Using Gradient Flow to Renormalise Matrix Elements for Meson Mixing and Lifetimes

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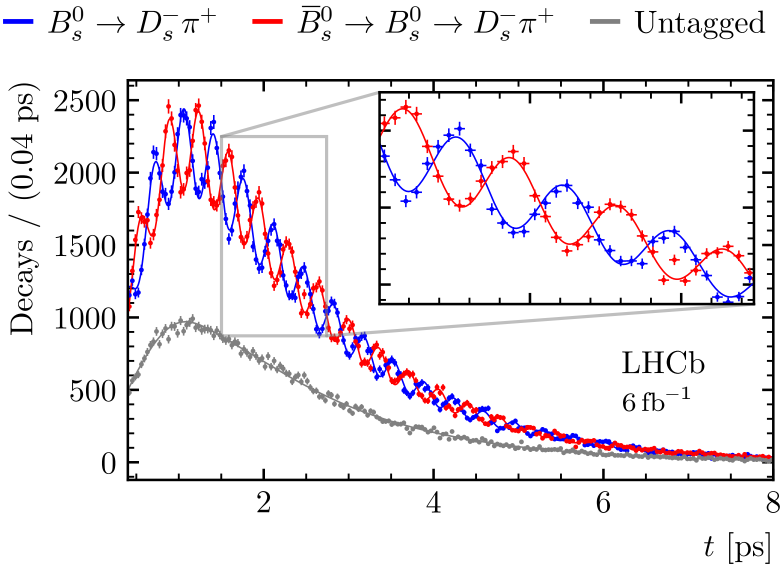
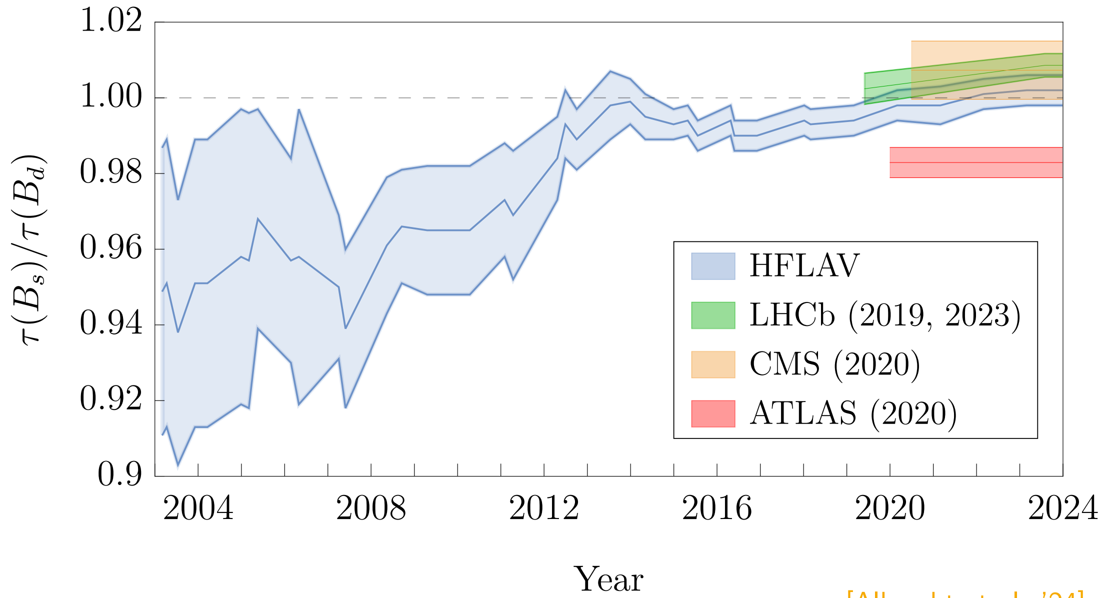
Introduction

- B mesons: bound QCD states of b quark and light antiquark (or charge conjugate)
- Large mass ($m_b \sim 4.2\,$GeV) and relatively long lifetime produce diverse phenomenology
  - ➥ Lifetime prediction enters the predictions of many processes
- CDF, D0, BaBar, Belle(II), LHCb, ATLAS, CMS brought about high-precision era for B physics
- Neutral B mesons $B_s^0 = (\bar{b}s)$, $\bar{B}_s^0 = (b\bar{s})$ have different mass eigenstates ➡ quark eigenstate "mixing" or oscillations



[LHCb '21 005]

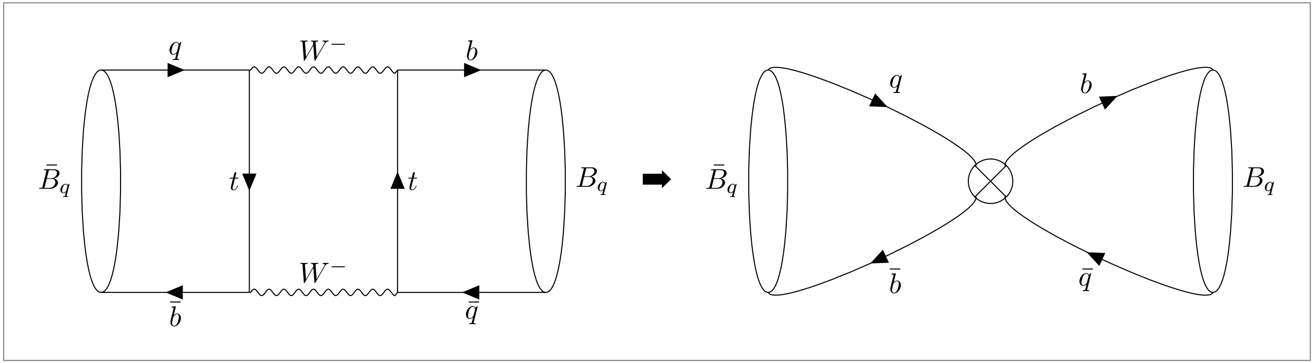
- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**



[Albrecht et al. '24]

- B-meson mixing and lifetimes are measured experimentally to high precision
  - Key observables for probing New Physics ➡ **high precision in theory needed!**
- For B lifetimes and mixing, we use the **Heavy Quark Expansion**

$$\Gamma_{B_q} = \Gamma_3 \langle \mathcal{O}_{D=3} \rangle + \Gamma_5 \frac{\langle \mathcal{O}_{D=5} \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_{D=6} \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_{D=6} \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_{D=7} \rangle}{m_b^4} + \ldots \right]$$



- Factorise observables into ➡ perturbative QCD contributions
  ➡ **Non-Perturbative Matrix Elements**

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing
  - ➥ preliminary $\Delta K = 2$ for Kaon mixing study with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago
  - ➥ contributions from gluon disconnected diagrams
  - ➥ mixing with lower dimension operators in renormalisation

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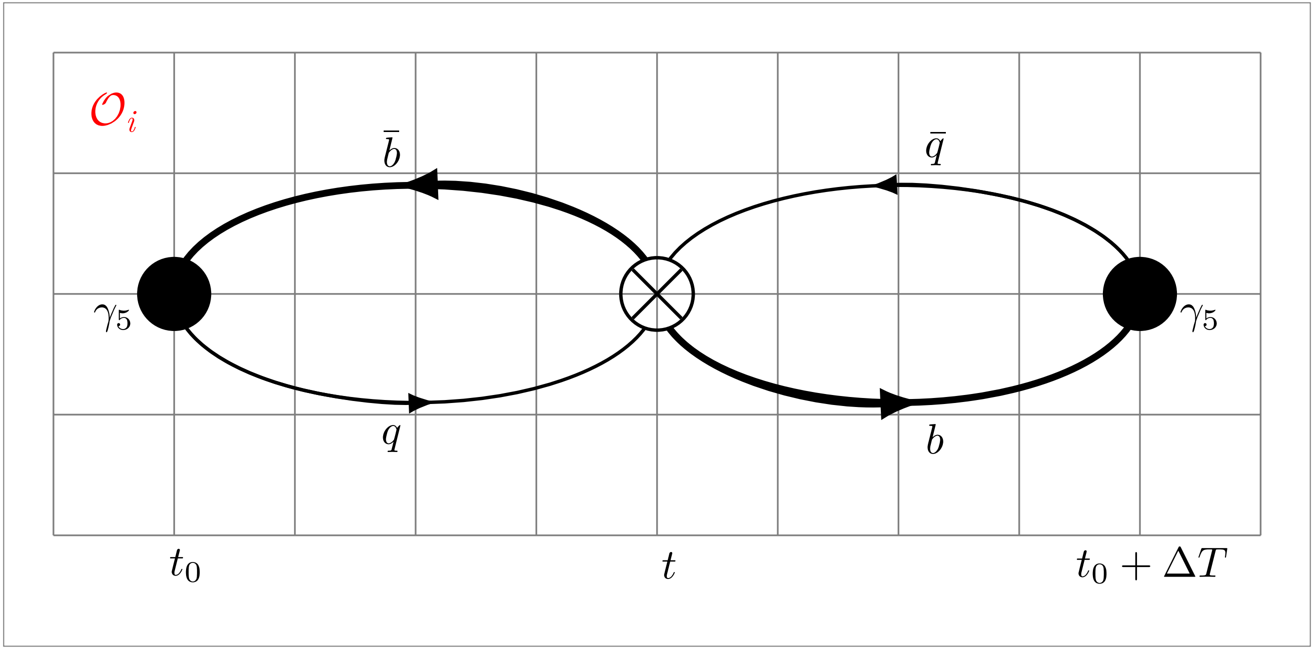
New Developments:

