Meson and baryon spectroscopy with charm quarks from lattice QCD

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Spectroscopy studies on the lattice

Motivation:

★ Postdiction of states that are well established experimentally.
  — Demonstration of lattice techniques.
  — (Precision) tests show systematics are under control.

★ Postdiction of states less well established experimentally.
  — Help with spin and parity assignments.
  — Whether a bound state/resonance exists.

★ Prediction of new states.
  — Expected from quark model.
  — Non-conventional, $q\bar{q}q\bar{q}$, pentaquarks, hybrids.

★ Testing theoretical descriptions.

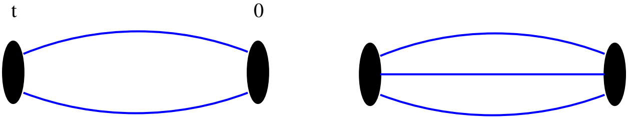
★ Investigating the internal structure of non-standard candidates.

Overview

- ★ Lattice details and challenges.
- ★ Lower lying hadron spectroscopy.
- ★ Open charm: positive parity D and D_s mesons, $QQ\bar{q}\bar{q}$ tetra-quarks, T_{cc}, and T_{bc}.
- ★ Closed charm: charmonium, $c\bar{c}qqq$ pentaquarks.
- ★ Internal structure.
- ★ Summary.

Not exhaustive!

Extracting hadron masses on the lattice



Construct two-point correlation functions using interpolators $\mathcal{O}_h$.

$$C_{2pt}(t) = \langle \mathcal{O}_h(t)\mathcal{O}_h^\dagger(0)\rangle$$
$$= \sum_n \langle\Omega|\mathcal{O}_h|n\rangle \frac{e^{-E_n t}}{2E_n V_3}\langle n|\mathcal{O}_h^\dagger|\Omega\rangle = |Z_0|^2 e^{-E_0 t} + |Z_1|^2 e^{-E_1 t} + \ldots$$

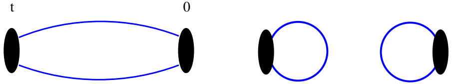
Spectral decomposition includes all states with the same quantum numbers as $\mathcal{O}_h$. **Challenges:**

- It can be difficult to isolate the states of interest, e.g. if want E_n, $n > 0$, in particular, if the spectrum is dense.
- Often use a large basis of interpolators $\mathcal{O}_h^i$ and compute $C^{ij}(t)$ and solve the generalised eigenvalue problem. Eigenvalues fall off as $\lambda^i(t) \sim e^{-E_i t}\left[1 + e^{-\Delta E\, t} + \ldots\right]$.

Extracting hadron masses on the lattice

Challenges:

- Evaluation of quark line diagrams for flavour singlet mesons, non-standard hadrons, hadrons close to or above strong decay thresholds.



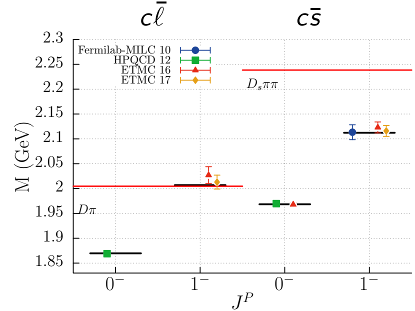
- Euclidean time $\rightarrow$ cannot directly extract properties of hadrons close to or above strong decay thresholds.
- Reduced symmetry on the lattice means it is difficult to identify continuum spin associated with E_n. Hadron at rest, $SU(2) \rightarrow {}^2O$ with PC, further reduced for a hadron in flight.
- Discretisation effects can be significant for hadrons containing charm quarks. $O(a^n m_c^n)$ with $0.2 \lesssim am_c \lesssim 0.7$ for $0.04 < a < 0.1$ fm.
- Simulating at unphysical u/d quark masses.

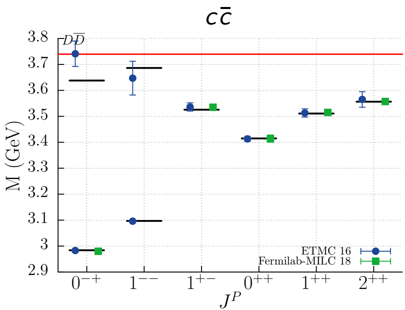
Required: $a \rightarrow 0$, $m_{u/d} \rightarrow m_{u/d}^{phys}$, $(V \rightarrow \infty)$.

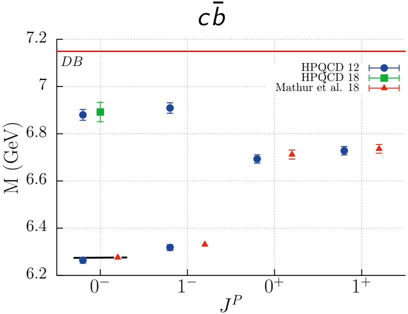
Usually simulate in (electrically neutral) isospin limit: $m_u = m_d = m_\ell$.

Only isospin conserving strong decays are relevant.

Low lying meson spectra







Systematics under control (through continuum, chiral extrapolation): general agreement with the experimental spectrum.

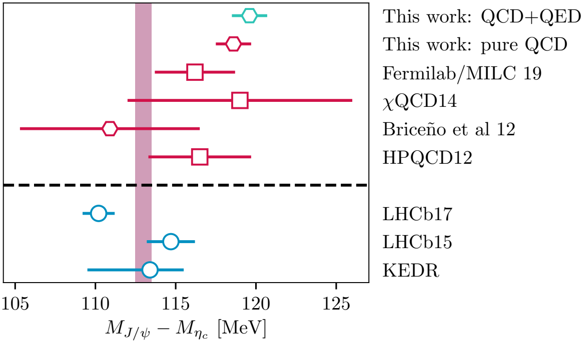
Predictions: B_c states. Recent measurements of $B_c(2S)$ **and** $B_c^*(2S)$ masses by [CMS,1902.00571]
$m_{B_c} = 6.871(2)$ GeV and $\Delta m = 29(2)$ MeV and [LHCb,1904.00081]
$m_{B_c} = 6.841(1)$ GeV and $\Delta m = 31(1)$ MeV.

Charm23: J-A. Urrea Nino, 7/18, "Towards the physical charmonium spectrum with improved distillation".

