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ON THE COMPOSITION OF EXOTIC HADRON RESONANCES

BASED ON WORK IN COLLABORATION WITH: A. ESPOSITO, D. GERMANI, A. GLIOTI, L. MAIANI, F. PICCININI, A. PILLONI, R. RATTAZZI, V. RIQUER.

EXOTIC HADRON RESONANCES

What can we learn from *exotic* hadrons?
(a few names: $X(3872), Z(4430), Z_c, Z'_c, Z_b \ldots$)

For most of them it is clear that standard interpretations in terms of $q\bar{q}$ mesons or qqq baryons are not viable.

Are they multiquark states, i.e. 'elementary' hadrons, or sort of mesonic nuclei, composite hadrons?

This subject is forcing us to think to *compositeness* and *fine-tuning* in a context rich of solid experimental discoveries.

ELEMENTARY VS COMPOSITE PARTICLES

The fields of **elementary** particles appear in $\mathscr{L}$.

As opposite, a **composite** particle is one whose field $\mathbf{\Phi}$ does not appear in $\mathscr{L}$: it can be created/destroyed by operators constructed by (functions of) other fields, e.g. those appearing in $\mathscr{L}$.

Consider the complete propagator for $\mathbf{\Phi}$ which may, or may not, be elementary

$$\Delta'(p) = \int_0^\infty \frac{\rho(\mu^2)}{p^2 + \mu^2 - i\epsilon} d\mu^2$$

where the spectral function is defined by ($\boldsymbol{\rho = 0}$ for $\boldsymbol{p^2 > 0}$)

$$\theta(p_0)\,\rho(-p^2) = \sum_n \delta^4(p - p_n)\,|\langle 0\,|\,\Phi(0)\,|\,n\rangle\,|^2$$

and $|\,n\rangle = |\,\boldsymbol{k}\rangle$ or multiparticle state $|\,\boldsymbol{k}_1, \boldsymbol{k}_2\rangle\ldots$

ELEMENTARY VS COMPOSITE PARTICLES

Let $|\boldsymbol{k}\rangle$ be a one-particle state with mass $\boldsymbol{m}$.
Suppose $\langle \boldsymbol{k}|$ has a non-zero amplitude with $\Phi^\dagger(0)\,|\,0\rangle$.
Then, according to a general result, the complete propagator $\Delta'(p)$ of the bare field Φ has a **pole** at $-m^2$ with residue $Z = |N|^2 > 0$ where (Lorentz)

$$\langle 0\,|\,\Phi(0)\,|\,\boldsymbol{k}\rangle = \frac{N}{\sqrt{2E}} \qquad E = \sqrt{\boldsymbol{k}^2 + m^2}$$

As a consequence of this, it must be $\rho(\mu^2) = Z\,\delta(\mu^2 - m^2)$

$$\Delta'(p) = \frac{Z}{p^2 + m^2 - i\epsilon}$$

ELEMENTARY VS COMPOSITE PARTICLES

However the spectral function also includes multiparticle states in $|n\rangle$. The contribution of states like $|\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots\rangle$ is incorporated in the function $\sigma \geq 0$

$$\rho(\mu^2) = Z\,\delta(\mu^2 - m^2) + \sigma(\mu^2)$$

Consider the case $Z = 0$ which corresponds to non-zero amplitudes of $\langle \boldsymbol{k}_1, \boldsymbol{k}_2, \ldots |$ with $\Phi^\dagger(0)|0\rangle$ only. Then

$$\Delta'(p) = \int_0^\infty \frac{\sigma(\mu^2)}{p^2 + \mu^2 - i\epsilon}\, d\mu^2$$

The complete propagator is described only by the coupling of Φ to **multi-particle states**, namely $\int_0^\infty \sigma(\mu^2)\, d\mu^2$

ELEMENTARY DEUTERON

Say that the Lagrangian $\mathscr{L}$ of the nuclear theory contains only the elementary fields of the proton $\boldsymbol{p}$ and the neutron $\boldsymbol{n}$.

Add to $\mathscr{L}$ another elementary field, $\mathfrak{d}$ (it can be composite in terms of quarks, but not in terms of $\boldsymbol{p}, \boldsymbol{n}$). Call it *elementary deuteron*.

Assume that $\langle \boldsymbol{k} |$ is a one-particle state of mass $\boldsymbol{m}$ having non-zero amplitude with $\mathfrak{d}^\dagger(0)|0\rangle$. It can't be $\langle n, \boldsymbol{k} |$ nor $\langle p, \boldsymbol{k} |$ – must be the elementary deuteron one-particle state.

The complete propagator of $\mathfrak{d}$ has a pole at $-m^2$ with residue Z: the manifestation of the elementary deuteron.

COMPOSITE DEUTERON

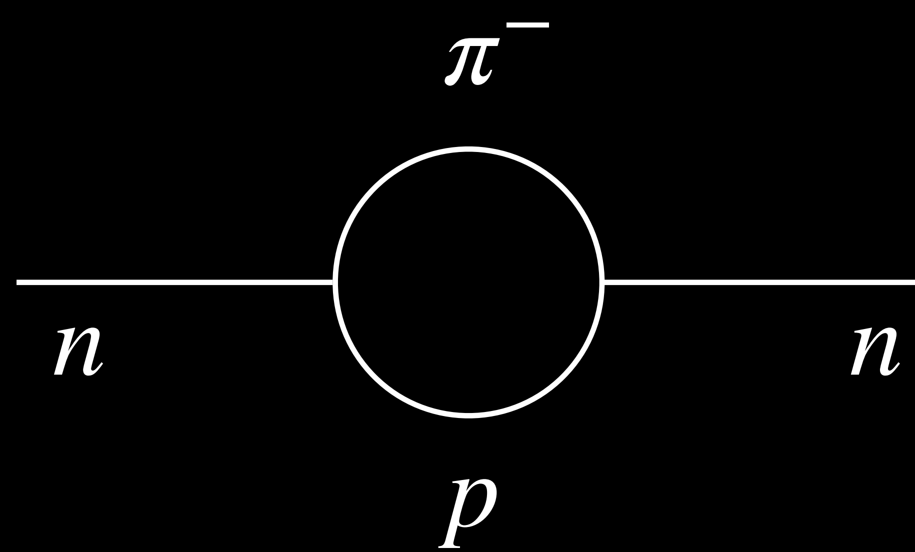
If $Z = 1$ we are making the case of the free theory, $\Delta'(p) = \Delta(p)$.

(Trivial case: if there is an elementary deuteron it must interact with n and p)

If $Z = 0$ we are in the case in which the complete propagator is due only to the coupling of $\mathfrak{d}$ to the np continuum, $|np, k_1, k_2\rangle$.

(Composite case: the $\mathfrak{d}$ field in $\mathscr{L}$ can be *substituted* by function $F(n,p)$ of the elementary fields n, p. We can introduce a field Φ for the composite deuteron by adding to $\mathscr{L}$ a term of the form $\Delta\mathscr{L} = \lambda(F(n,p) - \Phi)^2$ and integrating over Φ in the path integral. This opens the way (but does not correspond) to the description of deuteron as a np bound state.
Bound states can be counted with phase shifts in elastic scattering but their number N is $N = (\delta_\ell(0) - \delta_\ell(E = \infty))$. This formula is not `practical` since, at $E = \infty$, all the inelastic channels are open and Levinson theorem is proved for the elastic scattering only, and not even for shallow bound states.)

THE LEE MODEL



$$|n, \text{in}\rangle = \sqrt{Z}\,|n, \text{bare}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}}\,|p\,\pi^-(\boldsymbol{k})\rangle$$

$$Z + \int_{\boldsymbol{k}} |C_{\boldsymbol{k}}|^2 = 1$$

See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

The analysis is done in NRQM. The starting point is the same of that in the Lee model

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}}\,|np(\boldsymbol{k})\rangle$$

$$Z + \int_{\boldsymbol{k}} |C_{\boldsymbol{k}}|^2 = 1$$

Is it possible to extract $\boldsymbol{Z}$ from data?

See Weinberg Phys. Rev. 137, B672 (1965)

WEINBERG'S ANALYSIS OF THE DEUTERON

$$r_0 = -\frac{Z}{1-Z}R + O\left(\frac{1}{m_\pi}\right)$$ (effective range)

$$R = \frac{1}{\sqrt{2mB}}$$ ($B =$ binding energy)

$$a = \frac{2(1-Z)}{2-Z}R + O\left(\frac{1}{m_\pi}\right)$$ (scattering length > 0)

where the effective range expansion is

$$k\cot\delta \simeq -\frac{1}{a} + \frac{1}{2}r_0 k^2$$ ($\delta =$ phase-shift in pn)

THE SPECIAL ROLE OF $X(3872)$

- The binding energy of X is $B \lesssim 100$ keV, an outlier wrt most of the other exotic resonance observed.
- Does such a small B arise from a *tuning* of the strong interactions in the $D\bar{D}^*$ system ("molecule") making a large (and positive) and $B \sim 1/(2ma^2)$ small?
- From the X lineshape one can extract the effective range r_0, which for a molecule, like the deuteron, is expected to be $r_0 \sim 1/m_\pi \sim 1.5$ fm. But the X is *not like the deuteron* since it involves another coincidence: $m_{D^*} - m_D \simeq m_\pi$, whereas $m_n - m_p \ll m_\pi$ – the pion cannot be integrated out and we get a lower cutoff $\mu \equiv \sqrt{(m_D^* - m_D)^2 - m_\pi^2} \approx m_\pi/3$.
  Do pion interactions make a larger r_0? Positive or negative?
- Indeed, in the deuteron analysis, a compact deuteron would require a negative r_0 with $|r_0| > 1/m_\pi$.

