Quark-Hadron Duality Violation and Higher Order $1/m_b$ corrections in inclusive $B \to X_c \ell \bar{\nu}$

Quirks in Quark Flavour Physics 2024 - Zadar, Croatia

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in collaboration with
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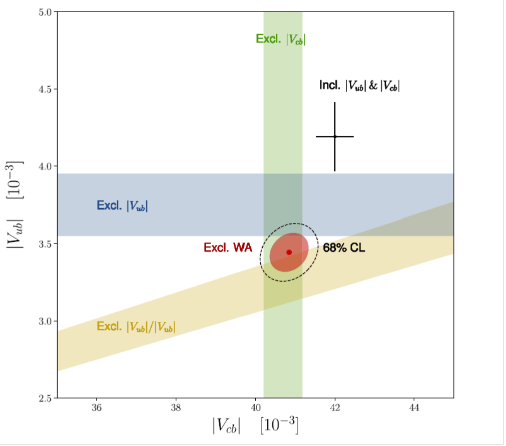
The V_{cb} puzzle

- Inclusive $B \to X_c \ell \bar{\nu}$ for extraction of V_{cb}
  $\to$ See talk by Keri Vos
- Heavy Quark Expansion (HQE) $\to$ power series in $\Lambda_{\mathrm{QCD}}/m_b$
  Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...
- Split b quark momentum as $p_b = m_b v + k$
  $\to$ Expand in $k \sim iD$



Bernlochner, Prim, Vos (*Eur. Phys. J. Spec. Top.* (2024))

Matrix elements

- Perform Operator Product Expansion (OPE)
  Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2} d\Gamma^{(5)} + \frac{1}{m_b^3} d\Gamma^{(6)} + \frac{1}{m_b^4} d\Gamma^{(7)} + \ldots \,, \qquad d\Gamma^{(n)} = \sum_i \mathcal{C}_i^{(n)} \langle B | \mathcal{O}_i^{(n)} | B \rangle$$

  - $d\Gamma^{(3)}$: Partonic result ($d\Gamma^{(4)} = 0$ due to Heavy Quark Symmetries)
  - $d\Gamma^{(5)}$: 2 parameters

$$2 m_B \mu_\pi^2 = -\langle B | \bar{b}_v (iD)^2 b_v | B \rangle$$
$$2 m_B \mu_G^2 = \langle B | \bar{b}_v (-i\sigma^{\mu\nu})(iD_\mu)(iD_\nu) b_v | B \rangle$$

  - $d\Gamma^{(6)}$: 2 parameters

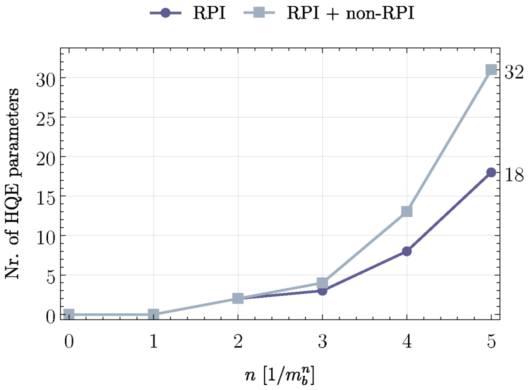
$$2 m_B \rho_D^3 = \langle B | \bar{b}_v [iD_\mu, [ivD, iD^\mu]] b_v | B \rangle / 2$$
$$2 m_B \rho_{LS}^3 = \langle B | \bar{b}_v \{iD_\mu, [ivD, iD_\nu]\} (-i\sigma^{\mu\nu}) b_v | B \rangle / 2$$

  - $d\Gamma^{(7)}$: 9 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]
  - $d\Gamma^{(8)}$: 18 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]

Counting Operators with Reparametrization Invariance

- In HQE, choice of v_μ is not unique
  Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen,...
- Lorentz invariance of QCD $\rightarrow$ Reparametrization Invariance (RPI) imposed by $v_\mu \rightarrow v_\mu + \delta v_\mu$
- RPI relates different orders in $1/m_b$ expansion Mannel, Vos [1802.09409]
  - $\rightarrow$ This allows us to find combinations of operators which are RPI
- Up to $1/m_b^4$: total of 8 independent parameters Mannel, Vos [1802.09409]
- At $1/m_b^5$, we find only 10 RPI operators