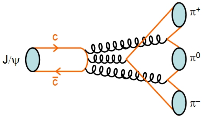
Charmonium 1S hyperfine splitting

High precision possible. Discrepancy possibly due to omission of $\bar{c}c$ annihilation diagrams.

[HPQCD,2005.01845]



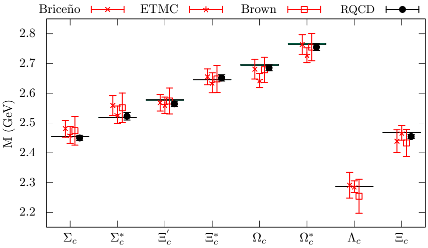
$c\bar{c}$ annihilation suppressed (OZI rule), $\Gamma_{J/\psi} \sim 93$ keV, $\Gamma_{\eta_c} = 32$ MeV. Mixing with light flavour singlet states and glueballs must be taken into account.

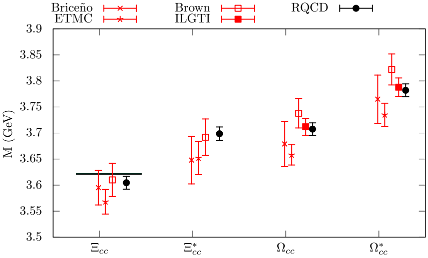


Charm23: Talk by T. Korzec 7/20 "Iso-Scalar States from Lattice QCD".

Talk by R. Höllwieser, 7/20 "Charmonium and glueballs including light hadrons".

Charmed baryons





[Briceno et al.,1207.3536], [ETMC,1406.4310], [Brown,1409.0497], [ILGTI,1211.6277], RQCD preliminary.

[BMWc,1406.4088] $\Xi_{cc}^{++} - \Xi_{cc}^{+} = 2.16(11)(17)$ MeV with -2.53(11)(06) MeV (QCD) and 4.69(10)(17) MeV (QED).

Ξ_{cc}^{++} baryon: [Liu et al.,0909.3294] $m_{\Xi_{cc}} = 3665 \pm 17 \pm 14^{+0}_{-78}$ MeV.

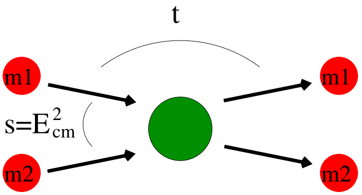
[LHCb,1707.01621] $m_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV.

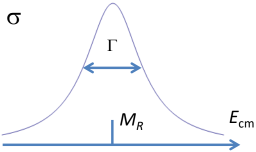
Earlier: [SELEX,hep-ex/0208014] $m_{\Xi_{cc}^{+}} = 3519 \pm 1$ MeV.

Note: Decay $\Xi_{cc}^* \to \Xi_{cc}\pi$ not possible as $\Delta M(\Xi_{cc}^* - \Xi_{cc}) < m_\pi$.

Also: predictions for ccc, $cb\ell$, ccb and cbb etc. See, e.g., [Brown et al.,1409.0497].

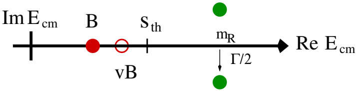
Near or above strong decay thresholds



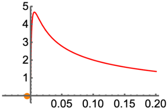


$$t_\ell(s) = \frac{1}{\rho(s)}\frac{1}{\cot\delta_\ell(s) - i} \qquad \sigma \propto |\rho t_\ell(s)|^2 = \frac{1}{\cot^2\delta + 1} \qquad t_\ell(s) = \frac{8\pi E_{cm}\Gamma}{m_R^2 - E_{cm}^2 - iE_{cm}\Gamma},$$

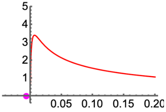
$$\rho(s) = |\vec{p}|/(8\pi\sqrt{s}) \qquad \Gamma \equiv g^2 p^{2\ell+1}/(6\pi E_{cm}^2).$$

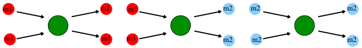


B: bound state



vB: virtual bound state





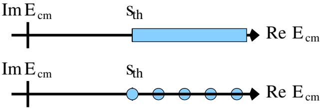
Coupled channels: t_{ℓ,M_1M_1}, t_{ℓ,M_2M_1}, t_{ℓ,M_2M_2}.

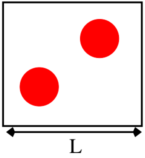
Near or above strong decay thresholds

Properties of resonances cannot be extracted directly on a (Euclidean) lattice.

Infinite volume: continuous spectrum.

Finite volume: discrete spectrum.





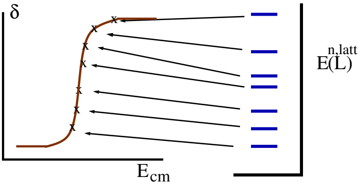
$$\det\left[\tilde{K}_\ell^{-1}(E_{cm})\delta_{\ell\ell'} - B_{\ell'\ell}^{\vec{P},\Lambda}(E_{cm})\right] = 0,$$

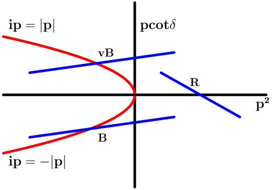
$$t_\ell^{-1} = \frac{2}{E_{cm}p^{2\ell}}\tilde{K}_\ell^{-1} - i\rho, \quad \tilde{K}_\ell^{-1} = p^{2\ell+1}\cot\delta.$$

From discrete E_n constrain t_ℓ^{-1} then analytically continue to the complex plane.

Elastic scattering:

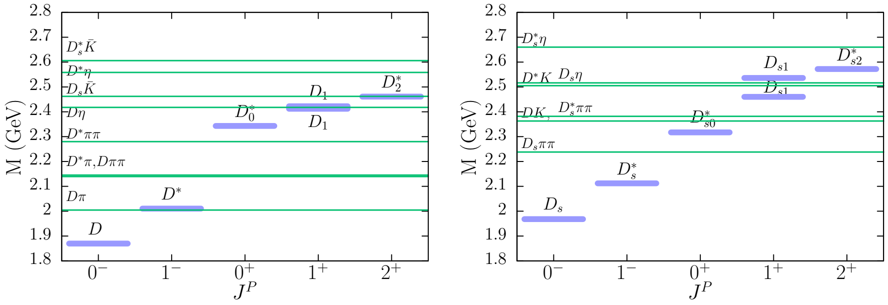
$E_n \to \delta$





Coupled channel scattering: under-constrained problem. Choose a parameterisation for $\tilde{K}^{-1}$ (t_ℓ^{-1}) and constrain the parameters using E_n. Three-body decays, see, e.g., review [Romero-Lopez,2212.13793]. Charm23 M. Hansen, 7/21, "Future Theory".

