LHCb anomalies, inclusive penguin decays and refactorisation

Tobias Hurth

Johannes Gutenberg Universität Mainz

PRISMA+

Quirks in Quark Flavour Physics

Zadar (Croatia), 18. June 2024

Plan of the Talk

- LHCb anomalies in the post R_K era and the problem of unknown power corrections to exclusive modes

- New physics reach of semileptonic penguin decays and nonlocal subleading corrections in inclusive modes

- Refactorisation in subleading $\bar{B} \to X_s \gamma$

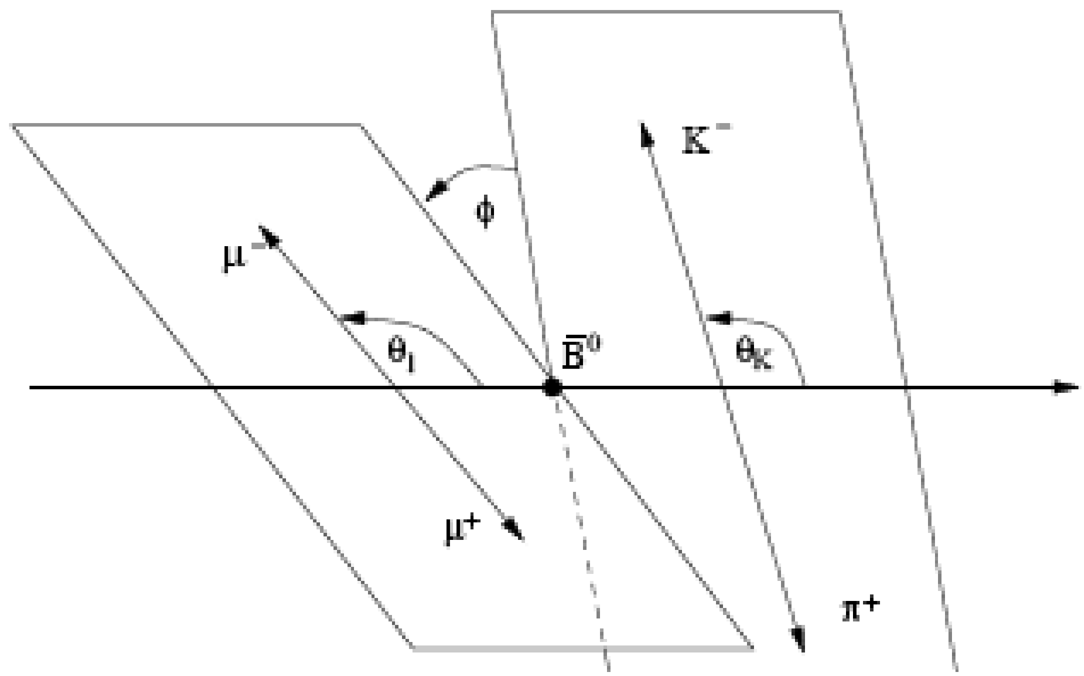
Prologue

The LHCb anomalies

Differential decay rate of $B \to K^* \ell\ell$

Assuming the $\bar{K}^*$ to be on the mass shell, the decay $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ described by the lepton-pair invariant mass, s, and the three angles θ_l, θ_{K^*}, ϕ.

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



$$J(q^2, \theta_l, \theta_K, \phi) =$$

$$= J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_l + J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi$$
$$+ J_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_K \sin\theta_l \cos\phi + (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_l$$
$$+ J_7 \sin 2\theta_K \sin\theta_l \sin\phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi$$

Large number of independent angular obervables

Careful design of theoretical clean angular observables

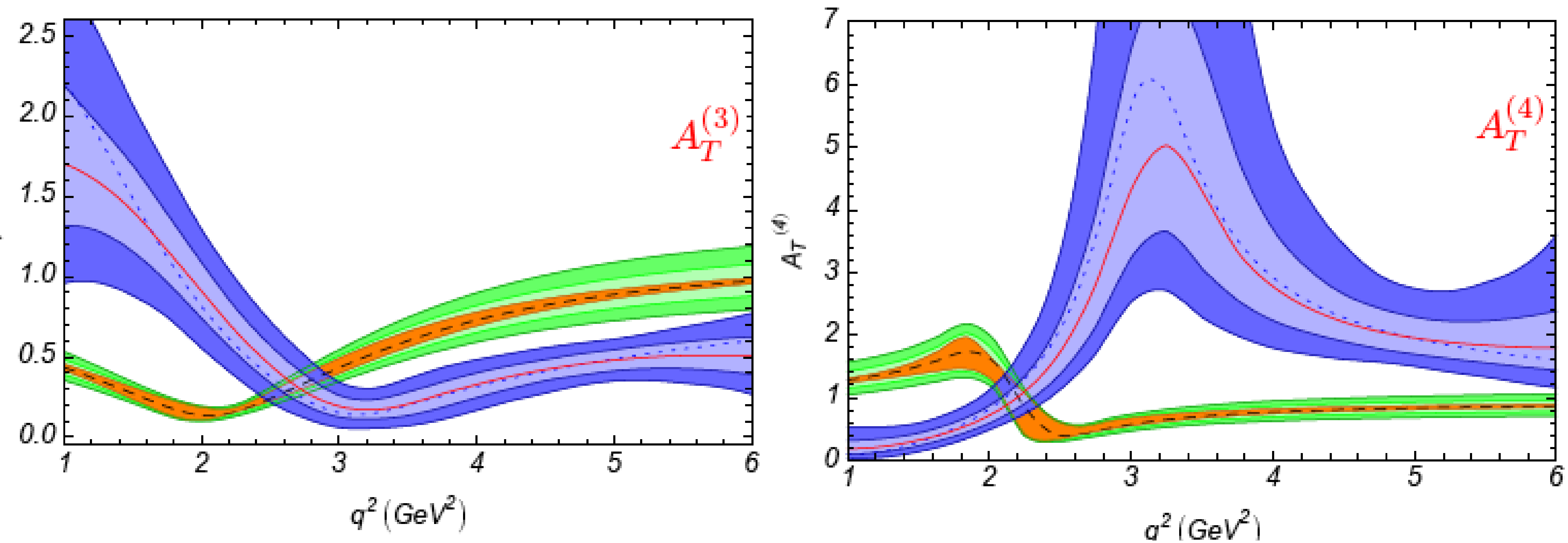
Egede,Hurth,Matias,Ramon,Reece,arXiv:0807.2589,arXiv:1005.0571

– Dependence of soft form factors, $\xi_\perp$ and $\xi_\parallel$, to be minimized !
form factors should cancel out exactly at LO, best for all s

– unknown Λ/m_b power corrections

$A_{\perp,\parallel,0} = A^0_{\perp,\parallel,0}\,(1 + c_{\perp,\parallel,0})$ vary c_i in a range of $\pm 10\%$ and also of $\pm 5\%$

Guesstimate



The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

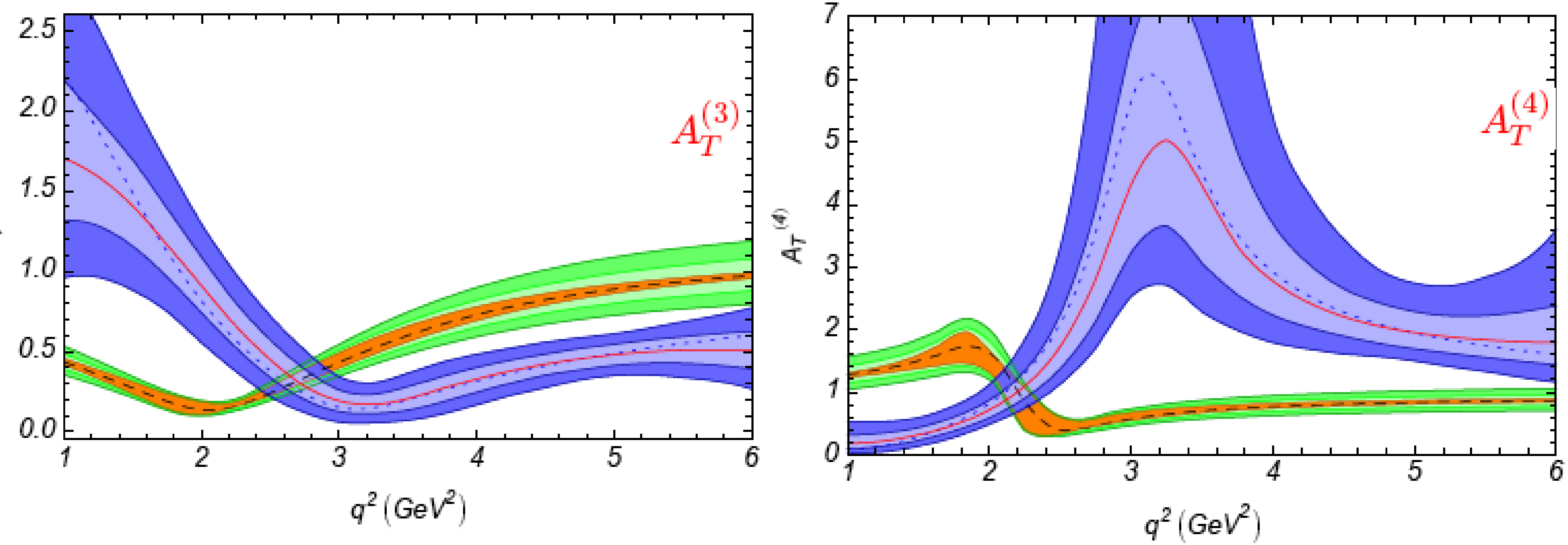
Careful design of theoretical clean angular observables

Egede,Hurth,Matias,Ramon,Reece,arXiv:0807.2589,arXiv:1005.0571

- Dependence of soft form factors, $\xi_\perp$ and $\xi_\parallel$, to be minimized !
  form factors should cancel out exactly at LO, best for all s
- unknown Λ/m_b power corrections

$A_{\perp,\parallel,0} = A^0_{\perp,\parallel,0}\left(1 + c_{\perp,\parallel,0}\right)$ vary c_i in a range of ±10% and also of ±5%

Guesstimate



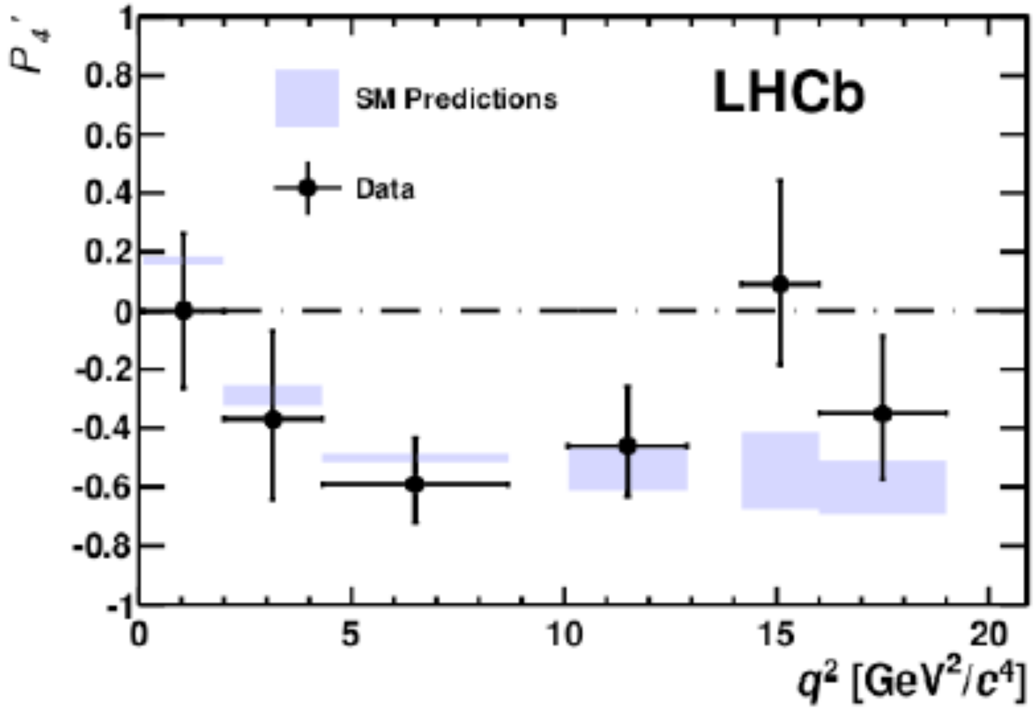
This was the dream in 2008

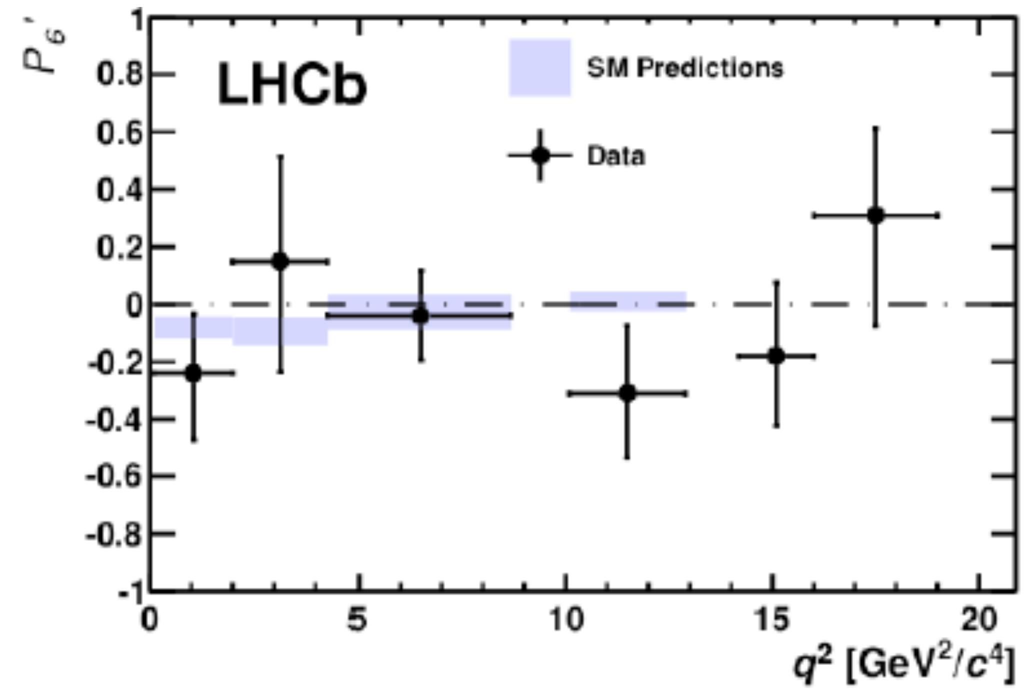
First measurements of new angular observables