BETHE/LANDAU-SMORODINSKY

Scattering in the presence of shallow bound states generated by *purely attractive potentials* in NRQM are characterized by

$$r_0 \geq 0$$

even if there is a repulsive core, but in a *very* narrow region around the origin. In this case $O(1/m_\pi) \geq 0$ once $Z = 0$.

Esposito et al. 2108.11413

So a nuclear deuteron would need an r_0 small (≈ 1 fm) and positive, whereas an elementary deuteron should involve an r_0 large ($\gg 1$ fm) and negative. Data on np scattering say

$$r_0^{\text{expt.}} = +1.74 \text{ fm}$$

THE CASE OF THE X(3872)

The vicinity of the X(3872) to $D\bar{D}^*$ threshold is considered by many authors as –the proof– of its nuclear nature: a loosely bound state of a D and a $\bar{D}^*$ meson. The term molecule is used.

No $D\bar{D}^*$ scattering experiments are possible, yet the experimental determination of r_0 can proceed through the `lineshape` of the X(3872) using the connection between scattering amplitude (S-wave, low k)

$$f = \frac{1}{k\cot\delta(k) - ik} = \frac{1}{-\frac{1}{a} + \frac{1}{2}r_0k^2 - ik}$$

and BW formula.

Assumption: the $D\bar{D}^*$ decay channel is the dominating one for the X.

THE CASE OF THE X(3872)

For small kinetic energies (and using LHCb analysis)

$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$\delta = m_{D^{*-}} + m_{D^+} - m_{\bar{D}^{*0}} - m_{D^0}$$

$$E = m_{J/\psi\pi\pi} - m_D - m_{\bar{D}^*}$$

and μ_+ is the reduced mass of the charged $D\bar{D}^*$ pair.

Esposito et al. 2108.11413, *Phys. Rev. D*105 (2022) 3, L031503

THE CASE OF THE X(3872)

For small kinetic energies

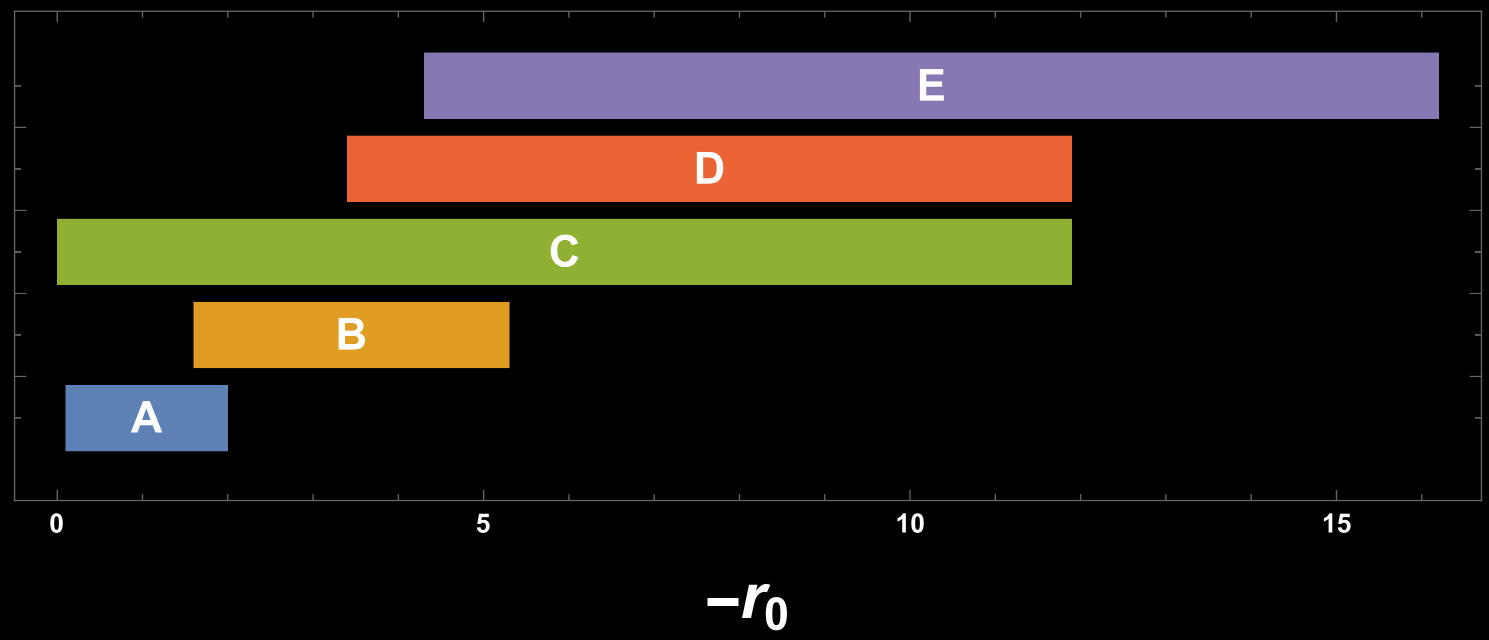
$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$-\frac{1}{a} = \frac{2m_X^0}{g} + \sqrt{2\mu_+\delta} \simeq -6.92 \text{ fm}$$ positive a

$$r_0 = -\frac{2}{\mu g} - \sqrt{\frac{2\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}$$ negative r_0

using $E = k^2/2\mu$, μ being the reduced mass of the neutral $D\bar{D}^*$ pair, and taking g (shaky…) and m_X^0 (stable determination) from the experimental analysis. Since g can be larger, $r_0 \leq -2$ fm.

$(-r_0)$ ACCORDING TO SOME ESTIMATES



A: Baru et al., 2110.07484

B: Esposito et al., 2108.11413

C: LHCb, 2109.01056

D: Maiani & Pilloni GGI-Lects

E: Mikhasenko, 2203.04622

COMPACT X

Having a negative r_0 means having a **finite** Z, which in turn means that there is an **elementary** X field in the Lagrangian.

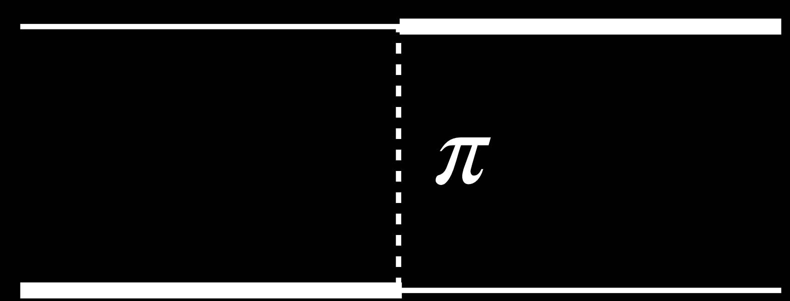
The X can interact as strongly as possible to the $D\bar{D}^*$ continuum, but the very fact that there is an elementary field of X, with whatever Z value, is an indication that it might be appropriate to work with an elementary X.

1) Does the Weinberg analysis apply to the X(3872)?
2) Can the Weinberg criterion be re-formulated in the framework of EFT?
3) Are there critical Z values to compare with?

MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\, \delta^3(\boldsymbol{r})$$

A perturbation to the $\delta^3(\boldsymbol{r})$ potential derives from



Potential = FT of the propagator in no-recoil approximation

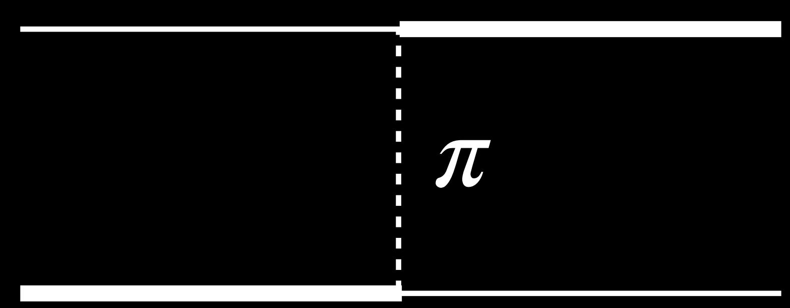
$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + m_\pi^2 - i\epsilon} d^3q \xrightarrow[\text{no rec.}]{} \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon} d^3q \approx \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon} d^3q$$

$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon} d^3q = -\frac{(2\pi)^3}{4\pi}\left(\frac{3\hat{r}_i\hat{r}_j}{r^3} - \frac{\delta_{ij}}{r^3} - \frac{4\pi}{3}\delta^3(\boldsymbol{r})\right)$$

MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\,\delta^3(\boldsymbol{r})$$

A perturbation to the –strong– $\delta^3(\boldsymbol{r})$ potential derives from



Potential $=$ FT of the propagator in no-recoil approximation

In S-wave we have to include the condition $\langle \hat{r}_i \hat{r}_j \rangle = \frac{1}{3}\delta_{ij}$

which, for $\mu = 0$, leaves only a –weak– $\delta^3(\boldsymbol{r})$ potential.

