Dipion distribution amplitudes from the $D \to \pi\pi\ell\nu_\ell$ decay

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work in progress
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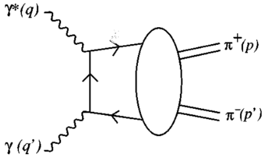
□ Dipion light-cone distribution amplitudes

- originally introduced and developed for $\gamma^*\gamma \to 2\pi$ processes

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)

M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)

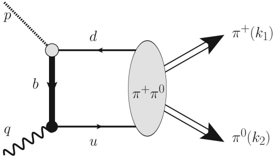
M. V. Polyakov, (1999)



- light-cone sum rules for $B \to \pi\pi$ form factors

Ch. Hambrock, AK, 1511.02509

S. Cheng, AK and J. Virto, 1709.00173



- factorization formulas in $B \to 3\pi$ decays,

S. Kränkl, T. Mannel and J. Virto, 1505.04111

- many interesting applications,

but very limited knowledge of these DAs!

□ What do we know about LCDAs

M. V. Polyakov, Nucl. Phys. B 555 (1999) 231.

- twist-2 DAs: (isospin $I = 1$ hereafter)

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\parallel}^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu} - k_{2\mu}k_{1\nu}}{2\zeta - 1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\perp}^{I=1}(u,\zeta,k^2)\,,$$

- ▶ the "angular" variable: $\zeta = k_1^+/k^+$, $1-\zeta = k_2^+/k^+$, $\zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}$.

  in dipion c.m. $(2\zeta - 1) = (1 - 4m_\pi^2/k^2)^{1/2} cos\theta_\pi$,

- ▶ normalization conditions → pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{l} \Phi_{\parallel}^{I=1}(u,\zeta,k^2) \\ \Phi_{\perp}^{I=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta - 1)\left\{ \begin{array}{ll} F_\pi^{em}(k^2) & \text{pion e.m. form factor} \\ F_\pi^{t}(k^2) & \text{pion “tensor” form factor} \end{array} \right.$$

- ▶ $F_\pi^{em}(0) = 1$, electric charge of the pion
- ▶ $F_\pi^t(0) = 1/f_{2\pi}^{\perp}$ unknown "tensor" charge of the pion

□ What do we know about LCDAs

M. V. Polyakov, Nucl. Phys. B 555 (1999) 231.

- double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_\perp(u,\zeta,k^2) = \frac{6u(1-u)}{f^\perp_{2\pi}} \sum_{n=0,2,..}^{\infty} \sum_{\ell=1,3,..}^{n+1} B^\perp_{n\ell}(k^2) C_n^{3/2}(2u-1)\beta_\pi P_\ell^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right),$$

  - $B^\perp_{n\ell}(k^2)$ - analogs of Gegenbauer moments for the pion DAs
  - $B^\perp_{01}(0) \equiv 1$, DA is asymptotic if only $B^\perp_{01}(k^2) \neq 0$
  - renormalization scale evolution - the same (ERBL)
  - $B^\perp_{n\ell}(k^2)$ - complex functions at $k^2 > 4m_\pi^2$

    (strong $\pi\pi$ phase, resonances)

- Can we determine or at least constrain Gegenbauer functions?
  - instanton vacuum model for $B^{\perp,\parallel}_{n\ell}$, $n = 0, 2, 4$, valid at small $k^2 \sim 4m_\pi^2$

    M. V. Polyakov and C. Weiss, (1999)
  - Omnes representation for the k^2-dependence

□ Employing the $D \to \pi\pi\ell\nu$ decays

- use universality of dipion DAs,
- calculate $D \to \pi\pi$ form factors from LCSRs in terms of DAs,
- follow the same strategy as for the single pion DAs:

  adopt an ansatz in terms of few first Gegenbauers (usually $n = 0, 2, 4$)
- model the k^2 dependence of $B_{n\ell}(k^2)$ using dispersion relations
- express observables for $D \to \pi\pi\ell\nu_\ell$ via these form factors

  (differential widths, angular observables)
- compare with data and fit the parameters of the $B_{n\ell}(k^2)$ functions
- an exploratory study, $D^0 \to \pi^-\pi^0\ell^+\nu_\ell$,

only the $I = 1$, twist-2 dipion DAs (P-wave and ρ dominance)