Open charm mesons: D and D_s spectrum



Only thresholds for QCD in the isospin limit shown.

Puzzles:

★ Very narrow states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ were predicted to lie much higher by early quark models, e.g. [Godfrey, Isgur,1985] and early lattice calculations, e.g. [Lewis, Woloshyn,hep-lat/0003011], [Hein et al.,hep-ph/0003130]. Similarly, for $D_0^*(2300)$.

★ Why is $M_{D_0^*}(2300) \sim M_{D_{s0}^*}(2317)$ and $M_{D_1}(2430) \sim M_{D_{s1}}(2460)$?

Narrow states close to a threshold: non-standard quark content, $c\bar{\ell}\ell\bar{q}$, $q \in \{\ell, s\}$.
Molecular states: weakly bound meson-meson $((c\bar{\ell}) - (\ell\bar{q}))$ states.
Tetraquark states: compact 4-quark $((c\ell) - (\bar{\ell}\bar{q}))$ states ...

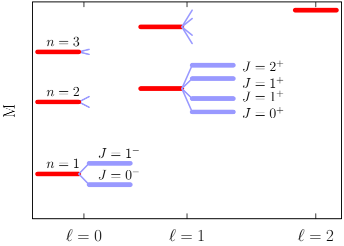
D and D_s spectrum

Heavy quark symmetry

$Q\bar{q}$ meson is hydrogen-like system, Q acts as a colour source.

Limit $m_Q \to \infty$
QNs: $j_q = \ell + s_q = \frac{1}{2}, \frac{3}{2}, \ldots,$

Finite m_Q
QNs: $J = \ell + S = 0, 1, 2, \ldots,$
$S = s_q + s_Q$, $P = -(-1)^\ell$



[Isgur, Wise,1991]:
$J^P = 0^+, 1^+$, $(m_Q \to \infty)$ $j_q = \frac{1}{2}$, decay in S-wave.
$J^P = 1^+, 2^+$, $(m_Q \to \infty)$ $j_q = \frac{3}{2}$, decay in D-wave.

SU(3) flavour symmetry:

$c\bar{q}$ $\quad \bar{\mathbf{3}}$

$c\bar{q}q\bar{q}$ $\quad \bar{\mathbf{3}} \otimes \mathbf{8} = \mathbf{15} \oplus \mathbf{6} \oplus \bar{\mathbf{3}}$

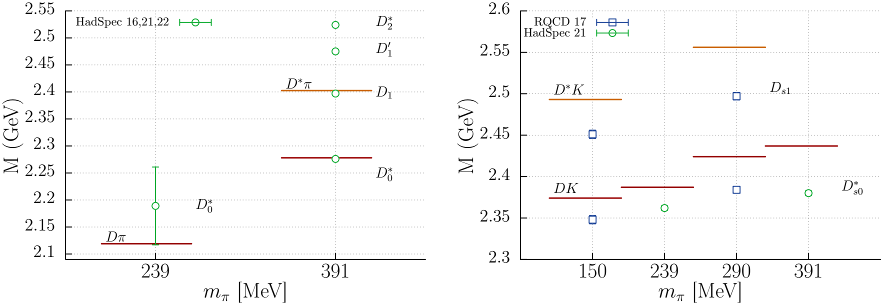
$\bar{\mathbf{3}} \otimes \mathbf{1} = \bar{\mathbf{3}}$

$q \in \{\bar{u}, \bar{d}, \bar{s}\}$

Some remnant of these symmetries expected in the observed spectrum.

D and D_s spectrum: $J^P = 0^+, 1^+, 2^+$

Finite lattice spacing, unphysical light quark masses (advantageous in terms of thresholds).



Elastic scattering:

[Mohler et al.,1308.3175,1403.8103] I=0 S-wave DK and D^*K, $m_\pi = 156$ and 266 MeV.
[RQCD,1706.01247] I=0 S-wave DK and D^*K, $m_\pi = 150$ and 290 MeV.
[HadSpec,2008.06432] S-, P- and D-wave I=0 DK and (I=0,1) $D\overline{K}$, $m_\pi = 391$, 239 MeV.

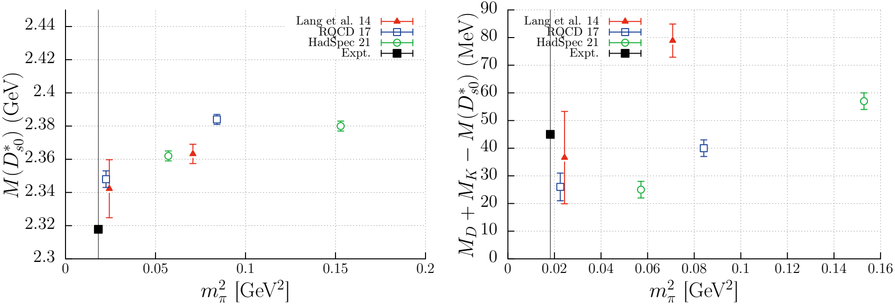
[Mohler et al.,1208.4059] I=1/2 S-wave $D\pi$ and $D^*\pi$, $m_\pi = 266$ MeV.
[HadSpec,2102.04973] I=1/2 S- and P-wave $D\pi$, $m_\pi = 239$ MeV.

Coupled channel:

[HadSpec,1607.07093] I=1/2 S-, P- and D-wave $D\pi$, $D\eta$, $D_s\bar{K}$, $m_\pi = 391$ MeV.

[HadSpec,2205.05026] I=1/2 S-, P- and D-wave $D^*\pi$, $D\pi$, $m_\pi = 391$ MeV.

$D_{s0}^*(2317)$: $J^P = 0^+$



D_{s0}^* sensitive to the light quark mass. Remains a bound state as $m_\pi \to m_\pi^{phys}$.

Some systematics not under control, e.g. discretisation effects.

Also: $J^P = 1^+$, $D_{s1}(2460)$ Expt: $m = 2.460$ GeV, $\Gamma < 3.5$ MeV.