However $\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \simeq 43$ MeV

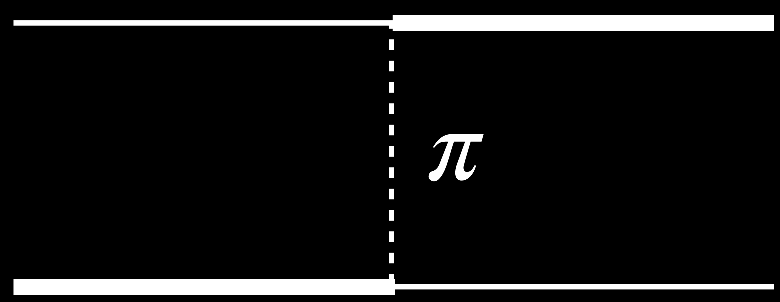
MOLECULAR PICTURE

Keep μ finite! Are the corrections to r_0 of the size $O(1/m_\pi)$ or $O(1/\mu)$? Notice that $(197\ \text{MeV fm})/\mu \sim 5$ fm which is right where the bars in the previous figure mostly fall.

In principle the π-exchange contribution to r_0 might be negative (it does not come from a purely attractive potential) and ≈ -5 fm, or smaller, the $D\bar{D}^*$ bound state being due to V_s only (not contributing to r_0).

If so the `Weinberg criterion`, which is fine for the deuteron, would just fail for the $X(3872)$. Difficult to judge without a calculation, even in consideration that V_w is small.

MOLECULAR PICTURE



Keep μ finite! Are the corrections to r_0 of the size $O(1/m_\pi)$ or $O(1/\mu)$?

$$\frac{g^2}{2f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2-\mu^2-i\epsilon}\,\frac{d^3q}{(2\pi)^3} = \underbrace{\frac{g^2}{6f_\pi^2}}_{\alpha}\left(\delta^3(r)+\mu^2\frac{e^{i\mu r}}{4\pi r}\right)\delta_{ij}$$

where the integral is decomposed by $A\delta_{ij} + B\, r^2\, n_i n_j$ and we use the S-wave relation

$$\langle n_i n_j\rangle = \frac{1}{3}\delta_{ij}$$

the contraction with non-rel. polarizations $e_i^{(\lambda)}\bar{e}_j^{(\lambda')}$ gives $\delta_{\lambda\lambda'}$

MOLECULAR PICTURE

So we have the case in which $\boldsymbol{V}$ itself is not small enough to be considered as a perturbation, but it can be divided in

$$V = V_s + V_w = -\left(\lambda_0 + 4\pi\alpha\right)\delta^3(\boldsymbol{r}) - \alpha\mu^2\frac{e^{i\mu r}}{r}$$

To compute any amplitude, all orders in $\boldsymbol{V_s}$ are needed, and possibly only the first order in $\boldsymbol{V_w}$.
The contribution deriving from $\boldsymbol{V_w}$ is calculated in the DWBA (Distorted-Wave-Born-Approximation) which amounts to use ($\pm =$ in/out)

$$T_{\beta\alpha} = \left(\Psi^{-}_{s\,\beta}, V_w \Psi^{+}_{s\,\alpha}\right)$$

THE IMAGINARY PART OF $V_w(r)$

How to take into account that there are unstable particles in the amplitudes T ? We should add `by hand` the D^* decay width to $V_s + V_w$, but a first principles derivation of this is possible.

$$-\frac{\nabla^2}{2m}\psi(r) - \left[\left(\lambda_0 + 4\pi\alpha\right)\delta^3(\boldsymbol{r}) + \alpha\mu^2\frac{e^{i\mu r}}{r} + i\frac{\Gamma}{2}\right]\psi(r) = E\,\psi(r)$$

Indeed the complex potential V_w alone will not allow any imaginary part in the positive spectrum $E > 0$ (exception made for ψs' exponentially blowing up).

$$\left(\lim_{r\to 0}\mathfrak{I}(V(r)) = \lim_{r\to 0}\mathfrak{I}\,\alpha\mu^2\frac{e^{i\mu r}}{r} = \frac{g^2\mu^3}{24\pi f_\pi^2} \equiv \frac{\Gamma}{2}\right)$$

Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini [draft]

CALCULATION OF r_0

$$f = \frac{1}{k\cot\delta(k) - ik} = f_s + f_w = \frac{1}{-\frac{1}{a} - ik} + f_w$$

$$f_w = -\frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr$$

Where $\chi_s(r)$ are scattering w.f. of the $\delta^3(\boldsymbol{r})$ potential, and $\boldsymbol{m}$ is the invariant $\boldsymbol{DD^*}$ mass. Thus $\boldsymbol{r_0}$ is determined by the $\boldsymbol{k}^2$ coefficient in the *double expansion* around $\boldsymbol{r_0 = 0}$ and $\boldsymbol{\alpha = 0}$ of the expression

$$f^{-1} = \left(\frac{1}{-\frac{1}{a} - ik} - \frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr\right)^{-1}$$

Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini [draft]

CALCULATION OF r_0

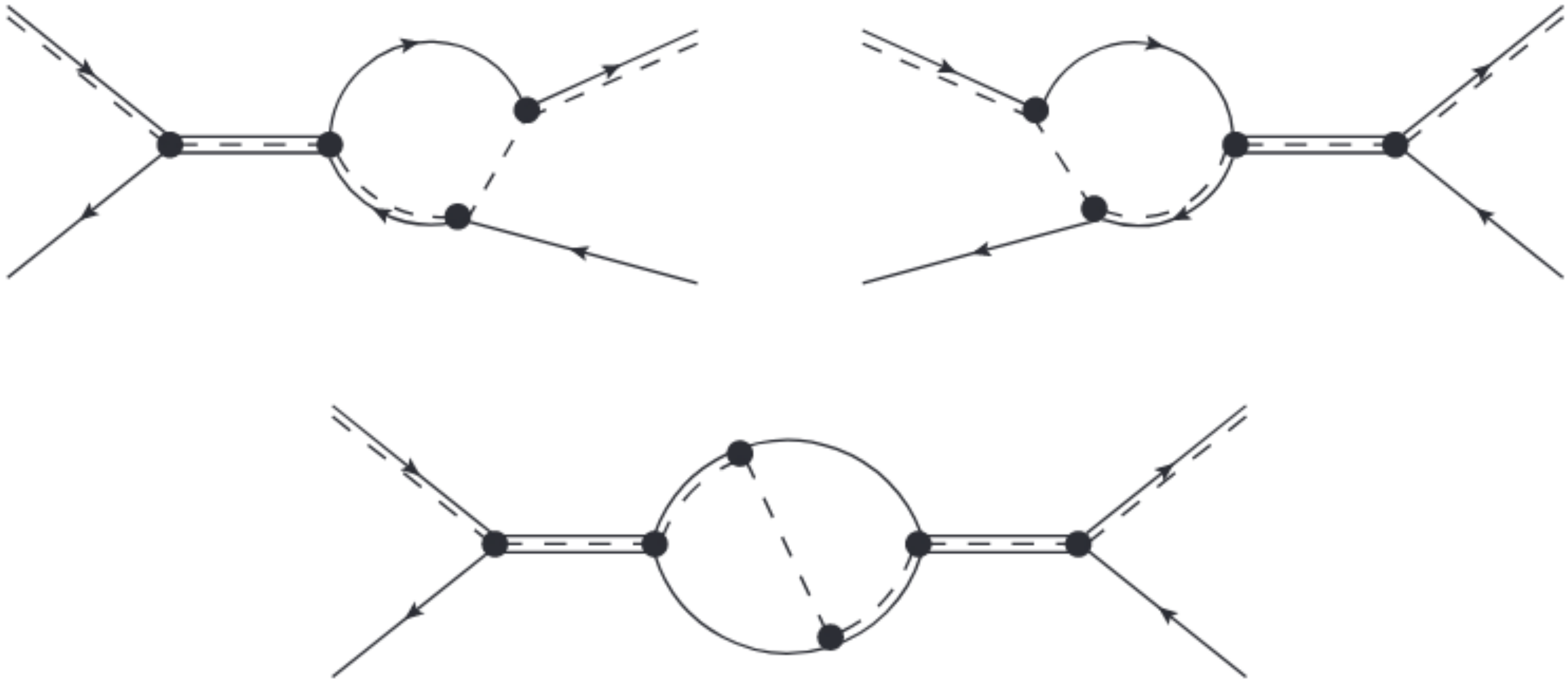
$$r_0 = 2m\alpha\left(\frac{2}{\mu^2 a^2} + \frac{8i}{3\mu a} - 1\right)$$

$$-0.20\ \text{fm} \lesssim \ \text{Re}\, r_0 \lesssim -0.15\ \text{fm}$$

$$0\ \text{fm} \lesssim \ \text{Im}\, r_0 \lesssim 0.17\ \text{fm}$$

$$\alpha = \frac{g^2}{24\pi f_\pi^2} = \frac{5\times 10^{-4}}{\mu^2}$$

These results agree, analytically, with what found by Braaten et al. using EFT. It turns out that the real part of r_0 is just a tiny (negative!) fraction of a Fermi. This confirms the fact that the Weinberg criterion can be extended to the $X(3872)$ too.



