Theoretical status and prospects in inclusive semileptonic beauty and charm decays

K. Keri Vos

Maastricht University & Nikhef

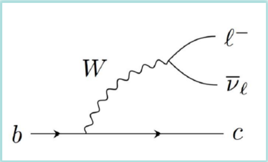
= Quirks 2024 =

The Beauty of Semileptonic Decays

Motivation:

- Theoretically relatively easy to describe: factorization of strong interaction effects

Quark level process



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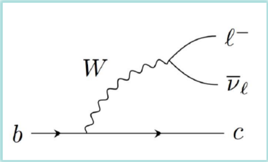
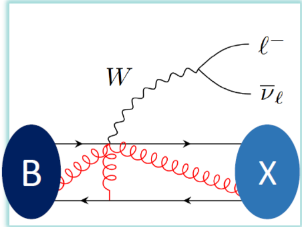


hadronization →

Reality: Bound state



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Two options:

- Exclusive decays: pick one final state with the desired quarks ($V_{cb} \to D^{(*)}$ and $V_{ub} \to \pi$)
- Inclusive decays: everything you can think of! (denoted with X_c or X_u)

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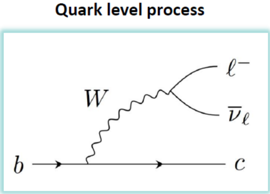

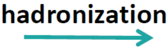

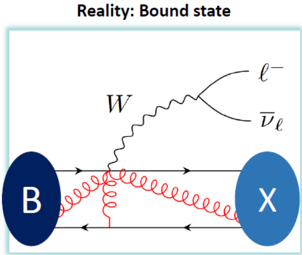

Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules = Exclusive Decays
  - Measurable: from data = Inclusive Decays

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}?$)
- Extract power corrections from data
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to Λ_{QCD}
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B | O_i^{(k)} | B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \ldots | B \rangle$ non-perturbative matrix elements $\to$ string of iD
- operators contain chains of covariant derivatives

HQE elements: $\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | \bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v | B \rangle$

- Extracted from data

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- Extracted from data
- Ideas for the lattice Juetner et al. [202305.14092]

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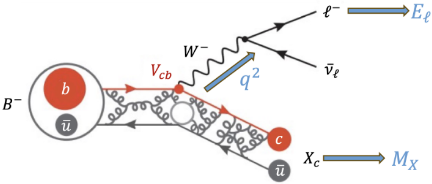
- Extracted from data

- Γ_2 :μ_π^2 and μ_G^2 at $1/m_b^2$
- Γ_3 :ρ_D^3 and ρ_{LS}^3 at $1/m_b^3$
- Many more at $1/m_b^{4,5}$ Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005. Pic from M. Fael

Non-perturbative matrix elements obtained from moments of differential rate



$$\langle O^n \rangle_{\text{cut}} = \frac{\int_{\text{cut}} dO\, O^n \frac{d\Gamma}{dO}}{\int_{\text{cut}} dO \frac{d\Gamma}{dO}}$$

$M_X^2 = (p_B - q)^2, E_\ell = v_B \cdot p_\ell$ and $q^2 = (p_\nu + p_\ell)^2$

hadronic mass, lepton energy and q^2 moments

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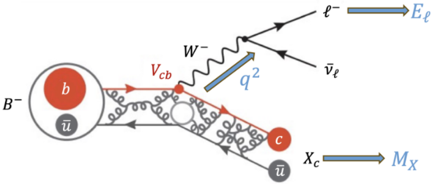

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hadronic mass, lepton energy and q^2 moments

- $\mu_\pi^2, \mu_G^2, \rho_D^3 + \cdots$ extracted from data $\rightarrow$ total rate $\rightarrow |V_{cb}|$

Quirks in $|V_{cb}|$

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} \left(\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)\right) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

Vices and Virtues:

- Systematic framework for power-corrections
- Higher precision: Include higher-order $1/m_b$ and α_s corrections in rate and moments!
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177, Mannel, Milutin, KKV [2311.1200]

- Standard lepton energy and hadronic mass moments are not RPI quantities
- Only RPI moments are q^2 moments
- Determinations from Belle and Belle II availabe Phys. Rev. D 104, 112011 (2022), Phys. Rev. D 107, 072002 (2023)