[Mohler et al.,1403.8103] S-wave D^*K, $m_\pi = 156$ MeV, $m = 2.484(11)$ GeV.

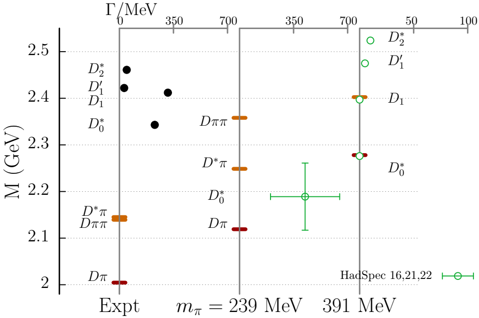
[RQCD,1706.01247] S-wave D^*K, $m_\pi = 150$ MeV, $m = 2.451(4)$ GeV.

$J^P = 2^+$, $D_{s2}(2573)$ Expt: $m = 2.569(1)$ GeV, $\Gamma = 16.9(7)$ MeV.

[HadSpec,2008.06432] D-wave DK, $m_\pi = 391$ MeV. $m = 2.583(3)$ GeV, $\Gamma = 3.4^{+1.7}_{-1.1}$ MeV.

[Mohler et al.,1403.8103] $c\bar{q}$, $m_\pi = 156$ MeV, $m = 2.596(11)$ GeV.

D spectrum: $J^P = 0^+, 1^+, 2^+$



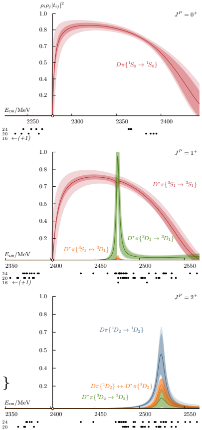
[HadSpec,1607.07093] I=1/2 S-, P- and D-wave $D\pi$, $D\eta$, $D_s\bar{K}$, $m_\pi = 391$ MeV.
[HadSpec,2205.05026] I=1/2 S-, P- and D-wave $D^*\pi$, $D\pi$, $m_\pi = 391$ MeV.
[HadSpec,2102.04973] I=1/2 S- and P-wave $D\pi$, $m_\pi = 239$ MeV.

$\rho_i \rho_j |t_{ij}|^2$

$J^P = 0^+$
$D\pi\{^1S_0 \leftrightarrow {}^1S_0\}$

$J^P = 1^+$
$D^*\pi\{^3S_1 \leftrightarrow {}^3S_1\}$
$D^*\pi\{^3S_1 \leftrightarrow {}^3D_1\}$
$D^*\pi\{^3D_1 \leftrightarrow {}^3D_1\}$

$J^P = 2^+$
$D\pi\{^1D_2 \leftrightarrow {}^1D_2\}$
$D\pi\{^1D_2\} \leftrightarrow D^*\pi\{^3D_2\}$
$D^*\pi\{^3D_2 \leftrightarrow {}^3D_2\}$



Above: $^{2S+1}\ell_J$.

★ Note the D^* is stable in these simulations.

★ D_0^* and D_1 at unphysical m_π are below experiment.

★ Note that the $D\pi\pi$ threshold opens as $m_\pi \to m_\pi^{phys}$.

★ Large coupling for D_1 to $D^*\pi$ suggests broad resonance as $m_\pi \to m_\pi^{phys}$.

★ HQS: $D\pi\{^1S_0\}$ and $D^*\pi\{^3S_1\}$ amplitudes are very similar. Decoupling of $J^P = 1^+$ states: the $D^*\pi\{^3S_1 \leftrightarrow {}^3D_1\}$ amplitude is consistent with zero and D_1' couples dominantly to $D^*\pi\{^3D_1\}$.

D and D_s spectrum

Unitarised chiral perturbation theory (UChPT) [Oller et al.,hep-ph/9803242],[Oller, Meißner,hep-ph/0011146]: near threshold states arise from the interactions between ground state charmed mesons and pseudo-Goldstone bosons. $T = 1/(V^{-1} - G)$.

Low energy constants fixed using lattice data [Liu et al.,1208.4535] for S-wave scattering lengths in I=3/2 $D\pi$, $D_s\pi$, D_sK, I=0 $D\bar{K}$ and I=1 $D\bar{K}$ channels. $a = 0.125$ fm and $m_\pi = 190 - 380$ MeV.

[Albaladejo et al.,1610.06727]: Reproduce the lattice finite volume $S = 0$, I=1/2 spectrum of [HadSpec,1607.07093], $m_\pi = 391$ MeV. Correspond to poles of $\tilde{T} = 1/(V^{-1} - \tilde{G})$.

At $m_\pi = m_\pi^{phys}$, no free parameters: [Albaladejo et al.,1610.06727], [Du et al.,1712.07957]:

$D^*_{s0}(2317)$	2315^{+18}_{-28} MeV	D^*_0	$2105^{+6}_{-8} - i102^{+10}_{-11}$ MeV	$2451^{+35}_{-26} - i134^{+7}_{-4}$ MeV
$D_{s1}(2460)$	2456^{+15}_{-21} MeV	D_1	$2247^{+6}_{-8} - i107^{+11}_{-10}$ MeV	$2555^{+47}_{-30} - i203^{+8}_{-9}$ MeV

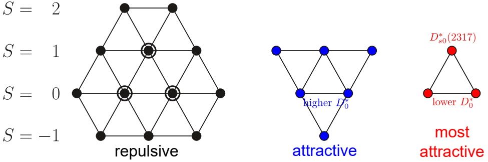
$J^P = 0^+$, I=$\frac{1}{2}$: second pole position not reliably extracted in [HadSpec,1607.07093]. [Asokan et al.,2212.07856]: SU(3) constraints can be imposed.

Experiment: analysis of LHCb data in [Du et al.,1712.07957,1903.08516,2012.04599]. See also [Du et al.,2012.04599].

D and D_s spectrum

$(c\bar{q})(q\bar{q})$

$\bar{3} \otimes 8 = 15 \oplus 6 \oplus \bar{3}$



[Guo et al.,Lattice 22]

For $J^P = 0^+$:

★ D_{s0}^* and lower D_0^* arise from the $\bar{3}$ interaction ($\rightarrow$ mass hierarchy). Higher D_0^* due to 6 interaction.