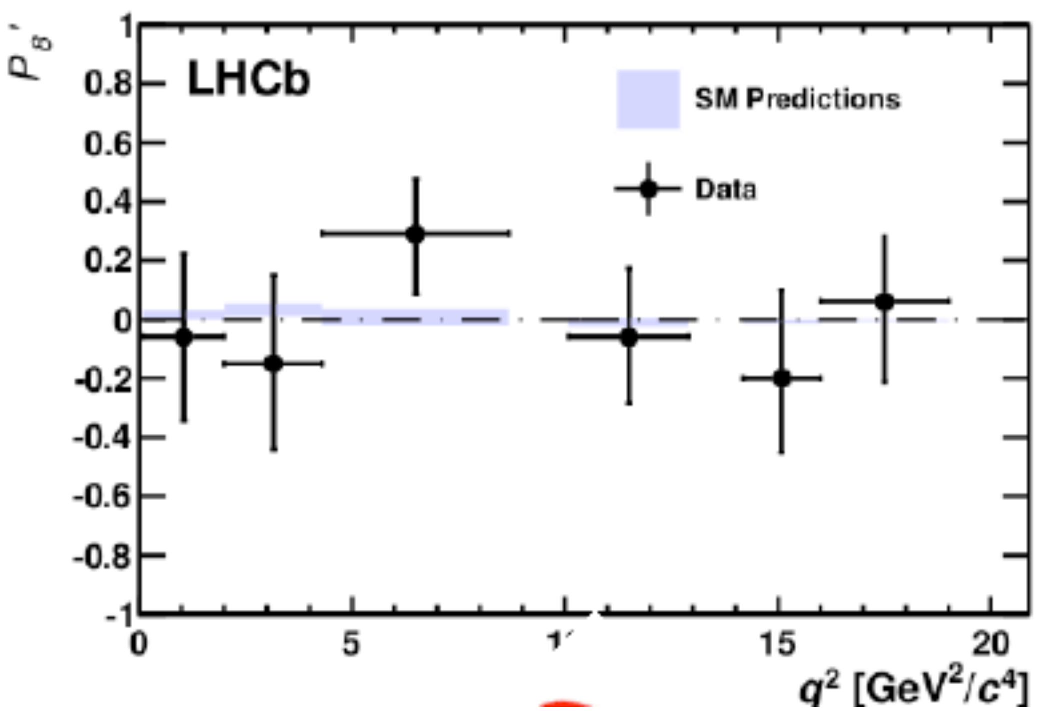
LHCb arXiv:1308.1707

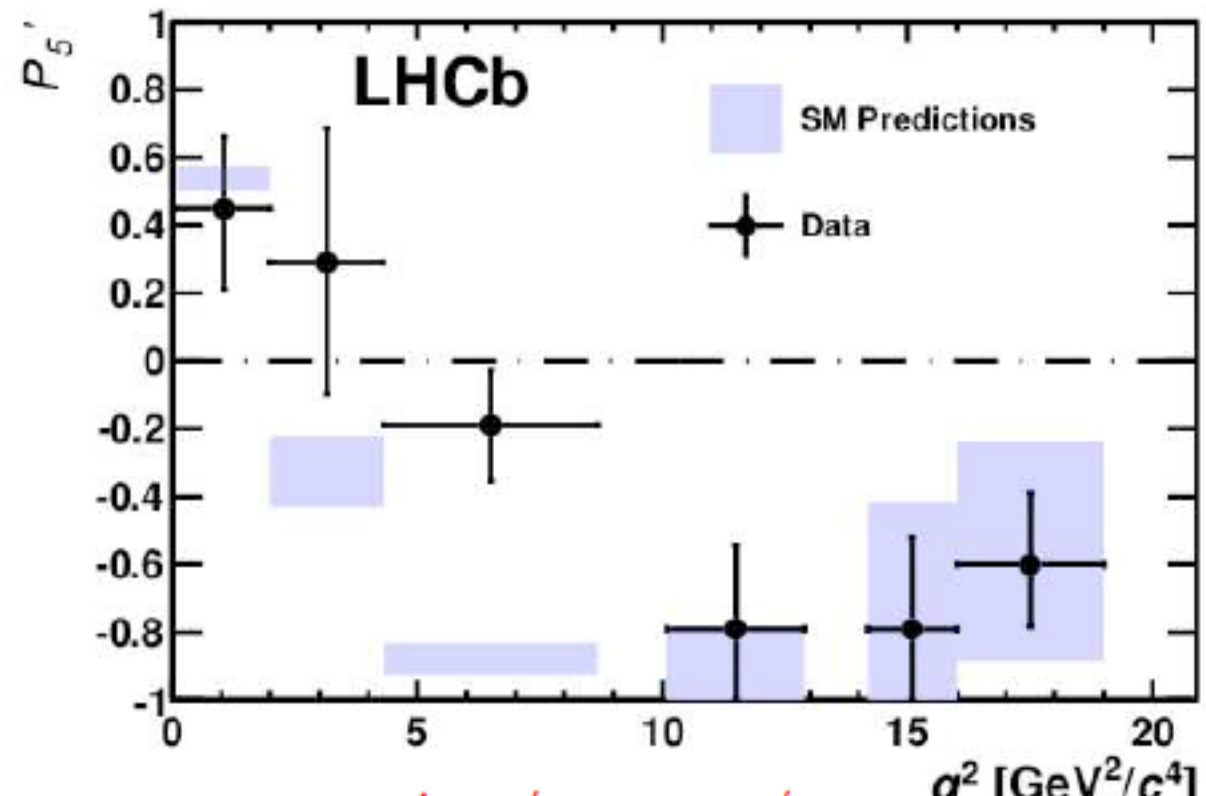
SM predictions

Descotes-Genon, Hurth, Matias, Virto arXiv:1303.5794









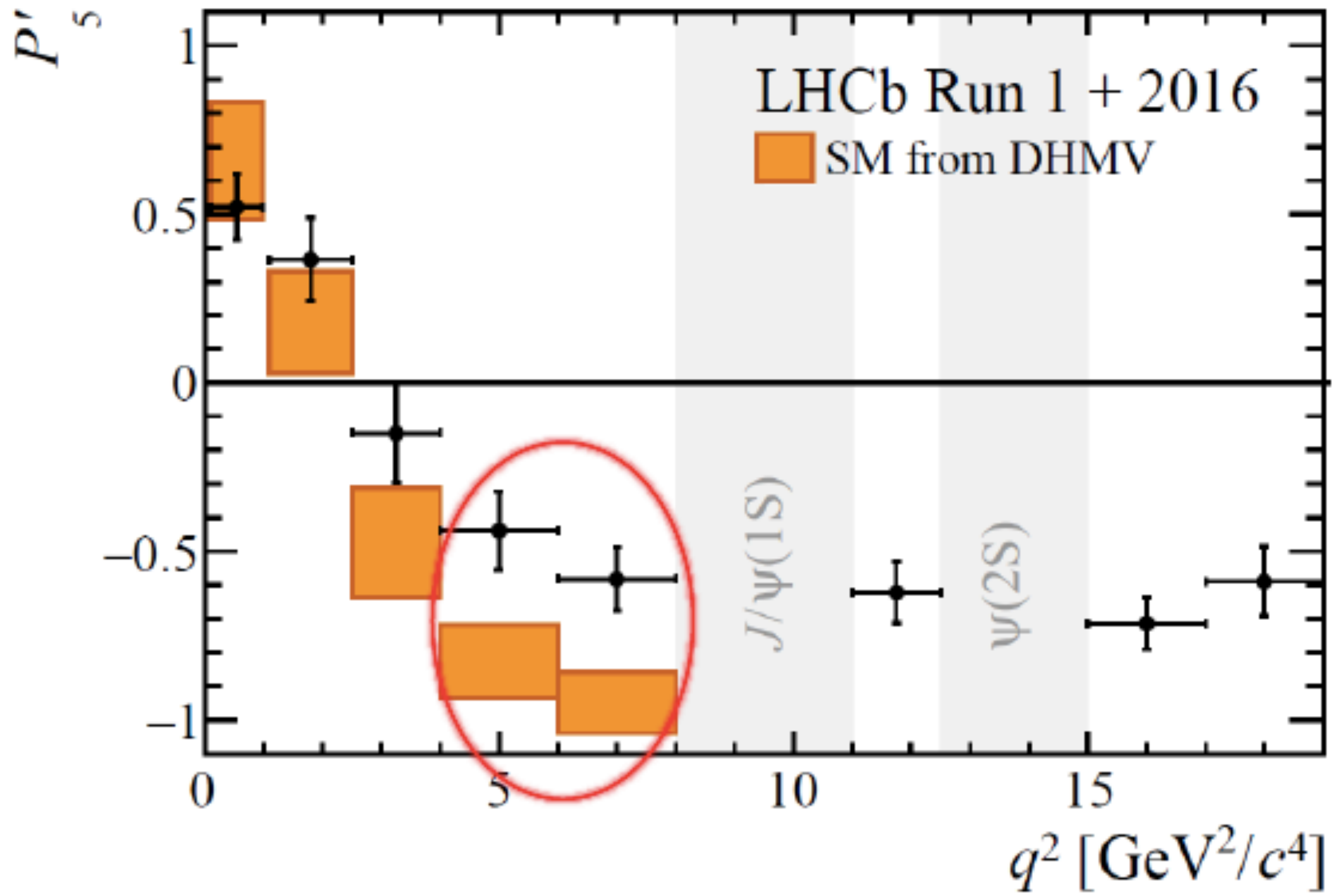
Good agreement with SM in P_4', P_6' and P_8',

but a 3.7σ deviation in the third bin in P_5'

Anomalies in $B \to K^*\mu^+\mu^-$ angular observables, in particular P_5' ; S_5

Long standing anomaly in the $B \to K^*\mu^+\mu^-$ angular observable P_5' / S_5 $\left(= P_5' \times \sqrt{F_L(1-F_L)}\right)$

- 2013 LHCb (1 fb^{-1})
- 2016 LHCb (3 fb^{-1})
- 2020 LHCb (4.7 fb^{-1})



[E. Smith CERN Seminar ’20
LHCb 2003.04831]

"$\approx 3\sigma$" local tension in P_5' with the respect SM predictions (DHMV)

Also deviations in other angular observables/bins and other decay modes

New Physics or underestimated hadronic uncertainties (form factors, power corrections) ?

Lepton flavour universality in $B \to K^{(*)}\ell^+\ell^-$

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$

Hiller,Kruger hep-ph/0310219

- Hadronic uncertainties cancel out

$\Longrightarrow$ *theoretically very clean $\mathcal{O}(1\%)$*

Jun. 2014	May. 2017	Mar. 2019	Mar. 2021	Oct. 2021
LHCb (1 fb^{-1}) 2.6σ in[1-6] GeV^2 of R_K	LHCb (3 fb^{-1}) 2.2σ in [0.045-1.1] GeV^2 2.5σ in [1.1-6] GeV^2 of R_{K^*}	LHCb (5 fb^{-1}) 2.5σ in[1.1–6] GeV^2 of R_K	LHCb (9 fb^{-1}) 3.1σ in[1.1–6] GeV^2 of R_K	LHCb (9 fb^{-1}) $< 1.5\sigma$ in[1.1–6] GeV^2 of $R_{K^{*+}}, R_{K_S^0}$

- Theoretical prediction very precise
- More than 4σ significance for New Physics

Would be a spectacular fall of the SM !

Lepton flavour universality in $B \to K^{(*)}\ell^+\ell^-$ -

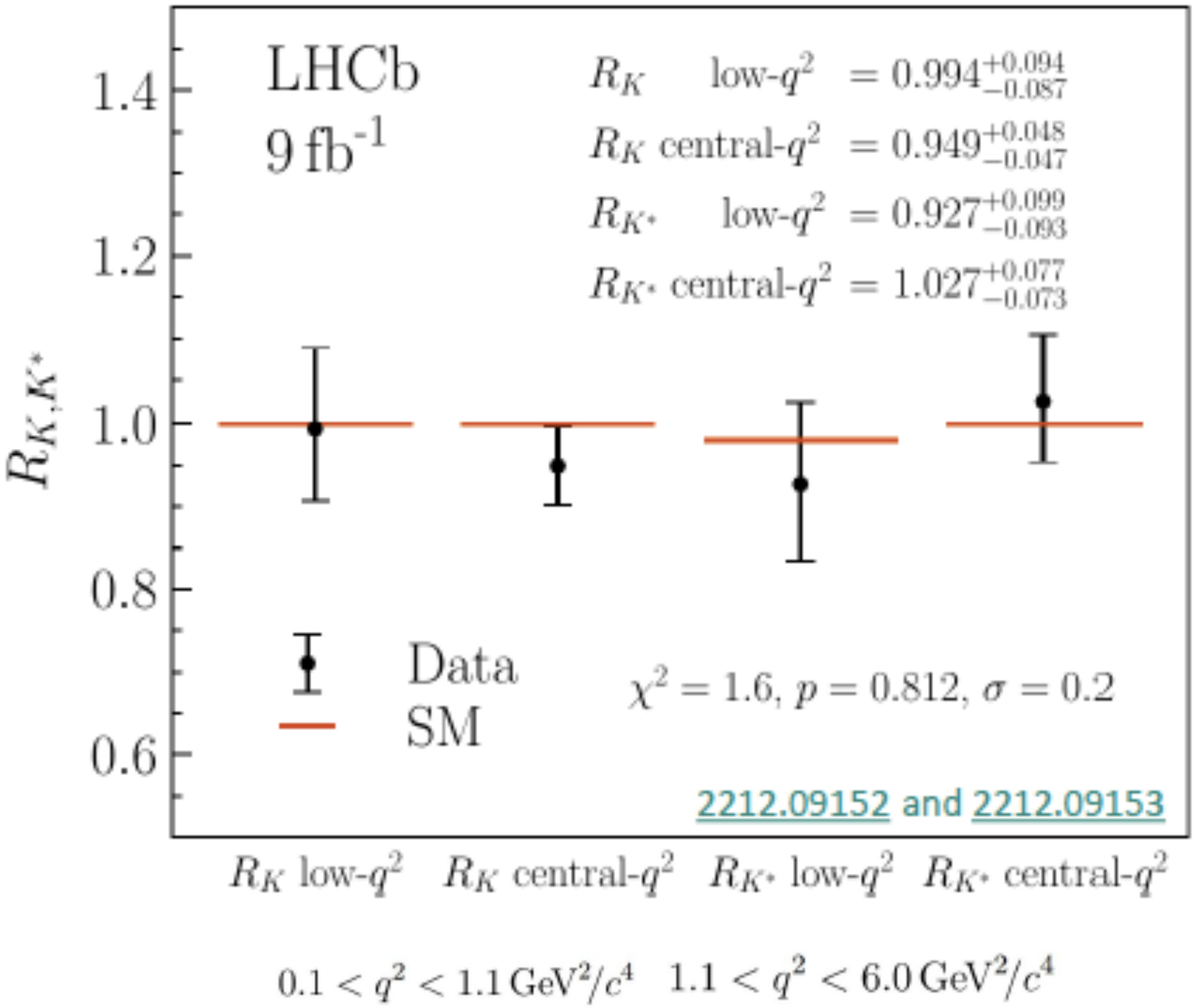
$$R_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$

Hiller,Kruger hep-ph/0310219

- Hadronic uncertainties cancel out

$\Longrightarrow$ *theoretically very clean $\mathcal{O}$(1%)*

December 20th update LHCb

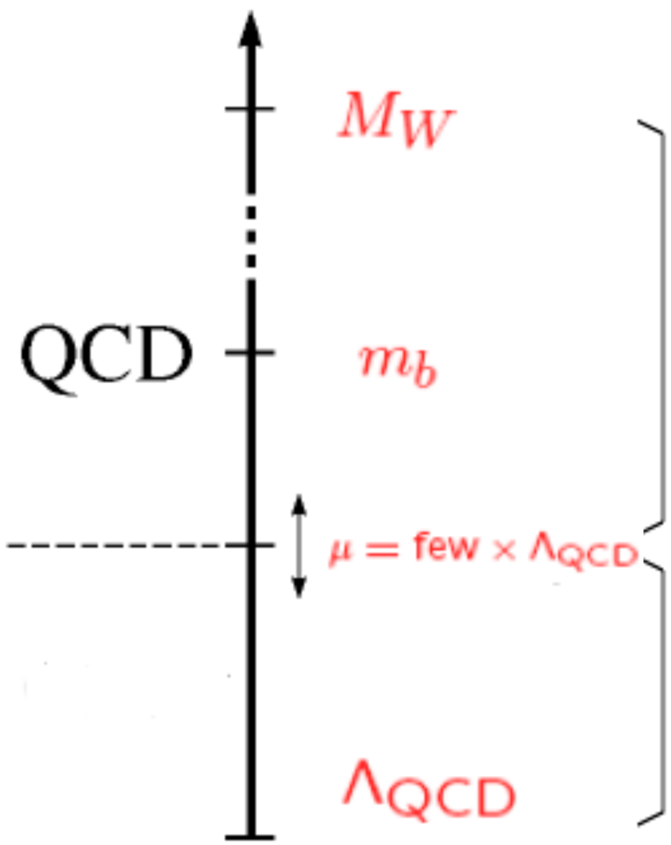


Compatible with SM with a simple χ^2 test on 4 measurement at 0.2σ.

The uncertainties reach 10% to 5% level.

Theoretical Framework

Theoretical tools for flavour precision observables



short-distance physics
perturbative

long-distance physics
nonperturbative

Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy})\ \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 >> M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

HQET, SCET, ...

Exclusive modes $B \to K^{(*)}\ell\ell$

Soft-collinear effective theory

$$\mathcal{T}_a^{(i)} = C_a^{(i)} \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

– Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

– Relations between formfactors in large-energy limit

– Limitation: insufficient information on power-suppressed Λ/m_b terms (breakdown of factorization: 'endpoint divergences')

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

Problem of nonfactorizable power corrections

- Crosscheck with $R_{\mu,e}$ ratios: **OPTION OUT !**

NP in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables (if there is a coherent picture)

- Ongoing efforts: Estimate of power corrections based on analyticity

van Dyk et al.: arXiv:2011.09813, 2206.03797

- In the long run: Solution with refactorization techniques *

New developments in the SCET community

Neubert et al., arXiv:2009.06779

- Crosscheck of the anomalies via inclusive modes

Problem of nonfactorizable power corrections

- Crosscheck with $R_{\mu,e}$ ratios: **OPTION OUT !**

NP in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables (if there is a coherent picture)

- Ongoing efforts: Estimate of power corrections based on analyticity

van Dyk et al.: arXiv:2011.09813, 2206.03797

- In the long run: Solution with refactorization techniques *

New developments in the SCET community

Neubert et al., arXiv:2009.06779

- Crosscheck of the anomalies via inclusive modes

*Caveat:
Many nonperturbative functions in exclusive modes at subleading order.