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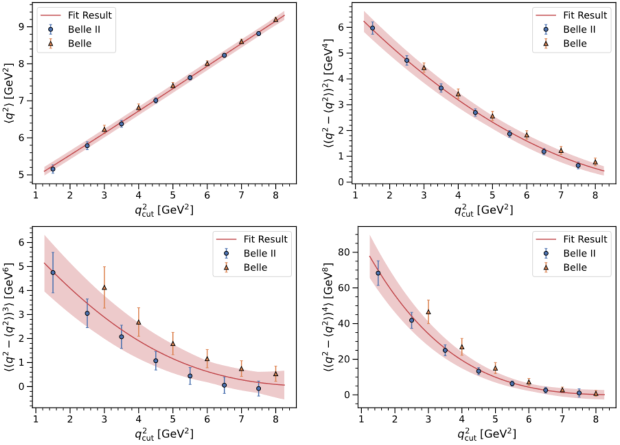
Quirks:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)
- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13) [Talk by Ilija]

q^2 moments

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]



Centralized moments as function of $q^2_{\rm cut}$

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- NNLO corrections to moments not included
- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1} \mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69) \mathrm{GeV}^4$$

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- Inputs for $B \to X_u \ell \nu$, B lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.
- Agreement with BCG extraction (differs due to branching ratio inputs)
  Bordone,Capdevila, Gambino [2021]

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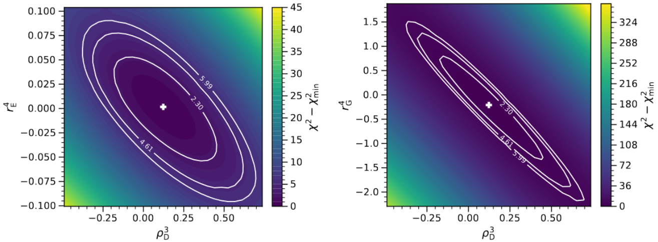

First combined fit

Gambino, Finauri [2310.20324]

- Includes terms up to $1/m_b^3$

NEW! Calculation of BLM α_s^2 corrections to q^2 moments

NEW! Includes QED corrections to the lepton moments Bigi, Bordone, Gambino, Haisch, Piccione, JHEP 11 (2023) 163

Combined E_ℓ, M_X, q^2 moments:

$$|V_{cb}|^{\rm GF}_{\rm incl,all} = (41.95 \pm 0.27|_{\rm exp} \pm 0.31|_{\rm th} \pm 0.25|_{\Gamma}) \times 10^{-3} = (41.95 \pm 0.48) \times 10^{-3}$$

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- Agrees with determination from three points at fixed cut

First combined Fit

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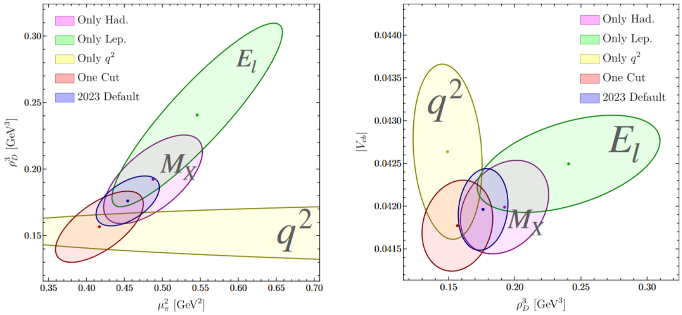
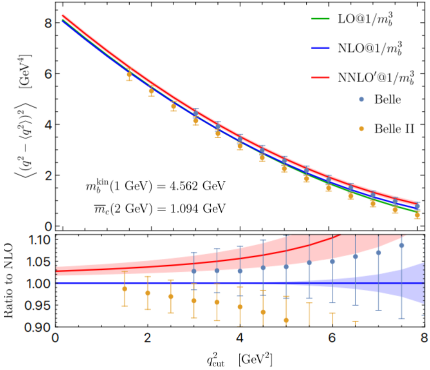


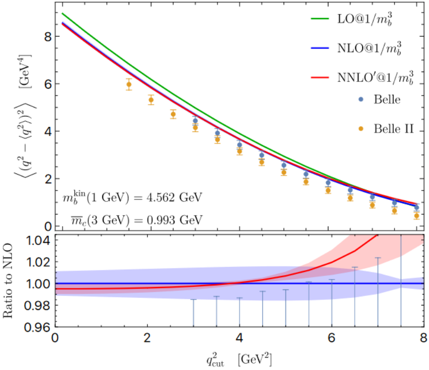
- Complementary between different measurements
- Extracted $\rho_D^3 = 0.176 \pm 0.019$ GeV^3
- Important input for lifetimes and $B \to X_u$, $B \to X_s$