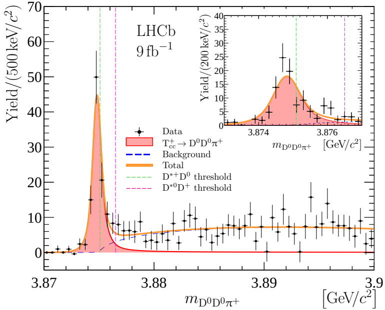
★ [Gregory et al.,2106.15391,2111.15544]: Lattice QCD simulations in SU(3) limit $m_\pi = 600 - 700$ MeV. Construct operators in 6 and 15 flavour representation.

Aim to show there is a state arising from the 6-rep and interaction in 15-rep is repulsive ($\rightarrow$ molecular picture).

$$M_{[6]} - (M_D + M_\pi) < 0, \qquad M_{[15]} - (M_D + M_\pi) > 0$$

$cc\bar{q}\bar{q}$, tetraquarks

[LHCb,2109.01038,2109.01056]



Very narrow structure T_{cc} in the $D^0D^0\pi^+$ invariant mass spectrum.

Just below the D^0D^{*+} threshold:
$M - (M_{D^0} + M_{D^{*+}}) = 0.36(4)$ MeV.

$\Gamma = 47.8$ keV, I=0 and $cc\bar{u}\bar{d}$ content suggested.

Phenomenological models: predictions with I=0, $J^P = 1^+$ within ± 100 MeV of the threshold, see e.g. [Karliner, Rosner,1707.07666], [Eichten,Quigg,1707.09575], [Janc, Rosner,hep-ph/0405208], [Carames et al.,2011].

[Padmanath and Prelovsek,2202.10110] DD^* scattering I=0 in S- and P-wave, $m_\pi = 280$ MeV.

[Chen et al.,2206.06185] DD^* scattering in S-wave I=0 and 1, $m_\pi = 350$ MeV.

[HALQCD,2302.04505] Scattering information obtained by determining the Nambu-Bethe-Salpeter wavefunction on the lattice and from this the I=0 S-wave DD^* interaction potential. $m_\pi = 146$ MeV.

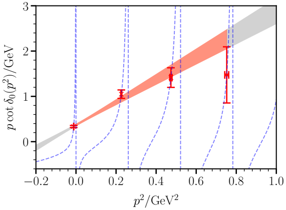
D^* is stable and energy region around DD^* is below $DD\pi$ (and D^*D^*). "Simpler" calculation.

$cc\bar{q}\bar{q}$, tetraquarks

Virtual bound state found.

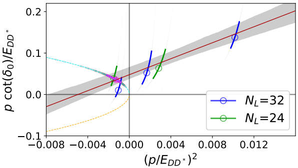
[Chen et al.,2206.06185]

$m_\pi = 350$ MeV



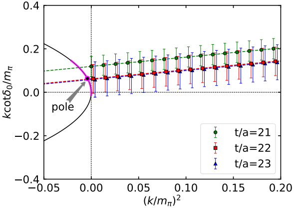
[Padmanath and Prelovsek,2202.10110]

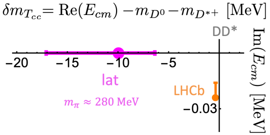
$m_\pi = 280$ MeV



[HALQCD,2302.04505]

$m_\pi = 146$ MeV.





Fit $p\cot\delta_0$ in the threshold region using the effective range expansion:

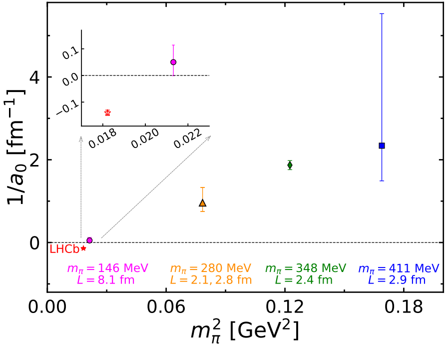
$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0p^2 + O(p^4)$$

[Du et al.,2303.09441]: effect of left hand cut needs to be investigated. Neglected so far.

Modifications to Lüscher formalism, work in progress, see, e.g. [Raposo,Hansen,2301.03981].

$cc\bar{q}\bar{q}$, tetraquarks

[HALQCD,2302.04505]



$\mathrm{Re}(1/a_0)^{expt}$ shown at $m_\pi = m_\pi^{phys} = 135$ MeV.

Simulation at m_π^{phys} challenging as $D^* \to D\pi$ and need to consider $T_{cc} \to DD\pi$ and isospin breaking.

[Chen et al.,2206.06185]: I=1 DD^* interaction is repulsive (I=0 is attractive). Consistent with DD^* interaction in a molecular picture via ρ exchange.

$QQ\bar{q}\bar{q}$, tetraquarks

★ $cc\bar{u}\bar{s}$, $bc\bar{u}\bar{d}$, $bc\bar{u}\bar{s}$ (and $bbq\bar{q}$) **states are also of interest.**

$cc\bar{u}\bar{s}$: Finite volume study of [Junnarkar et al.,1810.12285] found $E_{bind} = 8(8)$ MeV.

★ **Binding energy is found to increase with decreasing m_π and increasing m_Q:**

Finite volume lattice studies of $J = 1$ $bb\bar{u}\bar{d}$ ($E_{bind} = 100 - 150$ MeV) and
$bb\bar{u}\bar{s}$ ($E_{bind} = 70 - 100$ MeV),

see, e.g. [Junnarkar et al.,1810.12285], [Leskovec et al.,1904.04197], [Hudspith et al,2006.14294], [Meinel et al.,2205.13982],
also BB^* S-wave scattering study [Pflaumer et al.,2211.00951]

★ **Attractive interaction for DD^* and BB^*: so likely for DB^*.**

$bc\bar{q}\bar{q}$ in between $bb\bar{q}\bar{q}$ (possible compact diquark-anti-diquark tetraquark) and $cc\bar{q}\bar{q}$ (possible DD^* molecule).

Phenomenological models: both bound and unstable $bc\bar{q}\bar{q}$ states predicted, see, e.g., [Carlson et al.,1998], [Ebert et al.,0706.3853], [Karliner, Rosner,1707.07666], [Eichten,Quigg,1707.09575].

★ **Lattice studies of $bc\bar{u}\bar{d}$, $bc\bar{u}\bar{s}$:** so far finite volume studies do not show a clear picture, see, e.g., [Hudspith et al.,2006.14294], [Meinel et al.,2205.13982].

