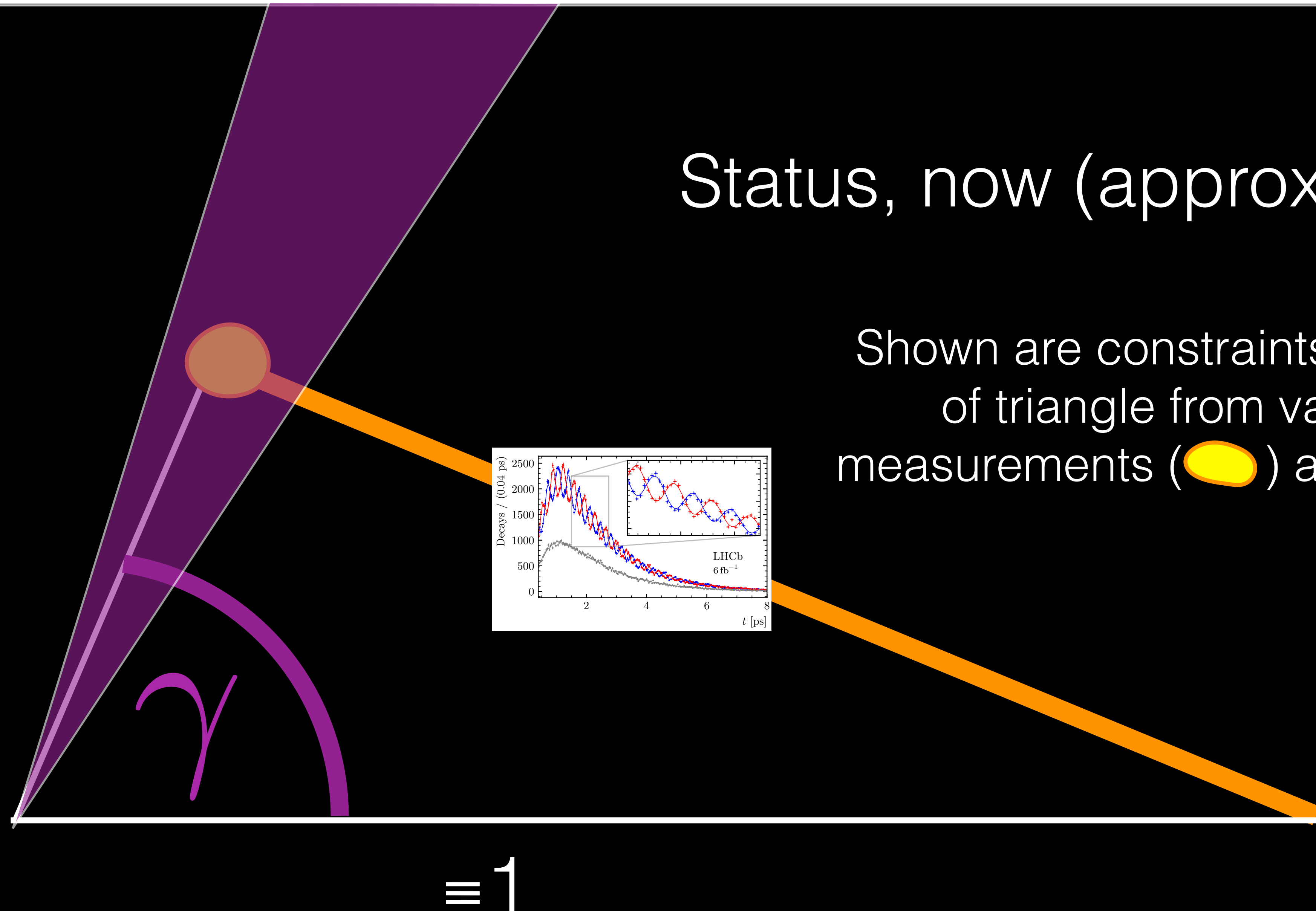
New unbinned method to combine $\psi(3770) \to D\overline{D}$ and $B \to DK$ data

Making γ more precise by optimising information usage.

E Gersabeck, J Lane, J Rademacker:
arXiv:2305.10787 (2023)

Unitarity triangle



Status, now (approximately)

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

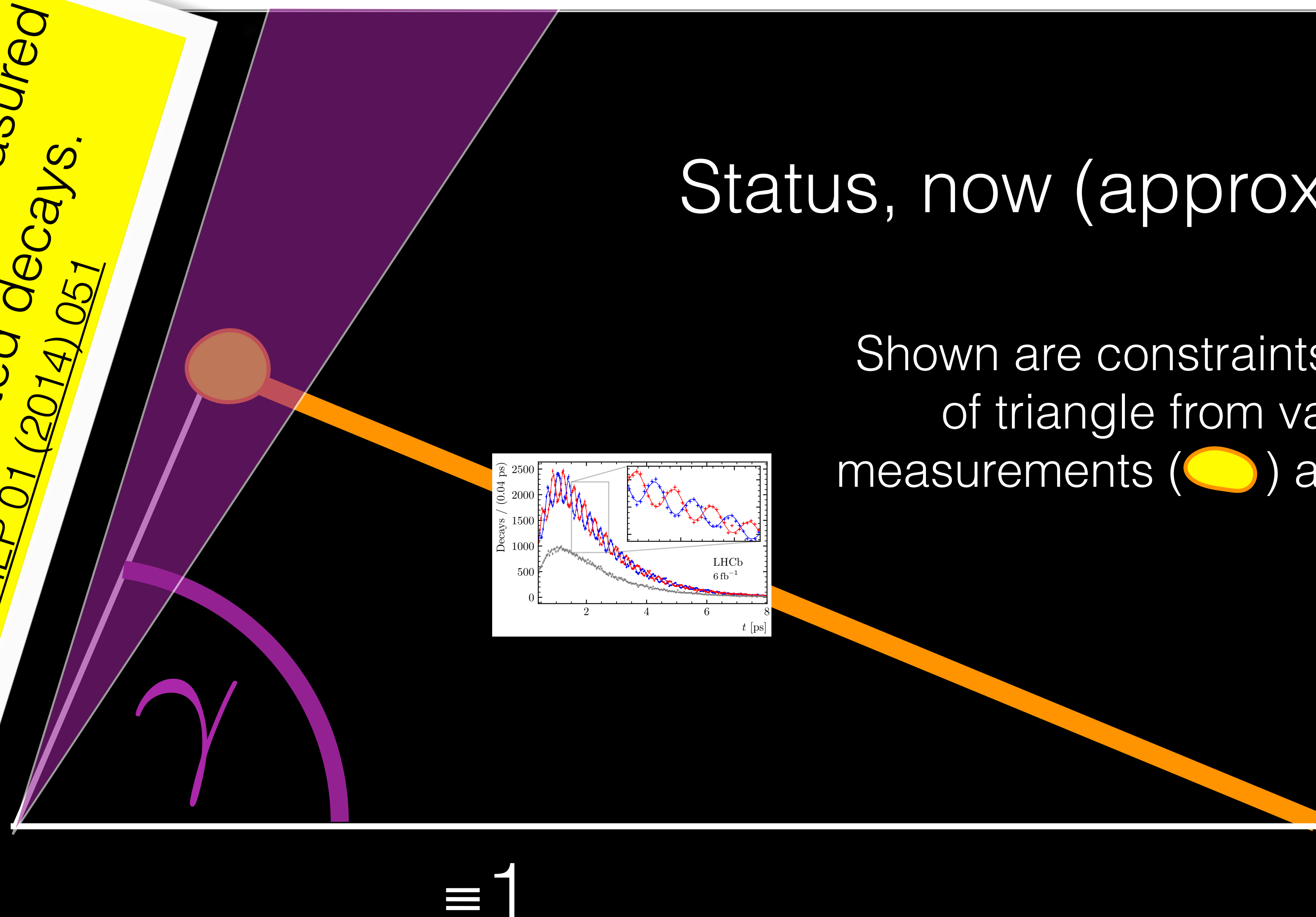
Unitarity triangle

Negligible theory uncertainty on γ measured in $B^{\pm} \to DK^{\pm}$ and related decays.

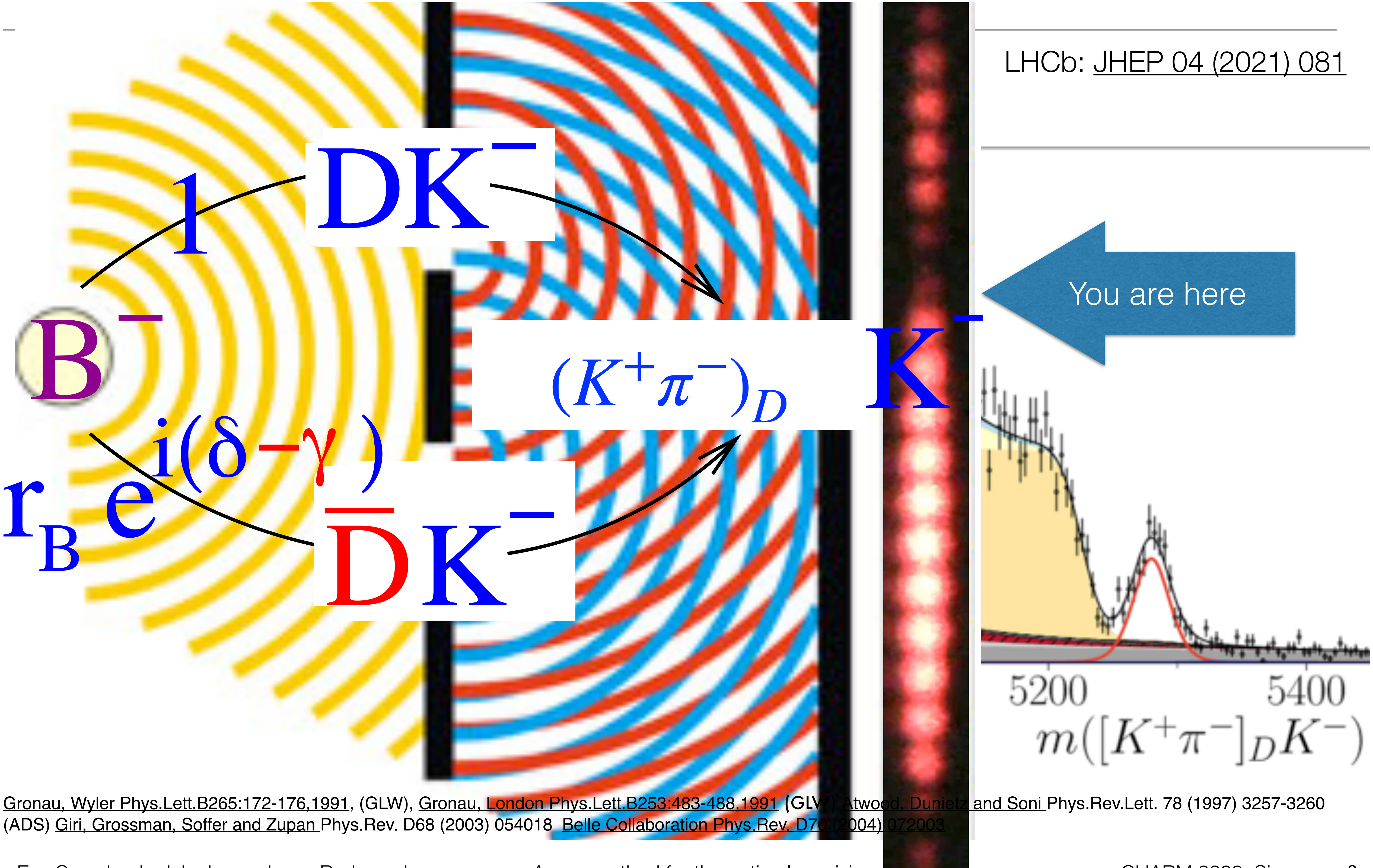
Brod, Zupan JHEP 01 (2014) 051

Status, now (approximately)

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )



CP violation is an interference effect



LHCb: JHEP 04 (2021) 081

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

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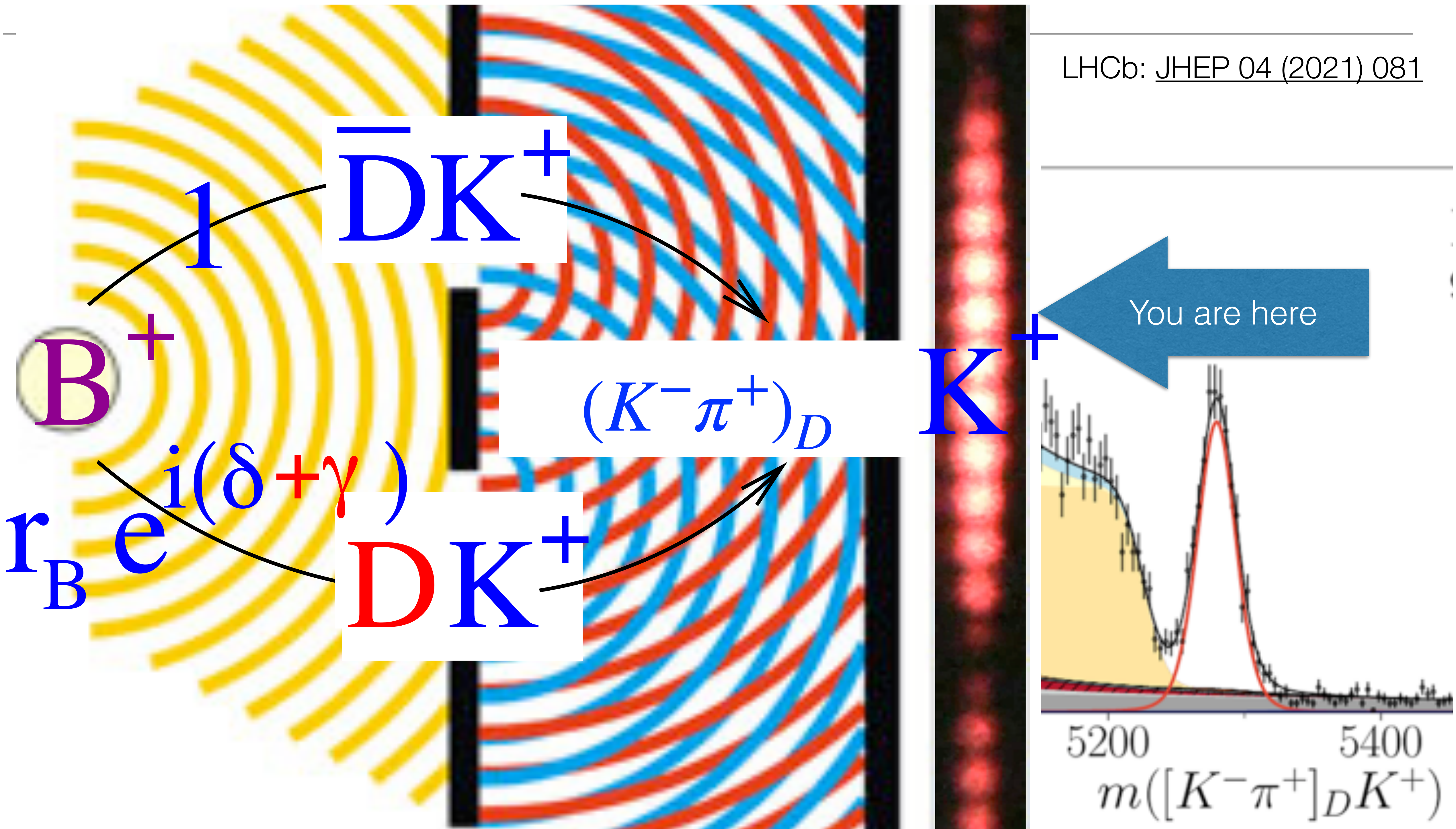

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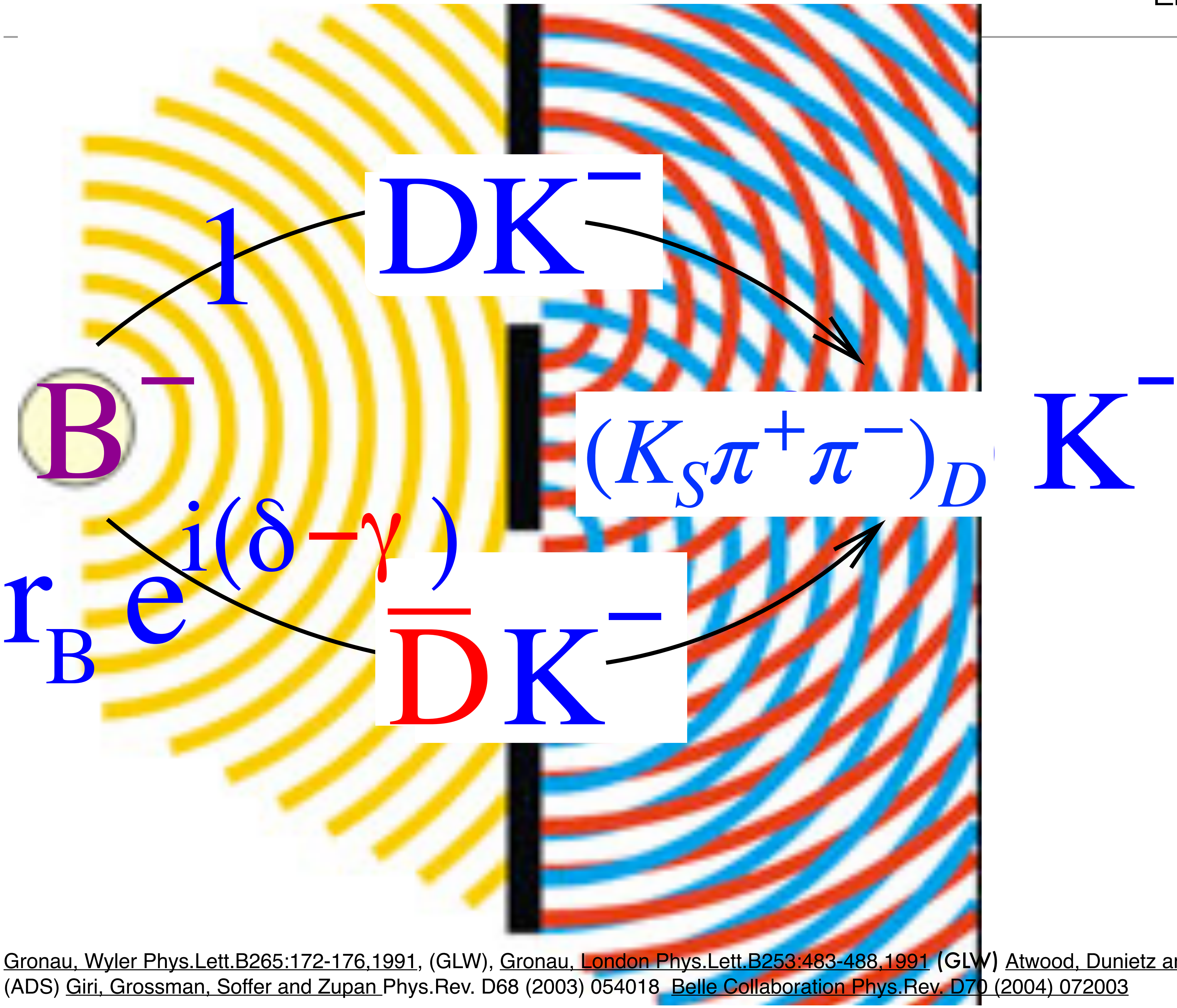
CP violation is an interference effect

LHCb: JHEP 02 (2021) 169



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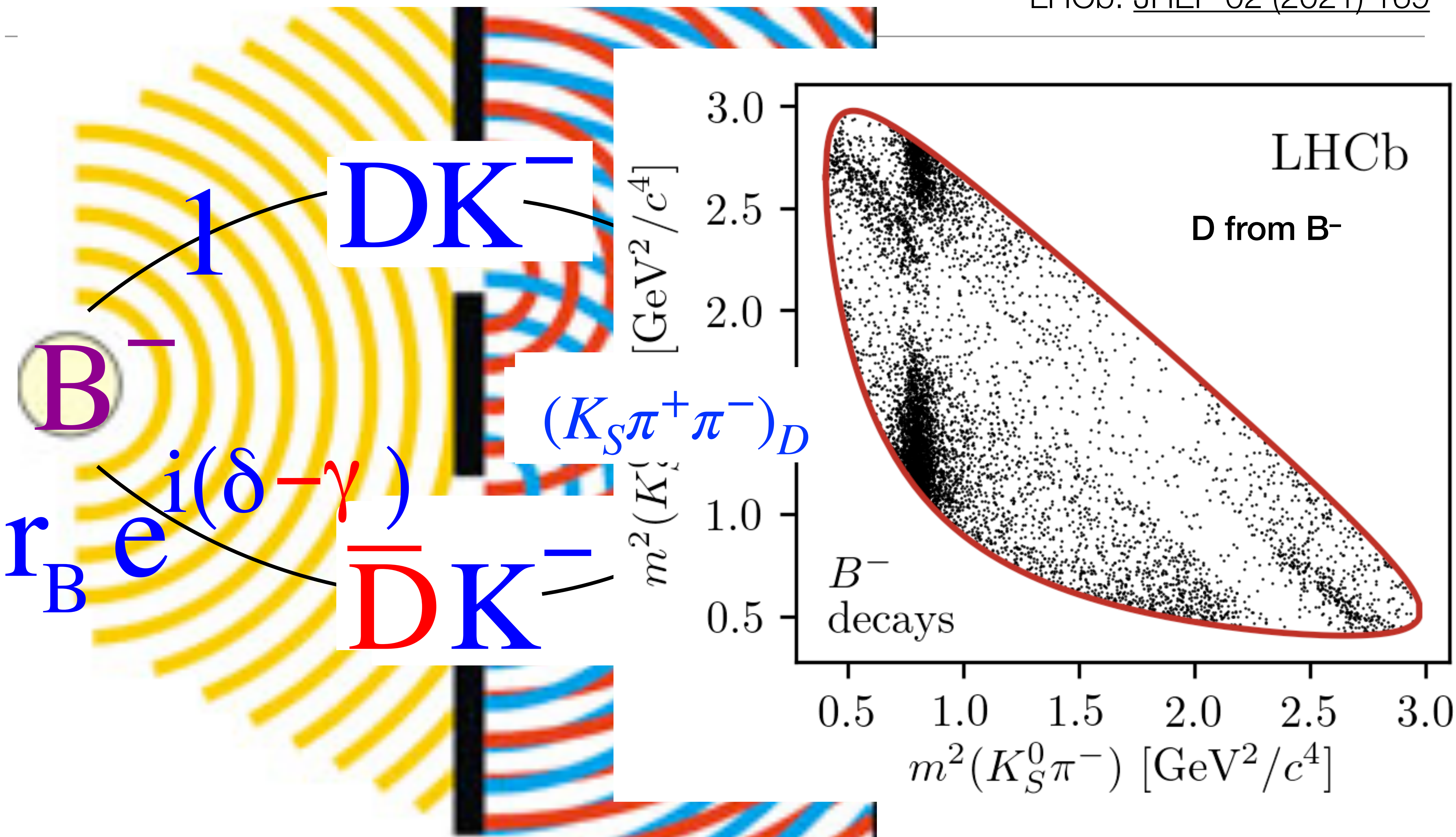
CP violation is an interference effect

LHCb: JHEP 02 (2021) 169



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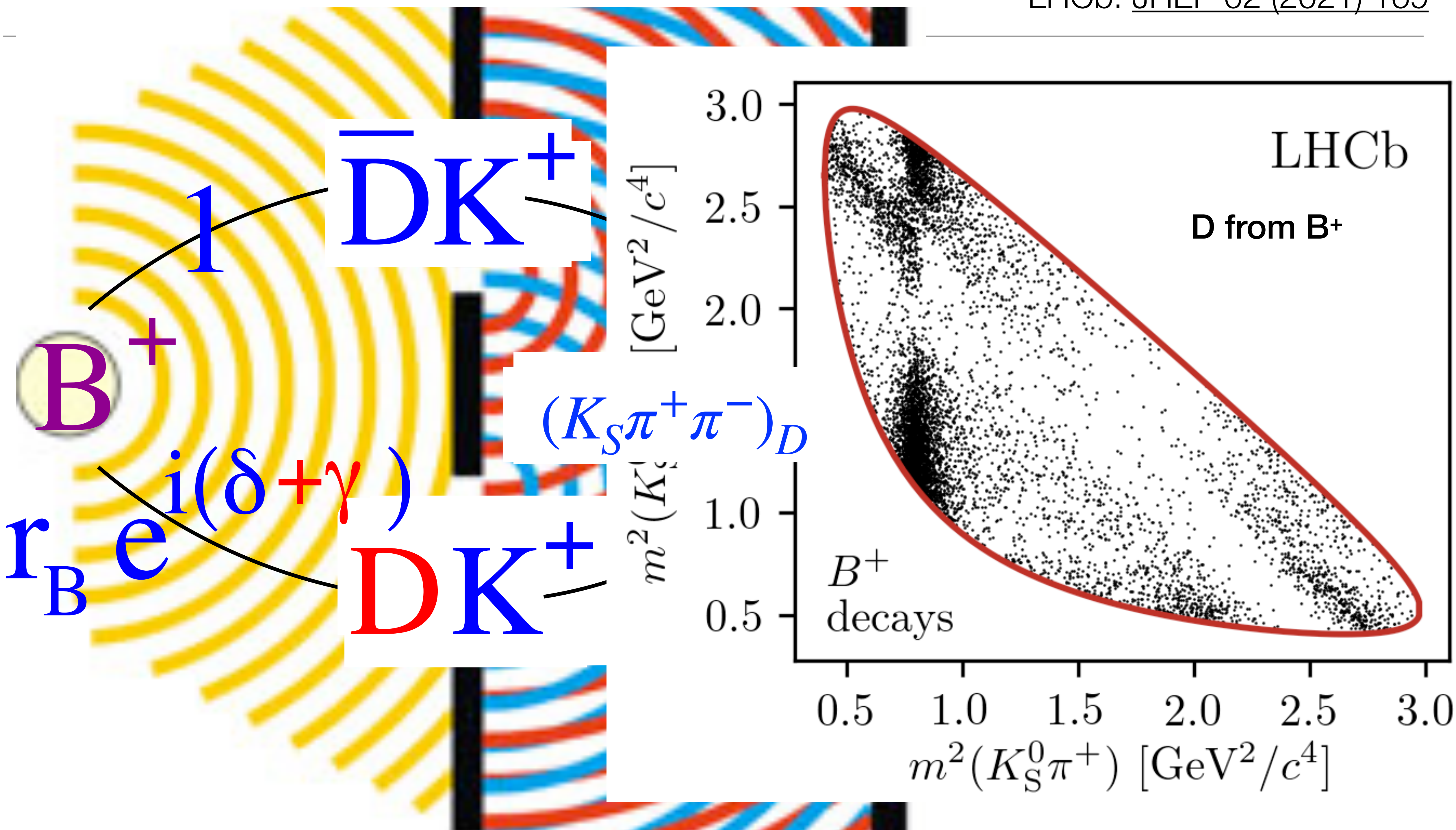
CP violation is an interference effect

LHCb: JHEP 02 (2021) 169



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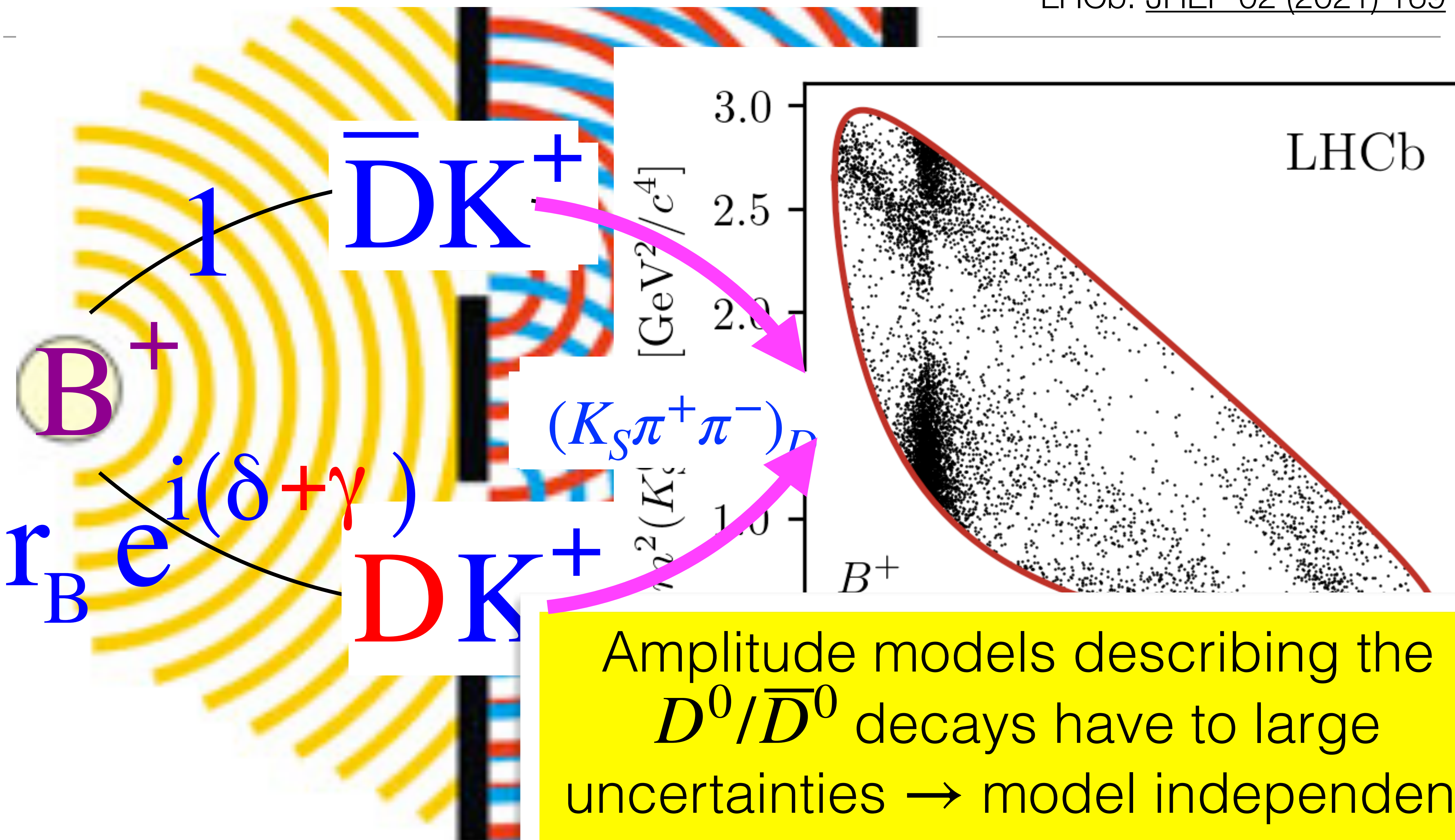

Amplitude models describing the $D^0/\overline{D}^0$ decays have to large uncertainties → model independent methods

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau,
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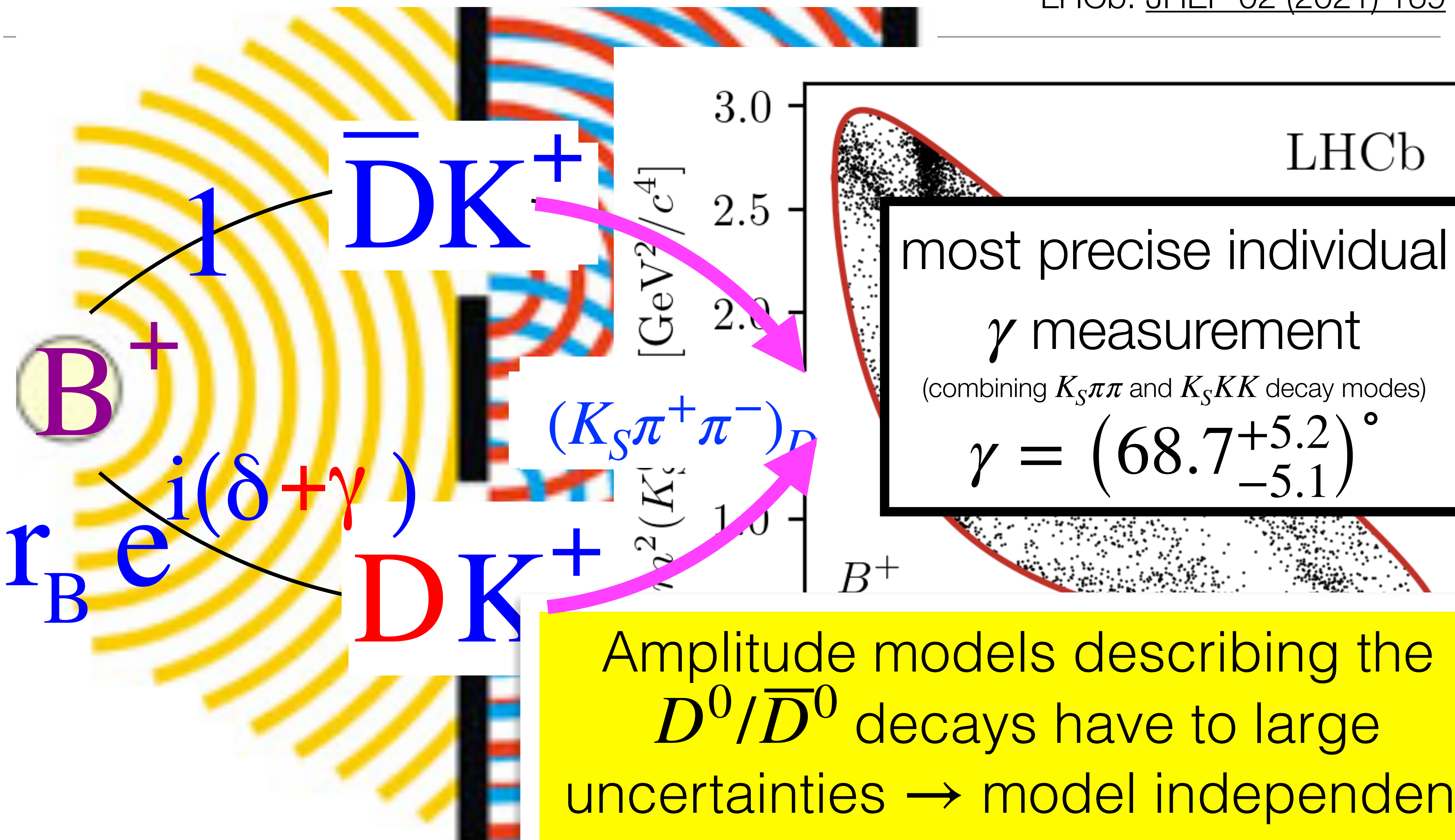
LHCb: JHEP 02 (2021) 169



most precise individual γ measurement

(combining $K_S\pi\pi$ and K_SKK decay modes)

$$\gamma = \left(68.7^{+5.2}_{-5.1}\right)^\circ$$

Amplitude models describing the $D^0/\overline{D}^0$ decays have to large uncertainties → model independent methods