- [Lin, Detmold, Meinel '22] ➡ spectator effects in b hadrons
  - ➥ focus on lifetime ratios for both B mesons and Λ_b baryon
  - ➥ isospin breaking, $\langle B|\mathcal{O}^d - \mathcal{O}^u|B\rangle$
  - ➥ position-space renormalisation + perturbative matching to $\overline{\mathrm{MS}}$
- this work, [Black et al. '23]
  - ➥ goal is individual $\Delta B = 0$ matrix elements for B mesons
  - ➥ non-perturbative gradient flow renormalisation
  - ➥ perturbative matching to $\overline{\mathrm{MS}}$ in short-flow-time expansion

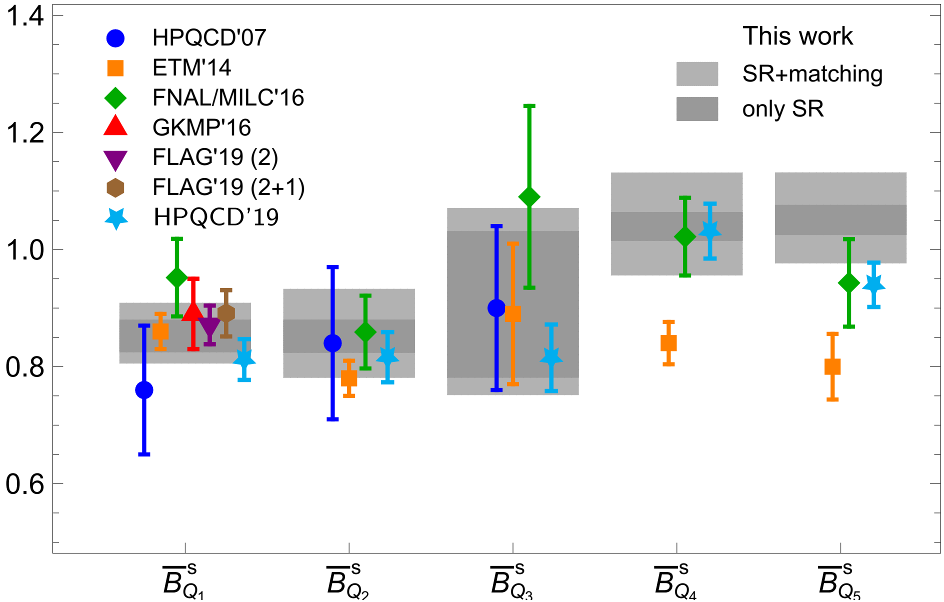
Operators and Current Status


$\mathcal{O}_i$
$\bar{b}$
$\bar{q}$
γ_5
γ_5
q
b
t_0
t
$t_0 + \Delta T$

- Mass difference of neutral mesons $\Delta M_q \,(q = d, s)$ governed by $\Delta B = 2$ four-quark operators
- General BSM basis has 5 dimension-six operators
- In the SM, only $\mathcal{O}_1^q$ contributes to ΔM

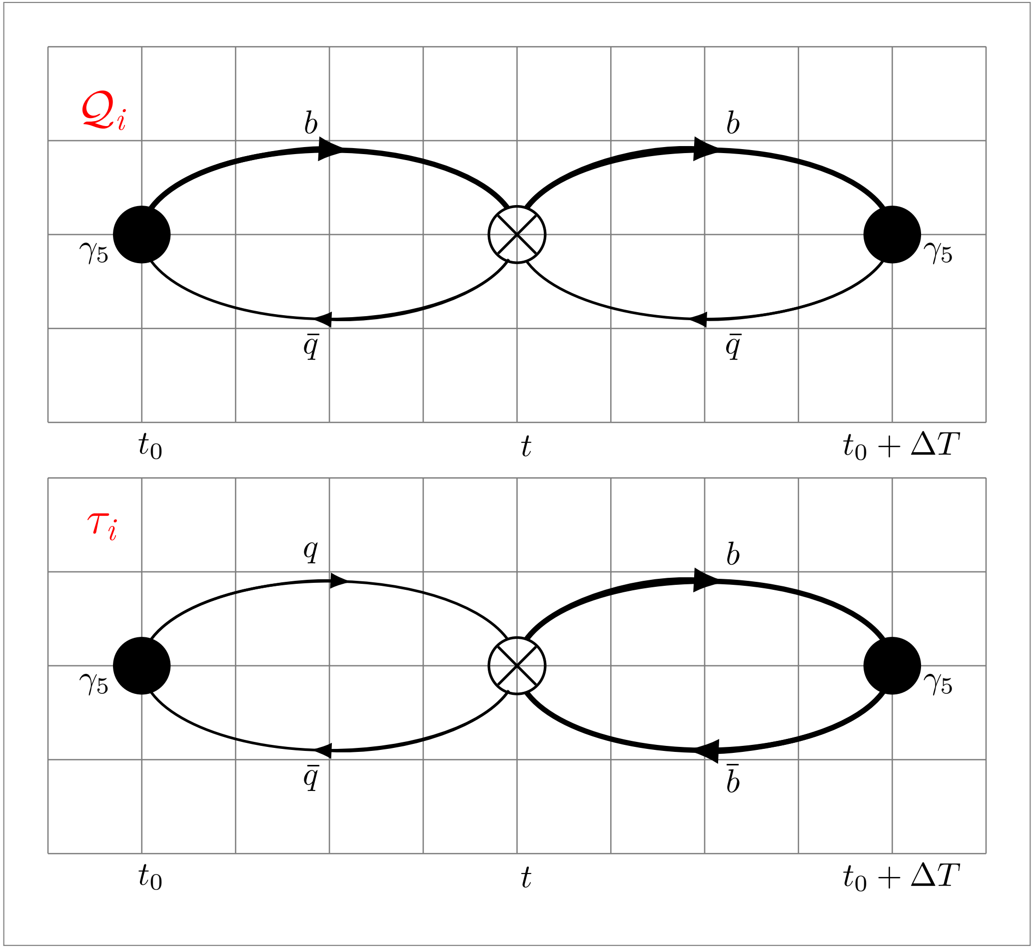
$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q$$

- Matrix elements parameterised in terms of **decay constant** f_{B_q} and **bag parameters** B_i^q
- HPQCD and FNAL/MILC choose perturbative renormalisation + matching schemes
- RBC-UKQCD set up a non-perturbative renormalisation (NPR)



[King '22]

- $\Delta B = 2$ Bag parameters well-studied on the lattice and with QCD sum rules
- see also ongoing work by RBC/UKQCD and JLQCD [Boyle et al '21] [Tsang, Lattice '23]
- dimension-7 matrix elements calculated for first time [HPQCD '19]



$\mathcal{Q}_i$
b
b
γ_5
γ_5
$\bar{q}$
$\bar{q}$
t_0
t
$t_0 + \Delta T$
τ_i
q
b
γ_5
γ_5
$\bar{q}$
$\bar{b}$
t_0
t
$t_0 + \Delta T$