  Mannel, ISM, Vos [2311.12002]



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- Dilepton invariant mass (q^2) moments are RPI and can be used to extract $|V_{cb}^{\mathrm{incl}}|$

  (q^2-cut needed due to experimental setup)

  Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\mathrm{cut}} = \frac{\int_{q^2 > q^2_{\mathrm{cut}}} \mathrm{d}q^2 \; (q^2)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}{\int_{q^2 > q^2_{\mathrm{cut}}} \mathrm{d}q^2 \; \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}$$

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- Data $\to$ values for reduced set of RPI parameters up to $1/m_b^4 \to Br(\bar{B} \to X_c \ell \bar{\nu}) \to$

$$|V_{cb}^{\mathrm{incl}}| = (41.69 \pm 0.63) \times 10^{-3}$$ Bernlochner, Vos, et al. [2205.10274]

Inclusive V_{cb} determination

- Want an RPI-observable $\to$ only depend on reduced set of RPI operators
- Dilepton invariant mass (q^2) moments are RPI and can be used to extract $|V_{cb}^{\rm incl}|$ (q^2-cut needed due to experimental setup)
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- First determination of V_{cb} up to $\mathcal{O}(1/m_b^4)$ and first extraction of $1/m_b^4$ matrix elements from data
- Agreement at $1-2\sigma$ level with previous $\mathcal{O}(1/m_b^3)$ determinations
  Finauri, Gambino [2310.20324]; Bordone, Capdevila, Gambino [2107.00604]; Alberti, Gambino, Healey, Nandi[1411.6560]; Gambino, Schwanda [1307.4551]

Where do we currently stand?

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
$1/m_b^2$	✓	✓		
$1/m_b^3$	✓	✓		
$1/m_b^4$	✓			
$m_b^{\mathrm{kin}}/\bar{m}_c$		✓	✓	✓

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$1/m_b^4$	✓			

- Green: known perturbative corrections Jezabek, Kuhn (1989); Melnikov (2008); Pak, Czarnecki (2008); Becher, Boos, Lunghi (2007); Alberti, Gambino, Nandi (2014); Mannel, Pivovarov, Rosenthal (2015); Gambino, Healey, Turczyk (2016); Mannel, Pivovarov (2020); Fael, Schonwald, Steinhauser (2020, 2021); Fael, Herren (2024)
- Next step for higher precision:

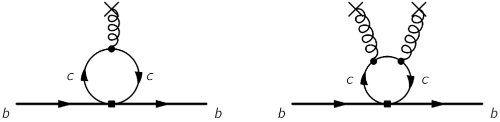
→ $1/m_b^5$

→ Quark-Hadron Duality Violation

Going higher in the $1/m_b$ expansion

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

"genuine" $\dfrac{1}{m_b^5}$ & Intrinsic Charm $\dfrac{1}{m_b^3 m_c^2}$



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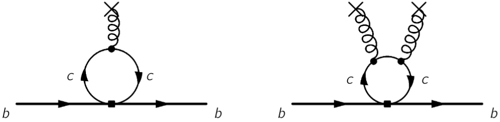

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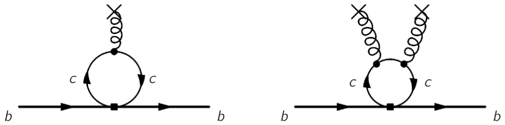

