On the potential of Light-Cone Sum Rules without semi-global Quark-Hadron Duality

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Status of B-~~anomalies~~ quirks in $b \to s\ell\ell$

- Anomalies in the clean observables $R_{K^{(*)}}, BR(B_{(s)} \to \mu\mu)$ are gone.
- Quirks are remaining in theoretically challenging observables: $P_5'(B \to K^*\mu\mu)$, $\mathrm{BR}(B \to K^+\mu\mu)$, ...
- *Are B-~~anomalies~~ quirks due to New Physics or misunderstood SM effect?*

Theoretical status of $B \to M\ell\ell$ ($b \to s\ell\ell$)

$$A(B \to M\ell^+\ell^-) = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[(A_\mu + T_\mu)\bar{u}_\ell\gamma^\mu v_\ell + B_\mu \bar{u}_\ell\gamma^\mu\gamma_5 v_\ell\right], \quad (1)$$

where

$$\begin{aligned} A_\mu &= C_9\langle \bar{M}|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle - C_7\frac{2im_b}{q^2} q^\nu \langle \bar{M}|\bar{s}\sigma_{\mu\nu}P_R b|\bar{B}\rangle, \\ B_\mu &= C_{10}\langle \bar{M}|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle, \end{aligned} \quad (2)$$

the $B \to M$ matrix elements are expressed in terms of (local) form factors. The remaining term is the non-local contribution

$$T_\mu = -\frac{4\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x}\langle \bar{M}|\, T j_\mu(x) H^c(0)|\bar{B}\rangle. \quad (3)$$

Non-local contributions do not seem to explain the anomalies (still debated by some):

- Unitarity bounds (Bobeth, Chrzaszcz, van Dyk, Virto 2017 [1], Gubernari, van Dyk, Virto 2020 [2]).
- q^2-independence of the fitted NP Wilson coefficients (e.g. Bordone, Isidori, Mächler, Tinari 2024 [3]). See also $B \to K^*\mu\mu$ LHCb 2024 (Eluned's talk)

Local form factors for $B \to$ light at low q^2

The status of the B-~~anomalies~~ quirks relies essentially on the calculation of the local form factors in the SM. In this talk I focus on the low-q^2 range where most of the anomalies are. There are three ways to compute the form factors in this region:

- LCSR with light-meson distribution amplitude (DA) (see e.g. Khodjamirian Russov 2017 [4])
- LCSR with B-meson DA (**this talk**) (e.g. Cui, Huang, Shen, Wang, Wang 2022 [5] in SCET, Gubernari, Kokulu, Van Dyk 2018 [6] in HQET). GKvD used e.g. in LHCb 2024 [7].
- Lattice QCD, at $q^2 = 0$ only $f_{+,T}^{B\to K}$ (HPQCD 2022 [8])

Establishing the sum rule

Khodjamirian, Mannel, Offen 2005 [9]

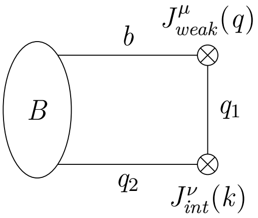
$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x}\langle 0|\, T\left\{J^{\nu}_{int}(x)J^{\mu}_{weak}(0)\right\}|\bar{B}(p_B = q+k)\rangle\,, \quad (4)$$

From analyticity (taking $k^2 < 0$) and unitarity

$$\Pi^{\mu\nu}(q,k) = \frac{\langle 0|J^{\nu}_{int}|M(k)\rangle\langle M(k)|J^{\mu}_{weak}|\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \int_{s_{cont}}^{+\infty} ds \frac{\rho^{\mu\nu}(q,s)}{s-k^2}, \quad (5)$$

From the identification of Lorentz structures in the definition of the form factors

$$\Pi_F(q^2,k^2) = Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2,s)}{s-k^2}, \quad (6)$$



Establishing the sum rule

We reproduce the results of Gubernari, Kokulu, van Dyk 2018 [6], with 2-particles up to twist-5 and and 3-particles up to twist-3. For 2-particles e.g.:

$$\Pi_F^{LCOPE} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{(s(\sigma) - k^2)^n}, \tag{7}$$

$$I_n^{F,(2\mathrm{p})}(\sigma, q^2) = \frac{f_B m_B}{(1-\sigma)^n} \sum_{\psi_{2\mathrm{p}}} C_n^{F,\psi_{2\mathrm{p}}}(\sigma, q^2)\, \psi_{2\mathrm{p}}(\sigma m_B) \tag{8}$$

where $\psi_{2p} = \phi_+, \phi_-, g_+, g_-$ are the distributions amplitudes. B-LCDAs from Braun, Ji, Manoshov 2017 [10] using the exponential model (using the alternative models yield a $0 - 10\%$ variation). *We derive the full expression for the twist-5 2-particle LCDA* g_-*. (Wandzura-Wilczek approximation in GKvD 2018)*