LCSRs for $D \to \pi\pi$ form factors

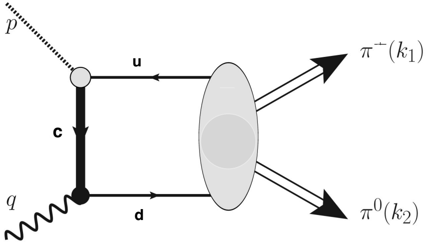
obtained transforming the sum rules for the $B \to \pi\pi$ form factors

Ch. Hambrock, AK, 1511.02509

- The correlation function:

$$\Pi_\mu(q, k_1, k_2) =$$
$$= i \int d^4x\, e^{iqx} \langle \pi^+(k_1)\pi^0(k_2)| T\{\bar{d}(x)\gamma_\mu(1-\gamma_5)c(x), im_c\bar{c}(0)\gamma_5 u(0)\}|0\rangle$$

- OPE near the light-cone $x^2 \sim 0$,
  - valid at $q^2 \ll m_c^2$ (c-quark virtual) and at $k^2 \lesssim 1$ GeV^2
  - the LO diagram: $\langle c(x)\bar{c}(0)\rangle \to$ the perturbative part
  - vacuum $\to$ on-shell dipion hadronic matrix elements of bilocal $\bar{u}(x)d(0)$ operators $\to$ dipion DAs $\Phi_{\perp,\parallel}$



Result for the correlation function

- at LO, twist-2 accuracy: $k = k_1 + k_2, \overline{k} = k_1 - k_2\,,$

$$\Pi_\mu(q,k_1,k_2) = i\sqrt{2}m_c \int_0^1 \frac{du}{(q+uk)^2 - m_c^2}\Big\{\Big[(q\cdot\overline{k})k_\mu - \Big((q\cdot k) + uk^2\Big)\overline{k}_\mu$$

$$+i\epsilon_{\mu\alpha\beta\rho}q^\alpha k_1^\beta k_2^\rho\Big]\frac{\Phi_\perp(u,\zeta,k^2)}{2\zeta-1} - m_c k_\mu \Phi_\parallel(u,\zeta,k^2)\Big\}\,.$$

- read off invariant amplitudes at independent Lorentz structures:
- transform to a form of dispersion integral in the variable p^2:

$s(u) = \frac{m_c^2 - q^2\bar{u} + k^2 u\bar{u}}{u}$

$$\Pi^{(r)}(p^2,q^2,k^2,\zeta) = \sum_{i=\parallel,\perp} f_i^{(r)}(q^2,k^2,\zeta)\int_{m_c^2}^{\infty}\frac{ds}{s-p^2}\left(\frac{du}{ds}\right)\Phi_i(u(s),\zeta,k^2)\,.$$

$q\cdot\bar{k} = \frac{1}{2}(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2)$

Hadronic dispersion relation

- the ground D-meson state contribution:

$$\Pi_\mu(q,k_1,k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|D^0(p)\rangle f_D m_D^2}{m_D^2 - p^2} + \dots ,$$

- expansion of $D \to \pi\pi$ matrix element in form factors:

$$i\langle \pi^-(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|D^0(p)\rangle = -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_D}}\,i\epsilon^{\mu\alpha\beta\gamma}\,q_\alpha\,k_{1\beta}\,k_{2\gamma}$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q^\mu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_D}}\left(k^\mu - \frac{k\cdot q}{q^2}q^\mu\right)$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}^\mu - \frac{4(q\cdot k)(q\cdot \bar{k})}{\lambda_D}\,k^\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_D}\,q^\mu\right),$$

- quark-hadron duality in the D-channel, $\Rightarrow$ effective threshold s_0, Borel transformation , $p^2 \to M^2$

Resulting expressions for the form factors

- in both sum rules only the chiral-odd twist-2 DA contributes:

$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_D}} = \frac{m_c}{\sqrt{2} f_D m_D^2 (1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u} \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_D^2}{M^2} - \frac{m_c^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}} ,$$

$$\frac{F_\parallel(q^2,k^2,\zeta)}{\sqrt{k^2}} = \frac{m_c}{\sqrt{2} f_D m_D^2 (1-2\zeta)} \int\limits_{u_0(s_0)}^{1} \frac{du}{u^2} \left(m_c^2 - q^2 + k^2 u^2\right) \Phi_\perp(u,\zeta,k^2)\, e^{\frac{m_D^2}{M^2} - \frac{m_c^2 - q^2\bar{u} + k^2 u\bar{u}}{uM^2}}$$

- an additional relation between the axial-current form factors:

$$\frac{1}{\sqrt{\lambda_D}}(m_D^2 - q^2 - k^2) F_0(q^2,k^2,\zeta) = F_t(q^2,k^2,\zeta) + 2\frac{\sqrt{k^2}\sqrt{q^2}(2\zeta-1)}{\sqrt{\lambda_D}} F_\parallel(q^2,k^2,\zeta)\Big] .$$

- to obtain F_0 use a separate sum rule for F_t from a different correlation function, contains only $\Phi_\parallel$

S. Cheng, AK and J. Virto, 1709.00173

Sum rules for partial waves

- The form factors expanded in partial waves:

$$F_{\perp,\parallel}(q^2,k^2,\zeta) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}F_{\perp,\parallel}^{(\ell)}(q^2,k^2)\frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi},$$

$\zeta \sim \cos\theta$, $P_l^{(m)}$ -the (associated) Legendre polynomials

- form factors of $D \to \{\pi\pi\}_\ell$ transitions

$$F_\perp^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2)J_n^\perp(q^2,k^2,M^2,s_0^B),$$

$$F_\parallel^{(\ell)}(q^2,k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{m_b^3}{m_B^2 f_B}e^{m_B^2/M^2}\sum_{n=0,2,4,..}\sum_{\ell'=1,3,..}^{n+1} I_{\ell\ell'}\, B_{n\ell'}^\perp(k^2)J_n^\parallel(q^2,k^2,M^2,s_0^B),$$

- $I_{\ell\ell'}$ - integrals over Legendre polynomials,
- $J_n^{\perp,\parallel}$ - the Borel-weighted integrals over $C_n^{3/2}(2u-1)$
- in the limit of asymptotic DA, ($B_{01} \neq 0$, $B_{n>0,\ell} = 0$),
  only P-wave form factors are $\neq 0$

Gegenbauer functions

- retaining the first three coefficients: $B_{01}(k^2)$, $B_{21}(k^2)$, $B_{23}(k^2)$
- the k^2 dependence:
  - the Omnes solution (double subtracted)

$$B^{\perp}_{n\ell}(k^2) = B^{\perp}_{n\ell}(0)\exp\left\{\sum_{r=1}^{N-1}\frac{(k^2)^r}{r!}\beta^{(r)}_{n\ell} + \frac{(k^2)^N}{\pi}\int\limits_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell(s)}{s^N(s-k^2-i\epsilon)}\right\},\quad n\ell = 01, 21\,,;\, N=2$$

$\delta_1(s)$ -the P-wave $\pi\pi$ scattering phase

G. Colangelo, M. Hoferichter and P. Stoffer, 1810.00007

  - parametrization for $B^{\perp}_{23}$:

double subtracted dispersion relation $\oplus$ $\rho_3(1690)$ resonance $\oplus$ expansion in k^2

$$B^{\perp}_{23}(k^2) = B^{\perp}_{23}(0) + k^2 B^{\perp\,\prime}_{23}(0) + (k^2)^2\left(\frac{g^{\perp}_{23}}{m^4_{\rho_3}(m^2_{\rho_3}-k^2-i\epsilon)} + R^{\perp}_{23}\right),$$

- altogether eight unknown parameters, including $f^{\perp}_{2\pi}$,

□ The pion tensor charge

- the local current hadronic matrix element

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\sigma_{\mu\nu}d|0\rangle = i/f_{2\pi}^{\perp}(k_{1\mu}k_{2\mu} - k_{2\mu}k_{1\nu})$$

- the instanton vacuum model (IVM) predicts $f_{2\pi}^{\perp} = 640$ MeV

- the ρ-dominance yields a value in the same ballpark

using a QCD sum rule estimate for the transverse ρ meson decay constant

$$f_{\rho}^{\perp} = 160 \pm 10 \text{ MeV}$$

P.Ball, V.Braun, (1996)

- in the default fit we adopt the IVM value allowing a 20% error

- is there a lattice QCD prediction?