NEW: NNLO corrections to q^2 momemts

Herren, Fael [2403.03976]



$\overline{m_c}(2\ \mathrm{GeV})$ not ideal choice



$\overline{m_c}(3\ \mathrm{GeV})$ better

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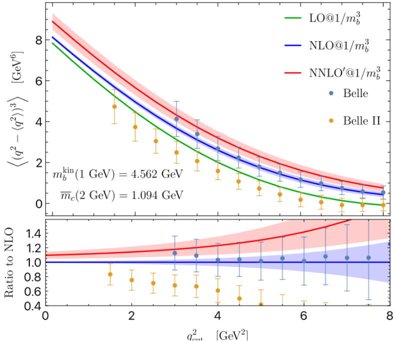

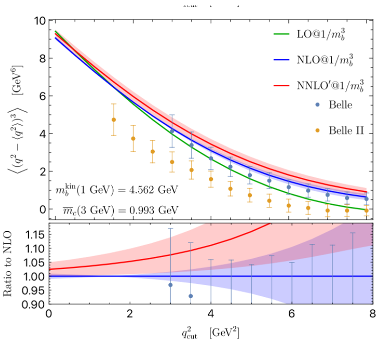

NNLO effects mainly re-absorbed in the fit into a shift of ρ_D^3, r_E^4 and r_G^4.
No major shift in $|V_{cb}|$.

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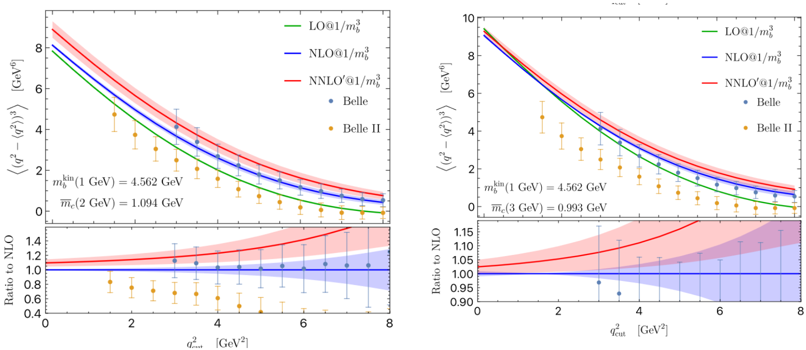

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Full combined analysis and updated q^2 fits in progress!

NEW: Inclusive decays: The Kolya package

Kolya package, Fael, Mulatin, KKV [in progress]

Open source Python package:
https://gitlab.com/vcb-inclusive/kolya

- Observables:
  - Centralized $\langle E_\ell \rangle$ moments
  - Centralized $\langle q^2 \rangle$ moments
  - Centralized $\langle M_X^2 \rangle$ moments
  - Total rate + branching ratio with kinematic cut
- Includes all known α_s corrections Herren, Steinhauser, Fael, Schonwald
- Includes power corrections up to $1/m_b^5$ Mannel, Milutin, KKV [2311.1200]

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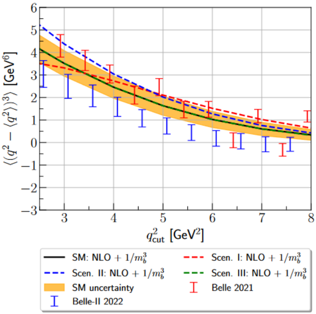
Quirks:

- Includes New Physics effects Fael, Rahimi, KKV [JHEP 02 (2023) 086]

New Physics?

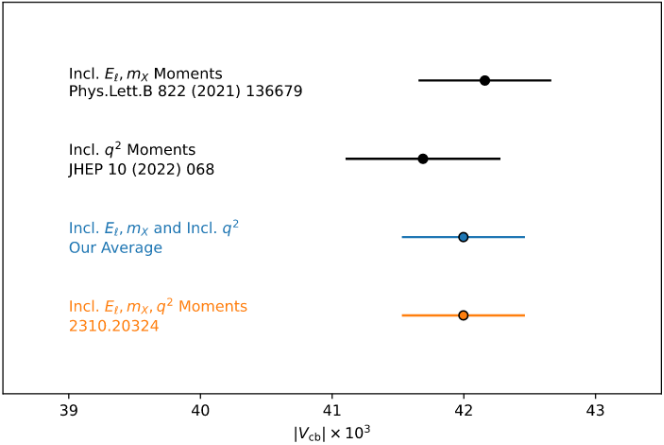
Fael, Rahimi, KKV [2208.04282]