Global Quark-Hadron duality

$$\Pi_F = Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2, s)}{s - k^2} \simeq \Pi_F^{LCOPE} = \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{(s(\sigma) - k^2)^n} \tag{9}$$

Applying the Borel transform to suppress the spectral density integral

$$\begin{aligned} \mathcal{B}_{M^2}\Pi_F &= Y_F F(q^2) e^{-m_M^2/M^2} + \int_{s_{cont}}^{+\infty} \rho(s) e^{-s/M^2} \\ &= \int_0^{+\infty} d\sigma \sum_{n=1}^{+\infty} \frac{I_n^F(\sigma)}{M^{2(n-1)}(n-1)!} e^{-s(\sigma)/M^2} \end{aligned} \tag{10}$$

Semi-global Quark Hadron Duality

Semi-Global Quark-Hadron Duality approximation

$$\int_{s_{cont}}^{+\infty} \rho(s) e^{-s/M^2} = \frac{1}{\pi} \int_{s_0}^{+\infty} \mathrm{Im}\Pi^{pert}(s) e^{-s/M^2} \tag{11}$$

Thus

$$Y_F F(q^2) e^{-m_M^2/M^2} = \mathcal{B}_{M^2}\Pi_F^{pert} - \frac{1}{\pi} \int_{s_0}^{+\infty} \mathrm{Im}\Pi^{pert}(s) e^{-s/M^2} \tag{12}$$

where the effective threshold is set to verify

$$m_M^2 = \frac{\frac{d}{d(-1/M^2)} \left[\mathcal{B}_{M^2}\Pi_F^{pert} - \frac{1}{\pi} \int_{s_0}^{+\infty} \mathrm{Im}\Pi^{pert}(s) e^{-s/M^2} \right]}{\mathcal{B}_{M^2}\Pi_F^{pert} - \frac{1}{\pi} \int_{s_0}^{+\infty} \mathrm{Im}\Pi^{pert}(s) e^{-s/M^2}} \tag{13}$$

Challenges of B-meson LCSR

- **Effective threshold**: For $B \to \pi, \rho, K$, the effective threshold s_0 cannot be determined using the meson mass sum rule. Thresholds from QCD sum rules are used in GKvD 2018.
- **Borel window**: In the literature $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$ (KMO '06, GKvD '18) or alternatively $M^2 \in [1, 1.5]$ GeV^2 (CHSWW '22, DKVV '23). *Up to 20% variations of the form factor between $M^2 = 0.5\,GeV^2$ and $M^2 = 1.5\,GeV^2$.*
- B-**LCDA** are only known asymptotically at low ω, full DAs are model dependent. Plagued by end-point divergences which can break the twist hierarchy.
- B-**LCDA parameters** are poorly known. We use our own average of the results of λ_B^{-1} Khodjamirian, Mandal, Mannel 2020 [11] $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV^{-1} (GKvD: $\lambda_B^{-1} = 2.2 \pm 0.6$ GeV^{-1})

Dependence on λ_B^{-1} and effective threshold determination strategy in B-meson LCSR

form factor	$B \to K^*$			$B \to \rho$		
	GKvD [6]	(i)	(ii)	GKvD [6]	(i)	(ii)
V	0.33 ± 0.11	$0.31^{+0.19}_{-0.15}$	$0.48^{+0.24}_{-0.20}$	0.27 ± 0.14	$0.16^{+0.12}_{-0.09}$	$0.27^{+0.16}_{-0.13}$
A_1	0.26 ± 0.08	$0.25^{+0.14}_{-0.12}$	$0.36^{+0.18}_{-0.15}$	0.22 ± 0.10	$0.14^{+0.09}_{-0.07}$	$0.21^{+0.11}_{-0.10}$
A_2	0.24 ± 0.09	$0.22^{+0.16}_{-0.12}$	$0.36^{+0.20}_{-0.17}$	0.19 ± 0.11	$0.11^{+0.10}_{-0.07}$	$0.20^{+0.14}_{-0.10}$
T_1	0.29 ± 0.10	$0.27^{+0.17}_{-0.13}$	$0.41^{+0.20}_{-0.17}$	0.24 ± 0.12	$0.15^{+0.10}_{-0.08}$	$0.24^{+0.13}_{-0.11}$
T_{23}	0.58 ± 0.13	$0.58^{+0.19}_{-0.20}$	$0.73^{+0.16}_{-0.21}$	0.56 ± 0.15	$0.43^{+0.16}_{-0.15}$	$0.56^{+0.16}_{-0.16}$
s_0 (GeV^2)	$[1.4, 1.7]$	$1.53^{+0.35}_{-0.09}$	$1.54^{+0.34}_{-0.10}$	1.6 ± 0.032	$1.03^{+0.08}_{-0.04}$	$1.05^{+0.09}_{-0.04}$