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

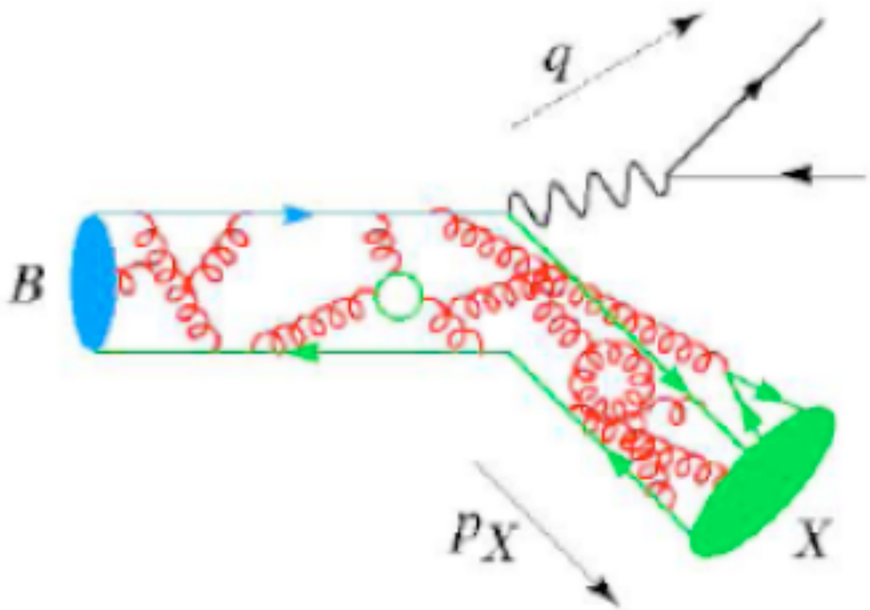
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

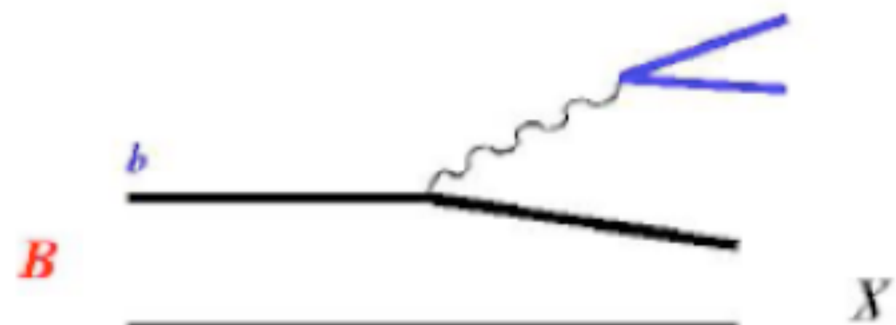
- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990





Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Old story:

- If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

Dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

$b \to s\gamma$: Benzke,Lee,Neubert,Paz,arXiv:1003.5012

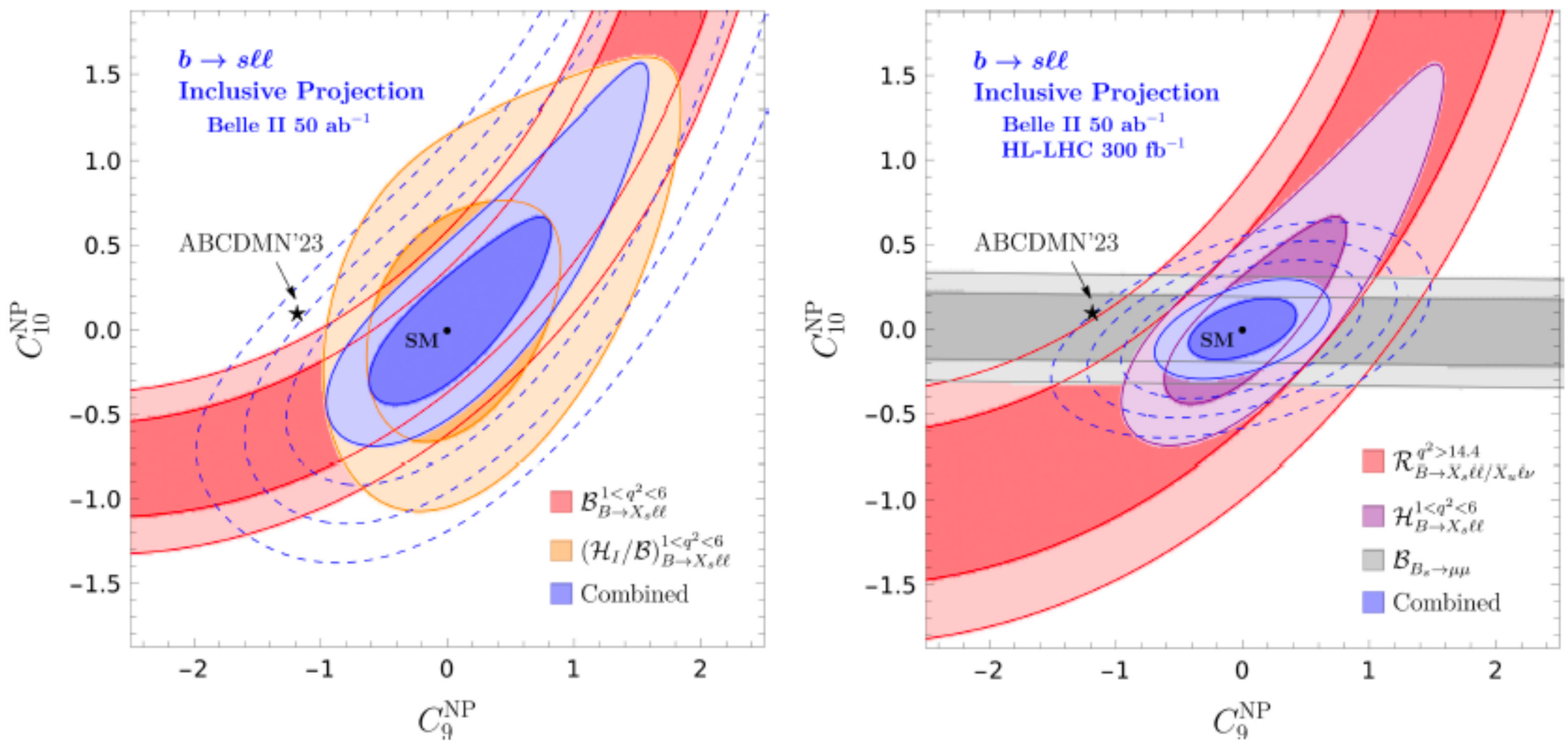
$b \to s\ell\ell$: Benzke,Hurth,Turczyk,arXiv:1705.10366

New Physics Reach of Semi-leptonic Penguin Decays

Assuming Belle II measures SM values

Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Update for post-R_K era arXiv:2404.03517



Belle-II Extrapolations

Akimasa Ishikawa, B2TiP

Error of Branching ratio $\bar{B} \to X_s \ell^+ \ell^-$

BF (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
$\geq$ 14.4	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

Error of Normalized Forward-Backward-Asymmetry

AFBn (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
$\geq$ 14.4	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

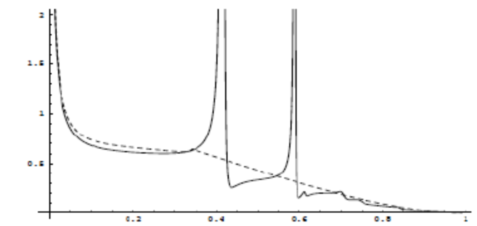
$B \to (\pi, \rho)\ell^+\ell^-$, semi-inclusive $\bar{B} \to X_d \ell^+ \ell^-$ at 50/ab
(uncertainties like $\bar{B} \to X_s \ell^+ \ell^-$ at 0.7/ab)

Nonlocal subleading contributions

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Benzke,Hurth,Turczyk,arXiv:1705.10366; Benzke,Hurth,arXiv:2006.00624

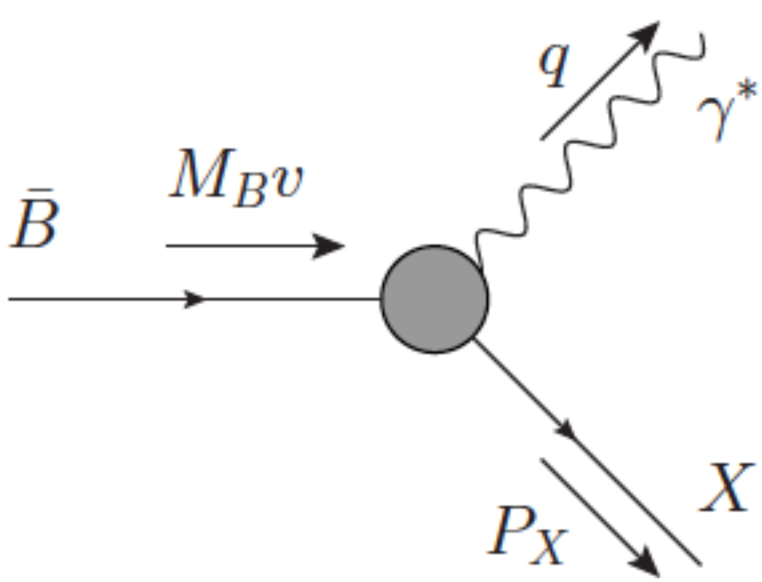
- Cuts in the dilepton mass spectrum necessary due to $c\bar{c}$ resonances



- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Kinematics: X_s is jetlike and $m_x^2 \leq m_b \Lambda_{QCD}$ (shapefunction region)

- Multiscale problem $\Rightarrow$ SCET with scaling Λ_{QCD}/m_b



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD} m_b \gg \Lambda_{\rm QCD}^2$$

Little calculation

- B meson rest frame $\quad q = p_B - p_X \qquad 2\, m_B\, E_X \, = m_B^2 + M_X^2 - q^2$

  X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

- $p_X^- p_X^+ = m_X^2 \qquad$ two light-cone components

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\rm QCD})$$

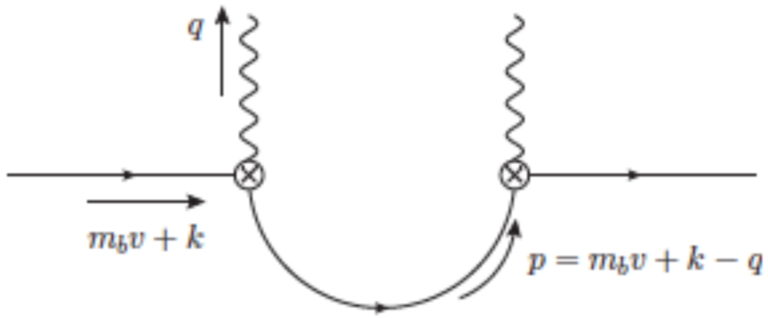
- $q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n} q = m_B - p_X^-$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

$$\lambda = \Lambda_{\rm QCD}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \dots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

Factorization theorem

$$d\Gamma \sim H \cdot J \otimes S$$

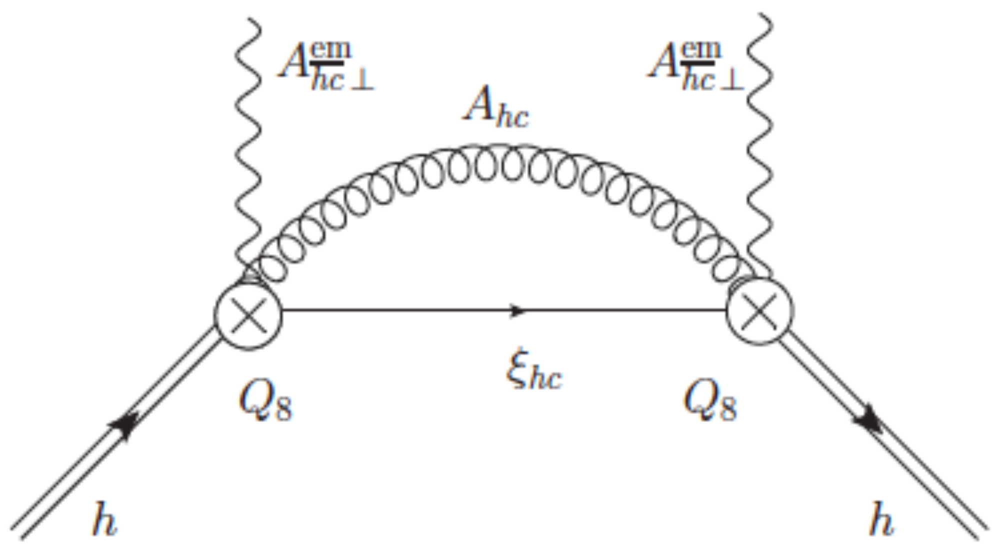
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

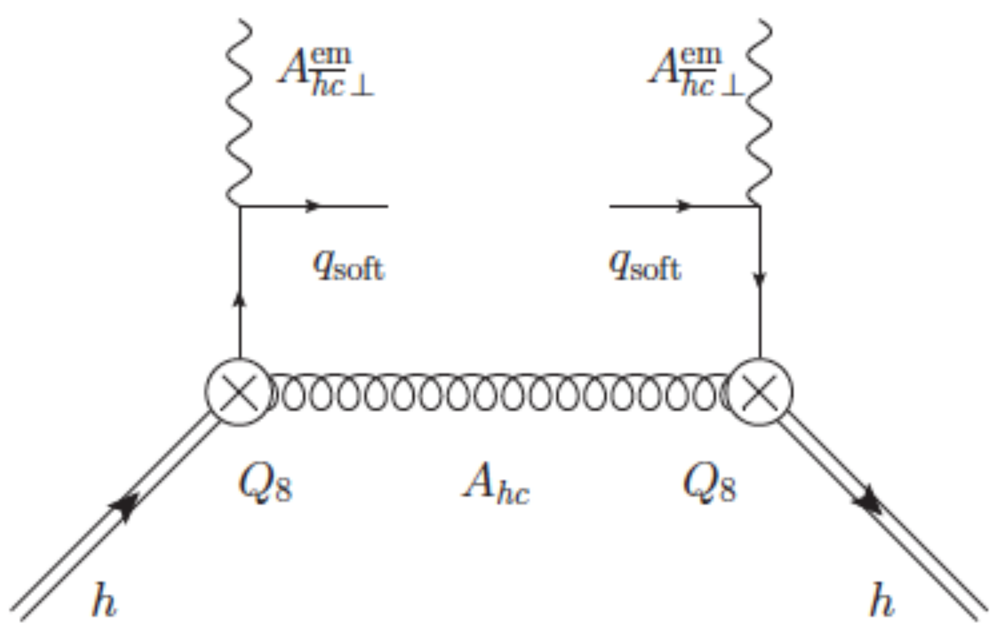
Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$



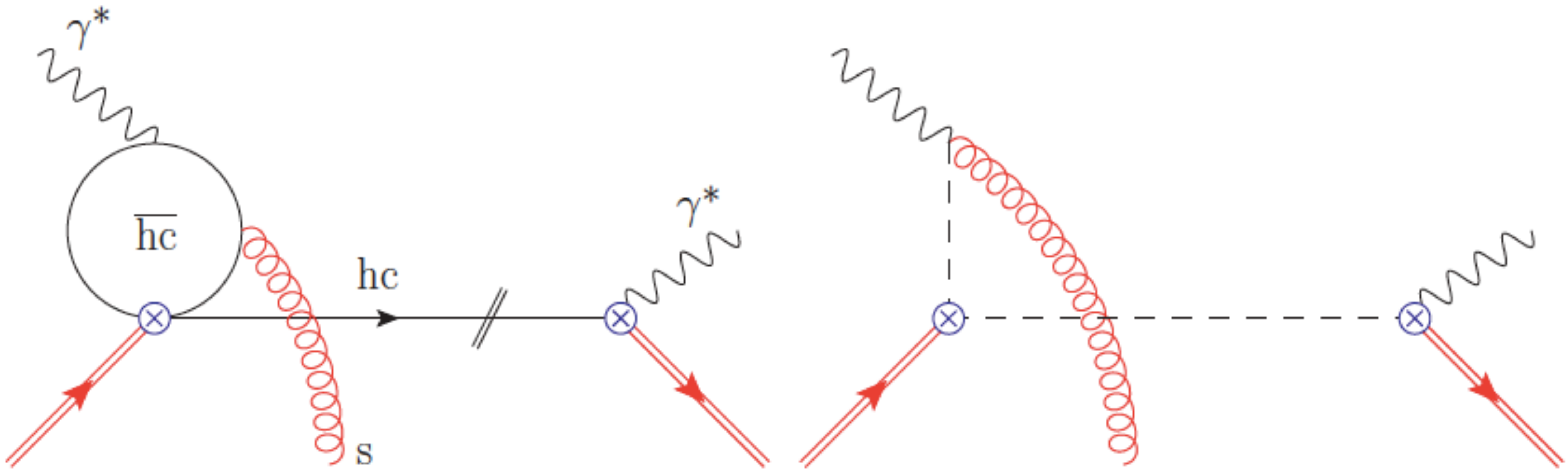
$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_1 and Q_7

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J}$$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.



