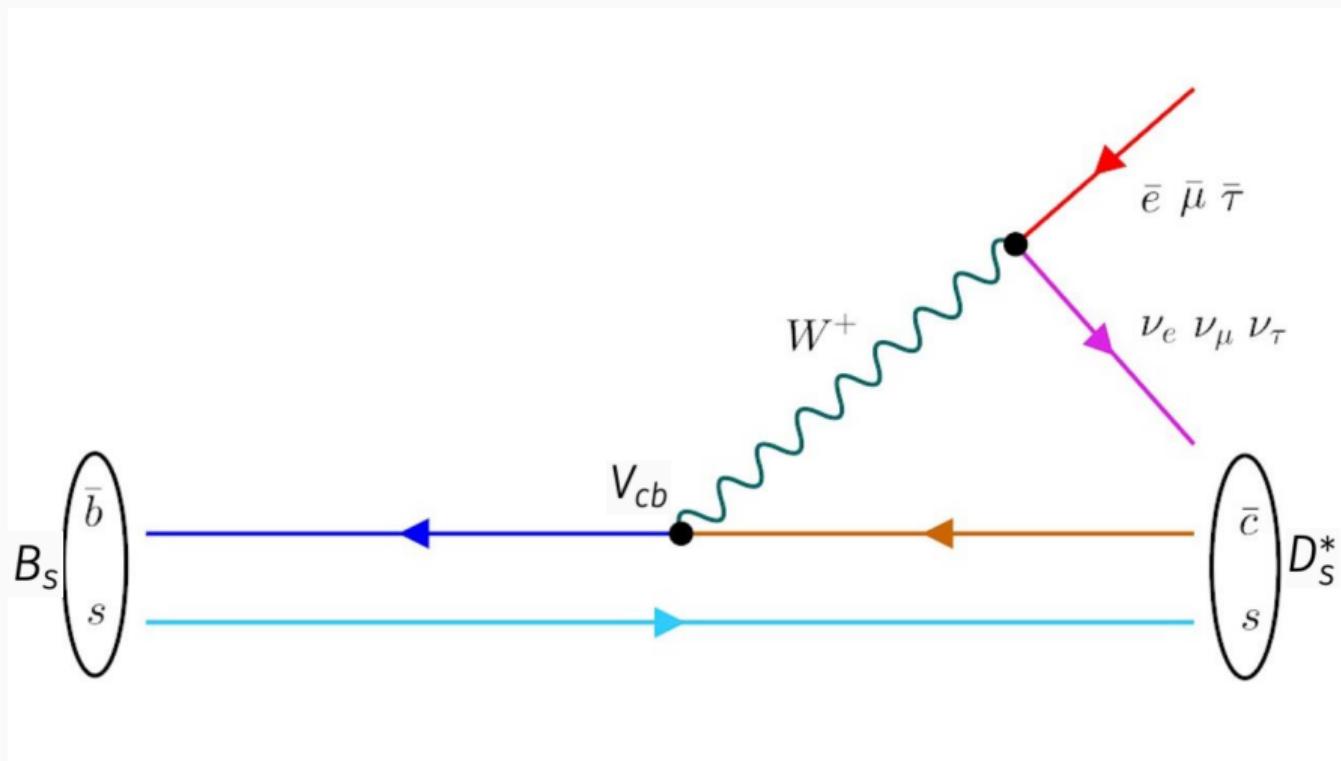
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## Semileptonic $B_s \rightarrow D_s^* l \nu_l$ Decays



