$B_c \to \eta_c$ form factors at large recoil: interplay of soft-quark and soft-gluon double logarithms

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based on work in progress with Guido Bell, Philipp Böer, Thorsten Feldmann and Vladyslav Shtabovenko [2309.08410, PoS]

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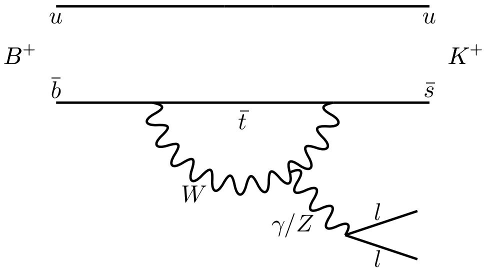
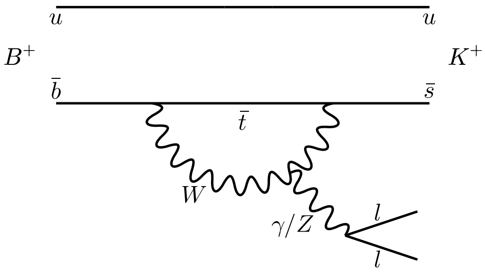
B-physics

- No significant(!) hint for new physics in direct searches yet
- High sensitivity to new physics contributions
- Allows to test CP violation and the CKM matrix
- More precision necessary in view of future experimental developments
- Use lattice for suitable objects
  - Heavy-to-heavy transitions
  - Heavy-to-light form factors at small recoil
- Resort to QCD sum rules or factorization techniques for heavy-to-light form factors at large recoil



B-physics

- Kinematics of heavy-to-light form factors at large recoil
  - Integrate out large momentum of heavy quark $\rightarrow$ soft QCD dynamics
  - At large recoil the light meson is very energetic $\rightarrow$ collinear
- Appropriate effective theory was worked out in the early 2000s
  - Soft-Collinear Effective Theory
    [Bauer et al. '01, '02; Beneke et al. '02; Beneke, Feldmann, '03]
- Conceptual challenges
  - Endpoint divergences appears generically
  - Spoil factorization, i.e. the separation of perturbative and non-perturbative effects



SCET at next-to-leading power

- Significant interest in SCET at next-to-leading power in recent years
  - Threshold resummation [Beneke et al. 18-20]
  - $H \to b\bar{b} \to \gamma\gamma$ and $gg \to b\bar{b} \to H$ [Neubert et al. 19-22]
  - Thrust distribution [Stewart et al. 19; Beneke et al. 22]
  - Leptonic B decays [Feldmann, Gubernari, Huber, Seitz 22; Cornella, König, Neubert 22]
  - Resolved contribution in $B \to X_s\gamma$ [T. Hurth, Szafron 22, Tobias' talk yesterday!]
  - $e\mu$ backward scattering [Bell, Böer, Feldmann 22]
- At next-to-leading power endpoint-divergent convolution integrals arise generically
- For some processes this problem has been solved via refactorisation identities [Böer 2018; Neubert, Liu 2019]

Exclusive B decays do not belong to this class

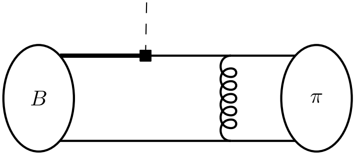
Exclusive B decays at large recoil

$$\left\langle \pi(p) \middle| \bar{q}\gamma^\mu b \middle| \bar{B}(p_B) \right\rangle = F_+(q^2)(p_B^\mu + p^\mu) + F_-(q^2) q^\mu$$

$$F_i(q^2) = C_i \xi_{B\to\pi}(q^2) + \phi_B \otimes T_i^I(q^2) \otimes \phi_\pi$$

$$\xi_{B\to\pi}(q^2) = \sum_i \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u \;\, \phi_B^i(\omega) T_{II}^i(u,\omega;q^2) \phi_\pi^i(u)$$

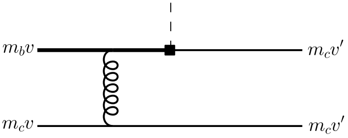
- Complicated interplay of soft and collinear dynamics
- Factorisation theorem has been derived in the early days of SCET [Bauer, Pirjol, Stewart 02; Beneke, Feldmann 03; Neubert, Lange 03]
- Factorisation of soft overlap is plagued by endpoint divergences