$$\frac{d\Gamma^{\text{res}}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+i\varepsilon}$$
$$\frac{1}{\omega_1}\Bigg[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right)-(\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)$$
$$+\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\Bigg]g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1) = \int\frac{dr}{2\pi}e^{-i\omega_1 r}\int\frac{dt}{2\pi}e^{-i\omega t}\frac{1}{M_B}\langle\bar{B}|\bar{h}(t\mathbf{n})\ldots G_s^{\alpha\beta}(r\bar{\mathbf{n}})\ldots h(0)|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
  - PT invariance implies that soft functions are real
  - Moments of shape functions are related to HQET parameters
  - Soft functions have no significant structure outside the hadronic range
  - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
  - PT invariance implies that soft functions are real
  - Moments of shape functions are related to HQET parameters
  - Soft functions have no significant structure outside the hadronic range
  - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^0 \, h_{17}(\omega_1, \mu) = 0.237 \pm 0.040 \, \mathrm{GeV}^2$$

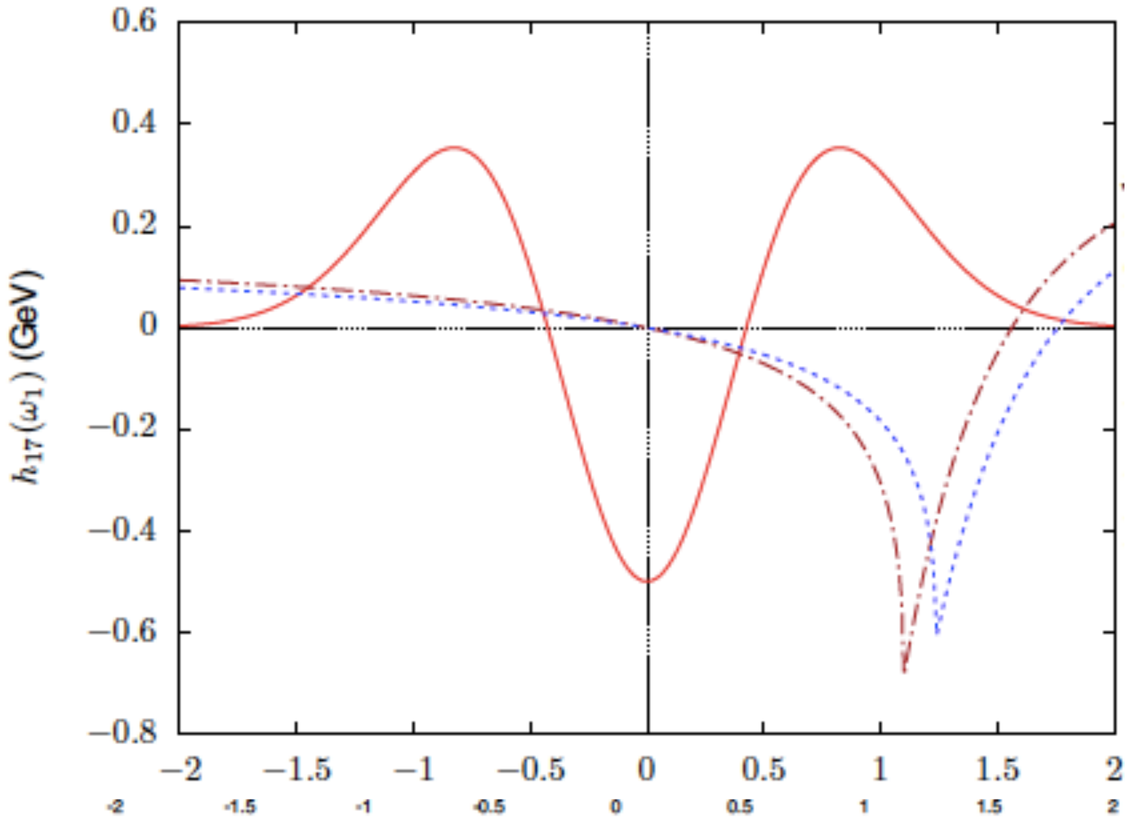
New input: $$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^2 \, h_{17}(\omega_1, \mu) = 0.15 \pm 0.12 \, \mathrm{GeV}^4$$

Paz et al. arXiv:1908.02812

Updated result for $\bar{B} \to X_s\gamma$

Benzke,Hurth,arXiv:2006.00624

Charm dependence of jet function:

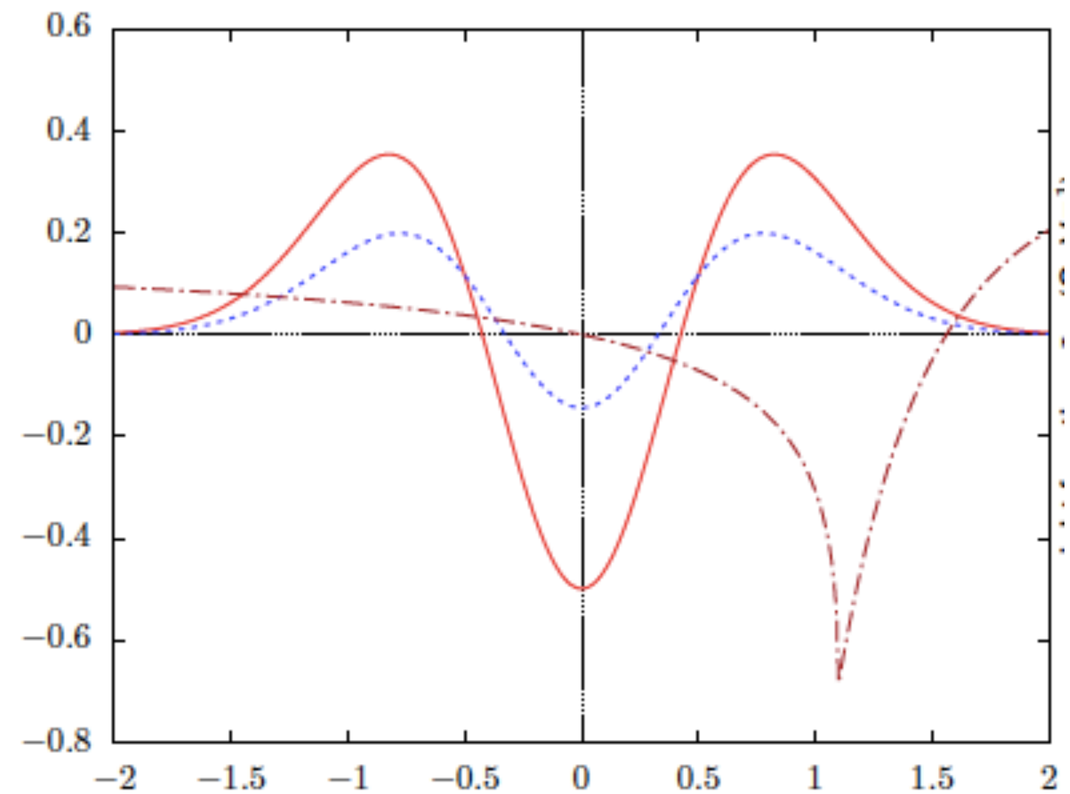


Benzke,Hurth,arXiv:2006.00624

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-3.7\%,\, 6.5\%]$$

Constraint on shape function:



Neubert et al., arXiv: 1003.5012

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-1.9\%,\, 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-5.2\%,\, 6.5\%]$$

(In addition: large scale dependence)

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \to X_s\gamma$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**
- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**
- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

Bartocci, Böer, Hurth, work in progress

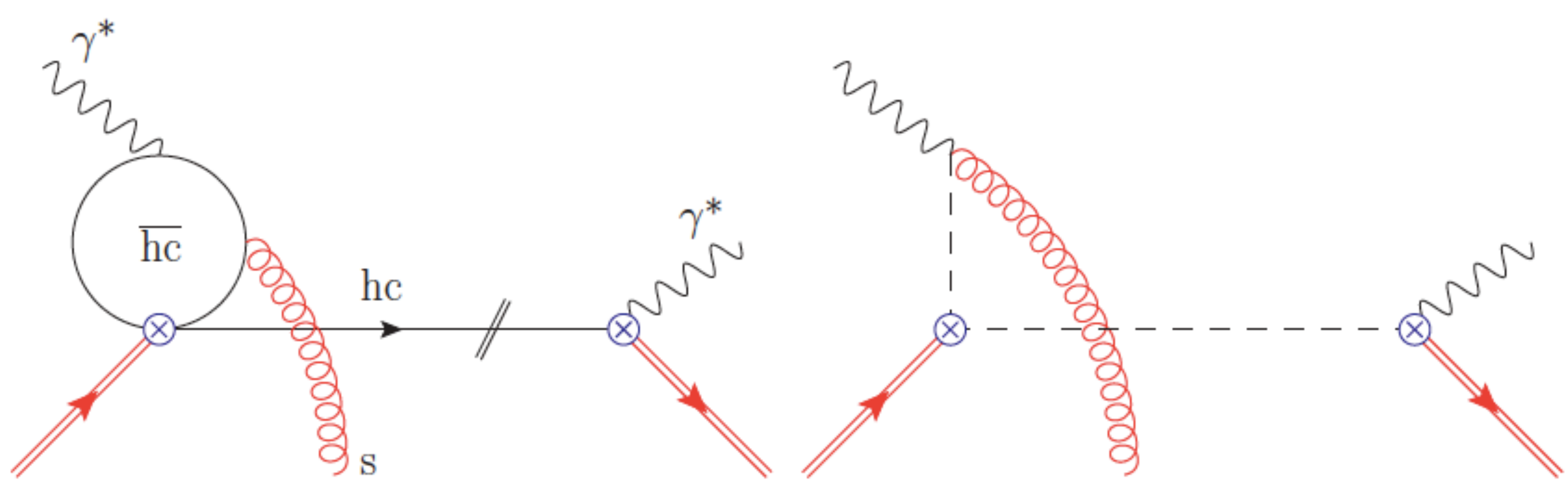
For NLL analysis we have to establish a factorisation theorem

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$

Steps of the NLL analysis

* analysis of renormalisation properties of the soft function

$$\mathcal{S}_{ren}(\omega, \omega_1) = \int_{-\infty}^{\bar{\Lambda}} d\omega' Z_{\mathcal{S}}(\omega, \omega', \omega_1, \omega_1') \, \mathcal{S}_{bare}(\omega', \omega_1').$$



Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**
- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

Bartocci, Böer, Hurth, work in progress

For NLL analysis we have to establish a factorisation theorem

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$

Steps of the NLL analysis

* analysis of renormalisation properties of the soft function

$$\mathcal{S}_{ren}(\omega, \omega_1) = \int_{-\infty}^{\bar{\Lambda}} d\omega' Z_{\mathcal{S}}(\omega, \omega', \omega_1, \omega_1') \, \mathcal{S}_{bare}(\omega', \omega_1').$$

* α_s corrections to anti-jet function
* α_s corrections to jet function
* use RG techniques to run various functions to a common scale.

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

- Voloshin term of +3% (shape function effect neglected) which is part of the resolved contributions has to be added:

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%] \quad \to \quad \mathcal{F}^{17}_{b\to s\gamma} = (5.45 \pm 2.55)\%$$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

- Voloshin term of +3% (shape function effect neglected) which is part of the resolved contributions has to be added:

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%] \quad \to \quad \mathcal{F}^{17}_{b\to s\gamma} = (5.45 \pm 2.55)\%$$

- Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 1.9\%] \qquad \qquad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%]$$

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%]$$

Reason for significantly smaller error is twofold:

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%]$$

Reason for significantly smaller error is twofold:

- For charm dependence only the parametric uncertainty was used

$$1.17\,\mathrm{GeV} \leq m_c \leq 1.23\,\mathrm{GeV}$$

We use scale variation of the hard-collinear scale

$$\mu_{\mathrm{hc}} \sim \sqrt{m_b\,\Lambda_{\mathrm{QCD}}} \quad \text{from} \quad 1.3\,\mathrm{GeV}\ \text{to}\ 1.7\,\mathrm{GeV}$$ and get

$$1.14\,\mathrm{GeV} \leq m_c \leq 1.26\,\mathrm{GeV}$$

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%]$$

Reason for significantly smaller error is twofold:

– For charm dependence only the parametric uncertainty was used

$$1.17\,\mathrm{GeV} \leq m_c \leq 1.23\,\mathrm{GeV}$$

We use scale variation of the hard-collinear scale

$\mu_{\rm hc} \sim \sqrt{m_b\,\Lambda_{\rm QCD}}$ from $1.3\,\mathrm{GeV}$ to $1.7\,\mathrm{GeV}$ and get

$$1.14\,\mathrm{GeV} \leq m_c \leq 1.26\,\mathrm{GeV}$$

– Numerically large $1/m_b^2$ term due to kinematic factors was dropped compared to the original analysis in 2010 Neubert et al., arXiv: 1003.5012

This kinematic $1/m_b^2$ term has a $1/m_b$ shape function, all other $1/m_b^2$ contributions have a shape function of order $1/m_b^2$. So no cancellation expected. Benzke,Hurth,arXiv:2303.06447

The large kinematic $1/m_b^2$ term can be used as conservative estimate of all $1/m_b^2$ contributions to resolved $\mathcal{O}_{7\gamma} - \mathcal{O}_1$.

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%, 4.7\%]$$

Reason for significantly smaller error is twofold:

– For charm dependence only the parametric uncertainty was used

$$1.17\,\mathrm{GeV} \leq m_c \leq 1.23\,\mathrm{GeV}$$

We use scale variation of the hard-collinear scale

$$\mu_{\mathrm{hc}} \sim \sqrt{m_b\,\Lambda_{\mathrm{QCD}}} \quad \text{from} \quad 1.3\,\mathrm{GeV}\ \text{to}\ 1.7\,\mathrm{GeV}$$ and get

$$1.14\,\mathrm{GeV} \leq m_c \leq 1.26\,\mathrm{GeV}$$

– Numerically large $1/m_b^2$ term due to kinematic factors was dropped compared to the original analysis in 2010 Neubert et al., arXiv: 1003.5012

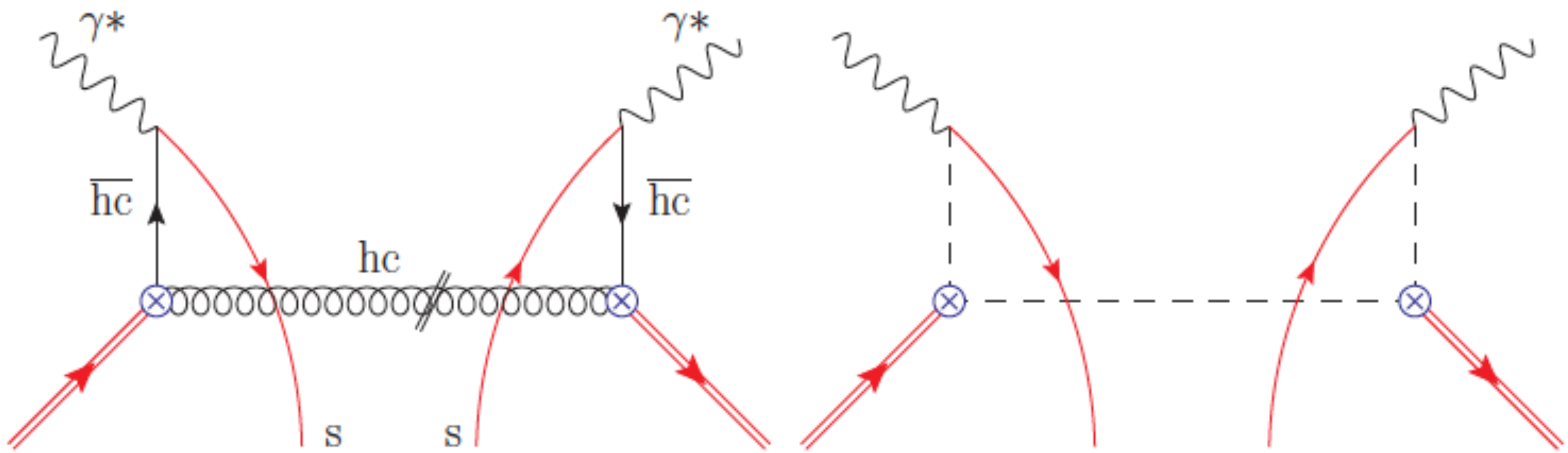
This kinematic $1/m_b^2$ term has a $1/m_b$ shape function, all other $1/m_b^2$ contributions have a shape function of order $1/m_b^2$. So no cancellation expected. Benzke,Hurth,arXiv:2303.06447

The large kinematic $1/m_b^2$ term can be used as conservative estimate of all $1/m_b^2$ contributions to resolved $\mathcal{O}_{7\gamma} - \mathcal{O}_1$.

Underestimation of the uncertainty due to the resolved contribution.