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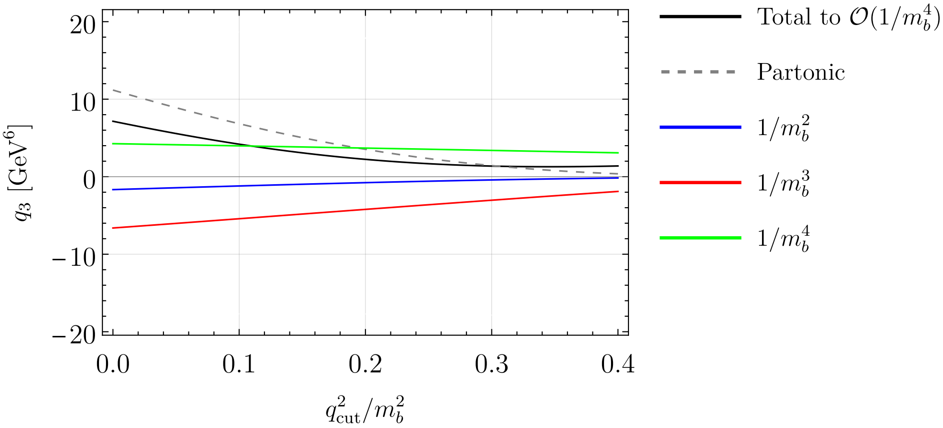
- We need $1/m_b^3 \times 1/m_c^2$ contributions to complete the calculation at $1/m_b^4$
  - We calculate the full dimension 8 contributions
  - We extract the Intrinsic Charm contribution
  - We find that only 1 combination of parameters describes the IC in the q^2-moments

    Mannel, ISM, Vos [2311.12002]

q^2-moments



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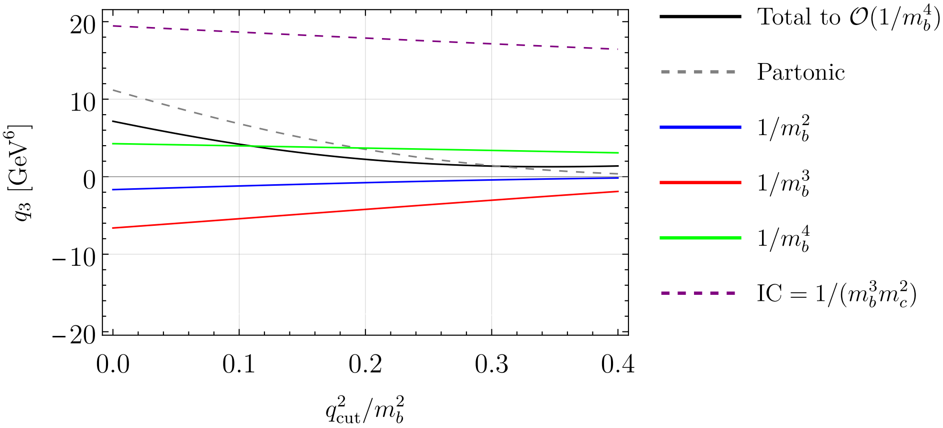

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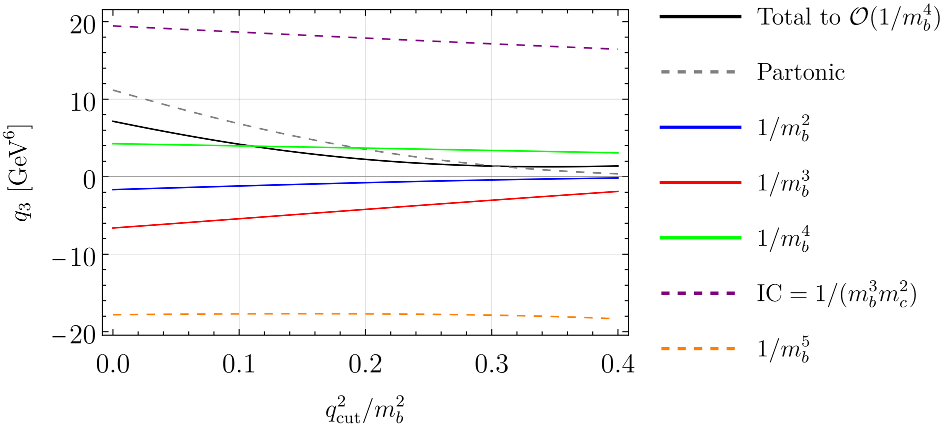