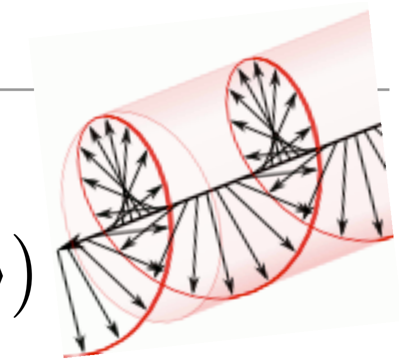
Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau,
(ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003

CLEO-c/BES III's unique data provide additional information on D mesons.

analogous to linear and circular polarised light

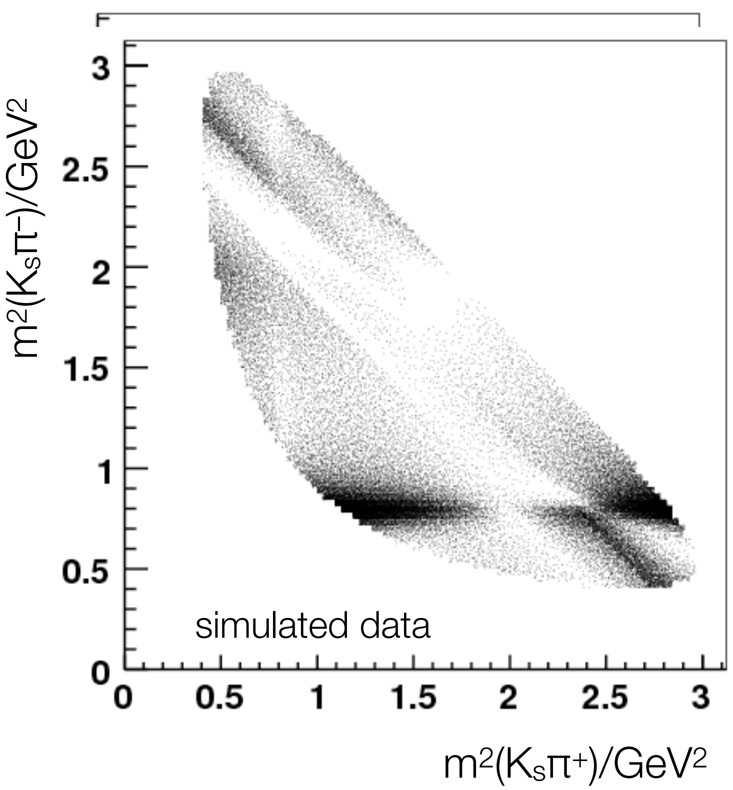
$|D\rangle$

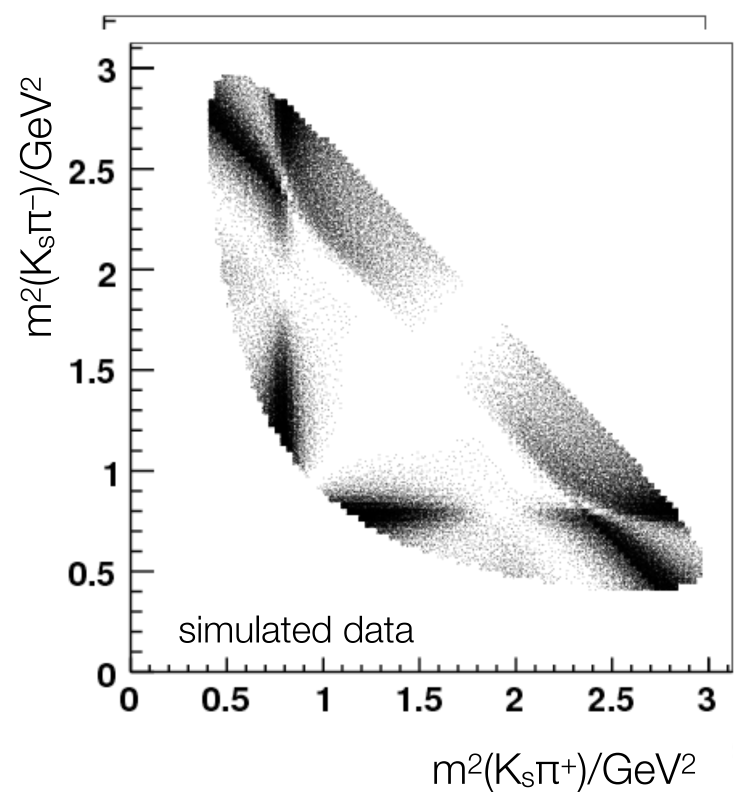
$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |\overline{D}\rangle\right)$$



$\to K_s\pi^+\pi^-$

$\to K_s\pi^+\pi^-$

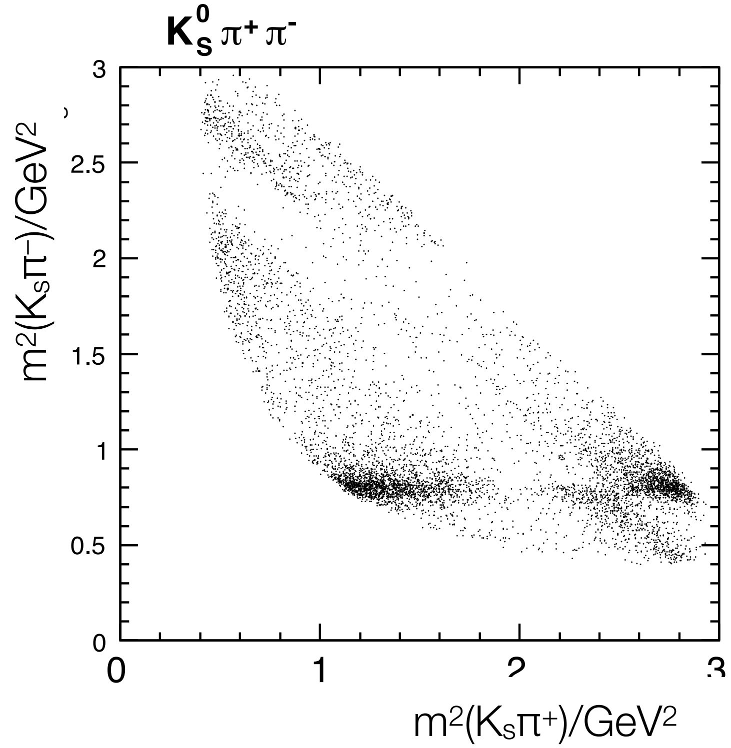




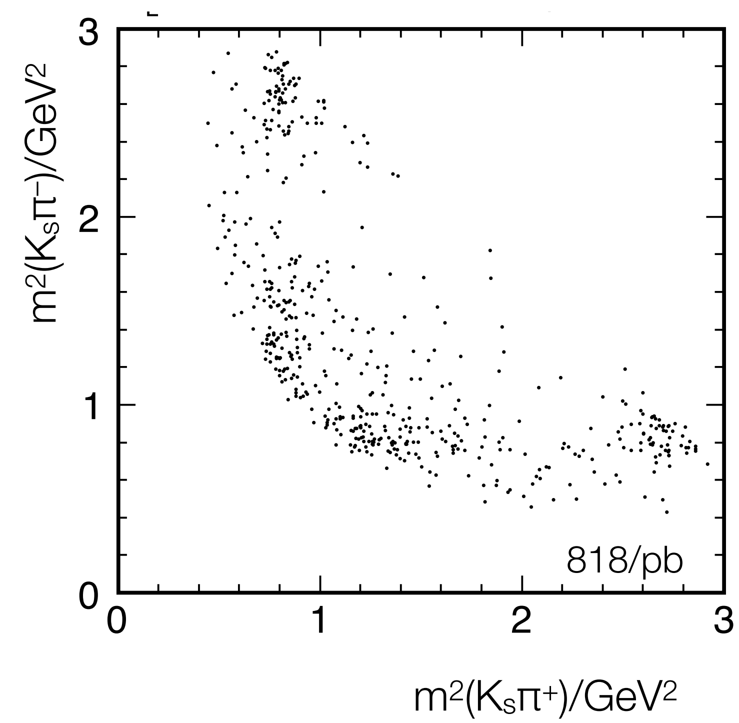
<u>Two</u> measurements for each point in Dalitz space - can extract magnitude <u>and</u> phase!

CLEO-c/BES III's unique data provide additional information on D mesons.

$$|D\rangle \rightarrow K_s\pi^+\pi^-$$



$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |\overline{D}\rangle\right) \rightarrow K_s\pi^+\pi^-$$



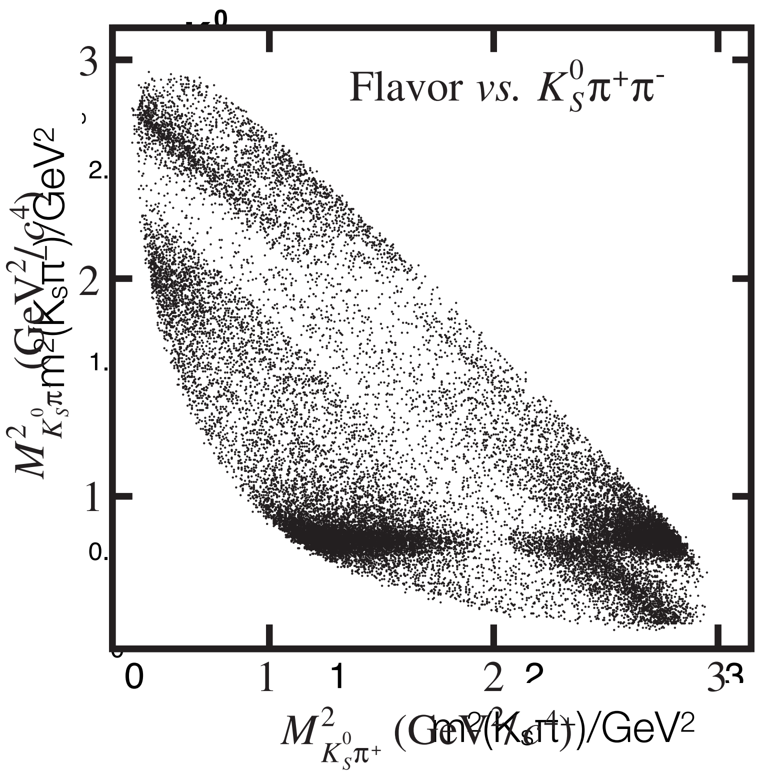
Two measurements for each point in Dalitz space - can extract magnitude and phase!

BESIII: PRL 124 (2020) 24, 241802

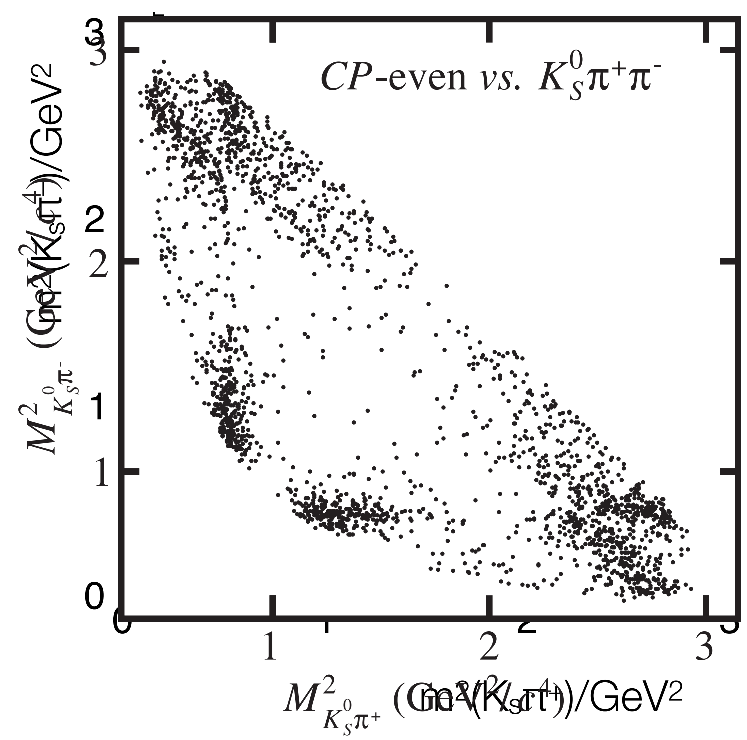
CLEO-c: Phys. Rev. D80, 032002 (2009), updated in Phys.Rev. D82 (2010) 112006

CLEO-c/BES III's unique data provide additional information on D mesons.

$$|D\rangle \to K_s \pi^+ \pi^-$$



$$|D_{CP-}\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |\overline{D}\rangle\right) \to K_s \pi^+ \pi^-$$

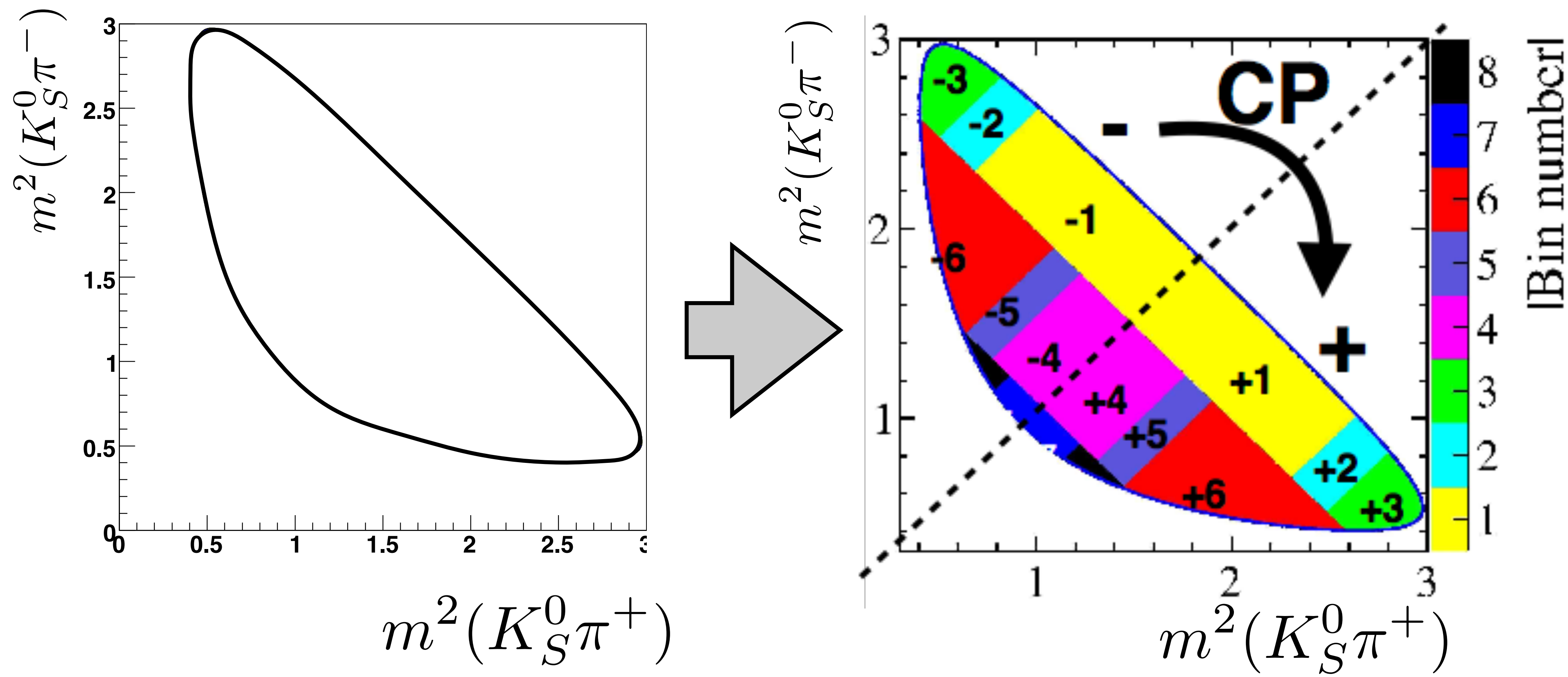


Two measurements for each point in Dalitz space - can extract magnitude and phase!

BESIII: PRL 124 (2020) 24, 241802

CLEO-c: Phys. Rev. D80, 032002 (2009), updated in Phys.Rev. D82 (2010) 112006

Model-independent, binned approach



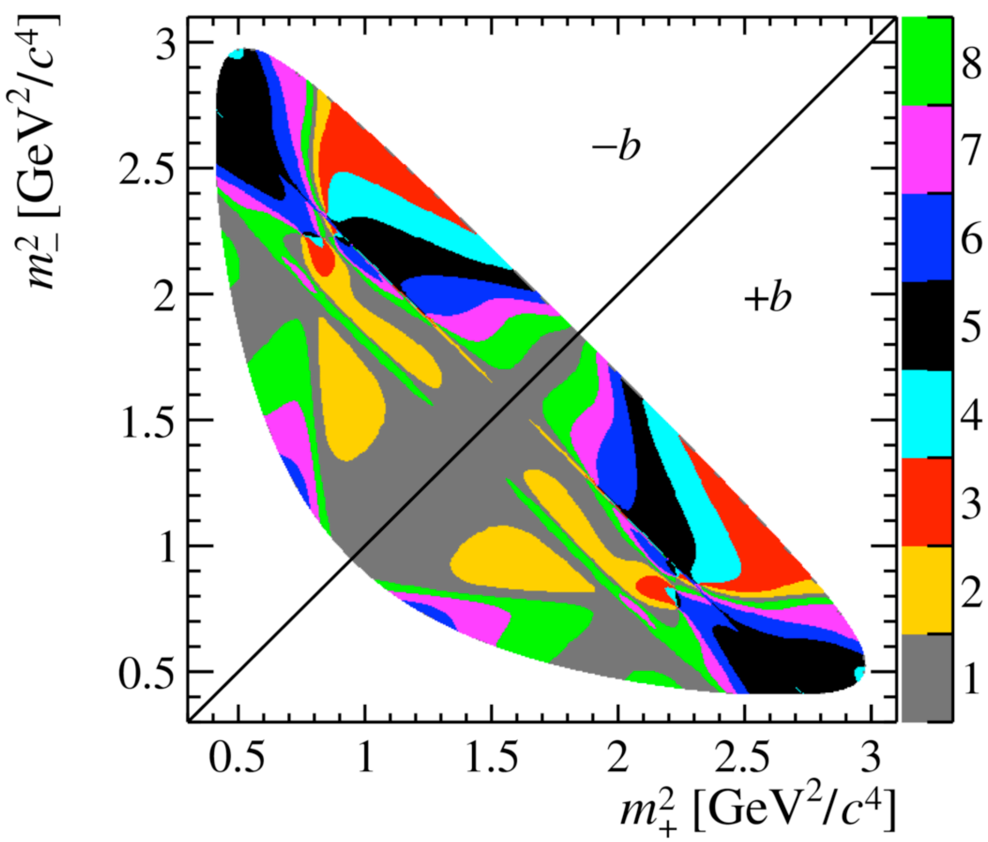
One complex number per bin-pair, $c_i + \mathrm{i}s_i$, contains the key information uniquely accessible at CLEO-c/BES III, which is related to the phases the D^0 and $\overline{D}^0$ decay amplitudes.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

New unbinned method

E Gersabeck, J Lane, JR:
arXiv:2305.10787 (2023)

Carefully optimised binning



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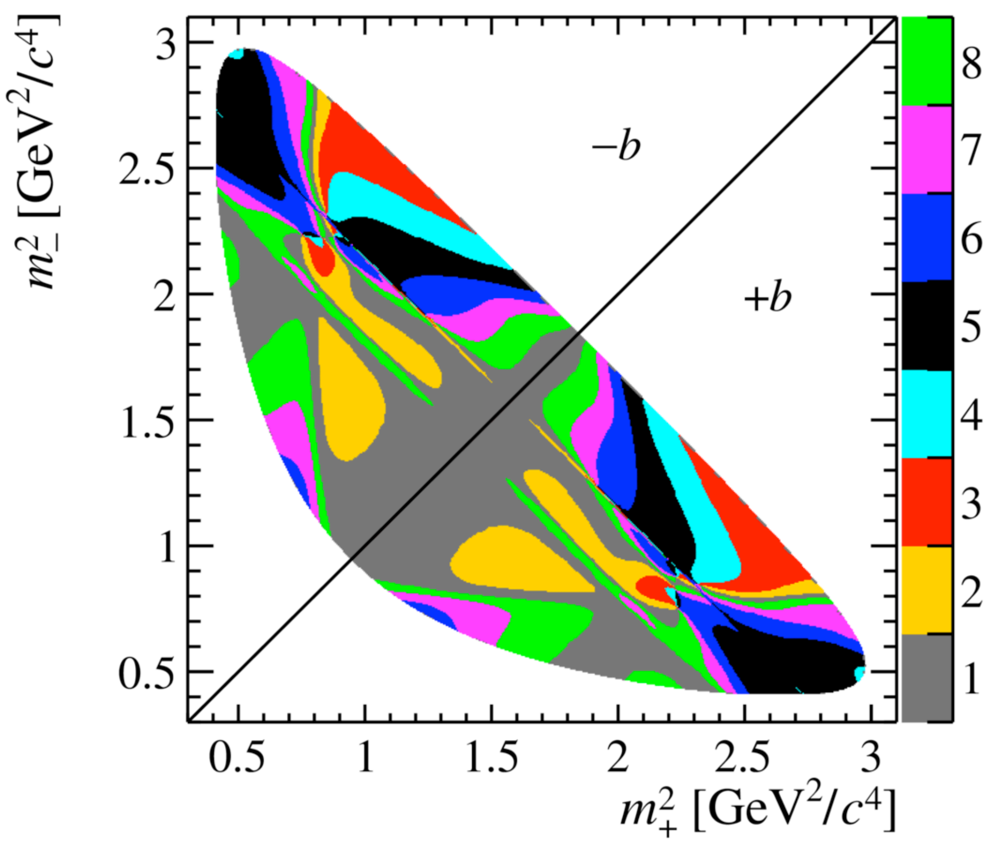

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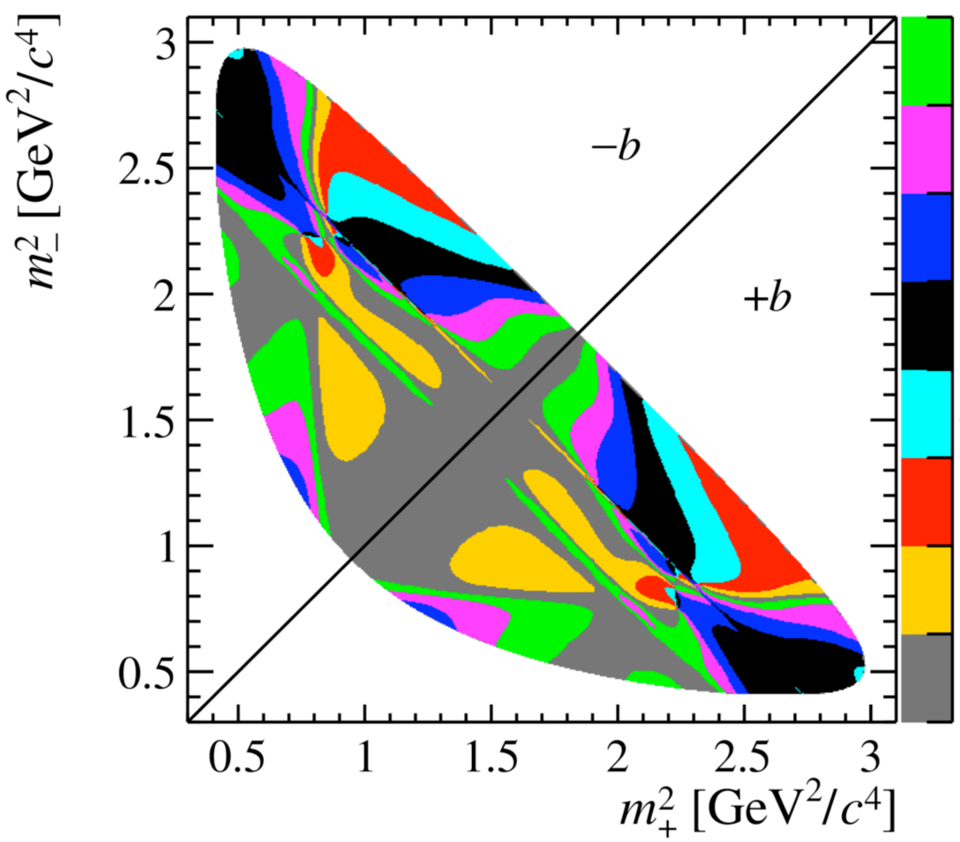
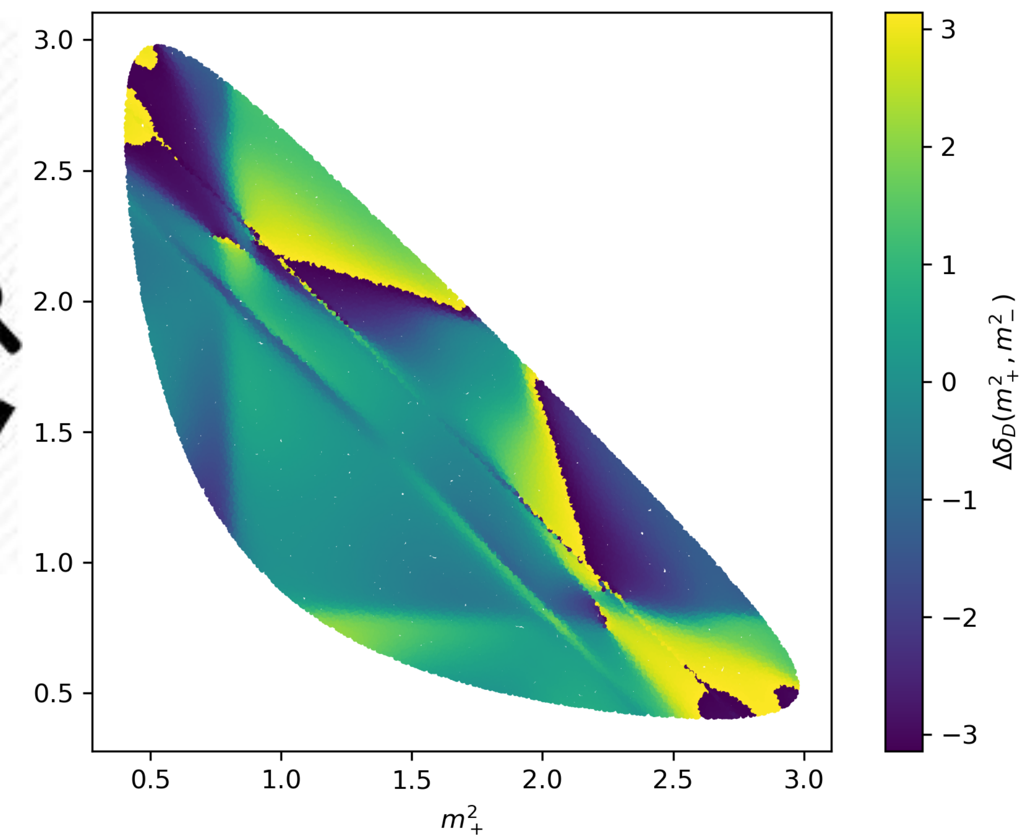


New, unbinned model-independent method



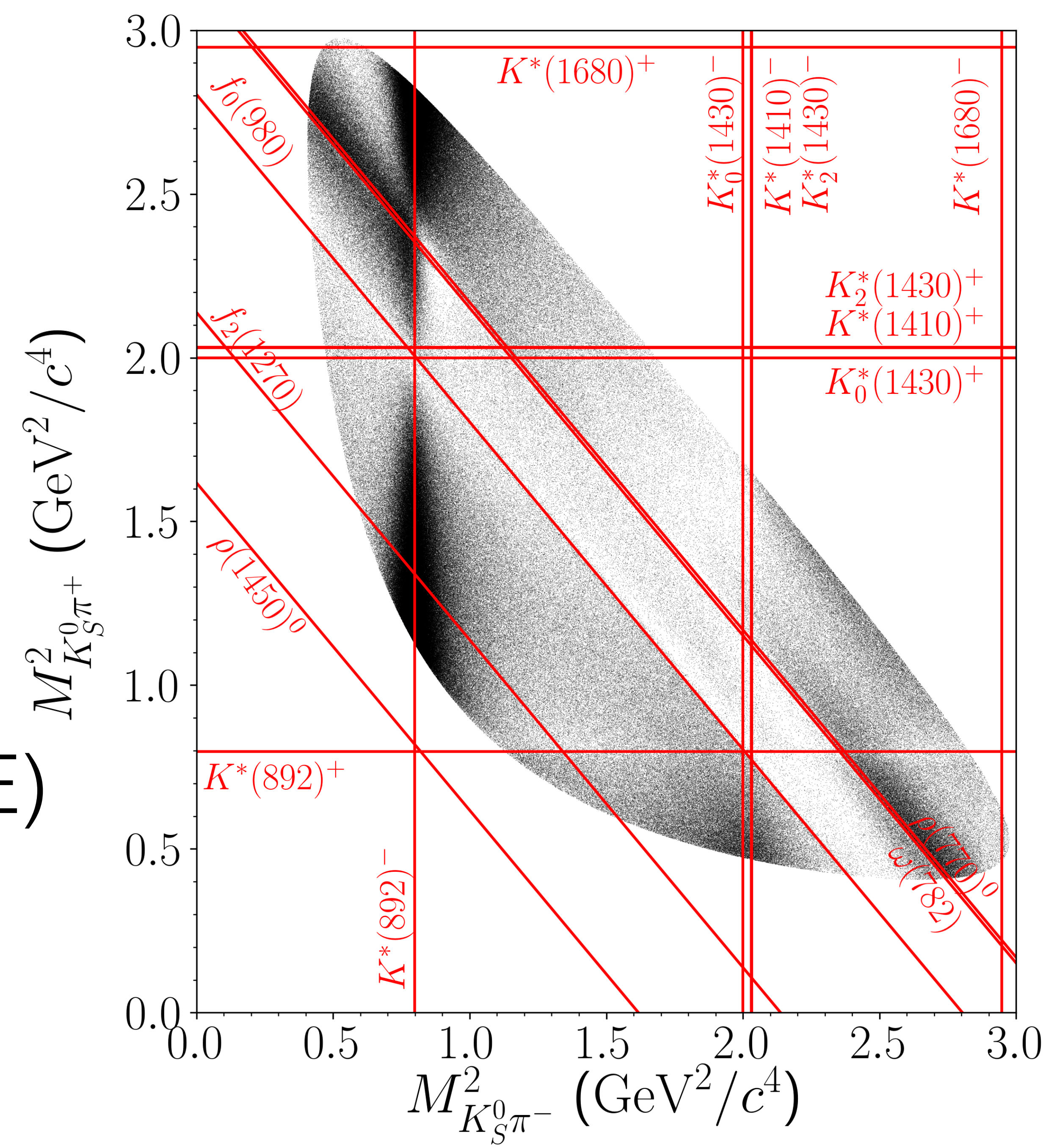
Other unbinned methods exist: Poluektov, Eur.Phys.J.C 78 (2018) 2, 121; Backus et al, arXiv:2211.05133. In contrast to these and the binned method, we do not do any integration, averaging or projection from 2D to 1D, and therefore do not suffer the associated information loss.