Braaten, Galilean invariant XEFT, Phys. Rev. D 103, 036014 (2021), arXiv:2010.05801 [hep-ph]

r_0 FROM LATTICE

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 2202.10110

Applying the lattice Lüscher method, the authors study the DD^* scattering amplitude and make a determination of the scattering length and of the effective range for $\mathcal{T}_{cc}$

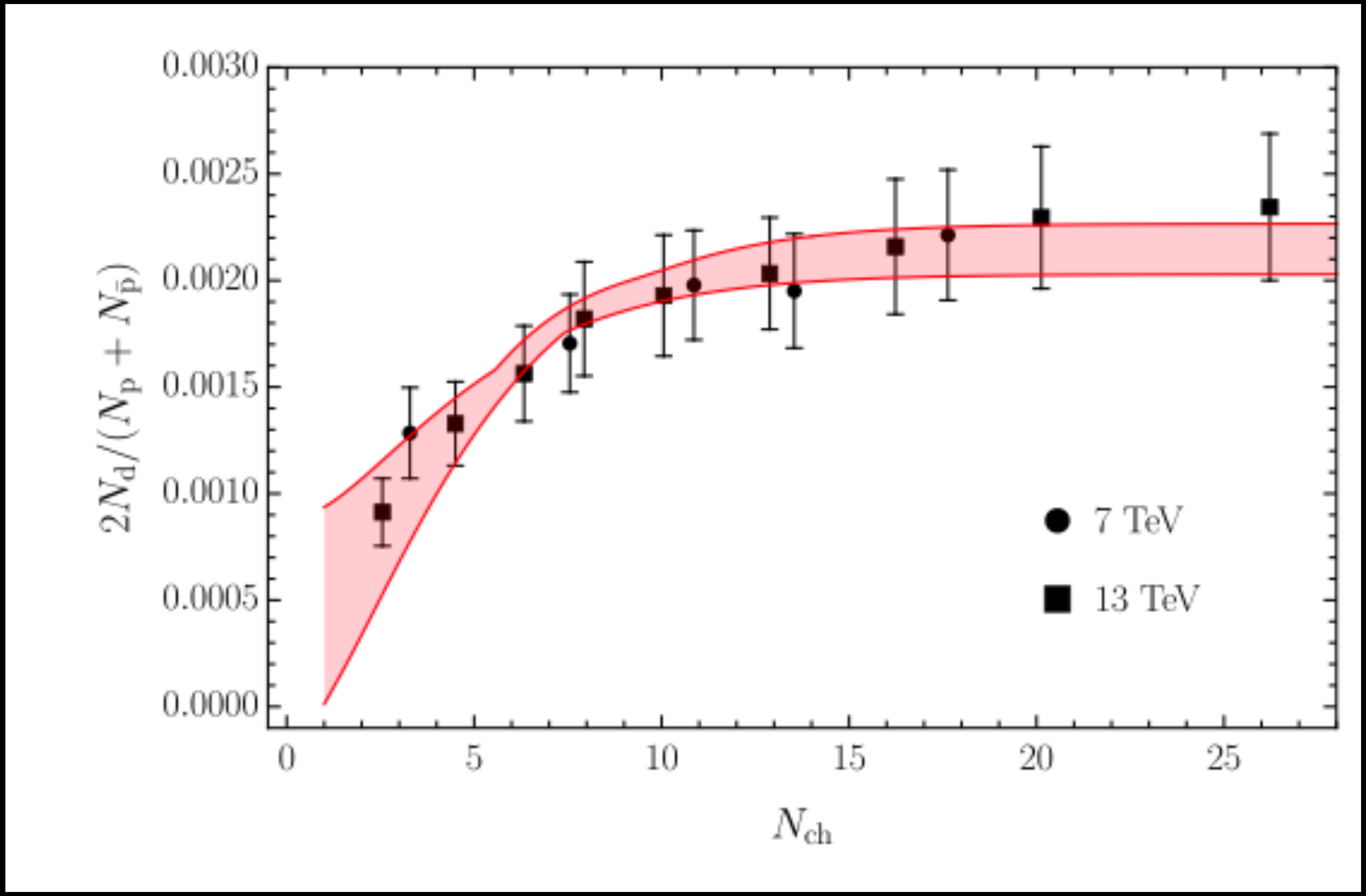
$$a = -1.04(29)\ \text{fm}$$
$$r_0 = +0.96^{+0.18}_{-0.20}\ \text{fm}$$

The mass of the pion is $m_\pi = 280$ MeV, to keep the D^* stable. This result, for the moment, is compatible with a *virtual state* because of the negative a – like the singlet deuteron. As for LHCb (2109.01056 p.12)

$$a = +7.16\ \text{fm}$$
$$-11.9 \leq r_0 \leq 0\ \text{fm}$$

DOES THE X(3872) BEHAVE AS THE DEUTERON?