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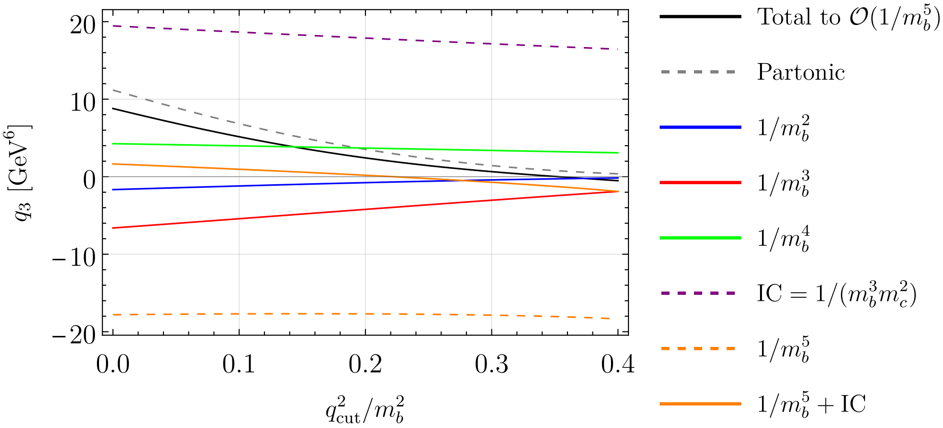

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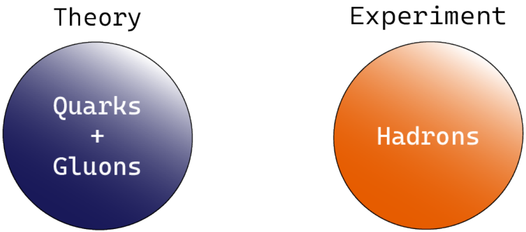
- Work in progress: Full $|V_{cb}|^{\mathrm{incl}}$ fit to $1/m_b^5$

Quark-Hadron Duality (QHD)



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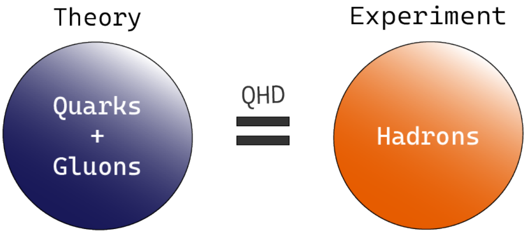

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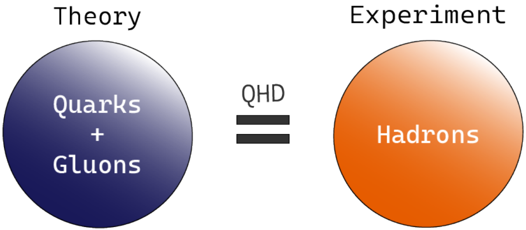

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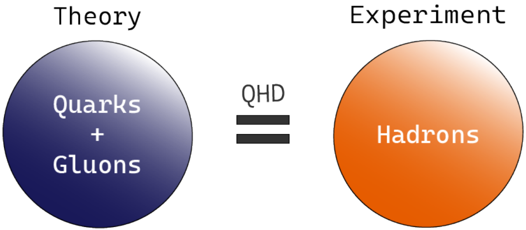

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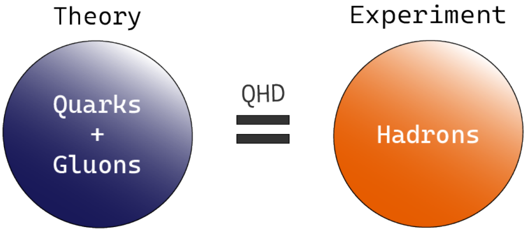

- QHD in OPE: well behaved Taylor series and analytic in Λ_{QCD}/Q where $Q = m_b v - q$
  - → data currently shows no indications of failure of HQE
  - → but factorially increasing number of HQE parameters hint for QHD Violation