- For lifetimes, the dimension-6 $\Delta B = 0$ operators are:

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

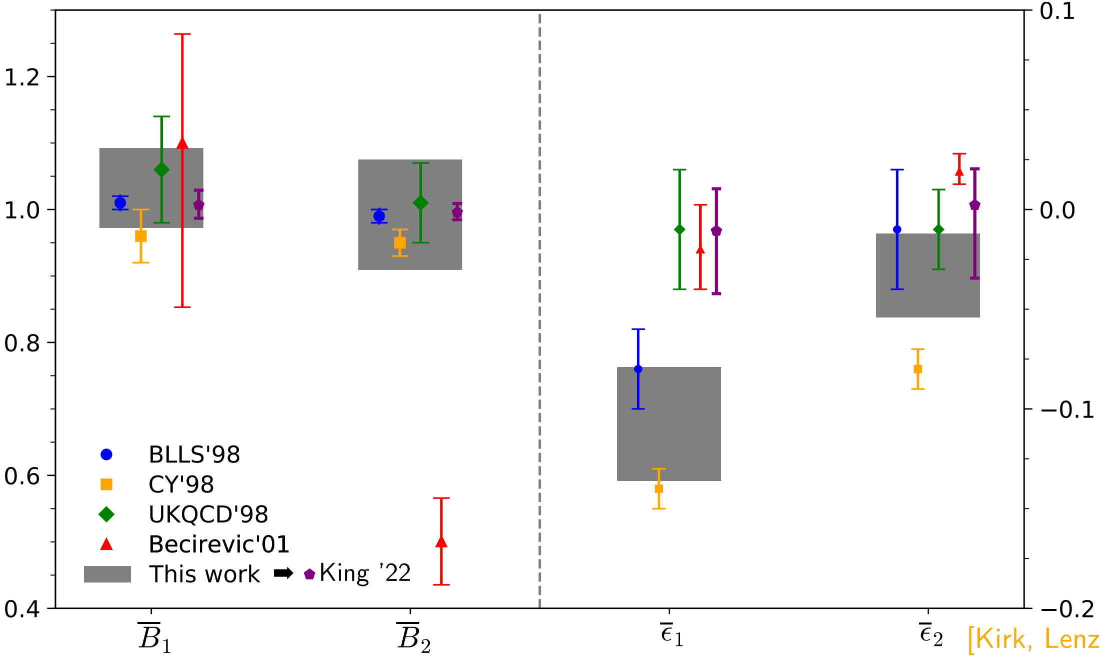
$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we want these to be colour-singlet operators:

$$\begin{aligned}
\mathcal{Q}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\
\mathcal{Q}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1+\gamma_5) b^\beta) \\
\tau_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\
\tau_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta
\end{aligned}
\qquad
\begin{pmatrix} Q_1^+ \\ Q_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \tau_1^+ \\ \tau_2^+ \end{pmatrix}$$



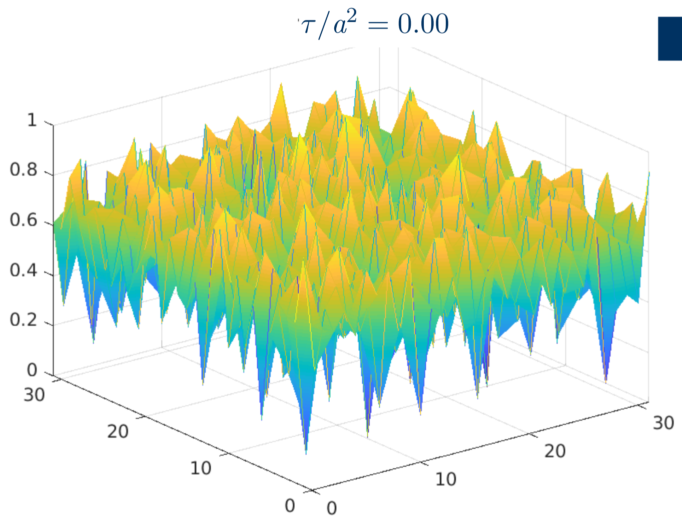
[Kirk, Lenz, Rauh '17]

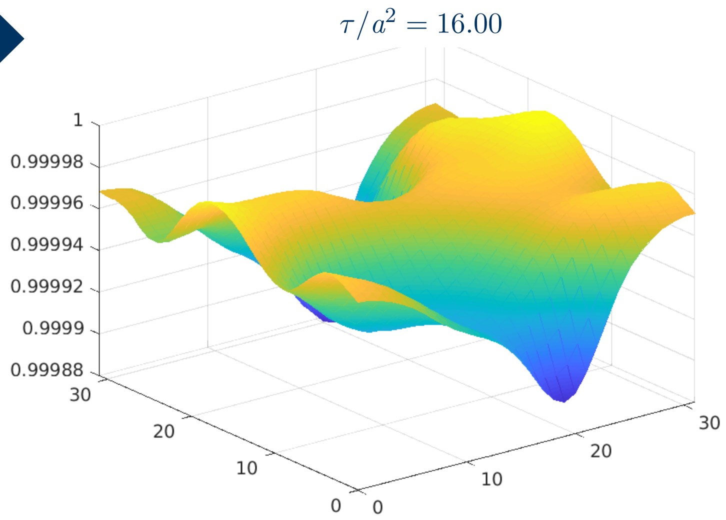
➤ Sum rules results taken in HQET limit

1. Complete exploratory studies in simplified setup without additional extrapolations
   - ➥ test case for gradient flow renormalisation and short-flow-time expansion procedure
   - ➥ simulate physical charm and strange ➡ consider charm-strange pseudoscalar meson
2. Run full-scale simulations for B meson mixing and lifetimes
3. Use $\Delta B = 2$ matrix elements for further validation of method
4. Pioneer connected $\Delta B = 0$ matrix element calculation
5. Tackle disconnected contributions

Gradient Flow

- Introduced by [Narayanan, Neuberger '06] [Lüscher '10] [Lüscher '13]
  - ➥ scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
- Introduce auxiliary dimension, **flow time** τ as a way to regularise the UV
- Well-defined smearing of gauge and fermion fields ➡ smoothens UV fluctuations





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  - ➥ scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
- Introduce auxiliary dimension, **flow time** τ as a way to regularise the UV
- Well-defined smearing of gauge and fermion fields ➡ smoothens UV fluctuations
- Extend gauge and fermion fields in flow time and express dependence with first-order differential equations:

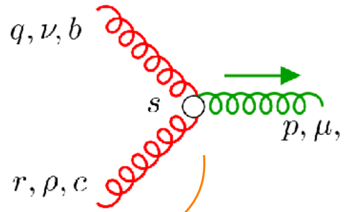
$$\partial_t B_\mu(\tau, x) = \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau, x), \quad B_\mu(0, x) = A_\mu(x),$$
$$\partial_t \chi(\tau, x) = \mathcal{D}^2(\tau)\chi(\tau, x), \quad \chi(0, x) = q(x).$$

- Fermionic Gradient Flow needed for renormalisation
- For use in renormalisation, there are two concepts:
  - ➥ Gradient flow as an RG transformation [Carosso et al. '18] [Hasenfratz et al. '22]
  - ➥ Short-flow-time expansion [Lüscher, Weisz '11] [Makino, Suzuki '14] [Monahan, Orginos '15]

- Well-studied for e.g. energy-momentum tensor [Makino, Suzuki '14] [Harlander, Kluth, Lange '18]

- Re-express effective Hamiltonian in terms of 'flowed' operators:

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau) \widetilde{\mathcal{O}}_n(\tau).$$



- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau) \mathcal{O}_m + O(\tau)$$

new Feynman diagrams

'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

matching matrix calculated perturbatively

Gradient Flow – Short-Flow-Time Expansion

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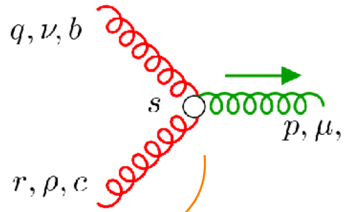