- $B \rightarrow D^* l \nu_l$  similar, just different spectator

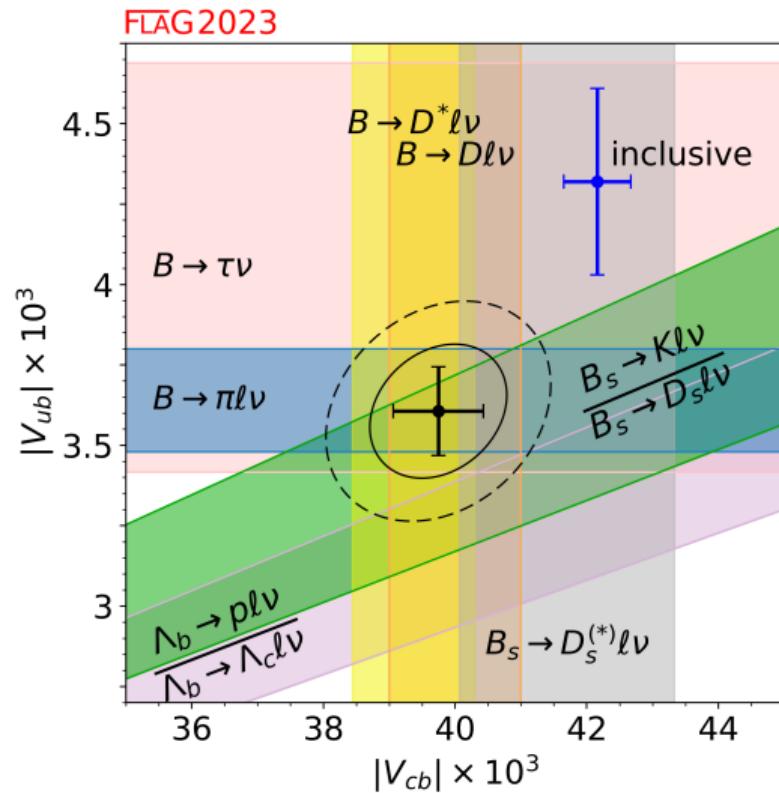
# The CKM Matrix

- The Standard Model has six quark flavours
- Probability for transition of one flavour to another
- Parameters can be determined from a combination of experiment and theory
- Hierarchical structure

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97373 \pm 0.00031 & 0.2243 \pm 0.0008 & (3.82 \pm 0.20) \times 10^{-3} \\ 0.221 \pm 0.004 & 0.975 \pm 0.006 & (40.8 \pm 1.4) \times 10^{-3} \\ (8.6 \pm 0.2) \times 10^{-3} & (41.5 \pm 0.9) \times 10^{-3} & 1.014 \pm 0.029 \end{pmatrix}$$

[PDG, Workman et al. PTEP (2022) 083C01]

# Motivation: Inclusive vs. Exclusive $V_{cb}$

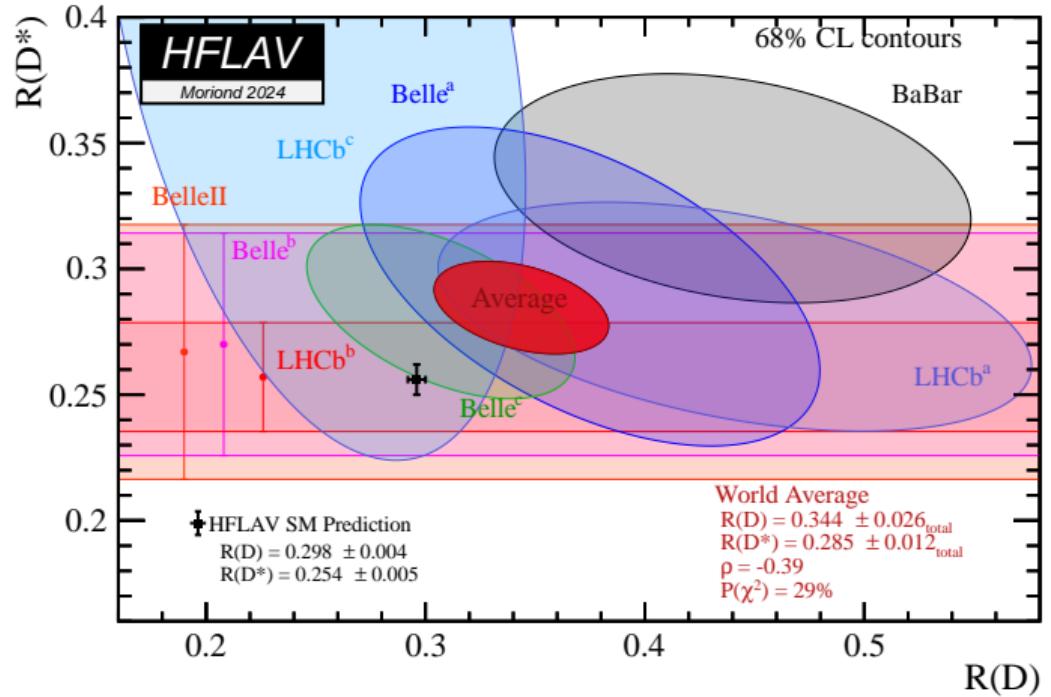


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- 2 – 3 $\sigma$  tension between inclusive and exclusive
- $V_{cb}^{incl} = (42.2 \pm 0.8) \times 10^3$
- $V_{cb}^{excl} = (39.4 \pm 0.8) \times 10^3$