$bc\bar{q}_1\bar{q}_2$, tetraquarks, $J^P = 1^+$

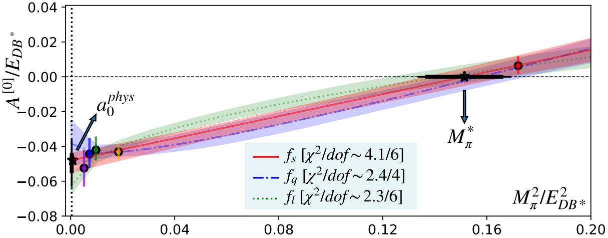
[Padmanath,Radhakrishnan,Mathur,2023]: DB^* S-wave scattering (D^*B and D^*B^* thresholds somewhat higher).

Lattice spacing ($a \sim 0.06 - 0.12$ fm) and quark mass dependence considered.

Range of $m_\pi = 0.5 - 3.0$ GeV includes $m_{q_1} = m_{q_2} \sim m_s$ and m_c.

Lowest finite volume energy level for each m_π used in the scattering analysis

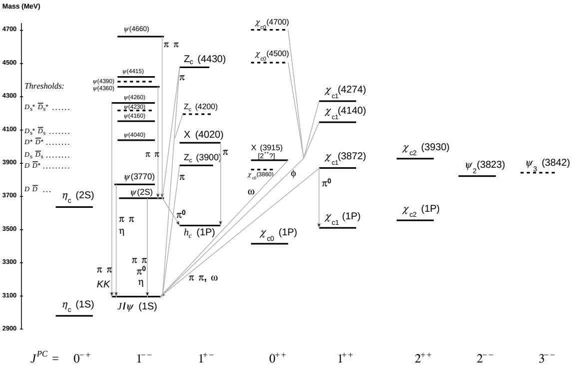
$$p \cot \delta_0 \approx 1/a_0 + O(a) = A^{[0]} + O(a).$$



$$M(T_{cb}) - (M(B^*) + M(D)) = -43(^{+6}_{-7})(^{+14}_{-24})\,\text{MeV} \qquad M_\pi^* = 2.73(21)(14)\,\text{GeV}$$

Charmonium and $c\bar{c}$ exotics

[PDG,2019]



Above the $D\overline{D}$ threshold, many states of interest including the $X(3872)$ ($\chi_{c1}(3872)$) and $Z_c^+(3900)$. Z_cs are no C eigenstates.

Challenges: Dense spectrum of states: a number of states with same/different J^{PC} in a narrow energy region. Multiple two-particle and three-particle decay channels can be open.

X(3872), $J^{PC} = 1^{++}$, I=0, no recent work (see [Padmanath et al.,1503.03257], elastic DD^* scattering, find a shallow bound state).
$Z_c^+(3900)$, $J^P = 1^+$, I=1, no evidence as yet via Lüschers method, see, e.g. [CLQCD,1907.03371].
[HALQCD,1602.03465] coupling between $D\overline{D}^*$ and $J/\psi\omega$ channels is responsible for the Z_c.

Charmonium

[Piemonte et al.,1905.03506] $J^{PC}=1^{--},3^{--}$ elastic $D\overline{D}$ scattering with $\ell=1,3$. Conventional states: 1^{--} channel, bound state $\psi(2S)$ below the $D\overline{D}$ threshold, $\psi(3770)$ resonance slightly above. $\psi(3770)$: g consistent with expt., $\Gamma=g^2p^3/(6\pi s)$. $m_{3^{--}}$ compatible with $X(3842)$ [LHCb,1903.12240]. BR($D\overline{D}$)$\sim$ 93%, $J/\psi\eta$ and 3 body decays ignored.

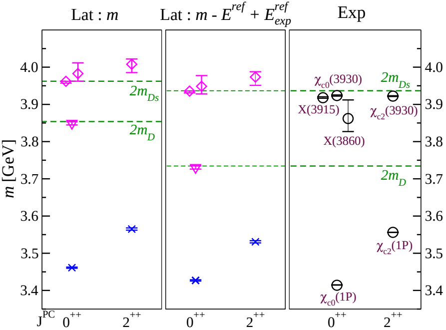
[Prelovsek et al.,2011.02542] Coupled channel $D\bar{D}$ and $D_s\bar{D}_s$ S- and D-wave scattering.
$J/\psi\omega$ and $\eta_c\eta$ channels ignored, $m_c>m_c^{phys}$.

$J^{PC}=0^{++}$: ★ State just below $D\overline{D}$ threshold ($m_\pi>m_\pi^{phys}$!), not yet observed in expt.

★ Narrow resonance just below $D_s\overline{D}_s$ which may be related to $X(3915)/\chi_{c0}(3930)$.

★ Broad resonance which may be related to $X(3860)$.

★ $J^{PC}=2^{++}$ similar to $\chi_{c2}(3930)$.



Expt: $J^{PC}=0^{++}$: X(3860) [Belle,1704.01872], the $\chi_{c0}(3930)$ [LHCb,2009.00025] and the X(3915) [BaBar,0711.2047], [Belle,0912.4451] below the $D_s\bar{D}_s$ threshold. Also the $X(3960)$ [LHCb,2210.15153].

Theory: additional shallow bound state suggested in [Gammermann et al.,hep-ph/0612179]. Partner to $X(3872)$ suggested in [Hildago Duque et al.,1305.4487], [Baru et al.,1605.09649]. See also [Danilkin et al.,2111.15033] and [Guo et al.,2212.00631].

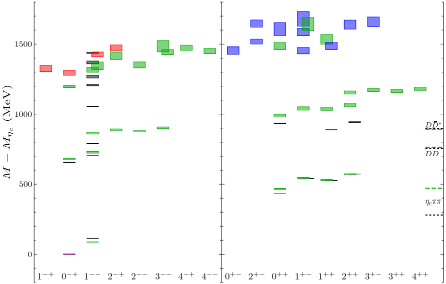
Charmonium hybrids

Charmonium states with an excited gluonic component.

Studies so far treat hybrids as stable.

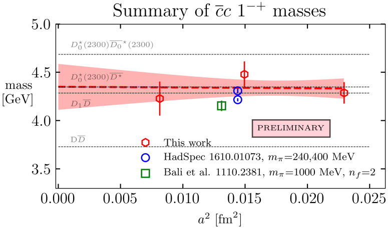
Non-quark model J^{PC}, e.g. 1^{-+}.