BaBar & BELLE $D^0 \to K_S\pi^+\pi^-$ amplitude analysis

PRD 98 (2018) 11, 112012

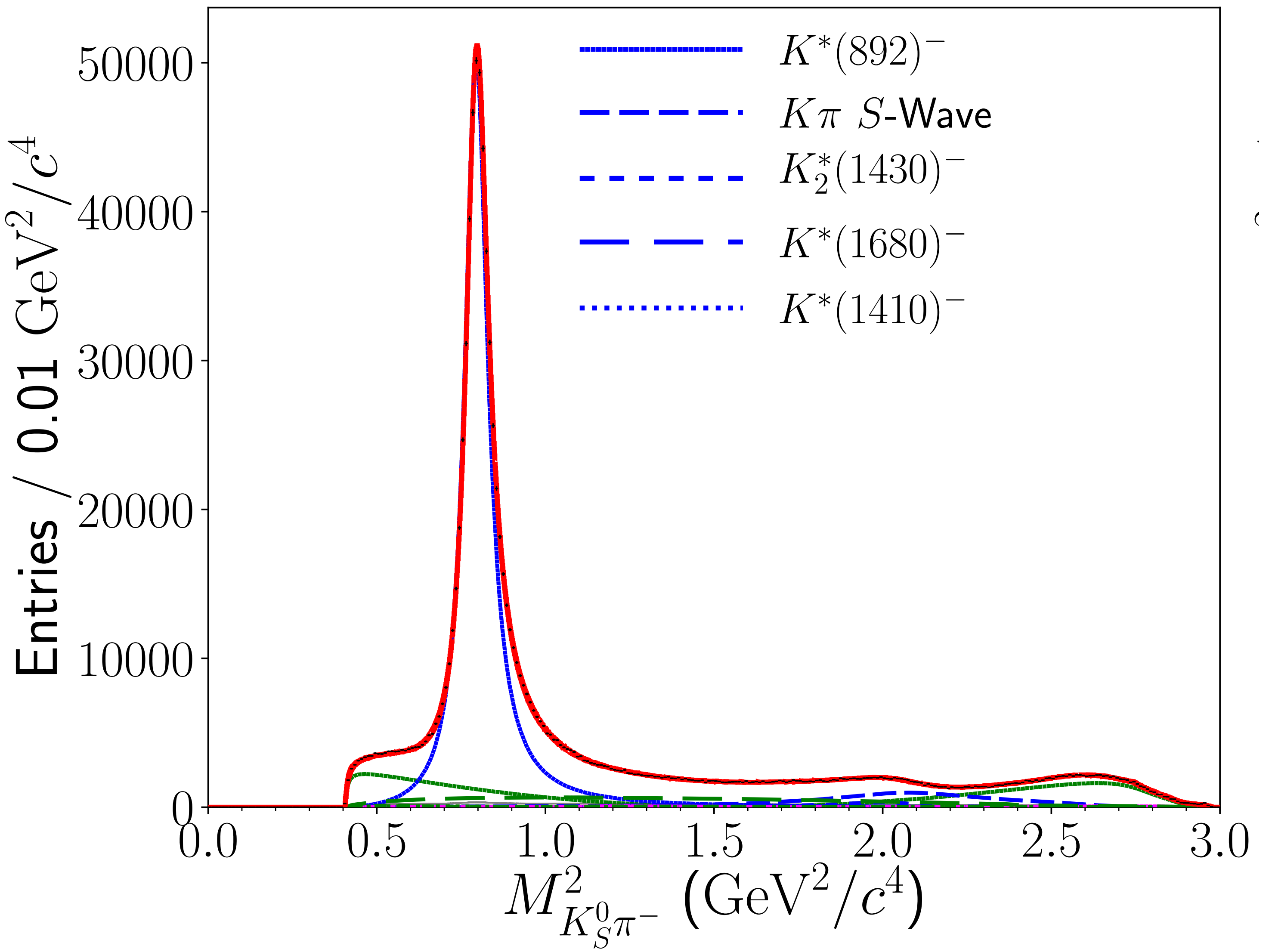
Best statistical precision on γ is achieved with an unbinned model-*dependent* method. So let's have a look at those models.

1.2M signal events (BELLE)
94% signal purity



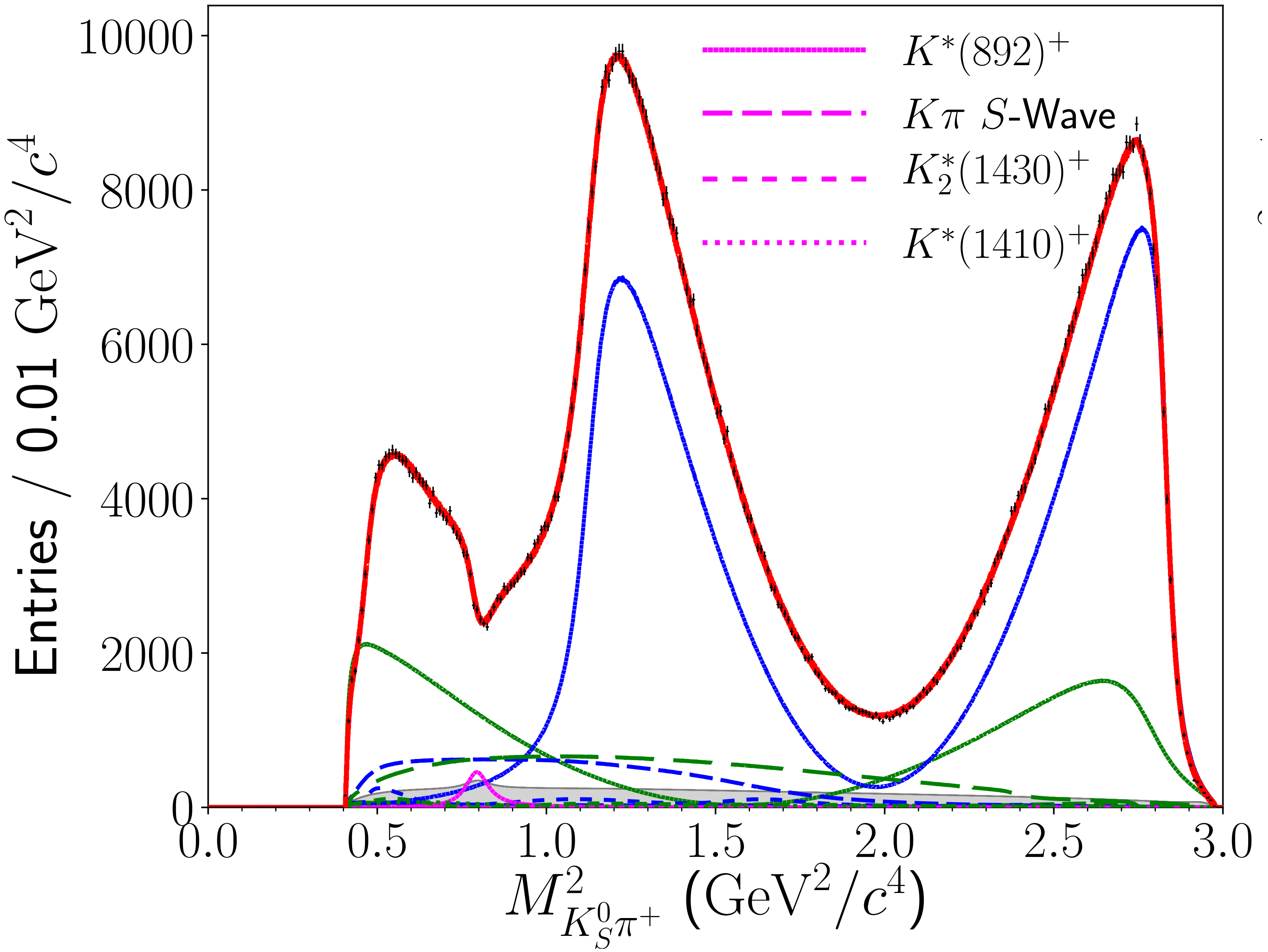
BaBar & BELLE $D^0 \to K_S\pi^+\pi^-$ amplitude analysis

PRD 98 (2018) 11, 112012



BaBar & BELLE $D^0 \to K_S\pi^+\pi^-$ amplitude analysis

PRD 98 (2018) 11, 112012



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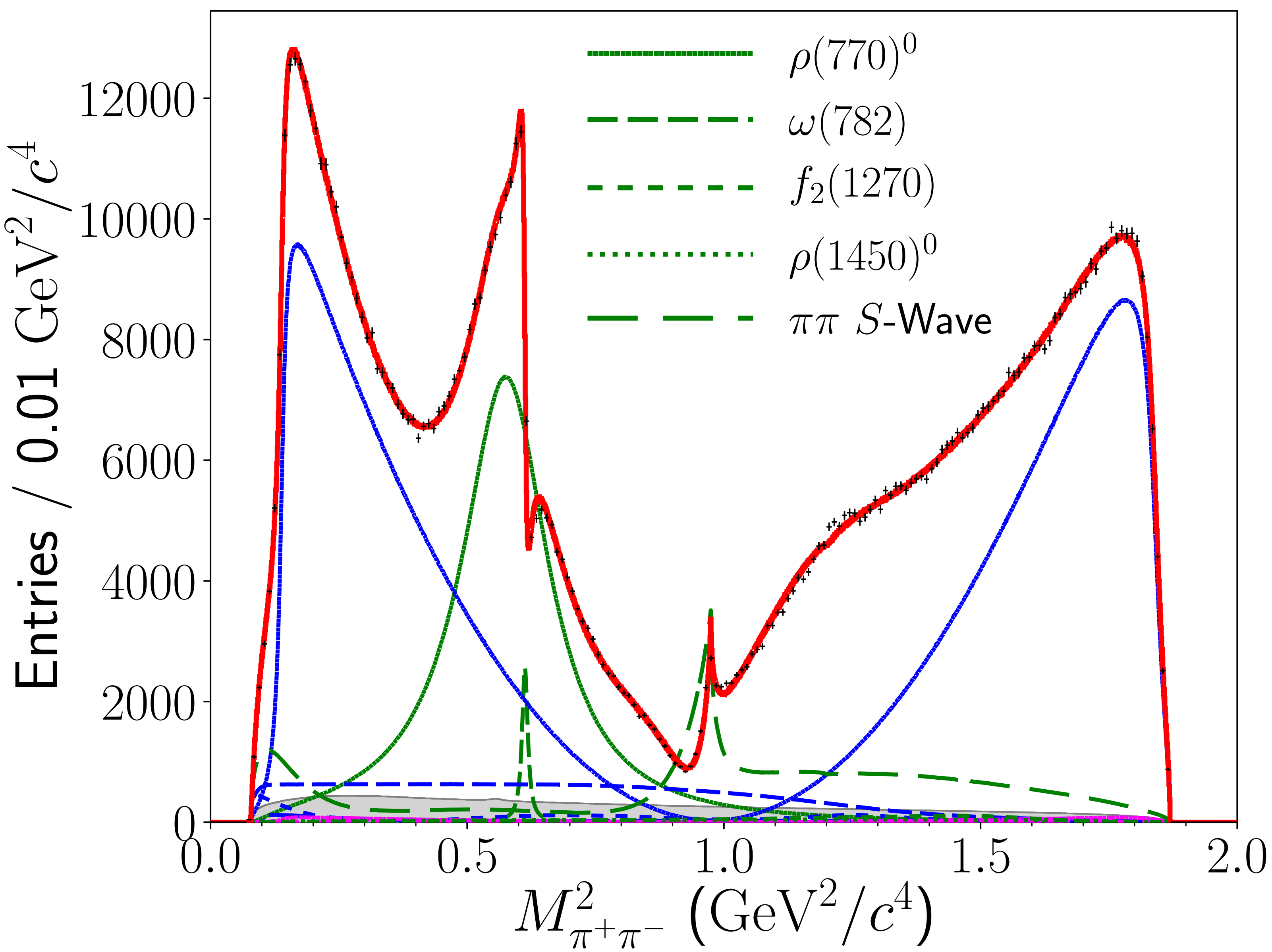
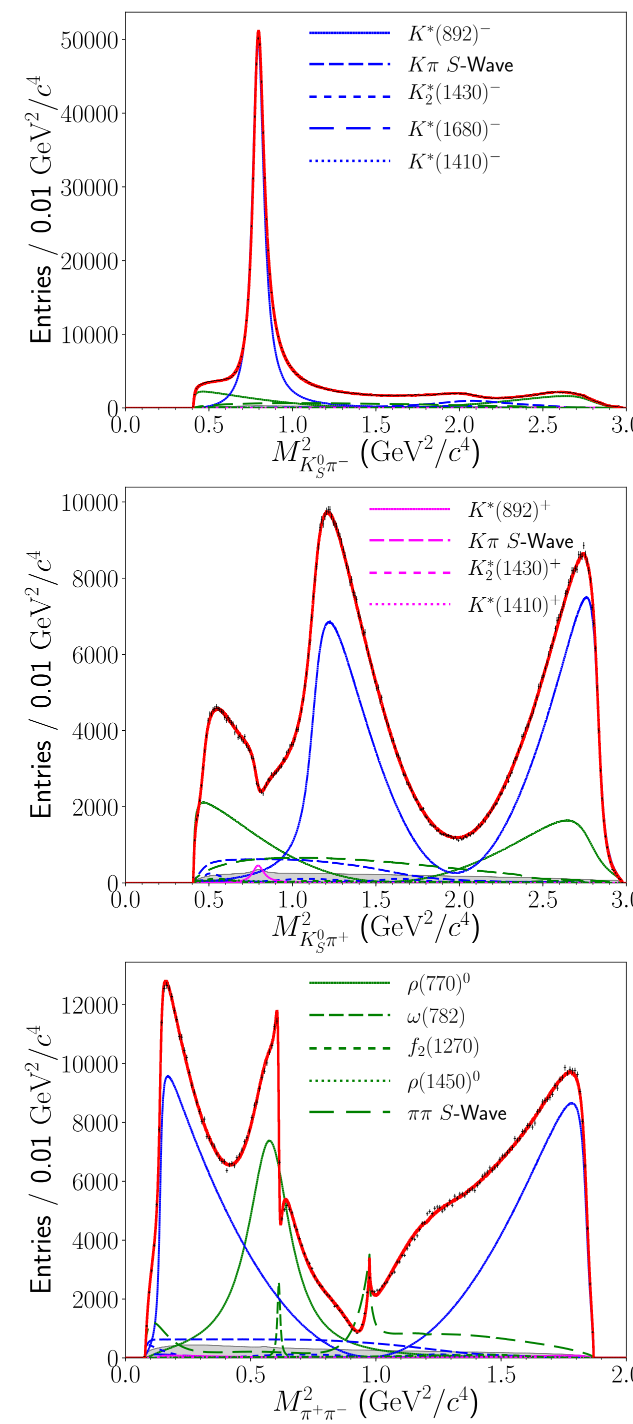
PRD 98 (2018) 11, 112012

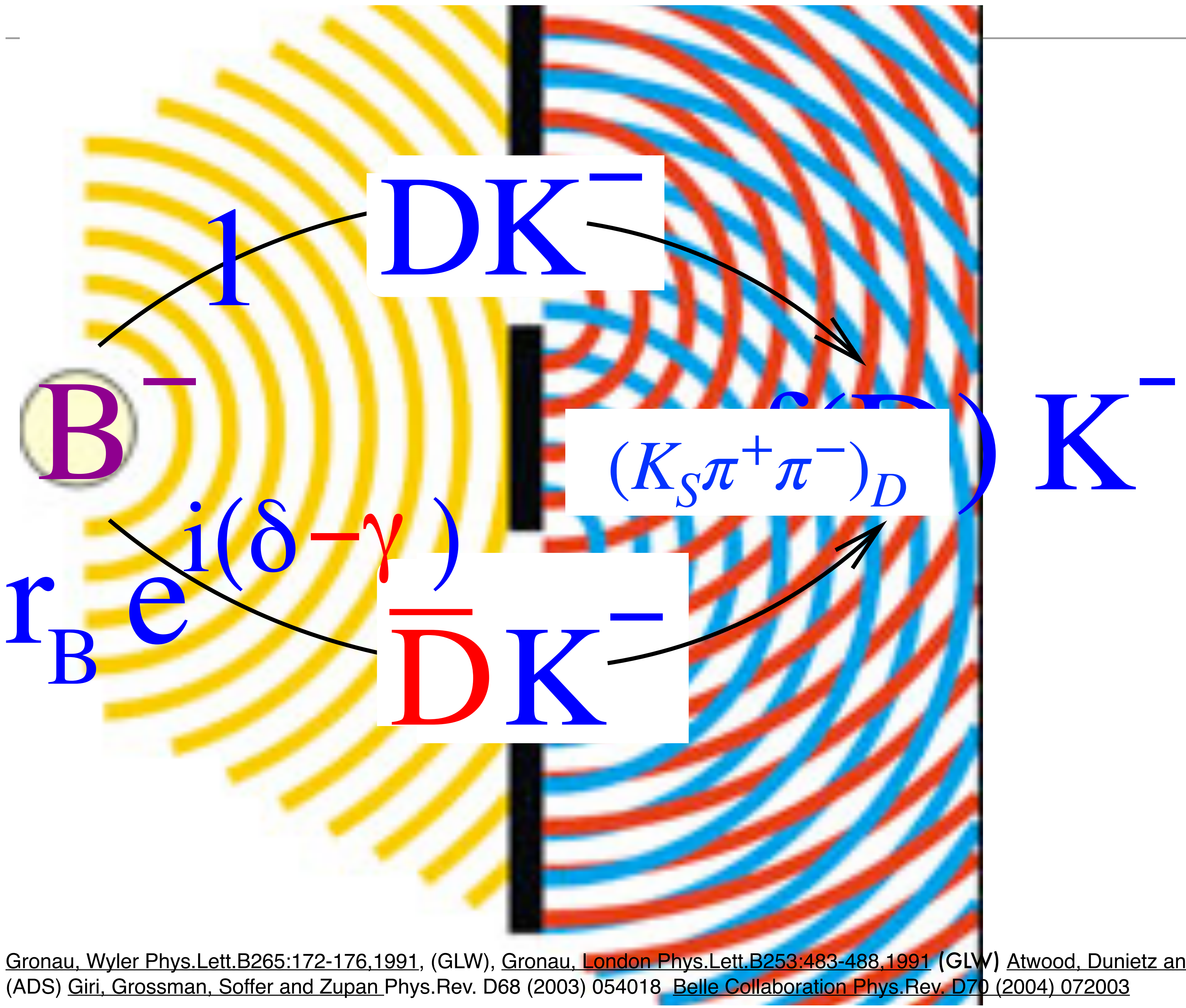


Unbinned quasi model-independent (QMI) method

- The magnitudes of amplitude models are OK and can be verified on data.
- Violation of unitarity and analyticity in models destroys link between magnitude and phase - the models' phases are uncertain.
- Idea: Keep models' magnitudes, but correct phases in model-independent way.



CP violation is an interference effect



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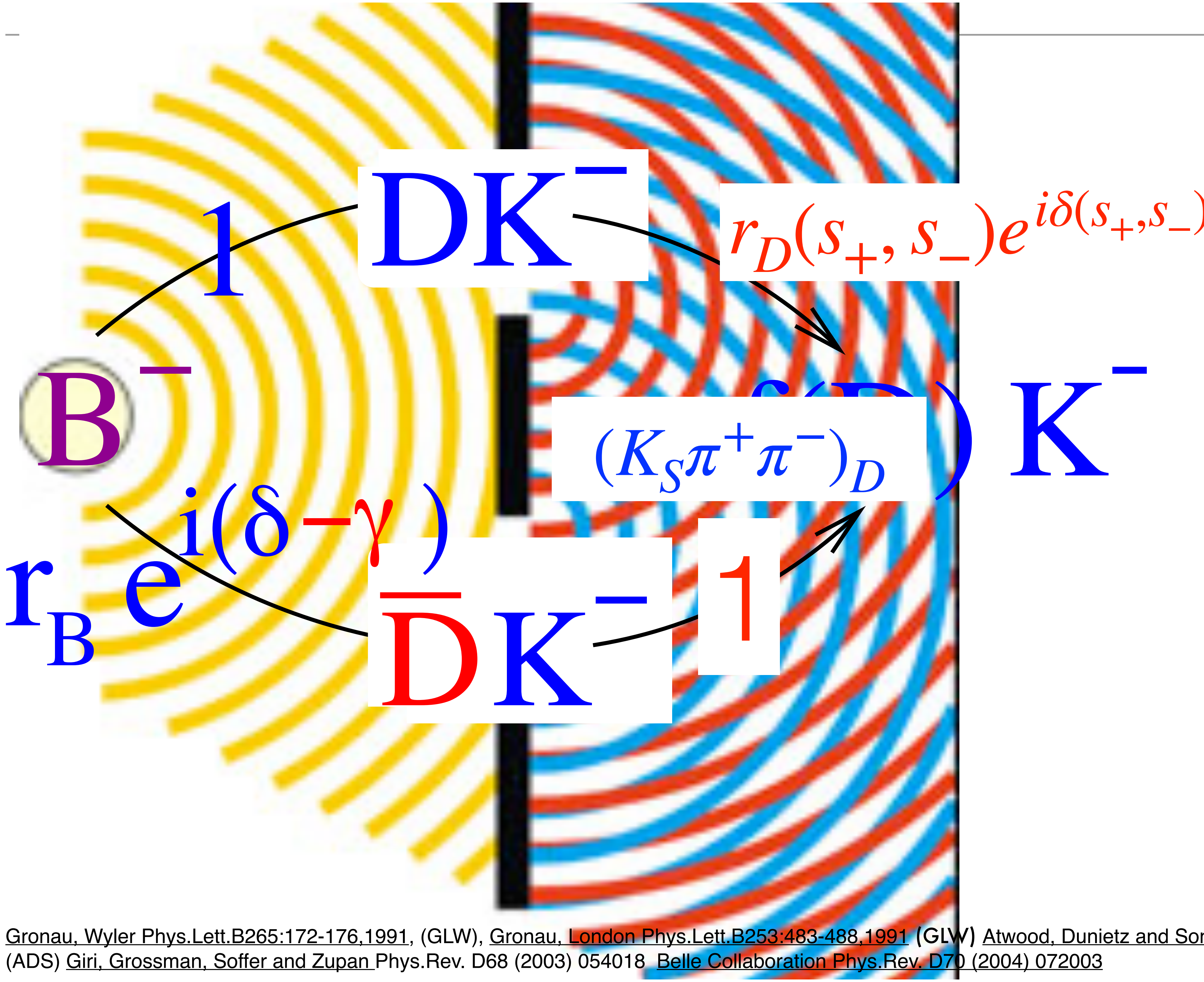

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Correct phase differences

arXiv:2305.10787 (2023)

In all relevant decay rates, at the charm threshold and in B decays, phases enter through interference terms between D^0 and $\overline{D}^0$ decay amplitudes, e.g. for D^0 from B^-

$$\Gamma^-(s_+, s_-) \propto r_D^2(s_+, s_-) + r_B^2 + 2r_D(s_+, s_-)r_B \cos\left(\delta_B - \gamma - \delta_D(s_+, s_-)\right)$$
$$\Gamma^+(s_-, s_+) \propto r_D^2(s_+, s_-) + r_B^2 + 2r_D(s_+, s_-)r_B \cos\left(\delta_B + \gamma - \delta_D(s_+, s_-)\right)$$

We correct this term, the phase difference of the D^0 and $\overline{D}^0$ decay amplitudes to the same phase space point:

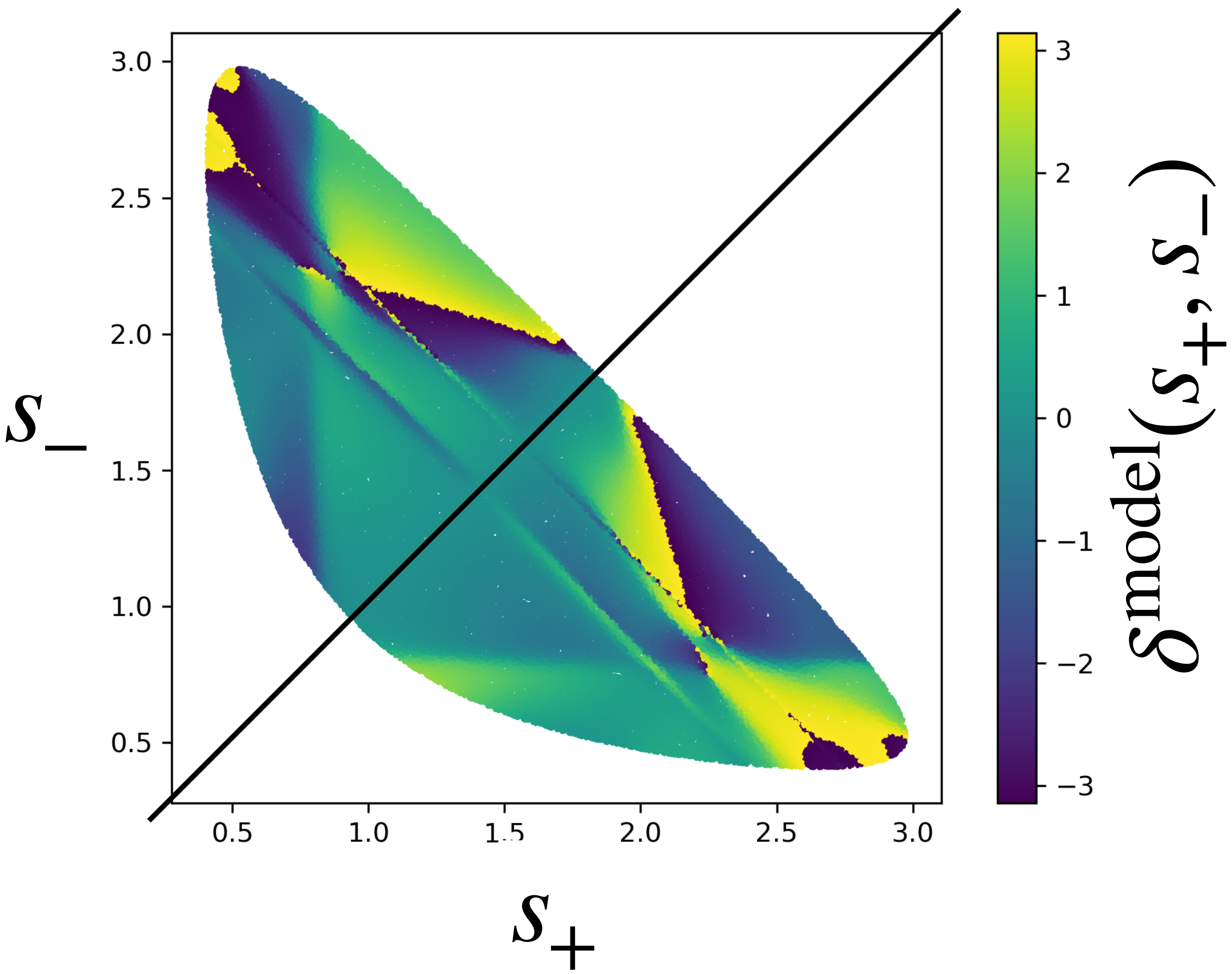
$$\delta_D = \delta_D^{\mathrm{model}} + \delta_D^{\mathrm{corr}}$$

Idea: Generic parametrisation of deviation of phase from model-prediction

arXiv:2305.10787 (2023)

$$\delta(s_+, s_-) = \delta^{\text{model}}(s_+, s_-) + \delta^{\text{corr}}(s_+, s_-)$$

$\delta^{\text{corr}}(s_+, s_-)$ = polynomial in s_+, s_-, determined in simultaneous fit to $B^\pm \to DK^\pm$ and $\psi(3770) \to D\bar{D}$ data



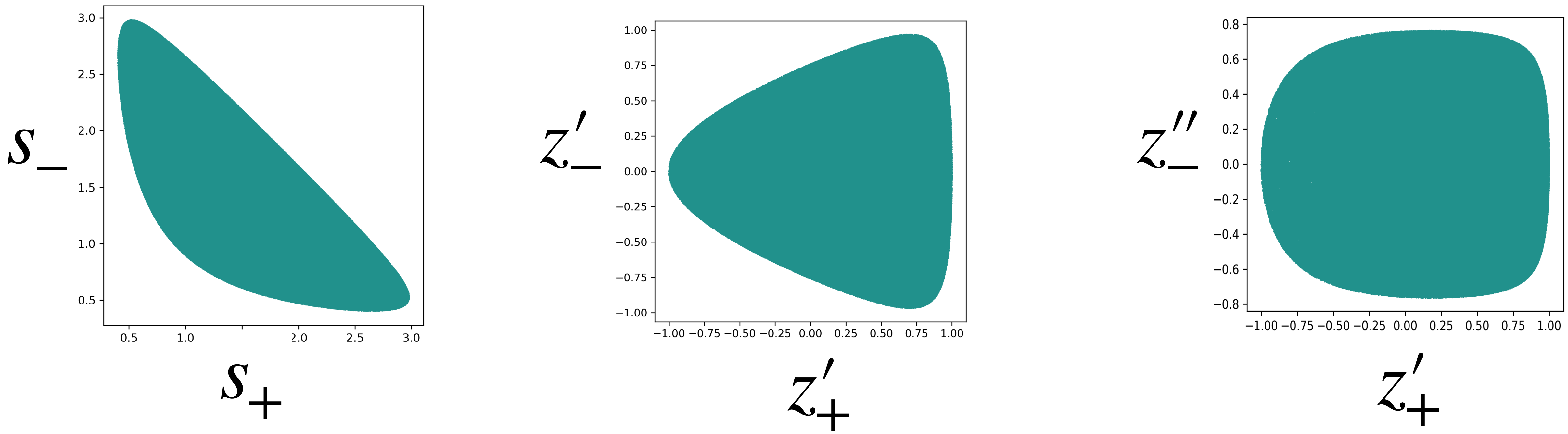
Symmetry:

$$\delta(s_+, s_-) = -\delta(s_-, s_+)$$

$$\delta^{\text{corr}}(s_+, s_-) = -\delta^{\text{corr}}(s_-, s_+)$$

Rotate and stretch

arXiv:2305.10787 (2023)



fit parameters

ith order Legendre polynomials

(2j+1)th order Legendre polynomials

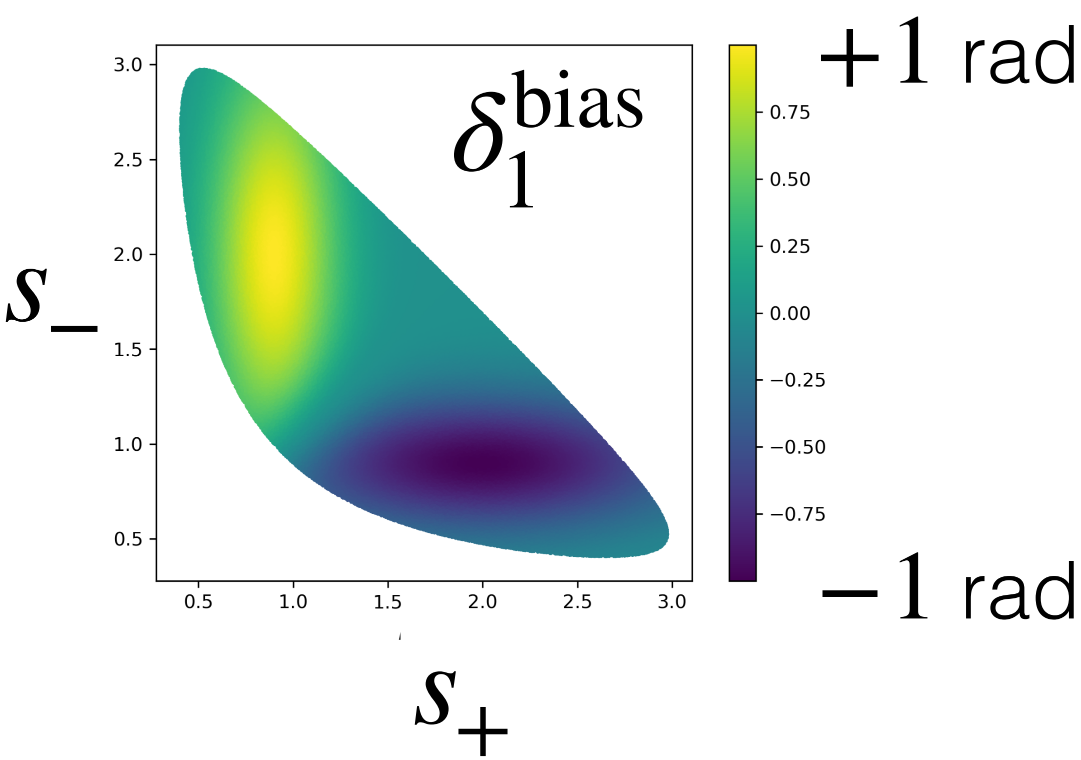
$$\delta^{\text{corr}} = \sum_{i=0}^{N} \sum_{j=0}^{\frac{N-i}{2}} C_{i,2j+1}\, P_i(z'_+)\, P_{2j+1}(z''_-)$$

only odd powers of $z''_- \propto s_- - s_+$, ensures that $\delta^{\text{corr}}(s_+, s_-) = -\,\delta^{\text{corr}}(s_-, s_+)$

Testing the method

arXiv:2305.10787 (2023)

Generate Babar & BELLE amplitude model with modified phase difference $\delta(s_+, s_-)$



$$\delta_1^{\text{bias}} = \text{erf}\left(\frac{s_+ - s_-}{\varepsilon}\right) g(s_+, s_-)$$

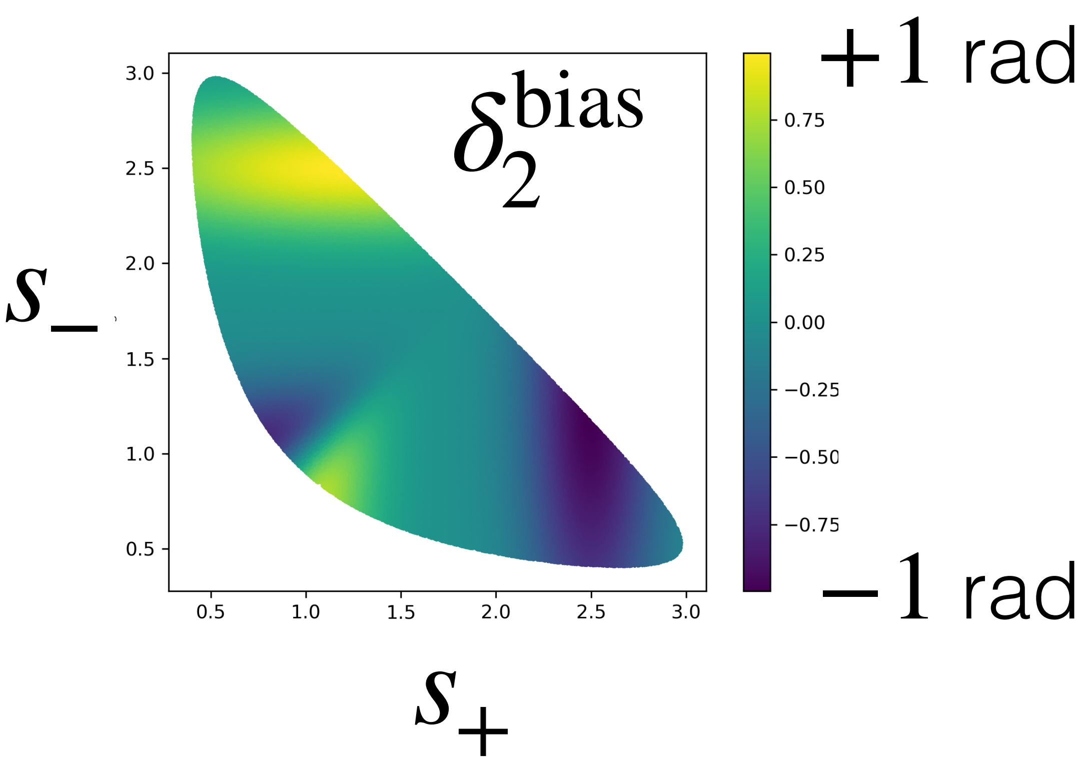
where g is a 2-D Gaussian, mirror reflected at $s_+ = s_-$

Fit starts from un-modified model - will it be able to find $\delta^{\text{corr}} \approx \delta_1^{\text{bias}}$ to a sufficient approximation?

