$Q_1 - Q_{7\gamma}$ resolved-photon contribution to $B \to X_s\gamma$ at $\mathcal{O}(\alpha_s)$

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work in progress

with R. Bartocci and T. Hurth

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Motivation

. . . this is a continuation of T. Hurth's talk. A short recap:

- CP-averaged photon-energy spectrum of $\bar{B} \to X_s\gamma$ decays:

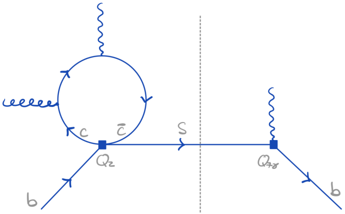
$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{2\pi^4} \overline{m}_b^2(\mu) E_\gamma^3 \Big[|H_\gamma(\mu)|^2 \int d\omega \, m_b J(m_b(\omega + p_+); \mu) S(\omega; \mu) + \frac{1}{m_b} \sum_{i \leq j} \mathrm{Re}\big[C_i^*(\mu) C_j(\mu)\big] F_{ij}(E_\gamma; \mu) + \dots \Big]$$

- $Q_1^{(q)}$-$Q_{7\gamma}$ interference contribution estimated to be $5.15\% \pm 2.55\%$ [Benzke, Hurth '20]
  $\rightarrow$ constitutes the largest uncertainty

- In addition, large m_c-dependence and scale ambiguity $\sim 40\%$ (not included in the above estimates)

  $\rightarrow$ NLO analysis and RG treatment cures scale ambiguity

Factorization at Leading Order



$+ \quad (\gamma \leftrightarrow g).$

Optical theorem:

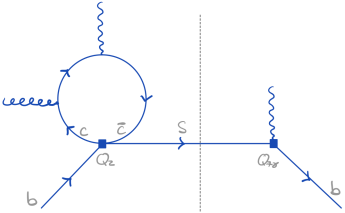
$$d\Gamma \sim \text{Disc}|_{\text{restr.}} \left[i \int d^4x \, \langle \bar{B} | \mathcal{H}^\dagger_{\text{eff}}(x) \, \mathcal{H}_{\text{eff}}(0) | \bar{B} \rangle \right]$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q \Big(C_1 Q_1^{(q)} + C_2 Q_2^{(q)} + C_{7\gamma} Q_{7\gamma} + \dots \Big)$$

Here: Interference of current-current operators $Q_{1,2}^{(q)}$ with $Q_{7\gamma}$ ("single-resolved photon contribution")

$$Q_1^{(q)} = [\bar{q}b]_{V-A}[\bar{s}q]_{V-A}\,, \quad Q_2^{(q)} = [\bar{q}_i b_j]_{V-A}[\bar{s}_j q_i]_{V-A}\,, \quad Q_{7\gamma} = -\frac{e m_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b]$$

Factorization at Leading Order



$+ \quad (\gamma \leftrightarrow g).$

Kinematic endpoint region $M_B - 2E_\gamma \sim \Lambda_{\rm QCD}$:

$\rightarrow$ large energy $E_X \sim m_B$ of hadronic final state X_s, but small inv. mass $M_X^2 = M_B(M_B - 2E_\gamma)$

$\rightarrow$ jet-like configuration

Factorization = scale separation [Benzke, Lee, Neubert, Paz '10]

$$d\Gamma(\bar{B} \to X_s\gamma) \sim \text{Re} \int_{-\infty}^{\bar{\Lambda}} d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{2E_\gamma \omega_1}\right)\right] g_{17}(\omega, \omega_1; \mu)\,, \quad F(x) = 4x\arctan^2\left(\frac{1}{\sqrt{4x-1}}\right)$$

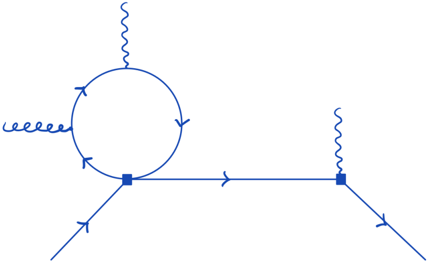
$\rightarrow$ $\mu^2 \sim m_b^2$: 1 = LO hard matching coefficients