Non-relativistic heavy-to-light form factors

- Process considered:
  - $B_c \to \eta_c$ at large recoil $\gamma \equiv v \cdot v' = \mathcal{O}(m_b/m_c)$
  - Non-relativistic approximation with $m_b \gg m_c \gg \Lambda_{\text{QCD}}$
  - Perturbative toy example for $B \to \pi$ form factors
- Factorisation theorems for heavy-to-light form factors have factorisable and non-factorisable terms
- Non-factorisable part is called soft-overlap contribution

$$F(\gamma) \equiv \frac{1}{2E_\eta} \langle \eta_c(p_\eta)|(\bar{c}\,\Gamma\, b)(0)|B_c(p_B)\rangle \qquad \text{with} \qquad \Gamma = \frac{\not{\bar{n}}\not{n}}{4}$$



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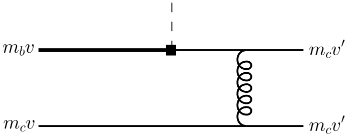
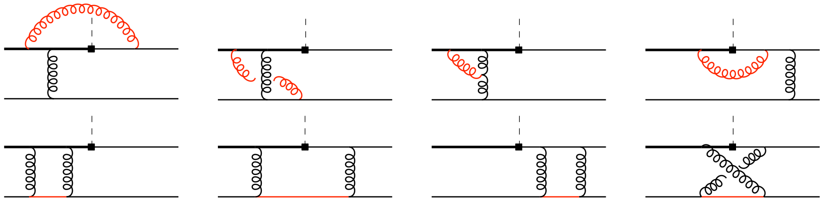


$$= F^{(0)}(\gamma) \equiv 20\,\xi_0, \qquad \xi_0 = \frac{\alpha_s C_F}{4\pi}\frac{\pi^2 f_\eta f_B m_B}{N_c E_\eta^2 m_\eta}$$

Non-relativistic heavy-to-light form factors

Goal of this work:

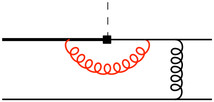
Diagrammatic resummation of leading double logs of the soft-overlap contribution



Double logs arise from both soft quarks and soft gluons

Non-relativistic heavy-to-light form factors

- A soft gluon diagram at 1-loop:



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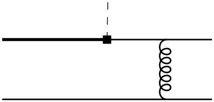
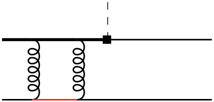


$$\times\ \frac{\alpha_s C_F}{4\pi}(-2)\int_{\ell_-}^{p_{\eta-}}\frac{dk_-}{k_-}\int_{\ell_+}^{k_-}\frac{dk_+}{k_+}$$

- A soft quark diagram at 1-loop:



$$\simeq \frac{\alpha_s C_F}{4\pi}\,\xi_0\,16\int_{\ell_-}^{p_{\eta-}}\frac{dk_-}{k_-}\int_{p_{\eta+}}^{\ell_+}\frac{dk_+}{k_+}\,\theta(k_+k_- - m_c^2)$$

- Total result @ 1-loop ($\gamma \equiv v\cdot v' = \mathcal{O}(m_b/m_c), L \equiv \ln 2\gamma$):

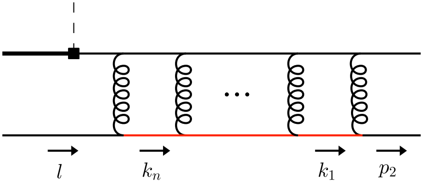
$$F^{(1)}(\gamma) \simeq \underbrace{-\frac{C_F L^2}{2}\times F^{(0)}(\gamma)}_{\text{soft-gluon corrections}} \quad + \quad \underbrace{\xi_0 L^2\,(18\,C_F - 2\,C_A)}_{\text{soft-quark corrections}}$$

Soft quark logs in $B_c \to \eta_c$ form factor

- In light-cone gauge ($\bar{n} \cdot A = 0$) all Abelian soft-quark logs are given by ladder diagrams:

$$\ell_- < k_{1-} < k_{2-} < \cdots < k_{n-} < p_{2-}$$

$$\ell_+ > k_{1+} > k_{2+} > \cdots > k_{n+} > p_{2+}$$



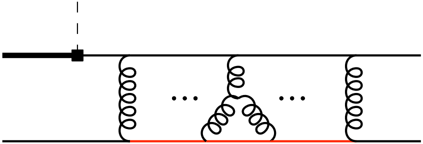
- Soft-quark logs are governed by recursive integral equations

$$f_m(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta -}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} f_m(k_+, k_-)$$

