CHARM HADRON LIFETIMES AND D^0-$\bar{D}^0$ MIXING

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LIFETIMES OF CHARMED HADRONS

WEAKLY DECAYING CHARMED HADRONS

c-MESONS

$$D^+(\bar{c}d)$$
$$D^0(\bar{c}u)$$
$$D_s^0(\bar{c}s)$$

Guberina, Nussinov, Peccei, Rückl, PLB 89 (1979) 111
Bilić, Guberina, Trampetić, NPB 248 (1984) 261
Khoze, Shifman, Sov. Phys. Usp. 26 (1983) 387
Shifman, Voloshin, Sov. J. Nucl. Phys. 41 (1985) 120

King, Lenz, Piscopo, Rauh, Rusov, Vlahos, 2109.13219

c-BARYONS

$$\Lambda_c^+(cud)$$
$$\Xi_c^+(cus)$$
$$\Xi_c^0(cds)$$

$$\Omega_c^0(css)$$

Guberina, Rückl, Trampetić, Z. Phys. C 33 (1986) 297
Shifman, Voloshin, Sov. Phys. JETP 64 (1986) 698
Guberina, Melic, 9704445
H-Y Cheng, 9704260
H-Y Cheng, 1807.00916
H-Y Cheng, C-W Liu, 2305.00665

Gratrex, Melic, Nisandzic, 2204.11935

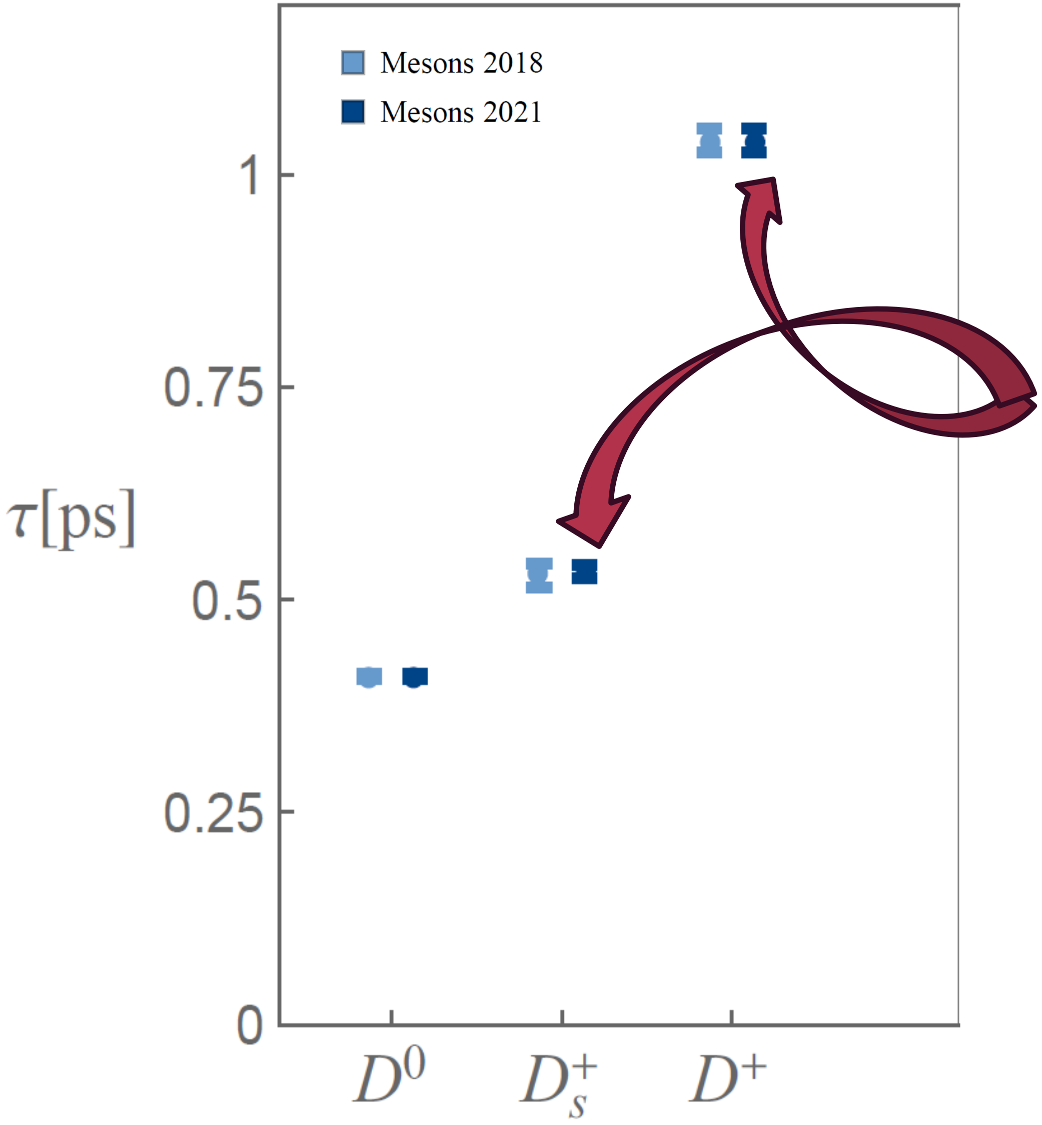
cc-BARYONS

$$\Xi_{cc}^{++}(ccs)$$
$$\Xi_{cc}^+(ccd)$$
$$\Omega_{cc}^+(ccs)$$

Kiselev, Likhoded, Onishchenko, 9807354
Guberina, Melic, Stefancic, 9901323
H-Y Cheng, Y-L Shi, 1809.08102

Dulibic, Gratrex, Melic, Nisandzic, 2305.02243

EXPERIMENTAL SITUATION – CHARMED MESONS



practically unchanged lifetime pattern since 1980‘s

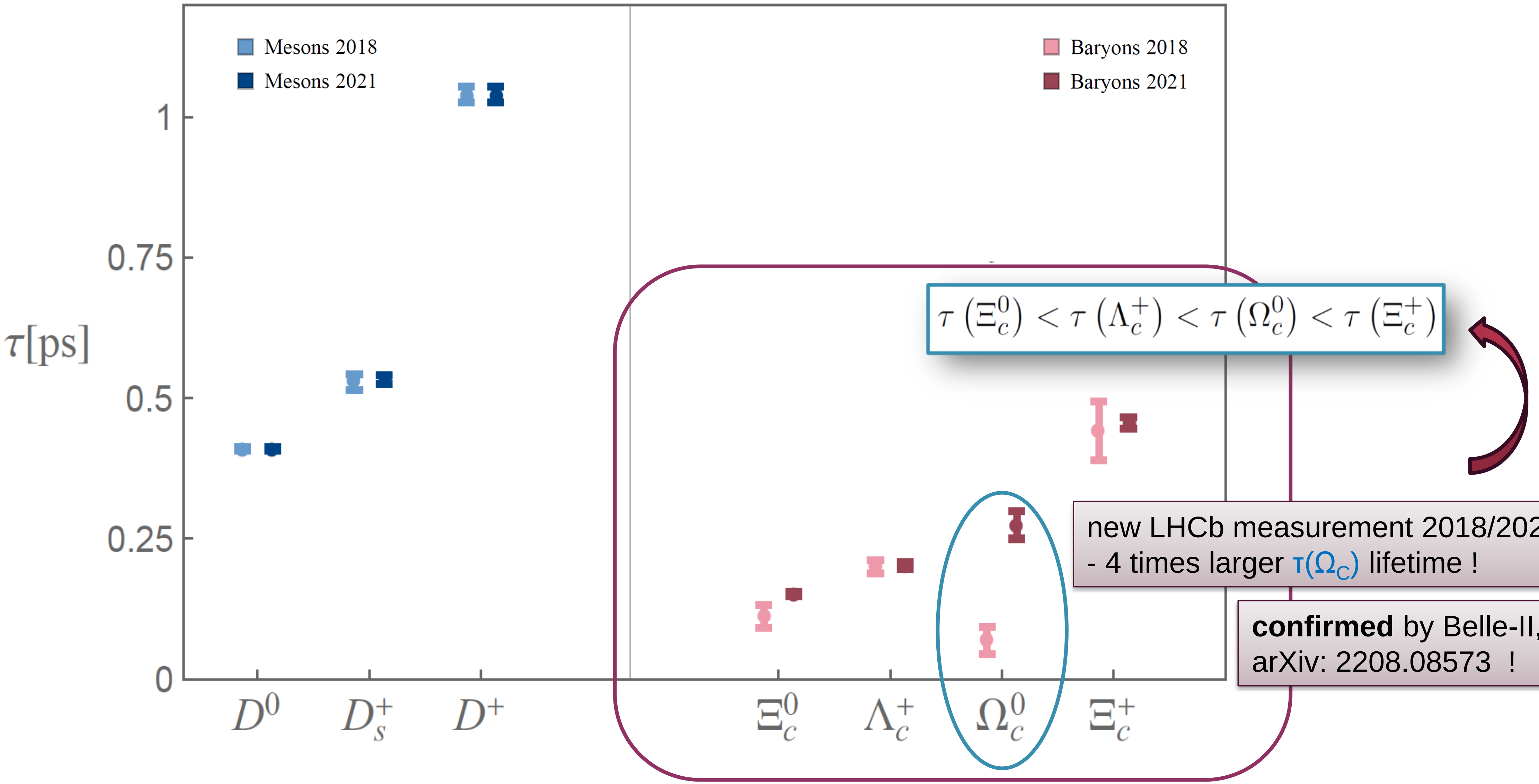
broad spread of lifetimes of singly CHARMED MESONS

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.54 \pm 0.02$$

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.23 \pm 0.01$$

EXPERIMENTAL SITUATION – CHARMED BARYONS

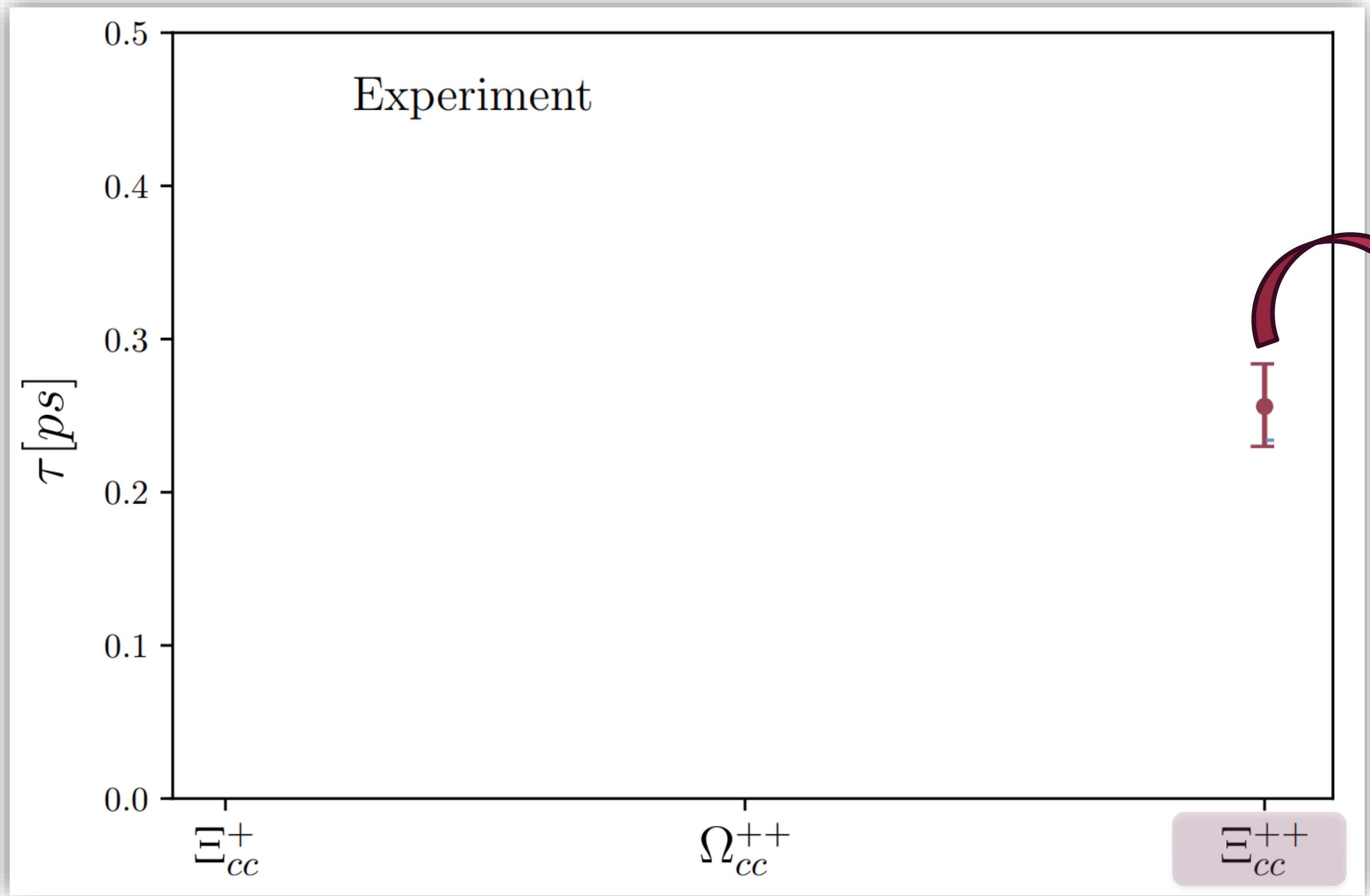
large spread among lifetimes of singly charmed hadrons:



new LHCb measurement 2018/2020
- 4 times larger $\tau(\Omega_C)$ lifetime !

confirmed by Belle-II,
arXiv: 2208.08573 !