Testing the method

arXiv:2305.10787 (2023)

Generate Babar & BELLE amplitude model with modified phase difference $\delta(s_+, s_-)$



This is a sum of two gaussian bias functions

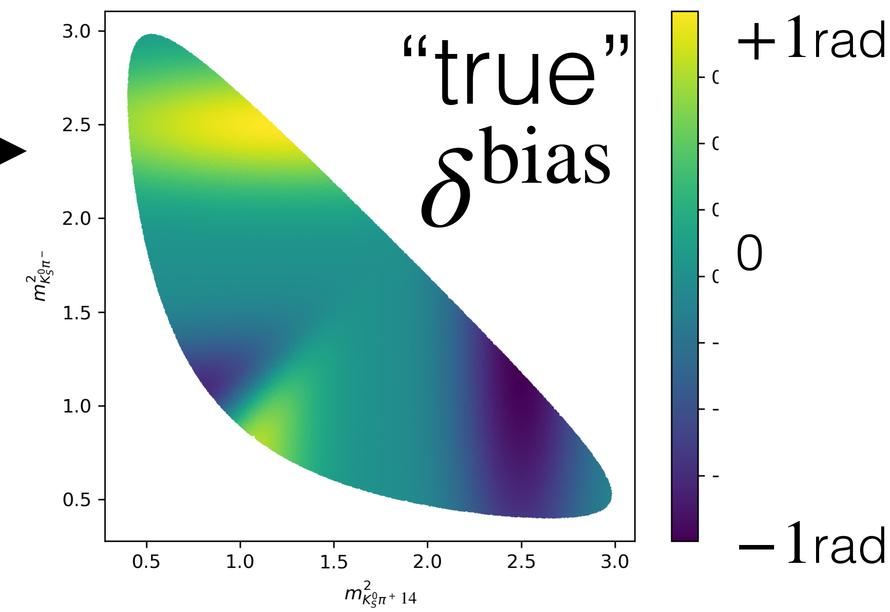
Fit starts from un-modified model - will it be able to find $\delta^{\text{corr}} \approx \delta_2^{\text{bias}}$ to a sufficient approximation?

arXiv:2305.10787 (2023)

Single fit, different-order correction polynomials

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



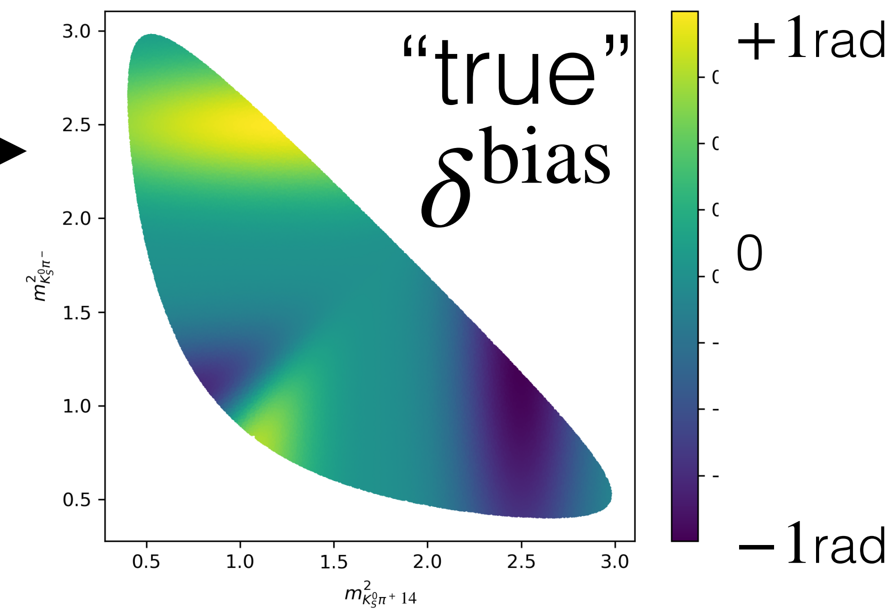
Order	$\Delta x_+ \cdot 100$	$\Delta y_+ \cdot 100$	$\Delta x_- \cdot 100$	$\Delta y_- \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$

LHCb yields as in JHEP 02 (2021) 169
BESIII yields as in: PRL 124 (2020) 24, 241802, PRD 101 (2020) 11200

Single fit, different-order correction polynomials

$$x_\pm = r_B \cos(\delta_B \pm \gamma)$$
$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



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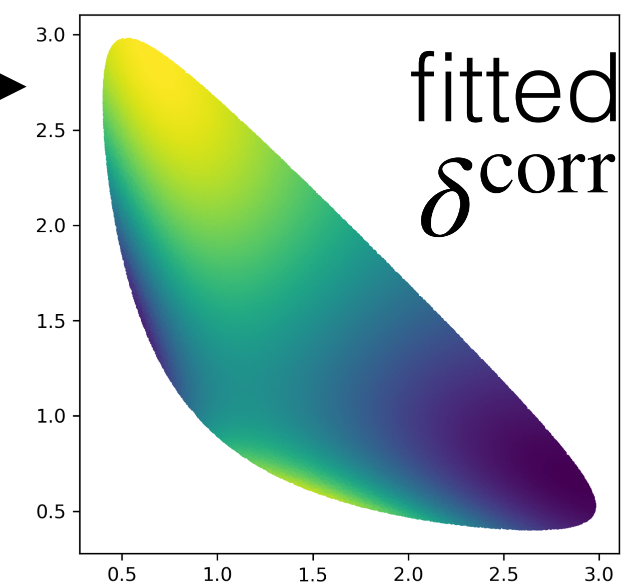
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$



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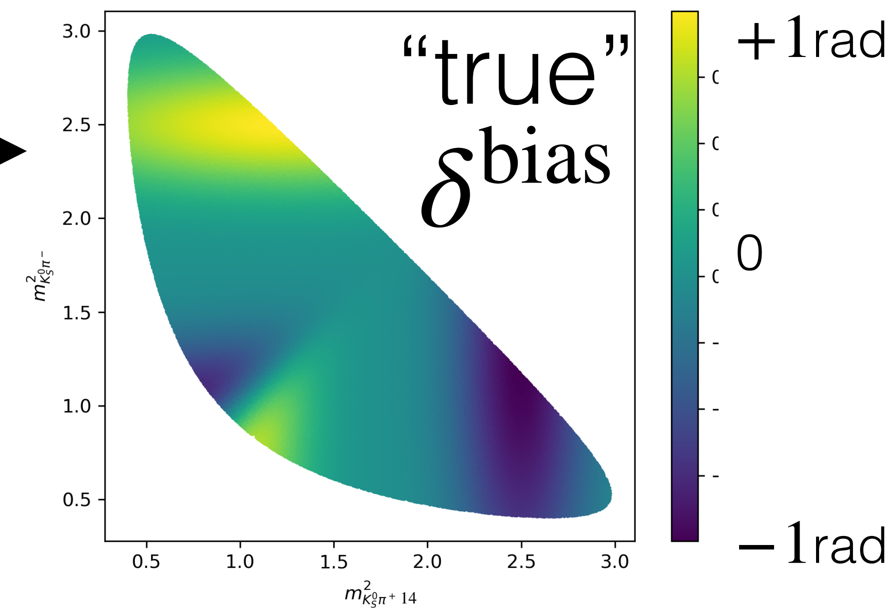
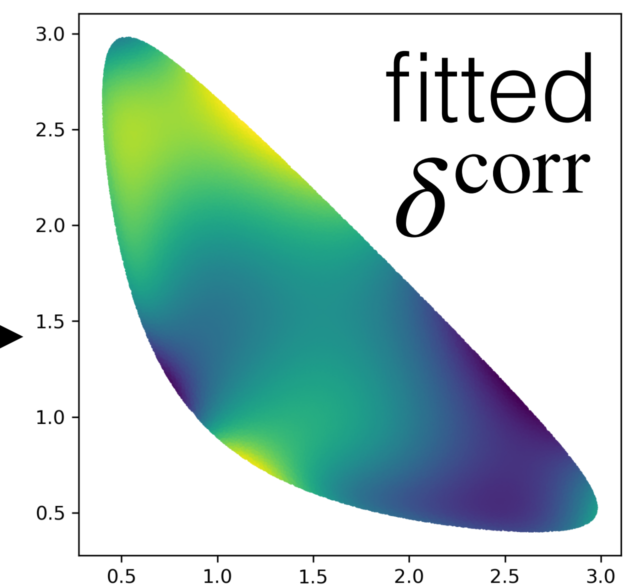
$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



Order	$\Delta x_+ \cdot 100$	$\Delta y_+ \cdot 100$	$\Delta x_- \cdot 100$	$\Delta y_- \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$



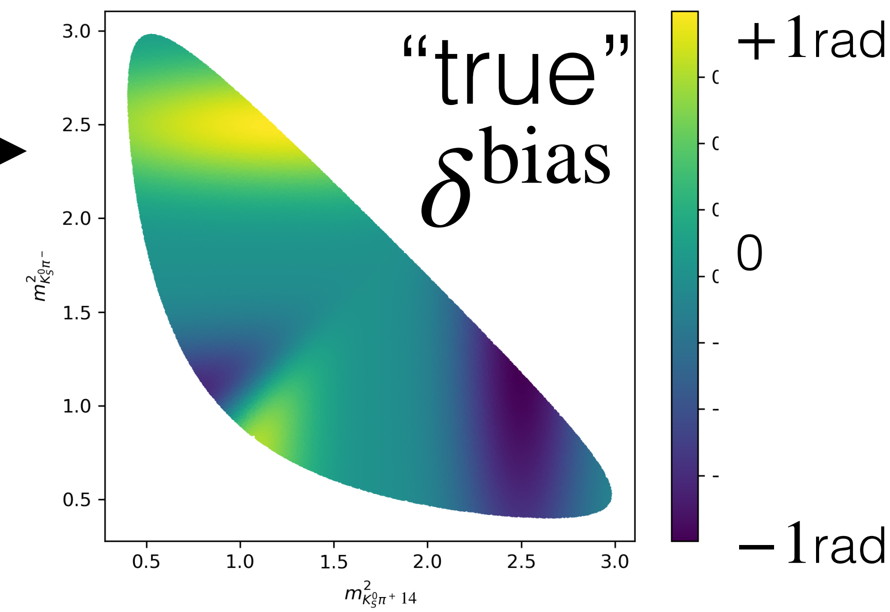
LHCb yields as in JHEP 02 (2021) 169
BESIII yields as in: PRL 124 (2020) 24, 241802, PRD 101 (2020) 11200

arXiv:2305.10787 (2023)

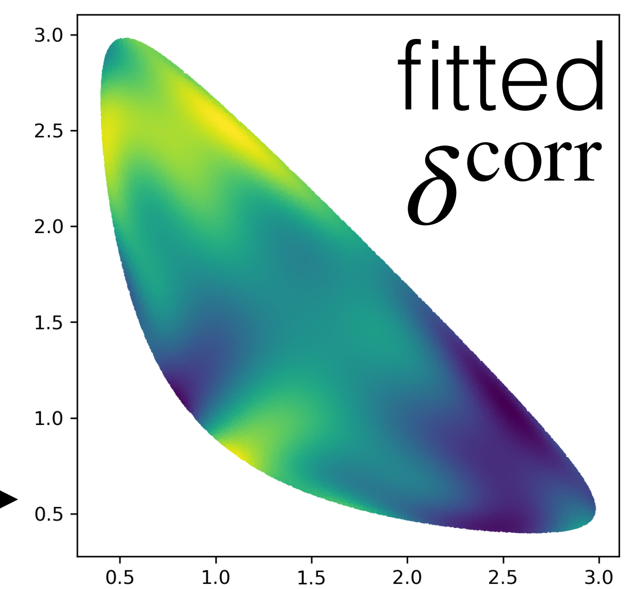
Single fit, different-order correction polynomials

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Deviation of δ from model in event generation →



Order	$\Delta x_{+} \cdot 100$	$\Delta y_{+} \cdot 100$	$\Delta x_{-} \cdot 100$	$\Delta y_{-} \cdot 100$
MD	$+1.3 \pm 0.8$	$+1.2 \pm 1.1$	-1.0 ± 1.3	-3.3 ± 1.3
1	$+1.1 \pm 0.8$	$+0.5 \pm 1.0$	-1.3 ± 0.8	-0.6 ± 1.0
2	$+0.5 \pm 0.9$	$+0.1 \pm 1.0$	-1.0 ± 0.8	$+0.4 \pm 1.0$
3	$+0.6 \pm 0.8$	0.0 ± 1.0	-1.2 ± 0.8	$+0.4 \pm 1.0$
4	$+0.3 \pm 0.8$	$+0.4 \pm 1.0$	-0.8 ± 0.8	$+0.3 \pm 1.0$
5	$+0.4 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.3 \pm 1.0$
6	$+0.3 \pm 0.8$	$+0.7 \pm 1.0$	-0.8 ± 0.8	$+0.4 \pm 1.0$
7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$



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Single fit, different-order correction polynomials

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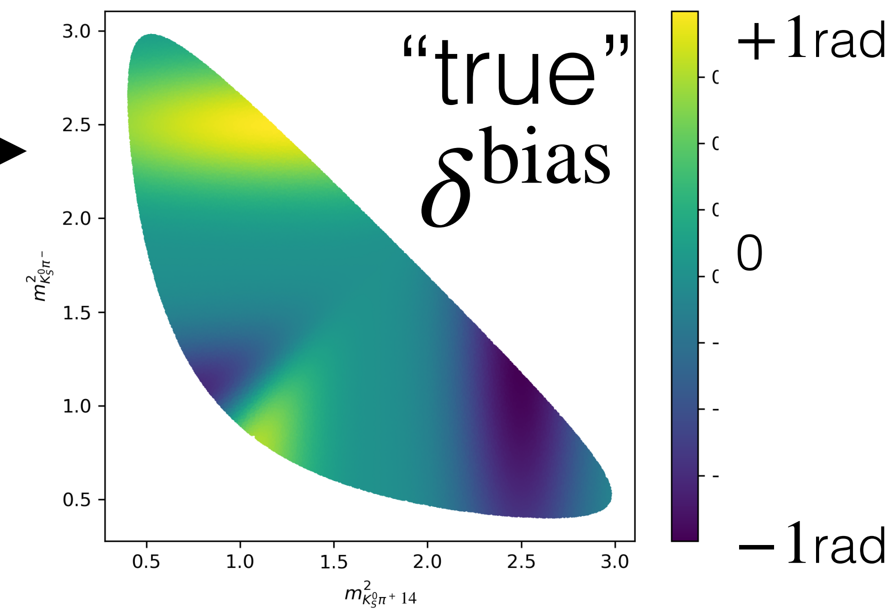
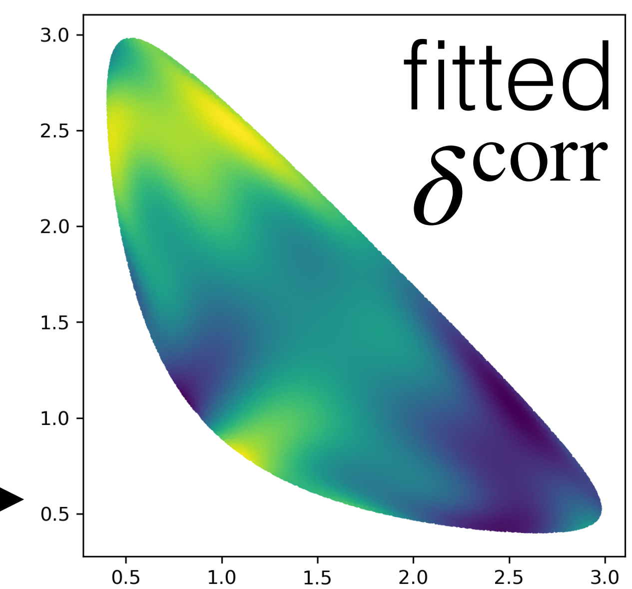
Deviation of δ from model in event generation →



While, here, our focus is on the use for γ, the model-independent determination of strong phases is also interesting for developing and testing amplitude models, and for the measurement of charm mixing.

7	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
8	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.9 ± 0.8	$+0.7 \pm 1.0$
9	$+0.3 \pm 0.8$	$+0.5 \pm 1.0$	-0.7 ± 0.8	$+0.7 \pm 1.0$

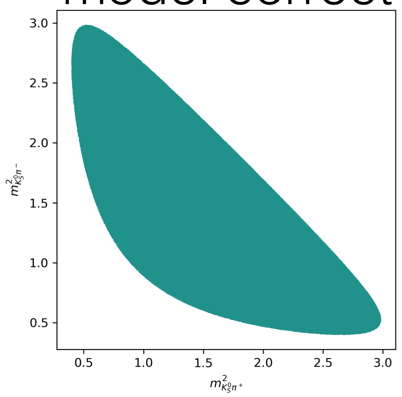


LHCb yields as in JHEP 02 (2021) 169
BESIII yields as in: PRL 124 (2020) 24, 241802, PRD 101 (2020) 11200

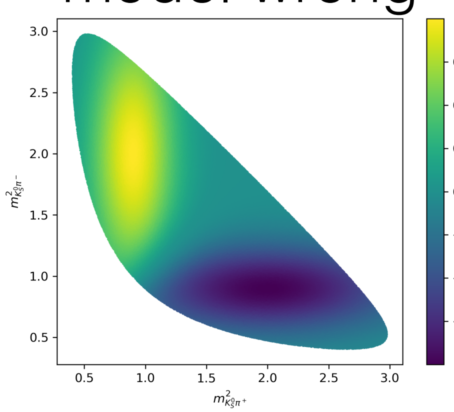
Pull studies, 100 fits

arXiv:2305.10787 (2023)

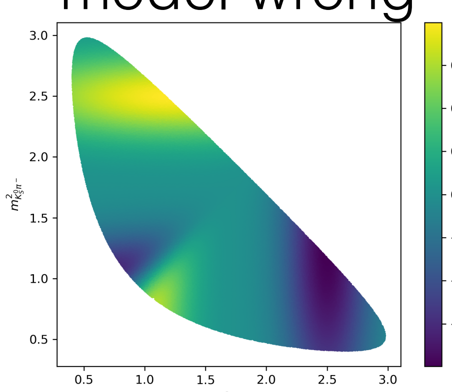
model correct



model wrong



model wrong



pull: $\dfrac{\text{fit} - \text{input}}{\sigma}$

Table shows (mean) ± (standard deviation) of pull distribution. Expect: 0 ± 1

Method	$\frac{\Delta x_+}{\sigma_{x_+}}$	$\frac{\Delta y_+}{\sigma_{y_+}}$	$\frac{\Delta x_-}{\sigma_{x_-}}$	$\frac{\Delta y_-}{\sigma_{y_-}}$
QMI	-0.12 ± 0.82	$+0.07 \pm 1.01$	-0.12 ± 1.11	-0.02 ± 1.06
MD	-0.08 ± 0.82	$+0.13 \pm 1.01$	-0.07 ± 1.06	-0.12 ± 1.01
QMI	$+0.22 \pm 0.89$	-0.07 ± 0.93	$+0.16 \pm 1.08$	$+0.07 \pm 1.28$
MD	$+1.10 \pm 0.85$	$+3.42 \pm 1.03$	$+0.36 \pm 1.03$	-3.85 ± 1.21
QMI	$+0.17 \pm 0.90$	$+0.07 \pm 0.94$	$+0.02 \pm 0.99$	$+0.13 \pm 1.01$
MD	$+2.04 \pm 0.87$	$+1.07 \pm 0.95$	-0.93 ± 1.16	-1.81 ± 1.24

Uncertainty on mean: ± 0.1, on standard deviation: ± 0.07

Precision on γ with $B^+ \to DK^+, D \to K_S\pi\pi$

arXiv:2305.10787 (2023)

		$\sigma_\gamma(^\circ)$		
Lumi		**new QMI**	**Model-dependent**	**8 bins, fixed* ci, si**
1xLHCb	**1xBESIII**	4.2	4.2	5.1
1xLHCb	**10xBESIII**	4.2		
100xLHCb	**1xBESIII**	0.45	0.42	0.52
100xLHCb	**10xBESIII**	0.43		

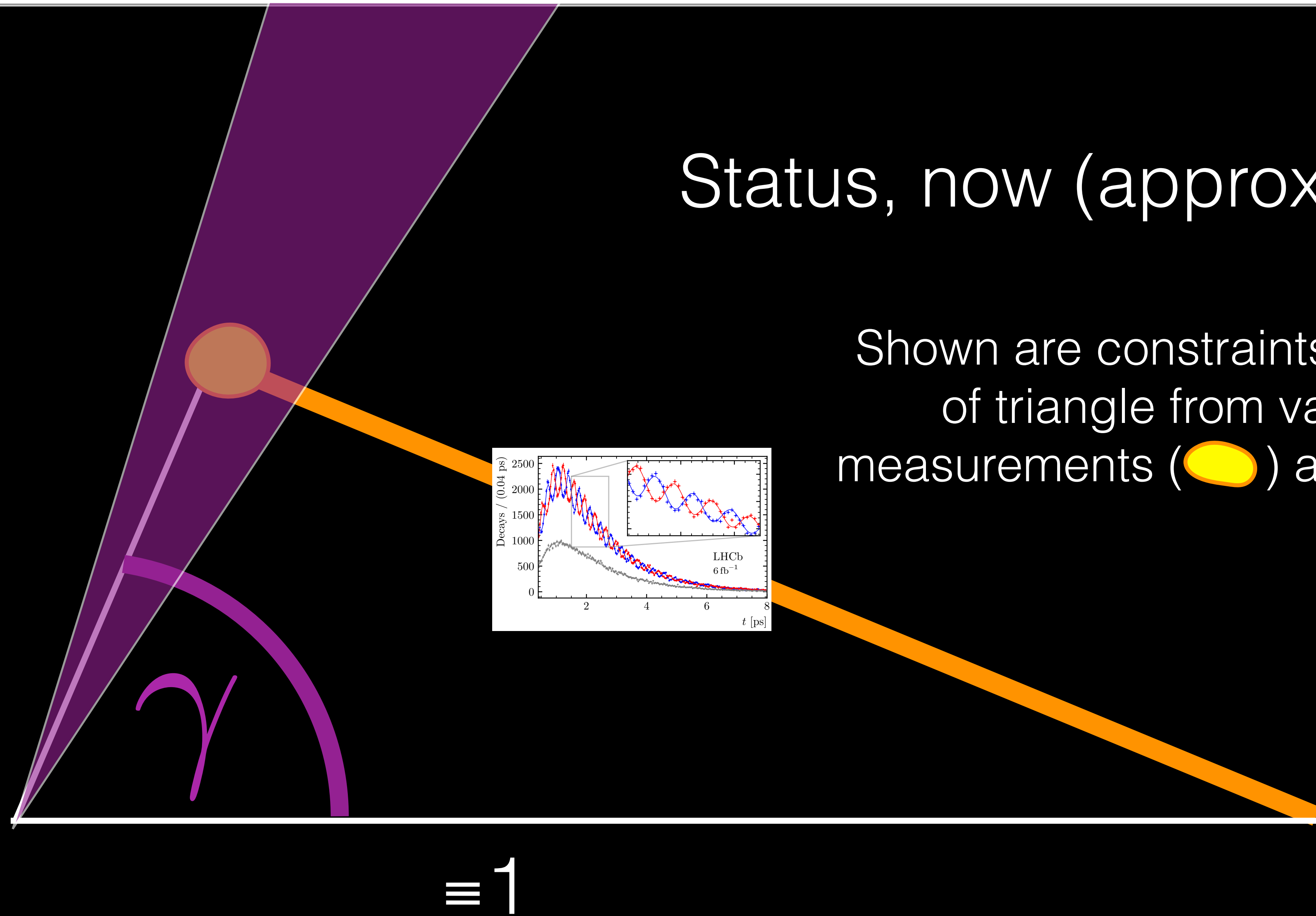
(average error reported in 100 pseudo experiments)

*) additional uncertainty on binned γ fit due to finite BESIII data for 1xBESIII: 1.2°
PRD 101 (2020) 11200

(BTW, ultimate precision on γ achieved through combining multiple decay modes)

Summary

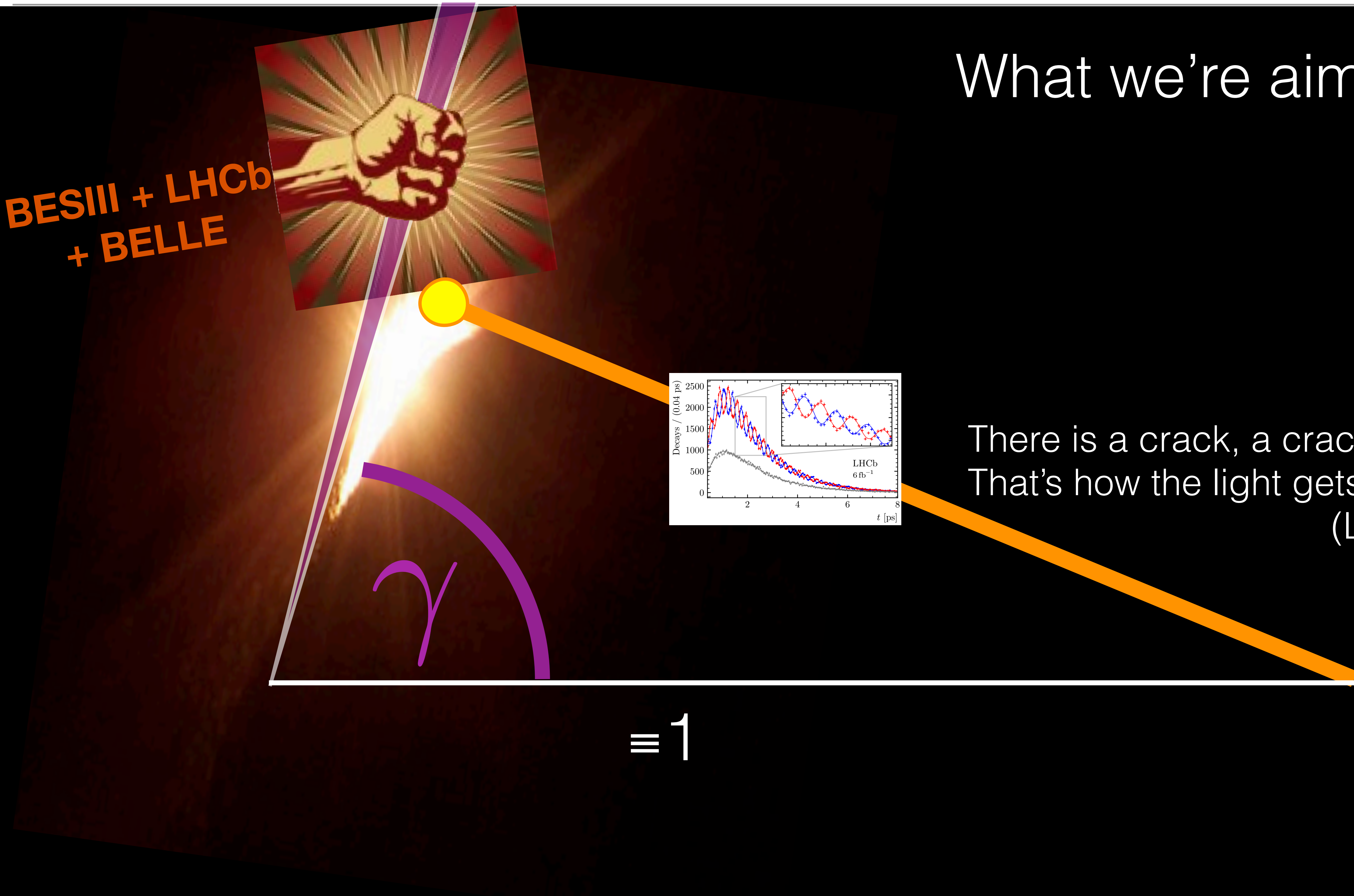
Status, now (approximately)



Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

Summary

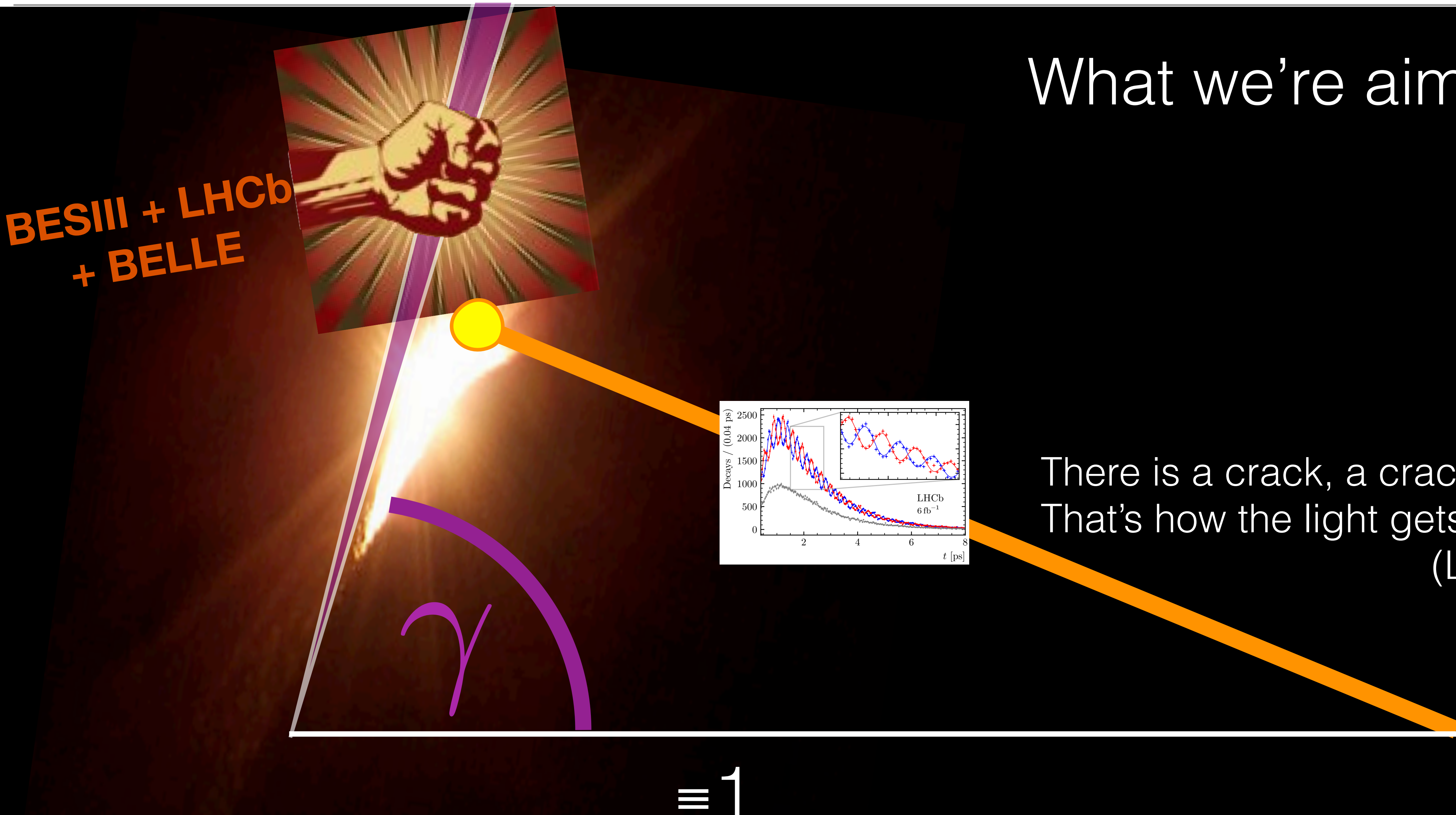
What we're aiming for



There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

Summary

What we're aiming for



There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

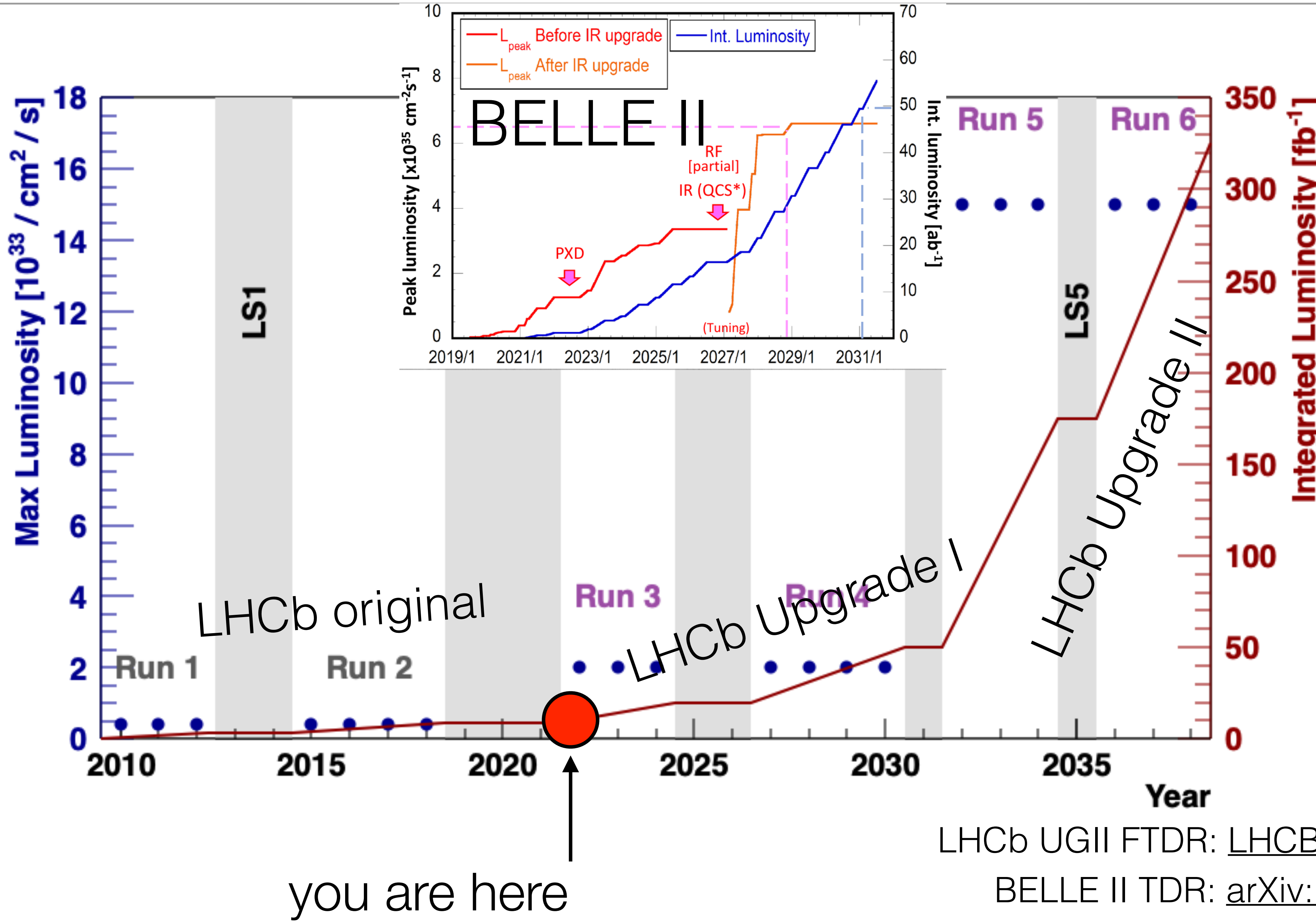
New unbinned method (arXiv:2305.10787) makes optimal use of the information contained in $\psi(3770) \to D\overline{D}$ and $B^{\pm} \to DK^{\pm}$ data for a better precision on γ.