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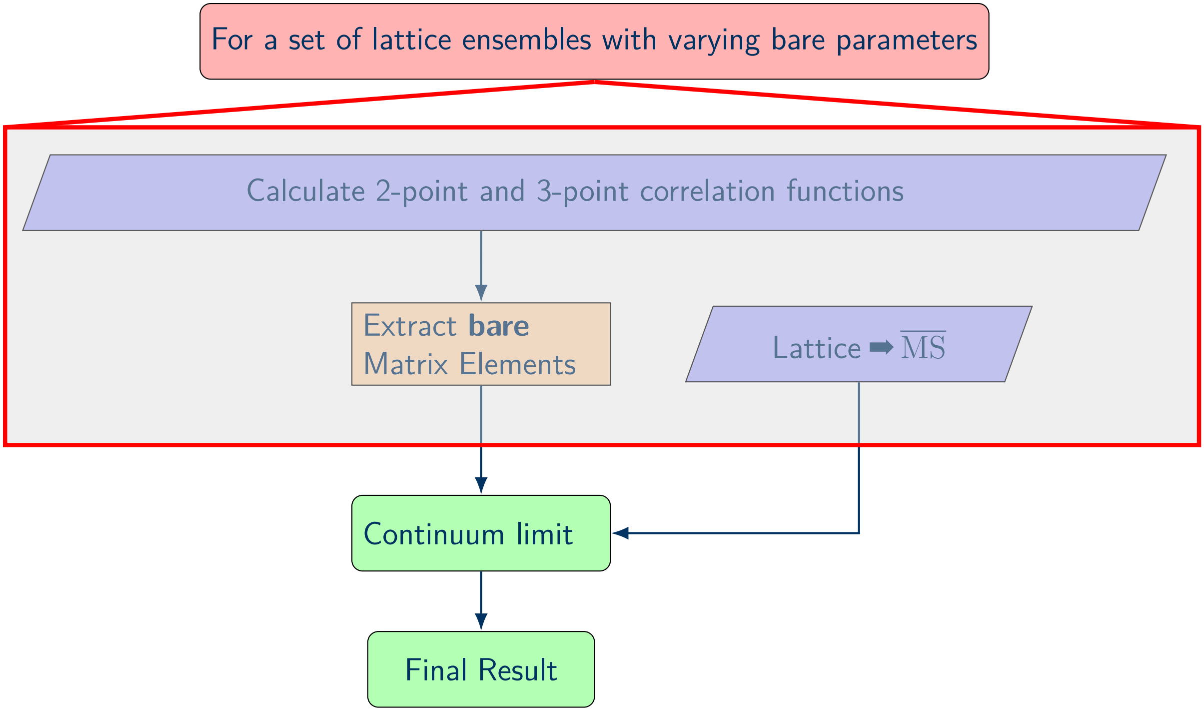
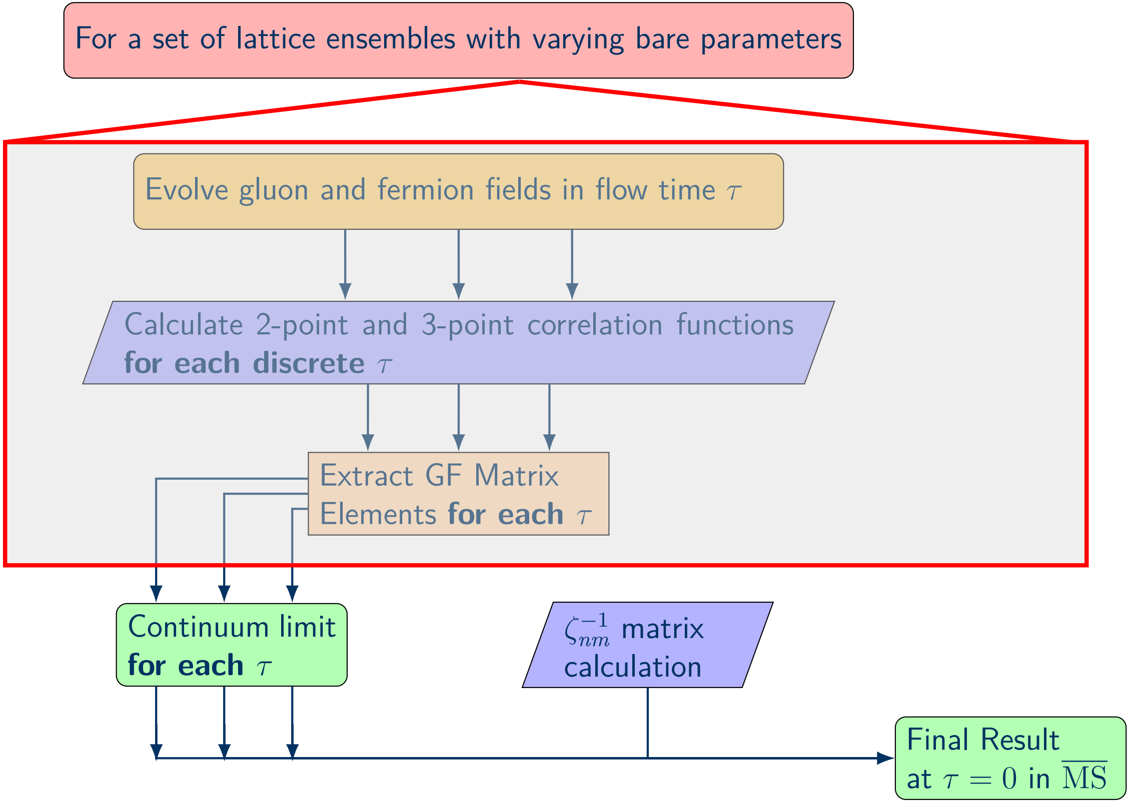
$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in $\overline{\text{MS}}$ found in $\tau \to 0$ limit ➡ 'window' problem
  - ➥ large systematic effects at very small flow times
  - ➥ large flow time dominated by operators $\propto O(\tau)$


For a set of lattice ensembles with varying bare parameters
Calculate 2-point and 3-point correlation functions
Extract **bare** Matrix Elements
Lattice ➡ $\overline{\mathrm{MS}}$
Continuum limit
Final Result


For a set of lattice ensembles with varying bare parameters
Evolve gluon and fermion fields in flow time τ
Calculate 2-point and 3-point correlation functions **for each discrete** τ
Extract GF Matrix Elements **for each** τ
Continuum limit **for each** τ
ζ_{nm}^{-1} matrix calculation
Final Result at $\tau = 0$ in $\overline{\mathrm{MS}}$

Lattice Simulation

- We use RBC/UKQCD's 2+1 flavour DWF + Iwasaki gauge action ensembles [Shamir '93] [Iwasaki, Yoshie '84] [Iwasaki '85]

	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs × N_{conf}
C1	24	64	1.7848	0.005	0.040	340	32 × 101
C2	24	64	1.7848	0.010	0.040	433	32 × 101
M1	32	64	2.3833	0.004	0.030	302	32 × 79
M2	32	64	2.3833	0.006	0.030	362	32 × 89
M3	32	64	2.3833	0.008	0.030	411	32 × 68
F1S	48	96	2.785	0.002144	0.02144	267	24 × 98

[Allton et al. '08] [Aoki et al. '10] [Blum et al. '14] [Boyle et al. '17]