EXPERIMENTAL SITUATION – DOUBLY CHARMED BARYONS



Naively:

$$\tau(H_{cc}) = \frac{1}{2}\tau(H_c)$$

But it is somewhat larger, since

$$\tau^{\exp}(H_c) = 0.15 - 0.45\ ps$$

THEORY : TOTAL DECAY WIDTH ➡ LIFETIMES

$$\frac{1}{\tau(H)} = \Gamma(H) = \frac{1}{2m_H}\langle H|\mathcal{T}|H\rangle$$

Shifman, Voloshin 85

$$\mathcal{T} = \mathrm{Im}\, i \int d^4x\, T\left[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right]$$

forward-scattering amplitude

$\mathcal{H}_{eff}$ = weak effective hamiltonian for a heavy Q decay

Buchalla, Buras, Lauternbacher 96

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\Big[\sum_{q,q'=d,s} V_{cq}V^*_{uq'}\left(C_1(\mu)Q_1^{(qq')} + C_2(\mu)Q_2^{(qq')}\right) - V_{ub}V^*_{cb}\sum_{k=3}^{6} C_k(\mu)Q_k$$

$$+ \sum_{\substack{q=d,s\\ \ell=e,\mu}} V_{cq}Q^{(q\ell)}\Big],$$

neglected for charm decays

non-leptonic(NL) and semileptonic (SL) decays included

WEAK HAMILTONIAN DIM6 and DIM7 OPERATORS

Dim 6 operators:

$$Q_1^{(qq')} = (\bar{c}^i \gamma_\mu (1-\gamma_5) q^j)(\bar{q}'^j \gamma^\mu (1-\gamma_5) u^i)\,,$$

$$Q_2^{(qq')} = (\bar{c}^i \gamma_\mu (1-\gamma_5) q^i)(\bar{q}'^j \gamma^\mu (1-\gamma_5) u^j)\,,$$

$$Q_{\mathrm{SL}}^{(q\ell)} = (\bar{c}\gamma_\mu (1-\gamma_5) q)(\bar{\ell}\gamma^\mu (1-\gamma_5)\nu_\ell)\,,$$

+ color-octet operators

+ μ-running and mixing

Dim 7 operators:

$$P_1^q = m_q (\bar{c}_i (1-\gamma_5) q_i)(\bar{q}_j (1-\gamma_5) c_j)\,,$$

$$P_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) D^\rho q_i)(\bar{q}_j \gamma^\mu (1-\gamma_5) c_j)\,,$$

$$P_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho q_i)(\bar{q}_j (1+\gamma_5) c_j)\,,$$

$$S_1^q = m_q (\bar{c}_i (1-\gamma_5) t^a_{ij} q_j)(\bar{q}_k (1-\gamma_5) t^a_{kl} c_l)\,,$$

$$S_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) t^a_{ij} D^\rho q_j)(\bar{q}_k \gamma^\mu (1-\gamma_5) t^a_{kl} c_l)\,,$$

$$S_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1-\gamma_5) t^a_{ij} D^\rho q_j)(\bar{q}_k (1+\gamma_5) t^a_{kl} c_l)\,.$$

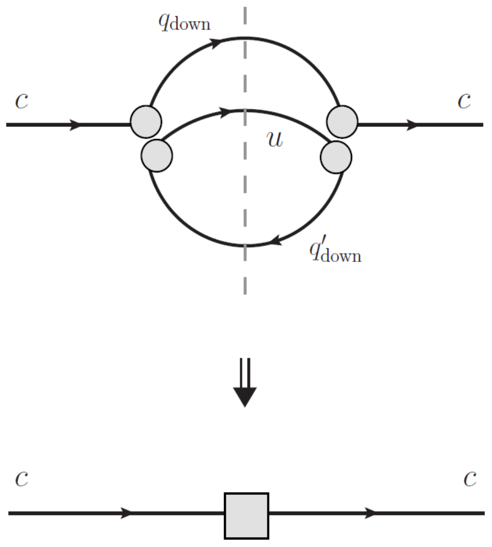
+ color-octet operators

+ non-local operators - *reabsorbed into* dim6 matrix elements (proven for mesons)

HEAVY QUARK EXPANSION (HQE) – systematic expansion in Λ_{QCD}/m_Q and α_s

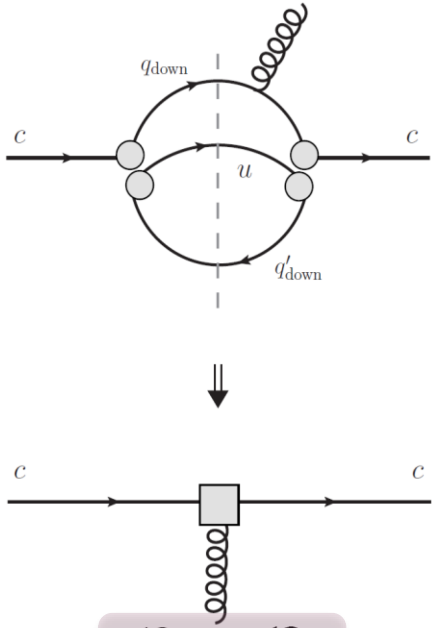
$$\mathcal{T} = \left(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2} \mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

LEADING NON-SPECTATOR CONTRIBUTION



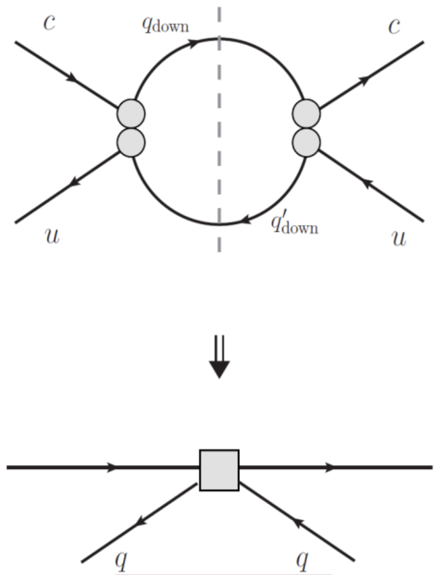
$\mathcal{O}_3$

NON-LEADING NON-SPECTATOR CONTRIBUTION **- from 90-ies; HQE**



$\mathcal{O}_5$, $\mathcal{O}_6$ etc

FOUR-QUARK SPECTATOR CONTRIBUTIONS **- ENHANCED**



$\tilde{\mathcal{O}}_6$, $\tilde{\mathcal{O}}_7$ etc

$$\mathcal{T} = \left(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2} \mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3} \mathcal{O}_6 + \ldots \right) + 16\pi^2 \left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \ldots \right)$$

WILSON COEFF. $\mathcal{C}_i = \mathcal{C}_i^{(0)}(\mu, \mu_0) + \mathcal{C}_i^{(1)}(\mu, \mu_0)\alpha_s(\mu) + \mathcal{C}_i^{(2)}(\mu, \mu_0)\alpha_s(\mu)^2 + \ldots,$

MATRIX ELEMENTS OF VARIOUS $\mathcal{O}$ OPERATORS ARE NEEDED

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \cdots + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} + \sum_i \frac{c_{7,i}^q \langle H | P_i^q | H \rangle}{m_Q^4} + \ldots \right) \right]$$

$$\Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3}$$

DECAY RATE has universal leading contribution to all hadrons (up to mass corrections in c_3) ~ m_Q^5

A BIT OF HISTORY

First **FLAVOUR ANOMALIES** were **connected with lifetimes** :

- 80' - $\tau(D^+)/\tau(D_0) \sim 2.1$ ✓ in 1980's
- 85' - $\tau(D_s)/\tau(D_0) \sim 1.5$ (when D_s was called F ☺) ✓ +/- in 1980's
- 90' - $\tau(\Lambda_b)/\tau(B) \sim 0.7\text{-}0.8$ ✓ NICE EXAMPLE OF AN „ANOMALY“ -> in 2011 "EXPERIMENTAL SOLUTION"
- 2000 – WA large → influence on V_{ub} inclusive ✓ / nonperturbative?
- 2018/2020-22 – $\tau(\Omega_c) \sim$ 3-4 times bigger then previously measured ✓ in 2023

"ANOMALIES" - 1nd CASE 1980's $\tau(D^+)/\tau(D^0) \sim 2.1$

$$\mathcal{T} = \left(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2} \mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3} \mathcal{O}_6 + \ldots \right) + 16\pi^2 \left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \ldots \right)$$

unknown pre-HQE 90' unknown

Guberina et al 79
Shifman et al 80

$m_Q = m_c$ - slow convergence; spectator **contributions ~ $1/m_c^3$ might BE IMPORTANT** - BUT WHY THERE WOULD BE SUCH DIFFERENCE IN D-MESON LIFETIMES?

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$$\mathcal{T} = \left(\mathcal{C}_3\mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6 + \ldots\right) + 16\pi^2\left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7 + \ldots\right)$$

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Guberina et al 79
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$m_Q = m_c$ - slow convergence; spectator **contributions ~ $1/m_c^3$ might BE IMPORTANT** - BUT WHY THERE WOULD BE SUCH DIFFERENCE IN D-MESON LIFETIMES?

$$\Gamma(H) = \Gamma_0\left[c_3 + \frac{16\pi^2}{2m_H}\left(\sum_{i,q}\frac{c^q_{6,i}\langle H|O^q_i|H\rangle}{m_Q^3}\right)\right]$$

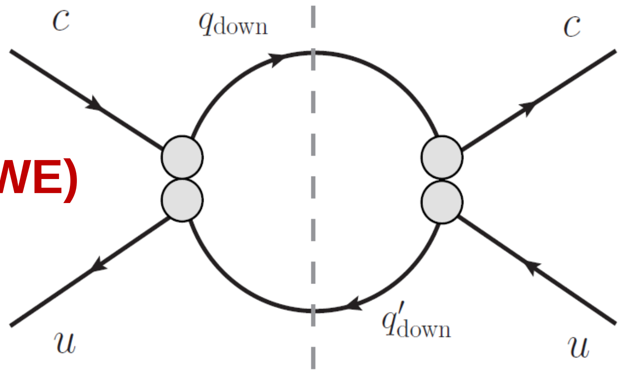
$\Gamma(D^0) >> \Gamma(D^+)$

$\tau(D^+) >> \tau(D^0)$

$D^0(c\bar{u})$

weak exchange (WE)

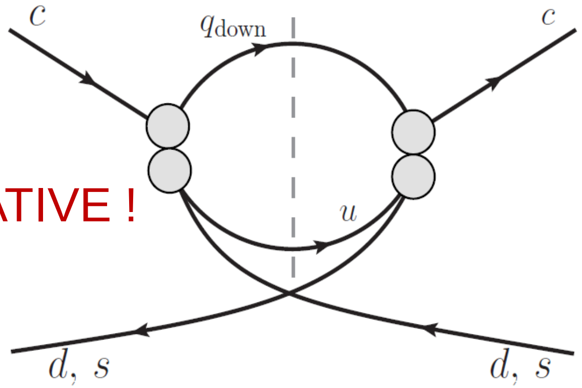
$\tilde{C}_6^{\rm WE}$ small



$D^+(c\bar{d})$

Pauli interference (PI)

$\tilde{C}_6^{\rm PI}$ LARGE AND NEGATIVE !