Modelling QHDV

- Optical theorem:

$$\mathrm{d}\Gamma \propto L^{\mu\nu}\,\mathrm{Im}\left[T_{\mu\nu}(vQ, Q^2)\right]$$

→ leptonic tensor $L^{\mu\nu}$
→ hadronic tensor as imaginary part of time-ordered product:

$$T_{\mu\nu}(Q) = \int \mathrm{d}^4x\, e^{-iQ\cdot x}\langle B(p)|\, T\{\bar{b}_v(x)\Gamma_\mu c(x)\, \bar{c}(0)\overline{\Gamma}_\nu b_v(0)\}|B(p)\rangle$$

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- Expand external field propagator for the charm quark:

$$\begin{aligned}-iS_{\mathrm{BGF}} &= \frac{1}{\not{Q} + i\not{D} - m_c} \\ &= \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c} + \ldots \\ &= \sum_{k=0}^{\infty}\left(\frac{1}{Q^2}\right)^{k+1}\not{Q}\left[-(i\not{D})\not{Q}\right]^k \quad (m_c = 0)\end{aligned}$$

Modelling QHDV continued

- Taking forward matrix element with B meson with velocity v:

$$T_{\mu\nu}(Q) = \sum_{k=0}^{\infty} \left(\frac{1}{Q^2}\right)^{k+1} \langle B(v)|\bar{b}_v \Gamma_\mu \not{Q}[-(i\not{D})\not{Q}]^k \overline{\Gamma}_\nu b_v(0)|B(v)\rangle$$

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- Order by order (schematically, all Lorentz indices suppressed):

$$\langle B(v)|\bar{b}_v \Gamma \not{Q} \overline{\Gamma} b_v|B(v)\rangle = a_0^{(i,0)}(vQ) \ ,$$

$$\langle B(v)|\bar{b}_v (-1)\Gamma \not{Q}(i\not{D})\not{Q} \overline{\Gamma} b_v|B(v)\rangle = \Lambda_{\mathrm{HQE}} \left(a_0^{(i,1)}(vQ)^2 + a_1^{(i,1)} Q^2\right) \ ,$$

$$\langle B(v)|\bar{b}_v \Gamma \not{Q}(i\not{D})\not{Q}(i\not{D})\not{Q} \overline{\Gamma} b_v|B(v)\rangle = \Lambda_{\mathrm{HQE}}^2 \left(a_0^{(i,2)}(vQ)^3 + a_1^{(i,2)}(vQ)Q^2\right) \ ,$$

$$\dots$$

$\rightarrow$ the index $i = 1, \dots, 5$ denotes the five scalar components of $T_{\mu\nu}$

Modelling QHDV continued

- Introduce dimensionless variables $r^2 = Q^2/\Lambda^2_{\mathrm{HQE}}$ and $t = vQ/\Lambda_{\mathrm{HQE}}$:

$$T_i(t, r^2) = \frac{1}{\Lambda_{\mathrm{HQE}}} \sum_{l=0}^{\infty} \left(\frac{1}{r^2}\right)^{l+1} P_l^{(i)}(t) \, , \qquad P_l^{(i)}(t) = \sum_{k=0}^{l+1} t^{l+1-k} a_k^{(i,k+l)}$$

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- We can study coefficients $a_k^{(i,k+l)}$ using known contributions to $1/m_b^5$
  - $\rightarrow$ model factorial growth as $P_l(t) = (2l)! p_l(t)$
  - $\rightarrow$ use known contributions to make ansatz for polynomials $p_l(t)$

$$p_l^{(1,4)}(t) = t^{l+1} + t^l + \ldots + t$$
$$p_l^{(2,3)}(t) = t^l + t^{l-1} + \ldots + t + 1$$
$$p_l^{(5)}(t) = t^{l+1} + t^l + \ldots + t^2$$

Borel Transform

- Now perform Borel Transform of $T_{\mu\nu}(r^2, t)$ w.r.t. $\lambda = 1/r$ and transform back
- Schematically:

$$F(\lambda) = \sum_n (2n)! \lambda^{2n}$$

$$B(M) = \sum_n M^{2n} = \frac{1}{1 - M^2} = \frac{1}{1 + M} \frac{1}{1 - M}$$

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- Finally, analytically continue to the Minkowskian case by $\lambda \to i\kappa = i\Lambda_{\mathrm{HQE}}/\sqrt{Q^2}$