- Analogous to $e\mu$ backward scattering [Bell, Böer, Feldmann 22]
- Final form factor with mixing due to complicated Dirac structure

Soft quark logs in $B_c \to \eta_c$ form factor

- All non-Abelian soft-quark logs are given by ladder diagrams with single insertions of a 3-gluon vertex:



- Recursive integral equations with all soft-quark contributions

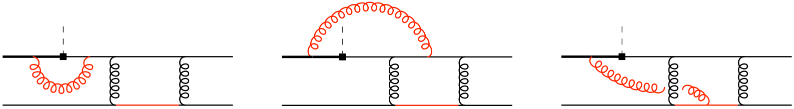
$$F(\gamma)\Big|_{\text{soft quark}} = \xi_0 \left(24\, f(l_+, l_-) - 4 + \frac{4C_A}{C_F}(1 - f_m(l_+, l_-)) \right)$$

$$f(l_+, l_-) = 1 + \frac{\alpha_s}{4\pi} \int_{l_-}^{p_{\eta-}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} \left(2C_F f(k_+, k_-) + \left(C_F - \frac{C_A}{2} \right) f_m(k_+, k_-) + \frac{C_A}{2} \right)$$

$$f_m(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta-}} \frac{\mathrm{d}k_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{\mathrm{d}k_+}{k_+} f_m(k_+, k_-)$$

Interplay with soft gluons

Sudakov logs account for soft gluon effects



$$F(\gamma) = \xi_0 \exp\left\{-\frac{\alpha_s C_F}{4\pi} L^2\right\} \times \left(24\, f(l_+, l_-) - 4 + \frac{4 C_A}{C_F}(1 - f_m(l_+, l_-))\right)$$

$$f(l_+, l_-) = 1 + \frac{\alpha_s}{4\pi} \int_{l_-}^{p_{\eta-}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} \exp\left\{-S\left(\frac{l_+}{k_+}, \frac{p_{\eta-}}{k_-}\right)\right\} \left(2 C_F f(k_+, k_-) - \left(\frac{C_A}{2} - C_F\right) f_m(k_+, k_-) + \frac{C_A}{2}\right)$$

$$f_m(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta-}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} \exp\left\{-S\left(\frac{l_+}{k_+}, \frac{p_{\eta-}}{k_-}\right)\right\} f_m(k_+, k_-)$$

$$S(x, y) \equiv \frac{\alpha_s C_F}{2\pi} \ln x \ln y$$

Order-by-order solution and asymptotics

- First few terms of an iterative solution are given by ($z \equiv \frac{\alpha_s C_F}{2\pi} \ln^2 2\gamma$):

$$F(z) = \xi_0 \, e^{-\frac{z}{2}} \left(20 + \frac{18\,C_F - 2\,C_A}{C_F} z + \frac{15\,C_F - 4\,C_A}{6\,C_F} z^2 - \frac{3\,C_F + 2\,C_A}{180\,C_F} z^3 + \frac{-51\,C_F + 22\,C_A}{10080\,C_F} z^4 + \frac{291\,C_F - 86\,C_A}{907200\,C_F} z^5 \right) + \dots$$

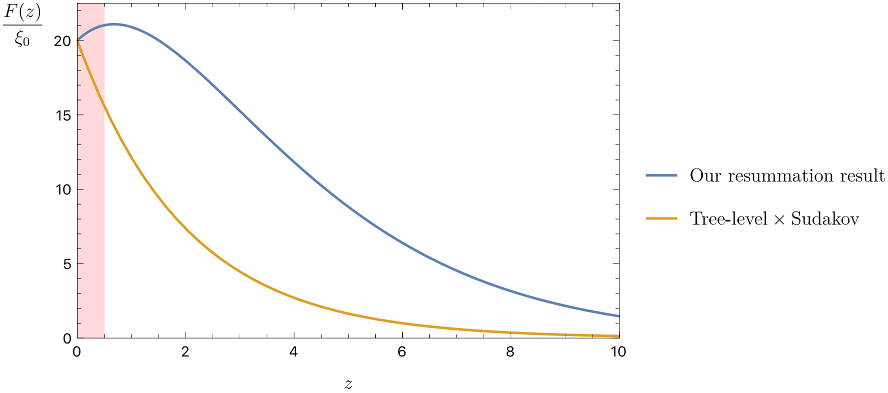
- We are able to determine the asymptotics in the limit

$$m_B \to \infty \quad \Rightarrow \quad z \to \infty$$

$$\Downarrow$$

$$F(z) \simeq \xi_0 \exp\left\{-\frac{z}{2}\right\} \left((z-1) \left(12 + \frac{16\,C_A}{C_F} - \frac{12\,C_{FA}}{C_F} (\ln z + \gamma_E) \right) - \frac{10\,C_A}{C_F} - 4 + \dots \right)$$