Table 1: Prediction of $B \to \rho, K^*$ form factors at $q^2 = 0$ following the calculation of GKvD 2018 using a different method for the determination of the effective threshold. $\lambda_B^{-1} = 2.2 \pm 0.6$ GeV^{-1} (i) and $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV^{-1} (ii).

Dependence on λ_B^{-1} and effective threshold determination strategy in B-meson LCSR

form factor	$B \to \pi$			$B \to K$		
	GKvD [6]	(iii)	(iv)	GKvD [6]	(iii)	(iv)
f_+	$0.21(7)$	$0.023(7)$	$0.26^{+0.08}_{-0.08}$	$0.27(8)$	$0.24(7)$	$0.34^{+0.09}_{-0.09}$
f_T	$0.19(7)$	$0.024(7)$	$0.24^{+0.06}_{-0.06}$	$0.25(7)$	$0.24(7)$	$0.31^{+0.06}_{-0.08}$
s_0 (GeV^2)	0.7 ± 0.014	$0.0393(1)$	0.7 ± 0.014	1.05 ± 0.021	$0.54^{+0.03}_{-0.02}$	1.05 ± 0.021

Table 2: Prediction of $B \to \pi, K$ form factors at $q^2 = 0$ following the calculation of GKvD 2018 [6]. Our results are obtained using $\lambda_B^{-1} = 2.72 \pm 0.66$ GeV^{-1} and s_0 obtained from a daughter sum rule (iii) and using the same threshold s_0 as in [6] (iv).

Method of the moments

We define

$$\widetilde{\Pi}_F^{(p)}(q^2,k^2) \equiv \frac{(m_M^2-k^2)^{p+1}}{p!\;Y_F}\Pi_F^{(p)}(q^2,k^2), \quad R_F(p,q^2,k^2) \equiv \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2-k^2}{s-k^2}\right)^{p+1},$$

and

$$\widetilde{m}_M^2(p,\ell,k^2) \equiv \left[\frac{p!}{(p-\ell)!}\frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}}\right]^{1/\ell} + k^2,$$

The sum rule becomes

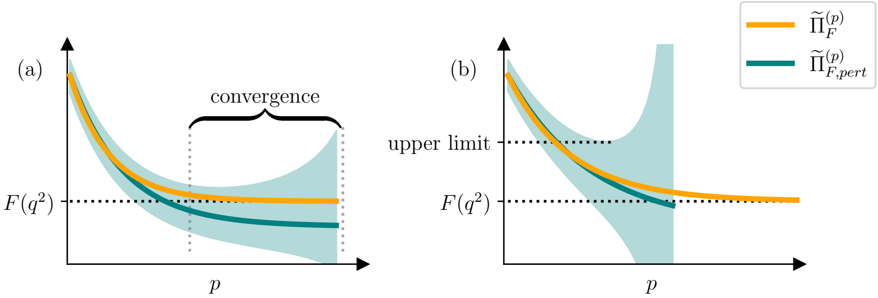
$$\widetilde{\Pi}_F^{(p)}(q^2,k^2) = F(q^2) + R_F(p), \tag{14}$$

and

$$\widetilde{\Pi}_F^{(p)}(q^2,k^2) \xrightarrow[p\to\infty]{} F(q^2), \quad R_F(p,q^2,k^2) \xrightarrow[p\to\infty]{} 0, \quad \widetilde{m}_M^2(p,\ell,k^2) \xrightarrow[p\to\infty]{} m_M^2. \tag{15}$$