“ANOMALIES” - 2nd CASE 1985's $\tau(D_s)/\tau(D_0) \sim 1.5$

$$\mathcal{T} = \left(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2} \mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3} \mathcal{O}_6 + \ldots \right) + 16\pi^2 \left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \ldots \right)$$

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Guberina,et al 79
Shifman et al 80

$m_Q = m_c$ - slow convergence; spectator **contributions ~ $1/m_c^3$ might BE IMPORTANT + SU(3) BREAKING**

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unknown pre-HQE 90'

unknown

Guberina,et al 79
Shifman et al 80

$m_Q = m_c$ - slow convergence; spectator **contributions ~ $1/m_c^3$ might BE IMPORTANT + SU(3) BREAKING**

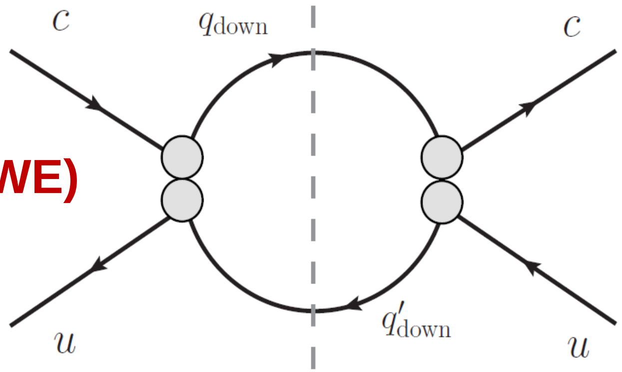
$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} \right) \right]$$

$\Gamma(D^0)_{WE} > \Gamma(D_s)_{WA}$

$\tau(D^0) < \tau(D_s)$

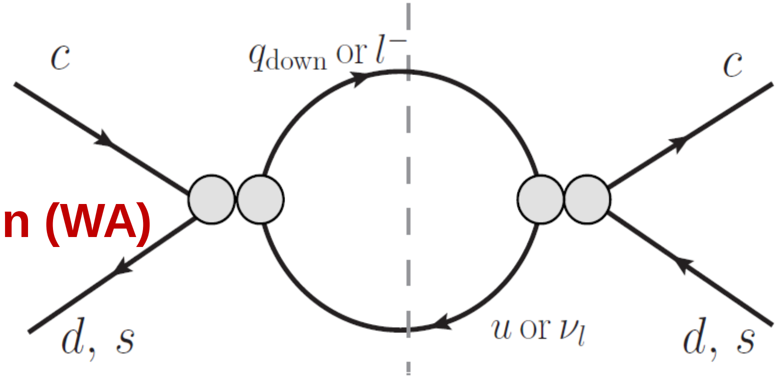
$D^0(c\bar{u})$

weak exchange (WE)



$D_s(c\bar{s})$

weak annihilation (WA)



GOING BACK TO THE PRESENT DEVELOPMENTS

$$\Gamma = \Gamma^{NL} + \Gamma^{SL}$$

$$\Gamma^{NL} = g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2}\left(g_\pi^{(0)} + g_G^{(0)}\right) + \frac{1}{m_c^3} g_{\text{Darwin}}^{(0)} + \frac{16\pi^2}{m_c^3}\left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c}\tilde{g}_7^{(0)}\right)$$

$g_G^{(1)}$ in NL decays - new !

Mannel, Moreno, Pivovarov 2304.08964

$$\Gamma^{SL} = g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2}\left(g_\pi^{(0)} + \alpha_s g_\pi^{(1)} + g_G^{(0)} + \alpha_s g_G^{(1)}\right) + \frac{1}{m_c^3} g_{\text{Darwin}}^{(0)}$$

$$+ \frac{16\pi^2}{m_c^3}\left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c}\tilde{g}_7^{(0)}\right)$$

Semileptonic (SL) modes	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 * ; Czakon, Czarnecki, Dowling '21
$\Gamma_3^{(2)}$	Czarnecki, Melnikov, v. Ritbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola '97-'13
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal '13-'15
$\Gamma_6^{(1)}$	Mannel, Pivovarov '19
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10

* see also talks by K. Schönwald and M. Fael

** Partial result

Non-leptonic (NL) modes	
$\Gamma_3^{(2)}$	Czarnecki, Slusarcyk, Tkachov '05 **
$\Gamma_3^{(1)}$	Ho-Kim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Gosdzinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh '84-'13
$\Gamma_5^{(0)}$	Bigi, Uraltsev, Vainshtein, Blok, Shifman '92
$\Gamma_6^{(0)}$	Lenz, MLP, Rusov, Mannel, Moreno, Pivovarov '20-'21
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste, Franco, Lubicz, Mescia, Tarantino, Rauh '02-'13
$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03-'04

alphaS and mass corrections taken into account – LO and existing NLO

CALCULATION OF MATRIX ELEMENTS

$$\mathcal{T} = \left(\mathcal{C}_3\mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6 + \ldots\right) + 16\pi^2\left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7 + \ldots\right)$$

$$\Gamma(H) = \frac{1}{2m_H}\langle H|\mathcal{T}|H\rangle$$

NON-SPECTATOR PART:

SPECTATOR PART:

$$\Gamma(H) = \Gamma_0\left[c_3 + \frac{c_\pi\mu_\pi^2 + c_G\mu_G^2}{m_Q^2} + \frac{c_\rho\rho_D^3}{m_Q^3} + \ldots\right] + \frac{16\pi^2}{2m_H}\left(\sum_{i,q}\frac{c_{6,i}^q\langle H|O_i^q|H\rangle}{m_Q^3} + \sum_i \frac{c_{7,i}^q\langle H|P_i^q|H\rangle}{m_Q^4} + \ldots\right)$$

$$\mu_\pi^2(H) = \frac{-1}{2m_H}\langle H|\bar{c}_v(iD)^2c_v|H\rangle\,,$$ kinetic parameter

$$\mu_G^2(H) = \frac{1}{2m_H}\langle H|\bar{c}_v\frac{1}{2}\sigma\cdot(g_sG)c_v|H\rangle\,,$$ chromomagnetic parameter

$$\rho_D^3(H) = \frac{1}{2m_H}\langle H|\bar{c}_v(iD_\mu)(iv\cdot D)(iD^\mu)c_v|H\rangle$$ Darwin term

$$\langle H|O_i^q|H\rangle$$

$$\langle H|P_i^q|H\rangle$$

four-quark matrix elements

CALCULATION OF NON-SPECTATOR MATRIX ELEMENTS

NON-SPECTATOR PART: - mainly universal – up to SU(3)$_f$ breaking and differences in spins of hadrons

μ_G^2 application of hadron mass formula:

$$m_H = m_c + \bar{\Lambda} + \frac{\mu_\pi^2(H)}{2m_c} - \frac{\mu_G^2(H)}{2m_c} + \mathcal{O}\left(\frac{1}{m_c^2}\right)$$

spin factor: $d_H = -2\left(S_H(S_H+1) - S_h(S_h+1) - S_l(S_l+1)\right)$

$$\mu_G^2(H) \equiv d_H \lambda_2 = d_H \frac{m_{H^*}^2 - m_H^2}{d_H - d_{H^*}}$$

H	D	D^*	$\Lambda_c^+, \Xi_c^+, \Xi_c^0$	Ω_c^0	Ω_c^{0*}
d_H	3	-1	0	4	-2

μ_π^2 HQET SR: $\mu_\pi^2 \geq \mu_G^2$

ρ_D^3 applying EOM of $G_{\mu\nu}$ and relating it to the dim6 operators:

$$2m_H \rho_D^3 = g_s^2 \langle H | \left(-\frac{1}{8} O_1^q + \frac{1}{24} \tilde{O}_1^q + \frac{1}{4} O_2^q - \frac{1}{12} \tilde{O}_2^q \right) | H \rangle + \mathcal{O}(1/m_c)$$

$$\rho_D^3(D_q) = \frac{g_s^2}{18} f_{D_q}^2 m_{D_q} + \mathcal{O}(1/m_c)$$

CALCULATION OF NON-SPECTATOR MATRIX ELEMENTS

NON-SPECTATOR PART:

	D^0	D^+	D_s^+	Λ_c^+	Ξ_c^+	Ξ_c^0	Ω_c^0
$\mu_G^2/\,\mathrm{GeV}^2$	0.41(12)	0.41(12)	0.44(13)	0	0	0	0.26(8)
$\mu_\pi^2/\,\mathrm{GeV}^2$	0.45(14)	0.45(14)	0.48(14)	0.50(15)	0.55(17)	0.55(17)	0.55(17)
$\rho_D^3/\,\mathrm{GeV}^3$	0.056(12)	0.056(22)	0.082(33)	0.04(1)	0.05(2)	0.06(2)	0.06(2)

+ 30% uncertainties

ρ_D^3 much smaller parameter but with a surprisingly large Wilson coefficient c_ρ

- sizable contribution of $1/m_c^3$; also sizable $SU(3)_F$ breaking effects

Lenz, Piscopo, Rusov 2004.09527
Mannel, Moreno, Pivovarov 2004.09485

CALCULATION OF SPECTATOR (FOUR-QUARK) MATRIX ELEMENTS

SPECTATOR PART FOR **MESONS**: - calculation of four-quark matrix elements

Dim 6 :

$$\langle D_q|\mathcal{O}_i^q|D_q\rangle = F_{D_q}(\mu)^2 m_{D_q} B_i^q \,,$$

$$\langle D_q|\mathcal{O}_i^{q'}|D_q\rangle = F_{D_q}(\mu)^2 m_{D_q} \delta_i^{q'q} \,, \quad q \neq q'$$

$$F_{D_q}(\mu)^2 \to f_{D_q}^2 m_{D_q} \left(1 + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\right)$$

HQET bag model parameters or lattice:

$$B_{1,2}^q \qquad \epsilon_{1,2}^q \equiv B_{3,4}^q \qquad \delta_i^{q'q}$$

Kirk, Lenz, Rauch, 1711.02100
King, Lenz, Rauch, 2112.03691
King et al, 2109.13219

Vacuum insertion approximation (VIA):

$$B_i^{P,R} = 1 \qquad \epsilon_i^{P,R} = 0$$

for color-octet operators

Dim 7 :

$$\langle D_q|\mathcal{P}_1^q|D_q\rangle = -m_q F^2 m_{D_q} B_1^P \,,$$

$$\langle D_q|\mathcal{P}_2^q|D_q\rangle = -\bar{\Lambda}_q F^2 m_{D_q} B_2^P \,,$$

$$\langle D_q|\mathcal{P}_2^q|D_q\rangle = -\bar{\Lambda}_q F^2 m_{D_q} B_2^P \,,$$

$$\langle D_q|\mathcal{R}_1^q|D_q\rangle = -F_{D_q}^2 m_{D_q} (\bar{\Lambda}_q - m_q) B_1^R \,,$$

$$\langle D_q|\mathcal{R}_2^q|D_q\rangle = F_{D_q}^2 m_{D_q} (\bar{\Lambda}_q - m_q) B_2^R \,,$$

Decay constants in the m_c -> infinity limit:

$$F_{D_q} \to f_{D_q}\sqrt{m_{D_q}}$$

CALCULATION OF SPECTATOR (FOUR-QUARK) MATRIX ELEMENTS

SPECTATOR PART FOR **BARYONS** : - calculation of four-quark matrix elements

NR CONSTITUENT QUARK MODEL

$$\frac{\langle \mathcal{B}_c | O_i^q | \mathcal{B}_c \rangle}{2 m_{\mathcal{B}_c}} \sim |\psi_{cq}^{\mathcal{B}_c}(0)|^2$$ and $$|\Psi(0)|^2_{ij} \sim \delta^3(0)$$

$$\mathcal{B}_c \sim c(q_1 q_2)$$

Rujula, Georgi, Glashow 1975

$$M_H = \sum_i m_i^H + \langle H_{\text{spin,H}} \rangle$$

$$H_{\text{spin, mesons}} = \frac{32\pi\alpha_s}{9} \frac{(\vec{s}_i \cdot \vec{s}_j)}{m_i^M m_j^M} \delta^3(\vec{r}_{ij})_M$$

$$H_{\text{spin, baryons}} = \sum_{i>j} \frac{16\pi\alpha_s}{9} \frac{(\vec{s}_i \cdot \vec{s}_j)}{m_i^{\mathcal{B}} m_j^{\mathcal{B}}} \delta^3(\vec{r}_{ij})_B$$

combining mass expressions for the hyperfine partners (e.g. 1/2+ and 3/2+):

$$|\Psi_{cq}^{\Lambda_c^+}(0)|^2 = \frac{4}{3} \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{D^*} - M_D} |\Psi_{cq}^{D_q}(0)|^2$$

e.g.