- NP would also influence the moments of the spectrum [Never tested!]
- Requires a simultaneous fit of hadronic parameters and NP

Summary of $|V_{cb}|$ inclusive

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

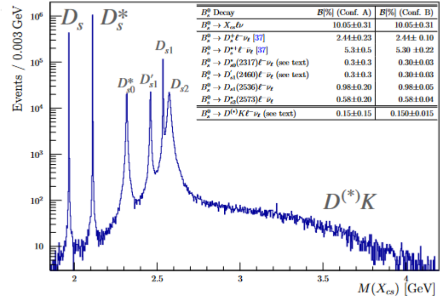


- Need new branching ratio measurements!

Inclusive B_s decays?

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael

First study of the possiblities using sum-over-exclusive technique



B_s^0 Decay	$\mathcal{B}$[%] (Conf. A)	$\mathcal{B}$[%] (Conf. B)
$B_s^0 \to X_{cs}\ell\nu$	10.05±0.31	10.05±0.31
$B_s^0 \to D_s^+\ell^-\nu_\ell$ [37]	2.44±0.23	2.44± 0.10
$B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37]	5.3±0.5	5.30 ±0.22
$B_s^0 \to D_{s0}^*(2317)\ell^-\nu_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D_{s1}(2536)\ell^-\nu_\ell$	0.98±0.20	0.98±0.05
$B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$	0.58±0.20	0.58±0.04
$B_s^0 \to D^{(*)}K\ell^-\nu_\ell$ (see text)	0.15±0.15	0.150±0.015

- B_s spectrum well-separated
- Only M_X^2 moments available
- Study $SU(3)$ breaking of HQE

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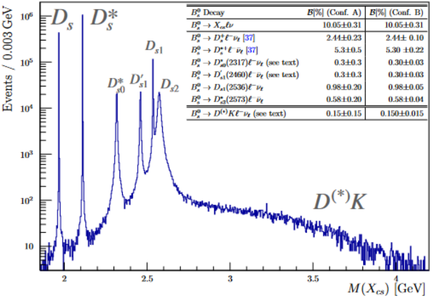

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- Improve knowledge D_s^{**} states
- Understand non-resonant contribution
- V_{cs} extraction requires Branching ratio from Belle II!

Heavy quark expansion for charm?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}

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  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\mathrm{QCD}}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \to s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Quirks:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

The HQE for charm

$D \to X_q \ell\nu$ is not a copy of $B \to X_c \ell\nu$!

OPE for $b \to c\ell\bar{\nu}$: $m_Q \sim m_q \gg \Lambda_{\rm QCD}$

- q is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v(iD^\alpha \cdots iD^\beta)Q_v$
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \dots\right]\frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

The HQE for charm

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- four-quark operators remain in OPE (weak annihilation)
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- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{\rm 4q}|D\rangle$
- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.
- RPI quantities depend on reduced set

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Let's test the HQE for charm on data!

What mass to use?

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\mathrm{kin}} = m_Q^{\mathrm{Pole}} - \left[\bar{\Lambda}\right]_{\mathrm{pert}} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\mathrm{pert}} + \ldots$$

$$[\bar{\Lambda}]_{\mathrm{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\mathrm{pert}} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\boldsymbol{\Lambda_{\rm QCD} < \mu < m_Q}$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??

    $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$

    $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Challenge: $\mu = 0.5$ GeV touches upon the non-perturbative regime?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344 Boushmelev, Mannel, KKV [2301.05607]

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Replace m_c by moments of the spectral density!

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- First study shows small improvement in pert. series

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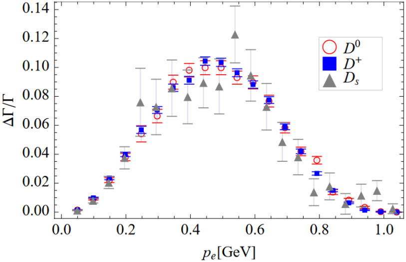
- Replace m_c by moments of the spectral density!