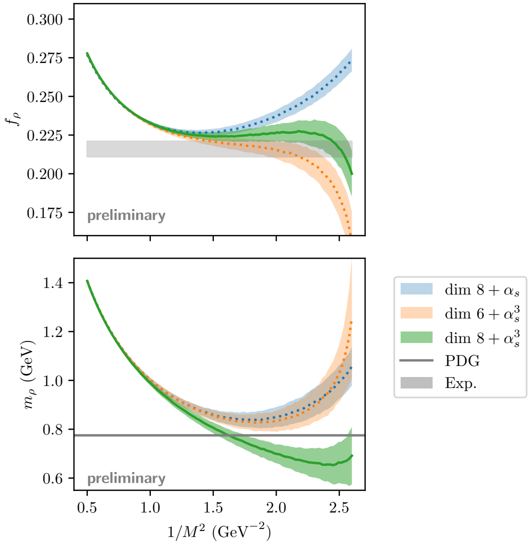
LCSR without semi-global QHD

Accounting for the truncation error, $\Pi_F^{(p)} = \Pi_{F,pert}^{(p)}$ within uncertainties. For p (M^2) large (low) enough, $R_F \ll F(q^2)$ and $R_F \ll \text{std}(\Pi_{F,pert}^{(p)})$, thus $F(q^2) = \Pi_{F,pert}^{(p)}(q^2)$ within uncertainties.



Same behaviour for $\widetilde{m}_M^2$, $\widetilde{m}_{M,pert}^2$ and m_M^2, thus the daughter sum rule can tell us when (if) we reach the convergence regime. $p \to \infty \leftrightarrow 1/M^2 \to \infty$

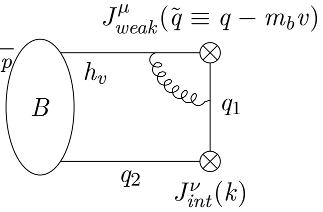
Check with two-point QCD sum rule for the ρ meson - WIP



- Shifman-Vainshtein-Zakharov [12] sum rule successfully predicts e.g. the ρ meson decay constant
- $\Pi_{SVZ} = \sum_d C_d(q^2) \langle O_d \rangle$ where $\langle O_d \rangle$ are quark-gluon vacuum condensates
- $\langle \bar{q} q \rangle$ from FLAG 2021, $\langle \bar{s} s \rangle$ from HPQCD [13], $\langle \alpha_s GG \rangle = 0.012(4)$ (SVZ)
- Dimension 8 contribution for the ρ sum rule extracted from $\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ [14]
- dim 0: α_s^3, dim 3: α_s^2, dim 6: α_s
- Caveat: This figure only includes parametric errors
- **We obtain f_ρ from SVZ sum rules without semi-global QHD**

QCD perturbativity

$$\Pi^{(p)}_{F,LCOPE}(q^2,k^2) = \int_0^{\sigma_{\max}} d\sigma \sum_{n=1}^{\infty} \frac{(n+p-1)!}{(n-1)!} \frac{I_n^{(F)}(\sigma)}{(s(\sigma)-k^2))^{n+p}}$$
$$\equiv \int_0^{\sigma_{\max}} d\sigma I_{\rm tot}^{(F,p)}(\sigma,k^2)$$

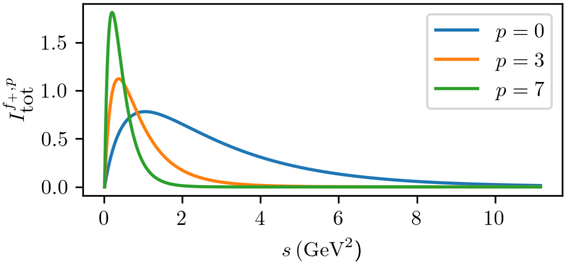


We define the average virtuality

$$\langle s \rangle = \frac{\int_0^{\sigma_{\max}} d\sigma |I_{\rm tot}^{(F,p)}(\sigma,k^2)| s(\sigma)}{\int_0^{\sigma_{\max}} d\sigma |I_{\rm tot}^{(F,p)}(\sigma,k^2)|}$$

To keep the radiative corrections under control we require

$$|\langle s \rangle - m_1^2| \gg \Lambda^2_{QCD}, \quad |k^2|, |\tilde{q}^2| \gg \Lambda^2_{QCD}.$$



QCD error model

For the characteristic QCD scale we take

$$\mu_{\rm QCD} \equiv \text{min}(\sqrt{\langle s\rangle - m_1^2}, \sqrt{|k^2|}, \sqrt{|\tilde{q}^2|}). \tag{16}$$