$\rightarrow$ $\mu^2 \sim m_b\Lambda_{\rm QCD}$: $\delta(\omega + p_+)$ = imag. part of the LO Quark jet function $(p_+ = m_b - 2E_\gamma)$

$\rightarrow$ $\mu^2 \sim m_b\Lambda_{\rm QCD}$: $[1 - F(\dots)]$ = LO "anti-jet" (penguin) function $(F(0) = 0,\, m_c^2 \sim \Lambda m_b)$

$\rightarrow$ $\mu^2 \sim \Lambda_{\rm QCD}^2$: g_{17} = subleading shape function

Including Radiative Corrections



$$d\Gamma(\bar{B} \to X_s\gamma) \sim H \times J \otimes g_{17} \otimes \bar{J}$$

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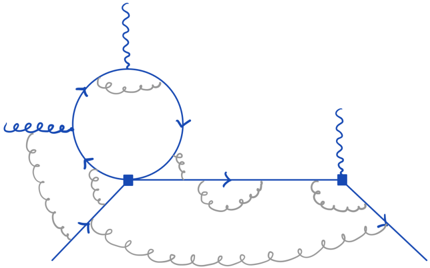

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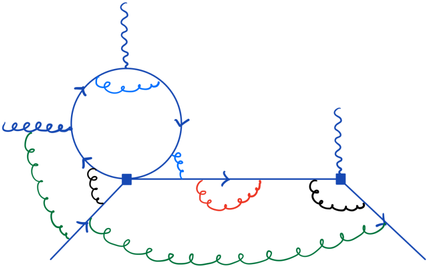


$$d\Gamma(\bar{B} \to X_s\gamma) \sim H \otimes J \otimes g_{17} \otimes \bar{J}$$

In Soft-Collinear EFT, we can compute individual gauge-inv. pieces separately

- ✓ **Hard** Matching known at $\mathcal{O}(\alpha_s)$
- ✓ Quark jet function known at $\mathcal{O}(\alpha_s)$
- **?** Task 1: $\mathcal{O}(\alpha_s)$ (2-loop) corrections to anti-jet function
- **?** Task 2: renormalization of g_{17} (bi-light-cone matrix element)

Elements of Soft-Collinear EFT

- Soft-Collinear Effective Theory (SCET) is designed to describe the long-distance physics in processes with energetic particles (jets)

$$h \sim (1,1,1)m_b$$
$$hc \sim (1,\sqrt{\lambda},\lambda)m_b$$
$$s \sim (\lambda,\lambda,\lambda)m_b$$
$$(\lambda = \Lambda/m_b)$$

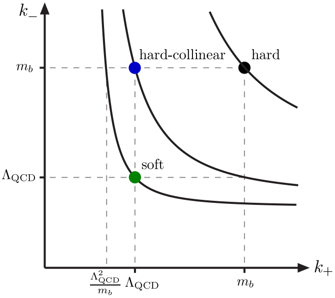
- relevant degrees of freedom (momentum regions):
  - → "**hard**" scale: $m_b = 4.2\text{GeV}$
  - → "soft" scale: $\Lambda \sim 0.5\text{GeV}$
  - → "hard-collinear" scale: $\sqrt{m_b \Lambda} \sim 1.5\text{GeV}$

- light-cone vectors: $n_\pm^\mu = (1,0,0,\pm 1)$

$$k^\mu = \frac{k_-}{2} n_-^\mu + k_\perp^\mu + \frac{k_+}{2} n_+^\mu = (k_-, k_\perp, k_+)$$

- two-step matching

$$\text{QCD} \xrightarrow{m_b \to \infty} \text{SCET}_\text{I} \xrightarrow{\sqrt{m_b\Lambda} \to \infty} \text{HQET}$$