[PDG, Workman et al. PTEP (2022) 083C01]  
FLAG 2023

# Motivation: Test Lepton Flavour Universality



$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}l\nu_l)}$$

with  $l = e, \mu$

HFLAV, Moriond 2024

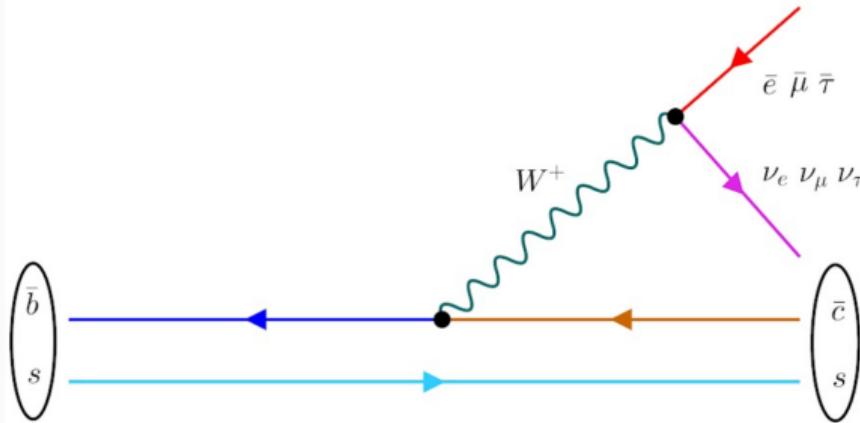
# Determining $V_{cb}$ from semileptonic Decays

$$\frac{d\Gamma(B_{(s)} \rightarrow D_{(s)}^* l \bar{\nu}_l)}{dq^2} = \mathcal{K}_{D^*}(q^2, m_l) \cdot |\mathcal{F}(q^2)|^2 \cdot |V_{cb}|^2$$

From Experiment	Known Factors	Nonpert. Input	CKM
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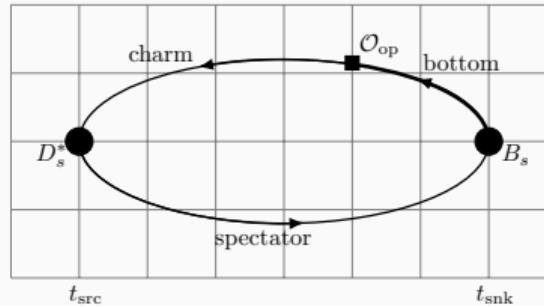
- Form factors from LQCD, LCSR
- Experiments: BaBar, BELLE, BELLE 2, LHCb
- Vector final states are experimentally favoured

# Exclusive $B_s \rightarrow D_s^*$ decay



- Use Narrow Width Approximation for  $D_{(s)}^*$ 
  - Treated as stable particles
- Focus on  $B_s$  and  $D_s^*$ 
  - LHCb data
  - Heavier spectator is easier to determine on lattice
  - In a later stage of this project: also  $B \rightarrow D^*$

# Relativistic Form Factors



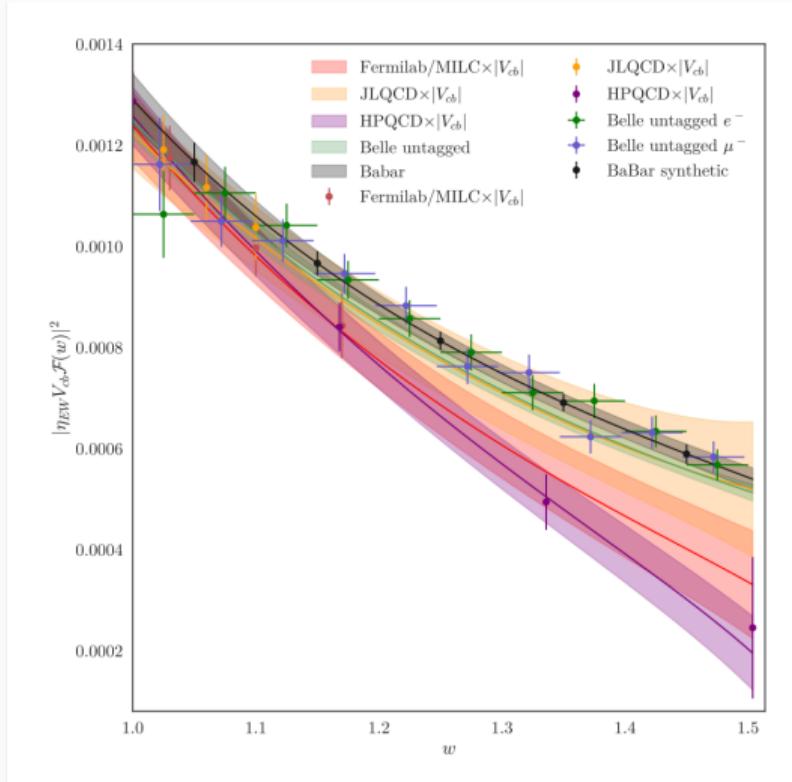
$$\begin{aligned}
 \langle D_{(s)}^*(k, \lambda) | \bar{c} \gamma^\mu b | B_{(s)}(p) \rangle &= V(q^2) \frac{2i\epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_{B_{(s)}} + M_{D_{(s)}^*}} \\
 \langle D_{(s)}^*(k, \lambda) | \bar{c} \gamma^\mu \gamma_5 b | B_{(s)}(p) \rangle &= A_0(q^2) \frac{2M_{D_{(s)}^*} \varepsilon^* \cdot q}{q^2} q^\mu \\
 &\quad + A_1(q^2) (M_{B_{(s)}} + M_{D_{(s)}^*}) \left[ \varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\
 &\quad - A_2(q^2) \frac{\varepsilon^* \cdot q}{M_{B_{(s)}} + M_{D_{(s)}^*}} \left[ k^\mu + p^\mu - \frac{M_{B_{(s)}}^2 - M_{D_{(s)}^*}^2}{q^2} q^\mu \right]
 \end{aligned}$$