Comparison to Sudakov



Standard Sudakov behavior is modified

$$z = \frac{\alpha_s C_F}{2\pi} \ln^2 2\gamma$$

Red band: physical values of z

Independent check at fixed order

- Naive factorization theorem can be used to perform checks at fixed order in perturbation theory
- To check the double log at $\mathcal{O}(\alpha_s^2)$ the only missing coefficient is the leading $1/\epsilon^4$ pole in the hard-collinear region
- We calculate this pole using two independent setups
- Both use typical tool chain (`qgraf`, `Form`, `Fire`, `pySecDec`, ...)

# of loops	# of master integrals
1	5
2	88
3	24732

- Confirms prediction from nested integral equations at $\mathcal{O}(\alpha_s^2)$
- Check at $\mathcal{O}(\alpha_s^3)$ is work in progress

Comparison to special cases

- $B_c \to \eta_c$ form factor at large recoil

$$F(\gamma) = \xi_0 \exp\left\{-\frac{\alpha_s C_F}{4\pi} L^2\right\} \times \left(24 f(l_+, l_-) - 4 + \frac{4C_A}{C_F}(1 - f_m(l_+, l_-))\right)$$

$$f(l_+, l_-) = 1 + \frac{\alpha_s}{4\pi} \int_{l_-}^{p_{\eta -}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} \exp\left\{-S\left(\frac{l_+}{k_+}, \frac{p_{\eta -}}{k_-}\right)\right\} \left(2C_F f(k_+, k_-) - \left(\frac{C_A}{2} - C_F\right) f_m(k_+, k_-) + \frac{C_A}{2}\right)$$

$$f_m(l_+, l_-) = 1 + \frac{\alpha_s C_F}{2\pi} \int_{l_-}^{p_{\eta -}} \frac{dk_-}{k_-} \int_{m_c^2/k_-}^{l_+} \frac{dk_+}{k_+} \exp\left\{-S\left(\frac{l_+}{k_+}, \frac{p_{\eta -}}{k_-}\right)\right\} f_m(k_+, k_-)$$

$$S(x, y) \equiv \frac{\alpha_s C_F}{2\pi} \ln x \ln y$$

- μe backward scattering [Bell, Böer, Feldmann 22]

$$F(l_-, l_+) = 1 + \frac{\alpha_{\text{em}}}{2\pi} \int_{l_-}^{p_-} \frac{dk_-}{k_-} \int_{m^2/k_-}^{l_+} \frac{dk_+}{k_+} F(k_-, k_+)$$

- $h \to b\bar{b} \to \gamma\gamma$ [Neubert et al. 19-22]

$$\mathcal{M}_b(h \to \gamma\gamma)\Big|_{DL} = \mathcal{M}_0 \int_{m_b^2/M_h}^{M_h} \frac{dl_+}{l_+} \int_{m_b^2/l_+}^{M_h} \frac{dl_-}{l_-} \exp\left\{-S\left(\frac{M_h}{l_+}, \frac{M_h}{l_-}\right)\right\}$$

Summary and outlook

- Resort to diagrammatic techniques
- Understood $B_c \to \eta_c$ form factor at large recoil in double log approximation
- Complicated interplay of soft gluon and soft quark logs
- Dirac structure leads to mixing
- Standard Sudakov behavior is modified
- We recover special cases of $h \to \gamma\gamma$ and $e\mu$ backward scattering
- Cross check at $\mathcal{O}(\alpha_s^3)$
- Develop EFT language
- Improve phenomenology

Iterative solution to arbitrary orders

$$f_m(z) = \sum_{n=0}^{\infty} c_n z^n \,, \qquad \hat{f}(z) = \sum_{n=0}^{\infty} d_n z^n \,, \qquad z = \frac{\alpha C_F}{2\pi} \ln^2(2\gamma) \,,$$