Elements of Soft-Collinear EFT

- operators in SCET are non-local along the light-cone
  - $\rightarrow$ convolution with matching coefficients
  - $\rightarrow$ dress fields with Wilson-lines to construct gauge-invariant building blocks, e.g.

$$\xi = \frac{\not{n}_-\not{n}_+}{4}\psi_C \to \chi \equiv W_C^\dagger \xi\,, \quad \text{with} \quad W_C(x) = \mathbf{P}\exp\left\{ig_s \int_{-\infty}^{0} \mathrm{d}s\, n_+ A_C(x+sn_+)\right\}$$

- Interaction terms between soft and collinear sectors can be removed from the leading-power SCET Lagrangian by a field redefinition ("decoupling transformation").
  - $\rightarrow$ soft-collinear Factorization

$$\xi(x) \to S_n(x_-)\xi^{(0)}(x) \quad \text{with} \quad S_n(x) = \mathbf{P}\exp\left\{ig_s \int_{-\infty}^{0} \mathrm{d}s\, n_- A_s(x+sn_-)\right\}$$

- "physical" (transversely polarized) soft gluons are power-suppressed in Λ/m_B. Need insertions of sub-leading Lagrangians

$$\mathcal{L}_\xi^{(1)} = \bar{\chi} i x_{\mu\perp}\left[i(n_-\partial)\mathcal{A}_{s\perp}^{\mu}\right]\frac{\not{n}_+}{2}\chi \quad \text{and} \quad \mathcal{L}_{\mathrm{YM}}^{(1)} = \frac{-1}{g_s^2}\mathrm{tr}\left(\left[n_+\partial\mathcal{A}_{\nu\perp}\right]\left[x_{\rho\perp} i n_-\partial\mathcal{A}_{s\perp}^{\rho}, \mathcal{A}^{\nu\perp}\right]\right)$$

Hard Matching ($m_b \to \infty$)

Tree-level matching:

$$Q_1^{(q)}(0) \to \mathcal{O}_1^{(q)}(0) = [\bar{\chi}_{\overline{\text{hc}}} h_v]_{V-A}\,[\bar{\chi}_{\text{hc}} \chi_{\overline{\text{hc}}}]_{V-A}\,, \qquad Q_2^{(q)}(0) \to \mathcal{O}_2^{(q)}(0) = [\bar{\chi}_{\overline{\text{hc}}i} h_{vj}]_{V-A}\,[\bar{\chi}_{\text{hc}j} \chi_{\overline{\text{hc}}i}]_{V-A}$$

$$Q_{7\gamma}(x) \to \mathcal{O}_{7\gamma}(x) = -\frac{em_b}{4\pi^2}\, e^{-im_b v\cdot x}\, \bar{\chi}(x)\,\frac{\not{n}_+}{2}\,\big[in_-\partial \not{A}_\perp^{\rm em}(x)\big](1+\gamma_5)h_v(x_-)$$

Hard Matching ($m_b \to \infty$)

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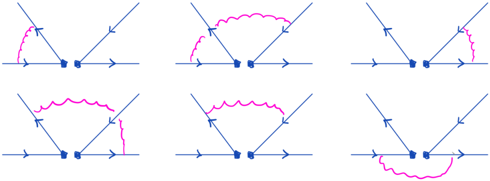


One-loop matching:

$$Q_1^{(q)}(0) \to \int_0^1 du\, \left(H_{11}(u;\mu)\,\tilde{\mathcal{O}}_1^{(q)}(u) + H_{12}(u;\mu)\,\tilde{\mathcal{O}}_2^{(q)}(u)\right)\,, \quad Q_2^{(q)}(0) \to \int_0^1 du\, \left(H_{21}(u;\mu)\,\tilde{\mathcal{O}}_1^{(q)}(u) + H_{22}(u;\mu)\,\tilde{\mathcal{O}}_2^{(q)}(u)\right)$$

$$Q_{7\gamma}(x) \to H_{7\gamma}(\mu)\,\mathcal{O}_{7\gamma}(x)$$

- singlet/octet mixing $\to$ non-vanishing contribution from Q_2 through hard loops (only!)
- hard loops delocalize charm-quark fields $\to$ convolution (similar to exclusive non-leptonics)

