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# Inclusive semileptonic $D$ decays

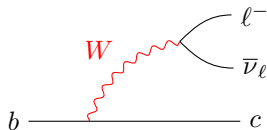
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K. Keri Vos

Maastricht University & Nikhef

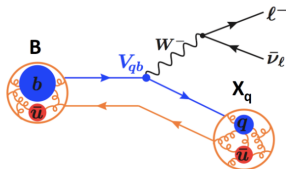
# Exclusive versus Inclusive Theory



Theory (Weak interaction): Transitions between **quarks/partons**

# Exclusive versus Inclusive Theory

Figure from Marzia Bordone



Theory (Weak interaction): Transitions between **quarks/partons**

Observation: Transitions between **hadrons**

## Challenge:

Dealing with QCD at large distances/small scales

Parametrize fundamental mismatch in non-perturbative objects

- Calculable: Lattice or Light-cone sumrules
- Measurable: from data

# Why inclusive decays?

Set up OPE and heavy quark expansion

Well established for  $B$  decays, precise framework

Extract important CKM parameters  $V_{cb}$  and  $V_{ub}$

Extract power corrections from data

Cross check of exclusive decays

# Inclusive $B \rightarrow X_c$ decays

# Inclusive Decays

## Inclusive $B \rightarrow X_c \ell \bar{\nu}$ : Heavy Quark Expansion (HQE)

$b$  quark mass is large compared to  $\Lambda_{\text{QCD}}$

Setting up the HQE: momentum of  $b$  quark:  $p_b = m_b v + k$ , expand in  $k/m_b$

Optical Theorem ! (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \dots \quad d\Gamma_i = \sum_k C_i^{(k)} \langle B | O_i^{(k)} | B \rangle$$

- $C_i^{(k)}$  perturbative Wilson coefficients
- $\langle B | O_i^{(k)} | B \rangle$  non-perturbative matrix elements ! string of  $1/m_b$
- operators contain chains of covariant derivatives

$$\langle B | O_i^{(n)} | B \rangle = \langle B | b \not{v} (i \not{D})^n | B \rangle$$

# Inclusive Decays

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- $\langle B | \mathcal{O}_i^{(k)} | B \rangle$  non-perturbative matrix elements ! string of  $iD$
- operators contain chains of covariant derivatives

$$\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | b \not{v} (iD) \dots (iD)_n b | B \rangle$$

HQE parameters extracted from **lepton energy**, **hadronic mass** and  $q^2$  moments

# Decay rate

$\Gamma_i$  are power series in  $O(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3$$

$\Gamma_0$ : decay of the free quark (partonic contributions),  $\Gamma_1 = 0$

$\Gamma_2$ :  $^2$  kinetic term and the  $^2_G$  chromomagnetic moment

$$2M_B \ ^2 = B j \bar{b}_v i D \ i D \ b_v j B$$

$$2M_B \ ^2_G = B j \bar{b}_v ( \ i \ ) i D \ i D \ b_v j B$$

$\Gamma_3$ :  $^3_D$  Darwin term and  $^3_{LS}$  spin-orbit term

$$2M_B \ ^3_D = \frac{1}{2} B j \bar{b}_v [ i D \ ; [ i v D ; i D \ ] ] b_v j B$$

$$2M_B \ ^3_{LS} = \frac{1}{2} B j \bar{b}_v \ i D \ ; \ i v D ; i D \ \ ( \ i \ ) b_v j B$$

$\Gamma_4$ : 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

$\Gamma_5$ : 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109



# Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

## Charged lepton energy

$$\langle E^n \rangle_{\text{cut}} = \frac{\int_{E > E_{\text{cut}}}^R dE \cdot E^n \frac{d\Gamma}{dE}}{\int_{E > E_{\text{cut}}}^R dE \cdot \frac{d\Gamma}{dE}}$$

## Hadronic invariant mass

$$\langle M_X^2 \rangle_{\text{cut}}^n = \frac{\int_{E > E_{\text{cut}}}^R dM_X^2 (M_X^2)^n \frac{d\Gamma}{dM_X^2}}{\int_{E > E_{\text{cut}}}^R dM_X^2 \frac{d\Gamma}{dM_X^2}}$$