# HQET Form Factors

$$\frac{\langle D_{(s)}^*(k, \varepsilon) | \mathcal{A}^\mu | B(p) \rangle}{\sqrt{M_{D^*} M_B}} = i \varepsilon_\nu^* \left[ g^{\mu\nu} (w + 1) h_{A_0}(w) - v_B^\nu (v_B^\mu h_{A_1}(w) + v_{D_{(s)}^*}^\mu h_{A_2}(w)) \right]$$
$$\frac{\langle D_{(s)}^*(k, \varepsilon) | \mathcal{V}^\mu | B(p_B) \rangle}{\sqrt{M_{D^*} M_B}} = \epsilon^{\mu\nu}_{\rho\sigma} \varepsilon_\nu^* v_B^\rho v_{D_{(s)}^*}^\sigma h_V(w), \quad \text{with} \quad w = v_{B_s} \cdot v_{D_{(s)}^*}$$

- Heavy to heavy transitions  $\rightarrow$  HQET form factors

# Existing Results for Form Factors



- $V_{cb}^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$
- $V_{cb}^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$
- $V_{cb}^{\text{FNAL-MILC}} = 38.17(85) \times 10^{-3}$

[HPQCD, Harison et al. (2022), PRD 105.094506]  
[JLQCD, Aoki et al. (2023), PRD 109.074503]  
[FNAL-MILC, Bazavov et al. (2022), EPJC 81.1141]

## Our Work

## Lattice Set Up

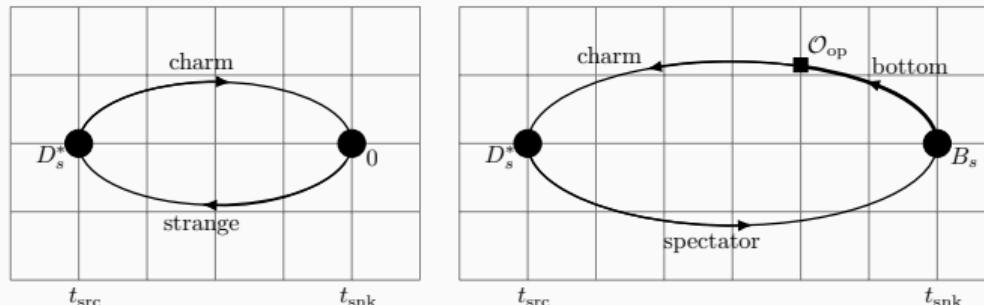
- RBC/UKQCD's 2+1 flavour gauge field ensembles
- Dynamical up/down and strange quarks in the sea and light sector using chiral domain-wall fermions
- Specifically optimized heavy domain-wall fermions for charm
- Relativistic heavy quark (RHQ) action for bottom
- Bottom, charm and strange close to physical

	L	T	$a^{-1}$ GeV	$am_l^{sea}$	$am_s^{sea}$	$M_\pi / \text{MeV}$	$\text{srcs} \times N_{\text{conf}}$
C1	24	64	1.7848	0.005	0.040	340	$1 \times 1636$
C2	24	64	1.7848	0.010	0.040	433	$1 \times 1419$
M1	32	64	2.3833	0.004	0.030	302	$2 \times 628$
M2	32	64	2.3833	0.006	0.030	362	$2 \times 889$
M3	32	64	2.3833	0.008	0.030	411	$2 \times 544$
F1S	<b>48</b>	<b>96</b>	<b>2.785</b>	<b>0.002144</b>	<b>0.02144</b>	<b>268</b>	<b><math>24 \times 98</math></b>

# Extract Form Factors

- Define 3pt - 2pt ratios
- Different combinations of polarizations, operators and momenta give access to formfactors

$$R_{B_s \rightarrow D_s^*}^\Gamma(t, t_{sink}) = \frac{C_{B_s \rightarrow D_s^*}^{3pt, \Gamma, \mu}(t, t_{sink}, k)}{\frac{1}{3} \sqrt{\sum_i C_{D_s^*}^{2pt}(t, k) C_{B_s}^{2pt}(t_{sink} - t, p)}} \sqrt{\frac{4 E_{D_s^*} M_{B_s} \sum_{\lambda, j} \varepsilon_j(k, \lambda) \varepsilon^{*j}(k, \lambda)}{e^{-E_{D_s^*}} e^{-M_{B_s}(t_{sink} - t)}}}$$
$$\xrightarrow[t_{sink} \rightarrow \infty]{t \rightarrow \infty} \sum_\lambda \varepsilon^\mu(k, \lambda) \langle D_s^*(k, \lambda) | \bar{c} \Gamma b | B_s(p) \rangle$$