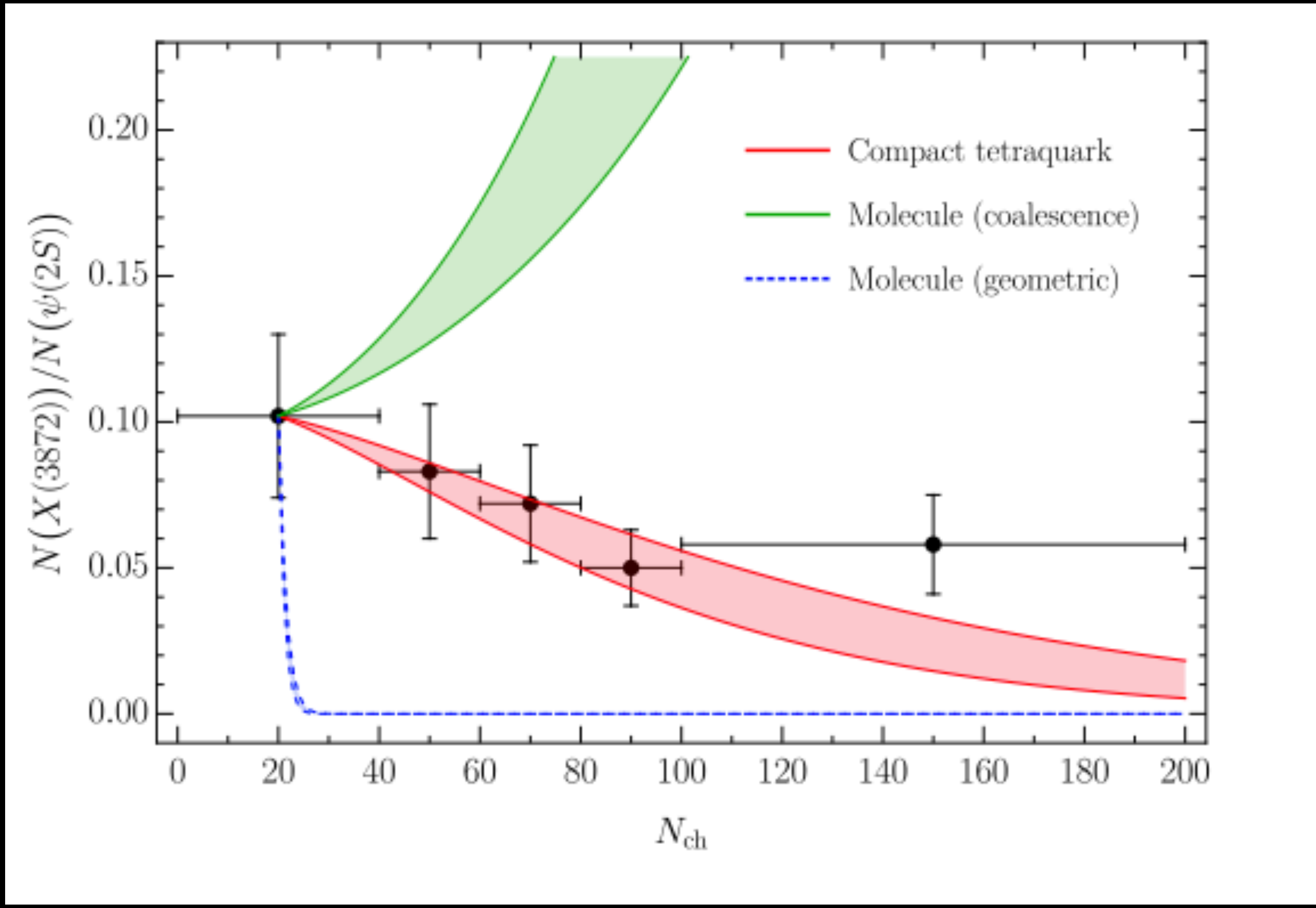
ALICE: 1902.09290; 2003.03184



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

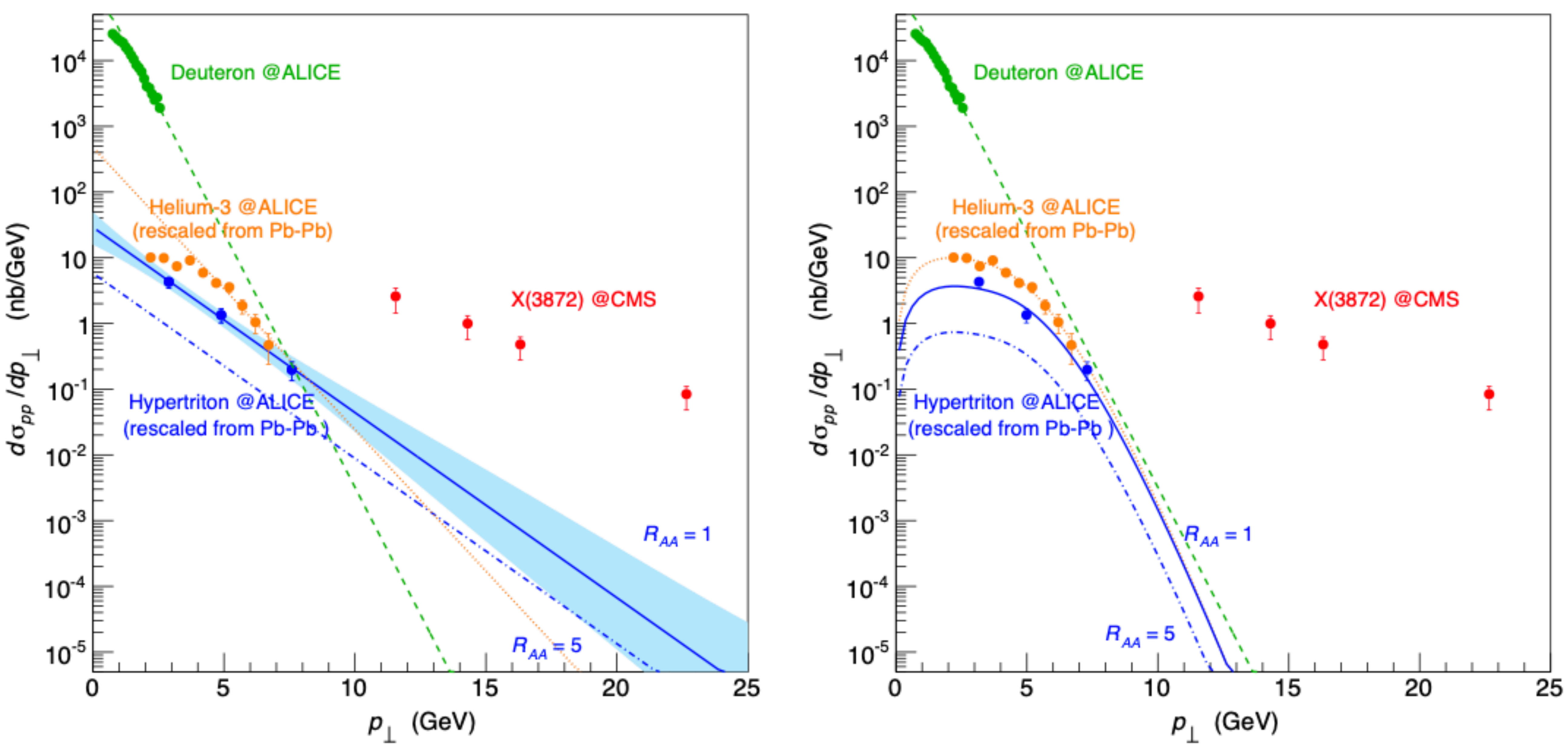
Number of deuterons as a function of the multiplicity computed with Boltzmann equation in a coalescence model.

DOES THE X(3872) BEHAVE AS THE DEUTERON?



The coalescence picture predicts a behavior (green band) qualitatively different from data.

NUCLEI AT HIGH p_T ?



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

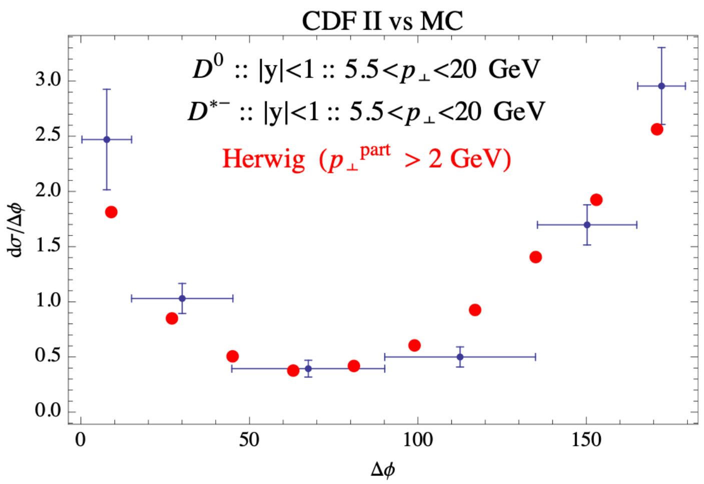


FIG. 1: The D^0D^{*-} pair cross section as function of $\Delta\phi$ at CDF Run II. The transverse momentum, $p_\perp$, and rapidity, y, ranges are indicated. Data points with error bars, are compared to the leading order event generator Herwig. The cuts on parton generation are $p_\perp^{\rm part} > 2$ GeV and $|y^{\rm part}| < 6$. We have checked that the dependency on these cuts is not significative. We find that we have to rescale the Herwig cross section values by a factor $K_{\rm Herwig} \simeq 1.8$ to best fit the data on open charm production.

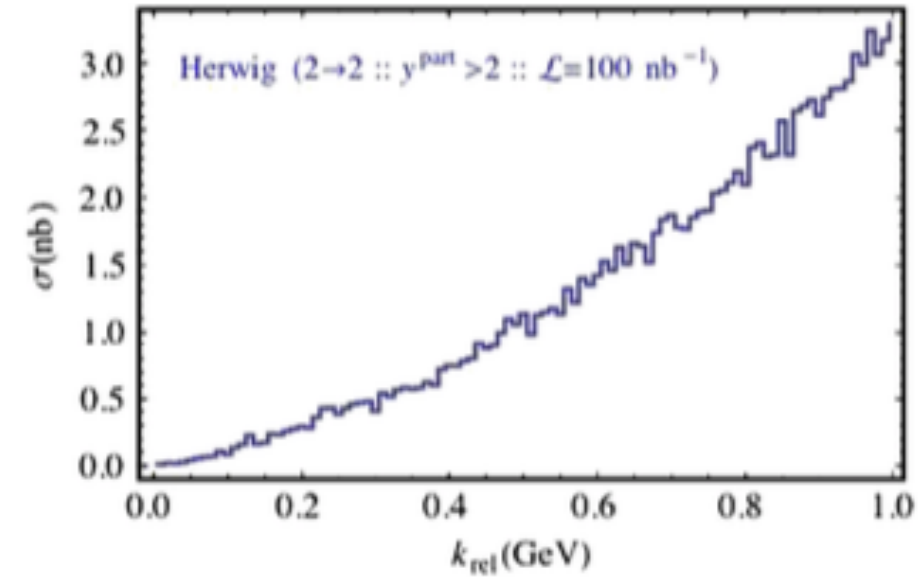
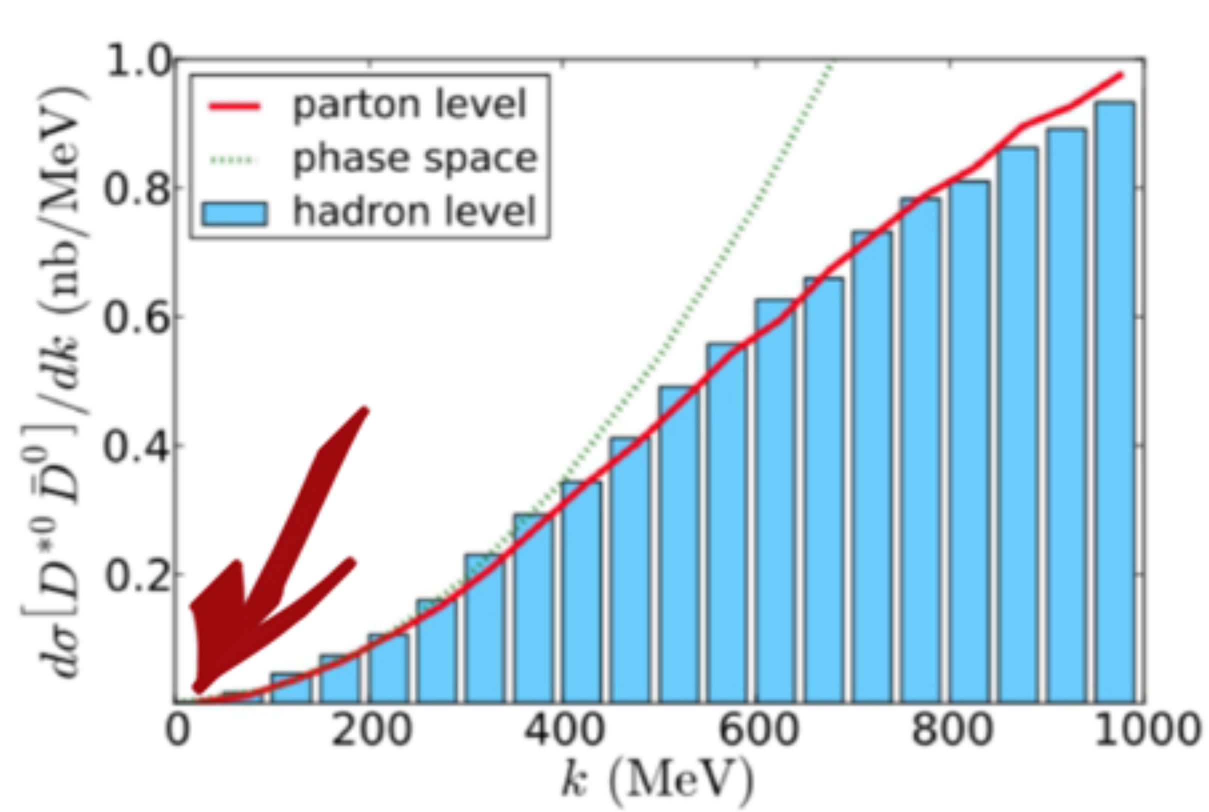