$$|\Psi_{cq}^{D_q}(0)|^2 = \tfrac{1}{12} f_{D_q}^2 m_{D_q}$$

dim 7 operators are expressed similarly, in terms on dim 6 operators as above; e.g. for triplet of baryons:

$$\langle \mathcal{T}_c | P_1^q | \mathcal{T}_c \rangle \simeq \frac{1}{2} m_q |\Psi_{cq}^{\mathcal{T}_c}(0)|^2 \qquad \langle \mathcal{T}_c | P_2^q | \mathcal{T}_c \rangle \simeq -\Lambda_{\text{QCD}} |\Psi_{cq}^{\mathcal{T}_c}(0)|^2$$

DIM 7 OPERATORS AND HQET/QCD BASIS OF OPERATORS

In HQET basis of operators there are additional **NON-LOCAL OPERATORS** at order $1/m_Q{}^4$: G_1 and G_2

$$G_1 \sim i \int d^4x\, T\left\{\mathcal{O}_1^q(0), \bar{h}_v(x)\frac{g_s}{2}\sigma \cdot G h_v(x)\right\}$$

IN MESONS:

One **can show** that they get exactly reabsorbed at O($1/m_Q{}^4$) in the decay constant to ***renormalize*** the HQET (static) decay constant to the QCD one:

$$\underbrace{F_D^2}_{\text{HQET}}\left(1 - \frac{\bar{\Lambda}}{m_c} + \underbrace{\frac{2G_1}{m_c} + \frac{12G_2}{m_c}}_{\text{non-local}}\right) = \underbrace{f_D^2 m_D}_{\text{QCD}} + O(1/m_c^2)$$

IN BARYONS:

Non-local matrix elements are not calculable – NO proof for such a relation

$$|\Psi_{bq}^{\Lambda_b^0}(0)|^2 \sim F_{B_q}^2$$

For charm baryons – we stay in the ***QCD basis of operators*** since the convergence of $1/m_c$ expansion is slow

- dim 7 operators contribute up to 50% of dim 6 operators

PECULARITIES OF DOUBLY-CHARMED BARYONS

Difference to singly-charmed baryons:

- counting contributions (two c quarks decaying)
- choice of hadronic parameters

- diquark system – cc-pair (instead the diquark q_1q_2-pair in singly-charmed baryons) $\mathcal{B}_{cc} \sim (cc)q$

Additional contributions to some of the matrix elements, e.g :

$$\mu_G^2(\mathcal{B}_{cc}) \to \mu^2_{G(D-q)}(\mathcal{B}_{cc}) + \mu^2_{G(c-c)}(\mathcal{B}_{cc})$$

, similarly for $\mu_\pi^2(\mathcal{B}_{cc})$

Additional contributions accessed by NRQCD expansion (up to $O(v^7)$), Ψ_c = 2-component NR spinor):

$$\bar{c}c = \bar{\Psi}_c\Psi_c - \frac{1}{2m_c^2}\bar{\Psi}_c(i\vec{D})^2\Psi_c + \frac{3}{8m_c^4}\bar{\Psi}_c(i\vec{D})^4\Psi_c - \frac{1}{2m_c^2}\bar{\Psi}_c\, g_s\vec{\sigma}\cdot\vec{B}\Psi_c - \frac{1}{4m_c^3}\bar{\Psi}_c\, g_s(\vec{\mathcal{D}}\cdot\vec{E})\Psi_c + \ldots$$

matrix elements, e.g.:

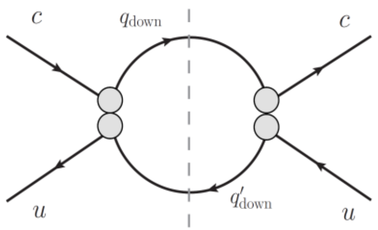
$$\frac{\langle\mathcal{B}_{cc}|\Psi_c^\dagger g_s\vec{\sigma}\cdot\vec{B}\Psi_c|\mathcal{B}_{cc}\rangle}{2M_{\mathcal{B}_{cc}}}\Big|_{c-c} = \frac{4}{9}\frac{g_s^2}{m_c}|\Psi_{cc}(0)|^2$$

where $|\Psi_{cc}(0)|^2 \neq \left(|\Psi_{\bar{c}c}(0)|^2 \equiv |\Psi_{J/\psi}(0)|^2\right)$

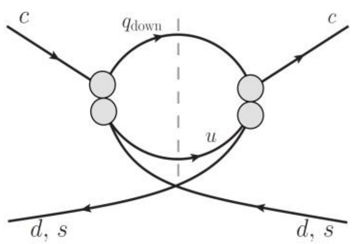
SPECTATOR (u,d,s) FOUR-QUARK CONTRIBUTIONS ARE IMPORTANT :

one-loop i.e 16 π^2 enhanced, although $1/m^3$ (dim6), $1/m^4$ (dim7) suppressed

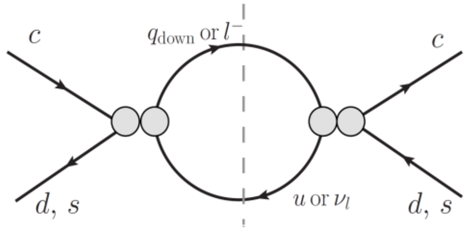
MESONS



WE

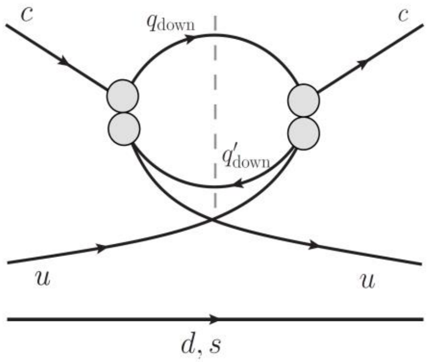


PI

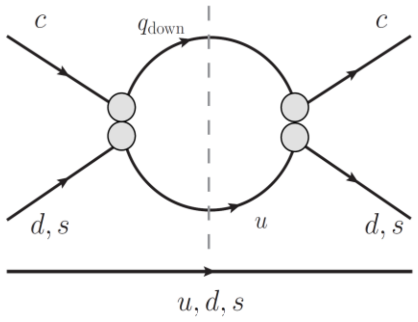


WA

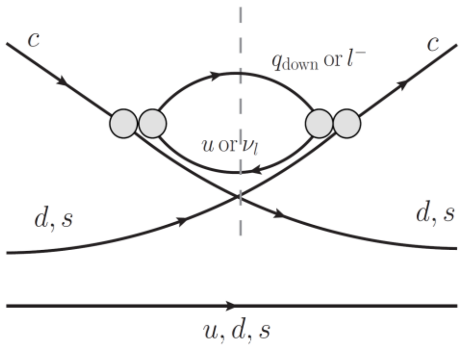
BARYONS



int-



exc



int+

decay / H_c	CE NL $c \to s\bar{d}u$	CE SL $c \to s\bar{l}\nu_l$
$\bar{D}^0(u\bar{c})$	$\tilde{\Gamma}_{\rm WE}$	-
$D^-(d\bar{c})$	$\tilde{\Gamma}_{\rm PI}$	-
$D_s^-(s\bar{c})$	$\tilde{\Gamma}_{\rm WA}$	$\tilde{\Gamma}^{\rm SL}_{\rm WA}$
$\Lambda_c^+(udc)$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int-}$	-
$\Xi_c^+(usc)$	$\tilde{\Gamma}_{\rm int-} + \tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$
$\Xi_c^0(dsc)$	$\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$
$\Omega_c^0(ssc)$	$\tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$

CE = leading; Cabibbo enhanced

* effects are different in different mesons
* effects are different in different baryons
* no helicity suppression for baryons
* effects in SL decays – **different BR(SL) !**

HEAVY QUARK MASS

$$\Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3}$$

POLE mass:

$$m_c^{\rm pole} = \overline{m}_c(\overline{m}_c)\left[1 + \frac{4}{3}\frac{\alpha_s(\overline{m}_c)}{\pi} + 10.3\left(\frac{\alpha_s(\overline{m}_c)}{\pi}\right)^2 + 116.5\left(\frac{\alpha_s(\overline{m}_c)}{\pi}\right)^3 + \ldots\right]$$

$$= \overline{m}_c(\overline{m}_c)(1 + 0.16 + 0.15 + 0.21 + \ldots)\,,$$

IR renormalon – divergent series starting from the 3rd (5th)-loop for m_c (m_b)

renormalon-free mass definitions:

$$m_c^X(\mu_f) = m_c^{\rm pole} - \delta m_c^X(\mu_f)$$

$$= \overline{m}_c(\overline{m}_c) + \overline{m}_c(\overline{m}_c)\sum_{n=1}^{\infty}\left[c_n(\mu, \overline{m}_c(\overline{m}_c)) - \frac{\mu_f}{\overline{m}_c(\overline{m}_c)} s_n^X(\mu/\mu_f)\right]\alpha_s^n(\mu)$$

- subtraction of IR renomalons
- rearrangement of α_S expansion - relevant for α_S-corrections in c_3 and c_6 terms

CHARM QUARK MASS

$\overline{m}_c(\overline{m}_c) = 1.28$ GeV	1-loop	2-loop	3-loop	4-loop
m_c^{pole}	1.49	1.68	1.95	2.43
m_c^{kin}	1.36	1.39	1.40	-
m_c^{MSR}	1.33	1.35	1.36	1.36

we provide results for different mass schemes... no large differences in the final results – rearrangements among $1/m_c$ and α_S-expansion !

RESULTS

RESULTS FOR BARYONS

Lifetime ratios of a baryon $\mathcal{B}_c$

$$\frac{\tau(\mathcal{B}_c)}{\tau(\Lambda_c^+)} \equiv \frac{1}{1 + (\Gamma^{\rm th}(\mathcal{B}_c) - \Gamma^{\rm th}(\Lambda_c^+))\tau^{\rm exp}(\Lambda_c^+)}$$

- some uncertainties cancel in the ratios

Inclusive SL branching ratios (e only) for $\mathcal{B}_c$:

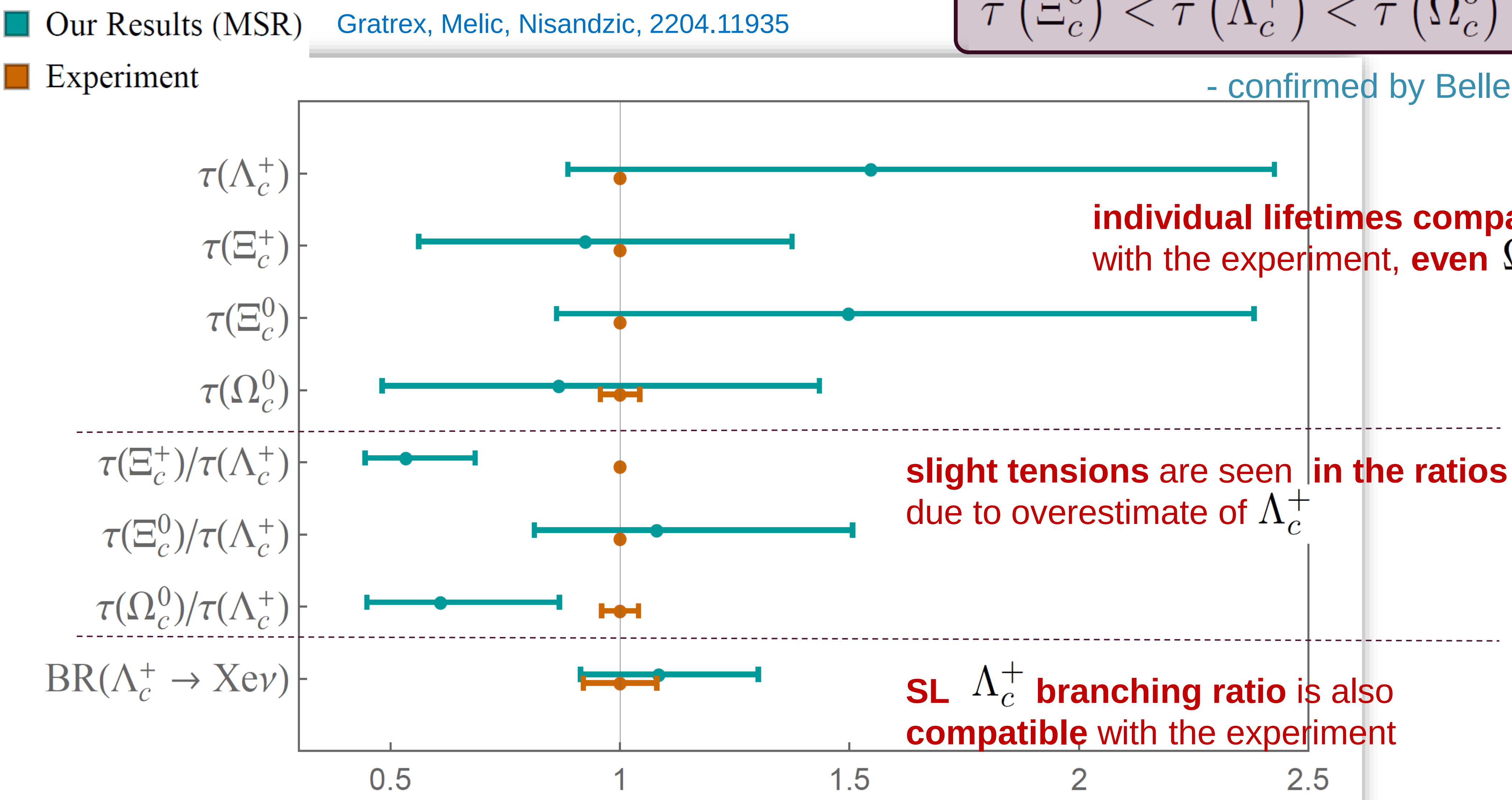
$$BR(\mathcal{B}_c \to X e \nu) \equiv \Gamma(\mathcal{B}_c \to X e \nu)\, \tau^{\rm exp}(\mathcal{B}_c)$$