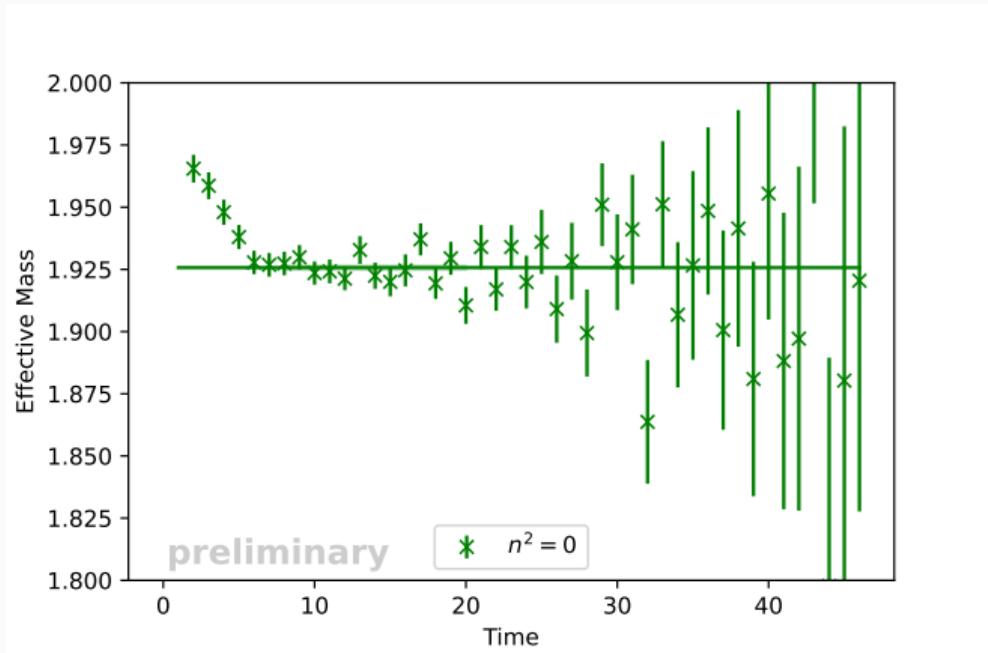
## Extract Form Factors

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- Define 3pt - 2pt ratios
- Different combinations of polarizations, operators and momenta give access to form factors
- Example for one **lattice** form factor

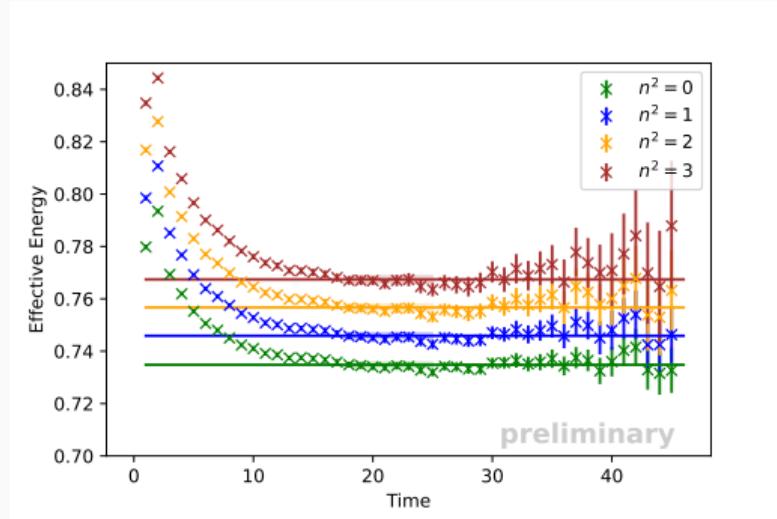
$$\tilde{A}_0(q^2) = \frac{1}{2} \frac{M_{D_s^*}}{E_{D_s^*} M_{B_s}} \frac{1}{k^\nu} q_\mu \sum_\lambda \varepsilon^\nu(k, \lambda) \langle D_s^*(k, \lambda) | \bar{c} \gamma^\mu \gamma_5 b | B_s(0) \rangle$$