□ Decay width in the P wave approximation

- the semileptonic decay amplitude

$$\mathcal{A}(D^0 \to \pi^- \pi^0 \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cd} \bar{u}_\nu \gamma_\mu (1-\gamma_5) v_\ell \times \langle \pi^-(k_1) \pi^0(k_2) | \bar{d} \gamma^\mu (1-\gamma_5) c | D^0(p) \rangle \,,$$

- form factors in the P-wave approximation (neglecting $\ell > 3$)

$$F_{\perp,\parallel}(k^2, q^2, q \cdot \bar{k}) \quad \to \quad \sqrt{3} F_{\perp,\parallel}^{(\ell=1)}(k^2, q^2) \frac{P_1^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi} \,,$$

$$F_0(k^2, q^2, q \cdot \bar{k}) \quad \to \quad \sqrt{3} F_0^{(\ell=1)}(k^2, q^2) P_1^{(0)}(\cos\theta_\pi) \,.$$

- double differential width and the k^2-distribution

$$\frac{d\Gamma(D^0 \to \pi^- \pi^0 \ell^+ \nu_\ell)}{dk^2 dq^2} = \frac{G_F^2 |V_{cd}|^2 q^2 \beta_\pi \sqrt{\lambda_D}}{3 \cdot 2^{10} \pi^5 m_D^3} \Big[|F_0^{(\ell=1)}(k^2, q^2)|^2 + \beta_\pi^2 \Big(|F_\perp^{(\ell=1)}(k^2, q^2)|^2 + |F_\parallel^{(\ell=1)}(k^2, q^2)|^2 \Big) \Big]$$

$$\frac{d\Gamma(D^0 \to \pi^- \pi^0 \ell^+ \nu_\ell)}{dk^2} = \int\limits_0^{(m_D - \sqrt{k^2})^2} dq^2 \, \frac{d\Gamma(D^0 \to \pi^- \pi^0 \ell^+ \nu_\ell)}{dk^2 dq^2} \,.$$

□ Fitting the BESS III data

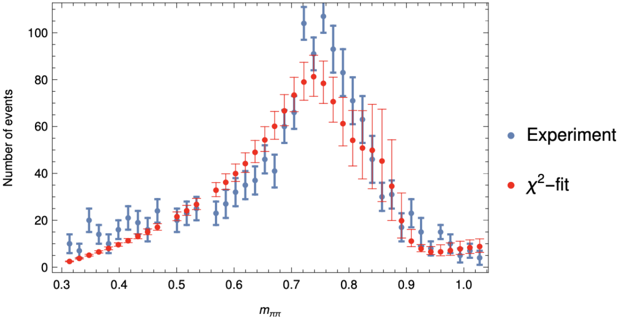
BESSIII collaboration, PRL 122, 062001 (2019), special thanks to Shulei Zhang

- only the k^2 distribution was used

(integration over q^2 done using z-expanded LCSR form factors)

Preliminary

$\frac{dN}{d\sqrt{k^2}}(D^0 \to \pi^-\pi^0 e^+\nu_e)$



- the fit returns $B_{21}^{\perp}(0) = 6.40 \pm 1.29$ and $B_{23}^{\perp}(0) = -5.85 \pm 1.22$

much larger than IVM values but with opposite signs and with a large correlation $\simeq 70\%$,

□ Future tasks and perspectives

- Improving LCSRs and input
  - a lattice-QCD or LCSR calculation of the pion tensor charge
  - twist-3,4 and $\bar{q}qG$ DAs and their double expansions,
    to be worked out with the methods used previosly for vector meson DAs
  - NLO gluon radiative corrections
- extend to $D \to \pi^+\pi^-, \pi^0\pi^0$ form factors, dipion in S, D-waves
  need $I = 0$ DAs
- experiment: a total angular analysis of $D \to \pi\pi\ell\nu_\ell$ desirable
  optimisically, will allow one to separate the form factors