CHARMED BARYONS

Gratrex, Melic, Nisandzic, 2204.11935

$$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right)$$

- confirmed by Belle-II , aug.2022



individual lifetimes compatible with the experiment, **even** Ω_c^0 **!**

slight tensions are seen **in the ratios** due to overestimate of Λ_c^+

SL Λ_c^+ **branching ratio** is also **compatible** with the experiment

CHARMED BARYONS - SL BRs

MSR scheme:

$BR(\Lambda_c^+ \to Xe\nu)/\%$	$4.28^{+0.47+0.39}_{-0.37-0.30}$
$BR(\Xi_c^+ \to Xe\nu)/\%$	$14.95^{+2.66+1.59}_{-2.45-1.50}$
$BR(\Xi_c^0 \to Xe\nu)/\%$	$5.06^{+0.91+0.54}_{-0.84-0.51}$
$BR(\Omega_c^0 \to Xe\nu)/\%$	$11.19^{+3.01+1.94}_{-2.89-2.09}$

SL decays are important to assess the validity of HQE in charmed baryons

- experimental measurements of $BR_{SL}(\Xi_c^+)$, $BR_{SL}(\Xi_c^0)$ and $BR_{SL}(\Omega_c^0)$ are needed

CHARMED MESONS

Lifetime ratios :

$$\frac{\tau(D^+_{(s)})}{\tau(D^0)} = 1 + \left(\Gamma^{\rm th}(D^0) - \Gamma^{\rm th}(D^+_{(s)})\right)\tau^{\rm exp}(D^+_{(s)})$$

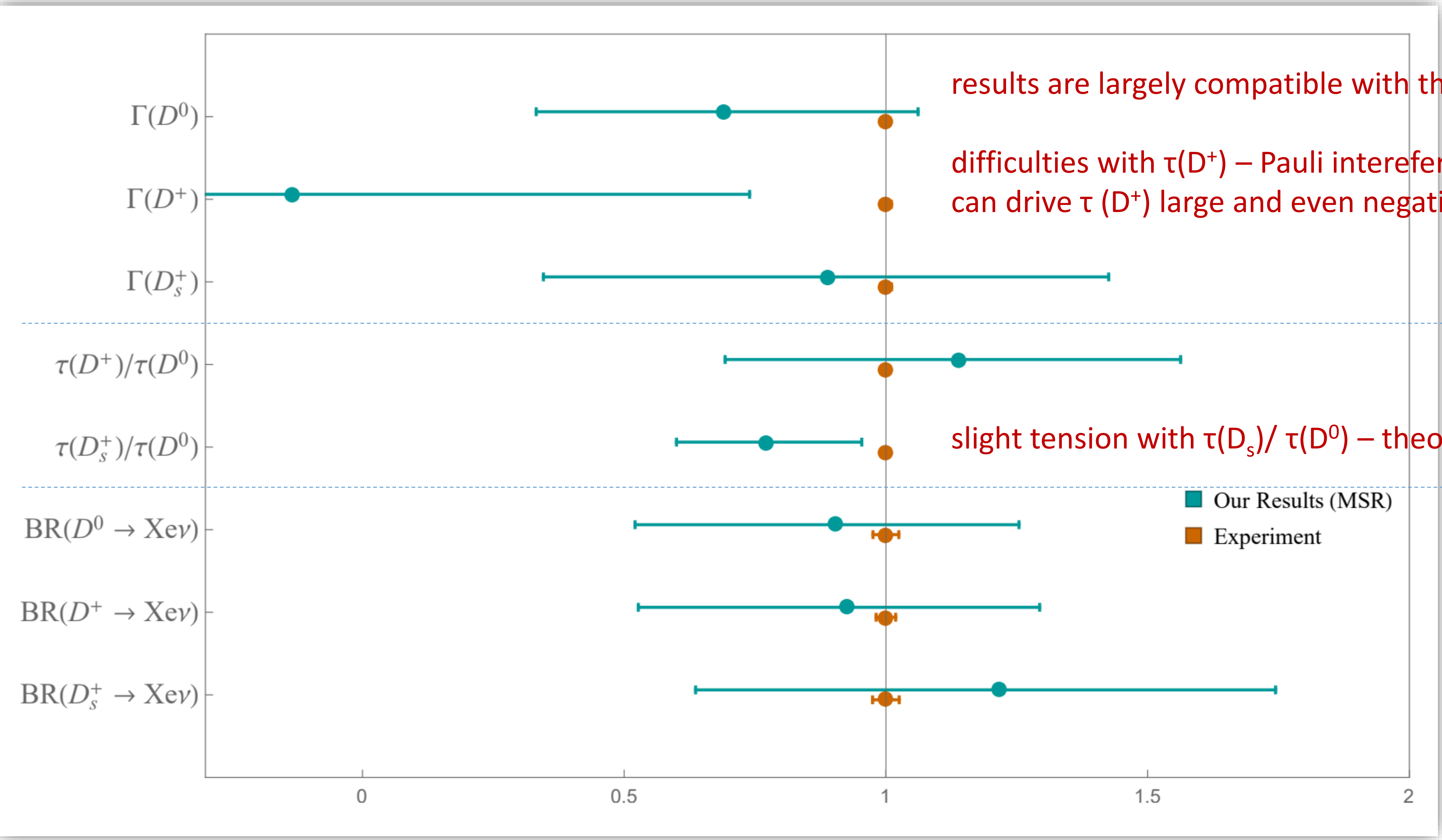
- some uncertainties cancel in the ratios

Inclusive SL branching ratios (e only) :

$$BR^{(e)}(D) = \Gamma^{(e)}(D)\tau^{\rm exp}(D)$$

$$\frac{\Gamma^{(e)}(D^+_{(s)})}{\Gamma^{(e)}(D^0)} = 1 + (\Gamma^{(e)\,{\rm th}}(D^+_{(s)}) - \Gamma^{(e)\,{\rm th}}(D^0))\left(\frac{\tau(D^0)}{BR^{(e)}(D^0)}\right)^{\rm exp}$$

King, Lenz, Piscopo, Rauh, Rusov, 2109.13219
Gratrex, Melic,Nisandzic, 2204.11935



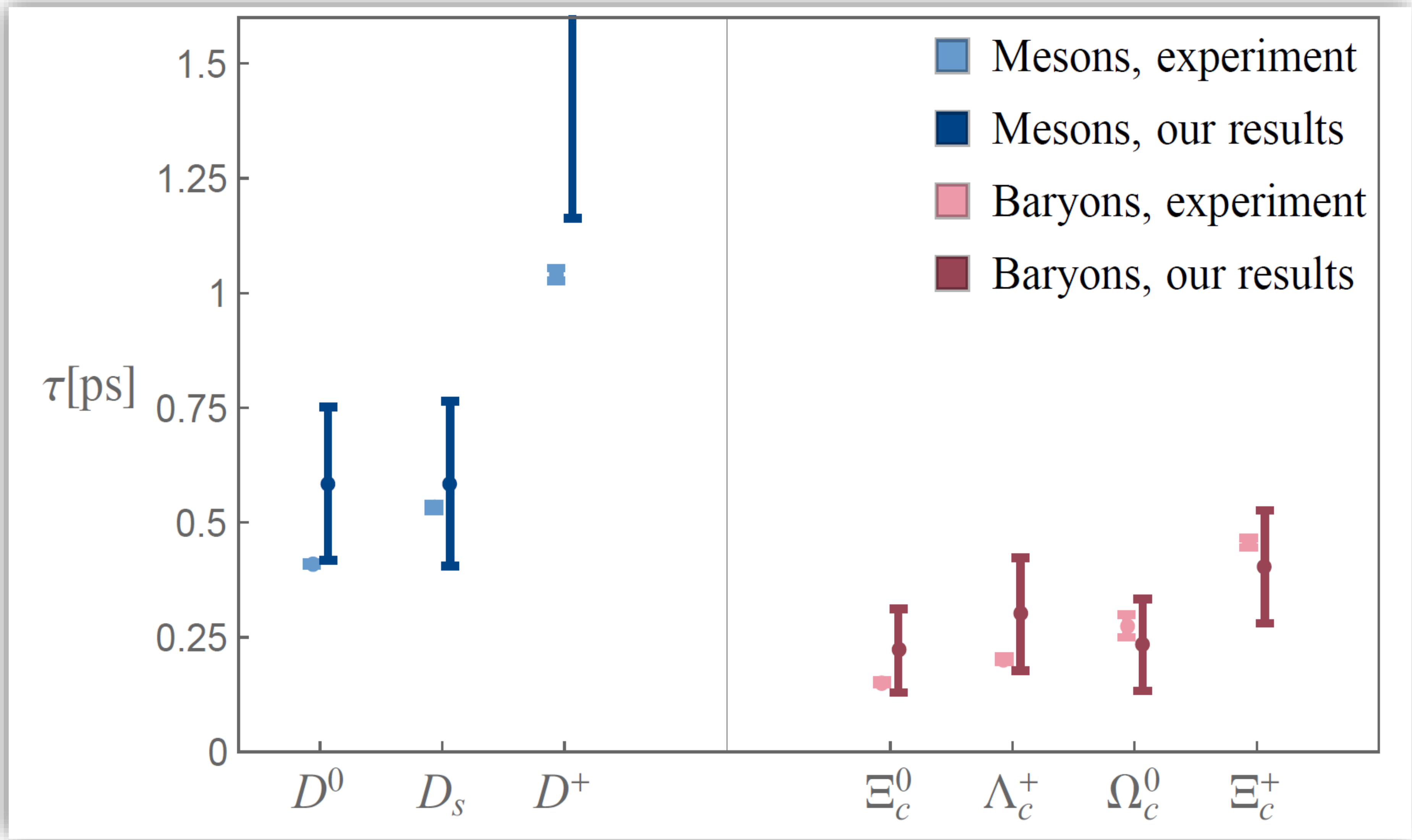
results are largely compatible with the experiment

difficulties with $\tau(D^+)$ – Pauli intereference term can drive τ (D^+) large and even negative!