[HadSpec,1610.01073]

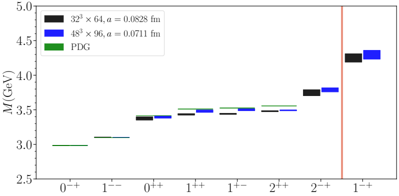


See also [HadSpec,1204.5425] and [χQCD,1202.2205].

[Ray and McNeile,2110.14101]



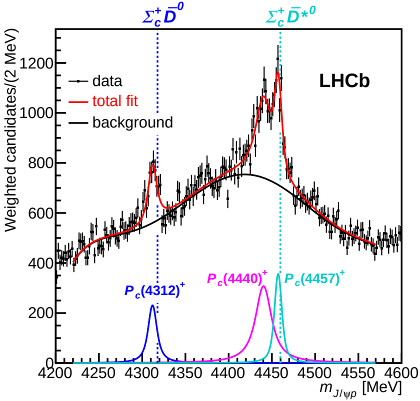
[Sun et al.,2012.06228]



Pentaquarks

A number of hidden charm penta-quarks have been discovered by [LHCb,1507.03414,1904.03947] in the $J/\psi p$ channel. $P_c^+(4380)$ and $P_c^+(4450) \to P_c^+(4312)$, $P_c^+(4440)$ and $P_c^+(4457)$.

Three narrow states close to $\Sigma_c^+\overline{D}^0$ and $\Sigma_c^+\overline{D}^{0*}$ thresholds.



Molecular picture: $P_c^+(4312)$ is a $J^P = \frac{1}{2}^+$ $\Sigma_c\overline{D}$ bound state.

$P_c^+(4440)$ and $P_c^+(4457)$ are $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^+$ (or visa versa) $\Sigma_c\overline{D}^*$ bound states.

Compact pentaquark states and hadrocharmonia interpretations also considered.

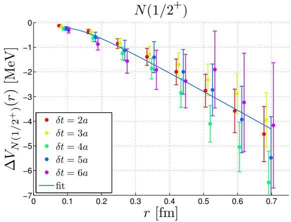
Pentaquarks $c\bar{c}qqq$

Other pentaquark states with different flavour content expected.

[Alberti et al.,1608.06537]: Investigated the hadro-quarkonium model [Dubynskiy and Voloshin,0803.2224] in the static approximation.

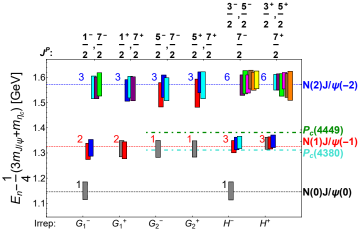
Study the modification of the static potential in the presence of a variety of light mesons as well as of octet and decuplet baryons at $m_\pi = 223$ MeV.

Binding energies of a few MeV are seen.



Lattice study complicated due to the number of possible meson (M) and baryon (B) decay channels and the spins of the Ms and Bs.

First scattering study: [Skerbis and Prelovsek,1811.02285]



NJ/ψ and $N\eta_c$ scattering
$m_\pi = 266$ MeV.

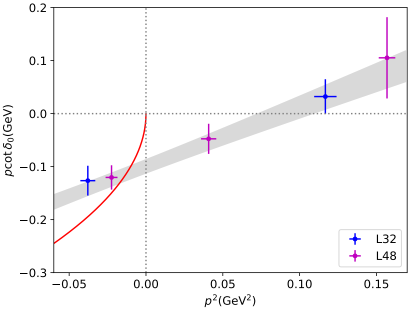
No significant energy shifts with respect to the non-interacting charmonium-nucleon energies.

No indication of a resonance or bound state.

Pentaquarks $c\bar{c}qqq$

[Xing et al.,2210.08555]: S-wave scattering of $\Sigma_c\overline{D}$ and $\Sigma_c\overline{D}^*$ with $J^P = \frac{1}{2}^-$, $m_\pi = 294$ MeV.

Bound states poles in both channels.



$\Sigma_c\overline{D}$ channel:

$M_{P_c} - (M_{\Sigma_c} + M_D) = 6(2)(2)$ MeV

($P_c(4312)$, $\Delta M \sim 9$ MeV).

$\Sigma_c\overline{D}^*$ channel:

$M_{P_c} - (M_{\Sigma_c} + M_{D^*}) = 6(2)(2)$ MeV

($P_c(4440)/P_c(4457)$).

Challenge: need to also consider $J/\psi N$, $\eta_c N$, $\Lambda_c\overline{D}$, $\Lambda_c\overline{D}^*$.

Internal structure

★ In addition to the mass spectrum, information on the internal structure of hadrons can be extracted on the lattice.

★ Compute decay constants $\langle \Omega | J(0) | X \rangle$.

★ Compute matrix elements $\langle X'(p') | J(0) | X(p) \rangle \rightarrow$ e.g. form factors ($X = X'$, define radii) and transition form factors.

★ Compare results for conventional and exotic states and to model predictions.

★ As a first step, for hadrons with charm, hadron treated as stable.

For resonances for $0 \rightarrow 2$, $1 \rightarrow 2$, $2 \rightarrow 2$, see, e.g., [Bernard et al.,1205.4642], [Briceño, Hansen,1509.08507], [Baroni et al.,1812.10504] and [Lozano et al.,2205.11316].

Internal structure

[RQCD,1706.01247]: D_s **decay constants**

$$J^P = 0^+ \qquad \textbf{Vector} \qquad \langle \Omega | \bar{s}\gamma_\mu c | D^*_{s0}(\boldsymbol{p}) \rangle = \mathbf{f_V^{0^+}}\, p_\mu$$

$$J^P = 1^+ \qquad \textbf{Axial-vector} \qquad \langle \Omega | \bar{s}\gamma_\nu\gamma_5 c | D_{s1}(\boldsymbol{p}, \epsilon) \rangle = \mathbf{f_A^{1^+}}\, m_{D_{s1}} \epsilon_\nu$$

$$f_V^{0^+} = 114(2)(0)(+5)(10)\ \text{MeV}, \qquad f_A^{1^+} = 194(3)(4)(+5)(10)\ \text{MeV}$$

(analogous to the pseudoscalar leptonic decay constant f_{D_s}, $\Gamma(D_s \to \ell\bar{\nu}) \propto f_{D_s}^2 |V_{cs}|^2$.)