The End

Summary

- The CKM angle γ allows a beautifully “clean” measurement with negligible theory uncertainty.
- Future datasets will allow an exquisitely precise measurement of γ.
- Model-independent methods *required* to truly benefit from these data. These rely on combining BES III and $B \to DK$ data.
- New unbinned method (arXiv:2305.10787) exploits the information contained in both types of data optimally for better precision on γ.

The future

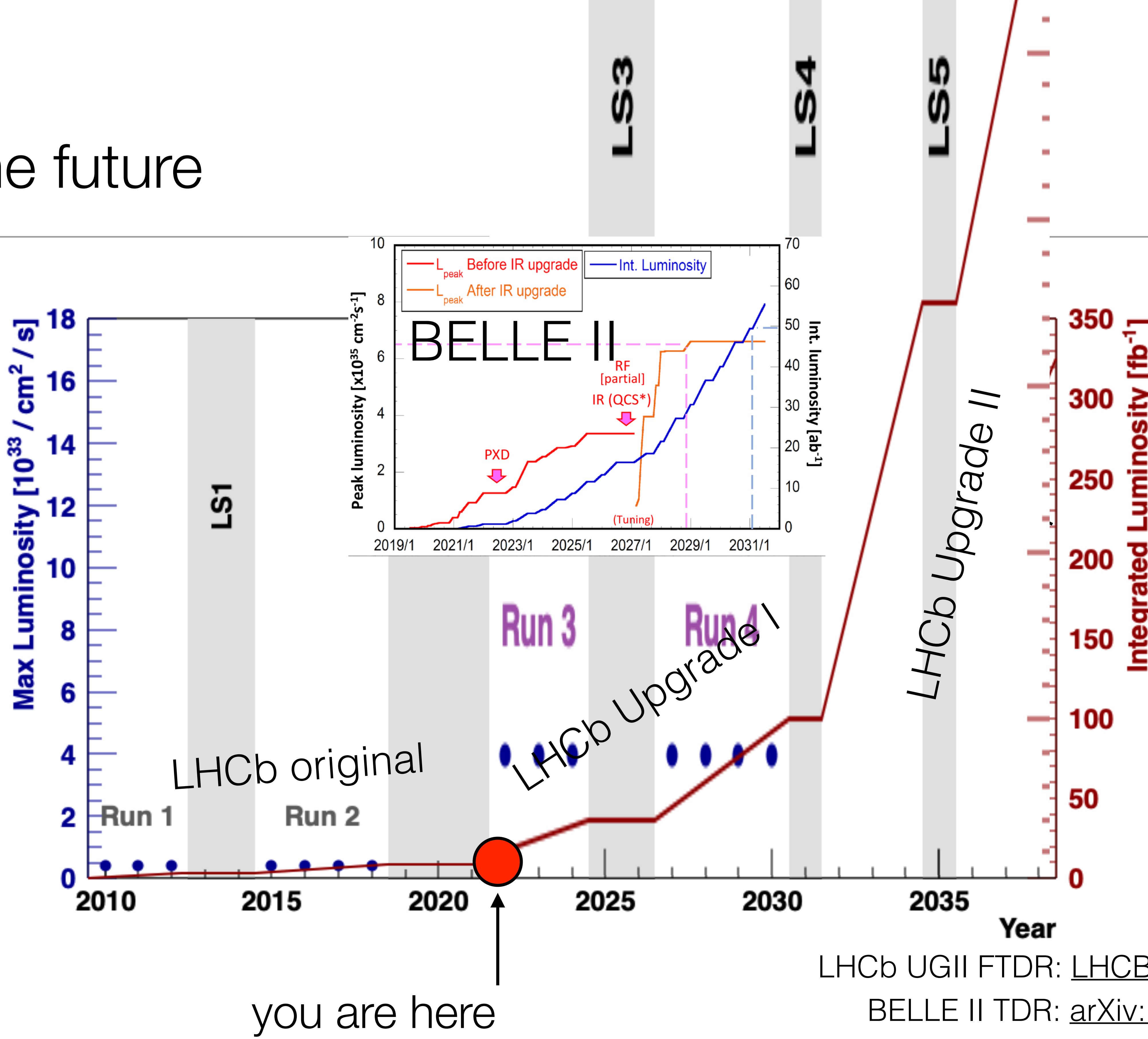


LHCb UGII FTDR: LHCB-TDR-023

BELLE II TDR: arXiv:1011.0352

LHCb UGII physics: arXiv:1808.08865

The future



LHCb UGII FTDR: LHCB-TDR-023

BELLE II TDR: arXiv:1011.0352

LHCb UGII physics: arXiv:1808.08865

$B^- \to DK^-; D \to \pi^+\pi^-\pi^+\pi^-, K^+K^-\pi^+\pi^-$

- Also expect good precision from other four-body modes.

- CP-even fractions measured by BES III.
  $F_+(D^0 \to K^+K^-\pi^+\pi^-) = 0.730 \pm 0.037 \pm 0.021$ BESIII: PRD 107 (2023) 3, 032009
  $F_+(D^0 \to \pi^+\pi^-\pi^+\pi^-) = 0.735 \pm 0.015 \pm 0.005$ BESIII: PRD 106 (2022) 9, 092004

- Recently studied at LHCb: arXiv:2301.10328. Expect excellent sensitivity with binned c_i, s_i.

- Binned c_i, s_i measured by CLEO-c for $D^0 \to 4\pi$, but not for $D^0 \to KK\pi\pi$ (lack of statistics).

- Precise measurements from BESIII for both modes would have significant impact.

B- decay rate

$$\Gamma^{-}(s_+, s_-) \propto |A_D|^2 + r_B^2 |\overline{A}_D|^2 + 2\mathrm{Re}\left(A_D \overline{A}_D^* r_B e^{-i(\delta_B - \gamma)}\right)$$

$$\propto |A_D|^2 + r_B^2 |\overline{A}_D|^2 + 2 |A_D| |\overline{A}|_D r_B \cos\left(\delta_B - \gamma - \delta_D(s_+, s_-)\right)$$

Similarly for CP-conjugate process, but with $\gamma \rightarrow -\gamma$

We correct this term, the phase difference of the D^0 and $\overline{D}^0$ decay amplitudes to the same phase space point:

$$\delta_D = \delta_D^{\mathrm{model}} + \delta_D^{\mathrm{corr}}$$

The Big Picture

LHCb UGII physics: arXiv:1808.08865

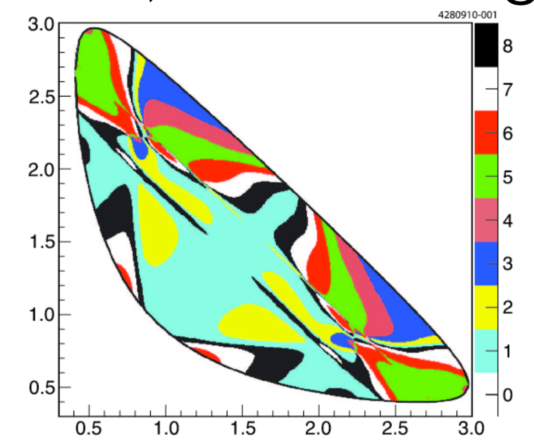
Observable	Current LHCb	LHCb 2025	Belle II	Upgrade II	ATLAS & CMS
EW Penguins					
R_K $(1 < q^2 < 6\,\mathrm{GeV}^2 c^4)$	0.1 [274]	0.025	0.036	0.007	–
R_{K^*} $(1 < q^2 < 6\,\mathrm{GeV}^2 c^4)$	0.1 [275]	0.031	0.032	0.008	–
R_ϕ, R_{pK}, R_π	–	0.08, 0.06, 0.18	–	0.02, 0.02, 0.05	–
CKM tests					
γ, with $B_s^0 \to D_s^+ K^-$	$(^{+17}_{-22})^\circ$ [136]	4°	–	1°	–
γ, all modes	$(^{+5.0}_{-5.8})^\circ$ [167]	1.5°	1.5°	0.35°	–
$\sin 2\beta$, with $B^0 \to J/\psi K_S^0$	0.04 [609]	0.011	0.005	0.003	–
ϕ_s, with $B_s^0 \to J/\psi \phi$	49 mrad [44]	14 mrad	–	4 mrad	22 mrad [610]
ϕ_s, with $B_s^0 \to D_s^+ D_s^-$	170 mrad [49]	35 mrad	–	9 mrad	–
$\phi_s^{s\bar{s}s}$, with $B_s^0 \to \phi\phi$	154 mrad [94]	39 mrad	–	11 mrad	Under study [611]
a_{sl}^s	33×10^{-4} [211]	10×10^{-4}	–	3×10^{-4}	–
$\lvert V_{ub}\rvert / \lvert V_{cb}\rvert$	6% [201]	3%	1%	1%	–
$\boldsymbol{B_s^0, B^0 \to \mu^+\mu^-}$					
$\mathcal{B}(B^0 \to \mu^+\mu^-)/\mathcal{B}(B_s^0 \to \mu^+\mu^-)$	90% [264]	34%	–	10%	21% [612]
$\tau_{B_s^0 \to \mu^+\mu^-}$	22% [264]	8%	–	2%	–
$S_{\mu\mu}$	–	–	–	0.2	–
$\boldsymbol{b \to c\ell^-\bar{\nu}_l}$ **LUV studies**					
$R(D^*)$	0.026 [215, 217]	0.0072	0.005	0.002	–
$R(J/\psi)$	0.24 [220]	0.071	–	0.02	–
Charm					
$\Delta A_{CP}(KK - \pi\pi)$	8.5×10^{-4} [613]	1.7×10^{-4}	5.4×10^{-4}	3.0×10^{-5}	–
A_Γ $(\approx x \sin\phi)$	2.8×10^{-4} [240]	4.3×10^{-5}	3.5×10^{-4}	1.0×10^{-5}	–
$x \sin\phi$ from $D^0 \to K^+\pi^-$	13×10^{-4} [228]	3.2×10^{-4}	4.6×10^{-4}	8.0×10^{-5}	–
$x \sin\phi$ from multibody decays	–	$(K3\pi)$ 4.0×10^{-5}	$(K_S^0\pi\pi)$ 1.2×10^{-4}	$(K3\pi)$ 8.0×10^{-6}	–

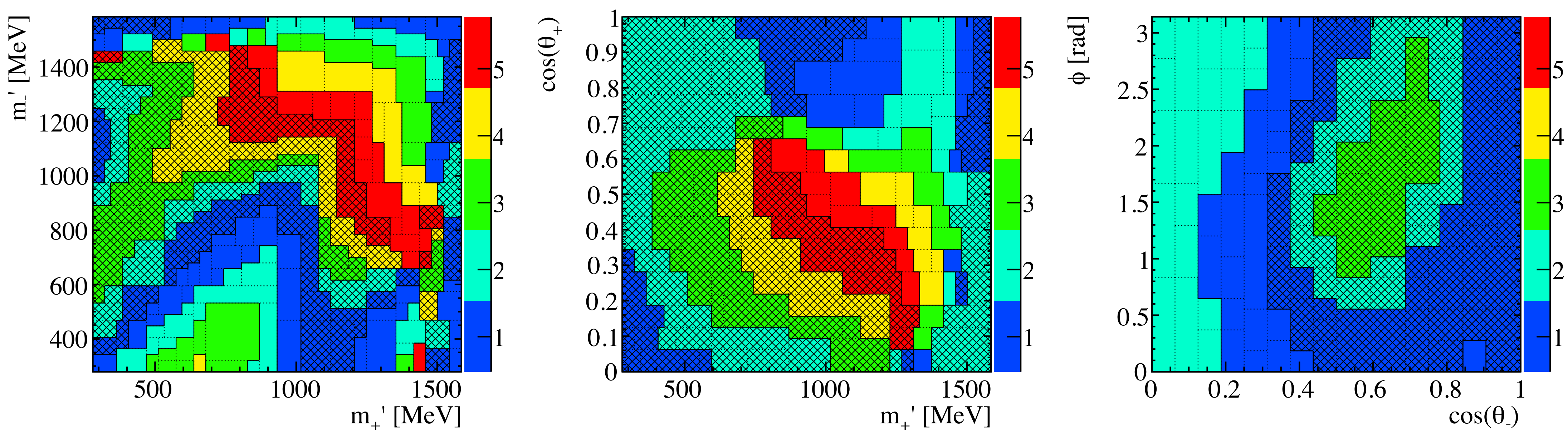
Winning by binning in 5 dimensions… e.g. D→ππππ

Harnew et al, using CLEO-c data: JHEP 1801 (2018) 144

Binning based on phase difference between D° and D̄° amplitudes going to same point in phase space, like optimised binning for $K_{S,L}$ ππ.
This approach requires a model.
Examples of 2-D slices through 5-D phase space based on D→ππππ amplitude model in JHEP 1705 (2017) 143.

$K_{S,L}$ ππ binning

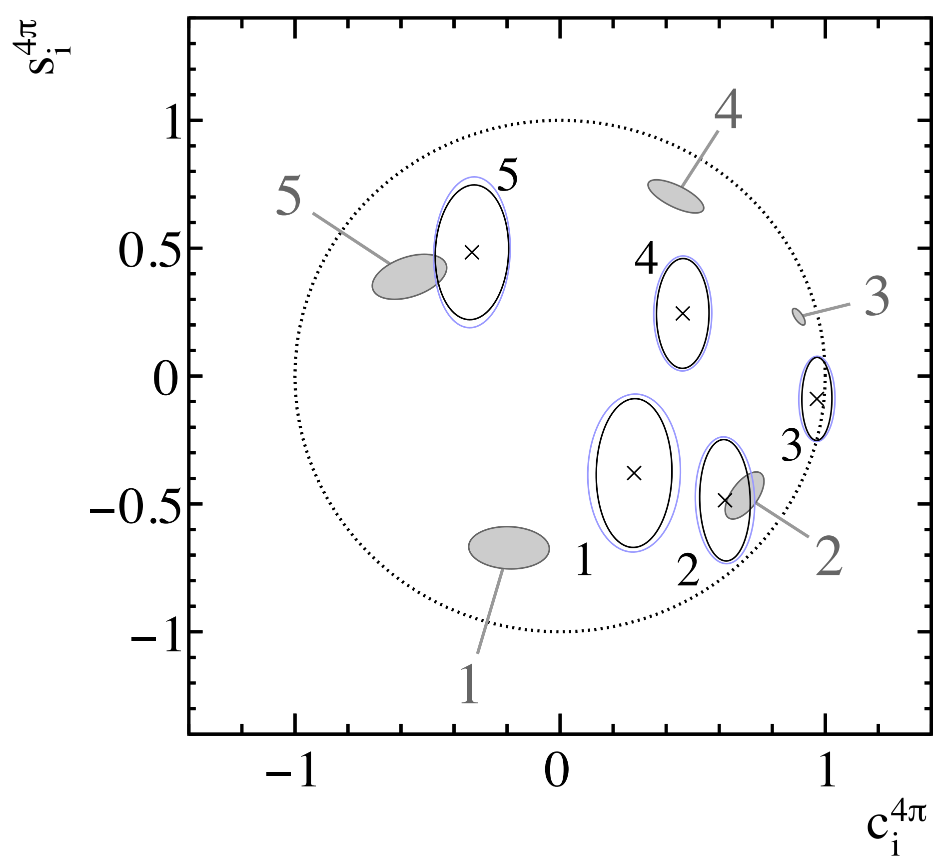




Results for 4-body c_i, s_i.

$D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$

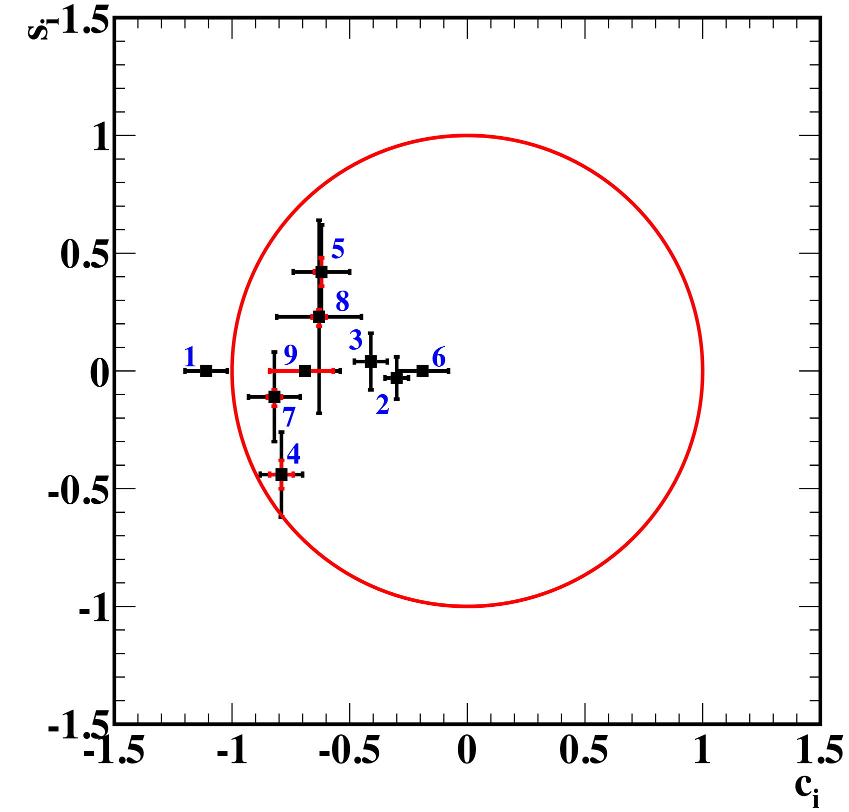
(measurements × and model expectation ⬭)



CLEO-c data: JHEP 1801 (2018) 144

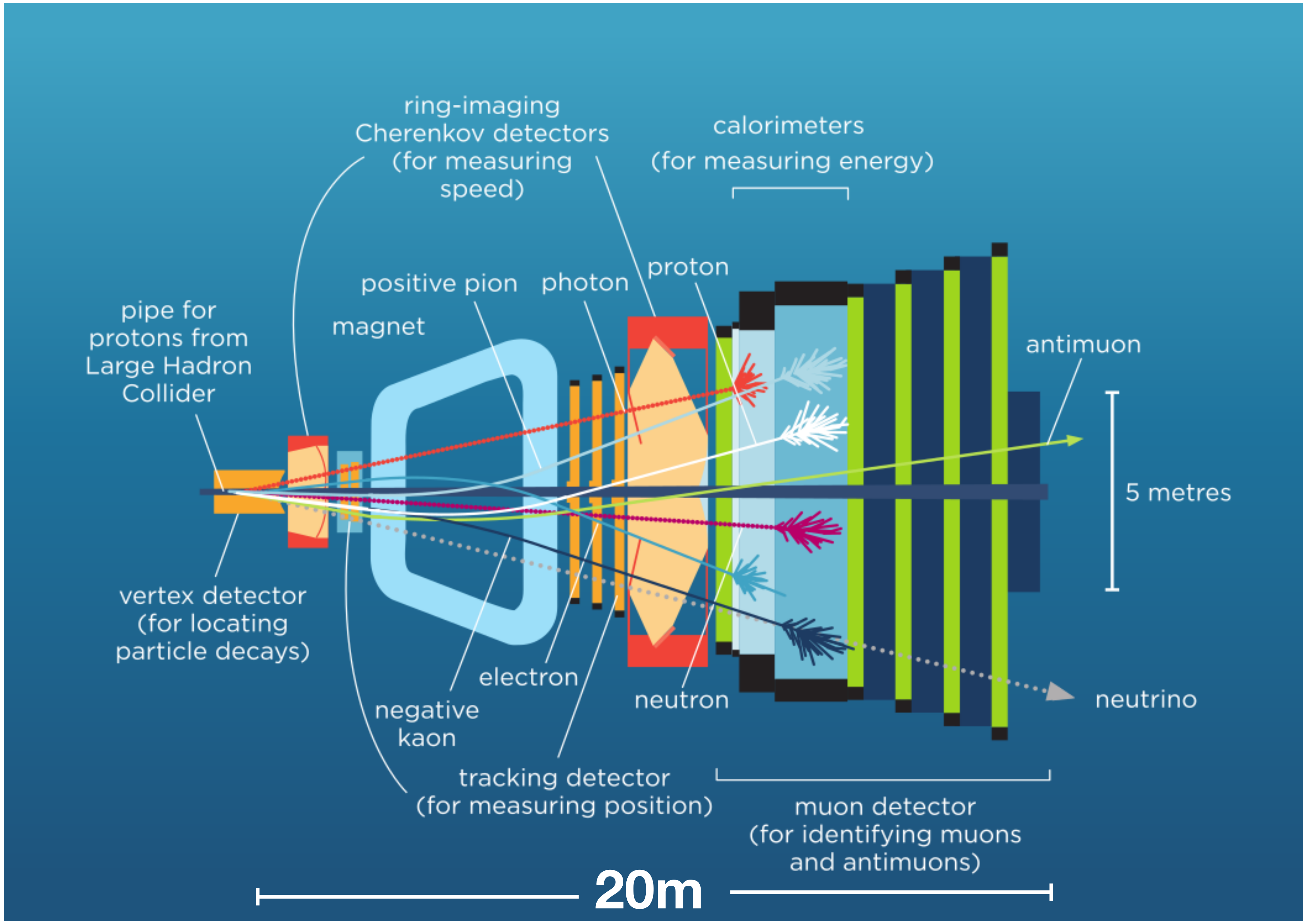
$D^0 \rightarrow K_S\pi^-\pi^+\pi^0$

no model (binning around resonance)



CLEO-c data: JHEP 1801 (2018) 082

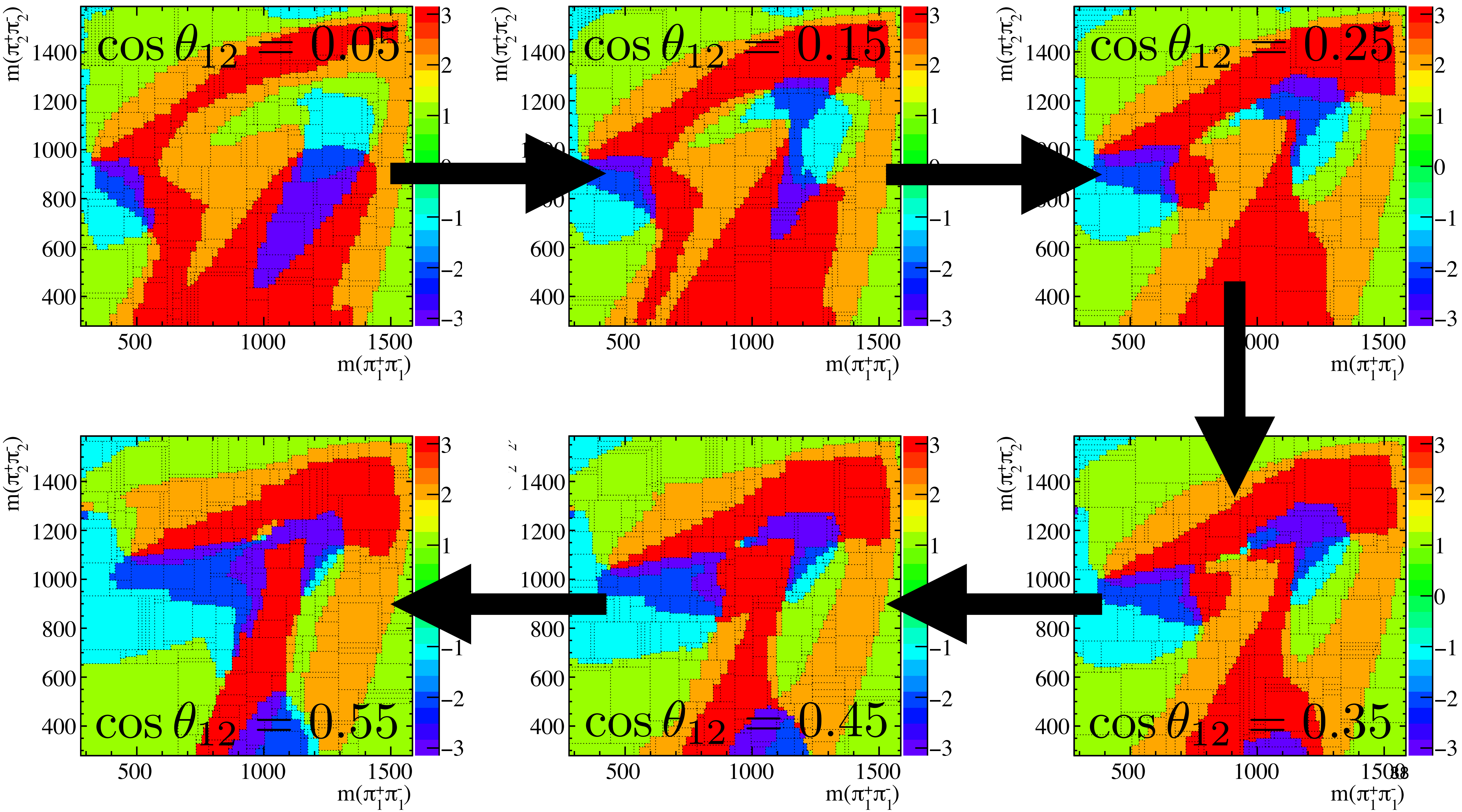
The LHCb Detector



Recent results for $D^o \to \pi^+\pi^-\pi^+\pi^-$ (2-D slices through 5D)

JHEP 1801 (2018) 144
Phys.Lett. B747 (2015) 9-17

$\cos\theta_{34} = 0 \qquad \phi = \pi/2$



Comparing methods

arXiv:2305.10787 (2023)

LHCb	$\sigma_{x_+} \cdot 10^2$		$\sigma_{y_+} \cdot 10^2$		$\sigma_{x_-} \cdot 10^2$		$\sigma_{y_-} \cdot 10^2$		σ_γ (°)	
Lumi	MD	bin	MD	bin	MD	bin	MD	bin	MD	bin
$\times 1$	0.780	0.886	1.081	1.482	0.878	1.189	0.939	1.328	4.23	5.09
$\times 100$	0.078	0.089	0.108	0.149	0.088	0.118	0.093	0.134	0.42	0.52

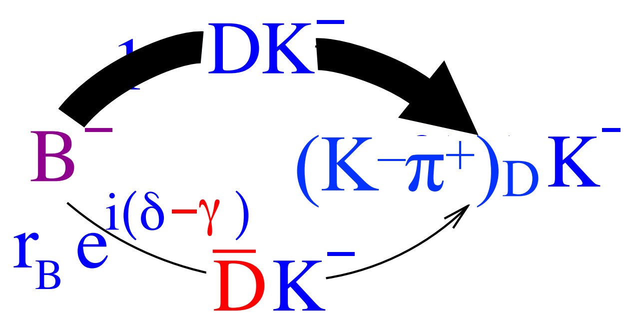
Lumi scenario: LHCb	BES III	$\sigma_{x_+} \cdot 10^2$	$\sigma_{y_+} \cdot 10^2$	$\sigma_{x_-} \cdot 10^2$	$\sigma_{y_-} \cdot 10^2$	σ_γ (°)
$\times 1$	$\times 1$	0.780	1.091	0.877	0.945	4.21
$\times 1$	$\times 10$	0.773	1.062	0.866	0.924	4.18
$\times 100$	$\times 1$	0.079	0.122	0.090	0.104	0.45
$\times 100$	$\times 10$	0.078	0.115	0.089	0.099	0.43

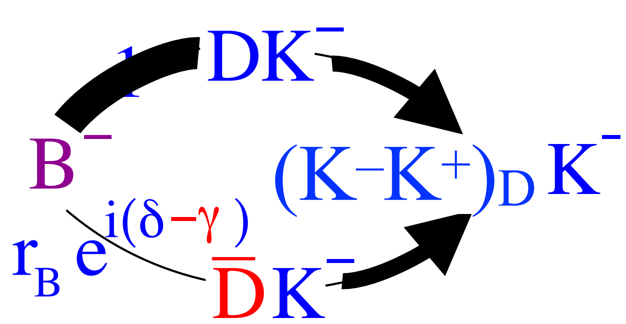
CP violation in 2-body modes.