The QHDV model

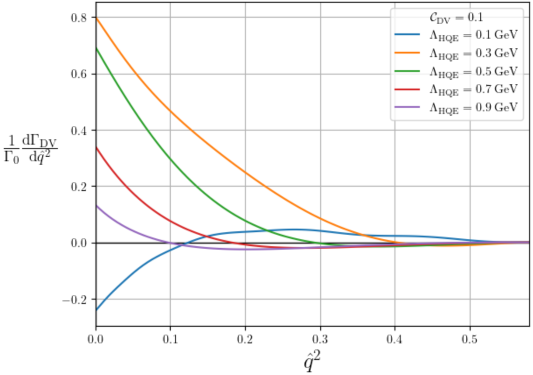
- This will lead to: Mannel, ISM, Verkade, Vos [WIP]

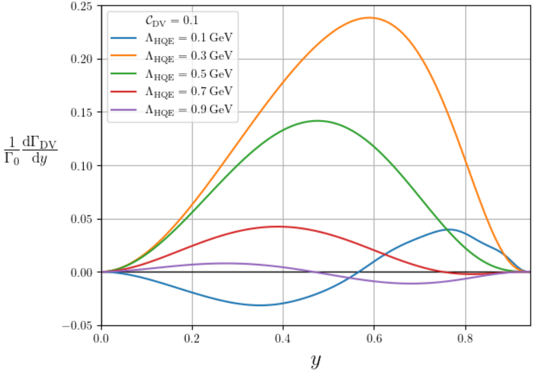
$$-\frac{1}{\pi}\hat{\Delta}_{DV}\mathrm{Im}\left[T_{1,4}(vQ, Q^2)\right] =$$

$$\mathcal{C}_{\mathrm{DV}}\frac{N}{\Lambda_{\mathrm{HQE}} - vQ}\frac{vQ}{\sqrt{Q^2}}\left(\sin\left(\frac{\sqrt{Q^2}}{\Lambda_{\mathrm{HQE}}}\right) - \sqrt{\frac{vQ}{\Lambda_{\mathrm{HQE}}}}\sin\left(\frac{1}{\sqrt{\Lambda_{\mathrm{HQE}}}}\sqrt{\frac{Q^2}{vQ}}\right)\right)$$

→ similar expressions for $T_{2,3,5}$
→ N is a normalisation and $\mathcal{C}_{\mathrm{DV}}$ characterises "strength" of QHDV
→ Λ_{HQE}=0.5 GeV as default (based on HQE parameters)

Differential rates for QHDV





- Γ_0: partonic rate, $\hat{q}^2 = q^2/m_b^2$, $y = 2E_\ell/m_b$

Conclusions and outlook

- We identified the 10 RPI operators at $1/m_b^5$
- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions
- We find an unexpectedly small overall contribution of the dimension-8 operators
- q^2, E_ℓ, M_X^2 moments to $1/m_b^5$ available in open-source library `kolya`
- We provide so-called trace formulae for determinations of observables to $\mathcal{O}(1/m_b^5)$ for $B \to X_c \ell \bar{\nu}$ Mannel, ISM, Vos [2311.12002]