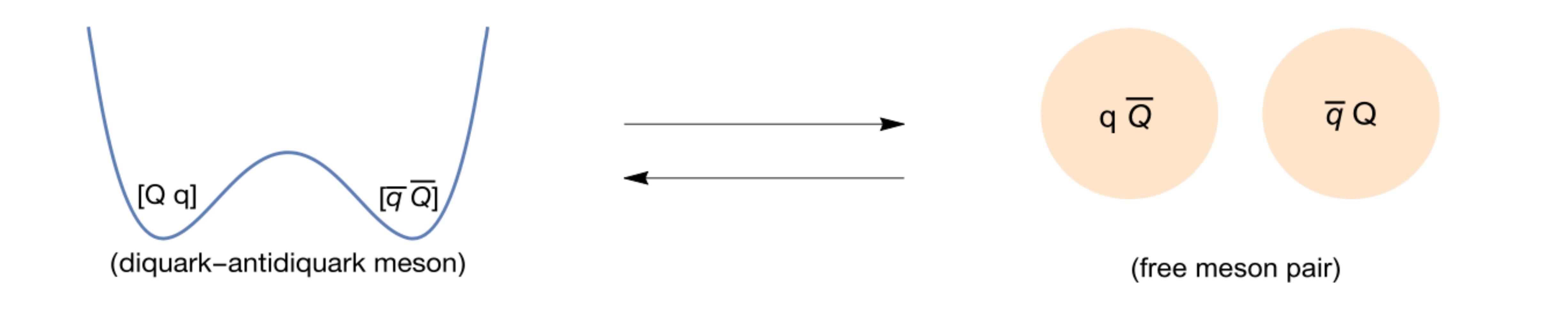
FIG. 3 (color online). The integrated cross section obtained with HERWIG as a function of the center of mass relative momentum of the mesons in the $D^0\bar{D}^{*0}$ molecule. This plot is obtained after the generation of 55×10^9 events with parton cuts $p_\perp^{\rm part} > 2$ GeV and $|y^{\rm part}| < 6$. The cuts on the final D mesons are such that the molecule produced has a $p_\perp > 5$ GeV and $|y| < 0.6$.

Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001



Braaten and Artoisenet, PRD81103 (2010) 114018

`SEGREGATED` DIQUARKS



Maiani, ADP, Riquer PLB 778 (2018) 247

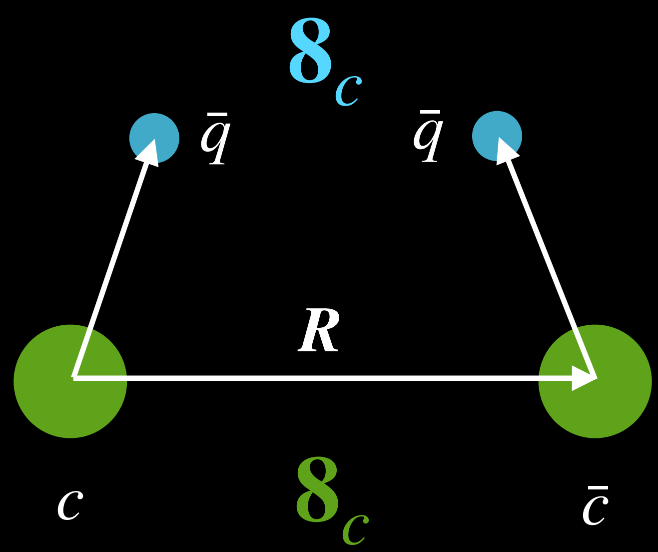
Maiani, Piccinini, ADP, Riquer PRD71 (2005) 014028

If $X^{\pm}$ is degenerate with X^0 it can't decay in $D^{\pm}\bar{D}*$ – it is forced to decay in $J/\psi\rho^{\pm}$, tunneling the heavy quark at a higher price in rate.

The $X^{\pm}$ might still be hiding in $J/\psi\rho^{\pm}$ decays.

This picture of `segregated diquarks` inspired the idea of `segregated hevay-quarks`, kept away by color repulsion in the octet.

THE BORN-OPPENHEIMER PICTURE



The fast motion of light quarks, in the field of heavy quarks (slow), generates an effective potential $\boldsymbol{V(R)}$ which in turn regulates the slower motion of heavy quarks – and can be used to calculate the spectrum.

The same picture might work for the $\mathcal{T}_{cc}$ and $\mathcal{T}_{bb}$ states, and for the pentaquarks!

Maiani, ADP, Riquer, *Phys.Rev.D* 100 (2019) 1, 014002; *Phys.Rev.D* 100 (2019) 7, 074002; EPJC83 (2023) 5, 378

Maiani, Pilloni, ADP, Riquer, PLB836 (2023) 137624 (on $\mathcal{T}_{cc}$ in B.O.)

Esposito, Papinutto, Pilloni, ADP, Tantalo, Phys Rev D88 (2013) 5, 054029 (on $\mathcal{T}_{cc}$ prediction)

THE $\mathcal{T}_{QQ}$ CASE

Assume this is the ground state

$$T = \left|(QQ)_{\bar{\mathbf{3}}}, (\bar{q}\bar{q})_{\mathbf{3}}\right\rangle_{\mathbf{1}} = \sqrt{\frac{1}{3}}\left|(\bar{q}Q)_{\mathbf{1}}, (\bar{q}Q)_{\mathbf{1}}\right\rangle_{\mathbf{1}} - \sqrt{\frac{2}{3}}\left|(\bar{q}Q)_{\mathbf{8}}, (\bar{q}Q)_{\mathbf{8}}\right\rangle_{\mathbf{1}}$$

The potential inside a single orbital is given by

$$V(r) = \frac{\lambda_{Q\bar{q}}}{r} + k_{Q\bar{q}}\, r + V_0 = -\frac{1}{3}\frac{\alpha_s}{r} + \frac{1}{4}k\, r + V_0$$

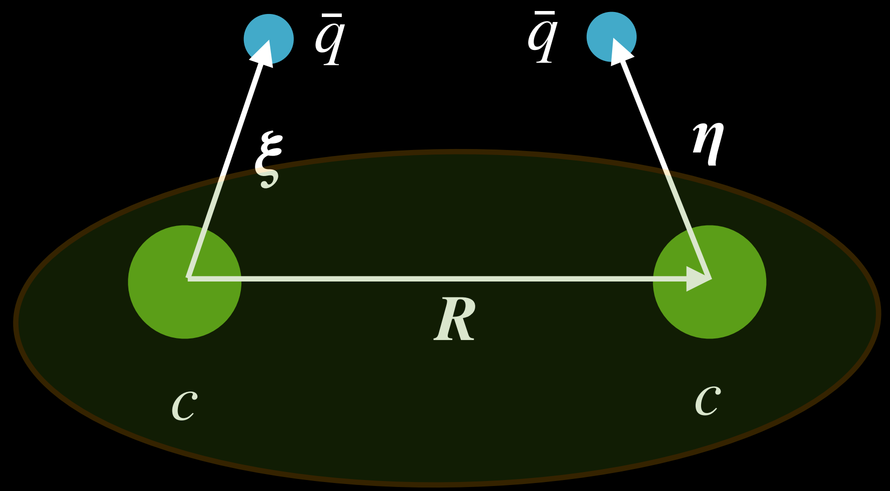
$$\lambda_{Q\bar{q}} = \left[\frac{1}{3}\times\frac{1}{2}\left(-\frac{8}{3}\right) + \frac{2}{3}\times\frac{1}{2}\left(3 - \frac{8}{3}\right)\right]\alpha_s = -\frac{1}{3}\alpha_s$$

using the diagonalization formula ($R_1 \otimes R_2 = S_1 \oplus S_2 \oplus \dots$)

$$R_1 \otimes R_2 = \bigoplus_j \frac{1}{2}(C_{S_j} - C_{R_1} - C_{R_2})\mathbf{1}_{S_j}$$

Maiani, Pilloni, ADP, Riquer, PLB836 (2023) 137624 (on $\mathcal{T}_{cc}$ in B.O.)