But used in recent $b \to s\gamma$ analysis. Misiak, Rehman, Steinhauser, arXiv:2002.01548v2

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\text{res}}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int \frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle \bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\text{F.T.}}$$

- Subtlety in the Q_8-Q_8 contribution: convolution integral is UV divergent
  - This implies that there is no complete proof of the factorization formula yet.
  - Nevertheless one shows that scale dependence of direct and resolved contribution cancel. Benzke,Lee,Neubert,Paz,arXiv:1003.5012
  - Refactorization methods allow to resolve the problem and reestablish factorization formula.

Refactorisation

Refactorisation in subleading $\bar{B} \to X_s\gamma$

Hurth, Szafron, arXiv:2301.01739

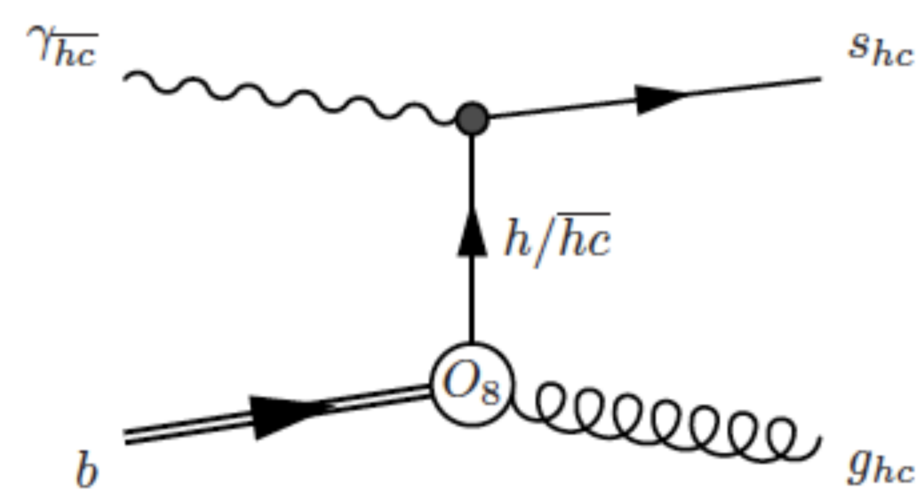
- Naive factorisation theorem with anti-hardcollinear Jet functions $\overline{J}$

$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

- Contribution of the gluon dipole operator does not factorise

$$O_{8g} = -\frac{g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu}(1+\gamma_5) G^{\mu\nu} b$$

$$q^\mu = E_\gamma \bar{n}^\mu \quad \text{and} \quad p_B^\mu = M_B v^\mu$$



Refactorisation in subleading $\bar{B} \to X_s\gamma$

Hurth,Szafron,arXiv:2301.01739

- Naive factorisation theorem with anti-hardcollinear Jet functions $\overline{J}$

$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$

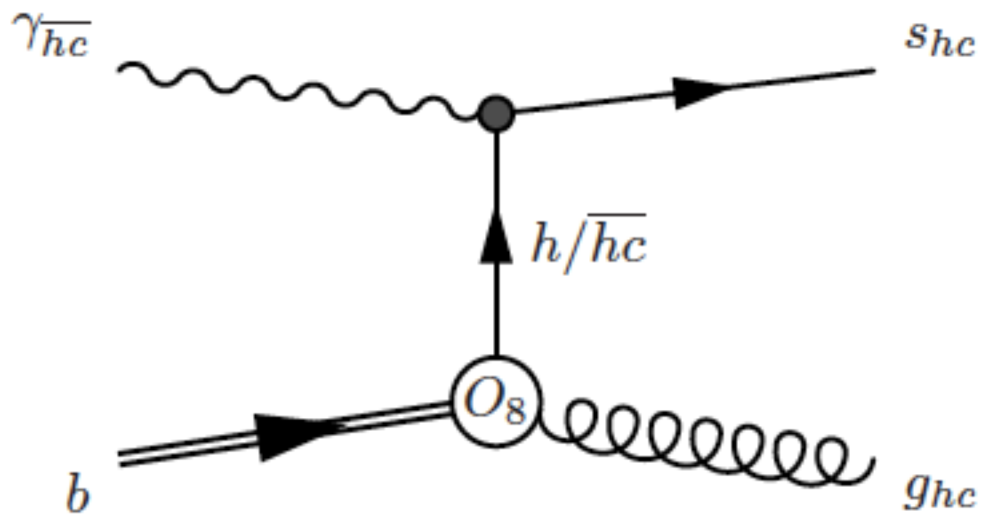
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

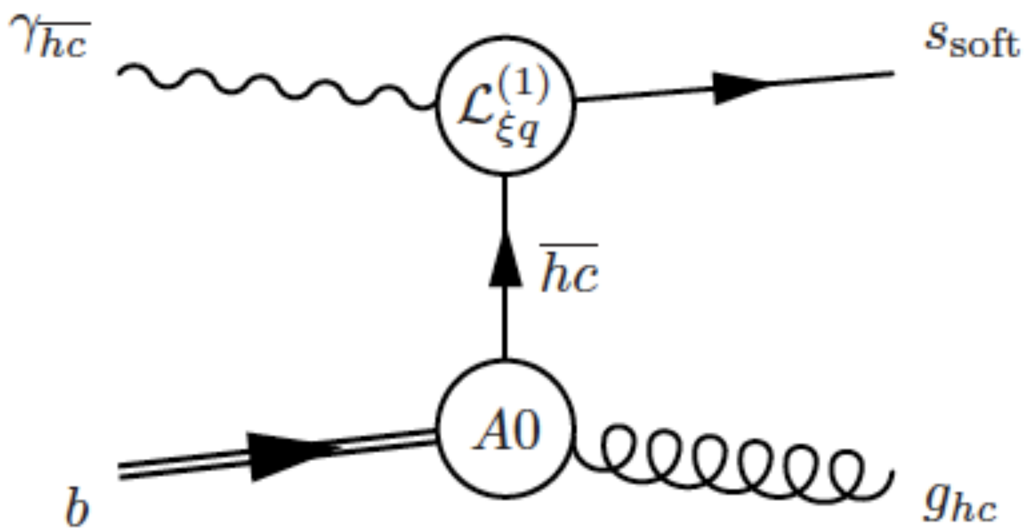
- Contribution of the gluon dipole operator does not factorise
- One can identify divergences in resolved *and* direct contribution in SCET-I as endpoint-divergences
- One can use refactorisation techniques developed in collider examples

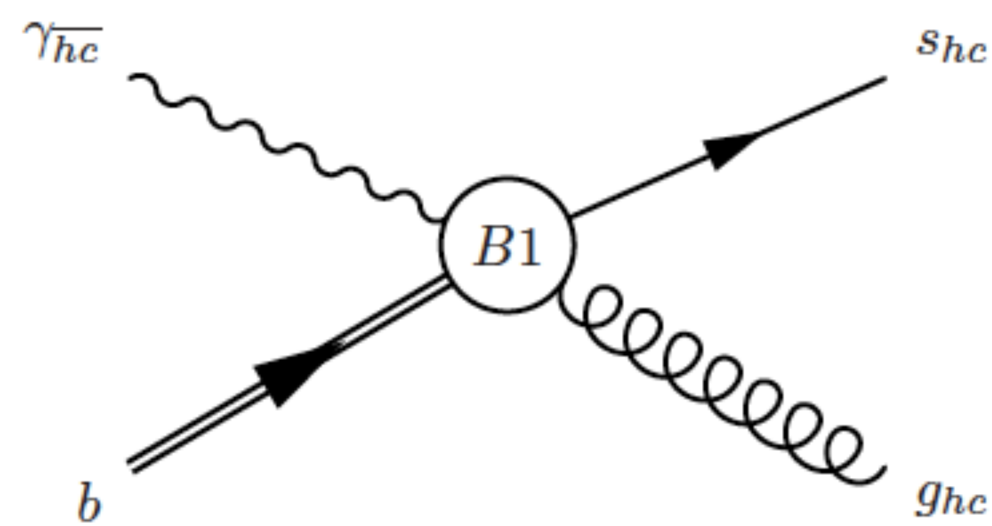
  Neubert et al.,arXiv:2009.06779

- First QCD application with nonperturbative objects in flavour physics

Degeneracy in EFT leads to endpoint divergences







$$\mathcal{O}^{A0}_{8g}(0) = \overline{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

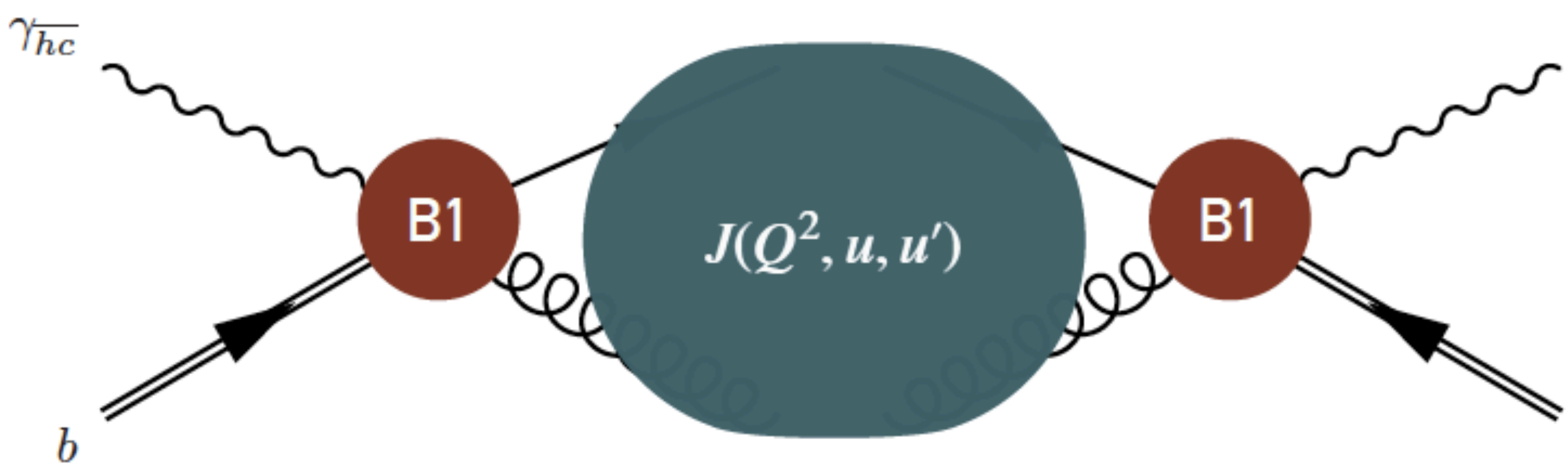
$$\mathcal{O}^{B1}_{8g}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

Factorisation of direct contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, \mathbf{C^{B1}}\left(\mathbf{m_b}, \mathbf{u}\right) \int_0^1 du'\, \mathbf{C^{B1*}}\left(\mathbf{m_b}, \mathbf{u'}\right) \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J}\left(\mathbf{M_B}\left(\mathbf{p_+} + \omega\right), \mathbf{u}, \mathbf{u'}\right) \mathcal{S}\left(\omega\right)$$

$$\mathbf{J}\left(\mathbf{p^2}, \mathbf{u}, \mathbf{u'}\right) = \frac{(-1)}{2N_c} \frac{1}{2\pi} \int \frac{dt dt'}{(2\pi)^2}\, d^4x\; e^{-im_b(ut-u't')+ipx}$$

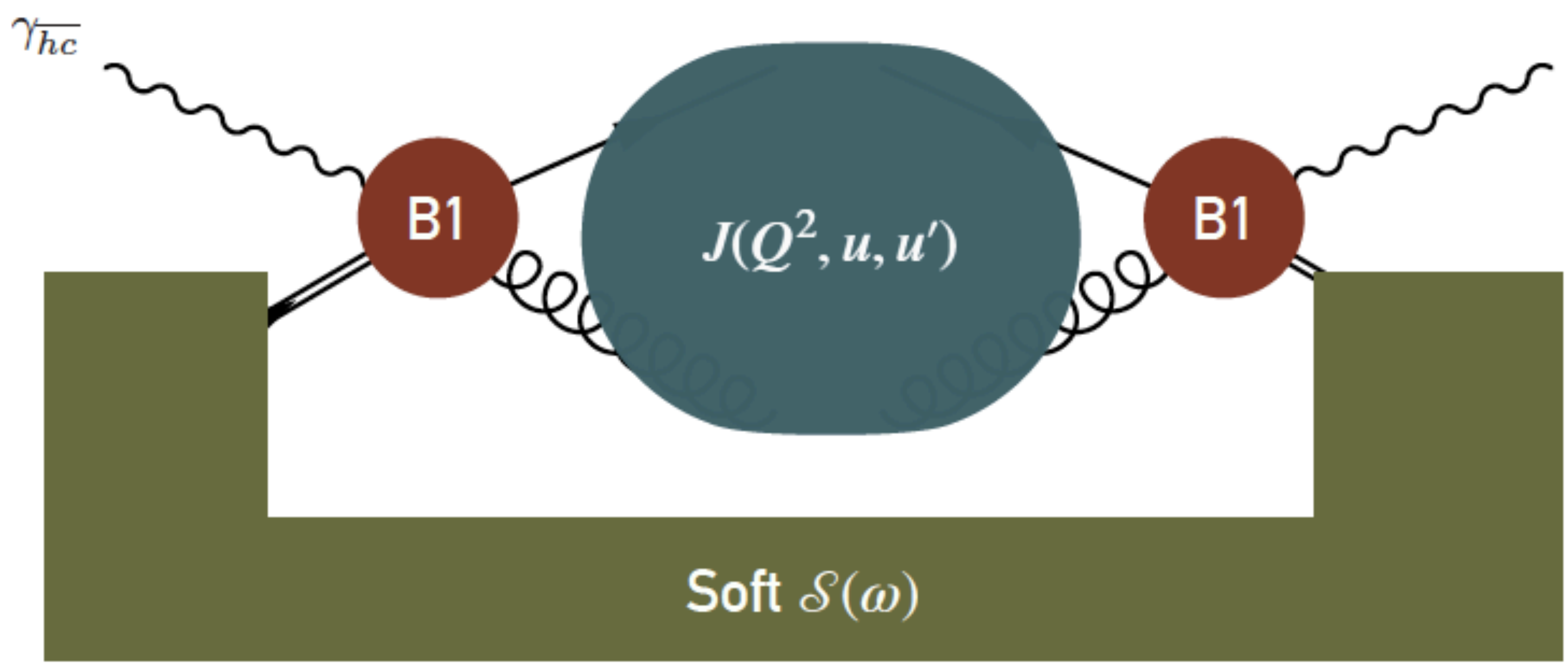
$$\text{Disc}\left[\langle 0|\, tr\left[\frac{1+\not{v}}{2}(1-\gamma_5)\mathcal{A}_{hc\perp}(x)\,\gamma_\perp^\nu \chi_{hc}(t'\bar{n}+x)\,\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\mathcal{A}_{hc\perp}(0)\,(1+\gamma_5)\right] |0]\rangle\right]$$



Factorisation of direct contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, \mathbf{C^{B1}}\left(\mathbf{m_b}, \mathbf{u}\right) \int_0^1 du'\, \mathbf{C^{B1*}}\left(\mathbf{m_b}, \mathbf{u'}\right) \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J}\left(\mathbf{M_B}\left(\mathbf{p_+} + \omega\right), \mathbf{u}, \mathbf{u'}\right) \mathcal{S}\left(\omega\right)$$

$$\mathcal{S}\left(\omega\right) = \frac{1}{2m_B} \int \frac{dt}{2\pi} e^{-i\omega t} \left\langle B \right| h\left(tn\right) S_n\left(tn\right) S_n^\dagger\left(0\right) h\left(0\right) \left| B \right\rangle$$



Endpoint divergence in direct contribution at leading order

Hard matching coefficients

$$\mathbf{C_{LO}^{B1}}\left(\mathbf{m_b},\mathbf{u}\right)=(-1)\frac{\overline{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t\,C_{8g}=(-1)\frac{\overline{u}}{u}C_{LO}^{A0}\left(m_b\right)$$

convoluted with jet function

$$\mathbf{J}\left(\mathbf{p^2},\mathbf{u},\mathbf{u'}\right)=C_F\frac{\alpha_s}{4\pi\,m_b}\theta(p^2)\,A(\epsilon)\,\delta(u-u')u^{1-\epsilon}(1-u)^{-\epsilon}\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$$