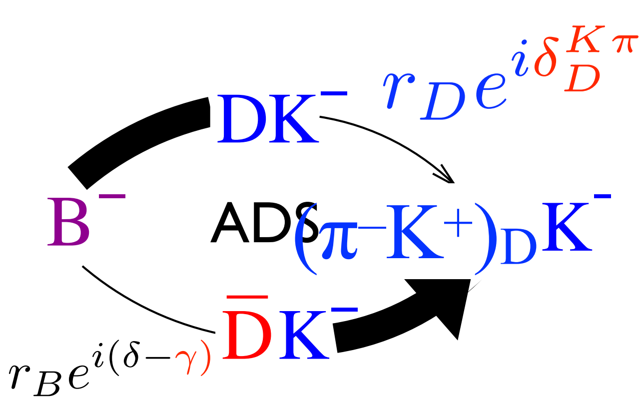
JHEP 04 (2021) 081

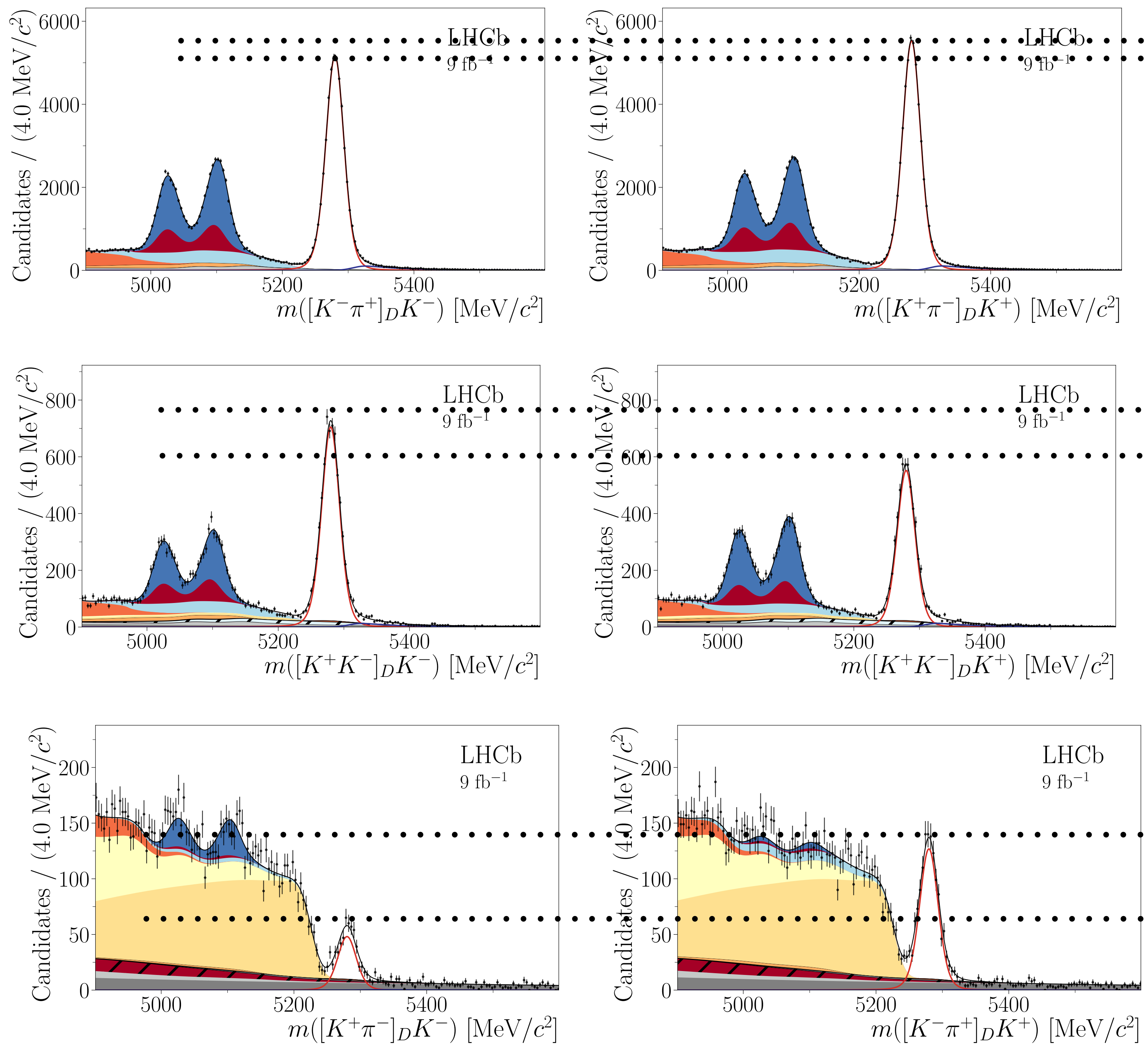
Phys.Lett. B777 (2018) 16-30

Phys.Lett. B760 (2016) 117-131









Parameter counting

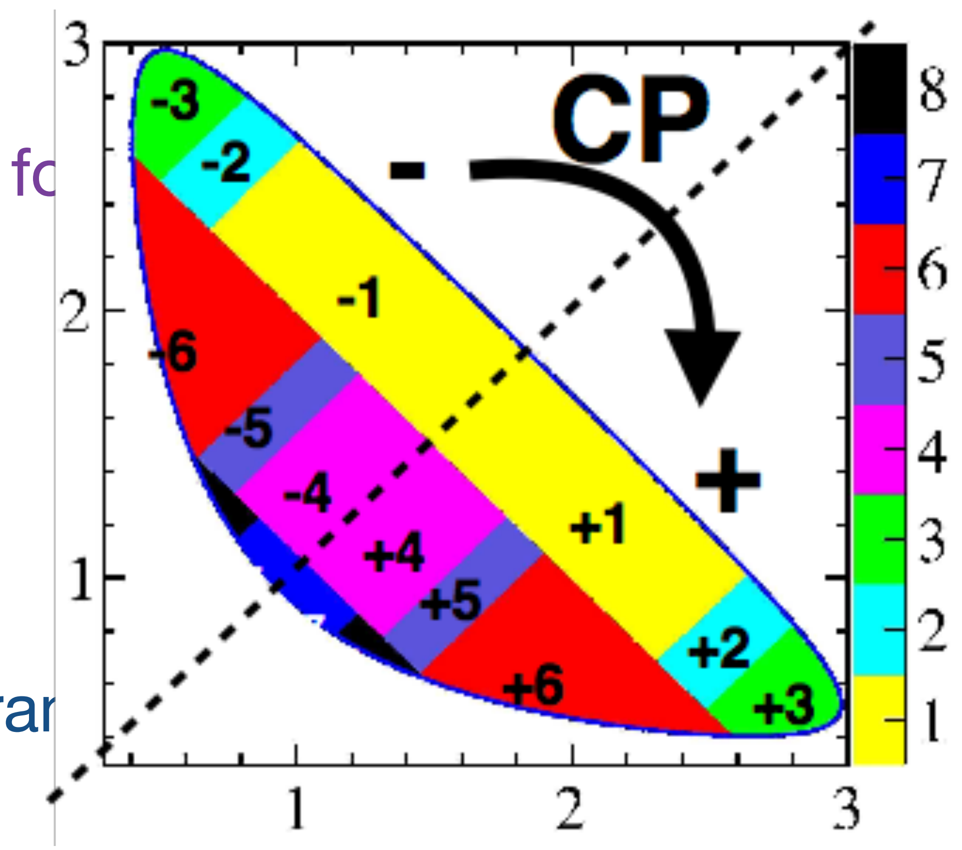
Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$

- Parameter counting:
  Number of bins:
  Dividing Dalitz plot into N bin pairs gives 4N bins (2N fo

  Number of parameters:
  3 global (r_B, δ_B, γ)
  2 N $(c_i, s_i$ one per each bin pair)

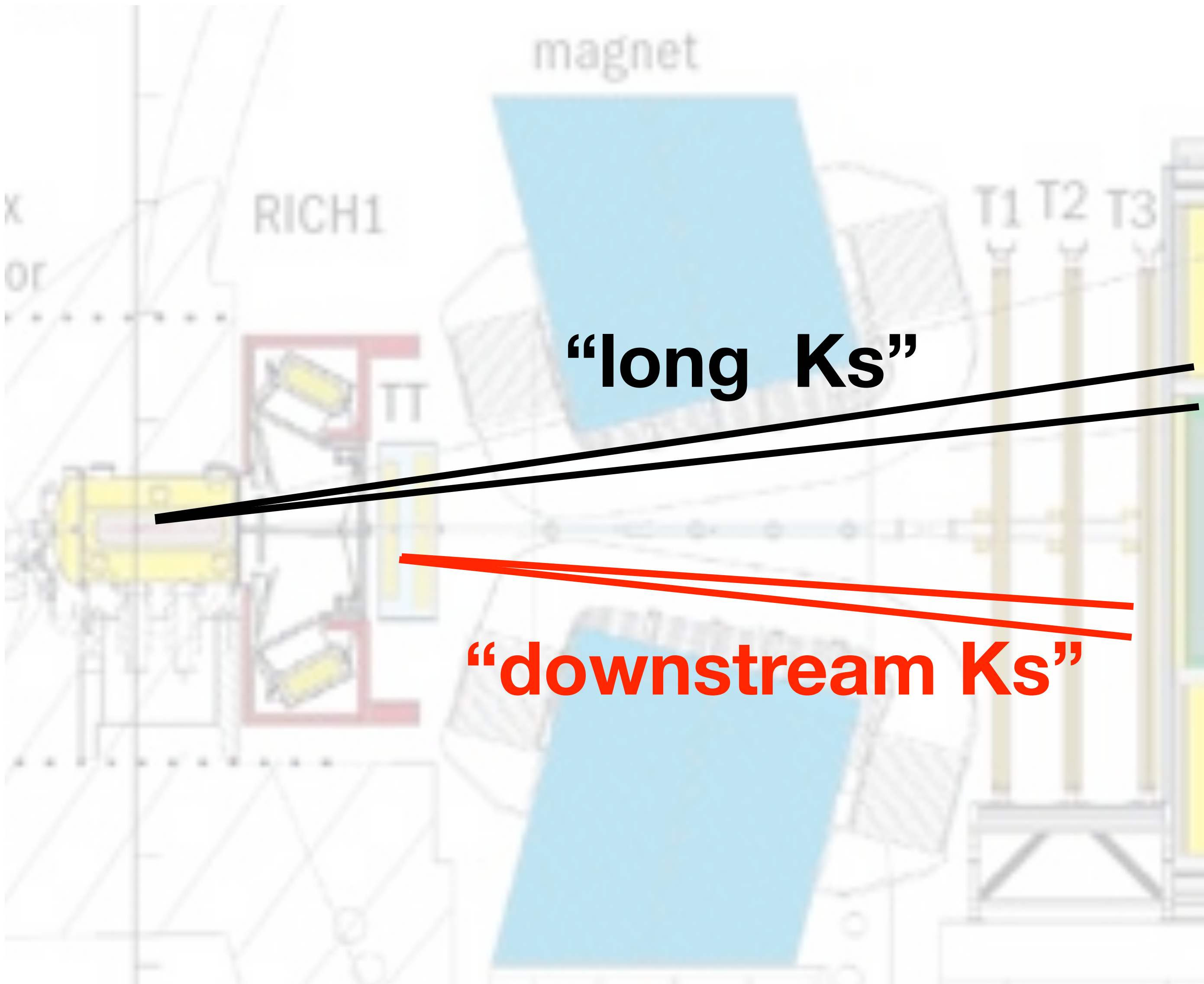
  Theoretically, if 2N+3 $\leq$ 4N (i.e. $N \geq 2$), can fit all parar
  from B decays.



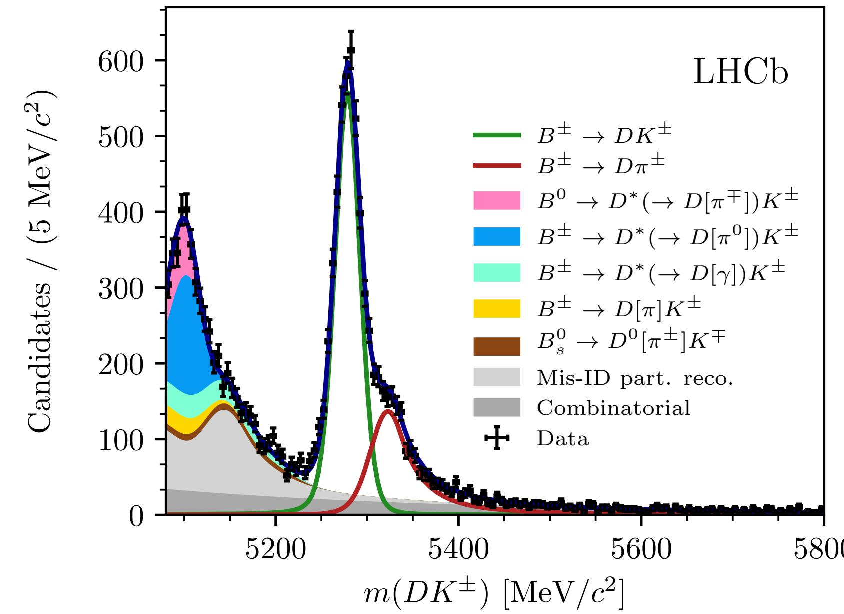
- In practice, to achieve good precision on γ, input from threshold to constrain c_i, s_i is absolutely critical.

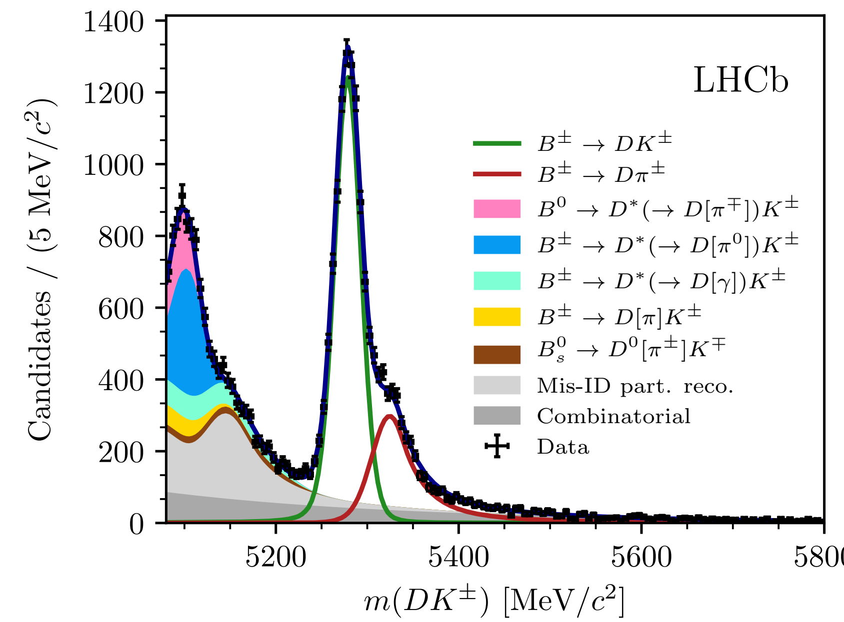
$B^\pm \to DK^\pm, D \to K_S\pi^+\pi^-$ at LHCb

LHCb: JHEP 02 (2021) 169



$B^\pm \to DK^\pm$





12.5k signal events

Input to LHCb γ combination

LHCb-CONF-2022-003

JHEP 12 (2021) 141

LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^\pm \to Dh^\pm$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^\pm \to Dh^\pm$	$D \to h^+\pi^-\pi^+\pi^-$	[30]	Run 1	As before
$B^\pm \to Dh^\pm$	$D \to K^\pm\pi^\mp\pi^+\pi^-$	[18]	Run 1&2	**New**
$B^\pm \to Dh^\pm$	$D \to h^+h^-\pi^0$	[19]	Run 1&2	**Updated**
$B^\pm \to Dh^\pm$	$D \to K^0_S h^+h^-$	[31]	Run 1&2	As before
$B^\pm \to Dh^\pm$	$D \to K^0_S K^\pm\pi^\mp$	[32]	Run 1&2	As before
$B^\pm \to D^*h^\pm$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^\pm \to DK^{*\pm}$	$D \to h^+h^-$	[33]	Run 1&2(*)	As before
$B^\pm \to DK^{*\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[33]	Run 1&2(*)	As before
$B^\pm \to Dh^\pm\pi^+\pi^-$	$D \to h^+h^-$	[34]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to h^+h^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to h^+\pi^-\pi^+\pi^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	[36]	Run 1	As before
$B^0 \to D^\mp\pi^\pm$	$D^+ \to K^-\pi^+\pi^+$	[37]	Run 1	As before
$B^0_s \to D^\mp_s K^\pm$	$D^+_s \to h^+h^-\pi^+$	[38]	Run 1	As before
$B^0_s \to D^\mp_s K^\pm\pi^+\pi^-$	$D^+_s \to h^+h^-\pi^+$	[39]	Run 1&2	As before
D decay	Observable(s)	Ref.	Dataset	Status since Ref. [14]
$D^0 \to h^+h^-$	ΔA_{CP}	[24, 40, 41]	Run 1&2	As before
$D^0 \to K^+K^-$	$A_{CP}(K^+K^-)$	[16, 24, 25]	Run 2	**New**
$D^0 \to h^+h^-$	$y_{CP} - y^{K^-\pi^+}_{CP}$	[42]	Run 1	As before
$D^0 \to h^+h^-$	$y_{CP} - y^{K^-\pi^+}_{CP}$	[15]	Run 2	**New**
$D^0 \to h^+h^-$	ΔY	[43–46]	Run 1&2	As before
$D^0 \to K^+\pi^-$ (Single Tag)	$R^\pm$, $(x'^\pm)^2$, $y'^\pm$	[47]	Run 1	As before
$D^0 \to K^+\pi^-$ (Double Tag)	$R^\pm$, $(x'^\pm)^2$, $y'^\pm$	[48]	Run 1&2(*)	As before
$D^0 \to K^\pm\pi^\mp\pi^+\pi^-$	$(x^2+y^2)/4$	[49]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x, y	[50]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[51]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[52]	Run 2	As before
$D^0 \to K^0_S\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_{CP}, Δx, Δy	[17]	Run 2	**New**

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
$B^\pm \to DK^{*\pm}$	$\kappa^{DK^{*\pm}}_{B^\pm}$	LHCb	[33]	As before
$B^0 \to DK^{*0}$	$\kappa^{DK^{*0}}_{B^0}$	LHCb	[53]	As before
$B^0 \to D^\mp\pi^\pm$	β	HFLAV	[13]	As before
$B^0_s \to D^\mp_s K^\pm(\pi\pi)$	ϕ_s	HFLAV	[13]	As before
$D \to K^+\pi^-$	$\cos\delta^{K\pi}_D$, $\sin\delta^{K\pi}_D$, $(r^{K\pi}_D)^2$, x^2, y	CLEO-c	[27]	**New**
$D \to K^+\pi^-$	$A_{K\pi}$, $A^{\pi\pi\pi^0}_{K\pi}$, $r^{K\pi}_D\cos\delta^{K\pi}_D$, $r^{K\pi}_D\sin\delta^{K\pi}_D$	BESIII	[28]	**New**
$D \to h^+h^-\pi^0$	$F^+_{\pi\pi\pi^0}$, $F^+_{KK\pi^0}$	CLEO-c	[54]	As before
$D \to \pi^+\pi^-\pi^+\pi^-$	$F^+_{4\pi}$	CLEO-c+BESIII	[26, 54]	**Updated**
$D \to K^+\pi^-\pi^0$	$r^{K\pi\pi^0}_D$, $\delta^{K\pi\pi^0}_D$, $\kappa^{K\pi\pi^0}_D$	CLEO-c+LHCb+BESIII	[55–57]	As before
$D \to K^\pm\pi^\mp\pi^+\pi^-$	$r^{K3\pi}_D$, $\delta^{K3\pi}_D$, $\kappa^{K3\pi}_D$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
$D \to K^0_S K^\pm\pi^\mp$	$r^{K^0_S K\pi}_D$, $\delta^{K^0_S K\pi}_D$, $\kappa^{K^0_S K\pi}_D$	CLEO	[58]	As before
$D \to K^0_S K^\pm\pi^\mp$	$r^{K^0_S K\pi}_D$	LHCb	[59]	As before

Input to LHCb γ combination

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to Dh^\pm$	$D \to h$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to Dh^\pm$	$D \to K$			
$B^\pm \to D^* h^\pm$	$D \to h$			
$B^\pm \to DK^{*\pm}$	$D \to h$			
$B^\pm \to DK^{*\pm}$	$D \to h$			
$B^\pm \to Dh^\pm\pi^+\pi^-$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to h$			
$B^0 \to DK^{*0}$	$D \to K$			
$B^0 \to D^\mp\pi^\pm$	$D^+ \to$			
$B_s^0 \to D_s^\mp K^\pm$	$D_s^+ \to$			
$B_s^0 \to D_s^\mp K^\pm\pi^+\pi^-$	$D_s^+ \to$			

D decay	Observ
$D^0 \to h^+h^-$	ΔA_{CP}
$D^0 \to K^+K^-$	$A_{CP}(K$
$D^0 \to h^+h^-$	$y_{CP} -$
$D^0 \to h^+h^-$	$y_{CP} -$
$D^0 \to h^+h^-$	ΔY
$D^0 \to K^+\pi^-$ (Single Tag)	$R^\pm$, $(x$
$D^0 \to K^+\pi^-$ (Double Tag)	$R^\pm$, $(x$
$D^0 \to K^\pm\pi^\mp\pi^+\pi^-$	$(x^2 + y$
$D^0 \to K_S^0\pi^+\pi^-$	x, y
$D^0 \to K_S^0\pi^+\pi^-$	x_{CP}, y_C
$D^0 \to K_S^0\pi^+\pi^-$	x_{CP}, y_C
$D^0 \to K_S^0\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_C

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
		LHCb	[33]	As before
		LHCb	[53]	As before
		HFLAV	[13]	As before
		HFLAV	[13]	As before
		CLEO-c	[27]	**New**
	$\delta_D^{K\pi}$	BESIII	[28]	**New**
		CLEO-c	[54]	As before
		CLEO-c+BESIII	[26, 54]	**Updated**
		CLEO-c+LHCb+BESIII	[55–57]	As before
		CLEO-c+LHCb+BESIII	[49, 55–57]	As before
		CLEO	[58]	As before
		LHCb	[59]	As before

Too much for one slide to be readable. Key point: It's a lot. And there's more, e.g.

BES III measurements on $D \to$

$K_S\pi^+\pi^-\pi^0$ arXiv:2305.03975 (2023)

$K^+K^-\pi^+\pi^-$ PRD 107 (2023) 3, 032009

LHCb measurements with

$B^\pm \to DK^\pm,\ D \to K^+K^-\pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-$

arXiv:2301.10328 (2023)

LHCb γ combination

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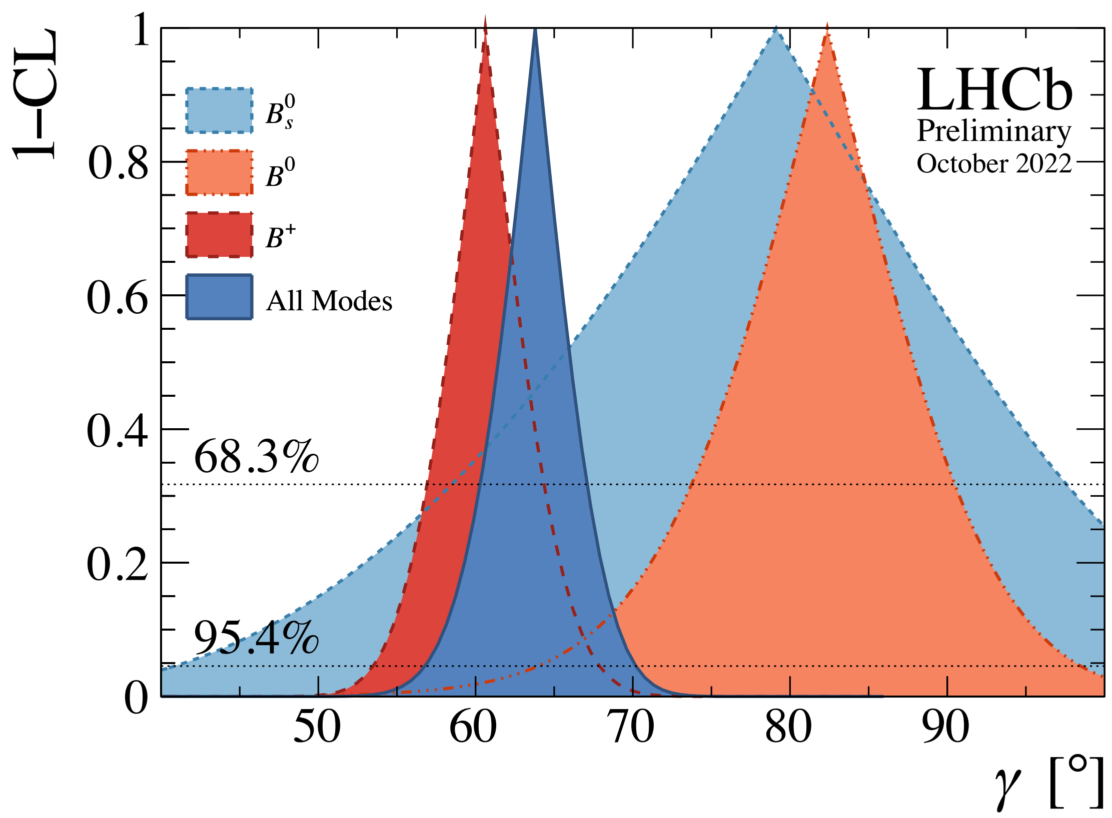
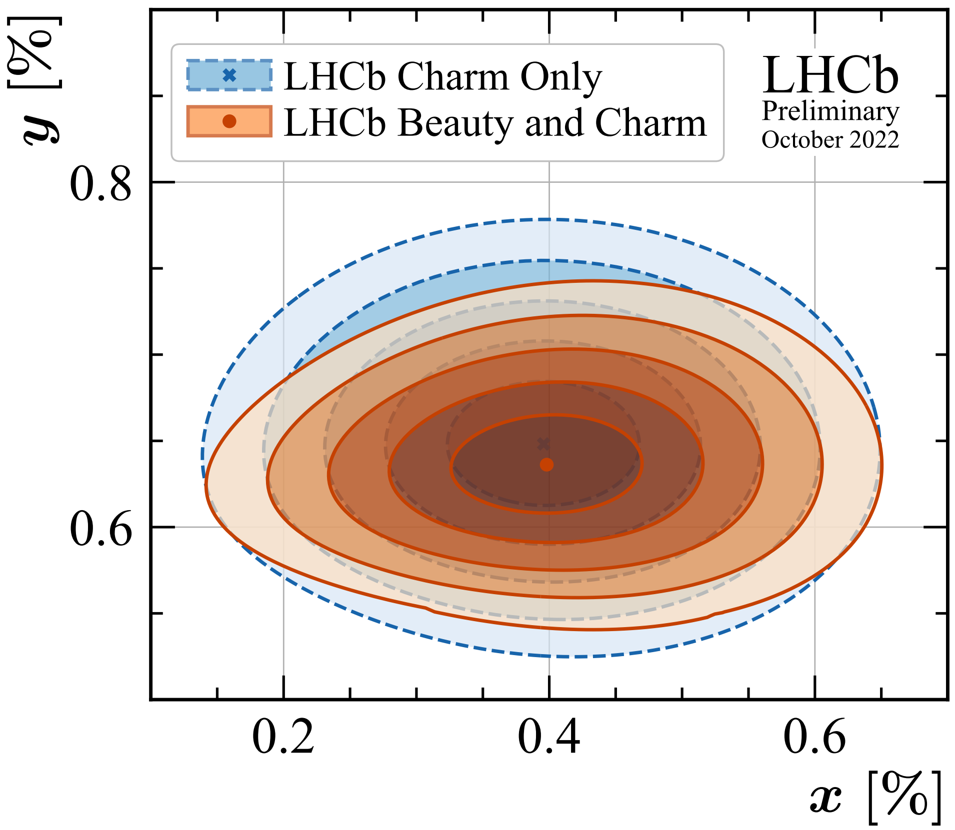
impact on charm mixing





$$\gamma = 63.8^{\circ}{}^{+3.5^{\circ}}_{-3.7^{\circ}}$$

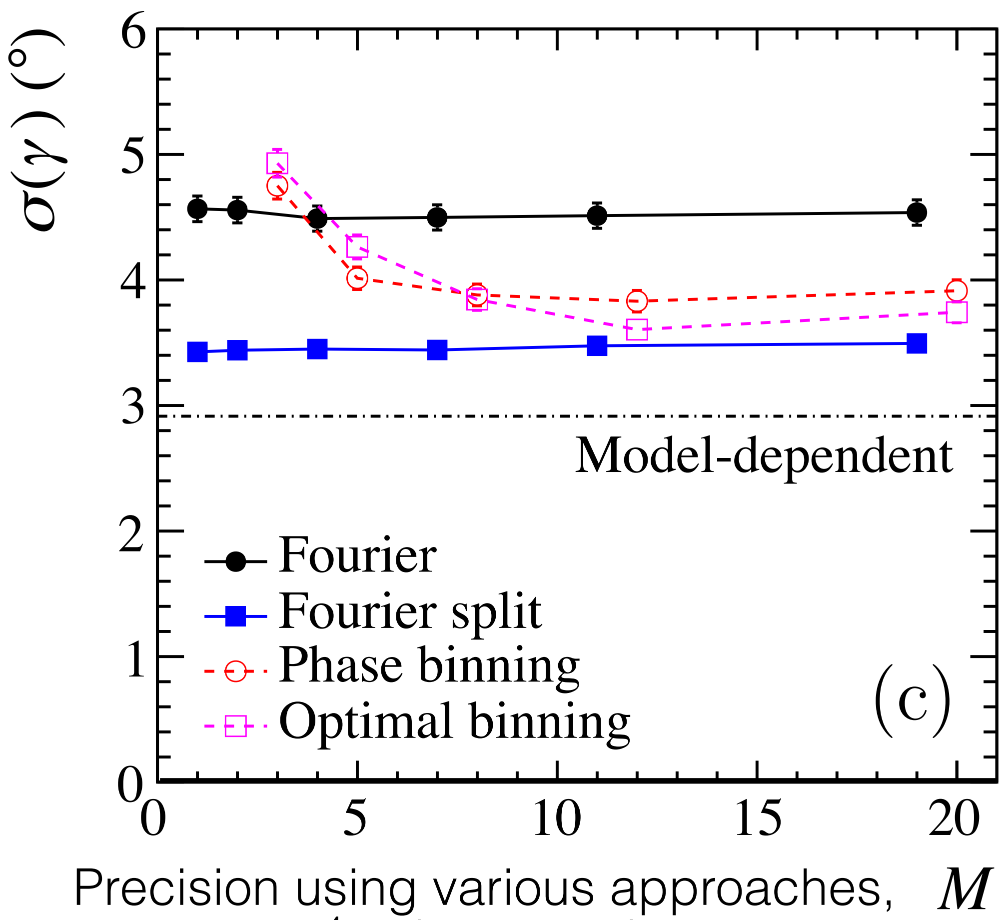
$$x = \left(3.98^{+0.50}_{-0.49}\right) \cdot 10^{-3}$$

$$y = \left(6.36^{+0.20}_{-0.19}\right) \cdot 10^{-3}$$

Unbinned model-independent method.

Anton Poluektov: Eur.Phys.J.C 78 (2018) 2, 121

- Project 2-D Dalitz plot onto 1D.
- Use amplitude model to associate each point in phase space to a phase difference δ^{model}.
- $c_i, s_i \to C(\delta^{\text{model}}), S(\delta^{\text{model}})$, functions C, S parameterised in a generic way (Fourier series)



Precision using various approaches, with 2×10^4 $B^{\pm} \to DK^{\pm}$ events and 10^4 $D\overline{D}$ events

Expect precision between binned and model-dependent approach.

Other unbinned methods exists

- Anton Poluektov: Eur.Phys.J.C 78 (2018) 2, 121. Projects 2-D Dalitz plot onto 1D. Achieves precision between binned and model-dependent approach.

- Jeffrey V. Backus et al, arXiv:2211.05133, integrate over the 2-D Dalitz plot in an unbinned way. Get a precision of ~5° for similar data set sizes we use, however, comparison is difficult due to different assumptions on the values of γ and δ_B, and implementation differences in amplitude model.

- In contrast to these methods and the binned method, we do not do any integration, averaging or projection and therefore do not suffer the associated information loss.