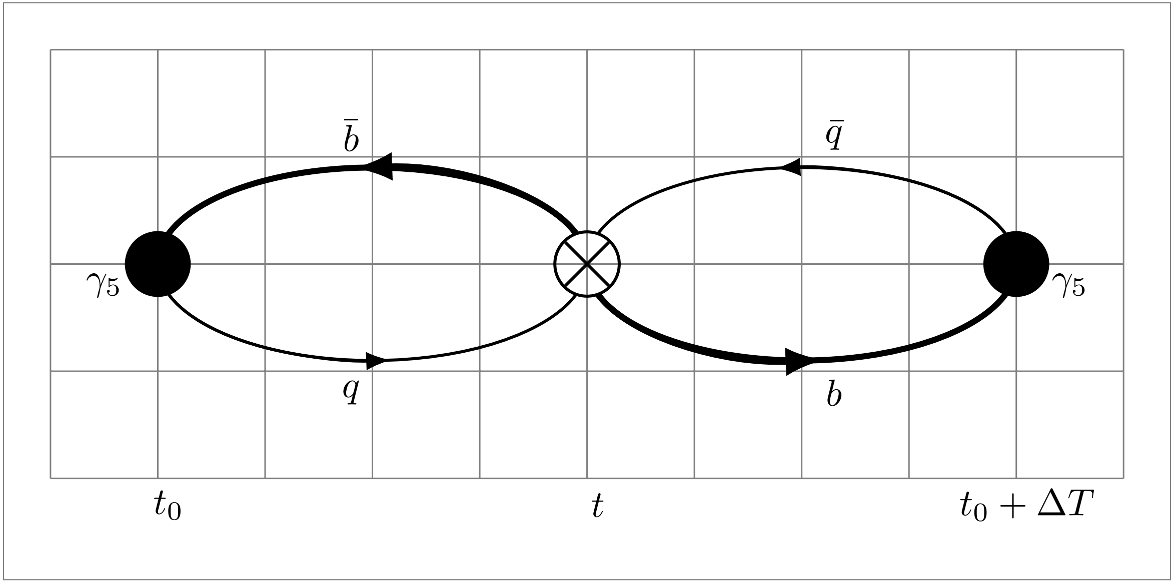
- For strange quarks tuned to physical value, $am_q \ll 1$ ✔
  - ➥ Shamir DWF
- For heavy b quarks, $am_q > 1$ ➡ large discretisation effects ✗
  - ➥ manageable for physical c quarks instead
  - ➥ stout-smeared Möbius DWF [Morningstar, Peardon '03] [Brower, Neff, Orginos '12]
- Exploratory setup using physical charm and strange quarks
  - ➥ $\Delta B = 0, 2$ ➡ $\Delta Q = 0, 2$, for generic heavy quark Q
  - ➥ neutral charm-strange meson mixing ➡ proxy to short-distance D^0 mixing up to spectator effects

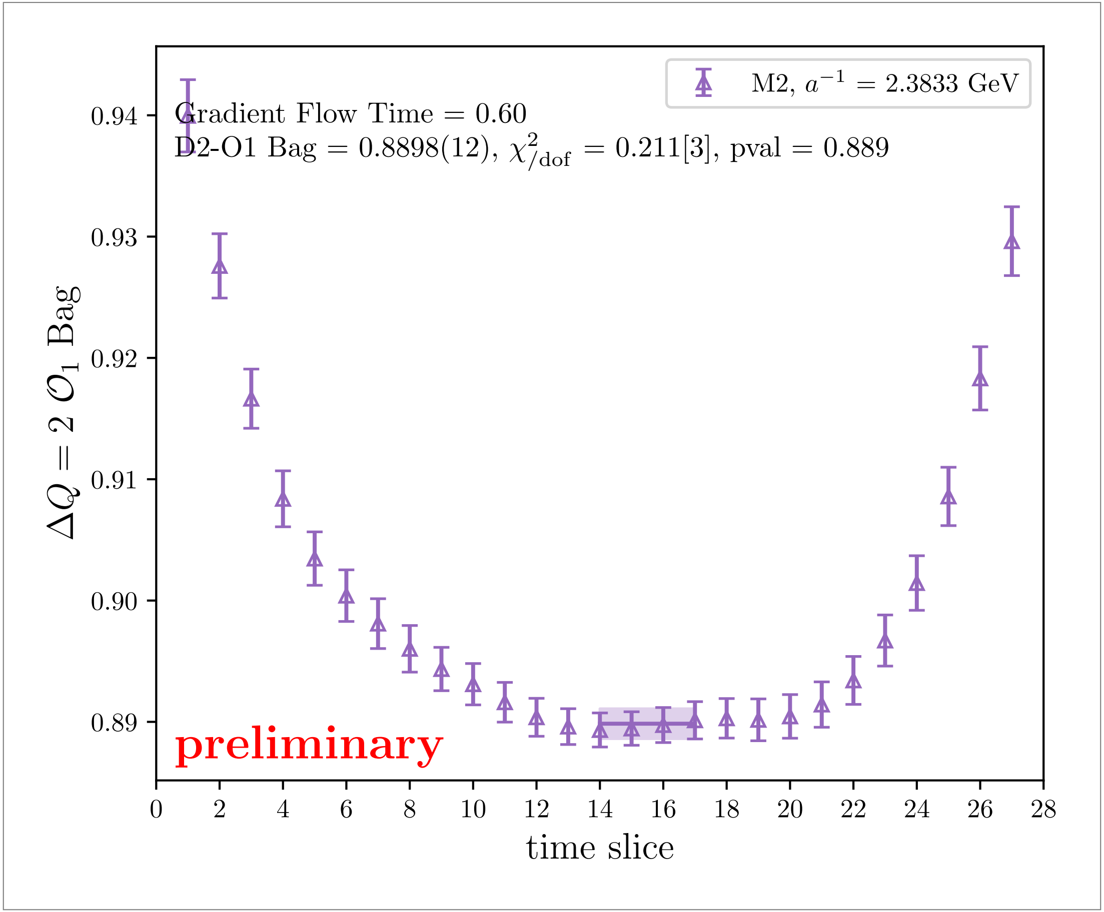
Data Analysis and Results – $\Delta Q = 2$

- Three-point correlation function:

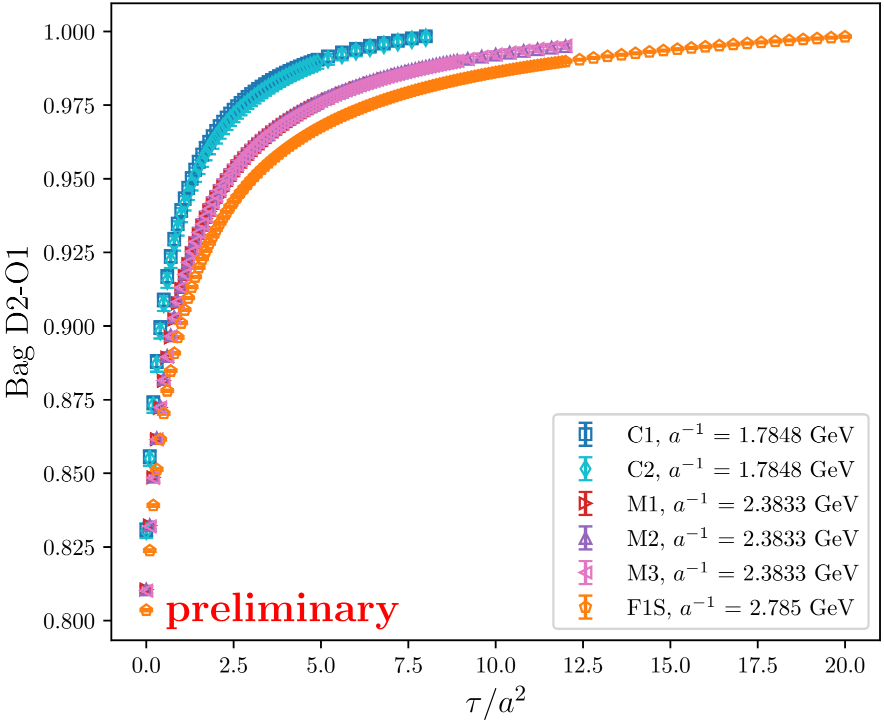
$$C^{3\text{pt}}_{\mathcal{Q}_i}(t, \Delta T, \tau) = \sum_{n,n'} \frac{\langle P_n | \mathcal{Q}_i | P_{n'} \rangle(\tau)}{4 M_n M_{n'}} e^{-(\Delta T - t) M_n} e^{-t M_{n'}} \underset{t_0 \ll t \ll t_0 + \Delta T}{\Longrightarrow} \frac{\langle P \rangle^2}{4 M^2} \langle \mathcal{Q}_i \rangle(\tau) e^{-\Delta T M}$$