slight tension with $\tau(D_s)$/ $\tau(D^0)$ – theoretically closer to 1

SINGLY CHARMED HADRON LIFETIMES - CONCLUSIONS

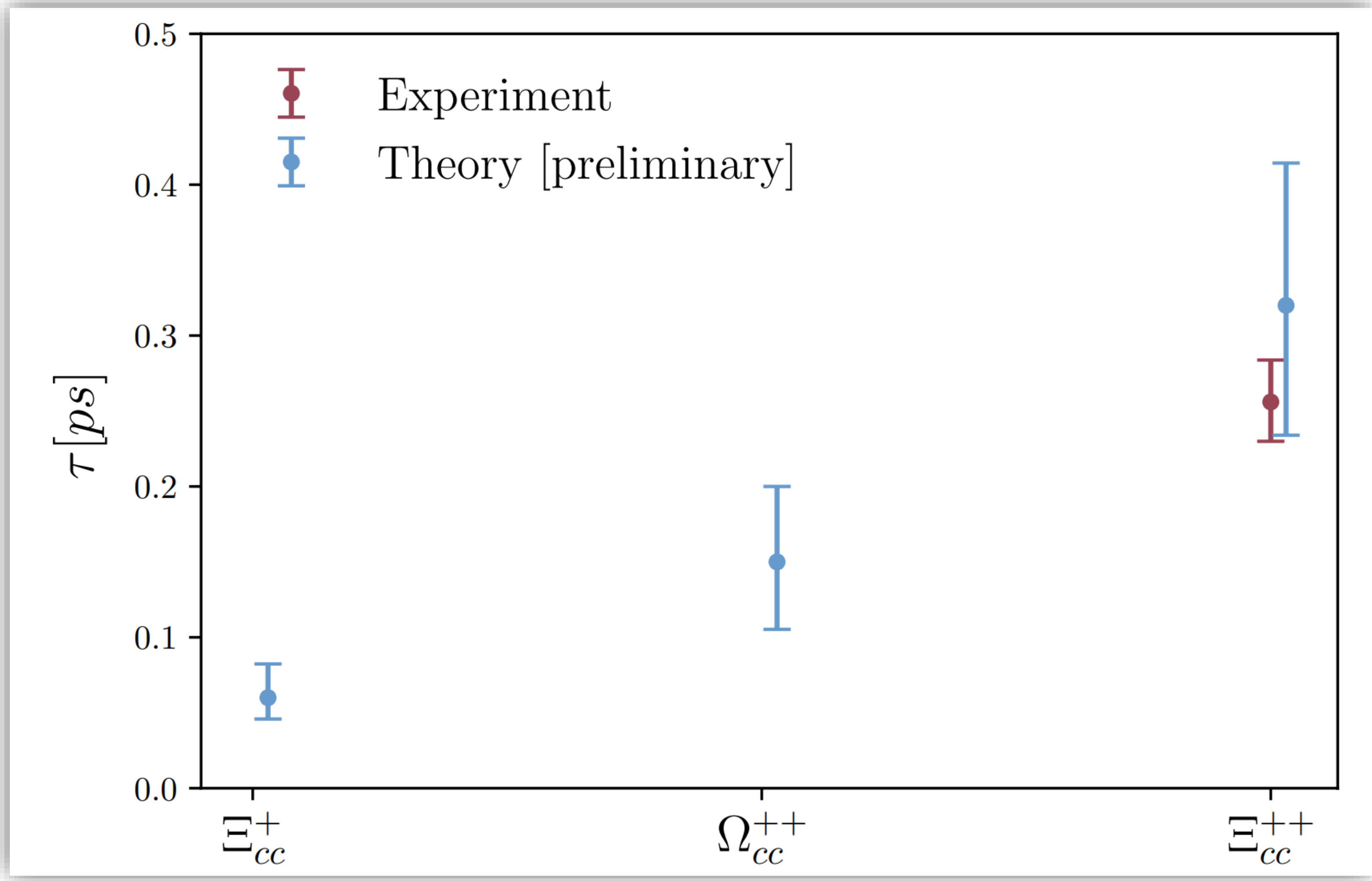
King, Lenz, Piscopo, Rauh, Rusov, 2109.13219
Gratrex, Melic, Nisandzic 2204.11935



satisfactory agreement with the experiment!

inclusion of newly calculated NLO corrections to μ_G^2 (Mannel, Moreno, Pivovarov 2304.08964) would probably significantly reduce uncertainty

DOUBLY CHARMED HADRON LIFETIMES - CONCLUSIONS



Dulibic, Gratrex, Melic, Nisandzic 2305.02243

$\tau(\Xi_{cc}^{++})$ is the only measured doubly-charmed baryon lifetime (LHCb 2018)

- good agreement

$\tau(\Xi_{cc}^{+})$ and $\tau(\Omega_{cc}^{+})$ measurement at LHCb Run-3 is feasible

CONCLUSIONS – CHARM HADRON LIFETIMES

- up-to-date results for lifetimes of weakly decaying hadrons with a single charm quark, with most complete set of contributions provided

- results compatible with experiment, albeit with large uncertainties, and favouring recent LHCb (2018/20) and Belle-II (8/2022) result for $\tau(\Omega^0_c)$ lifetime (~ 4× bigger than old measurements)

- difficulty in predicting $\tau(D^+)$ – only marginally compatible – huge negative Pauli interference contribution

- predictions for unmeasured $BR_{SL}(H)$ are important for complete assessment

- conclusions above are largely independent of the charm mass scheme

- **HQE seems to work for charm**

OUTLOOK

extending available contributions in $1/m_Q$ and α_s series

large uncertainties mean theory cannot compete with experiment – **more control of hadronic parameters** needed :

I. lattice determination of $\langle \tilde{\mathcal{O}}_6 \rangle$ planned (U Siegen)
II. higher α_s corrections planned (KIT) – NLO of 4q-dim7, NNLO of NL-dim3 etc..
III. exp. (BESIII, Belle II...) determination of the kinetic, chromomagnetic and Darwin parameter from SL decays? Too sensitive to four-quark "leakage"?

question of applicability of heavy quark approach to charm remains open

$\Rightarrow$ $\alpha_s(m_c) = 0.33$, $\Lambda_{QCD}/m_c = 0.30$ too large? (vs $\alpha_s(m_b) = 0.22$, $\Lambda_{QCD}/m_b = 0.10$)

- spectator contributions dominate over the leading free charm decay

theoretical improvements:

- revisiting formulation of HQE in charm mass?

(Mannel et al 2103.02058 - treating 4-q contributions as a part of the leading term?)

- testing quark-hadron duality violation? (seems to work for beauty)

$D^0 - \overline{D^0}$ MIXING – STATUS

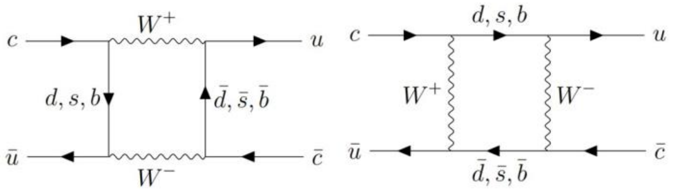
- an incomplete, personal look -

BASICS

neutral mesons mix:

$$i\frac{\partial}{\partial t}\begin{pmatrix}D^0\\ \bar{D}^0\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}D^0\\ \bar{D}^0\end{pmatrix} = \left(\begin{pmatrix}M_{11} & M_{12}\\ M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{12}^* & \Gamma_{11}\end{pmatrix}\right)\begin{pmatrix}D^0\\ \bar{D}^0\end{pmatrix}$$

off-shell states contribute   on -shell states contribute



parameters:

$$x_{12} = \frac{2\,|M_{12}|}{\Gamma_{D^0}} \qquad y_{12} = \frac{2\,|\Gamma_{12}|}{\Gamma_{D^0}}$$

+ possible (indirect) CPV : $\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)$

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

$$\Delta M \equiv M_1 - M_2, \quad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2.$$

$$\Delta M_D = 2|M_{12}^D| \cdot (1 + \mathcal{O}((\phi_{12}^D)^2))$$
$$\Delta\Gamma_D = 2|\Gamma_{12}^D| \cdot (1 + \mathcal{O}((\phi_{12}^D)^2))$$

$$x \approx x_{12} = 2\frac{|M_{12}|}{\Gamma_D} \qquad y \approx y_{12} = \frac{|\Gamma_{12}|}{\Gamma_D}$$

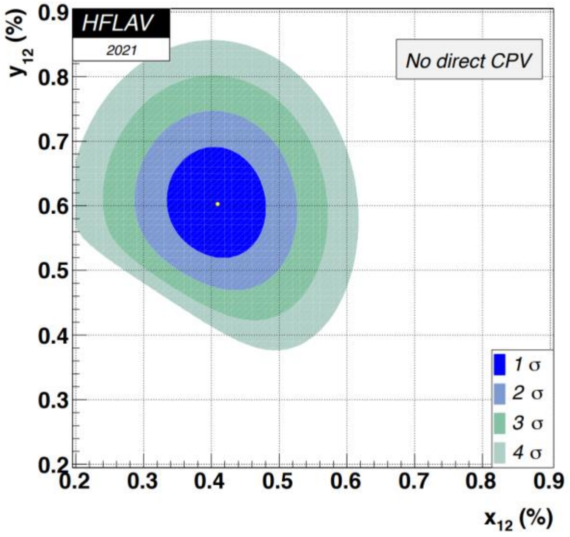
$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}$$

$$x = \frac{\Delta M_D}{\Gamma_D} \qquad y = \frac{\Delta\Gamma_D}{2\Gamma_D}$$

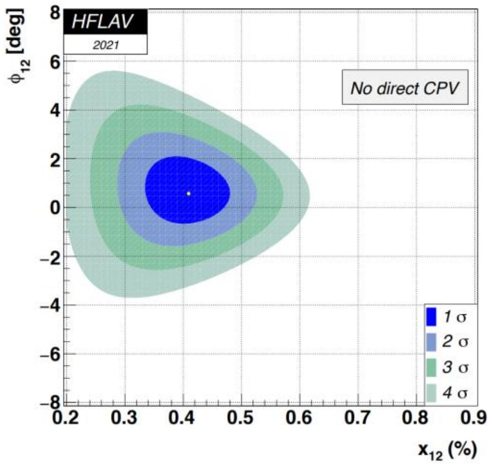
- more general approach with two phases Kagan, Silvestrini, 2001.07207 :

$$\phi_{12} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = \phi^M - \phi^\Gamma$$

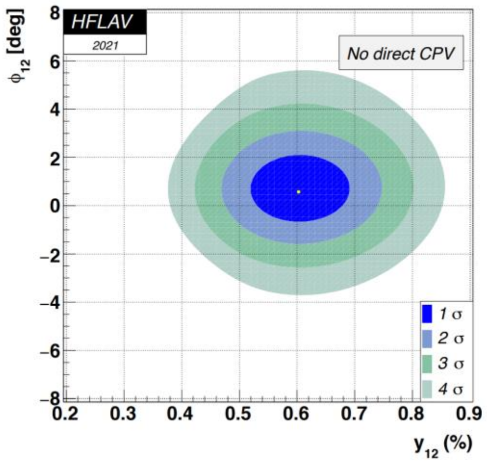
HFLAV fits, 2206.07501 - clear evidence for D^0-$\bar{D}^0$ mixing - no-mixing point x=y= 0 is excluded at >11.5 σ

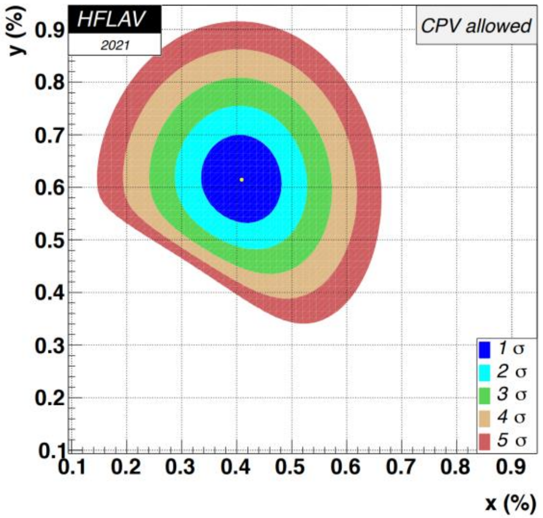


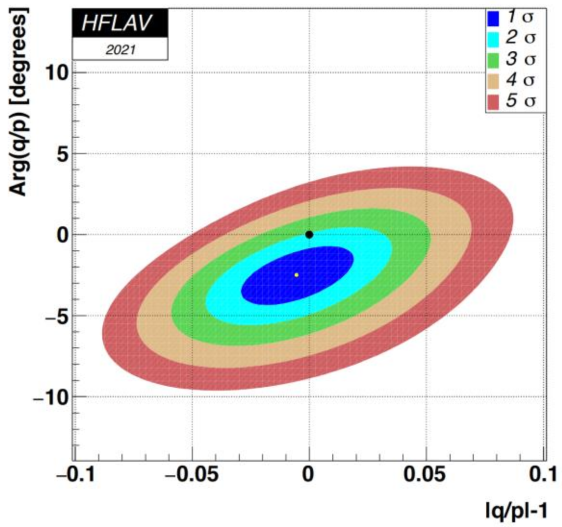
no direct evidence for CPV :



x







$$x = \frac{\Delta M}{\Gamma_D} = 0.409^{+0.048}_{-0.049}\ \%$$

$$y = \frac{\Delta \Gamma}{\Gamma_D} = 0.615^{+0.056}_{-0.055}\ \%$$

$$\phi(^\circ) = -2.5 \pm 1.2$$

$$2y_{CP} = (|q/p| + |p/q|)\, y \cos\phi - (|q/p| - |p/q|)\, x \sin\phi$$

$$y_{CP} = (0.719 \pm 0.113)\%$$

CP asymmetries in $D^0, \overline{D}^0$ meson decays:

$$A_{CP}(i \to f) \equiv \frac{|\langle f|T|i\rangle|^2 - |\langle \bar{f}|T|\bar{i}\rangle|^2}{|\langle f|T|i\rangle|^2 + |\langle \bar{f}|T|\bar{i}\rangle|^2} = \Sigma_j [p_j \sin(\Delta\delta_j)\,\sin(\Delta\phi_j)]_{i\to f}$$

strong weak

$$A_\Gamma \equiv \frac{\tau(\overline{D}^0 \to h^+h^-) - \tau(D^0 \to h^+h^-)}{\tau(\overline{D}^0 \to h^+h^-) + \tau(D^0 \to h^+h^-)}$$

where h^+h^- can be K^+K^- or $\pi^+\pi^-$

$$\Rightarrow \quad A_\Gamma = -a_{CP}^{\rm ind} - a_{CP}^{\rm dir} y_{CP}$$

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$$

$$\Rightarrow \quad \Delta A_{CP} \approx \Delta a_{CP}^{\rm dir}\left(1 + y_{CP}\frac{\overline{\langle t\rangle}}{\tau}\right) + a_{CP}^{\rm ind}\frac{\Delta\langle t\rangle}{\tau}$$