Interestingly, $\langle s\rangle > -k^2/p \approx M^2$ when p becomes large.

$$\Pi_F^{(p)} = \Pi_{F,LO}^{(p)}\left[1 + w_{\alpha_s}(\mu)\sum_{n=1}\left(\frac{\alpha_s(\mu)}{\pi}\right)^n\right] = \Pi_{F,LO}^{(p)}\left[1 + w_{\alpha_s}(\mu)\,\frac{\alpha_s(\mu)/\pi}{1-\alpha_s(\mu)/\pi}\right], \tag{17}$$

where $w_{\alpha_s}(\mu \sim m_B) \sim 1$ and $w_{\alpha_s}(\mu \to \Lambda_{QCD}) = 0$. We take $\mu = \mu_{QCD} > 0.8$ GeV and

$$w_{\alpha_s} \in [-1.5, 1.5] \tag{18}$$

LCOPE truncation error

$$\Pi_{F,LCOPE}(q^2,k^2) = \underbrace{\Pi_{2p}^{twist-2,3} + \Pi_{3p}^{twist-3,4}}_{\propto (x^2)^0:\ \text{LT}} + \underbrace{\Pi_{2p}^{twist-4,5} + \Pi_{3p}^{twist-5,6}}_{\propto x^2:\ \text{NLT}} + \ldots \tag{19}$$

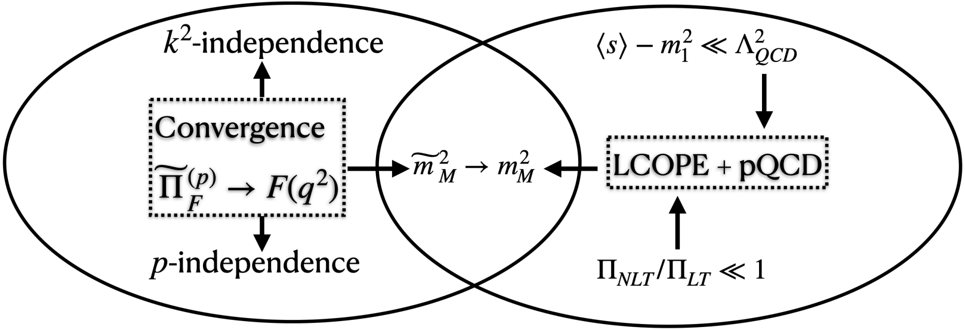
$$\begin{aligned} \Pi_F^{(p)} &= \sum_{t\geq 2} \Pi_{twist=t}^{(p)} = \Pi_{\rm LT}^{(p)} + \Pi_{\rm NLT}^{(p)} + \sum_{t\geq 6} \Pi_{twist=t}^{(p)} \\ &\equiv \Pi_{\rm LT}^{(p)} + \Pi_{\rm NLT}^{(p)} + w_{LCOPE} \times \frac{(\Pi_{\rm NLT}^{(p)})^2}{|\Pi_{\rm LT}^{(p)}| - |\Pi_{\rm NLT}^{(p)}|}. \end{aligned} \tag{20}$$