THE BORN-OPPENHEIMER POTENTIAL



$$\delta V = \lambda_{Q\bar{q}}\left(\frac{1}{|\boldsymbol{\xi}-\boldsymbol{R}|}+\frac{1}{|\boldsymbol{\eta}+\boldsymbol{R}|}\right)+\frac{\lambda_{q\bar{q}}}{|\boldsymbol{\xi}-\boldsymbol{R}-\boldsymbol{\eta}|}$$

$$V_{BO}(R) = -\frac{2}{3}\alpha_s\frac{1}{R} + (\Psi(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{R}), \delta V\, \Psi(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{R}))$$

$$M(\mathcal{T}^+_{cc})_{\text{th.}} = 3871\ \text{MeV} \quad M(\mathcal{T}^+_{cc})_{\text{exp.}} = 3875\ \text{MeV}$$

$$M(\mathcal{T}_{bb})_{\text{th.}} = 10552\ \text{MeV}$$

THE $\mathcal{T}_{QQ}$ GROUND STATE

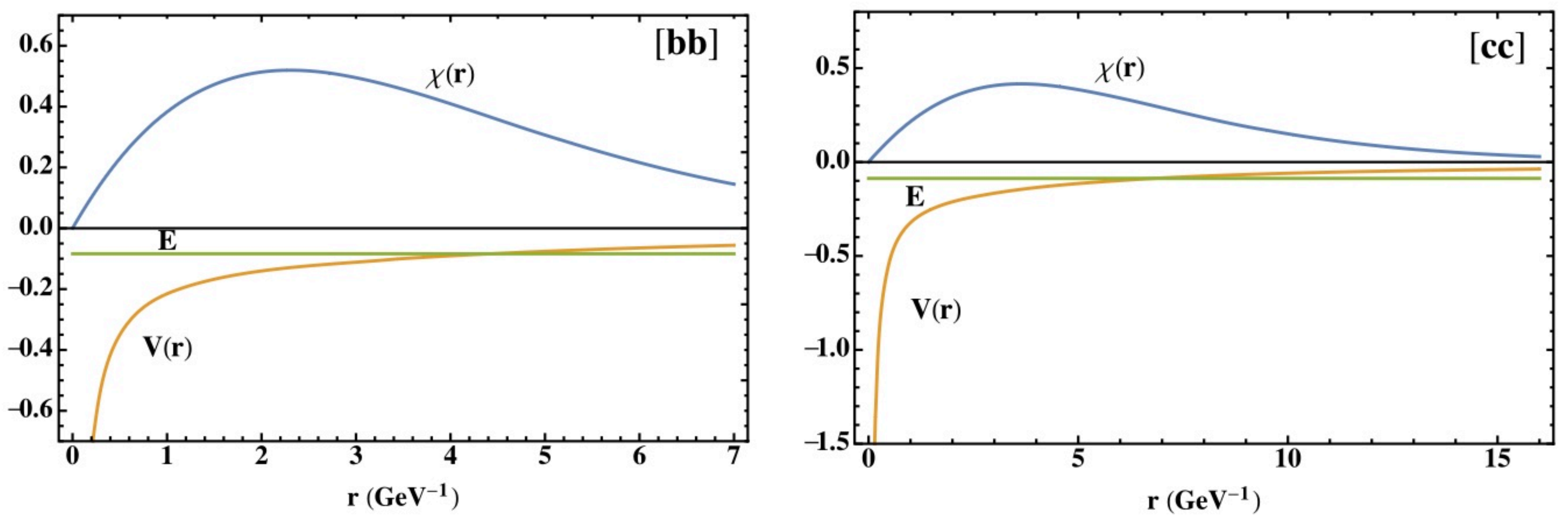


FIG. 4. Left panel: BO potential, eigenfunction, and eigenvalue $(bb)_{\bar{3}}\bar{q}\,\bar{q}$ tetraquark. Right panel: Same for $(cc)_{\bar{3}}\bar{q}\,\bar{q}$.

Maiani, ADP, Riquer, *Phys.Rev.D* 100 (2019) 1, 014002; *Phys.Rev.D* 100 (2019) 7, 074002; EPJC83 (2023) 5, 378

Rattazzi & al. observe that the $(QQ)_{\bar{3}}$ and $(QQ)_6$ mix [in preparation]. Also get two bound states, two types of tetraquarks.

THE $\mathscr{T}_{QQ}$ GROUND STATE

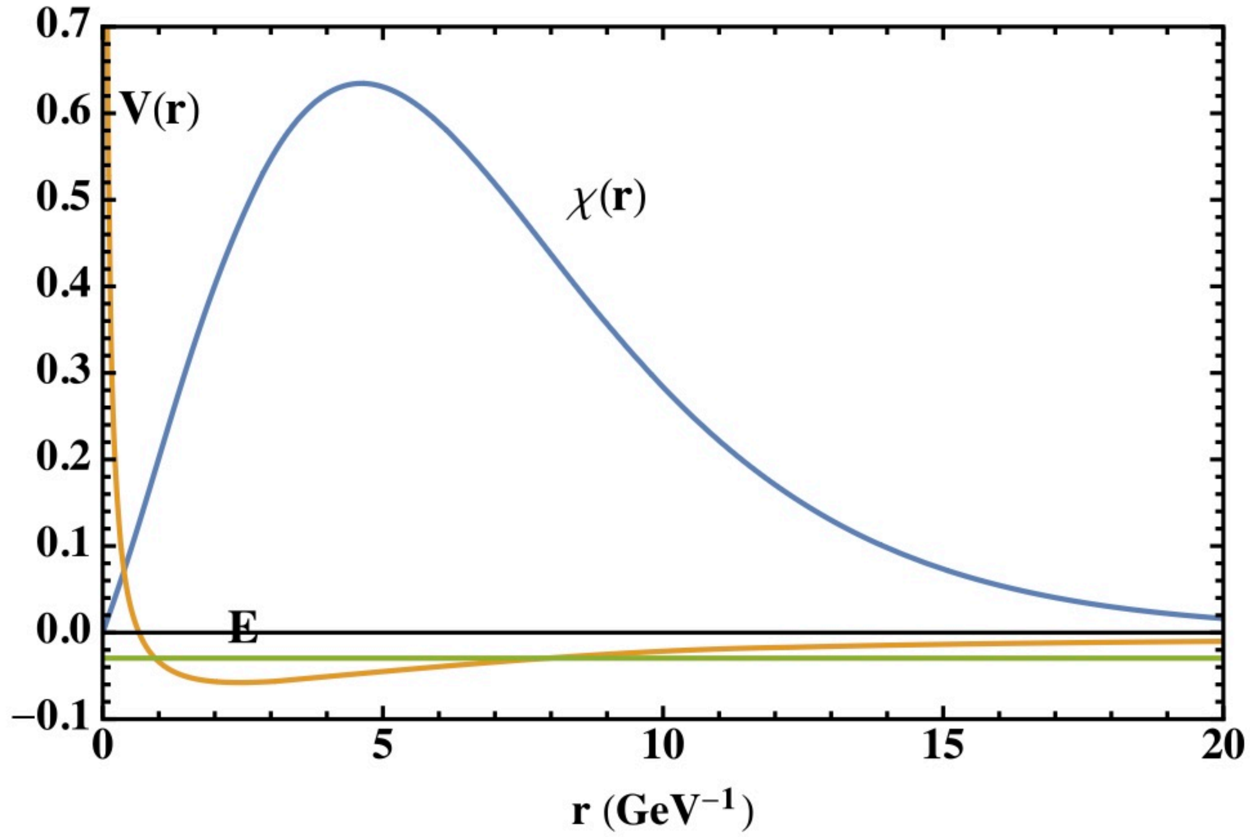
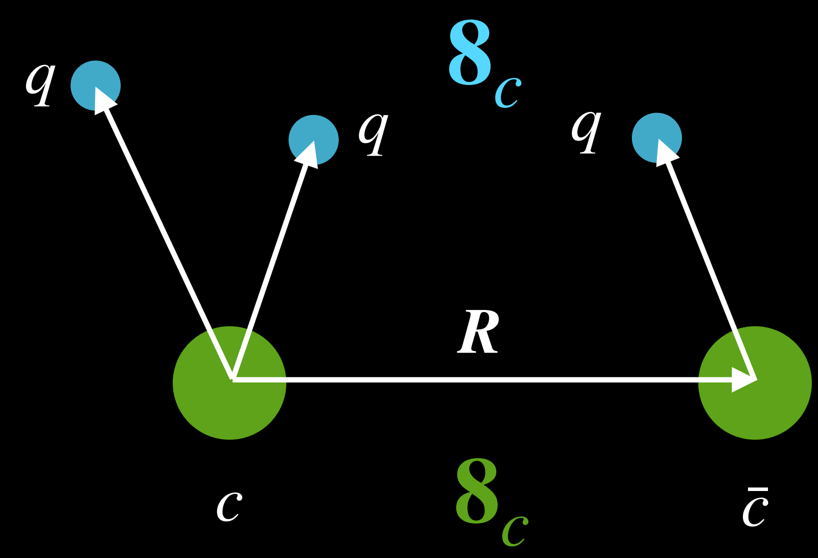


FIG. 5. A shallow bound state might be present in the color **6** channel.