Quark Jet Function

...known since the early days of SCET

→ appears e.g. in l.p. $\bar{B} \to X_s\gamma$ [Bauer et al. '02], or exclusive $B \to \gamma\ell\nu$ decays [Lunghi et al. '02]

Definition:

$$\langle 0|T\big(\chi^\alpha(x)\bar{\chi}^\beta(0)\big)|0\rangle = i\left(\frac{\not{n}}{2}\right)_{\alpha\beta}\int\frac{d^4k}{(2\pi)^4}\,e^{-ikx}\,J(n\cdot k,k^2;\mu)$$

Perturbative result:

$$J(n\cdot k,k^2;\mu) = \frac{1}{n\cdot k}\left[1+\frac{\alpha_s C_F}{4\pi}\left(2\ln^2\frac{-k^2}{\mu^2}-3\ln\frac{-k^2}{\mu^2}+7-\frac{\pi^2}{3}\right)\right]+\mathcal{O}(\alpha_s^2)$$

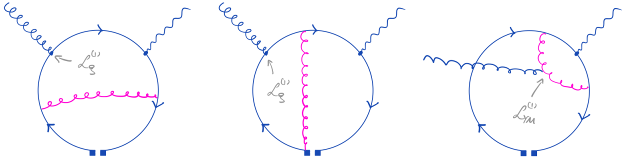
→ in inclusive decays need imag. part (cut)

Task 1: "Anti-Jet" Function at $\mathcal{O}(\alpha_s)$

Defined through T-products

$$\langle T(\mathcal{O}_1^{(q)} \mathcal{L}_\xi^{(1)}) \rangle \qquad \text{and} \qquad \langle T(\mathcal{O}_1^{(q)} \mathcal{L}_{\rm YM}^{(1)}) \rangle$$

Sample 2-loop diagrams:



→ delocalized quark-fields in $\mathcal{O}_1^{(q)}$ → Feynman Rule for operator is $\sim \delta(n \cdot k - 2E_\gamma u)$

→ contribution from $\mathcal{O}_2^{(q)}$ vanishes after summing all diagrams

→ expect smooth limit $m_c \to 0$

Task 2: Renormalization of Subleading Shape Function

Subleading shape function:

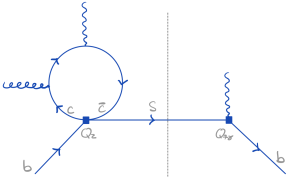
$$g_{17}(\omega,\omega_1;\mu) = \frac{-1}{2m_B}\int \frac{dr}{2\pi}\, e^{-i\omega_1 r}\int \frac{dt}{2\pi}\, e^{-i\omega t}$$
$$\langle \bar{B}|(\bar{h}_v S_n)(tn)\, \bar{\not{n}}(1+\gamma_5)\,(S_n^\dagger S_{\bar{n}})(0)(i\bar{n}\cdot\partial\not{\mathcal{A}}_{s\perp})(r\bar{n})(S_{\bar{n}}^\dagger h_v)(0)|\bar{B}\rangle$$

$$g_{17}(\omega,\omega_1;\mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega' \int_{-\infty}^{\infty} d\omega_1'\, Z_{17}(\omega,\omega',\omega_1,\omega_1';\mu)\, g_{17}^{\text{bare}}(\omega',\omega_1')$$

- g_{17} is real and fulfills

$$\int d\omega\, g_{17}(\omega,\omega_1;\mu) = \int d\omega\, g_{17}(\omega,-\omega_1;\mu)$$

- sensitivity to two distinct light-cone directions n and $\bar{n}$
  - $\rightarrow$ renormalization of such objects first understood in QED in [Beneke, PB, Toelstede, Vos '22] (see also [Huang et al. '23])
  - $\rightarrow$ modified plus-distributions $[\dots]_\oplus$ and $[\dots]_\ominus$ mix positive and negative ω-values