To be conservative we take a uniformly distributed w_{LCOPE}

$$w_{LCOPE} \in [-2, 2] \tag{21}$$

and keep $\Pi_{NLT}^{(p)}/\Pi_{LT}^{(p)} < 20\%$

Strategy summary



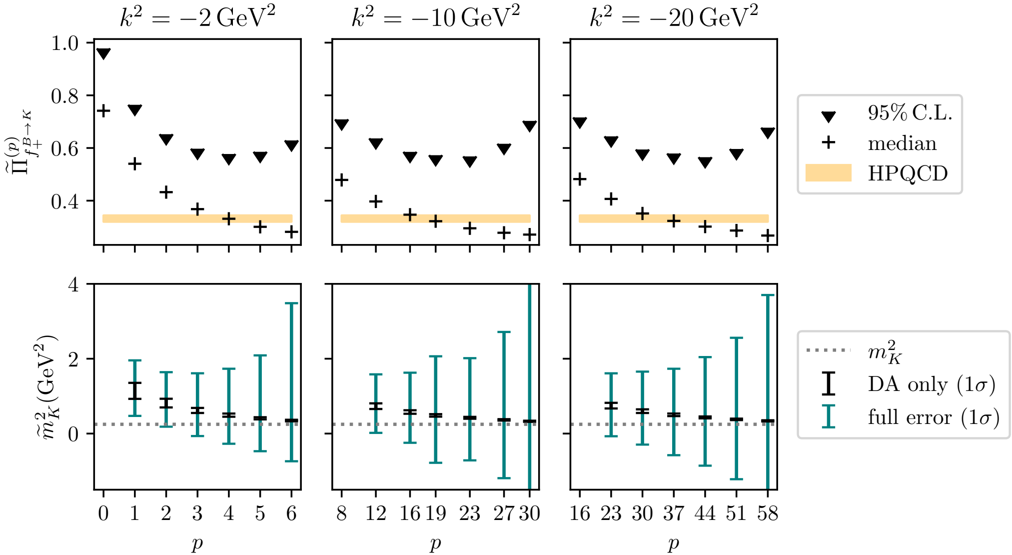
$-k^2/p$-dependence of $V^{B\to K^*}$

$-k^2/p$ (GeV^2)	$20/13$	$20/16$	$20/20$	$20/25$	$20/30$	$20/35$
$R_{V^{B\to K^*}}$	$0.50^{+0.15}_{-0.13}$	$0.37^{+0.11}_{-0.11}$	$0.25^{+0.08}_{-0.08}$	$0.15^{+0.05}_{-0.04}$	$0.08^{+0.03}_{-0.02}$	$0.03^{+0.01}_{-0.01}$
$V^{B\to K^*}$	$0.43^{+0.24}_{-0.19}$	$0.45^{+0.25}_{-0.20}$	$0.47^{+0.28}_{-0.22}$	$0.49^{+0.30}_{-0.21}$	$0.52^{+0.31}_{-0.23}$	$0.53^{+0.35}_{-0.24}$
$\widetilde{\Pi}^{(p)}_{V^{B\to K^*}}$	$0.94^{+0.37}_{-0.33}$	$0.84^{+0.36}_{-0.31}$	$0.72^{+0.35}_{-0.30}$	$0.64^{+0.34}_{-0.26}$	$0.60^{+0.32}_{-0.25}$	$0.55^{+0.35}_{-0.25}$
s_0(GeV^2)	$1.43^{+0.026}_{-0.012}$	$1.48^{+0.03}_{-0.01}$	$1.56^{+0.04}_{-0.02}$	$1.68^{+0.06}_{-0.03}$	$1.87^{+0.1}_{-0.05}$	$2.21^{+0.25}_{-0.10}$

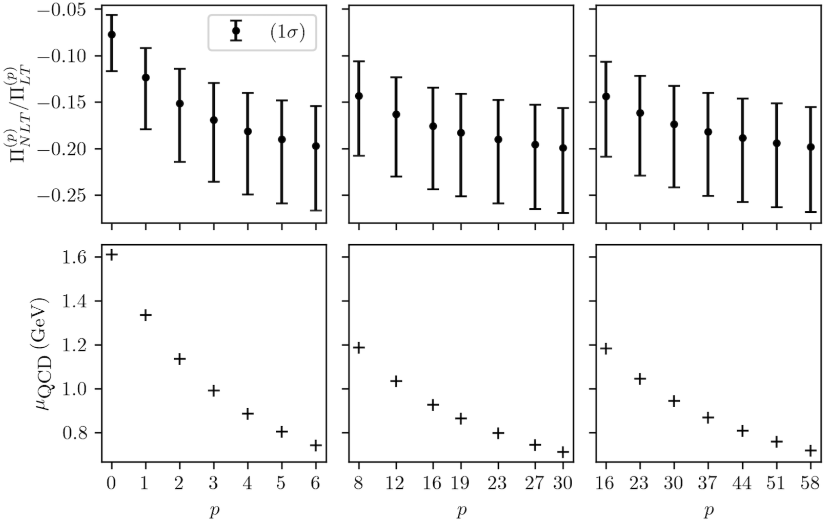
Table 3: Central values and 68% C.L. intervals of $\widetilde{\Pi}^{(p)}_{V^{B\to K^*}}$ $(q^2=0)$. The corresponding $R_{V^{B\to K^*}}$, $V^{B\to K^*}$ and s_0 are estimated using DSR and semi-global QHD.

At $-k^2/p$ $M^2 \sim 1.53\text{GeV}^2$ $R_F/\widetilde{\Pi}^{(p)}_F = 53\%$! We advocate for $-k^2/p$ $M^2 < 1$ GeV^2.
Variation of $V^{B\to K^*}$ of 20% between $M^2 = 0.5$ GeV^2 and $M^2 = 1.5$ GeV^2.