- In progress: Similar approach for the charm + power corrections

Extracting weak annihilation from data

CLEO data, Gambino, Kamenik [1004.0114]



- Lepton energy moments extracted from spectrum
- Kinetic mass for charm at $\mu = 0.5$ GeV threshold, HQE parameters as input
- Max 2% weak annihilation (WA) contribution to $B \to X_u \ell \nu$

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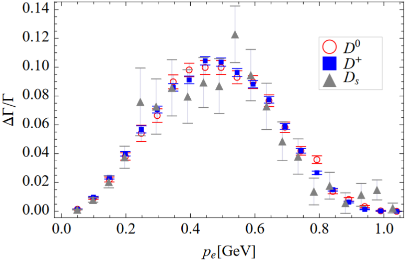

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- In progress: Feasibility study to measure q^2 moments at BESIII Bernlochner, Gilman, Malde, Prim, KKV, Wilkinson

It's Quirks all the way down...

Quirks in inclusive decays?

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Close collaboration between theory and experiment necessary!

Backup

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} \left(\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)\right) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

- Include terms up to $1/m_b^4$* see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

$$|V_{cb}|^{\rm BCG}_{\rm incl} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

Inclusive $B \to X_u$ semileptonic decays

Inclusive V_{ub}

Inclusive $B \to X_u \ell \nu$

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \to c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce non-perturbative shape functions ($\sim$ parton DAs in DIS)
- Different frameworks: BLNP, GGOU, DGE, ADFR

Recent update:

Belle [2102.00020]

$$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

Inclusive determinations need to be scrutinized

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
$\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

BLNP update - in progress -

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  - In progress: include higher-moments
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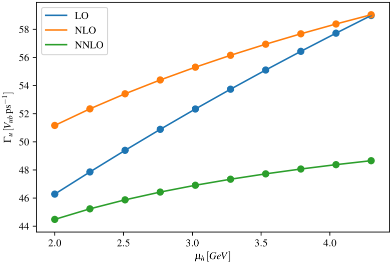
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization

Shape function parametrization

Preliminary! Olschewsky, Lange, Mannel, KKV [2306.xxxx]



- α_s^2 corrections give large corrections [see also Pezcjak 2019]
- Required to make precision predictions

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV [arXiv: 2105.02163]

Avoid background subtraction by calculating the full inclusive width:

$$d\Gamma(B \to X\ell) = d\Gamma(B \to X_c \ell \bar{\nu}) + d\Gamma(B \to X_u \ell \bar{\nu}) + d\Gamma(B \to X_c(\tau \to \ell \bar{\nu} \nu)\bar{\nu})$$

- $b \to u \ell \nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \to c(\tau \to \mu \nu \bar{\nu})\bar{\nu}$ contribution: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \to u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \to X_c\ell\nu$ by taking $m_c \to 0$ limit
- For V_{ub} determination
  - large charm background requires experimental cuts
  - reduces the inclusivity and local OPE no longer converges
  - spectrum described by non-local OPE
  - convolution of pert. coefficients with shape function

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2$+$1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \to X_c\ell\nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

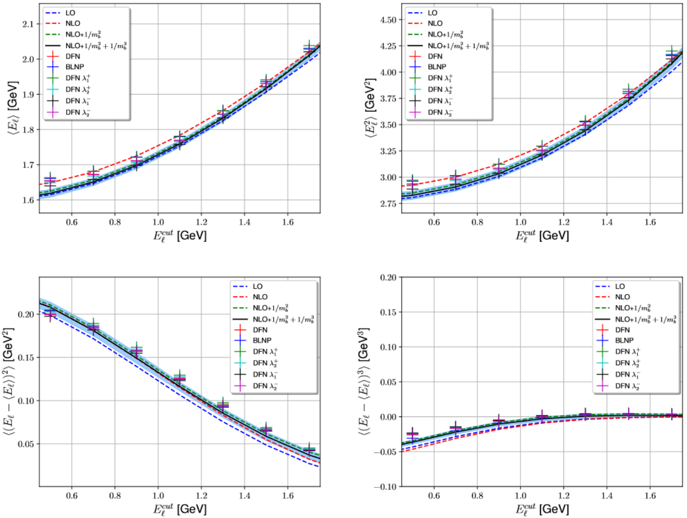
Monte Carlo:

- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
  - Inputs from $B \to X_c \ell \nu$ and $B \to X_s \gamma$ data using KN-scheme Kagan, Neubert 1998
  - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