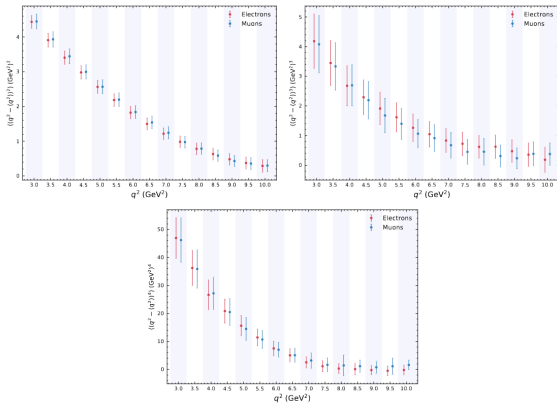
## Dilepton momentum

$$\langle q^2 \rangle_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2}^R dq^2 \frac{d\Gamma}{dq^2}}{\int_0^{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

Moments up to  $n = 3; 4$  and with several energy cuts available

Experimentally necessary to use some cut on the leptons

Belle Collaboration [2109.01685, 2105.08001]



Centralized moments as function of  $q_{\text{cut}}^2$

# Determining $V_{cb}$ and the HQE elements

$$hE_\ell^n i; h(M_X^2)^n i \quad h(q^2)^n i_{\text{cut}}$$

↓

$$m_b; m_c; \frac{2}{\pi}; \frac{2}{G}; \frac{3}{d}; r_E; r_G; S_E; S_B; S_{qB}; +$$

↓

$$\text{Br}(\bar{B} \rightarrow X_c \gamma) \sim \frac{jV_{cb}f^2}{B} \left[ \Gamma_{\mu_3} \frac{2}{3} + \Gamma_{\mu_G} \frac{2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\sim 3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{S_B} \frac{S_B^4}{m_b^4} + \Gamma_{S_E} \frac{S_E^4}{m_b^4} + \Gamma_{S_{qB}} \frac{S_{qB}^4}{m_b^4} \right] \#$$

↓  
 $V_{cb}$

# The advantage of $q^2$ moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

Standard **lepton energy** and **hadronic mass** moments are not RPI quantities

New  $q^2$  moments are RPI!

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## Reparametrization invariant quantities:

Setting up the HQE: momentum of  $b$  quark:  $p_b = m_b v + k$ , expand in  $k$   $iD$

Choice of  $v$  not unique: Reparametrization invariance (RPI)

$$v \neq v + v$$

$${}_{RP} v = v \quad \text{and} \quad {}_{RP} iD = m_b v$$

- links different orders in  $1/m_b$  ! reduction of parameters
- up to  $1/m_b^4$ : 8 parameters (previous 13)

# Heavy quark expansion for charm?

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# Why HQE for charm?

Expansion parameters  $s(m_c)$  and  $\alpha_{\text{QCD}}=m_c$  less than unity, but not so small  $\therefore$

Turn vice into virtue: more sensitive to higher  $1/m_Q$  corrections

Exploit the full physics potential of BES III, LHCb  $\therefore$

Constrain Weak Annihilation (WA) contributions

$$\int B_d \int s''$$

$$\int V_{ub}$$

[Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

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$$\frac{1}{\Gamma(B_d)} \frac{d\Gamma(B_d \rightarrow s \dots)}{ds}$$

$$\frac{1}{\Gamma(B_d)} \frac{d\Gamma(B_d \rightarrow V_{ub})}{ds}$$

[Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

Extraction of  $|V_{cs}|$  and  $|V_{cd}|$ ?



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## Challenges:

Valence and non-valence WA operators at higher orders

Scale for radiative corrections

Charm mass definition

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$$\begin{aligned} & \text{! } B_d \text{ ! } s \\ & \text{! } V_{ub} \end{aligned}$$

[Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

Extraction of  $|V_{cs}|$  and  $|V_{cd}|$ ?

## Challenges:

Valence and non-valence WA operators at higher orders

Scale for radiative corrections

Charm mass definition

In short: how to handle the charm?