Results - $f_+^{B\to K}$



Results - $f_+^{B\to K}$



Results - $B \to K$

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to K}$	$10/19$	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	$0.332(12)$ $0.27(8)$ $0.325(85)$ $0.395(33)$	HPQCD [8] GvDK [6]† CHSWW [5] KR [4]
$f_T^{B\to K}$	$10/8$	$0.03^{+0.06}_{-0.11}$	0.46	$0.34^{+0.08}_{-0.07}$	$0.332(21)$ $0.25(7)$ $0.381(27)$ $0.381(97)$	HPQCD [8] GvDK [6]† KR [4] CHSWW [5]

Table 4: Upper limits at the 95% confidence level and central value of $\widetilde{\Pi}_F^{(p)}$ for $B \to \pi, K$. We include the corresponding values of $-k^2$ and p as well as an estimate of $R_F(p,k^2)$ using quark-hadron duality.

Mass prediction

- The mass sum rule has huge error bars because we decorrelated the radiative corrections of the successive derivatives.
- The parametric uncertainties in the DA parameters cancel out when p increases

$$\widetilde{m}_M^2 = m_M^2 + (m_M^2 - k^2)\left[\frac{1}{\ell}\cdot\frac{R_F(p-\ell) - R_F(p)}{F(q^2)} + \mathcal{O}\left(\frac{R_F}{F}\right)^2\right], \qquad (22)$$

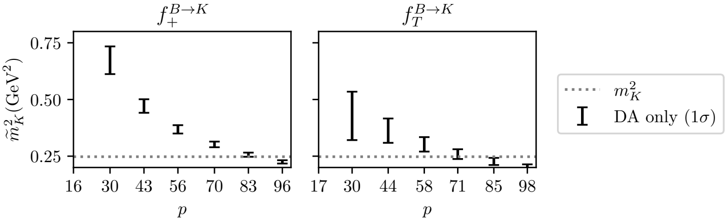
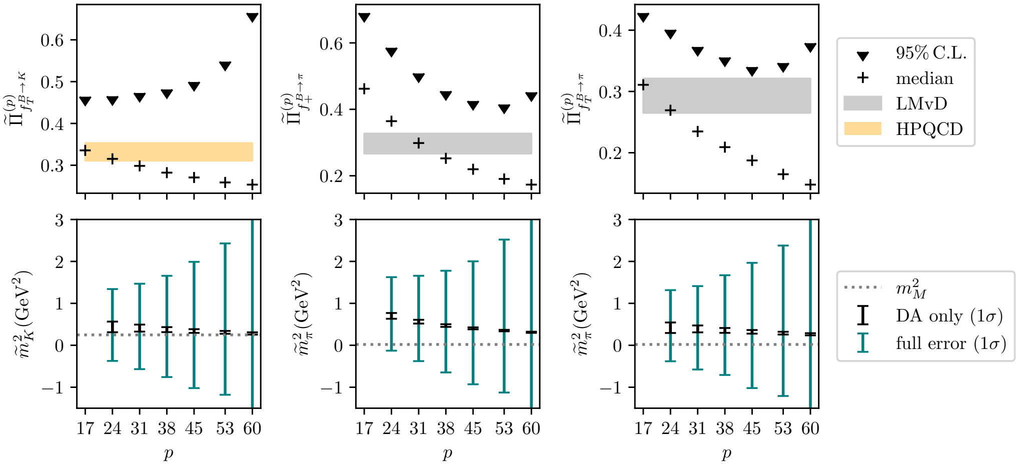
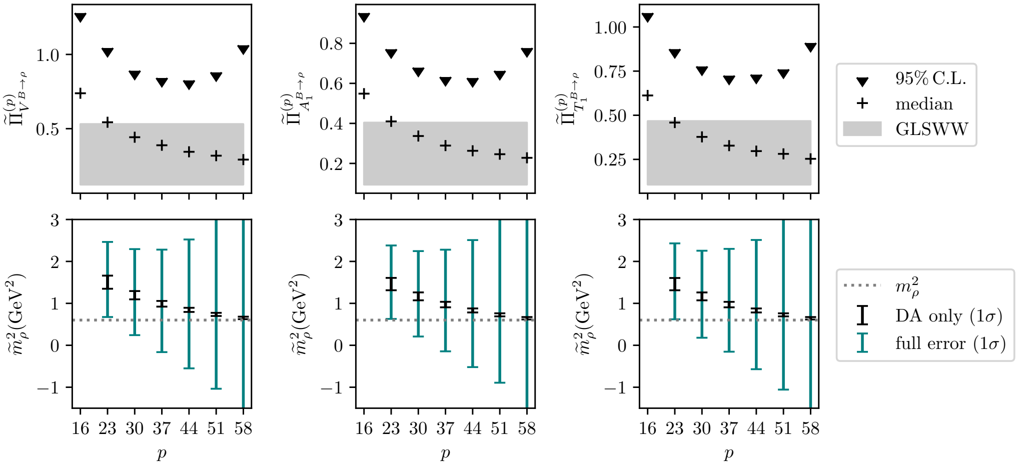