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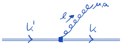
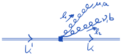


$$= g_s t^a \Gamma \left[-\bar{n}\cdot l \, \gamma_\perp^\mu + \bar{n}^\mu \not{l}_\perp \right] \times \delta(\omega - n\cdot k)\, \delta(\omega_1 - \bar{n}\cdot l)$$

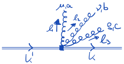


$$= i g_s^2 f^{abc} t^c \Gamma\, \delta(\omega - n\cdot k)\, \delta(\omega_1 - \bar{n}\cdot l_1 - \bar{n}\cdot l_2)\, \bar{n}^\mu \gamma_\perp^\nu$$

$$+ g_s^2 \Gamma \left[-\bar{n}\cdot l_1 \gamma_\perp^\mu + \not{l}_{1\perp} \bar{n}^\mu \right]$$

$$\times \Big\{ i f^{abc} t^c \delta(\omega - n\cdot k) \frac{\bar{n}^\nu}{\bar{n}\cdot l_2} \left(\delta(\omega_1 - \bar{n}\cdot l_1 - \bar{n}\cdot l_2) - \delta(\omega_1 - \bar{n}\cdot l_1) \right)$$

$$+ t^b t^a \delta(\omega_1 - \bar{n}\cdot l_1) \frac{n^\nu}{n\cdot l_2} \left(\delta(\omega - n\cdot k - n\cdot l_2) - \delta(\omega - n\cdot k) \right) \Big\} + (1 \leftrightarrow 2)$$



$$\ni -i g_s^3 f^{bcd} t^a t^d \Gamma \frac{n^\mu}{n\cdot l_1} \left(\delta(\omega - n\cdot k - n\cdot l_1) - \delta(\omega - n\cdot k) \right)$$

$$\times \Big\{ \delta(\omega_1 - \bar{n}\cdot l_2 - \bar{n}\cdot l_3) (\gamma_\perp^\nu \bar{n}^\rho - \gamma_\perp^\rho \bar{n}^\nu)$$

$$+ \frac{\bar{n}^\nu}{\bar{n}\cdot l_2} \left[-\bar{n}\cdot l_3 \gamma_\perp^\rho + \not{l}_{3\perp} \bar{n}^\rho \right] \left(\delta(\omega_1 - \bar{n}\cdot l_2 - \bar{n}\cdot l_3) - \delta(\omega_1 - \bar{n}\cdot l_3) \right)$$

$$- \frac{\bar{n}^\rho}{\bar{n}\cdot l_3} \left[-\bar{n}\cdot l_2 \gamma_\perp^\nu + \not{l}_{2\perp} \bar{n}^\nu \right] \left(\delta(\omega_1 - \bar{n}\cdot l_2 - \bar{n}\cdot l_3) - \delta(\omega_1 - \bar{n}\cdot l_2) \right) \Big\}$$

$$+ g_s^3 \Gamma n^\mu n^\nu \Big(\left[\frac{t^a t^b}{n\cdot l_2 (n\cdot l_1 + n\cdot l_2)} + \frac{t^b t^a}{n\cdot l_1 (n\cdot l_1 + n\cdot l_2)} \right] \delta(\omega - n\cdot k - n\cdot l_1 - n\cdot l_2)$$

$$- t^a t^b \frac{1}{n\cdot l_1\, n\cdot l_2} \delta(\omega - n\cdot k - n\cdot l_1) - t^b t^a \frac{1}{n\cdot l_1\, n\cdot l_2} \delta(\omega - n\cdot k - n\cdot l_2)$$

$$+ \left[\frac{t^a t^b}{n\cdot l_1 (n\cdot l_1 + n\cdot l_2)} + \frac{t^b t^a}{n\cdot l_2 (n\cdot l_1 + n\cdot l_2)} \right] \delta(\omega - n\cdot k) \Big)$$

$$t^c \left[-\bar{n}\cdot l_3 \gamma_\perp^\rho + \not{l}_{3\perp} \bar{n}^\rho \right] \delta(\omega_1 - \bar{n}\cdot l_3) \qquad + \text{permutations}$$