# The HQE for charm

**I:**  $m_Q \sim m_q \gg \Lambda_{\text{QCD}}$  OPE for  $b \rightarrow c$

$q$  is treated as a heavy degree of freedom

two-quarks operators:  $Q_V(iD \not{D})Q_V$

IR sensitivity to mass  $m_q$

$$\Gamma_{1=m_Q^3} = \frac{34}{3} + 8 \log \frac{m_q}{m_Q} + \dots + \frac{3}{m_Q^3}; \quad \text{with } \Gamma = (m_q = m_Q)^2$$

**II:**  $m_Q \gg m_q \gg \Lambda_{\text{QCD}}$  start with  $q$  dynamical

four-quark operators  $(Q_V q)(q Q_V)$

! removed when matching onto two-quark operators

RGE running gives  $\log(m_q = m_Q)$

**III:**  $m_Q \gg m_q \sim \Lambda_{\text{QCD}}$  OPE for  $c \rightarrow s$

$q$  dynamical degree of freedom

four-quark operators remain in OPE

no explicit  $\log(m_q = m_Q)$ : hidden inside new non-perturbative HQE

parameters

**IV:**  $m_Q \gg \Lambda_{\text{QCD}} \gg m_q$  for  $b \rightarrow u$  and  $c \rightarrow d$  transitions

# HQE for Charm revisited

The general structure of the expansion for  $D \rightarrow X_s \ell$ :

$$d = d_0 + d_{(2;1)} \frac{\text{QCD}}{m_c} + d_{(2;2)} \frac{m_s}{m_c} + d_3 \frac{\text{QCD}}{m_c} + d_{(4;1)} \frac{\text{QCD}}{m_c} + d_{(4;2)} \frac{m_s}{m_c} + \dots$$

Expansion parameters:

- $1 = m_c$
- $s$
- $m_s = m_c$

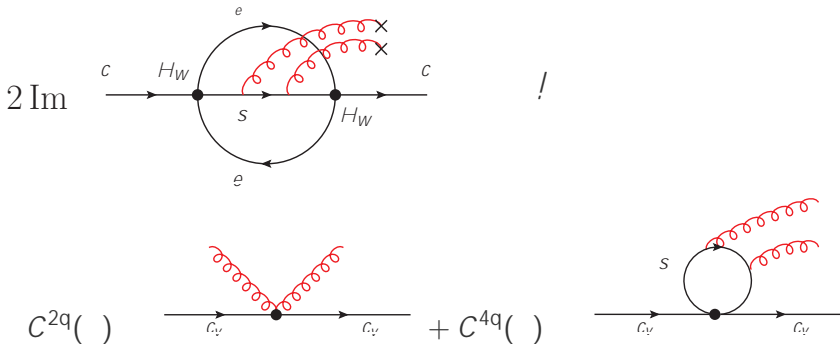
Fael, Mannel, KKV, hep-ph/1910.05234

# HQE for Charm revisited

Systematic treatment of four-quark operators order by order in  $1=m_Q$

Set up OPE directly for  $\text{tot}$  and  $\langle hM^{(n)} \rangle$

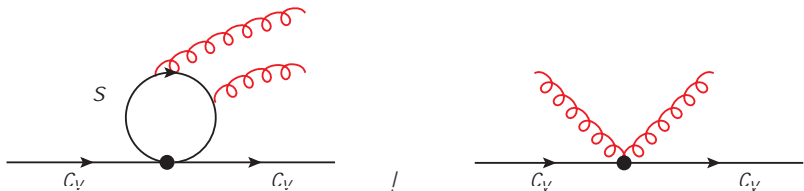
following the idea in [Bauer, Falk, Luke hep-ph/9604290](#)



# HQE for Charm revisited

$\log(m_c = m_b)$  in  $B \rightarrow X^0$  corresponds to  $\log(\mu = m_c)$  in  $D \rightarrow X^0$   
 caused by mixing of four-quark operators into two-quark operators:

$$C_i^{2q}(\mu) = C_i^{2q}(m_c) + \log \frac{\mu}{m_c} \sum_j \hat{\Lambda}_{ij}^T C_j^{4q}(m_c)$$



# HQE for Charm revisited

Fael, Mannel, KKV, hep-ph/1910.05234

Additional HQE parameters for  $c \rightarrow q$ :  $T_i = \frac{1}{2m_D} \langle h D j O_i^{4q} j D i \rangle$

Up to  $1/m_c^3$  only one extra HQE param:

$$\begin{aligned} \Gamma_0 = & 128 \pi^2 \left[ T_1(\mu) + T_2(\mu) + 2 \frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c} \right. \\ & \left. + \log \left( \frac{2}{m_c^2} \right) \left( 8 \tilde{r}_D^3 + \frac{1}{m_c} \left[ \frac{16}{3} r_G^4 + \frac{16}{3} r_E^4 + \frac{8}{3} S_E^4 + \frac{1}{3} S_{qB}^4 \right] - 12 m_s^4 \right) \right] \end{aligned}$$

Up to  $1/m_c^4$  only two extra HQE params:  $m$  and  $\epsilon$ .