Figure 1: Mass predictions accounting for DA parametric error only $k^2 = -20$ GeV^2

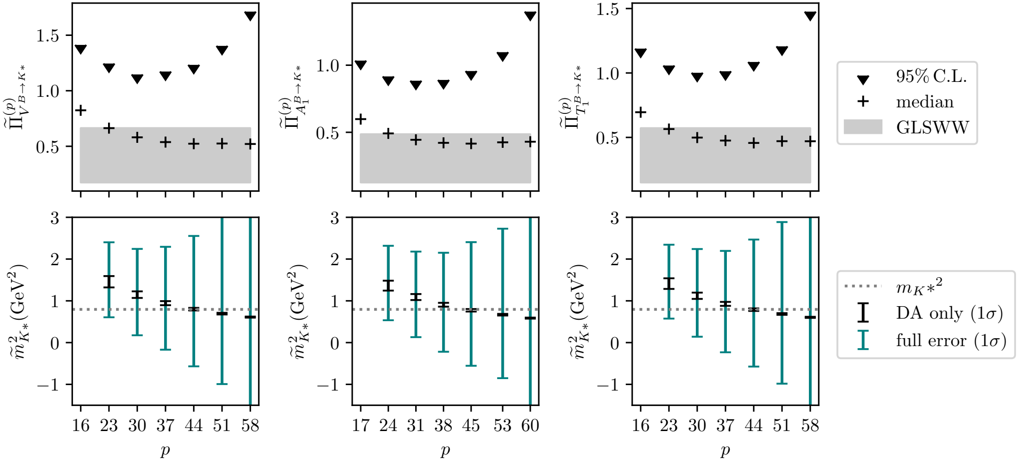
Results - $k^2 = -20\text{GeV}^2$



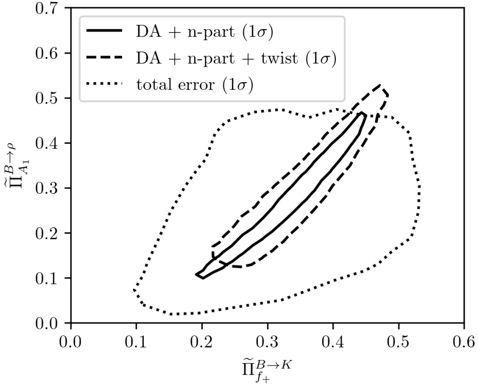
Results - $k^2 = -20\text{GeV}^2$

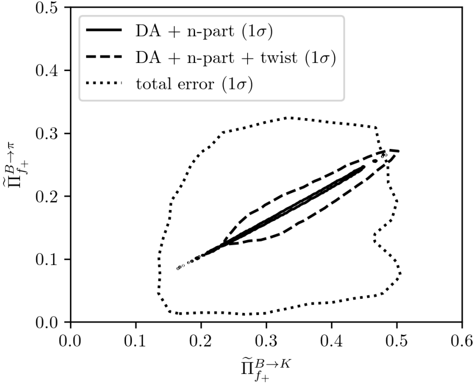


Results - $k^2 = -20\text{GeV}^2$



Correlation





Conclusion and prospects

- We propose to push the LCSR method to low Borel parameters (or $-k^2/p$) to suppress (virtually) entirely the spectral integral and avoid the semi-global QHD.
- We find that radiative corrections are potentially large but calculable at $0.4\ \text{GeV}^2 < M^2 < 0.5\ \text{GeV}^2$ and the LCOPE is under control for $B \to$ light channels. Simultaneously the spectral density integral seems to be negligible in this region.
- This approach should work well for LCSR with e.g. K-LCDAs for which the radiative corrections are known and higher twists have been calculated.
- We have started computing the radiative corrections for B-LCSR in HQET
- In this limit, we can potentially predict highly correlated form factors when including radiative corrections

5 References

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