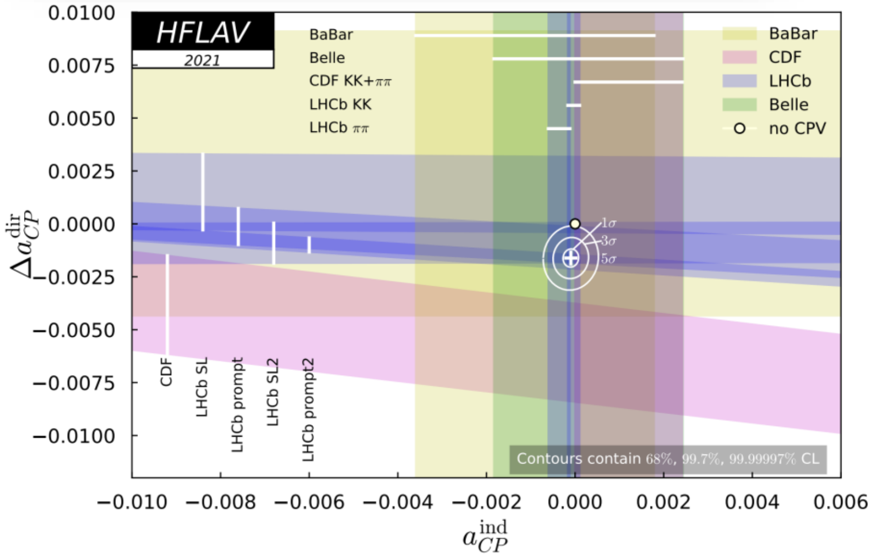
both terms are very small

$$a_{CP}^{\rm ind} = (-0.010 \pm 0.012)\%$$
$$\Delta a_{CP}^{\rm dir} = (-0.161 \pm 0.028)\%$$

THEORY : an order of magnitude smaller result -> LARGE NON-PERTURBATIVE CONTRIBUTIONS/FSI NEEDED

LCSR – $|P/T|_{\pi\pi, KK}$ calculation, arbitrary strong phase
Khodhjamirian, Petrov, 1706.07780

Dispersion relations – FSI/rescattering phases
Pich, Solomonidi, Silva, 2305.11951

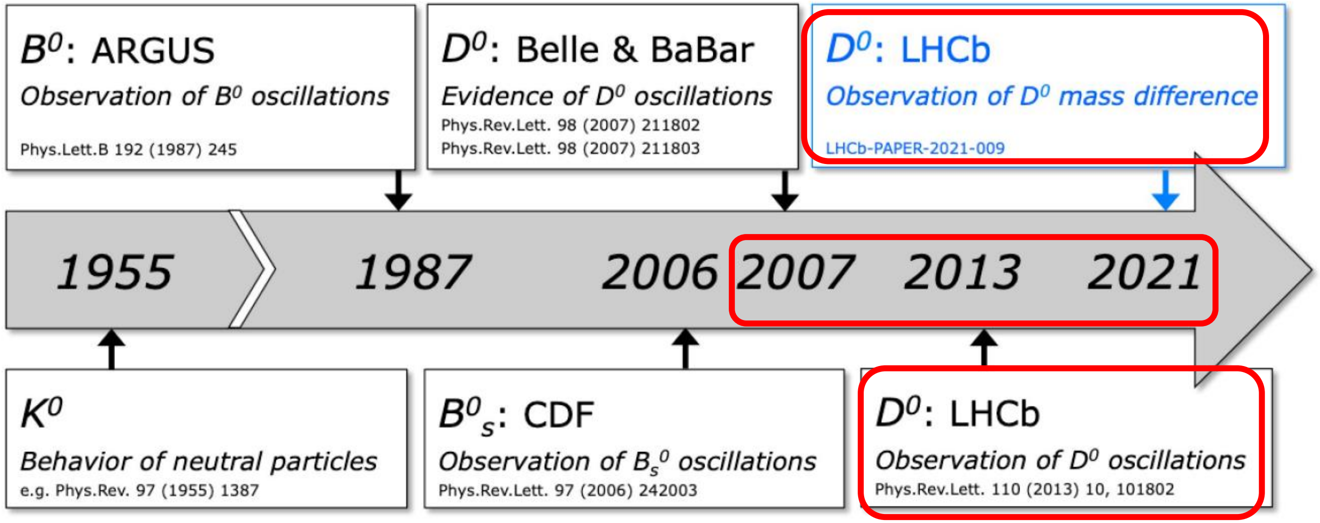


$$a_{CP}^{\rm dir} \equiv \frac{|\mathcal{A}_{D^0\to f}|^2 - |\mathcal{A}_{\overline{D}^0\to f}|^2}{|\mathcal{A}_{D^0\to f}|^2 + |\mathcal{A}_{\overline{D}^0\to f}|^2},$$

$$a_{CP}^{\rm ind} \equiv \frac{1}{2}\left[\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi - \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi\right]$$

$$y_{CP} = (0.719 \pm 0.113)\%$$

see also "Recent advances in charm mixing and CP violation at LHCb", T. Pajero, 2208.05769



LHCb 2021 at 10th CKM2020(2021), arXiv: 2106.03744

Angelo Carbone CHARM 2020, 31 May – 4 June

First observation of the mass difference between D^0 and $\overline{D^0}$:

m_1-m_2 = 6.4×10^{-6} eV

= 0.000000000000000000000000000000000000001 grams (1×10^{-38} g)

(m_1-m_2)/(D^0 mass) = 3×10^{-15}

B-$\overline{\mathrm{B}}$, B_s-$\overline{\mathrm{B}}_s$ and K- $\overline{\mathrm{K}}$ mixing are well under control – WHY IS SO DIFFICULT TO EXPLAIN D- $\overline{\mathrm{D}}$ MIXING?

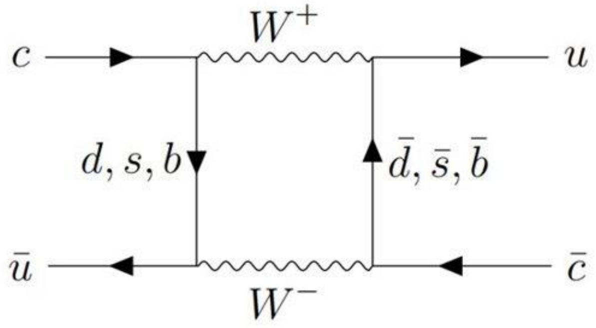
A LONG-STANDING PUZZLE – how to explain theoretically

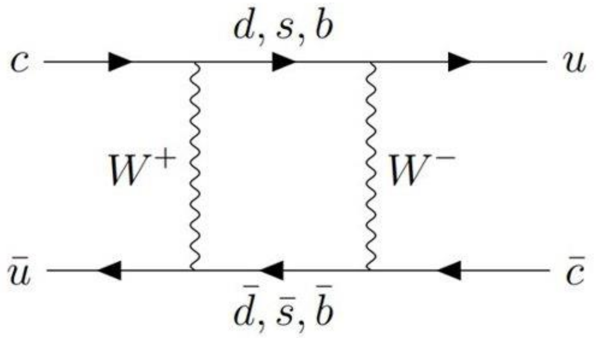
$$y = \frac{\Delta\Gamma}{\Gamma_D} = 0.615^{+0.056}_{-0.055}\ \%$$

$$x = \frac{\Delta M}{\Gamma_D} = 0.409^{+0.048}_{-0.049}\ \%$$

SM results are 4 orders of magnitude smaller than experimental results ?!

NAIVE HQE APPLICATION:





$$\Gamma_{12}^D = -\lambda_s^2 \left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd}^D - \Gamma_{dd}^D\right) - \lambda_b^2 \Gamma_{dd}^D$$

$$M_{12}^D = \lambda_s^2 \left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D\right] + 2\lambda_s\lambda_b \left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D\right] + \lambda_b^2 \left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D\right]$$

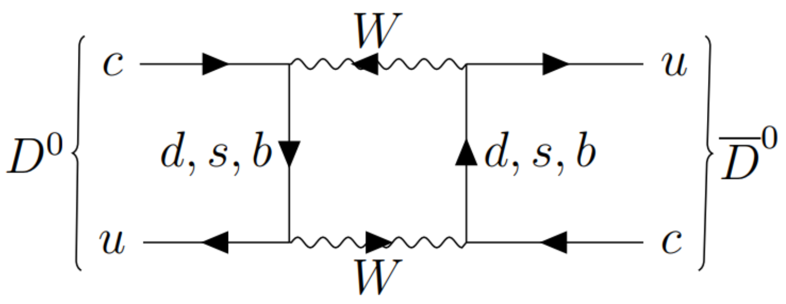
$$\begin{aligned}\Gamma_{12} = &\left(2.08 \cdot 10^{-7} - 1.34 \cdot 10^{-11} I\right) \text{(1st term)} \\ &- \left(3.74 \cdot 10^{-7} + 8.31 \cdot 10^{-7} I\right) \text{(2nd term)} \\ &+ \left(2.22 \cdot 10^{-8} - 2.5 \cdot 10^{-8} I\right) \text{(3rd term)}.\end{aligned}$$

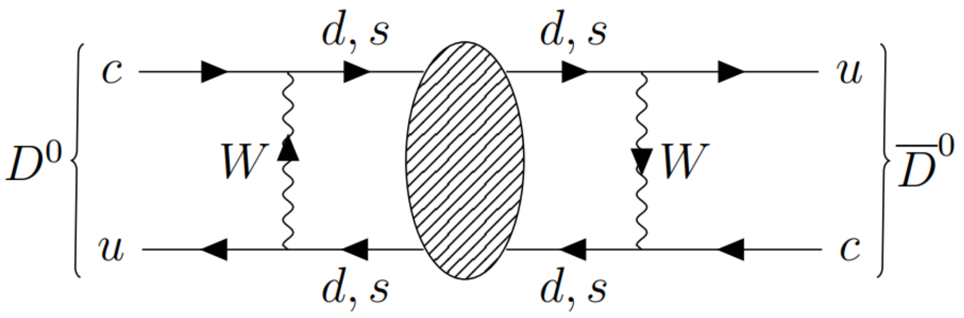
CKM dominant <-> GIM suppressed
CKM suppressed <-> GIM dominant

all three contributions are of the same size and SMALL
(although separate amplitudes are large: $\lambda_s^2 \Gamma_{ss}^D \tau_D \simeq 5.6\, y^{exp}$)

extreme GIM suppression !

$$y^{\text{naive HQE}} \sim \left(10^{-4}, 10^{-5}\right) y^{\text{exp}}$$





the matrix element :

$$2M_D\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)=\langle D^0|\mathcal{H}^{\Delta C=2}|\bar{D}^0\rangle+\sum_n\frac{\langle D^0|\mathcal{H}^{\Delta C=1}|n\rangle\langle n|\mathcal{H}^{\Delta C=1}|\bar{D}^0\rangle}{M_D-E_n+i0^+}$$

M_{12}, local contribution at $\mu \sim M_D$

M_{12}, Γ_{12} , intermediate states ($\pi\pi, \pi K, KK$ ...) contribution at $\mu \ll M_D$

INCLUSIVE (perturbative, HQE) APPROACH

EXCLUSIVE (nonperturbative) APPROACH

DISPERSIVE APPROACH – x and y are connected

LATTICE /HQET sum rules $\Delta C = 2$ operators only

lattice - Bazavov et al (Fermilab Lattice and MILC) 1706.04622
HQET sum rules - Kirk, Lenz , Rauch, 1711.02100

General solution to the problem in the HQE approach -> LIFTING THE GIM SUPPRESSION

INCLUSIVE HQE APPROACH

- SU(3) breaking by NLO and mass corrections
- inclusion of new, higher operators
- different renormalization scales in the process
- quark-hadron duality violation