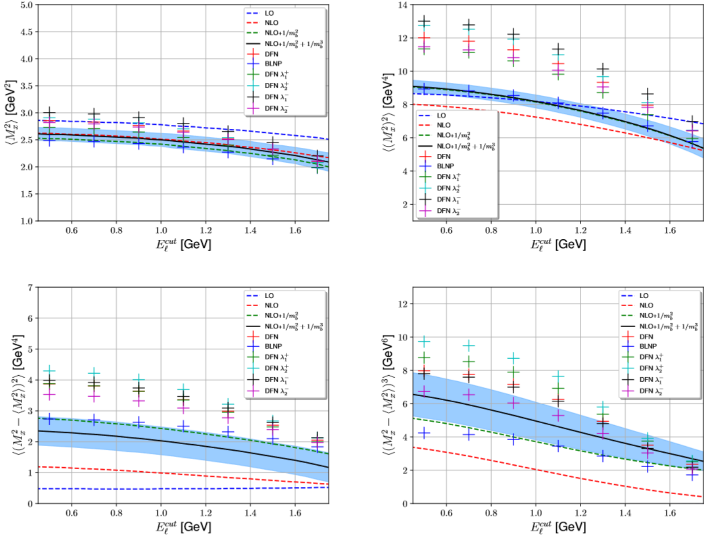
- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi \ell \bar{\nu}$ and $\bar{B} \to \rho \ell \bar{\nu}$

Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

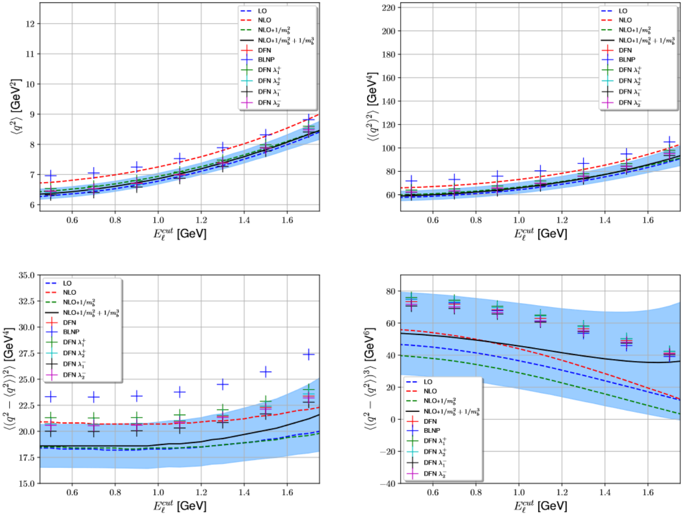
Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Wide spread between MC for higher moments

Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Monte Carlo versus local OPE: remarks

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
  - should include higher moments of the shape-function model?
  - include subleading shape functions?
- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

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- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0|T[j_\mu(x) j_\nu(0)]|0\rangle = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)$$

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$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right)$$

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- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s) \tag{1}$$

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- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n\cdot D)) b_v |B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y) - \omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

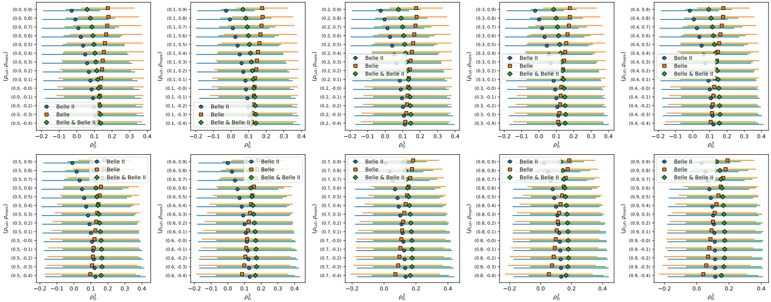
$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$
- Included by adding a penalty term to the χ^2
- Scan over large range of values
- V_{cb} constant w.r.t. theory correlations



Interplay between electrons and muons

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

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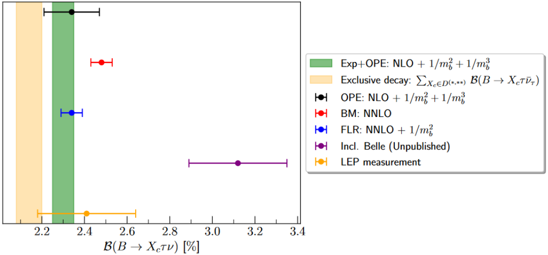

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- Next step ratios with τ! Need new measurements!

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