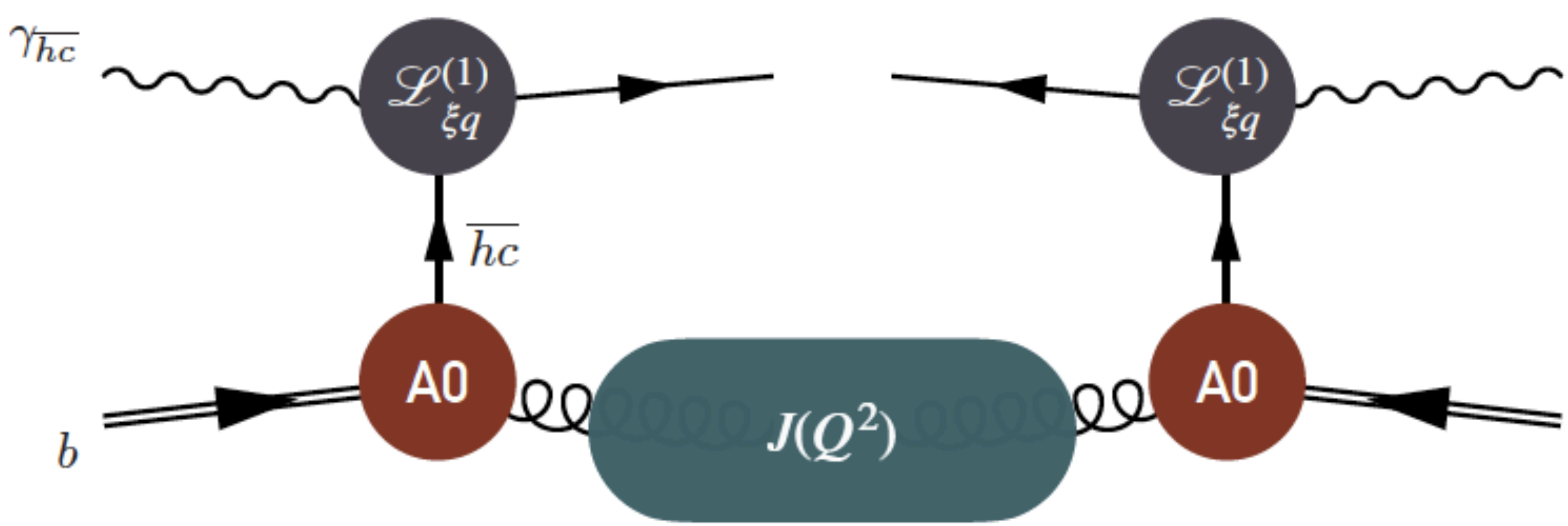
lead to endpoint divergence in the $u\to 0$ limit

$$\int_0^1 du\frac{1}{u}\int_u^1 du'\frac{1}{u'}u^{1-\epsilon}\delta(u-u')\sim\int_0^1 du\frac{1}{u^1+\epsilon}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\, J_g\left(m_b\left(p_+ + \omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{J}(\omega_1)\, \bar{J}^*(\omega_2)\, \mathcal{S}(\omega, \omega_1, \omega_2)$$

$$-g_s^2 \delta_{ab} g_\perp^{\mu\nu}\; J_g(p^2) = \frac{1}{2\pi i} \mathrm{Disc}\left[i \int d^4x e^{ipx} \langle 0| T\left[\mathcal{A}^{a\mu}_{hc\perp}(x), \mathcal{A}^{b\nu}_{hc\perp}(0)\right] |0\rangle\right]$$

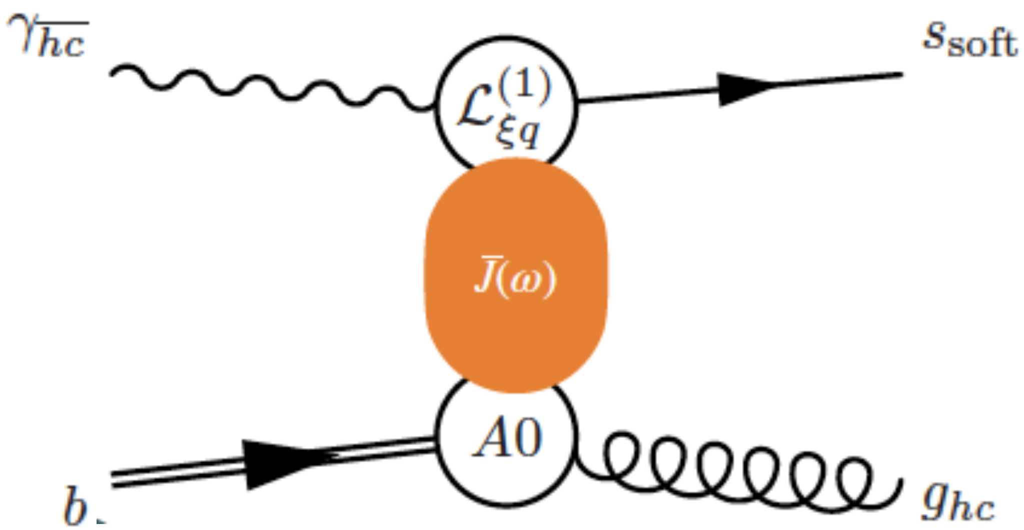


Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega \, J_g\left(m_b\left(p_+ + \omega\right)\right) \int d\omega_1 \int d\omega_2 \, \bar{J}\left(\omega_1\right) \bar{J}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

Anti-hardcollinear jet function $\overline{J}(\omega)$ is defined on the amplitude level.

$$O_{T\xi q} = i \int d^d x T \left[\mathcal{L}_{\xi q}(x), O_{8g}^{A0}(0)\right]$$



$$= \int d\omega \int \frac{dt}{2\pi} e^{-it\omega} \left[\overline{q_s}\right]_\alpha (tn) \left[\bar{J}(\omega)\right]_{\alpha\beta}^{a\,\nu\mu} Q_s \mathcal{B}^\nu_{\overline{hc}\perp}(0) \mathcal{A}^{\mu\,a}_{hc\perp}(0) \left[h(0)\right]_\beta$$

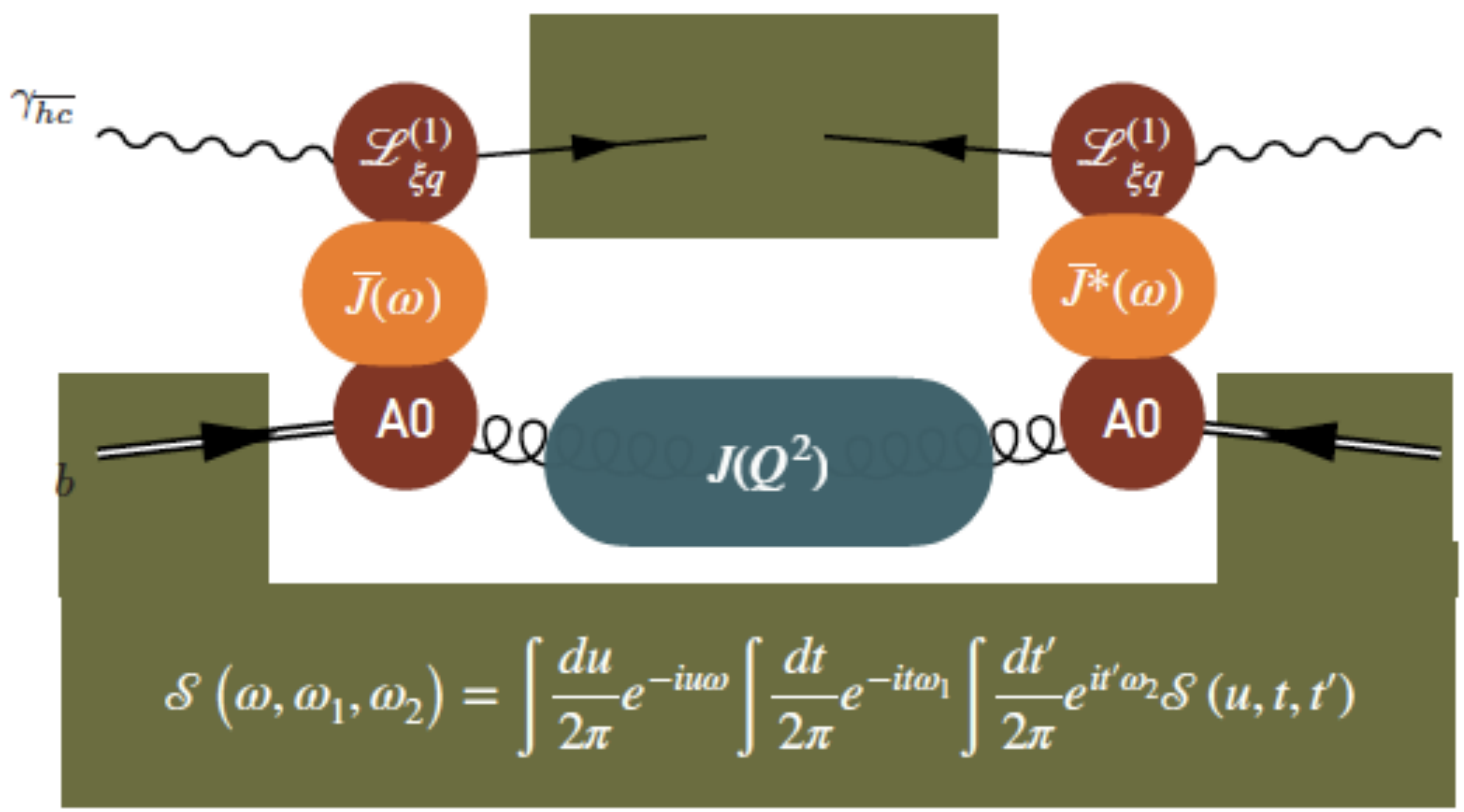
Decomposition to all orders: $\left[\bar{J}(\omega)\right]_{\alpha\beta}^{a\,\nu\mu} = \bar{J}(\omega)\, t^a \left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar{n}}\not{n}}{4}\right]_{\alpha\beta}$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \overline{\mathbf{J}}\left(\omega_1\right) \overline{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

Operatorial definition of the soft function in position space $\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t'}\right)$

$$\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t'}\right) = (d-2)^2 g_s^2 \langle B|\,\bar h\,(un)\,(1-\gamma_5)\left[S_n(un)\,t^a S_n^\dagger(un)\right] S_{\bar n}(un)\,S_{\bar n}^\dagger(t'\bar n+un)$$
$$\frac{\not{n}\bar{\not{n}}}{4} q_s(t'\bar n+un)\,\bar q_s(t\bar n)\,\frac{\bar{\not{n}}\not{n}}{4} S_{\bar n}(t\bar n)\,S_{\bar n}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right](1+\gamma_5)\,h(0)\,|B\rangle\,/\,(2m_B)$$



Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega \, J_g(m_b(p_+ + \omega)) \int d\omega_1 \int d\omega_2 \, \bar{J}(\omega_1) \, \bar{J}^*(\omega_2) \, \mathcal{S}(\omega, \omega_1, \omega_2)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

$\omega_{1,2} \to \infty$ corresponds to $t, t' \to 0$ and $q_s(un) \to S_n(un) q_{hc}(un)$, $\bar{q}_s(0) \to q_{hc} S_n^{+}(0)$

$$\mathcal{S}(u,t,t') = (d-2)^2 g_s^2 \langle B| \bar{h}(un)(1-\gamma_5)\left[S_n(un)\, t^a S_n^\dagger(un)\right] S_{\bar{n}}(un) S_{\bar{n}}^\dagger(t'\bar{n}+un)$$
$$\frac{\not{n}\not{\bar{n}}}{4} q_s(t'\bar{n}+un)\, \bar{q}_s(t\bar{n}) \frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}(t\bar{n}) S_{\bar{n}}^\dagger(0) \left[S_n(0)\, t^a S_n^\dagger(0)\right](1+\gamma_5) h(0) |B\rangle \,/\, (2m_B)$$

$$\mathcal{S}(u) = \langle B| \bar{h}(un)\, S_n(un)\, S_n^\dagger(0)\, h(0) |B\rangle \,/\, (2m_B)$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C}^{\mathbf{A0}}(\mathbf{m_b})\right|^2 \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J_g}(\mathbf{m_b}(\mathbf{p_+}+\omega)) \int d\omega_1 \int d\omega_2\, \overline{\mathbf{J}}(\omega_1)\, \overline{\mathbf{J}}^*(\omega_2)\, \mathcal{S}(\omega,\omega_1,\omega_2)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

More general:
Asymptotic $(\omega_1 \sim \omega_2 \leq \omega)$ soft function $\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)$ is a convolution of a perturbabtive kernel K and the leading power soft function.

$$\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2) = \int d\omega' K(\omega,\omega',\omega_1,\omega_2)\mathcal{S}(\omega')$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega \, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+} + \omega\right)\right) \int d\omega_1 \int d\omega_2 \, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

More general:

Asymptotic ($\omega_1 \sim \omega_2 \leq \omega$) soft function $\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$ is a convolution of a perturbabtive kernel K and the leading power soft function.

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = \int d\omega' K(\omega, \omega', \omega_1, \omega_2) \mathcal{S}(\omega')$$

Leading order in α_s:

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = C_F A(\epsilon) \frac{\alpha_s}{(4\pi)} \, \omega_1^{1-\epsilon} \delta(\omega_1 - \omega_2) \int_{\omega}^{\bar\Lambda} d\omega' \, \mathcal{S}(\omega') \, \left(\frac{(\omega' - \omega)}{\mu^2}\right)^{-\epsilon}$$

Refactorisation at leading order

$$\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0} = -\mathcal{N}\,\left|C_{LO}^{A0}(m_b)\right|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}\,A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega)\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}|_A^{\rm asy} = \mathcal{N}|C_{LO}^{A0}(m_b)|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega')\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

One verifies that

$$\frac{d\Gamma}{dE_\gamma}|_A^{\rm asy} = (-1)\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0}$$

Refactorisation conditions can be formulated on the operator level

Express the fact that in the limits $u \sim u' \ll 1$ and $\omega_1 \sim \omega_2 \gg \omega$ the two terms of the subleading $\mathcal{O}_8 - \mathcal{O}_8$ contribution have the same structure.

- $\llbracket C^{B1}(m_b, u) \rrbracket = (-1) C^{A0}(m_b)\, m_b \overline{J}(u m_b)$
  ($\llbracket g(u) \rrbracket$ only denotes the leading term of a function $g(u)$ in the limit $u \to 0$)
- $\widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ corresponds to $\mathcal{S}(\omega, \omega_1, \omega_2)$ in the limit $\omega_1 \sim \omega_2 \gg \omega$
  (In this limit: $q_s \to q_{sc}$ and higher power corrections in $\omega/\omega_{1,2}$ are neglected)
- $\int_{-p_+}^{\overline{\Lambda}} d\omega \, \llbracket J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega) \rrbracket = \int_{-p_+}^{\overline{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$
  (In this limit $\chi_{hc} \to q_{sc}$, brackets indicate again that the $u \to 0$ *and* $u' \to 0$ limits)

The refactorisation relations are operatorial relations that guarantee the cancellation of endpoint divergences between the two terms to all orders in α_s.

Finally we show that refactorisation and renormalisation commute.

Summary

- In the post-R_K era we still have signifcant tensions in exclusive $b \to s$ angular observables and branching ratios.

- The problem of unknown power corrections in the exclusive modes makes it difficult/impossible to distinguish between possible new physics effects and hadronic effects.

- Inclusive semi-leptonic decays require Belle-II for full exploitation, but are theoretically very clean and allow for crosschecks of the present tensions.

- Nonlocal power corrections presently belong to the largest uncertainties in the inclusive modes (5%) $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s \ell\ell$ (up to 5%) (higher moments of shape functions and α_s corrections needed).