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- We built a model for QHDV guided by known contributions to $1/m_b^5$
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  - We model the factorial growth of the number of HQE parameters
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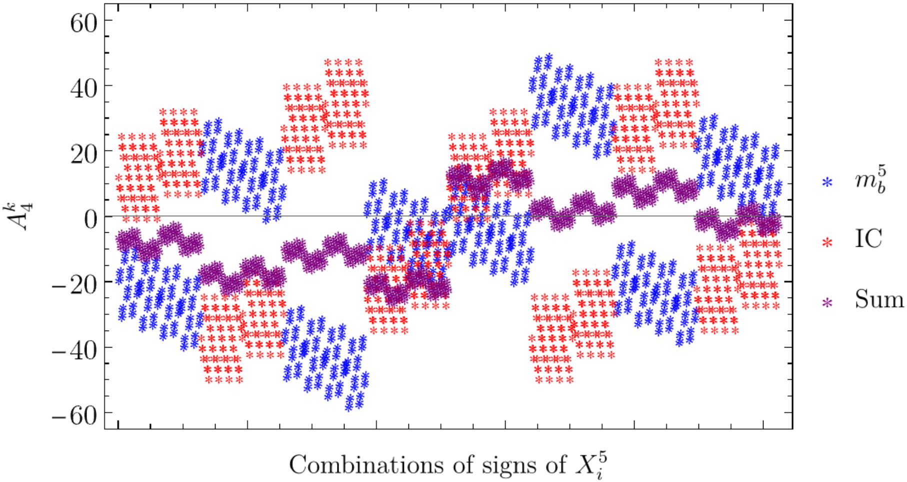
- Work in progress:
  - Full $|V_{cb}|^{\mathrm{incl}}$ fit to $1/m_b^5$
  - Effect of QHDV on kinematic moments
  - Constructing observables sensitive to QHDV $\to$ test our model on data

Back-up 1: Cancellation without LLSA

- Assume for all dimension 8 operators that $X_i^5 \sim \Lambda_{\mathrm{QCD}}^5$ and vary signs:



Back-up 2: Calculation of forward matrix element

- **Step 1:** expand charm propagator ($Q^\mu = m_b v^\mu$)

$$
\begin{aligned}
-iS_{\mathrm{BGF}} &= \frac{1}{\not{Q} + i\not{D} - m_c} \\
&= \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c} \\
&\quad + \frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c}(i\not{D})\frac{1}{\not{Q} - m_c} + \ldots
\end{aligned}
$$

Back-up 2: Calculation of forward matrix element

- **Step 2:** insert in forward matrix element

$$T = \left[\Gamma \frac{1}{\not{Q} - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha b_\beta \rangle$$

$$- \left[\Gamma \frac{1}{\not{Q} - m_c} \gamma^\mu \frac{1}{\not{Q} - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu) b_\beta \rangle$$

$$+ \left[\Gamma \frac{1}{\not{Q} - m_c} \gamma^\mu \frac{1}{\not{Q} - m_c} \gamma^\nu \frac{1}{\not{Q} - m_c} \Gamma^\dagger\right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu)(iD_\nu) b_\beta \rangle$$

$$+ \dots$$

Back-up 2: Calculation of forward matrix element

- **Step 3:** Determine Trace formula
  - Start at dimension 8 $\rightarrow$ lengthy, but systemically calculable
  - Compute dim-7, including $1/m_b$ correction through e.o.m.

$$(ivD)b_v = -\frac{1}{2m_b}(i\not{D})(i\not{D})b_v$$

  - ...
  - Compute dim-3, including corrections up to $1/m_b^5$

$$\langle \bar{b}_\alpha b_\beta \rangle = 2m_B \left(\frac{1+\not{v}}{4} + \frac{1}{8m_b^2}(\mu_G^2 - \mu_\pi^2) + \mathcal{O}(1/m_b^6) \right)_{\beta\alpha}$$

    - Need full (non-RPI) set of basic parameters up to $1/m_b^5$
- **Step 4:** Compute the trace with the geometric series

Back-up 3:LLSA

- At $1/m_b^5 \to$ too many HQE parameters to fit to data
- LLSA: Lowest-Lying-State Approximation to estimate through known parameters $\mu_\pi^2,\ \mu_G^2,\ \rho_D^3$ and ρ_{LS}^3
  Mannel, Turczyk, Uraltsev [1009.4622]

$$\langle B|\bar{b}\ AC\Gamma\ b(0)|B\rangle = \frac{1}{2m_B}\sum_n \langle B|\bar{b}\ A\ b(0)|n\rangle \cdot \langle n|\bar{b}\ C\Gamma\ b(0)|B\rangle$$