Heavy quark $m_Q \to \infty$ limit: (D_s, D_s^*), (D^*_{s0}, D_{s1}) form degenerate pairs.

$$m_c = m_c^{\text{ph}} < \infty: \qquad f_{D_s^*}/f_{D_s} = 1.10 - 1.26, \qquad f_{D_{s1}}/f_{D^*_{s0}} \sim 1.7,$$

Using [FLAG,2111.09849] for f_{D_s} and [Becirevic,1201.4039], [ETMC,1610.09671], [HPQCD,1312.5264] for $f_{D_s^*}$.

Nature of states: $P = +$ decay constants suppressed relative to $P = -$.

$$f_{D^*_{s0}}/f_{D_s} \approx 0.45, \qquad f_{D_{s1}}/f_{D_s^*} \approx 0.6 - 0.7$$

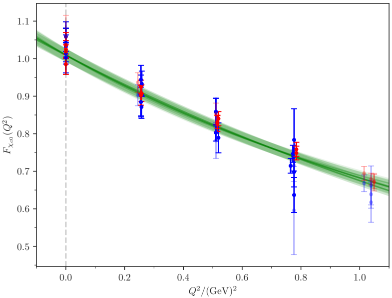
States are spatially more extended (in a non-relativistic $\bar{q}q$ picture $f \propto |\psi(0)|$)!

However, conventional mesons in the charmonium sector: roughly:
$\Gamma(\bar{c}c \to \gamma\gamma) \propto f_{\bar{c}c}^2/m_{\bar{c}c}$. From the expt. results: $f_{\chi_{c0}}/f_{\eta_c} = f_{0^{++}}/f_{0^{-+}} \sim 0.7$.

Internal structure

[HadSpec,2301.08213]: electromagnetic (transition) form factors of η_c, J/ψ, χ_{c0} and η_c'.

$$\langle h'_{J'}(\lambda', \vec{p}\,')|j^\mu|h_J(\lambda, \vec{p}\,)\rangle, \qquad \text{e.g. } \langle \chi_{c0}(\vec{p}\,')|j^\mu|\chi_{c0}(\vec{p}\,)\rangle = (p+p')^\mu F(Q^2).$$



Electromagnetic current j_μ.

Charge radius:

$$\langle r^2 \rangle = -6 \left. \frac{dF(Q^2)}{dQ^2} \right|_{Q^2=0}$$

$$\left\langle r^2_{\eta_c} \right\rangle^{\frac{1}{2}} < \left\langle r^2_{\chi_{c0}} \right\rangle^{\frac{1}{2}} < \left\langle r^2_{\eta_c'} \right\rangle^{\frac{1}{2}}$$

First step to computing radiative transitions involving exotic and excited charmonia.

See also e.g. [Dudek et al.,0902.2241], [CLQCD,1206.2086], [Becirevic et al.,1411.6426], [Schultz et al.,1501.07457], ...

Charm23: Talk by B. Colquhoun 7/18, "Precise determination of the decay rates of $\eta_c \to \gamma\gamma$, $J/\psi \to \gamma\eta_c$ and $J/\psi \to \eta_c e^+ e^-$ from lattice QCD"

Internal structure

[Sun et al.,2012.06228]: 1S, 1P and 1D charmonium states and exotic 1^{-+} (hybrid $c\bar{c}g$ or 4-quark $c\bar{c}q\bar{q}$ state).

Left: charm quark mass momentum fraction $\langle x \rangle_c$, where

$$1 = \langle x \rangle_{q=c} + \sum_{q \in \{u,d,s,\ldots\}} \langle x \rangle_q + \langle x \rangle_g$$

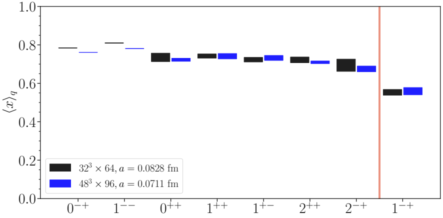
Right: mass contribution (determined from the charm quark sigma terms, $\langle h_J | c \mathbb{1} \bar{c} | h_J \rangle$).

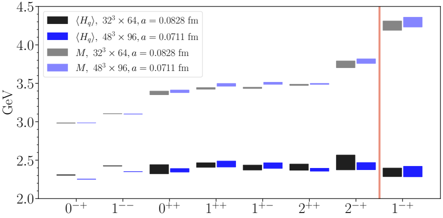
$$M \approx \langle H_q \rangle + \langle H_g \rangle$$

Valence charm quark contribution to the mass, $\langle H_q \rangle$, gluon contribution $\langle H_g \rangle \approx M - \langle H_q \rangle$.

Left: $\mu = 2$ GeV

Comparison with conventional state of similar mass?





Summary and outlook

Lattice studies can provide valuable information about mesons and baryons containing charm quarks. $c\bar{q}$, $c\bar{c}$, cqq, ccq, $c\bar{c}q\bar{q}$, $cc\bar{q}\bar{q}$, $c\bar{c}qqq$, $c\bar{c}g$, ... hadrons are being actively studied.

Lower lying hadrons (stable under strong decay):

- ★ Results with all systematics under control (discretisation effects, unphysical quark mass). Predictions in agreement with experiment.

Studies of near threshold states and resonances are challenging.

Results "consistent" with experiment or theory expectations.

- ★ D_s, $J^P = 0^+, 1^+, 2^+$. D, $J^P = 0^+, 1^+, 2^+$ (amplitudes consistent with the expectations of heavy quark symmetry).
- ★ T_{cc}: several studies find a virtual bound state. Scattering length increases ($1/a_0 \searrow$) as $m_\pi \to m_\pi^{phys}$.
- ★ T_{bc}: bound state found from first scattering study.
- ★ P_c^+: bound states found below $\Sigma_c \overline{D}$ and $\Sigma_c \overline{D}^*$ thresholds.

Some puzzles ($m_\pi > m_\pi^{phys}$!): the masses of the D_0^* and D_1 are below experiment for $m_\pi > m_\pi^{phys}$.
An extra state is found below $D\overline{D}$ threshold in the $J^{PC} = 0^{++}$ channel in charmonium.
Future: internal structure of states will be probed through the evaluation of matrix elements.