- Measure along positive flow time τ

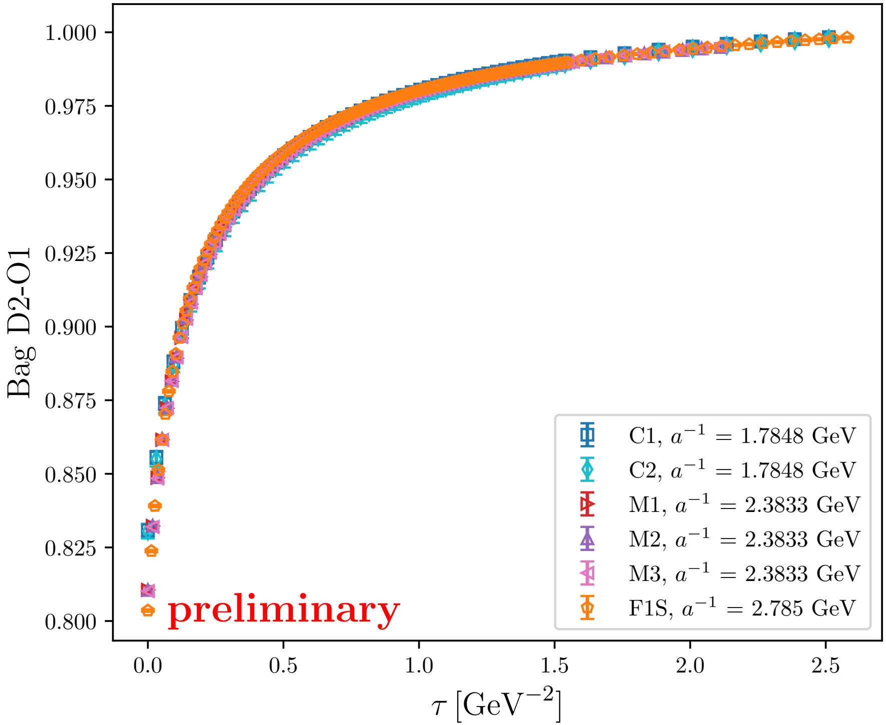




M2, $a^{-1} = 2.3833$ GeV
Gradient Flow Time = 0.60
D2-O1 Bag = 0.8898(12), $\chi^2_{/\mathrm{dof}} = 0.211[3]$, pval = 0.889
$\Delta Q = 2$ $\mathcal{O}_1$ Bag
0.94
0.93
0.92
0.91
0.90
0.89
preliminary
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
time slice



- operator is renormalised in 'GF' scheme as it is evolved along flow time
- data at same lattice spacing overlap ➡ no light sea quark effects



➤ different lattice spacings overlap in physical flow time ➡ mild continuum limit

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

matching matrix calculated perturbatively

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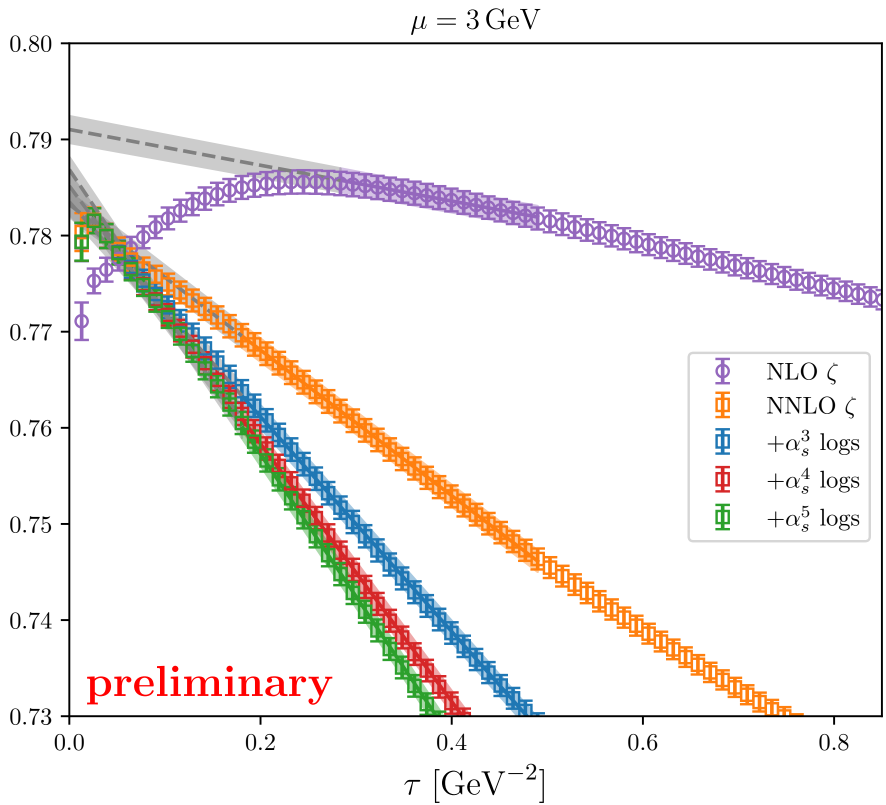
- Calculated at two-loop for $\mathcal{B}_1$ based on [Harlander, Lange '22] [Borgulat et al. '23]:

$$\zeta^{-1}_{\mathcal{B}_1}(\mu,\tau) = 1 + \frac{a_s}{4}\left(-\frac{11}{3} - 2L_{\mu\tau}\right) + \frac{a_s^2}{43200}\Big[-2376 - 79650L_{\mu\tau} - 24300L^2_{\mu\tau} + 8250n_f + 6000\,n_f\,L_{\mu\tau} + 1800\,n_f\,L^2_{\mu\tau} - 2775\pi^2 + 300\,n_f\pi^2 - 241800\log 2 + 202500\log 3 - 110700\,\mathrm{Li}_2\left(\frac{1}{4}\right)\Big]$$

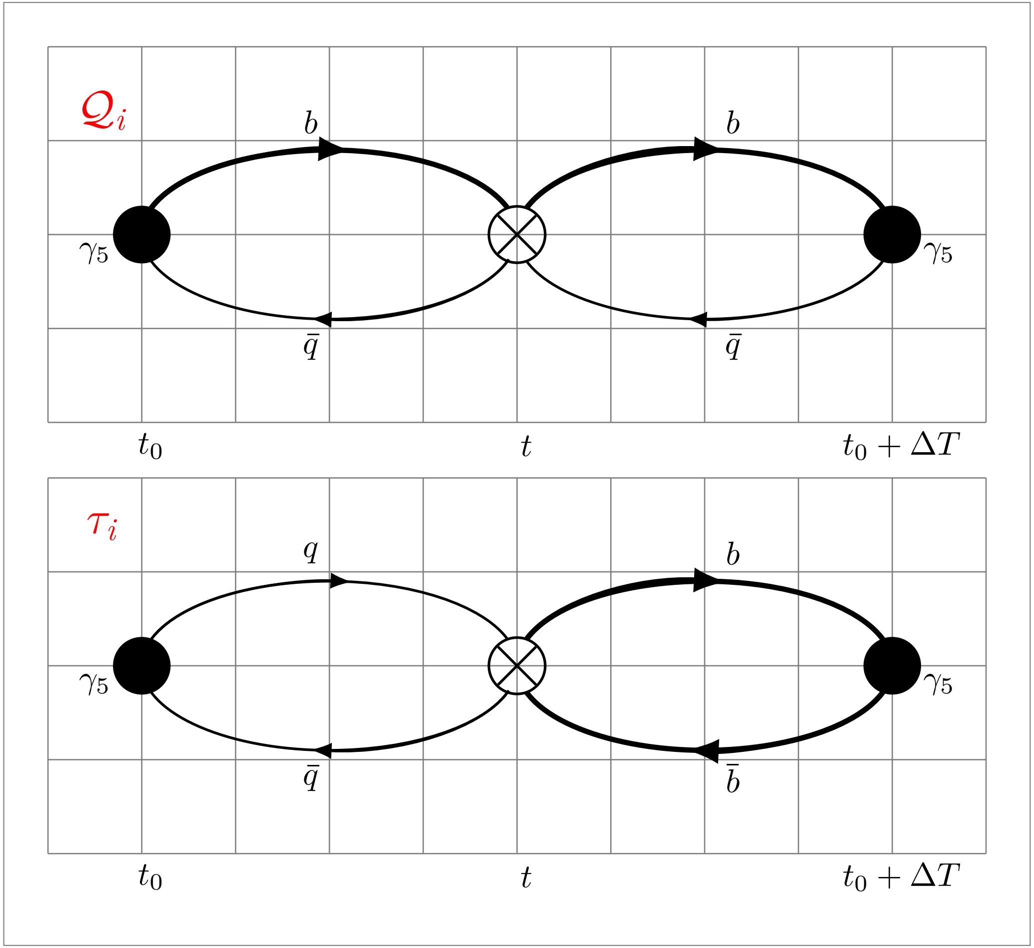
$$L_{\mu\tau} = \log(2\mu^2\tau) + \gamma_E, \quad \mu = 3\,\mathrm{GeV}$$