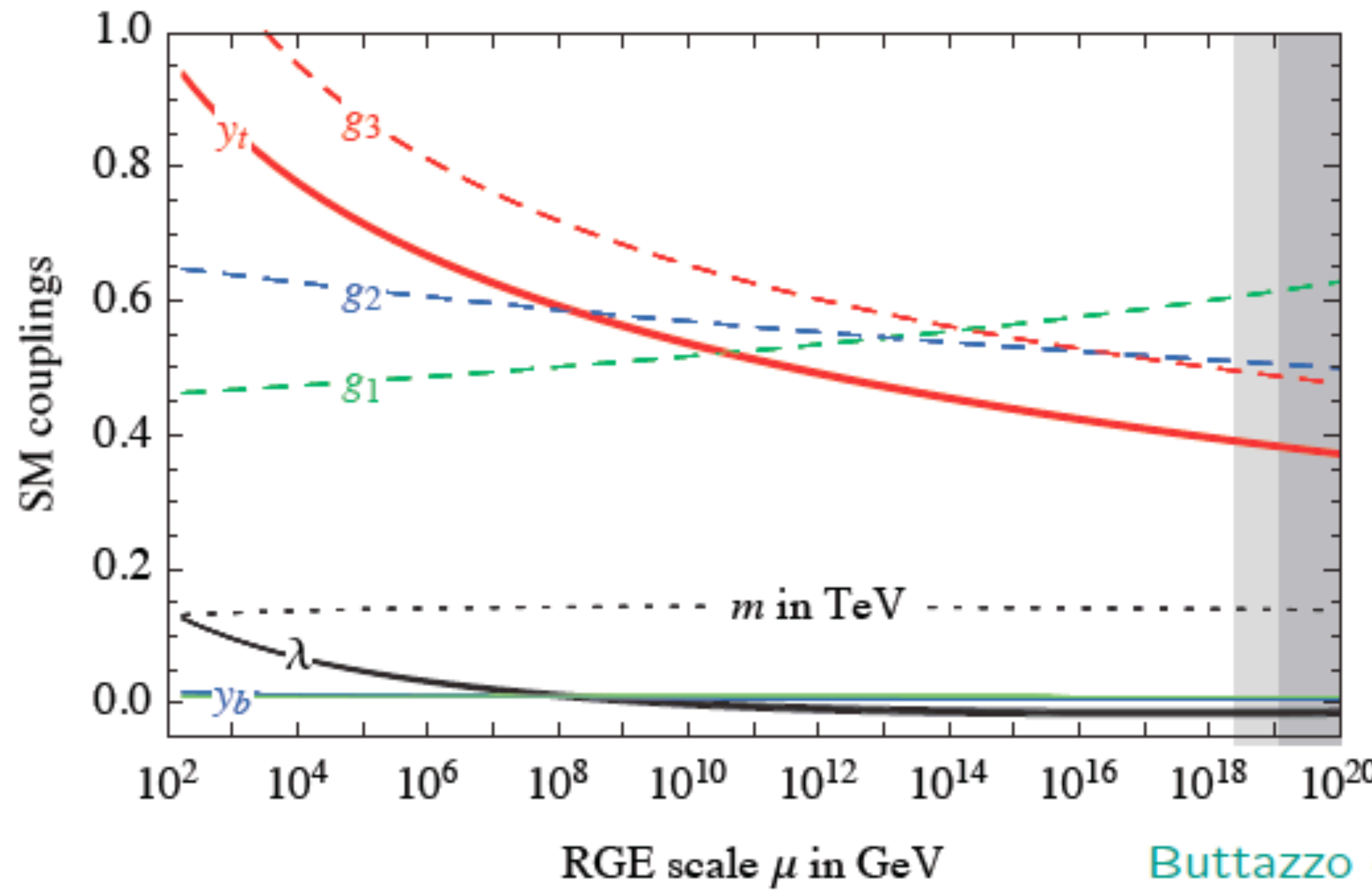
- Refactorisation techniques allow to solve the problem of endpoint divergences, in particular in subleading $\bar{B} \to X_s \gamma$.

Epilogue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

Michelangelo Mangano

- The days of "guaranteed" discoveries or no-lose theorems in particle physics are over, at least for the time being .....

- but the big questions of our field remain open (hierarchy problem. flavour, neutrinos, dark matter, baryogenesis,...)

- This simply implies that, more than for the past 30 years, future HEP's progress is to be driven by experimental exploration, possibly renouncing/reviewing deeply rooted theoretical bias.

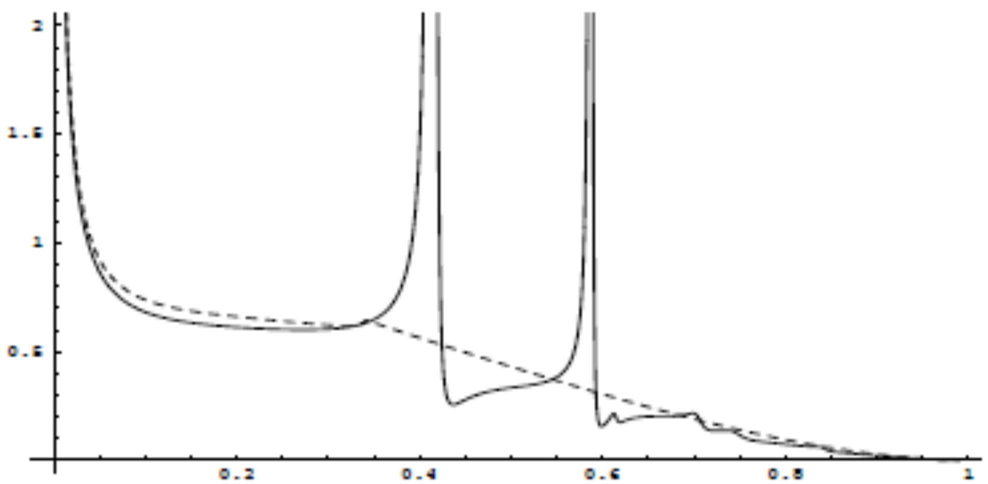
Spares

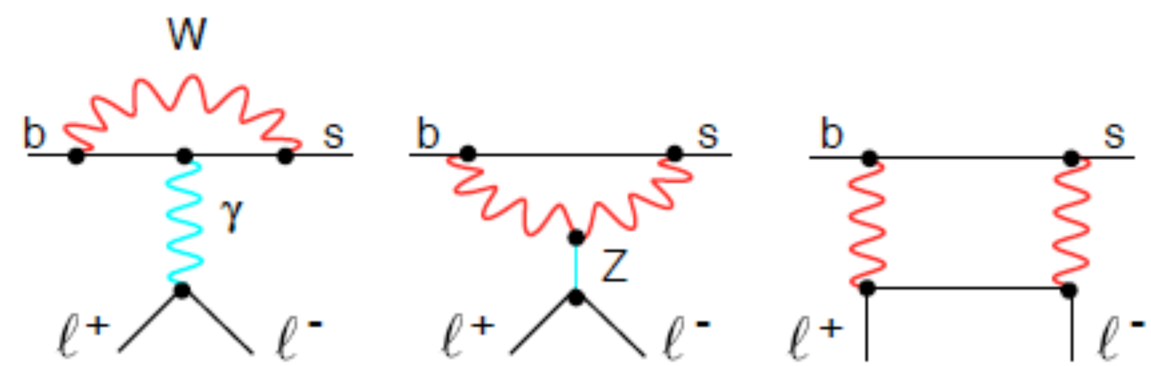
New Physics Reach of Semi-leptonic Penguin Decays

Review of previous calculations for $B \to X_s \ell\ell$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dilepton mass spectrum necessary :
  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$





$\hat{s} = q^2/m_b^2$

- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: dilepton mass spectrum
  Asatryan, Asatrian, Greub, Walker, hep-ph/0204341
  Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

$$BR(\bar{B} \to X_s \ell^+ \ell^-)_{Cut:\ q^2 \in [1GeV^2, 6GeV^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \to X_s l^+ l^-)_{Cut:\ q^2 > 14.4 GeV^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections $q^2 \in [1GeV^2, 6GeV^2]$

central value: -14%, perturbative error: $13\% \to 6.5\%$

Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

Huber,Hurth,Jenkins,Lunghi arXiv 2306.03134

- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Previous SCET calculation with some simplifications and certain problems with SCET scaling (q assumed to be hard)
  Uncertainty due to subleading shape functions estimated to 5 – 10%

  Lee,Ligeti,Stewart,Tackmann hep-ph/0512191

  Lee,Tackmann arXiv:0812.0001

- New Strategy to minimise uncertainty Huber,Hurth,Jenkins,Lunghi

  – Calculation of cut dependence using OPE for mild hadronic cuts

  – Analyse breakdown of OPE via λ_1 power corrections

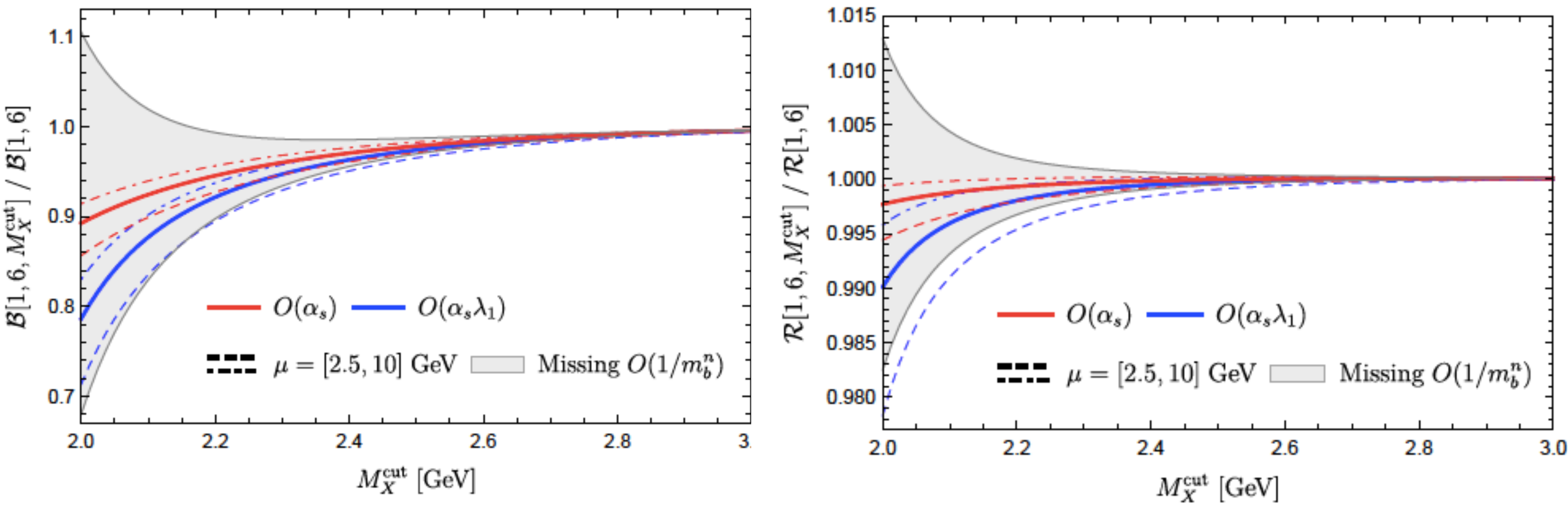
  – Try to interpolate betweeen SCET and OPE calculation

  – Use cut-independent ratios in OPE and SCET to analyse interpolation

Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

Huber, Hurth, Jenkins, Lunghi arXiv 2306.03134

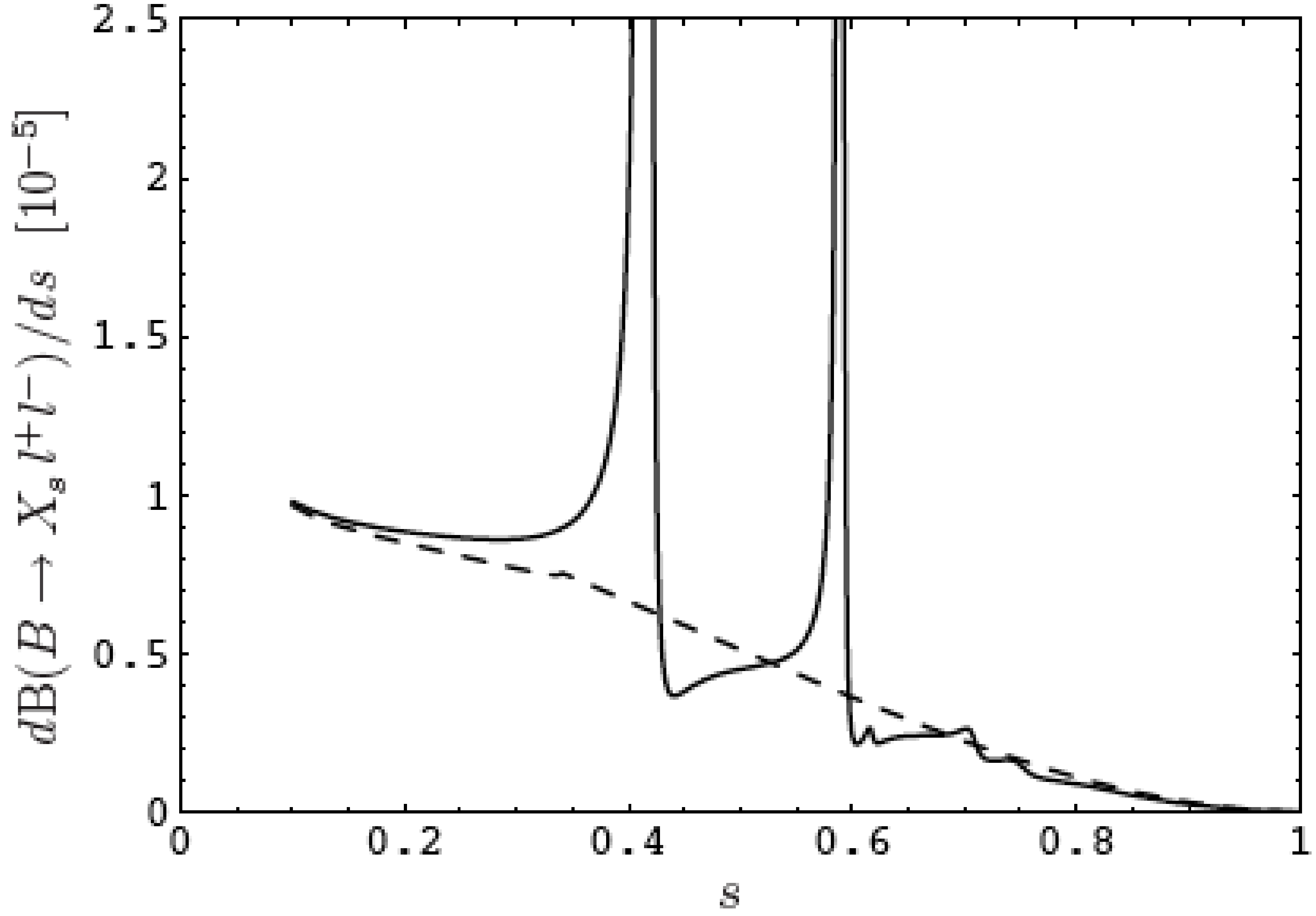
- We computed the fully differential distribution of $\bar{B} \to X_s \ell^+ \ell^-$ at $O(\alpha_s)$ in the OPE
- Also the three $\bar{B} \to X_s \ell^+ \ell^-$ angular observables, together with the $\bar{B} \to X_u \ell^- \nu$ branching fraction, all with the same hadronic mass cut
- We find effective Independence of the hadronic mass cut



Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

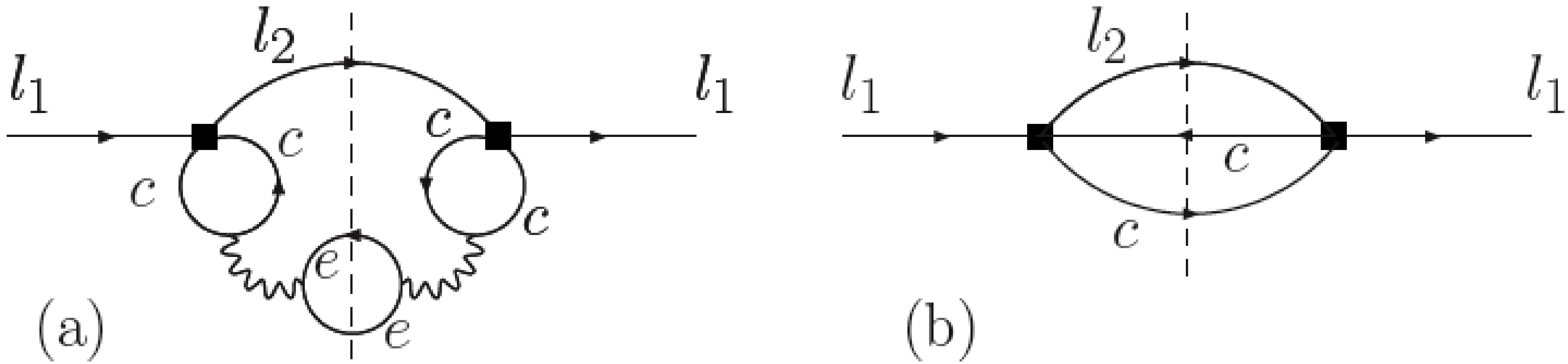
BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

Complete angular analysis of inclusive $B \to X_s \ell\ell$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$

- Dependence on Wilson coefficients

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\rm em}\,log(m_b^2/m_\ell^2) \qquad q^2 = (p_{\ell+} + p_{\ell-}) \Rightarrow q^2 = (p_{\ell+} + p_{\ell-} + p_\gamma)$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- In the ratio of the inclusive $b \to s\ell\ell$ decay rate in the high-q^2 region and the semileptonic decay rate large part of the nonperturbative effects cancel out:

Ligeti,Tackmann, arXiv:0707.1694

$$R^{(\ell)}_{\rm incl}(q_0^2) = \frac{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}}$$

Tensions in the inclusive high q^2 decay rate ??

Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612

$$R_{\rm incl}^{SM}(15) = \frac{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_s\bar{\ell}\ell)}{dq^2}}{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_u\bar{\ell}\nu)}{dq^2}} \quad x \quad \mathcal{B}(B\to X_u\bar{\ell}\nu)^{\rm exp}_{[15]} = (1.50\pm 0.24)\times 10^{-4}$$

Belle,arXiv:2107.13855

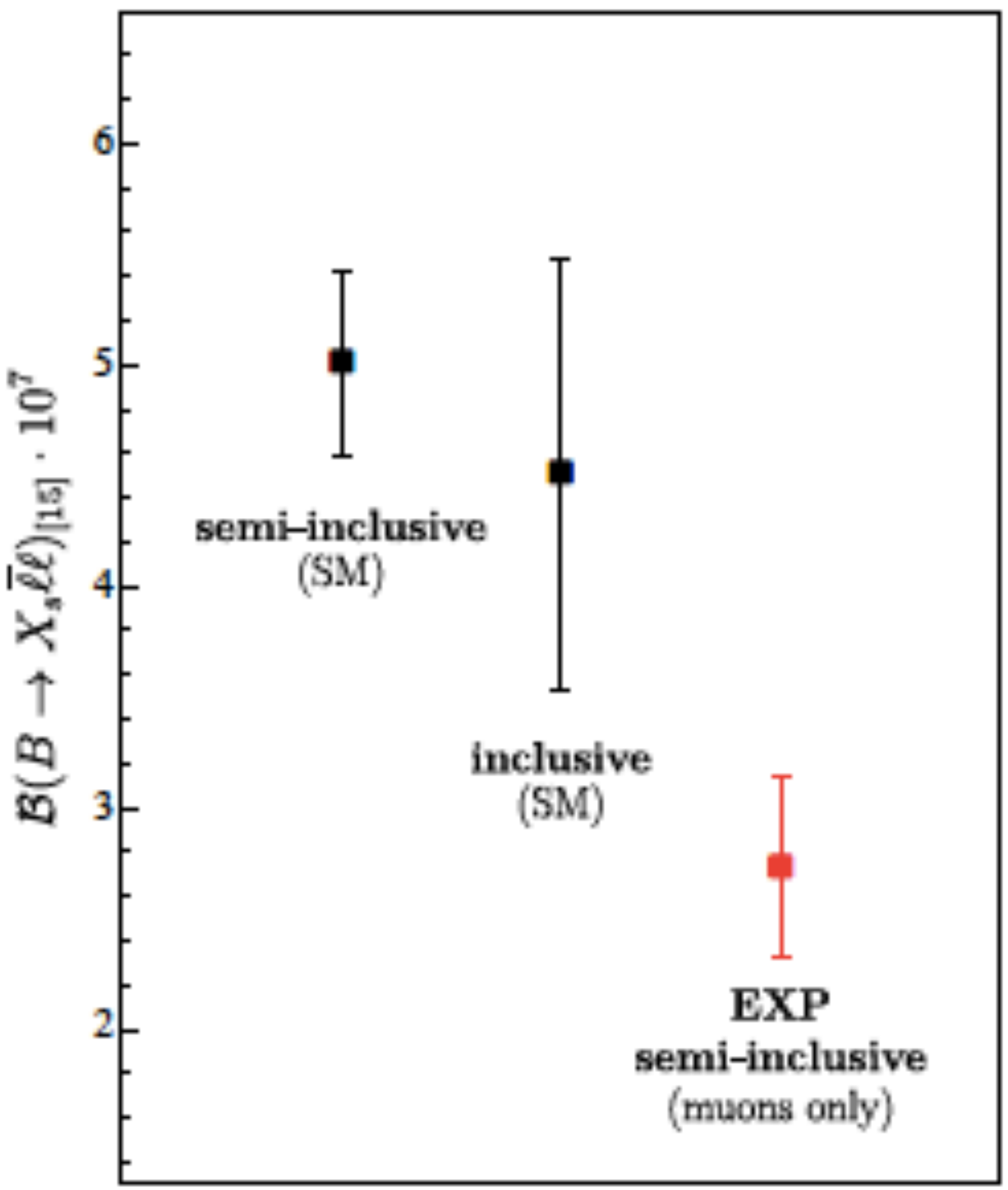
$$= \quad ''\mathcal{B}(B\to X_s\bar{\ell}\ell)^{SM}_{[15]}'' \quad \stackrel{!}{=} \quad \sum_i \mathcal{B}(B\to X_s^i\bar{\mu}\mu)^{\rm exp}_{[15]} = (2.74\pm 0.41)\times 10^{-7}$$

Isidori,Polonsky,Tinari,arXiv:2305.03076

- Experimental semi-inclusive rate is estimated by the sum of the $B\to K$ and $B\to K^*$ modes and a correction factor for the two-body final states $B\to K\pi$.

- Isidori et al. claim a tension up to 2σ – confirming analogous results in the exclusive modes.

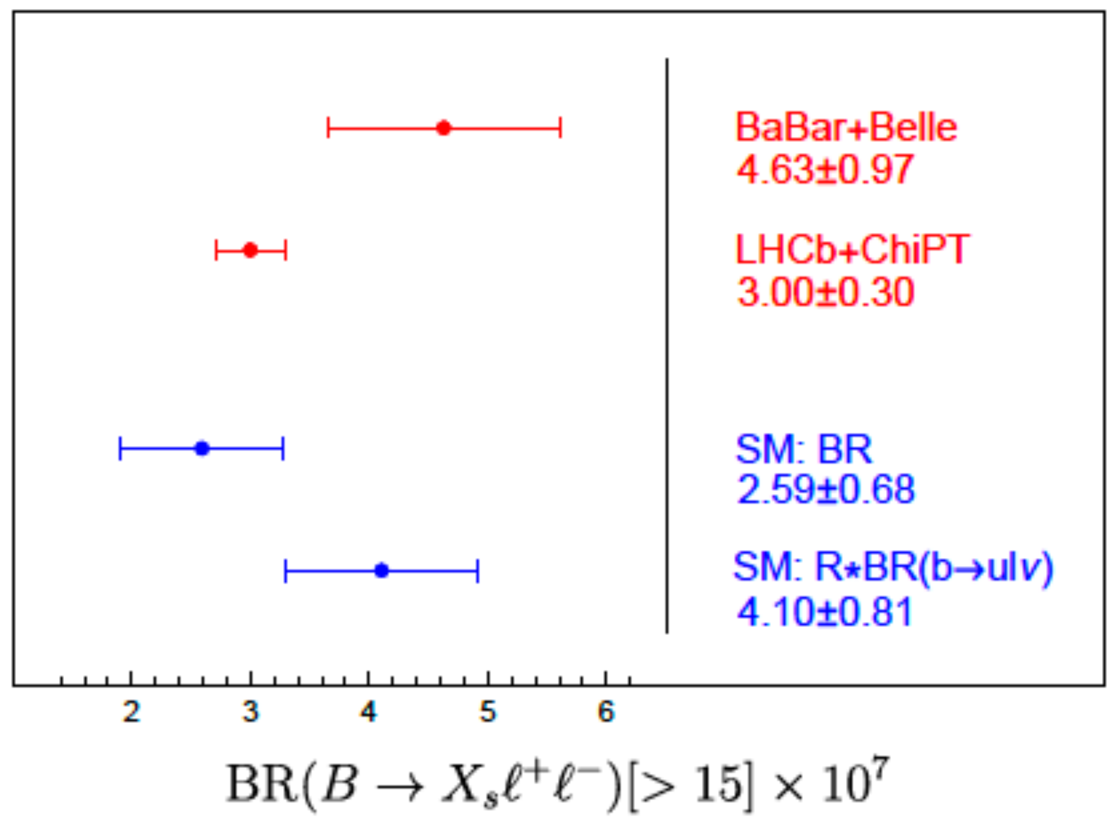
Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612



- We do not find any tension if we also consider our direct result for the branching $\mathcal{B}(B \to X_s \ell\ell)^{\text{SM}}_{[15]}$ and the Babar/Belle measurements.

Huber,Hurth,Jenkins,Lunghi,Qin,Vos arXiv:2007.04191

Talk by T.H. at FPCP23 and arXiv:2404.03517



- We find a slight tension between the two theoretical and also between the two experimental results. We have to be patient!

Intermezzo

Model independent Analysis of Anomalies

Hurth, Mahmoudi, Martinez Santos, Neshatpour arXiv:2104.10058
Hurth, Mahmoudi, Neshatpour arXiv:2310.05585 **Update 2023**

NP fits with a single operator

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Update 2023

Fit to all $b \to s\ell\ell$ observables

All observables post-$R_{K^{(*)}}$ update ($\chi^2_{\rm SM} = 271.0$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
δC_7	-0.02 ± 0.01	267.2	1.9σ
δC_{Q_1}	-0.04 ± 0.03	270.3	0.8σ
δC_{Q_2}	-0.01 ± 0.01	270.4	0.8σ
δC_9	-0.96 ± 0.13	230.7	6.3σ
δC_{10}	0.15 ± 0.15	270.0	1.0σ

Our guesstimate is 10% power corrections

All observables post-$R_{K^{(*)}}$ update ($\chi^2_{\rm SM} = 271.0$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
$\delta C_{\rm LL}$	-0.54 ± 0.12	249.1	4.7σ
$\delta C_{\rm LR}$	-0.42 ± 0.10	257.4	3.7σ
$\delta C_{\rm RL}$	0.00 ± 0.08	268.8	1.5σ
$\delta C_{\rm RR}$	0.21 ± 0.13	268.1	1.7σ

Our guesstimate is 10% power corrections

$C_{LL} \equiv C_9 = -C_{10}$ and $C_{RL} \equiv C'_9 = -C'_{10}$

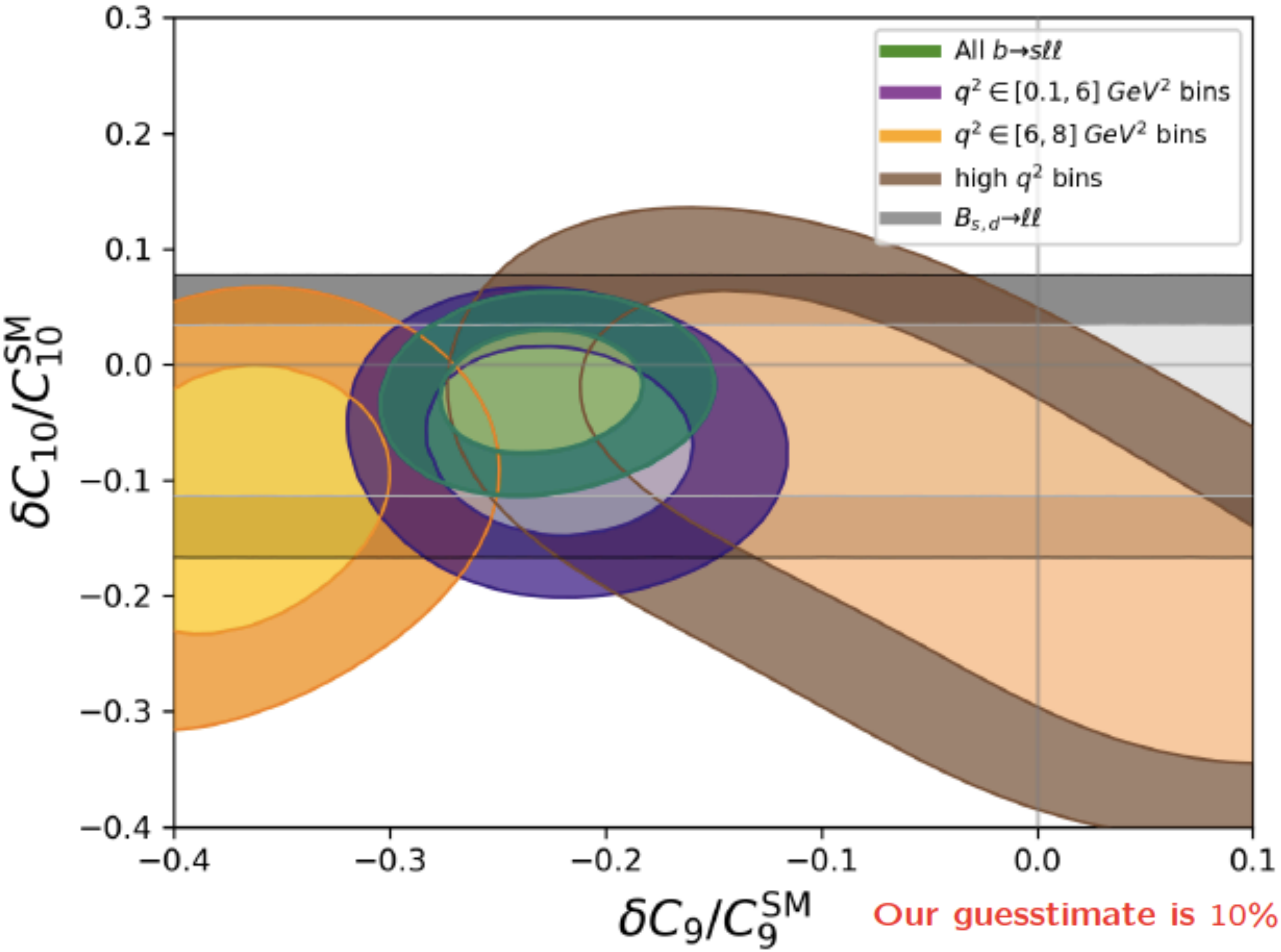
$C_{RR} \equiv C'_9 = C'_{10}$ and $C_{LR} \equiv C_9 = C_{10}$

NP fits with two operators

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Update 2023

Impact of the different sets of observables



Our guesstimate is 10% power corrections

Global fit to all $b \to s$ observables with the 12 universal operators

Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Update 2023

Considering only one or two Wilson coefficients may not give the full picture!

Issues:

LEE and method for eliminating insensitive parameters and flat directions
(Use profile of likehoods and correlation matrix....)

All observables with $\chi^2_{\rm SM} = 271.0$ post-$R_{K^{(*)}}$ update ($\chi^2_{\rm min} = 222.5$; Pull$_{\rm SM} = 4.7\sigma$)			
δC_7 0.07 ± 0.03		δC_8 -0.70 ± 0.50	
$\delta C_7'$ -0.01 ± 0.01		$\delta C_8'$ -0.50 ± 1.20	
δC_9 -1.18 ± 0.19	$\delta C_9'$ 0.06 ± 0.31	δC_{10} 0.23 ± 0.20	$\delta C_{10}'$ -0.05 ± 0.19
C_{Q_1} -0.30 ± 0.14	C_{Q_1}' -0.18 ± 0.14	C_{Q_2} 0.01 ± 0.02	C_{Q_2}' -0.03 ± 0.07

Our guesstimate is 10% power corrections

Spares II

Refactorised (endpoint finite) factorisation theorem

We subtract the two asymptotic terms

$$0 = 2\mathcal{N}\left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\Lambda} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \int_{m_b}^{\infty} d\omega_1 \overline{J}\left(\omega_1\right) \int_0^{\omega_1} d\omega_2 \overline{J}^*\left(\omega_2\right) \widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$$

$$+ 2\mathcal{N} \int_0^1 du \left[\!\left[C^{B1}(m_b, u)\right]\!\right] \int_u^1 du' \left[\!\left[C^{B1*}(m_b, u')\right]\!\right] \int_{-p_+}^{\overline{\Lambda}} d\omega \left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)\right]\!\right]$$

with

$$\left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)\right]\!\right] = J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

$$\left[\!\left[C^{B1}(m_b, u')\right]\!\right] = (-1) C^{A0}(m_b)\, m_b \overline{J}(u m_b)$$

from the all-order factorisation theorems we derived

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}\left|C^{A0}(m_b)\right|^2 \int_{-\infty}^{\infty} d\omega_1 \overline{J}\left(\omega_1\right) \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}^*\left(\omega_2\right) \int_{-p_+}^{\overline{\Lambda}} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

$$+ 2\mathcal{N} \int_0^1 du C^{B1}(m_b, u) \int_u^1 du' C^{B1*}(m_b, u') \int_{-p_+}^{\overline{\Lambda}} d\omega J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$
\begin{aligned}
\frac{d\Gamma}{dE_\gamma}\Big|_{A+B} = 2\mathcal{N} \int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ & J_g(m_b(p_+ + \omega)) \left|C^{A0}(m_b)\right|^2 \\
& \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}(\omega,\omega_1,\omega_2) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] \\
& + \int_0^1 du \int_u^1 du' \Big[C^{B1}_{LO}(m_b,u)\; C^{B1*}(m_b,u')\; J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega) \\
& - [\![C^{B1}(m_b,u)]\!]\, [\![C^{B1*}(m_b,u')]\!]\, [\![J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega)]\!] \Big] \Big\} ,
\end{aligned}
$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$
\begin{aligned}
\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N} \int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ & J_g(m_b(p_+ + \omega)) \left|C^{A0}\left(m_b\right)\right|^2 \\
& \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}\left(\omega,\omega_1,\omega_2\right) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] \\
& + \int_0^1 du \int_u^1 du' \left[C^{B1}_{LO}\left(m_b, u\right)\, C^{B1*}\left(m_b, u'\right)\, J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}\left(\omega\right) \right. \\
& \left. - \left[\!\left[C^{B1}\left(m_b, u\right) \right]\!\right] \left[\!\left[C^{B1*}\left(m_b, u'\right) \right]\!\right] \left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega) \right]\!\right] \right] \Big\},
\end{aligned}
$$

Finally we show that refactorisation and renormalisation commute.