# Effective Mass of $B_s$



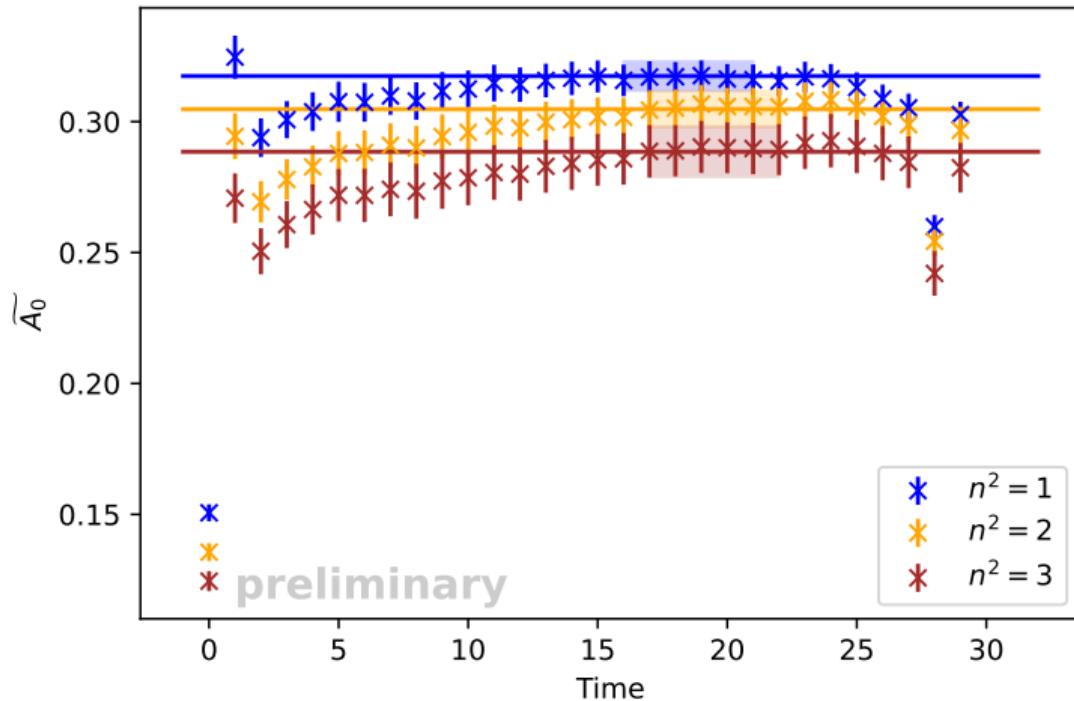
- Fit range: 8-20
- $M_{eff}^{B_s} = 1.92571(97)$
- In physical units:  $5.3631(27)$  GeV      (PDG:  $M_{B_s} = 5.36696(10)$  GeV)

# Effective Energy of $D_s^*$



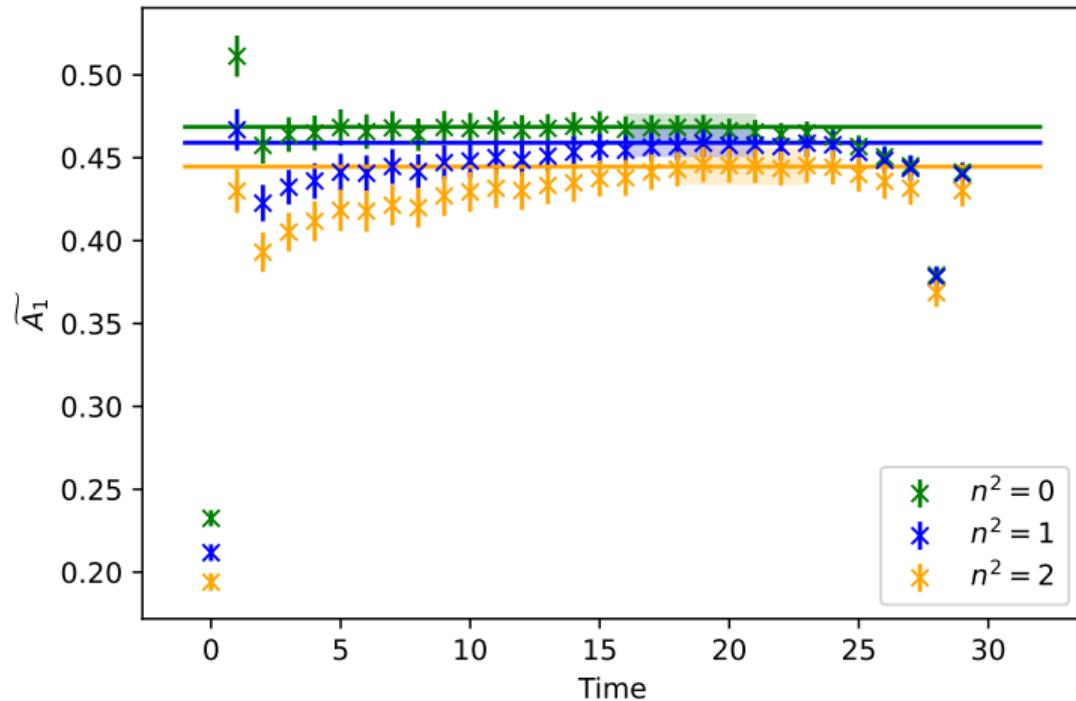
- $E_{eff}^{D_s^*}(n^2 = 0) = 0.7348(10)$
- $E_{eff}^{D_s^*}(n^2 = 1) = 0.7458(11)$
- $E_{eff}^{D_s^*}(n^2 = 2) = 0.7567(12)$
- $E_{eff}^{D_s^*}(n^2 = 3) = 0.7673(14)$
- In physical units: 2.0464(28) GeV
- PDG:  $M_{D_s^*} = 2.12212(4)$  GeV
- Fit ranges: 18-25