- Promising first signs of agreement
  - statistical errors only
- Different perturbative orders "in same ball park"
  - systematic errors needed for meaningful comparison
- Consider existing short-distance D^0 mixing results

[ETM '15]	0.757(27)
[FNAL/MILC '17]	0.795(56)

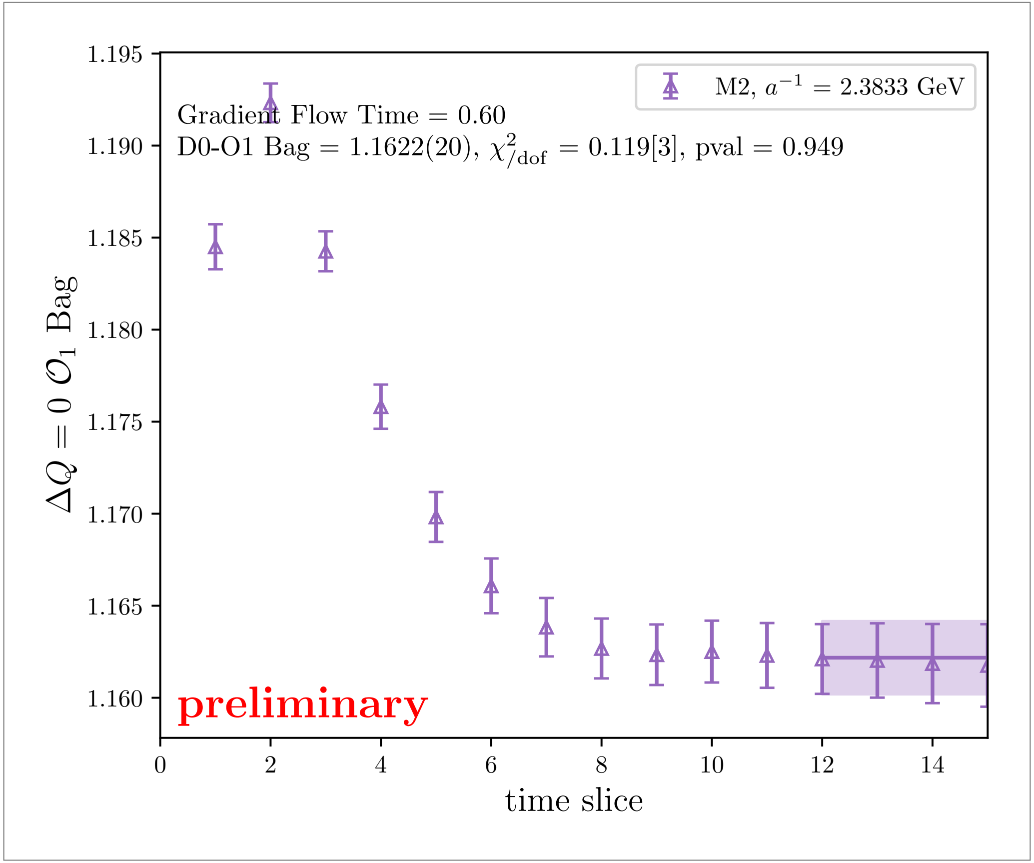


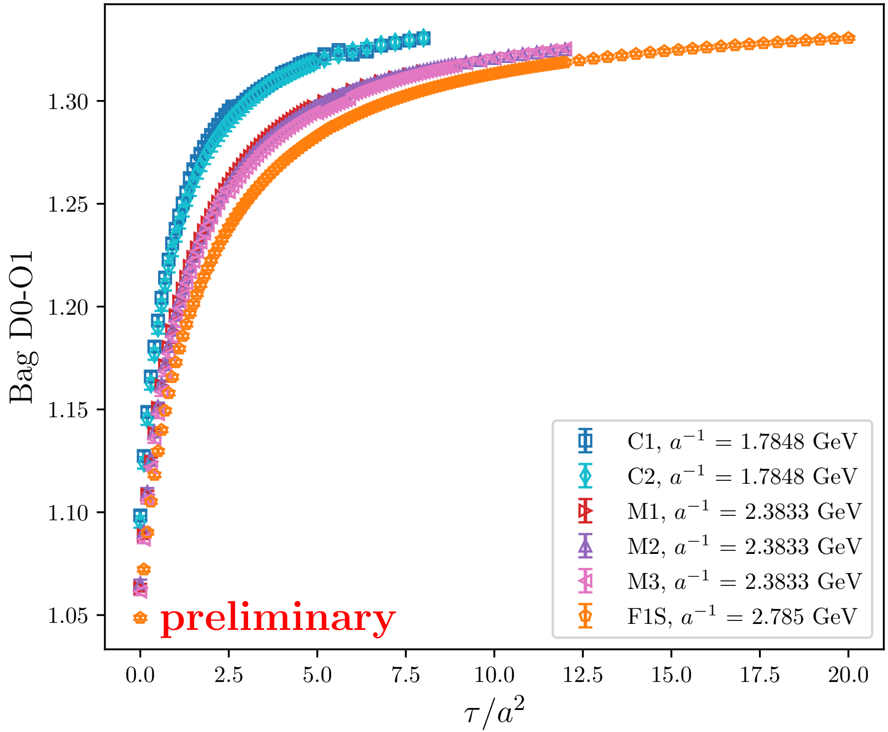
Data Analysis – $\Delta Q = 0$



$\mathcal{Q}_i$
b
b
γ_5
γ_5
$\bar{q}$
$\bar{q}$
t_0
t
$t_0 + \Delta T$
$\mathcal{T}_i$
q
b
γ_5
γ_5
$\bar{q}$
$\bar{b}$
t_0
t
$t_0 + \Delta T$