Input to LHCb γ combination

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^\pm \to Dh^\pm$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^\pm \to Dh^\pm$	$D \to h^+\pi^-\pi^+\pi^-$	[30]	Run 1	As before
$B^\pm \to Dh^\pm$	$D \to K^\pm\pi^\mp\pi^+\pi^-$	[18]	Run 1&2	**New**
$B^\pm \to Dh^\pm$	$D \to h^+h^-\pi^0$	[19]	Run 1&2	**Updated**
$B^\pm \to Dh^\pm$	$D \to K^0_S h^+h^-$	[31]	Run 1&2	As before
$B^\pm \to Dh^\pm$	$D \to K^0_S K^\pm\pi^\mp$	[32]	Run 1&2	As before
$B^\pm \to D^*h^\pm$	$D \to h^+h^-$	[29]	Run 1&2	As before
$B^\pm \to DK^{*\pm}$	$D \to h^+h^-$	[33]	Run 1&2(*)	As before
$B^\pm \to DK^{*\pm}$	$D \to h^+\pi^-\pi^+\pi^-$	[33]	Run 1&2(*)	As before
$B^\pm \to Dh^\pm\pi^+\pi^-$	$D \to h^+h^-$	[34]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to h^+h^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to h^+\pi^-\pi^+\pi^-$	[35]	Run 1&2(*)	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	[36]	Run 1	As before
$B^0 \to D^\mp\pi^\pm$	$D^+ \to K^-\pi^+\pi^+$	[37]	Run 1	As before
$B^0_s \to D^\mp_s K^\pm$	$D^+_s \to h^+h^-\pi^+$	[38]	Run 1	As before
$B^0_s \to D^\mp_s K^\pm\pi^+\pi^-$	$D^+_s \to h^+h^-\pi^+$	[39]	Run 1&2	As before
D decay	Observable(s)	Ref.	Dataset	Status since Ref. [14]
$D^0 \to h^+h^-$	ΔA_{CP}	[24, 40, 41]	Run 1&2	As before
$D^0 \to K^+K^-$	$A_{CP}(K^+K^-)$	[16, 24, 25]	Run 2	**New**
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[42]	Run 1	As before
$D^0 \to h^+h^-$	$y_{CP} - y_{CP}^{K^-\pi^+}$	[15]	Run 2	**New**
$D^0 \to h^+h^-$	ΔY	[43–46]	Run 1&2	As before
$D^0 \to K^+\pi^-$ (Single Tag)	$R^\pm$, $(x'^\pm)^2$, $y'^\pm$	[47]	Run 1	As before
$D^0 \to K^+\pi^-$ (Double Tag)	$R^\pm$, $(x'^\pm)^2$, $y'^\pm$	[48]	Run 1&2(*)	As before
$D^0 \to K^\pm\pi^\mp\pi^+\pi^-$	$(x^2+y^2)/4$	[49]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x, y	[50]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[51]	Run 1	As before
$D^0 \to K^0_S\pi^+\pi^-$	x_{CP}, y_{CP}, Δx, Δy	[52]	Run 2	As before
$D^0 \to K^0_S\pi^+\pi^-$ (μ^- tag)	x_{CP}, y_{CP}, Δx, Δy	[17]	Run 2	**New**

BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
$B^\pm \to DK^{*\pm}$	$\kappa_{B^\pm}^{DK^{*\pm}}$	LHCb	[33]	As before
$B^0 \to DK^{*0}$	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[53]	As before
$B^0 \to D^\mp\pi^\pm$	β	HFLAV	[13]	As before
$B^0_s \to D^\mp_s K^\pm(\pi\pi)$	ϕ_s	HFLAV	[13]	As before
$D \to K^+\pi^-$	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[27]	**New**
$D \to K^+\pi^-$	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[28]	**New**
$D \to h^+h^-\pi^0$	$F^+_{\pi\pi\pi^0}$, $F^+_{KK\pi^0}$	CLEO-c	[54]	As before
$D \to \pi^+\pi^-\pi^+\pi^-$	$F^+_{4\pi}$	CLEO-c+BESIII	[26, 54]	**Updated**
$D \to K^+\pi^-\pi^0$	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[55–57]	As before
$D \to K^\pm\pi^\mp\pi^+\pi^-$	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
$D \to K^0_S K^\pm\pi^\mp$	$r_D^{K^0_S K\pi}$, $\delta_D^{K^0_S K\pi}$, $\kappa_D^{K^0_S K\pi}$	CLEO	[58]	As before
$D \to K^0_S K^\pm\pi^\mp$	$r_D^{K^0_S K\pi}$	LHCb	[59]	As before

$$\gamma = 63.8^{\circ}{}^{+3.5^\circ}_{-3.7^\circ}$$

Input to LHCb γ combination

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LHCb

B decay	D decay	Ref.	Dataset	Status since Ref. [14]
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^{\pm} \to$				
$B^0 \to$				
$B^0 \to$				
$B^0 \to$				
$B^0 \to$				
$B_s^0 \to$				
$B_s^0 \to$				

D deca				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				
$D^0 \to$				

Too much for one slide to be readable. Key point: It's a lot. And there's more, e.g.

BES III measurements on $D \to$

$K_S\pi^+\pi^-\pi^0$ arXiv:2305.03975 (2023)

$K^+K^-\pi^+\pi^-$ PRD 107 (2023) 3, 032009

LHCb measurements with $B^{\pm} \to DK^{\pm}$, $D \to K^+K^-\pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-$ arXiv:2301.10328 (2023)

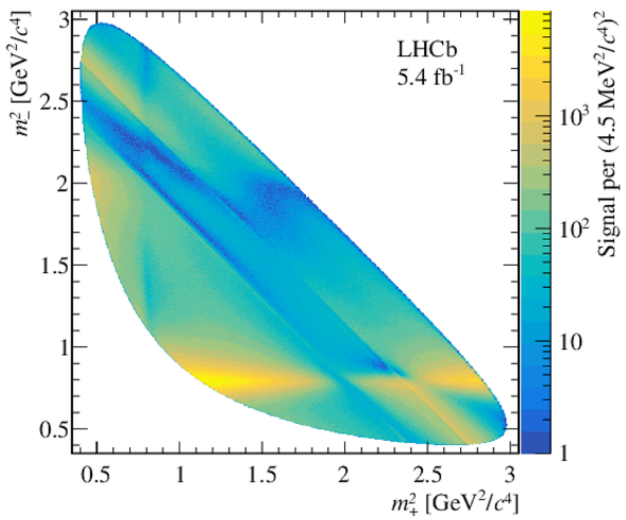
BES III and others

Decay	Parameters	Source	Ref.	Status since Ref. [14]
	$\kappa_{B^{\pm}}^{DK^{*\pm}}$	LHCb	[33]	As before
	$\kappa_{B^0}^{DK^{*0}}$	LHCb	[53]	As before
	β	HFLAV	[13]	As before
…)	ϕ_s	HFLAV	[13]	As before
	$\cos\delta_D^{K\pi}$, $\sin\delta_D^{K\pi}$, $(r_D^{K\pi})^2$, x^2, y	CLEO-c	[27]	**New**
	$A_{K\pi}$, $A_{K\pi}^{\pi\pi\pi^0}$, $r_D^{K\pi}\cos\delta_D^{K\pi}$, $r_D^{K\pi}\sin\delta_D^{K\pi}$	BESIII	[28]	**New**
	$F_{\pi\pi\pi^0}^+$, $F_{KK\pi^0}^+$	CLEO-c	[54]	As before
	$F_{4\pi}^+$	CLEO-c+BESIII	[26, 54]	**Updated**
	$r_D^{K\pi\pi^0}$, $\delta_D^{K\pi\pi^0}$, $\kappa_D^{K\pi\pi^0}$	CLEO-c+LHCb+BESIII	[55–57]	As before
	$r_D^{K3\pi}$, $\delta_D^{K3\pi}$, $\kappa_D^{K3\pi}$	CLEO-c+LHCb+BESIII	[49, 55–57]	As before
	$r_D^{K_S^0K\pi}$, $\delta_D^{K_S^0K\pi}$, $\kappa_D^{K_S^0K\pi}$	CLEO	[58]	As before
	$r_D^{K_S^0K\pi}$	LHCb	[59]	As before

$$\gamma = 63.8^{\circ}{}^{+3.5^{\circ}}_{-3.7^{\circ}}$$

LHCb model-independent mixing with $D^0 \to K_S \pi^+ \pi^-$

Same BES III input also critical for charm mixing



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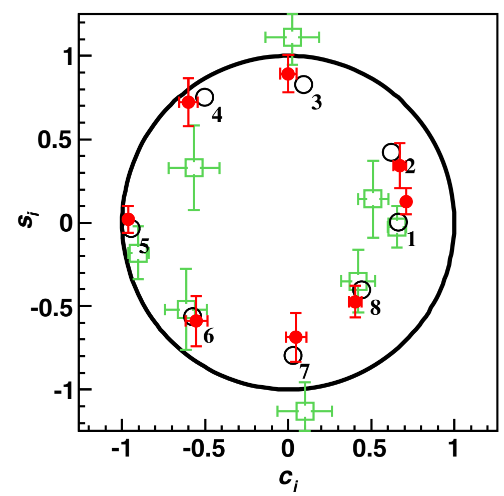


3.1×10^7

$D^0 \to K_S^0 \pi^+ \pi^-$

BESIII: PRL 124 (2020) 24, 241802

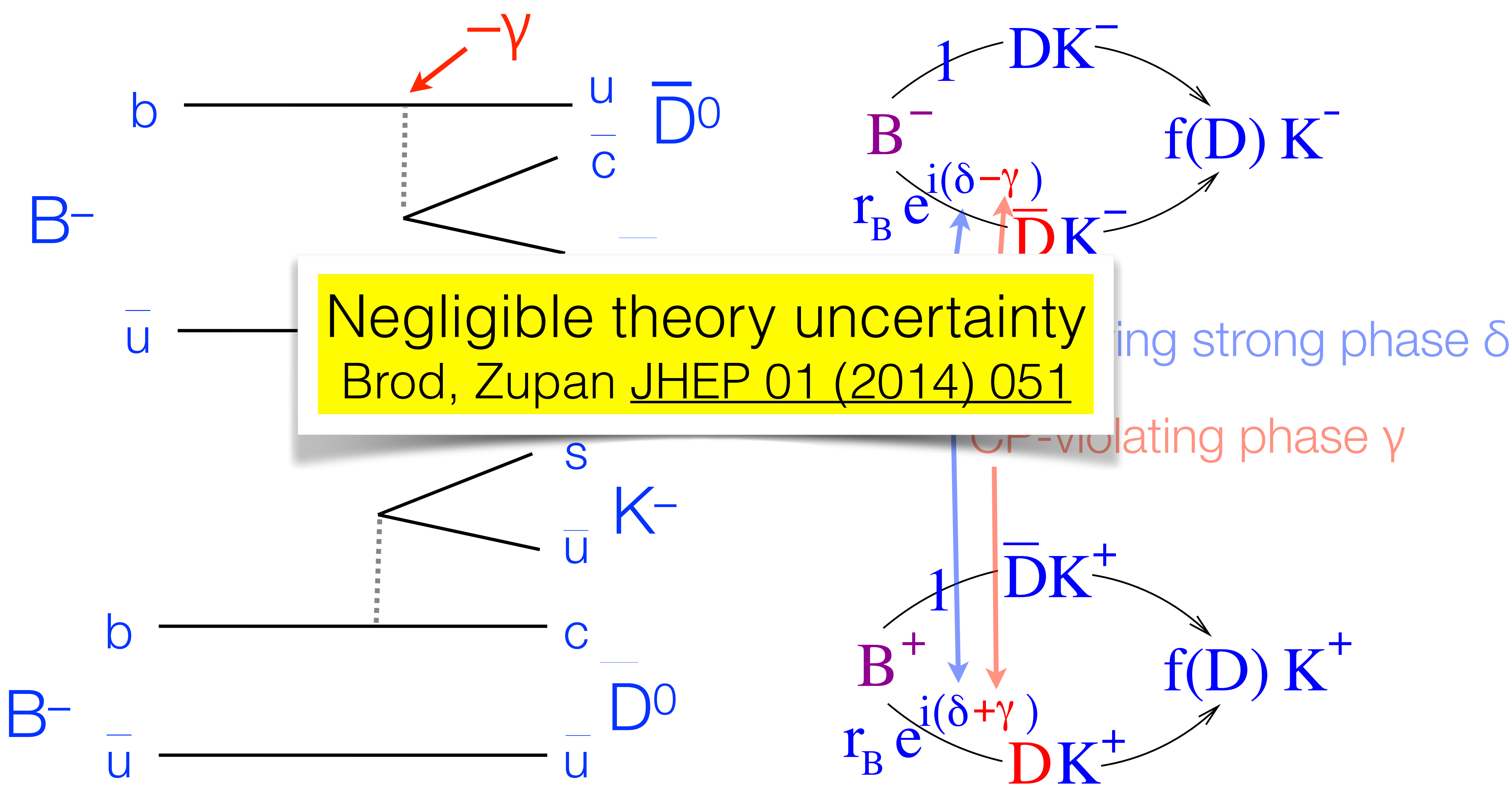
LHCb: PRL 127 (2021) 11, 111801

Method: Phys.Rev. D99 (2019) no.1, 012007

first observation of non-zero $x = \frac{\Delta m}{\Gamma}$, i.e. of a mass difference between the two charm mass eigenstates.

$$x = (3.98^{+0.56}_{-0.54}) \times 10^{-3},$$
$$y = (\ 4.6^{+1.5}_{-1.4}\ ) \times 10^{-3},$$
$$|q/p| = 0.996 \pm 0.052,$$
$$\phi = 0.056^{+0.047}_{-0.051}.$$

$B^{\pm} \rightarrow DK^{\pm}$

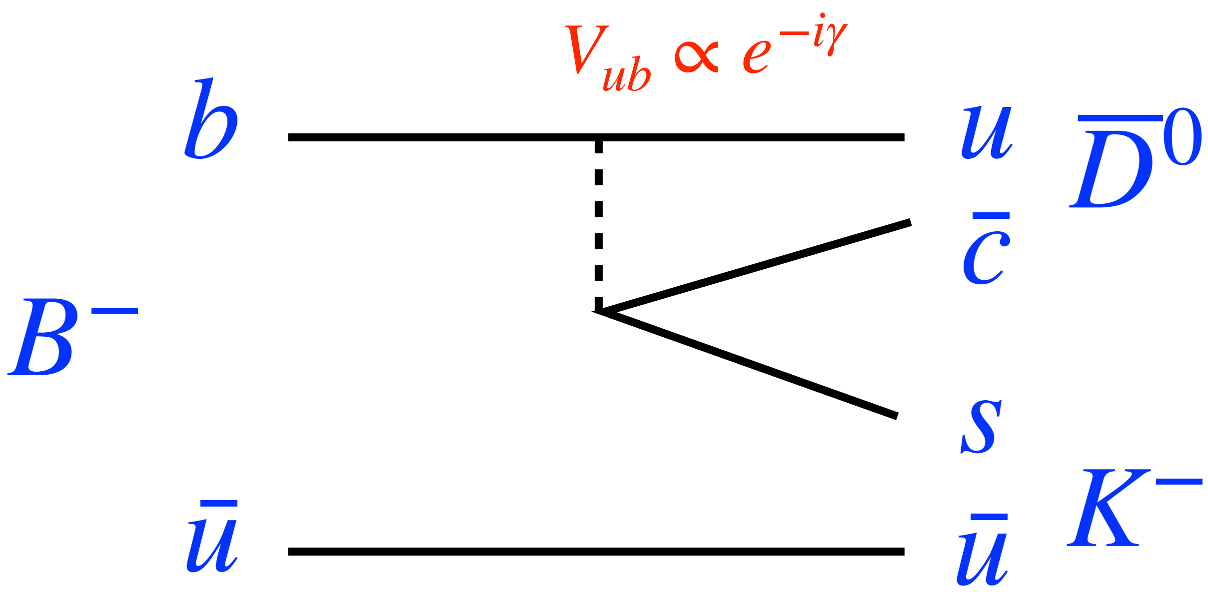


Negligible theory uncertainty
Brod, Zupan JHEP 01 (2014) 051

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

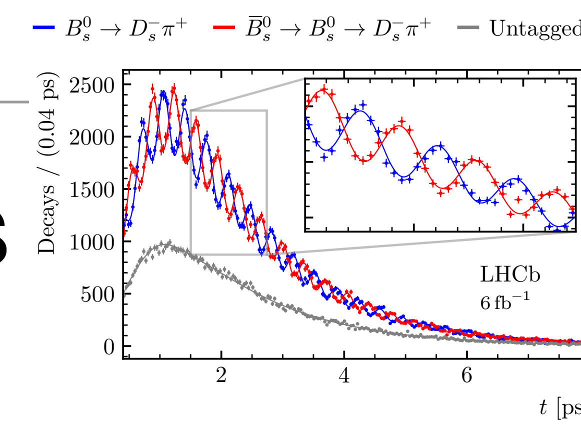
Trees and loops

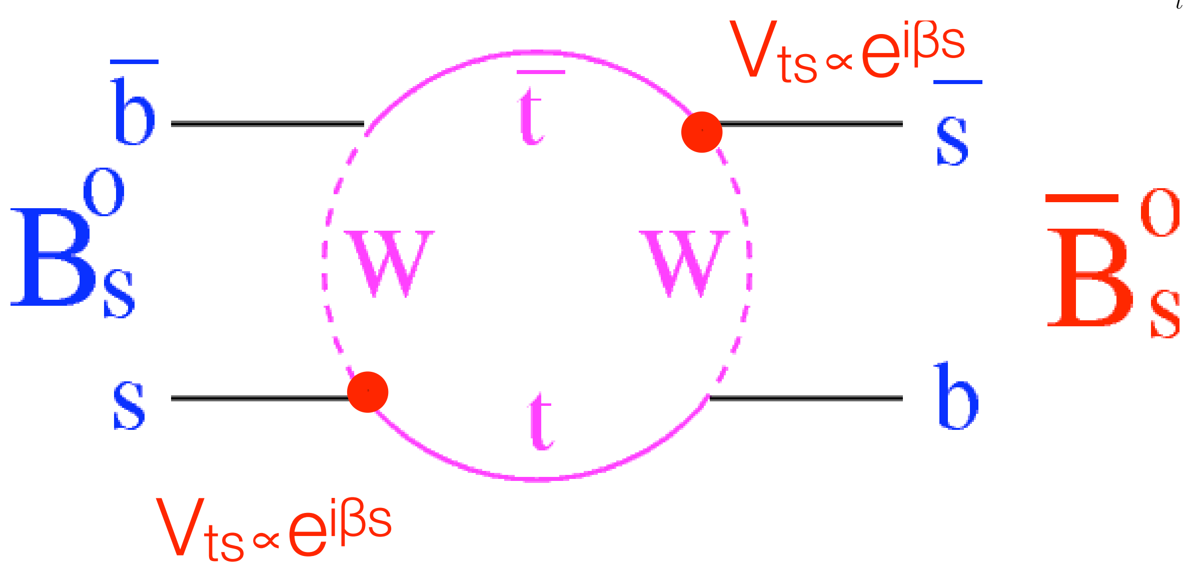
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5



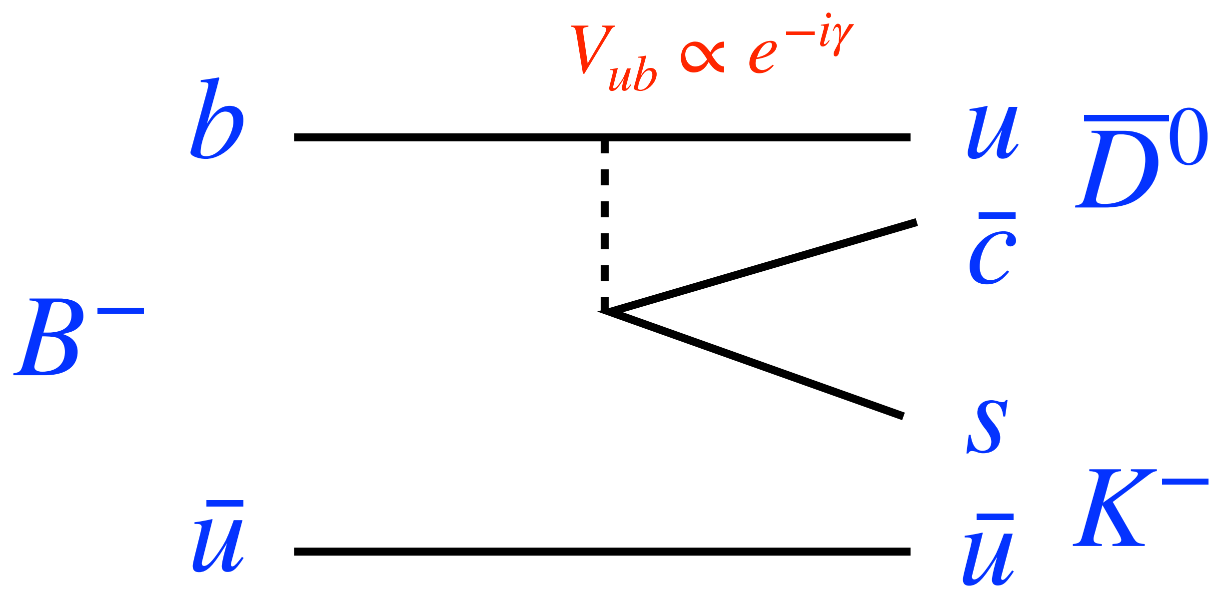


"New Physics" in trees generally seen as less likely than in loops. (Lenz et al find however that there is room for NP in trees that could affect γ by several degrees.) In any case: would like to compare tees and loops.

JHEP 06 (2014) 040, JHEP 07 (2020) 177

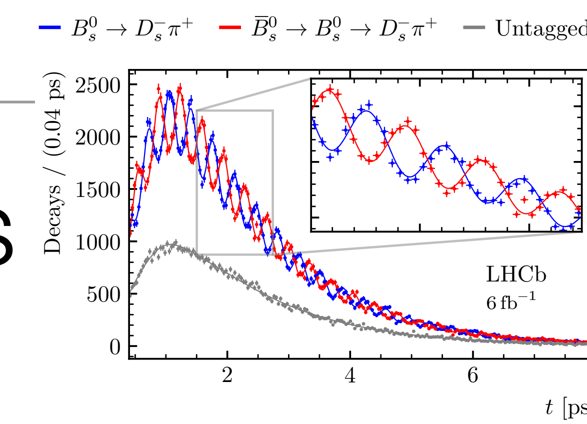
Trees and loops

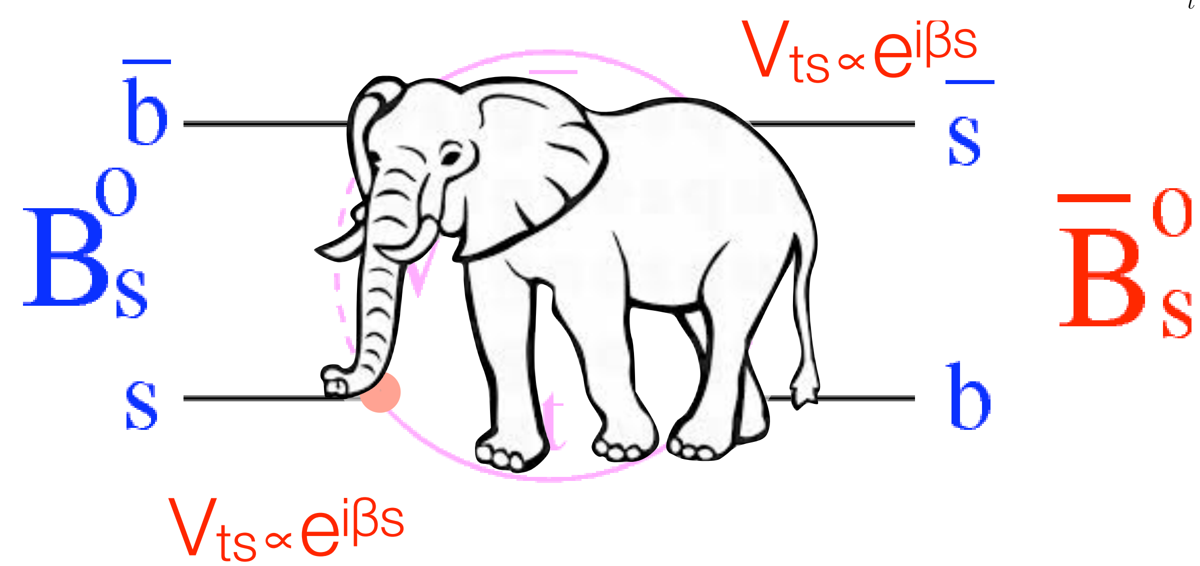
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5



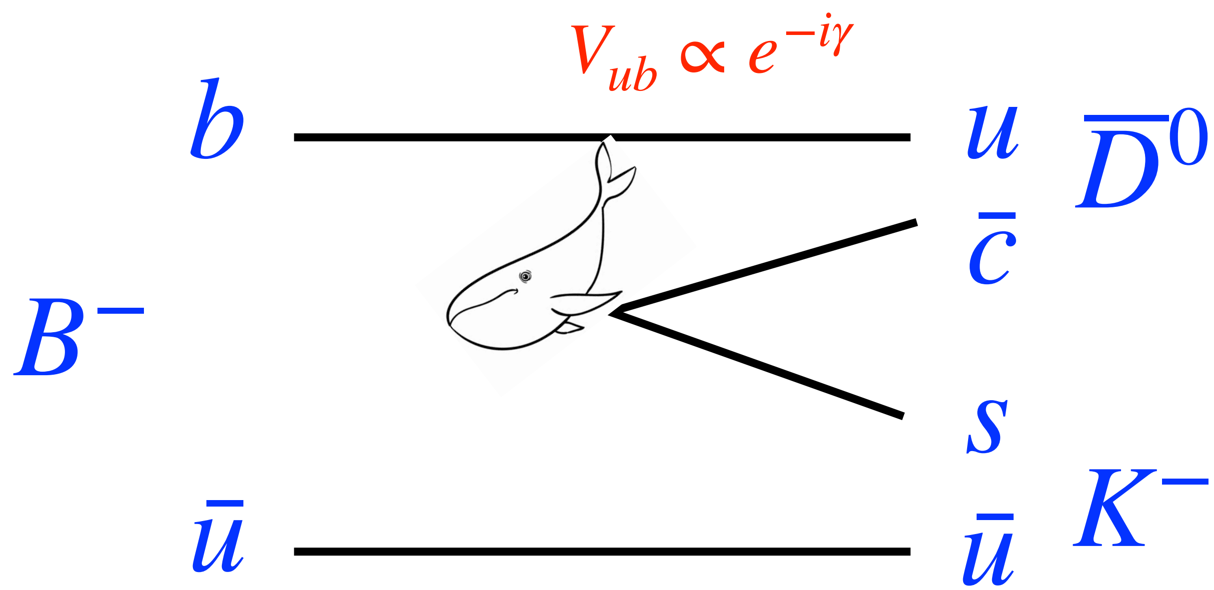


"New Physics" in trees generally seen as less likely than in loops. (Lenz et al find however that there is room for NP in trees that could affect γ by several degrees.) In any case: would like to compare tees and loops.

JHEP 06 (2014) 040, JHEP 07 (2020) 177

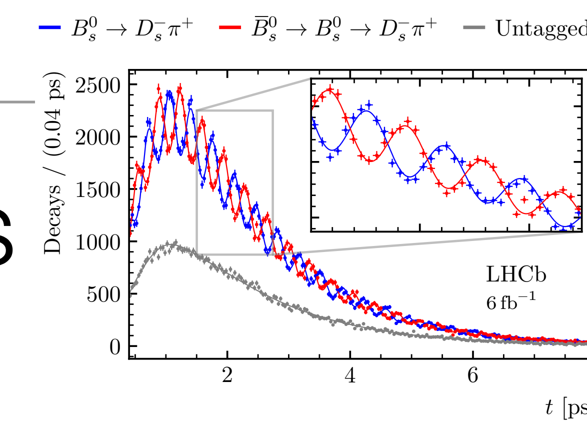
Trees and loops

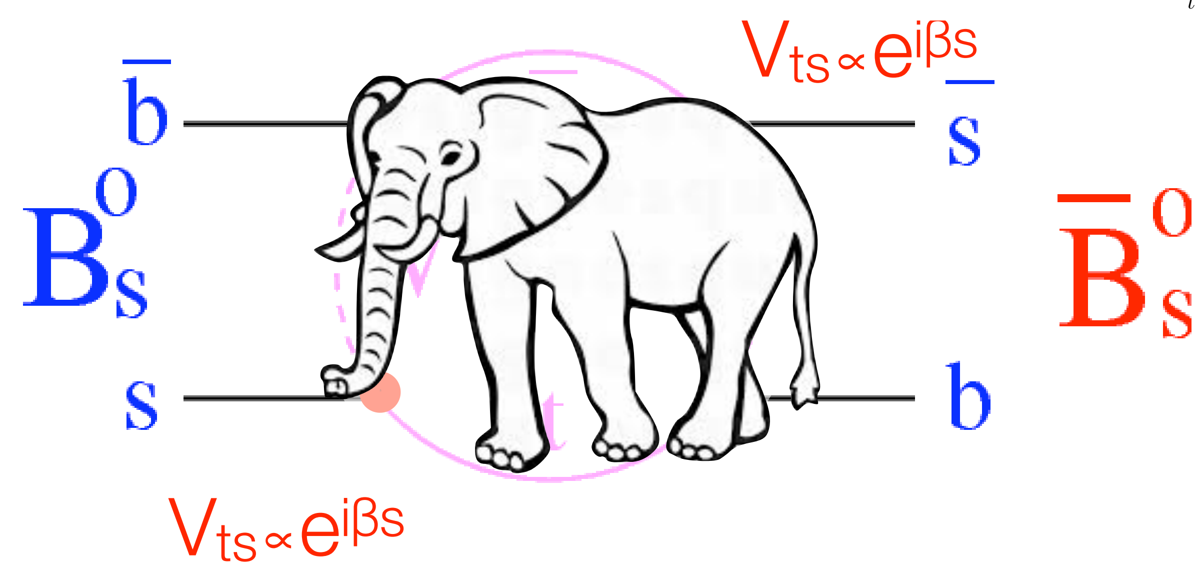
Trees



Loops

LHCb: Nature Phys. 18 (2022) 1, 1-5



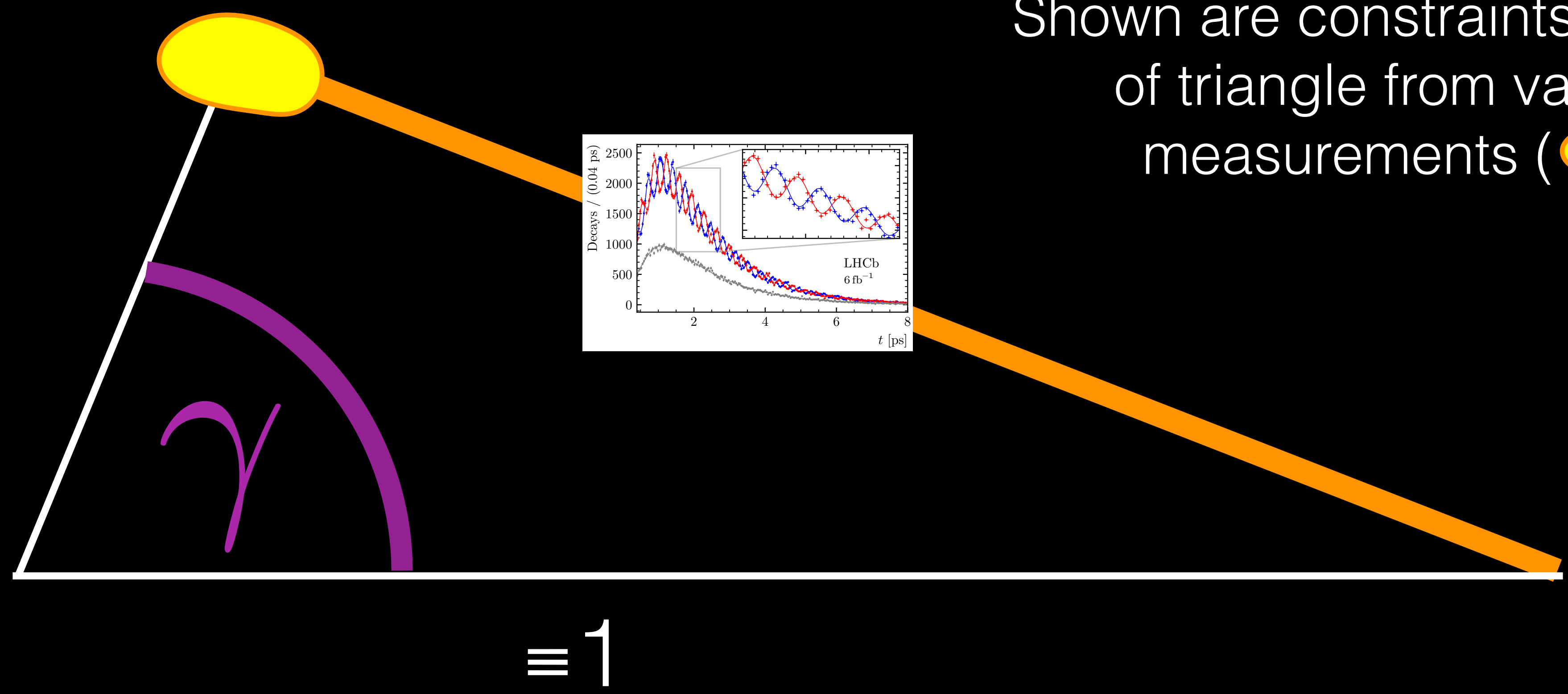


"New Physics" in trees generally seen as less likely than in loops. (Lenz et al find however that there is room for NP in trees that could affect γ by several degrees.) In any case: would like to compare tees and loops.