# Form Factor $\tilde{A}_0$



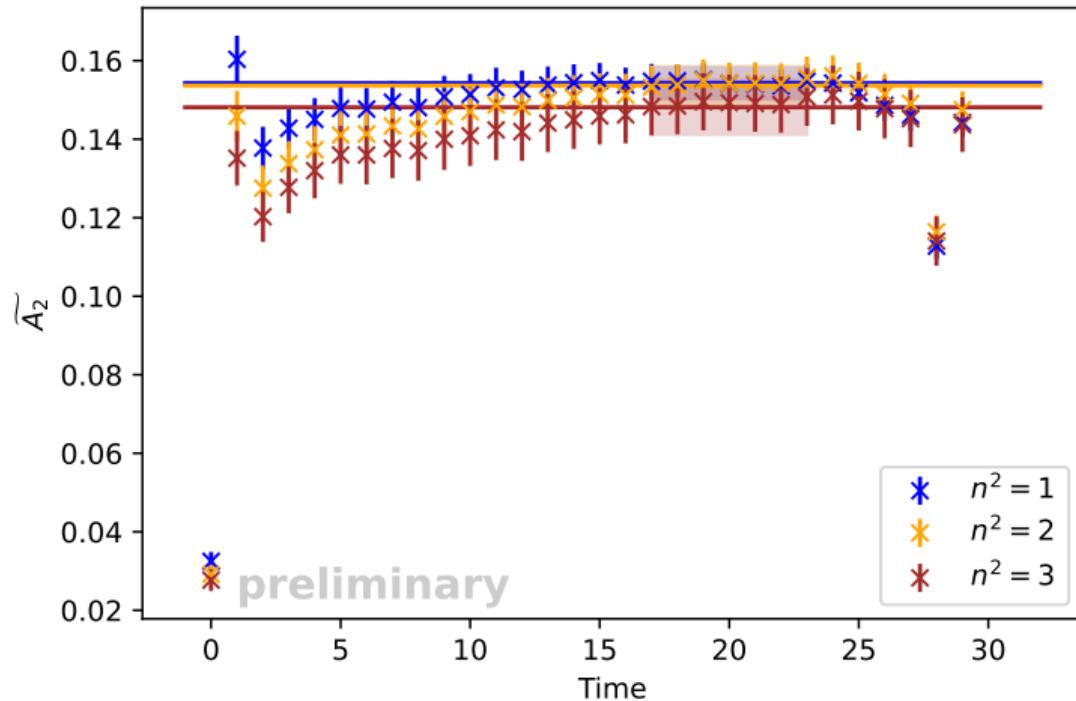
$$\begin{aligned}\tilde{A}_0(n^2 = 1) &= 0.3174(55) \\ \tilde{A}_0(n^2 = 2) &= 0.3047(70) \\ \tilde{A}_0(n^2 = 3) &= 0.2885(95)\end{aligned}$$

# Form Factor $\tilde{A}_1$



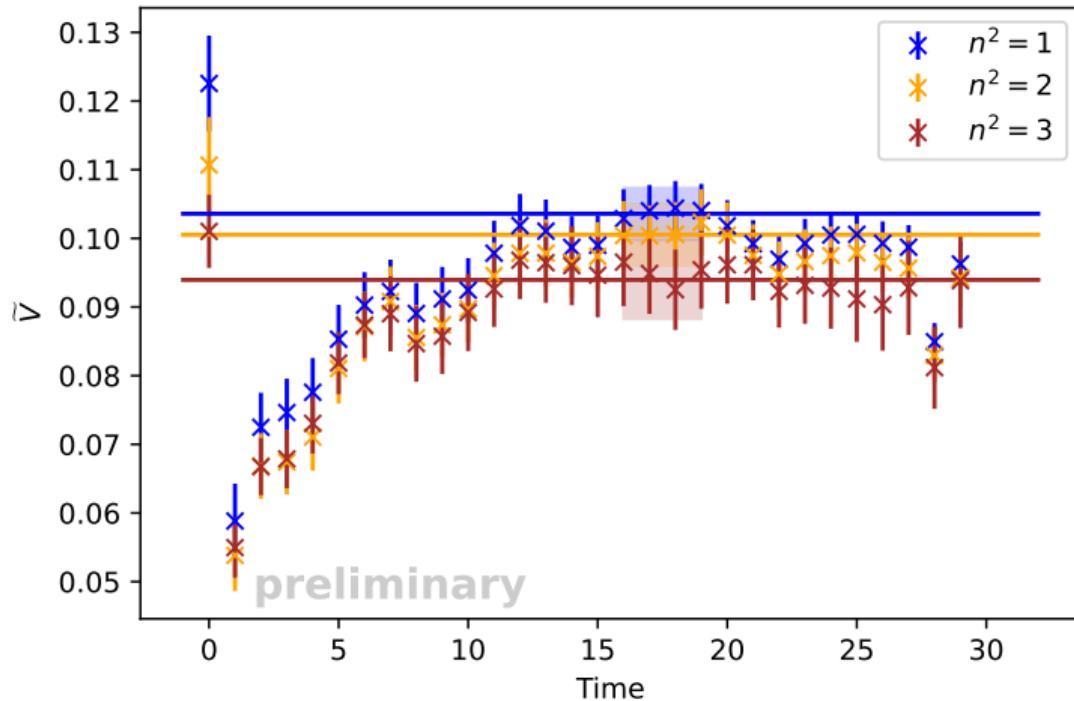
$$\begin{aligned}\tilde{A}_1(n^2 = 0) &= 0.4686(72) \\ \tilde{A}_1(n^2 = 1) &= 0.4591(78) \\ \tilde{A}_1(n^2 = 2) &= 0.445(10)\end{aligned}$$