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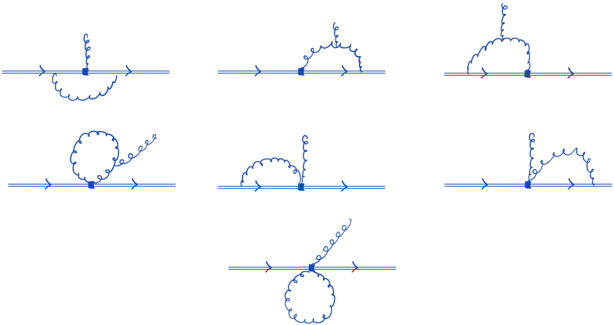

Status of the Calculation

Task 2: Renormalization of g_{17}:

? Certain terms seem to violate special properties of the forward matrix element
(for example, the partonic matrix element contains rescattering phases)

$\rightarrow$ Pole-cancellation of all $1/\varepsilon$ singularities in the full process should help to understand these terms

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Task 1: 2-loop anti-jet function:

$\rightarrow$ To understand the subtleties in g_{17}, we first study the simpler case of $m_c \rightarrow m_u = 0$.
(in progress)

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Once all conceptual issues are resolved:

- include m_c dependence
- evolve all functions to a common scale with RG techniques (cures scale ambiguity)
- improved numerical estimate of power-corrections

Summary and Outlook

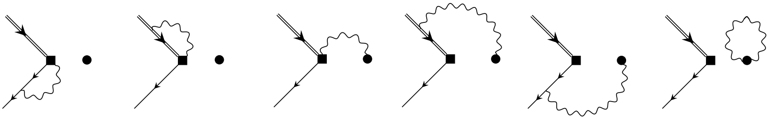
- $Q_1^{(c)}$-$Q_{7\gamma}$ interference provides dominant uncertainty in $\bar{B} \to X_s\gamma$.
  $\to$ improve estimates through NLO calculation which reduces scale ambiguity
- Two main challenges:
  1.) 2-loop calculation of anti-jet (penguin) function
  2.) renormalization of sub-leading shape function ("bi-light cone" matrix element)
- First step: establish consistent framework (pole cancellation/scale evolution) for $m_c \to 0$.
- Second step: include m_c dependence, pheno implications
- Outlook: generalize to $\bar{B} \to X_s\ell^+\ell^-$, applications to exclusive modes, ...

Backup-Slides

QED-generalized B-Meson LCDA

$$iF_{\rm stat}(\mu)\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\Phi_{B,+-}(\omega;\mu) = \frac{1}{R_c R_{\bar c}}\langle 0|\,\bar q_s^{(d)}(tn_-)[tn_-,0]^{(d)}\not{n}_-\gamma_5 h_v(0)\,(S^\dagger_{n_+}S^\dagger_{n_-})(0)\,|\bar B_v\rangle$$

- different objects compared to standard QCD B-meson LCDA $\Rightarrow$ “Soft functions”
  - $\rightarrow$ process dependent as soft photons feel charge+direction of final-state particles
  - $\rightarrow$ imag. parts from final-state rescattering
  - $\rightarrow$ different support properties: $\omega \in (-\infty,\infty)$ if anti-coll. meson is charged



$$\gamma_{+-}(\omega,\omega') = \frac{\alpha_s C_F}{\pi}\left[\left(\ln\frac{\mu}{\omega-i0}-\frac{1}{2}\right)\delta(\omega-\omega') - H_+(\omega,\omega')\right]$$
$$+\frac{\alpha_{\rm em}}{\pi}\left[\left(-\frac{5}{9}\ln\frac{\mu}{\omega-i0}-\frac{5}{36}-\frac{2\pi i}{3}\right)\delta(\omega-\omega') - \frac{1}{9}H_+(\omega,\omega') + \frac{1}{3}H_-(\omega,\omega')\right]$$

- modified plus-distributions $[\ldots]_\oplus$ and $[\ldots]_\ominus$ mix positive and negative ω