Maiani, ADP, Riquer, *Phys.Rev.D* 100 (2019) 1, 014002; *Phys.Rev.D* 100 (2019) 7, 074002; EPJC83 (2023) 5, 378

Rattazzi & al. observe that the $(QQ)_{\bar{3}}$ and $(QQ)_{6}$ mix [in preparation]. Also get two bound states, two types of tetraquarks.

PENTAQUARKS AND FERMI STATISTICS



The three light quarks in the pentaquark have to be in a color-octet configuration (a mixed representation).

We show that Fermi statistics applied to the complex of the three light quarks requires three $SU(3)_f$ octets, two with spin 1/2 and one with spin 3/2. Additional lines corresponding to decays into $J/\psi + \Sigma$ and $J/\psi + \Xi$ are predicted.

Maiani, ADP, Riquer, Eur. Phys. J. C 83 (2023) 5, 378

THE EQUAL SPACING RULE

In the vector mesons octet

$$K^* \approx (\phi + \rho)/2$$

The analog of ϕ in the hidden charm tetraquarks is

$$X(1^{++}) = [cs][\bar{c}\bar{s}] \qquad X(4140) \text{ seen in } J/\psi\phi$$

To first order in SU(3) flavor symmetry breaking we might predict

$$Z_{cs} \stackrel{!}{=} (X(4140) + X(3872))/2 = 4009 \text{ MeV}$$

A Z_{cs} has been observed at 4003 MeV.

Maiani, ADP, Riquer, Sci. Bulletin 66, 1616 (2021)

Z_{cs} AND NEGATIVE CHARGE CONJUGATION

Observed by LHCb in the decay

$$B^+ \to \phi + Z_{cs}^+(4003) \to \phi + K^+ + J/\psi$$

In the diquark-antidiquark model we predict that $M(X(1^{++})) = M(Z(1^{+-}))$. Using the same spacing rules, given the $Z(3900)$ and the recently discovered $Z_{cs}(3985)$ we predict a $Z_{ss}(\simeq 4076)$

Maiani, ADP, Riquer, Sci. Bulletin 66, 1616 (2021)

CONCLUSIONS

- It would be useful to have new comparative studies on the r_0 of the X(3872) and of the $\mathcal{T}_{QQ}$ particles, and to agree on the way to extract information from data (not easy).
- It would be of great relevance to learn more, on the experimental side, about deuteron production at high p_T .
- Some states are produced promptly in pp collisions, some are not. There is no clear reason why.
- Are there loosely bound molecules $B\bar{B}*$? Can we formulate more stringient bounds on $X^{\pm}$ particles?
- Derive Weinberg criterium in a modern language.
- More basically: are we on the right questions?

BACKUP

THE EFFECTIVE RANGE EXPANSION

$$f = \frac{1}{k \cot \delta(k) - ik}$$

$$k \cot \delta = -\frac{1}{a} + \underbrace{\frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^{n+1}}_{r(\Lambda)} = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \ldots$$

In NN scattering $|1/a| \ll \Lambda$ where we assume that baryons interact through a scalar particle with mass Λ and $|r_n| \sim 1/\Lambda$.

From the lineshape of the X one finds $1/a \sim 28$ MeV $< \mu < m_\pi$.

In doing a low momentum expansion we need $ak < 1$ or $k < 1/a$, i.e. much below the cutoff μ.

Better to expand in (k/Λ) retaining ka

$$f = -\frac{1}{(1-x)\left(\frac{1}{a} + ik\right)} = -\frac{(1 + x + x^2 + \ldots)}{\left(\frac{1}{a} + ik\right)}, \qquad x = \frac{r(\Lambda)}{\left(\frac{1}{a} + ik\right)}$$

A DERIVATION OF THE DWBA FORMULA

$$f_{\text{Born}} = -\frac{m}{2\pi}\int V(r)\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} d^3r$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_{\ell=0}^{\infty} i^{\ell} j_{\ell}(kr)(2\ell+1)P_{\ell}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{r}})$$

Expand

$$e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}} = \sum_{\ell=0}^{\infty} i^{\ell} j_{\ell}(k'r)(2\ell+1)(-1)^{\ell}P_{\ell}(\hat{\boldsymbol{k}}'\cdot\hat{\boldsymbol{r}})$$

$$\int P_{\ell}(\boldsymbol{n}_1\cdot\boldsymbol{n}_2)P_{\ell'}(\boldsymbol{n}_1\cdot\boldsymbol{n}_3)d\Omega_1 = \delta_{\ell\ell'}\frac{4\pi}{(2\ell+1)}P_{\ell}(\boldsymbol{n}_2\cdot\boldsymbol{n}_3)$$

$(-1)^{\ell} i^{2\ell} = +1$ for every ℓ, and $k = k'$ for elastic collisions

A DERIVATION OF THE DWBA FORMULA

So we get

$$f = -2m \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos\theta) \int V(r)(j_\ell(kr))^2 r^2 dr$$

To be compared with Holtsmark formula

$$f = \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos\theta) \frac{e^{i\delta}\sin\delta}{k}$$

giving

$$\frac{e^{i\delta}\sin\delta}{k} = -2m \int V(r)(j_\ell(kr))^2 r^2 dr$$

A DERIVATION OF THE DWBA FORMULA

$$\chi^{(0)}(r) = 2kr\, j_\ell(kr)$$

$$\frac{e^{i\delta}\sin\delta}{k} = -\frac{2m}{4k^2}\int V(r)\,\chi^{(0)}(r)^2\,dr$$

Now consider $V = V_s + V_w$. DWBA consists in computing $T_{\beta\alpha} = \left(\Psi^-_{s\,\beta}, V_w \Psi^+_{s\,\alpha}\right)$ with the in/out states of V_s, $\Psi^\pm_{s\alpha}$. Thus

$$f = \frac{e^{i\delta_s}\sin\delta_s}{k} + \frac{e^{i\delta_w}\sin\delta_w}{k}$$

$$f_w = \frac{e^{i\delta_w}\sin\delta_w}{k} = -\frac{2m}{4k^2}\int_0^\infty V_w(r)\,\chi_s^2(r)\,dr$$

Where we substituted $\chi^{(0)} \rightarrow \chi_s$

ONE RECIPE TO COMPUTE r_0

- Use $e^{-\mu r}$ in place of $e^{i\mu r}$ and in the final expression set $\mu \to -i\mu$
- Use the regularized* $\chi_s^I(r) = 2kr\left(\frac{e^{i\delta}\sin(kr+\delta)}{kr} - \frac{e^{i\delta}\sin\delta}{kr}\right)$

  for $r \in [0,\lambda]$ and $\chi_s^{II}(r) = 2kr\left(\frac{e^{i\delta}\sin(kr+\delta)}{kr}\right)$ for $r \in [\lambda, \infty]$
- The integral is finite. Use* $\delta = \cot^{-1}\left(-\frac{1}{ka_s}\right)$
- Double-expand the result around $k = 0$ and $\alpha = 0$.
- Take the $\lambda \to 0$ limit
- Set $\mu \to -i\mu$

*R. Jackiw, `Delta Function Potentials in two- and three- dimensional quantum mechanics` in Diverse Topics in Theoretical and Mathematical Physics, World Scientific.
See also Gosdzynsky, Tarrach (https://doi.org/10.1119/1.16691) — suggested by Adam Szczepaniak.

THE SCATTERING LENGTH

The scattering length in the formula of r_0 is taken from data:
it is a *renormalized scattering length* $a = a_R$.
The renormalization is required by the UV divergences appearing in the calculation of r_0 – due to scales $r < \epsilon$ cutoff.

$$\frac{a_s}{a_R} = 1 - (2\alpha\mu\mu_r)\left[\frac{1}{a_R\mu} + \gamma_E\mu a_R + 2i + \mu a_R\left(\log(\epsilon\mu) - i\frac{\pi}{2}\right)\right]$$

where $k\cot\delta = -1/a_s$

THE VICINITY TO THRESHOLD

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+28$	$+6.7$	$+5$	$+1.8$

(MeV)

SUM RULE IN KÄLLÉN-LEHMAN

An elementary deuteron would not correspond to $Z = 1$ but to whatever $0 < Z < 1$. Strictly speaking, only the case $Z = 0$ corresponds to the exclusively composite state.

Indeed it can be shown that the following sum rule holds

$$\int_0^\infty \rho(\mu^2)\, d\mu^2 = 1$$

which corresponds to

$$Z + \int_0^\infty \sigma(\mu^2)\, d\mu^2 = 1$$

`NUCLEAR DEMOCRACY`

"A proton could be obtained from a neutron and a pion, or from a Λ and a K, or from two nucleons and one anti-nucleon, and so on. Could we therefore say that a proton consists of continuous matter? [. . .] *There is no difference in principle between elementary particles and compound systems.*"

–WERNER HEISENBER, 1975 TALK AT GERMAN PHYSICAL SOCIETY