# Form Factor $\tilde{A}_2$



$$\begin{aligned}\tilde{A}_2(n^2 = 1) &= 0.1542(40) \\ \tilde{A}_2(n^2 = 2) &= 0.1537(51) \\ \tilde{A}_2(n^2 = 3) &= 0.1481(70)\end{aligned}$$

# Form Factor $\tilde{V}$



$$\begin{aligned}\tilde{V}(n^2 = 1) &= 0.1036(37) \\ \tilde{V}(n^2 = 2) &= 0.1005(45) \\ \tilde{V}(n^2 = 3) &= 0.0940(57)\end{aligned}$$

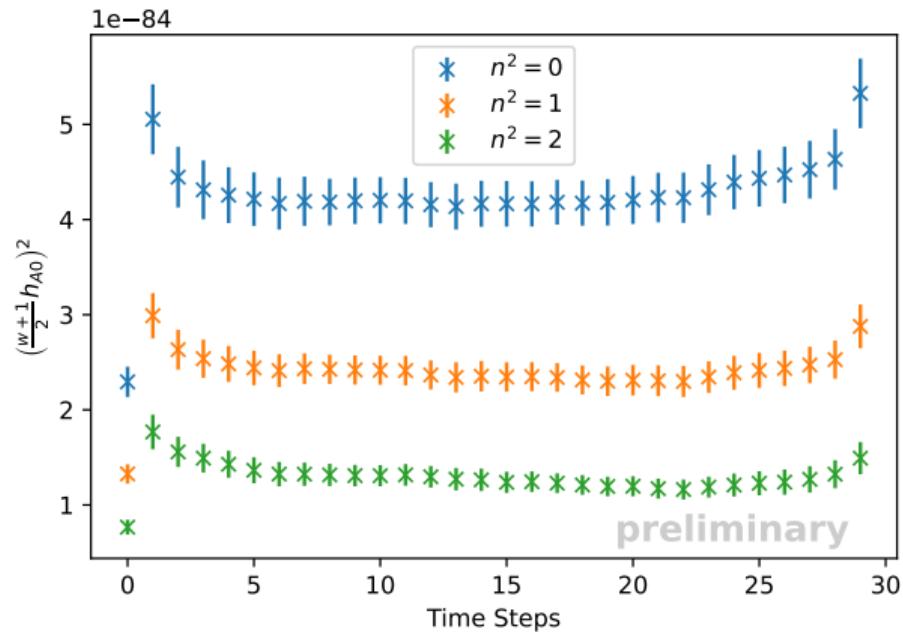
## Extract HQET Form Factors

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- Different approach than relativistic form factors: Double ratios of 3pt functions
- So far: Denominator set to 1 as blinding factor

$$\frac{\langle D^*(p_\perp) | A_j | B(0) \rangle \langle B(0) | A_j | D^*(p_\perp) \rangle}{\langle D^*(0) | V^4 | D^*(0) \rangle \langle B(0) | V^4 | B(0) \rangle} \sim \left[ \frac{w+1}{2} h_{A_0}(w) \right]^2$$

# First Results for HQET Form Factors



# Next Steps

Short term:

- Analyse other ensembles
- Include other charm masses
- Include order  $a$  improvement terms
- Perform excited state fits
- Determine renormalization factors
- Explore improved set-up

Long term:

- Combine into global fit: remove discretization artifacts
- Extra- / interpolate to physical quark mass
- Take continuum limit
- Kinematical extrapolation
- Estimate systematic effects
- Analyse  $B \rightarrow D^*$