  - $A = iD_{\mu_1}...iD_{\mu_k},\ C = iD_{\mu_{k+1}}...iD_{\mu_n}$

Back-up 4: QHDV model

$$-\frac{1}{\pi}\hat{\Delta}_{DV}\mathrm{Im}\left[T_{2,3}(vQ,Q^2)\right] =$$

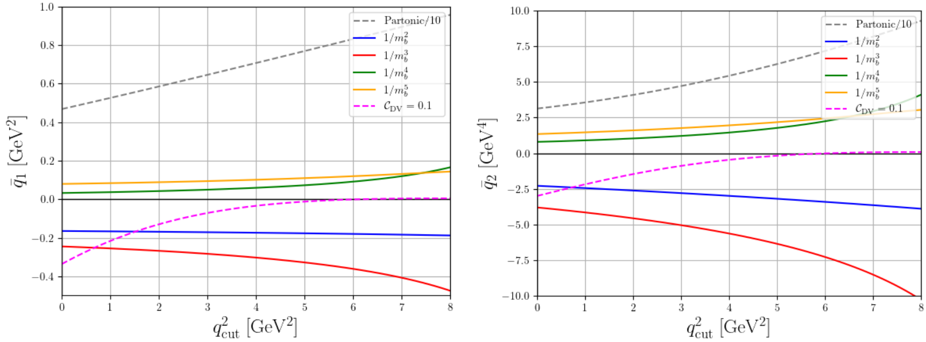
$$\mathcal{C}_{\mathrm{DV}}\frac{N}{\Lambda_{\mathrm{HQE}}-vQ}\frac{\Lambda_{\mathrm{HQE}}}{\sqrt{Q^2}}\left(\sin\left(\frac{\sqrt{Q^2}}{\Lambda_{\mathrm{HQE}}}\right)-\sqrt{\frac{vQ}{\Lambda_{\mathrm{HQE}}}}\sin\left(\frac{1}{\sqrt{\Lambda_{\mathrm{HQE}}}}\sqrt{\frac{Q^2}{vQ}}\right)\right)$$

$$-\frac{1}{\pi}\hat{\Delta}_{DV}\mathrm{Im}\left[T_{5}(vQ,Q^2)\right] =$$

$$\mathcal{C}_{\mathrm{DV}}\frac{N}{\Lambda_{\mathrm{HQE}}-vQ}\frac{(vQ)^2}{\Lambda_{\mathrm{HQE}}\sqrt{Q^2}}\left(\sin\left(\frac{\sqrt{Q^2}}{\Lambda_{\mathrm{HQE}}}\right)-\sqrt{\frac{\Lambda_{\mathrm{HQE}}}{vQ}}\sin\left(\frac{1}{\sqrt{\Lambda_{\mathrm{HQE}}}}\sqrt{\frac{Q^2}{vQ}}\right)\right)$$

Back-up 5: QHDV in q^2 moments

- Preliminary



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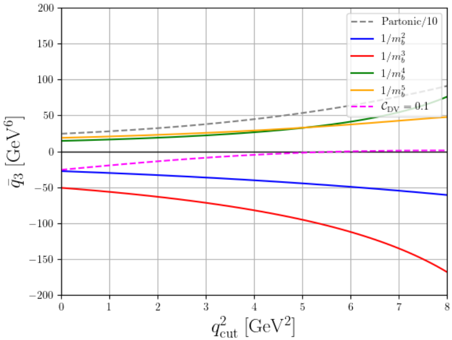

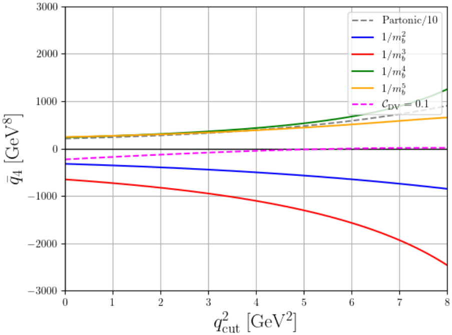

Back-up 6: Dependence on $\Lambda_{\rm HQE}$

- Preliminary