JHEP 06 (2014) 040, JHEP 07 (2020) 177

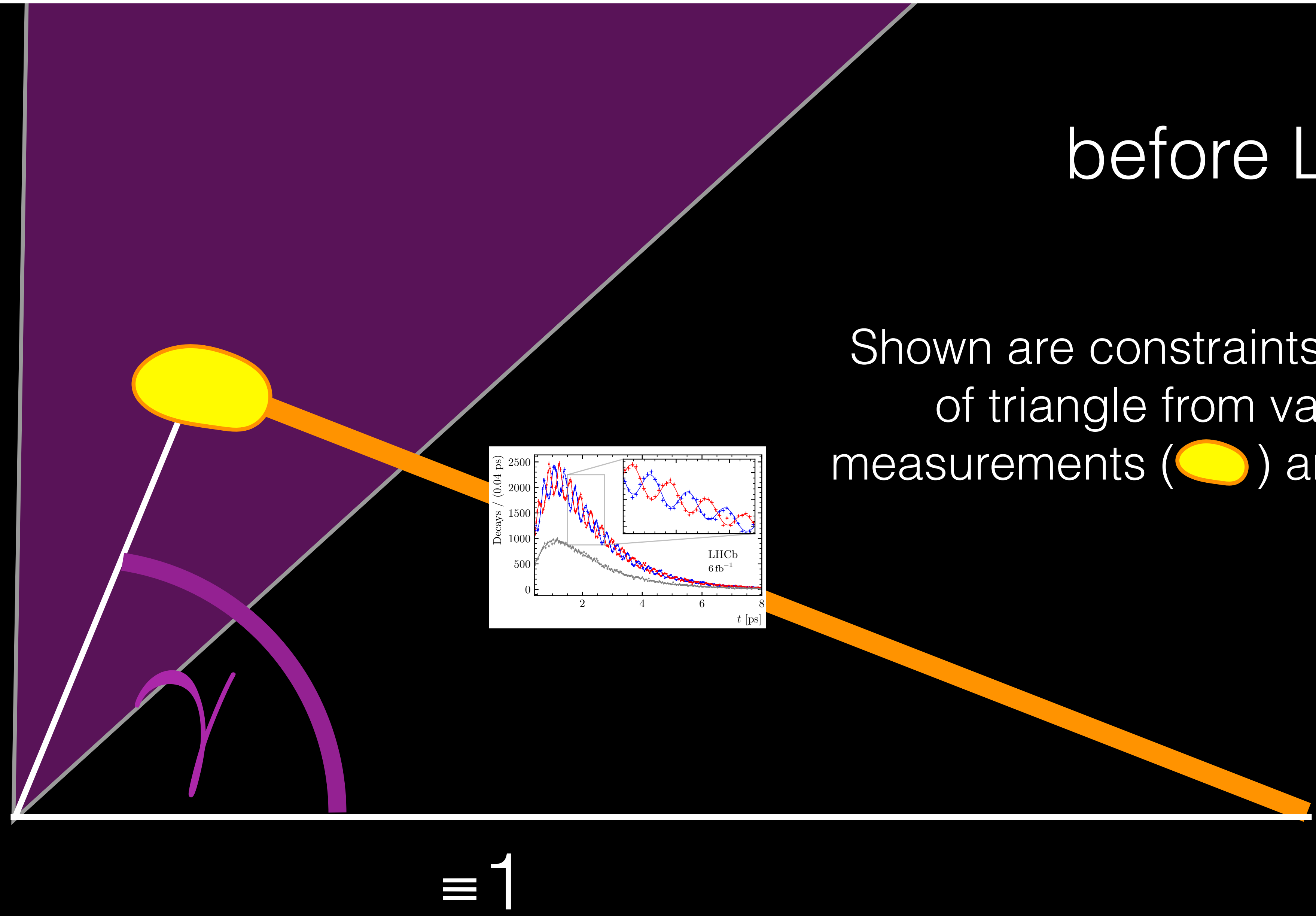
Unitarity triangle

geometric representation of Standard Model constraints



Shown are constraints on apex of triangle from various measurements ( )

Unitarity triangle

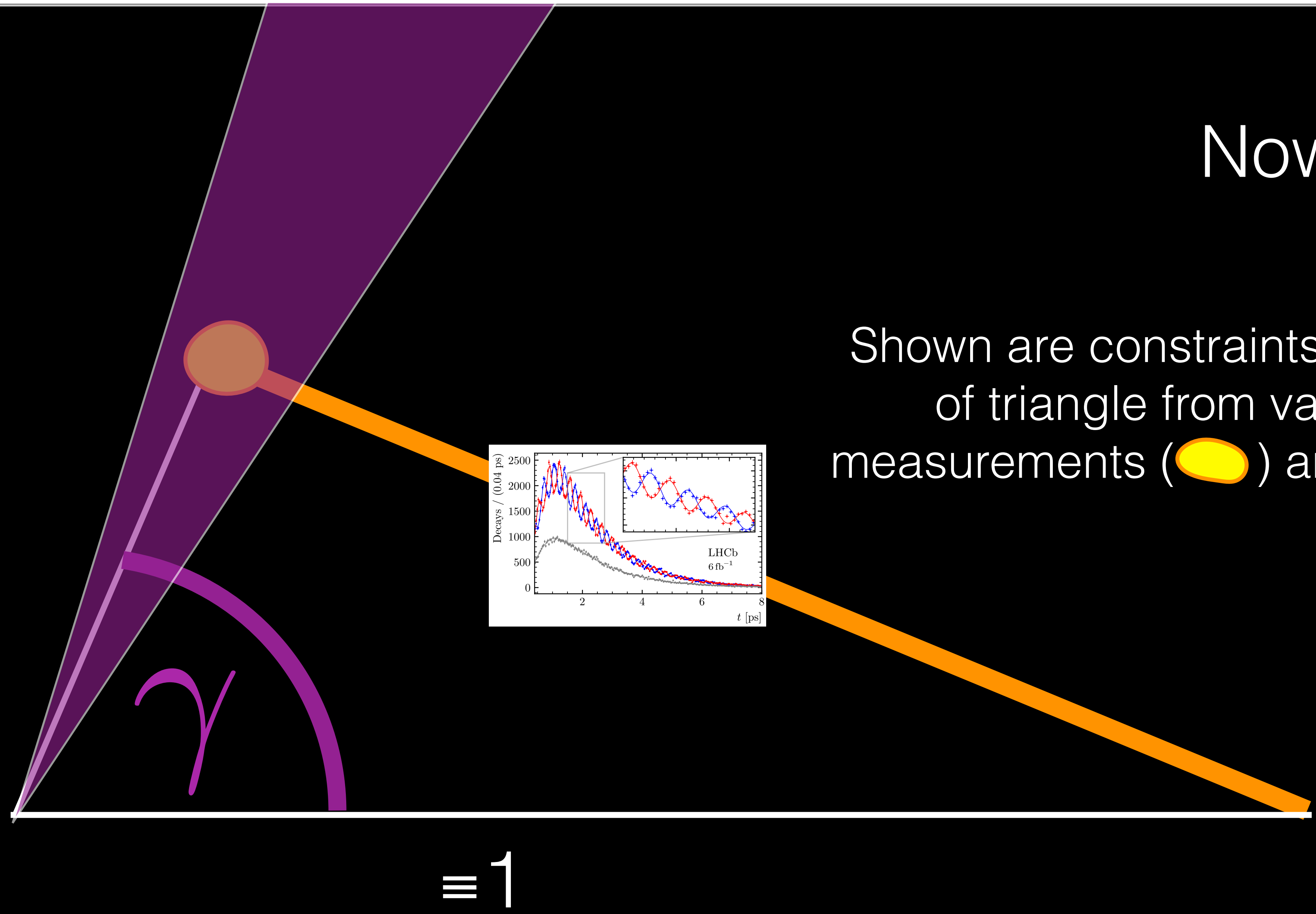


before LHCb

Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

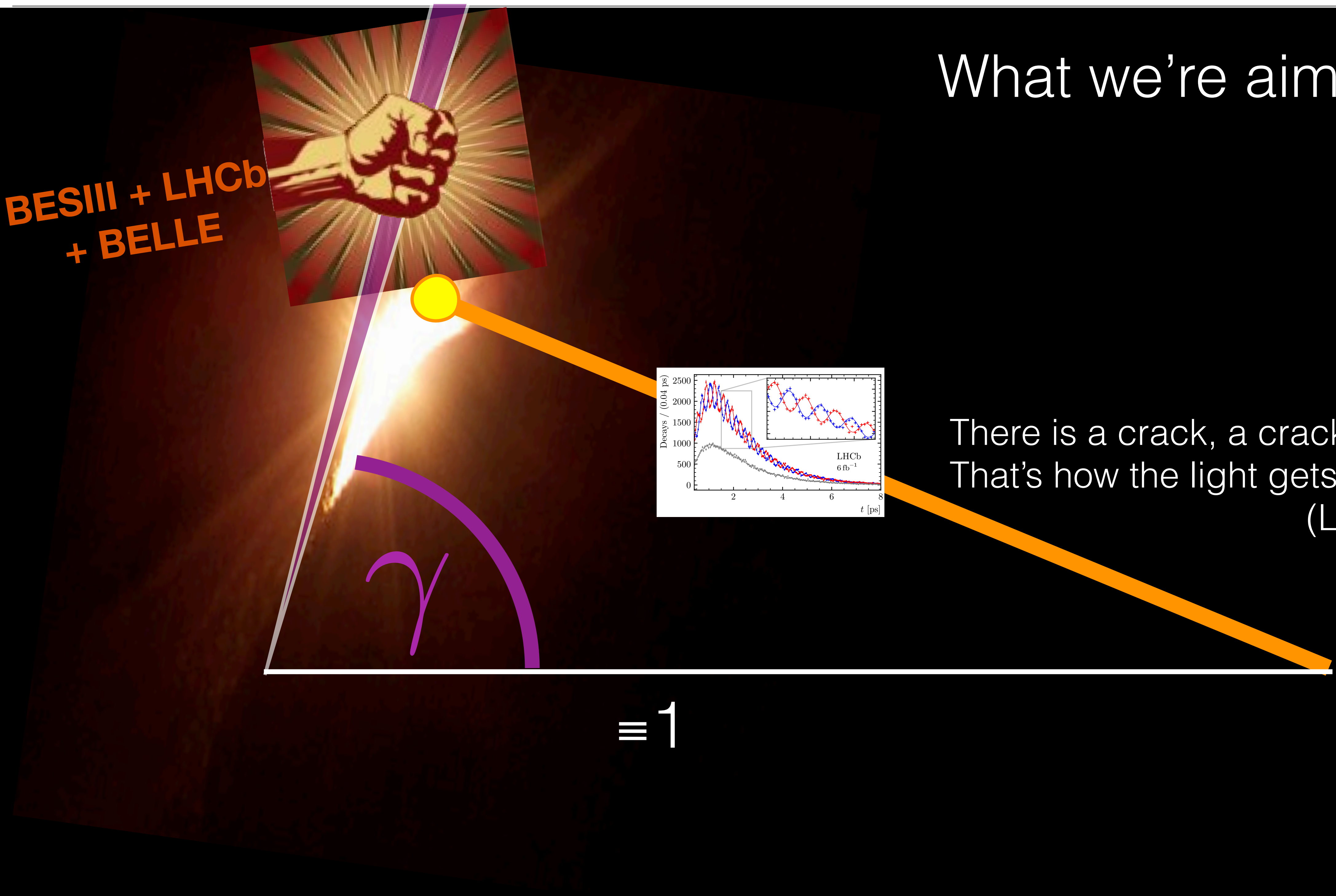
Unitarity triangle

Now



Shown are constraints on apex of triangle from various measurements ( ) and γ ( )

Unitarity triangle



What we're aiming for

There is a crack, a crack in everything
That's how the light gets in.
(Leonard Cohen)

Measurment of γ with LHCb & BES III data with model-independent binned method.

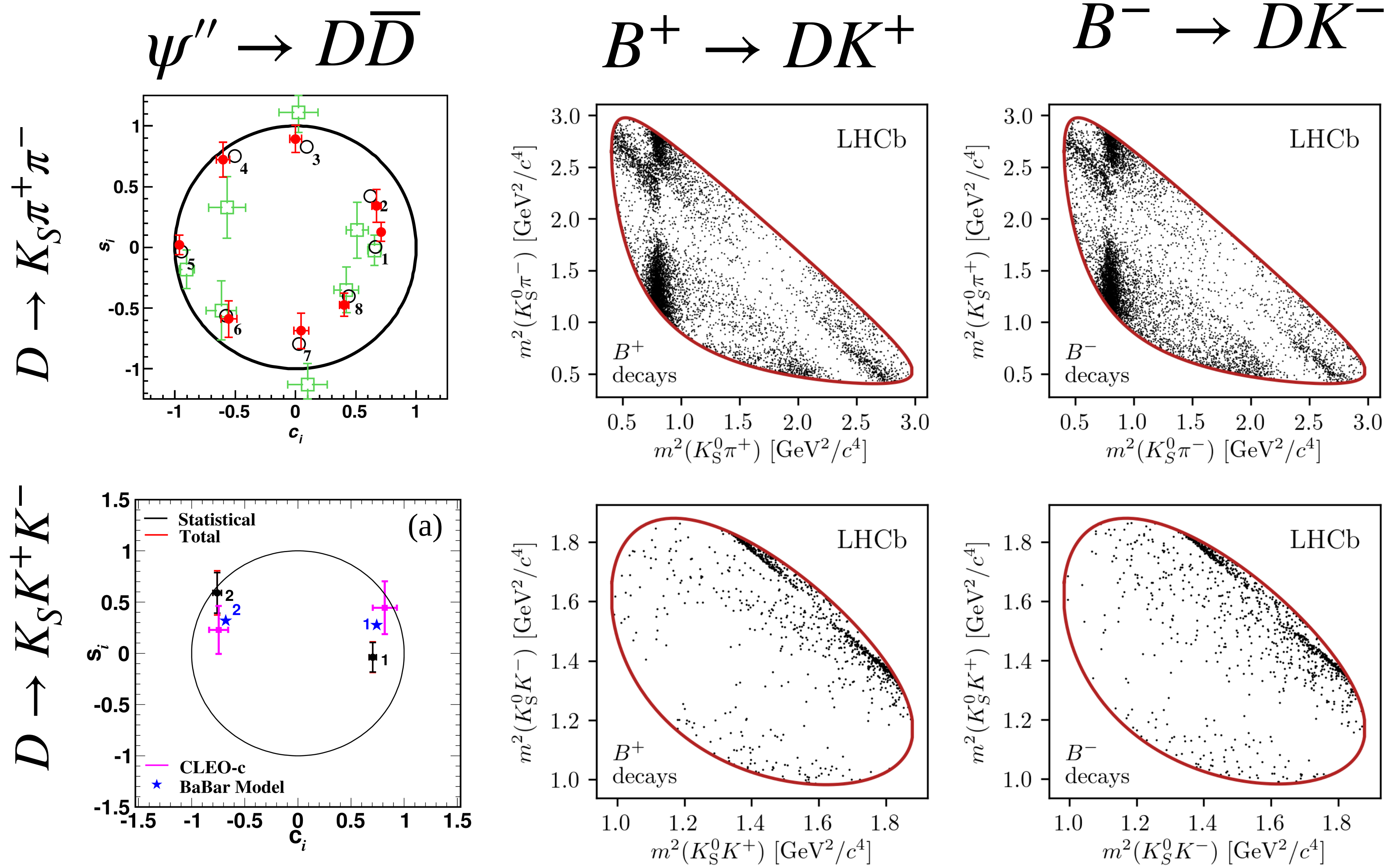
BESIII: PRL 124 (2020) 24, 241802

BESIII: PRD 102 (2020) 5, 052008

LHCb: JHEP 02 (2021) 169

$$\gamma = \left(68.7^{+5.2}_{-5.1}\right)^{\circ}$$

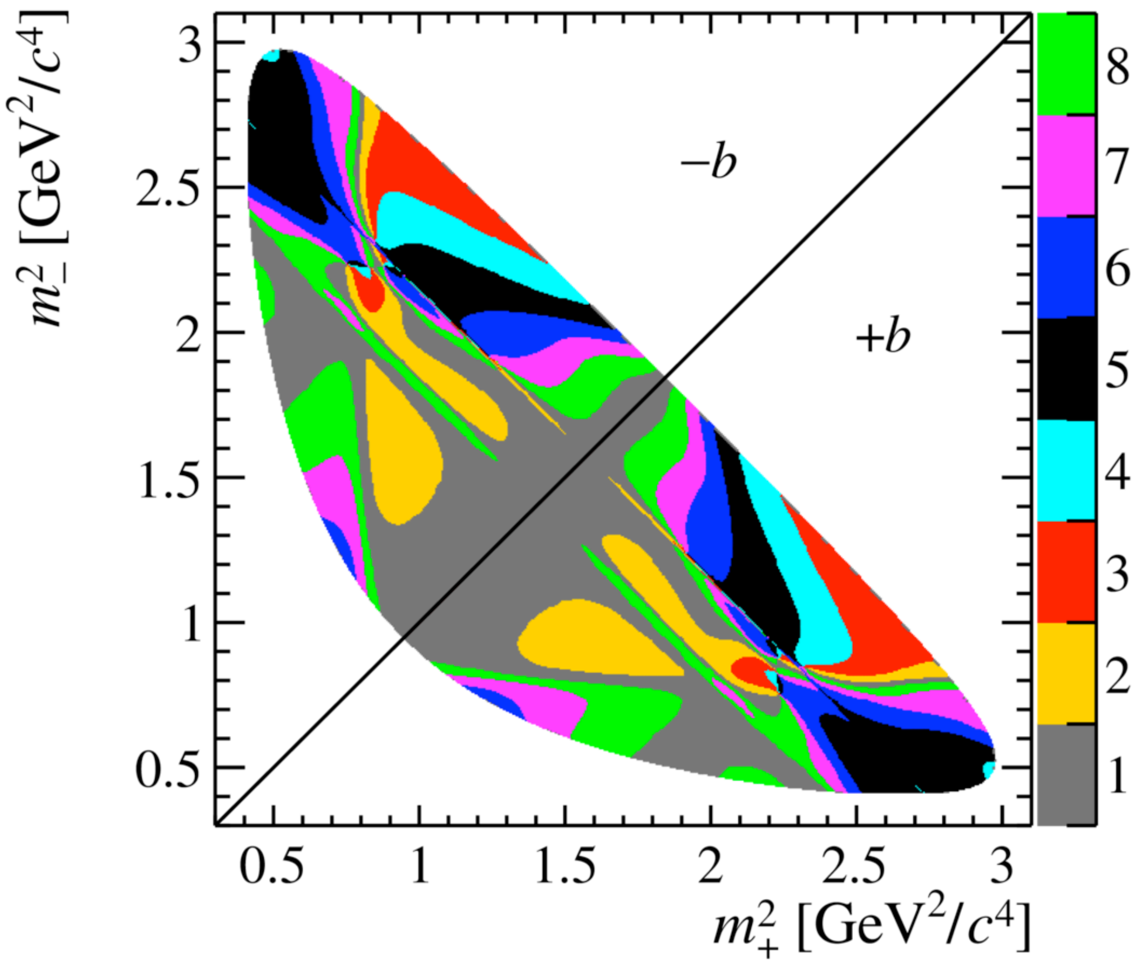
best individual measurement of γ



Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 **(GLW)** Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

Measurements of c_i, s_i at BES III

Model-informed, optimised binning

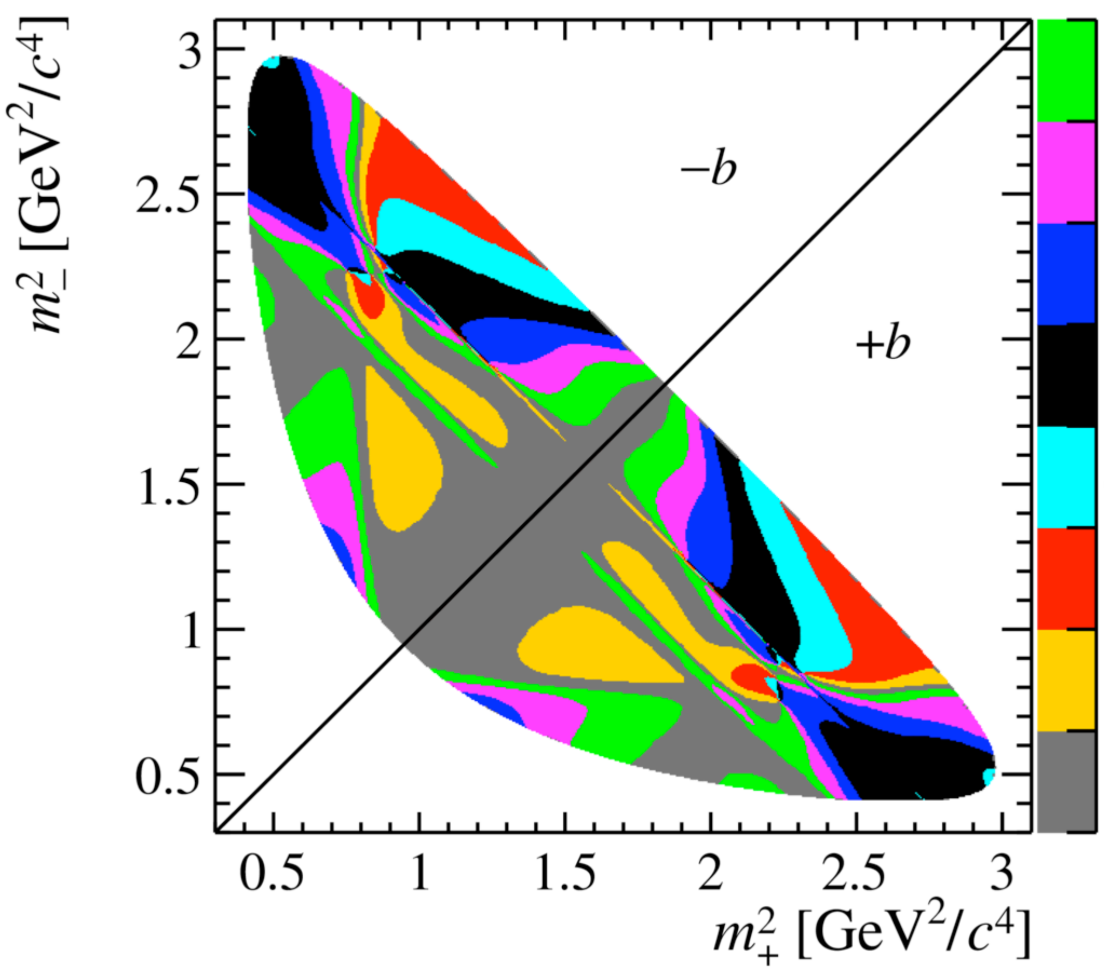


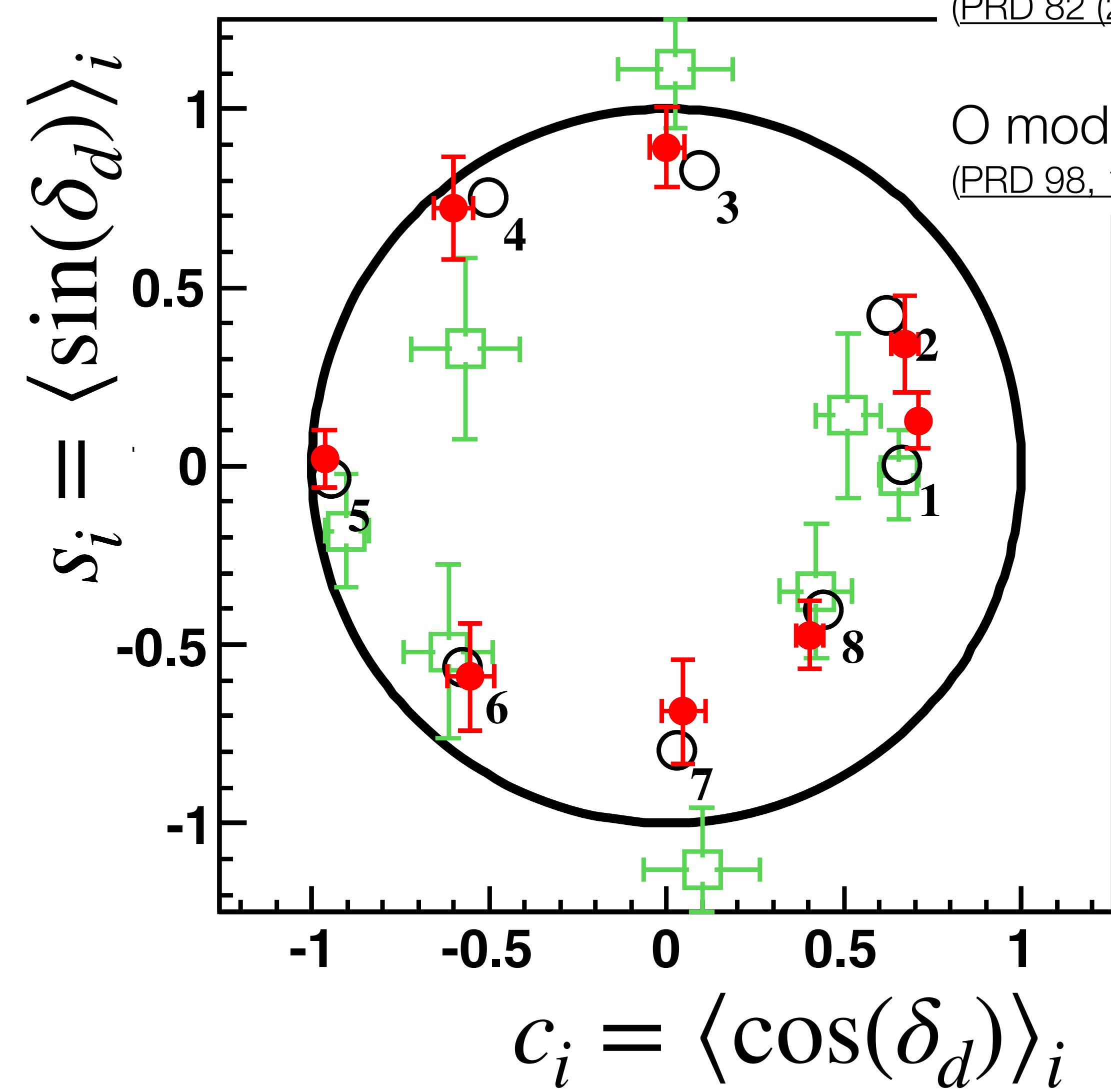
Measurements of c_i, s_i at BES III

Model-informed, optimised binning

BESIII: PRL 124 (2020) 24, 241802

in $D^0 \to K_S\pi^+\pi^-$





● BESIII
(PRL 124 (2020) 24, 241802)

□ CLEO-c
(PRD 82 (2010) 112006)

O model
(PRD 98, 112012)

Model independent, binned γ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binned decay rate:

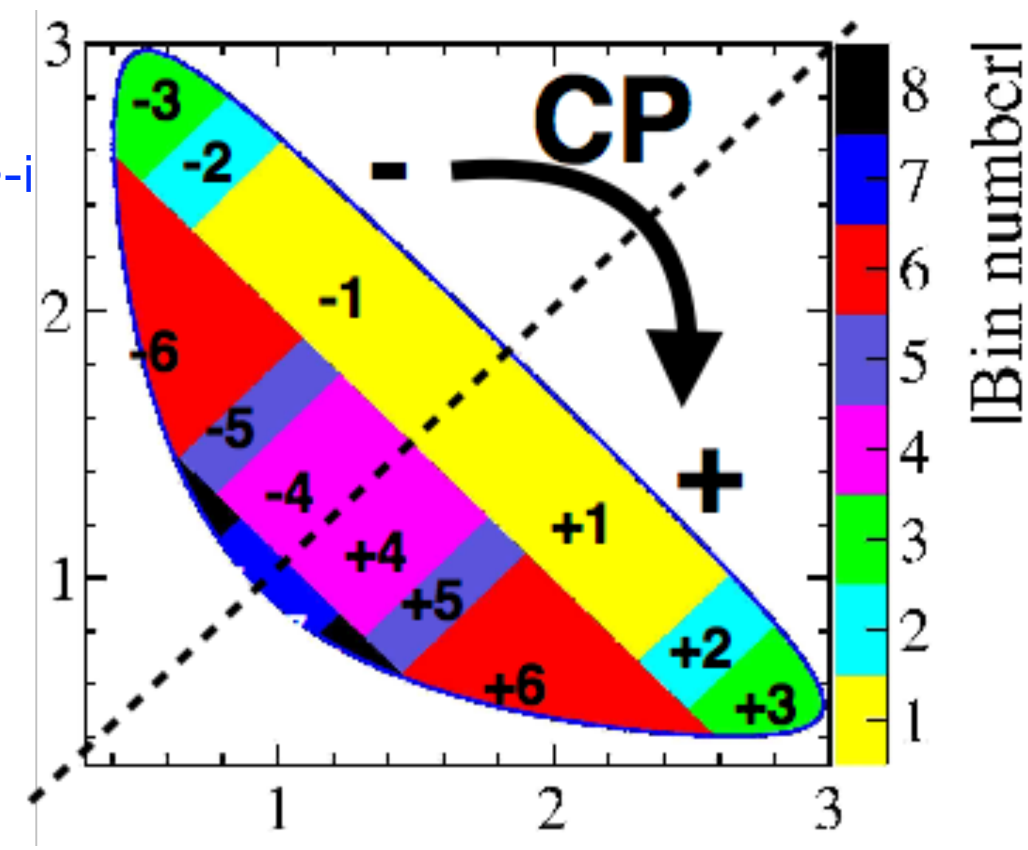
$$\Gamma\left(B^{\pm} \to D(K_s\pi^+\pi^-)K^{\pm}\right)_i =$$

$$\mathcal{T}_i + r_B^2\mathcal{T}_{-i} + 2r_B\sqrt{\mathcal{T}_i\mathcal{T}_{-i}}\left\{c_i\cos\left(\delta \pm \gamma\right) + s_i\sin\left(\delta \pm \gamma\right)\right\}$$

$\mathcal{T}_i$ known from flavour-specifc D decays

(weighted) average of $\cos(\delta_D)$ and $\sin(\delta_D)$ over bin i, where δ_D = phase difference between D→Ksππ and Dbar→Ksππ

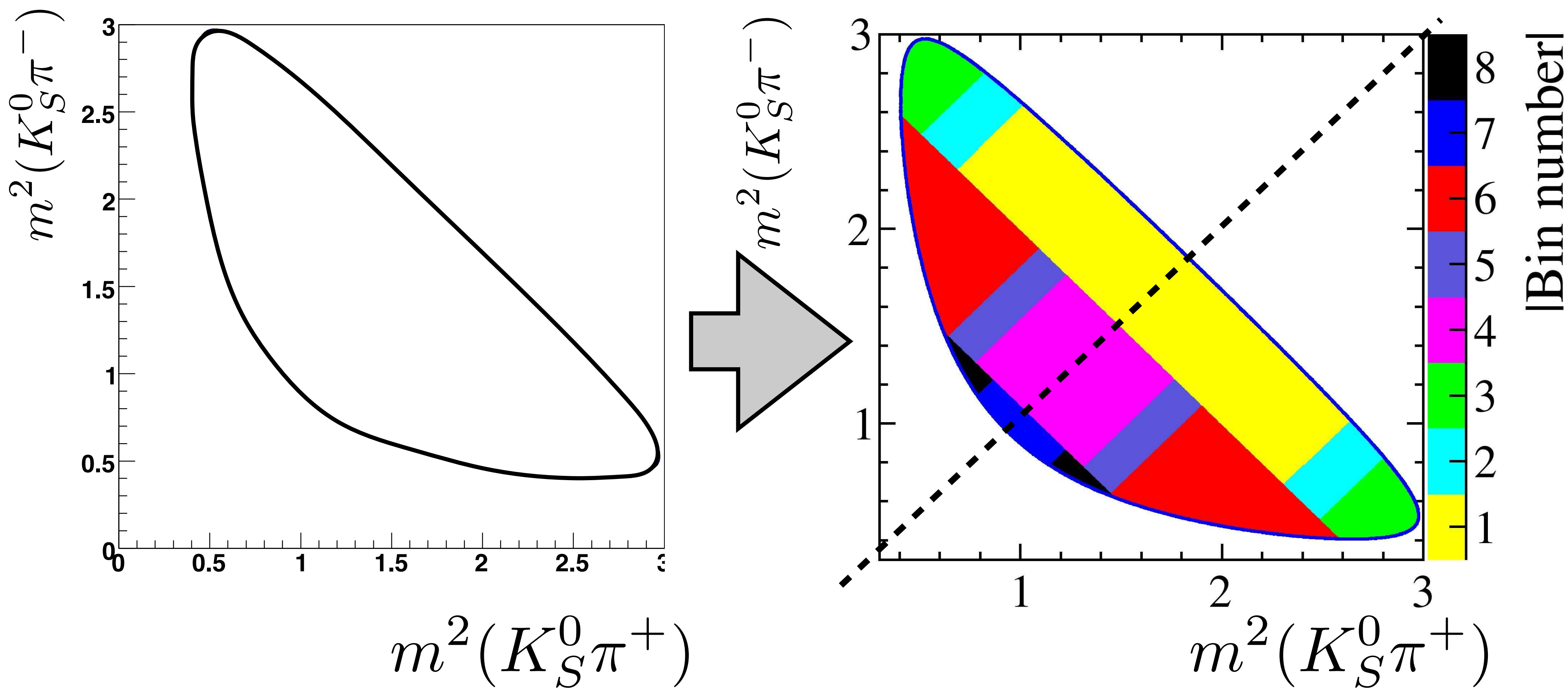
- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$



- Distribution sensitive to c_i, s_i, r_B, δ and γ.

- c_i, s_i, measured at charm threshold.

Model-independent, binned approach



One complex number per bin-pair, $c_i + \mathbf{i}s_i$, contains the key information uniquely accessible at CLEO-c/BES III, which is related to the phases the D^0 and $\overline{D}^0$ decay amplitudes.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).