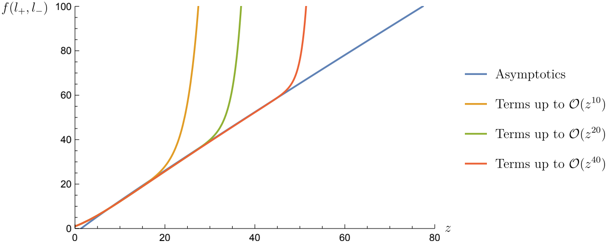
$$c_n = \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1}\,\Gamma(n+k+1)}{\Gamma(2n+1)} \; c_k \,,$$

$$d_n = \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1}\,\Gamma(n+k+1)}{\Gamma(2n+1)} \left\{ d_k - \frac{C_{FA}}{2C_F} \left(\mathcal{H}_{n-k-1} + \frac{1}{n-k} \right) c_k \right\} ,$$

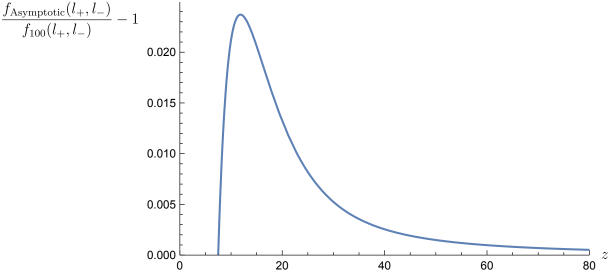
with $n \geq 1$, $c_0 = 1$, $d_0 = 1 + C_A/(4C_F)$, $\mathcal{H}_n$ is the n-th harmonic number and $f(z) = \hat{f}(z) - C_A/(4C_F)$

Convergence of the order-by-order solution

Convergence of f up to different powers of z:



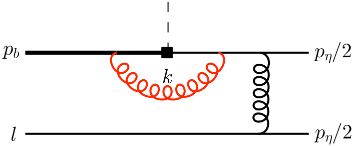
Difference of the asymptotic result for f and it's expansion up to $\mathcal{O}(z^{100})$:



Non-relativistic heavy-to-light form factors

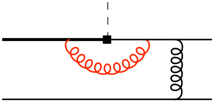
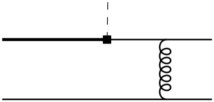
A soft gluon diagram at 1-loop:

$$p_{b+} \gg l_+ \gg \quad p_{\eta+}$$
$$p_{\eta-} \sim p_{b-} \gg l_-$$



Fermion denominators in the loop (soft gluon is on-shell):

$$\begin{aligned}
(p_b - k)^2 - m_b^2 &= (p_{b+} - k_+)(p_{b-} - k_-) + k_\perp^2 - m_b^2 \\
&\to -p_{b+}k- \\
(p_\eta - l - k)^2 - m_c^2 &= (p_{\eta+} - l_+ - k_+)(p_{b-} - l_- - k_-) + k_\perp^2 - m_c^2 \\
&\simeq (-l_+ - k_+)(p_{b-} - k_-) + k_\perp^2 - m_c^2 \\
&\to -p_{b-}k+
\end{aligned}$$

 $\simeq$  $\times \; \dfrac{\alpha_s C_F}{4\pi}(-2)\displaystyle\int_{\ell_+}^{p_{\eta-}} \frac{dk_-}{k_-}\int_{\ell_+}^{k_-}\frac{dk_+}{k_+}$

Non-relativistic heavy-to-light form factors

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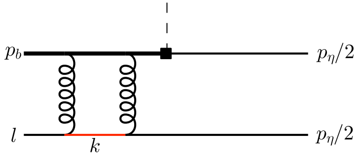
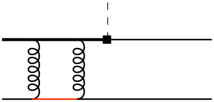


Gluon denominators in the loop (soft quark is on-shell):

$$\begin{aligned}(l-k)^2 &= (l_+ - k_+)(l_- - k_-) + k_\perp^2 \\ &\to -l_+ k- \\ (k - p_\eta/2)^2 &= (k_+ - p_{\eta+}/2)(k_- - p_{\eta-}/2) + k_\perp^2 \\ &\to -p_{\eta-} k+\end{aligned}$$



$$\simeq \frac{\alpha_s C_F}{4\pi} \xi_0 \, 16 \int_{\ell_-}^{p_{\eta-}} \frac{dk_-}{k_-} \int_{p_{\eta+}}^{\ell_+} \frac{dk_+}{k_+} \theta(k_+ k_- - m_c^2)$$