Golowich, Petrov, 0506185 - NLO corrections
Bobrowski, Lenz, Riedl, Rohrwild, 0904.3971 - alphaS and mass corrections
Bobrowski, Lenz, Riedl, Rohrwild, 1002.4794
Bigi, Uraltsev, 0005089 – quark-hadron duality; suggestion for higher dim operators
Bobrowski, Lenz, Rauh, 1208.6438 - higher dim operators - dim 9
Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 1603.07770 - quark-hadron duality violation
Umeeda, 2106.06215 - quark-hadron duality violation in the t'Hooft model
Lenz, Piscopo, Vlahos, 2007.03022 - different scales in the process

EXCLUSIVE APPROACH

- SU(3) breaking
- inclusion of multi-body states
- quark-hadron duality violation
- topological amplitude approach

Falk, Grossmann, Ligeti, Petrov, 0110317 - SU(3) breaking
H-Y Cheng, Chiang, 1005.1106
Jiang, Yu, Qiu, H-n Li, C-D Lu, 1705.07335 - topological amplitudes
Gershon, Libby, Wilkinson, 1506.08594 - inclusion of multi-body states

DISPERSIVE APPROACH

- SU(3) breaking through physical thresholds of different D meson decay channels for y(s)

Falk, Grossmann, Ligeti, Nir, Petrov, 0402204 - from dispersion relation in HQET limit
H-n. Li, Umeeda, Xu, Yu, 2001.04079 - inverse problem
H-n. Li, 2208.14798

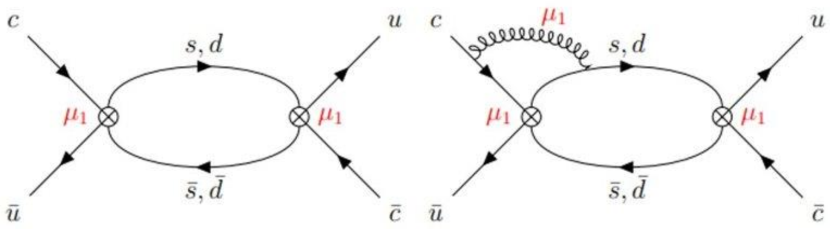
A BRIEF DISCUSSION FO DIFFERENT APPROACHES

INCLUSIVE APPROACH in general gives the mixing parameters *x* and/or *y* still far below the current data

- **large NLO corrections?**
  large mass corrections?

$\alpha_s(m_c) \approx 0.34$

$\frac{\Lambda_{QCD}}{m_c} \approx 3\frac{\Lambda_{QCD}}{m_b}$

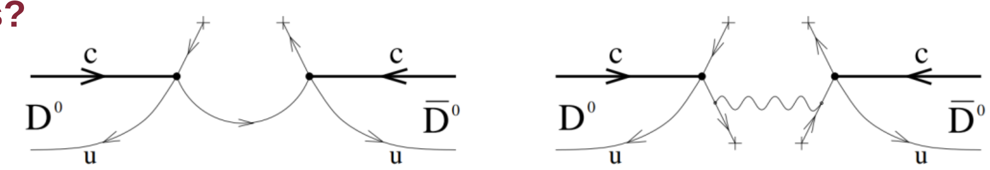


Golowich, Petrov, 0506185 - NLO corrections
Bobrowski, Lenz, Riedl, Rohrwild, 0904.3971 -NLO and mass corr.
Bobrowski, Lenz, Riedl, Rohrwild, 1002.4794

$y_D \sim x_D \simeq 6 \cdot 10^{-7}$

QCD corrections lower the GIM suppression of the first term by on power of z= m_s^2/m_c^2 (from z^3 to z^2)

- **higher dimensional operators?**



Bigi, Uraltsev, 0005089 suggestion for higher dim operators
Bobrowski, Lenz, Rauh, 1208.6438 - higher dim operators - dim 9

-> an enhancement by a factor of 10 by still below the observation

SU(3) suppression is softened by cutting one or two quark lines -> dim=9 , dim=12 operators -> this requires information on a large number of nonperturbative matrix elements

- **quark-hadron duality violation?**

  a simple model for duality violation

Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 1603.07770

-> 20% duality violation could explain the width difference

- **renormalization scale setting?**

different internal quark pairs contribute different channels and their renormalization scale need not to be equal -> $\mu_1^{q_1 q_2}$ instead $\mu_x^{ss} = \mu_x^{sd} = \mu_x^{dd} = \mu$

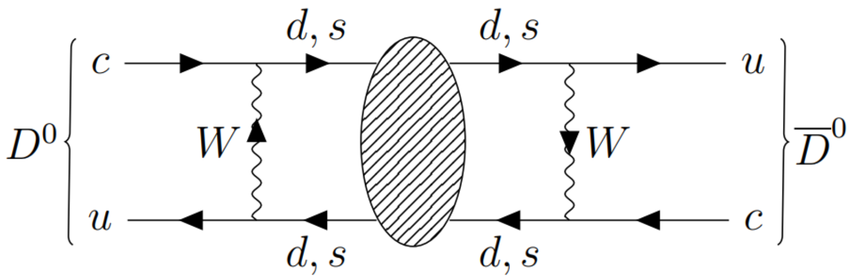
Lenz, Piscopo, Vlahos, 2007.03022

-> specific choice could give experimental values

EXCLUSIVE APPROACH

$$\Gamma_{12}^D = \sum_n \rho_n \langle \overline{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle ,$$

$$M_{12}^D = \sum_n \langle \overline{D}^0 | \mathcal{H}_{eff.}^{\Delta C=2} | D^0 \rangle + P \sum_n \frac{\langle \overline{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2}$$



- n= pi pi, pi K, K K, ….
  pi pi pi, pi pi K, pi K K, K K K, pi pi pi pi,….

Falk, Grossmann, Ligeti, Petrov, 0110317 - SU(3) breaking
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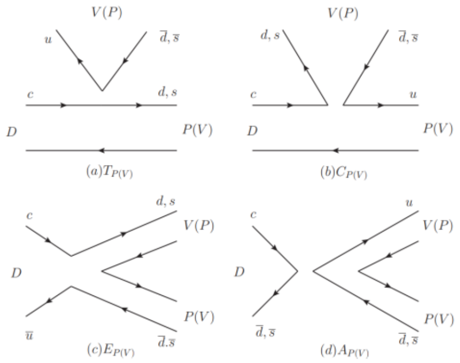
-> experimental bounds can be satisfied

- based on topological parametrization of the amplitudes

D0 -> PP, PV, (VV-negligible) modes



Jiang, Yu, Qiu, H-n Li, C-D Lu, 1705.07335 - topological amplitudes

-> cannot resolve the problem: y(obtained) ~ 1/3 y(exp)

$$y_{PP} = (0.10 \pm 0.02)\%,\ y_{PV} = (0.11 \pm 0.07)\%$$

$$y_{VV} = (-0.42 \pm 0.34) \times 10^{-3}$$

topological amplitudes : color-favored tree-emission diagram T
color-suppressed tree-emission diagram C
W-exchange diagram E
W-annihilation diagram A

naive factorization + nonfactorizable contributions (FSI) are parametrized and determined from the global fit to the data (H-n Li et al, 1203.3120, 1305.7021) + SU(3) breaking

DISPERSIVE APPROACH

Use of the dispersion relation between Δm and $\Delta\Gamma$ (x and y)

dispersive approach in HQET limit

Falk, Grossmann, Ligeti, Nir, Petrov, 0402204

correlator: $$\Sigma_{p_D}(q) = i \int \mathrm{d}^4 z \, \langle \overline{D}(p_D) | \, T\left[\mathcal{H}_w(z)\, \mathcal{H}_w(0)\right] | D(p_D) \rangle \, e^{i(q-p_D)\cdot z}$$

$$-\frac{1}{2m_D}\Sigma_{p_D}(p_D) = \left(\Delta m - \frac{i}{2}\Delta\Gamma\right)$$

general $\Sigma_{p_D}(q)$: ➡ $$\Delta m = -\frac{1}{2\pi} \mathrm{P} \int_{2m_\pi}^{\infty} \mathrm{d}E \left[\frac{\Delta\Gamma(E)}{E - m_D} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{E}\right)\right]$$

with models for y (E) , it is possible to get x -> x~ y
however. the derivation was in HQET limit

dispersive approach as an inverse problem - the nonperturbative observables at low mass are solved with the perturbative inputs from high mass.

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s - q^2 - i\varepsilon},$$

$$\mathrm{Re}[\Pi(s)] = \frac{1}{\pi} P \int_0^{\infty} \frac{\mathrm{Im}\left[\Pi(s')\right]}{s - s'} ds'$$

$$M_{12}(s) - \frac{i}{2}\Gamma_{12}(s) = \langle D^0(s) | \mathcal{H}_w^{\Delta C=2} | \bar{D}^0(s) \rangle$$

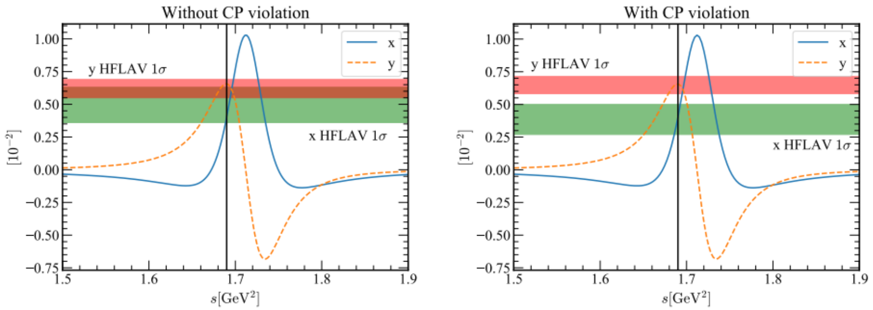
(V-A)(V-A), (S-P)(S-P) operators

H-n. Li, Umeeda, Xu, Yu, 2001.04079
H-n. Li, 2208.14798

it is possible to find a solutions $\{x(m_D), y(m_D)\}$ which accomodates the data: $y(m_D)=0.52\%$, $x(m_D) = 0.21\%$

$$\int_0^{\Lambda} ds' \frac{y(s')}{s - s'} = \pi x(s) - \int_{\Lambda}^{\infty} ds' \frac{y(s')}{s - s'} \equiv \omega(s)$$

s=fictitious D-meson mass



RHS - PERTURBATIVE PART FROM BOX DIAGRAMS

INVERSE PROBLEM

y(s) ➡ x(s) from $M_{12}(s) = \frac{P}{2\pi}\int_0^{\infty} ds' \frac{\Gamma_{12}(s')}{s - s'}$

different physical thresholds of various channels introduce SU(3) breaking; the channel with KK states is a major source of the needed enhancement from the (S-P)(S-P) eff. operator (confirmed by the lattice) – 4 orders of magnitudes larger $y(m_D)$ is obtained which then explains the data

possible caveat: inverse problem of a dispersion relation is ill –posed (unstable solutions)– it needs regularization Xiong, Wei, F-S Yu, 2211.13753

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H-n. Li, Umeeda, Xu, Yu, 2001.04079
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$$M_{12}(s) - \frac{i}{2}\Gamma_{12}(s) = \langle D^0(s)|\mathcal{H}_w^{\Delta C=2}|\bar{D}^0(s)\rangle$$

$$\mathrm{Re}[\Pi(s)] = \frac{1}{\pi}P\int_0^{\infty}\frac{\mathrm{Im}\,[\Pi(s')]}{s-s'}ds'$$

(V-A)(V-A), (S-P)(S-P) operators

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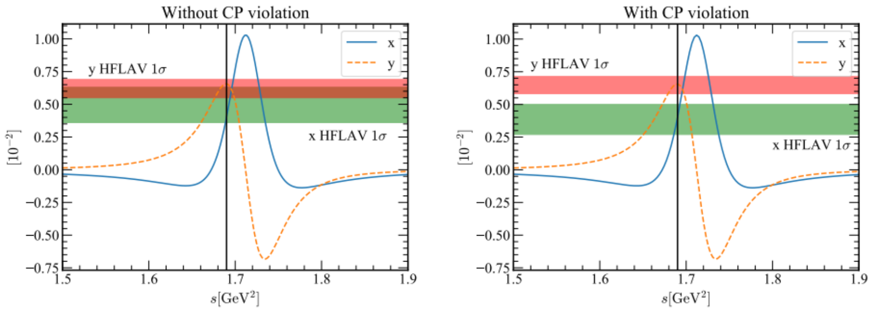

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Conclusion: $D^0 - \overline{D^0}$ MIXING PROBLEM

– STILL LOT OF WORK TO DO -

THANK YOU !