- Bag parameters for $\mathcal{Q}_i$ extracted as for $\Delta B = 2$ operators





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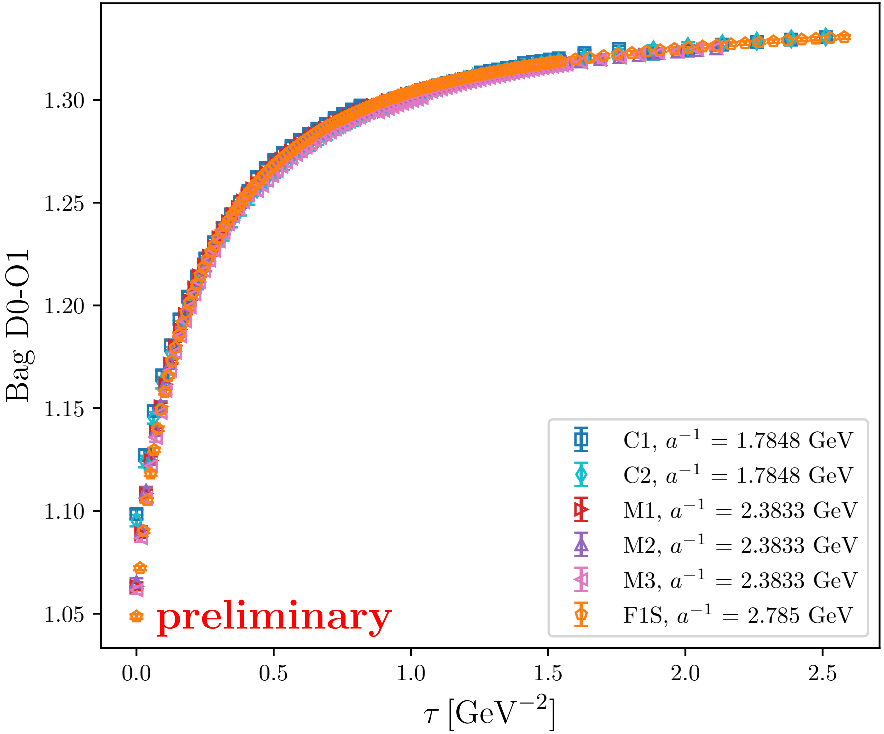
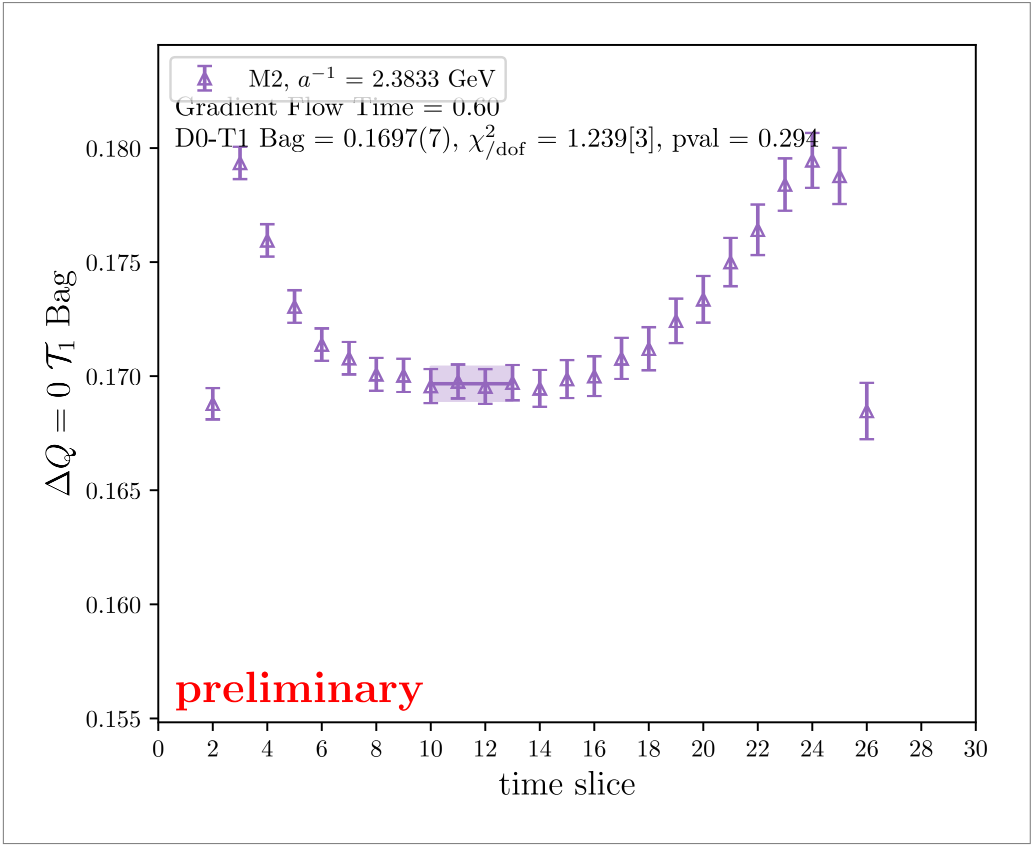


➤ different lattice spacings overlap in physical flow time ➡ mild continuum limit

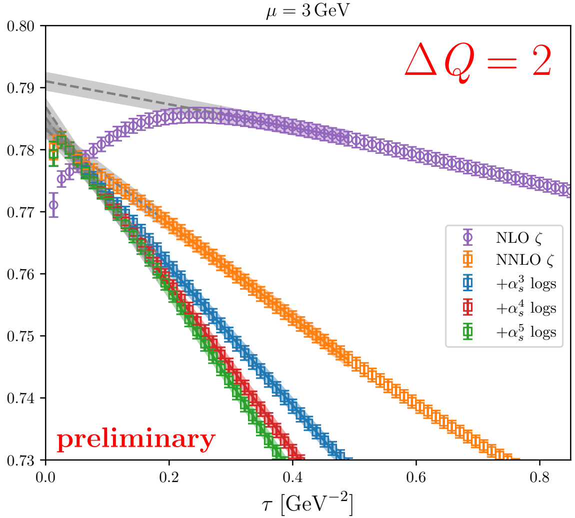
- Three-point functions for τ_i have different functional form

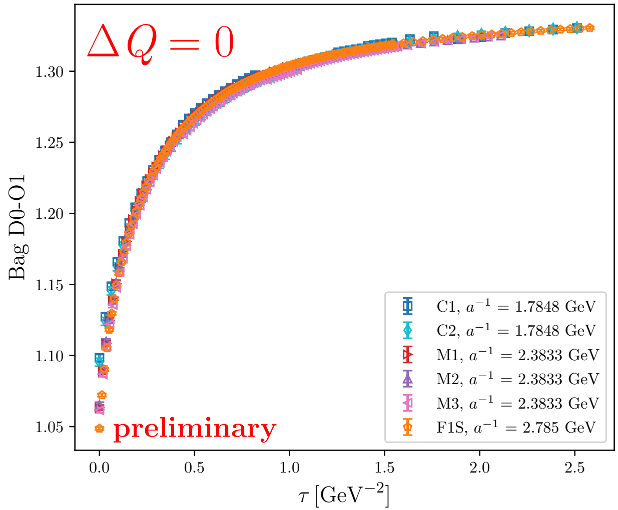
- asymmetric signal: $(b\bar{b}) \to (s\bar{s})$
- O_1 and T_1 mix in renormalisation
  - need both for preliminary results
- work in progress



Summary and Outlook

- $\Delta B = 0$ four-quark matrix elements are strongly-desired quantities
  - Standard renormalisation introduces mixing with operators of lower mass dimension
  - We aim to use the fermionic gradient flow as a non-perturbative renormalisation procedure
- We calculate $\Delta Q = 2$ matrix elements as a test case for the short-flow-time expansion
- Shown first analysis for short-distance charm-strange mixing and charm-strange lifetimes
- Preliminary $\Delta Q = 2$ results show promising consistency with literature





- Complete exploratory work with physical charm-strange meson
  - GF→ $\overline{\mathrm{MS}}$ analysis for all 5 dimension-six $\Delta Q = 2$ operators
  - Validation against literature
  - First GF→ $\overline{\mathrm{MS}}$ analysis for dimension-six $\Delta Q = 0$ operators (connected pieces)
- Perturbative matching needed for complete $\Delta B = 2$ basis and all $\Delta B = 0$ operators
- Complete full-scale simulations for B meson mixing and lifetimes
  - multiple heavier-than-charm masses ➡ extrapolate to physical b mass
  - further comparisons to literature with $\Delta B = 2$ results
  - first results for $\Delta B = 0$ operators
- Consider gluon disconnected contributions

Join us for... **Lattice Meets Continuum**

Siegen, 30th September – 4th October 2024

https://indico.physik.uni-siegen.de/event/158/

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- Full BSM basis:

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \qquad \langle \mathcal{O}_2^q \rangle = \langle \bar{B}_q | \mathcal{O}_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$\mathcal{O}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1-\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_3^q \rangle = \langle \bar{B}_q | \mathcal{O}_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$\mathcal{O}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \qquad \langle \mathcal{O}_4^q \rangle = \langle \bar{B}_q | \mathcal{O}_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b+m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$\mathcal{O}_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \; \bar{b}^\beta (1+\gamma_5) q^\alpha, \qquad \langle \mathcal{O}_5^q \rangle = \langle \bar{B}_q | \mathcal{O}_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b+m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

- Transformed basis (colour singlets only)

$$\begin{aligned}
\mathcal{Q}_1^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \\
\mathcal{Q}_2^q &= \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1+\gamma_5) q^\beta, \\
\mathcal{Q}_3^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta, \\
\mathcal{Q}_4^q &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta, \\
\mathcal{Q}_5^q &= \frac{1}{4} \bar{b}^\alpha \sigma_{\mu\nu} (1-\gamma_5) q^\alpha \; \bar{b}^\beta \sigma_{\mu\nu} (1-\gamma_5) q^\beta
\end{aligned}$$

$$\begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ \mathcal{O}_3^+ \\ \mathcal{O}_4^+ \\ \mathcal{O}_5^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \mathcal{Q}_3^+ \\ \mathcal{Q}_4^+ \\ \mathcal{Q}_5^+ \end{pmatrix}$$

- Advantages for both lattice calculation and the NPR procedure
- We are only concerned with parity-even components which then can be transformed back to SUSY basis