# HQE for charm revisited

$$= m_s^2 = m_c^2$$

Fael, Mannel, KKV, hep-ph/1910.05234

$$\frac{(D \rightarrow X_s \gamma)}{0} = 1 - 8 \frac{10^{-2}}{3} + \left( 2 - 8 \right) \frac{2}{m_c^2} + 6 \frac{\tilde{D}}{m_c^3} \\ + \frac{16}{9} \frac{r_G^4}{m_c^4} + \frac{32}{9} \frac{r_E^4}{m_c^4} + \frac{34}{3} \frac{s_B^4}{m_c^4} + \frac{74}{9} \frac{s_E^4}{m_c^4} + \frac{47}{36} \frac{s_{qB}^4}{m_c^4} + \frac{0}{m_c^3}$$

RPI quantities ( $q^2$  moments) depend on reduced set

Data required to test description

Comparison of extracted HQE parameters with  $B$  decays



$$= m_s^2 = m_c^2$$

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$$\frac{(D \rightarrow X_s \gamma)}{0} = 1 - 8 \frac{10}{m_c^2} + 3 + \left( 2 - 8 \right) \frac{r_G^2}{m_c^2} + 6 \frac{r_D^3}{m_c^3} \\ + \frac{16}{9} \frac{r_G^4}{m_c^4} + \frac{32}{9} \frac{r_E^4}{m_c^4} - \frac{34}{3} \frac{s_B^4}{m_c^4} + \frac{74}{9} \frac{s_E^4}{m_c^4} + \frac{47}{36} \frac{s_{qB}^4}{m_c^4} + \frac{0}{m_c^3}$$

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Key question: HQE indeed applicable to inclusive charm decays?

$$= m_s^2 = m_c^2$$

Fael, Mannel, KKV, hep-ph/1910.05234

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Key question: How to handle the charm mass?

# How to handle the charm mass?

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# Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

Renormalon issues require short-distance mass

$\overline{MS}$  for scales above heavy quark mass

Kinetic mass: relating hadron versus quark mass

QCD corrections using hard cut o

$$m_Q(\mu)^{\text{kin}} = m_Q^{\text{Pole}} \left[ 1 + \frac{h_1}{m_Q} + \frac{h_2}{2m_Q^2} + \dots \right]_{\text{pert}}$$
$$[h_1]_{\text{pert}} = \frac{4}{3} C_F \frac{s(m_c)}{m_Q} \quad [h_2]_{\text{pert}} = C_F \frac{s(m_c)}{m_Q^2}$$

Higher-order terms in the HQE generate corrections  $(\frac{s}{m_Q})^n = m_Q^{-n}$ .

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Higher-order terms in the HQE generate corrections  $(s/m_Q)^n = m_Q^n$ .

$\Lambda_{\text{QCD}} < \mu < m_Q$ : expansion parameters  $s/m_Q$

- Well established for  $m_B$ :  $s/m_B \sim 0.2$
- Charm??

$$\mu = 1 \text{ GeV} \quad s/m_c \sim 1$$

$$\mu = 0.5 \text{ GeV} \quad s/m_c \sim 0.4$$

Putting all power corrections to zero!

$$m_c^{\text{kin}}(1 \text{ GeV}) = 1.16 \text{ GeV} \quad (m_s \rightarrow 0 \text{ limit})$$

$$(c \rightarrow s)_{\text{kin}} = 0 \quad 1 + 7.7 \frac{s(m_c)}{m_c} + 69 \frac{s(m_c)}{m_c^2} \#$$

$$m_c^{\text{kin}}(0.5 \text{ GeV}) = 1.4 \text{ GeV} \quad (m_s \rightarrow 0 \text{ limit})$$

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= 0.5 GeV touches upon the non-perturbative regime?

# Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

$m_c$  not observable ! no physical meaning

Extracted from data: moments of the spectral density in  $e^+e^- \rightarrow \text{hadrons}$

$$R(s) = \frac{(e^+e^- \rightarrow \text{hadrons})}{(e^+e^- \rightarrow \mu^+\mu^-)}$$



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Start from vacuum correlator

$$\int d^4x e^{iqx} \langle 0 | T [j(x) j(0)] | 0 \rangle = (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \chi(q^2)$$

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Expand around  $q^2 = 0$ : ( $C_n = C_n^{(0)} + \frac{s(\dots)}{2} C_n^{(1)} + \dots$ )

$$(q^2) = (0) + \frac{4}{9} \frac{3}{16} \sum_{n=1}^{\infty} C_n \frac{q^2}{4m_c^2}$$

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$C_n$  known up to  $\frac{2}{5}$  and related to moments

$$C_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \sum_{s=2}^{\infty} \frac{ds}{s^{n+1}} R(s) \quad (1)$$

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Replace  $m_c$ :

$$m_c = \frac{1}{2} \frac{C_n}{M_n} \quad 1=(2n)$$

Chetyrkin, Kuehn, Steinhauser [hep-ph/9705254], Penin, Pivovarov [hep-ph/9805344]

Boushmelev, Mannel, KKV [2301.05607]

$$\begin{aligned}
 \Gamma(c \rightarrow s \gamma) &= \frac{G_F^2 V_{cs}^2}{192 \cdot 3} \left( \frac{1}{2} \left( \frac{\bar{C}_n}{M_n} \right)^{1=2} \right)^5 \left( 1 + \frac{s(\cdot)}{a_1} a_1 + \left( \frac{s(\cdot)}{a_2} \right)^2 a_2 + \dots \right) \\
 &= \frac{G_F^2 V_{cs}^2}{6144 \cdot 3} \left( \frac{\bar{C}_n^{(0)}}{M_n} \right)^{5=2} \left( 1 + \frac{s(\cdot)}{a_1} \left[ a_1 + \frac{5}{2n} \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right] \right. \\
 &\quad \left. + \left( \frac{s(\cdot)}{a_2} \right)^2 \left[ a_2 + \frac{5}{2n} a_1 \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} + \frac{5}{2n} \frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}} + \frac{5}{4n} \left( \frac{5}{4n} - 1 \right) \left( \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right)^2 \right] + \dots \right)
 \end{aligned}$$

Conclusion for  $B$ : pert. series improves a bit

Scale at which  $\frac{2}{s}$  vanishes rather low:  $0.7 m_b$

In progress: Similar approach for the charm + power corrections

Extrapolate data to  $p_e \rightarrow 0$  and convert from lab frame to  $D$  meson rest frame  
Kinetic mass for charm at  $\sqrt{s} = 0.5$  GeV threshold, HQE parameters as input  
Obtain strong bounds on weak annihilation (WA) contribution  
Max 2% WA contribution to  $B \rightarrow X_u \gamma$

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Max 2% WA contribution to  $B \rightarrow X_u \gamma$   
**My wish:** Extract HQE and WA directly from  $q^2$  moments at BESIII



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Close collaboration between theory and experiment necessary!

# Backup

Avoid background subtraction by calculating the full inclusive width:

$$\Gamma(B \rightarrow X) = \Gamma(B \rightarrow X_c) + \Gamma(B \rightarrow X_u) + \Gamma(B \rightarrow X_c(\text{hadrons}))$$

$b \rightarrow u$  contribution: suppressed by  $V_{ub} = V_{cb}$

$b \rightarrow c(\text{hadrons})$  contribution: phase space suppressed

QED effects

Quark-hadron duality violation?

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:

estimate how much this description would improve  $V_{cb}$  determination

Can be analyzed in local OPE  $\mathcal{B} \rightarrow X_c \ell$  by taking  $m_c \rightarrow 0$  limit

For  $V_{ub}$  determination

- large charm background requires experimental cuts
- reduces the inclusivity and local OPE no longer converges
- spectrum described by non-local OPE
- convolution of pert. coefficients with shape function

### Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

$$\text{NLO} + 1 = m_b^2 + 1 = m_b^3$$

In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)

First study: no  $\mathcal{S}$  for  $1 = m_b^2$ , no additional uncert. due to missing higher orders

Inputs HQE parameters from  $\mathcal{B} \rightarrow X_c \ell$  study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]



Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

### Monte Carlo:

BLNP: specific shape function input parameters shape function parameters  $s_b = 3 : 95$  and  $s_c = 0 : 72$

DFN:  $s_c$  corrections convoluted with the exponential shape function model

- Inputs from  $B \rightarrow X_c$  and  $B \rightarrow X_s$  data using KN-scheme Kagan, Neubert 1998
- $(s_1^+; s_2^+; s_1^-; s_2^-)$  are obtained by varying and  $s_c^2$  within 1 Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collaboration [arXiv:2102.00020.]

convolution with hadronization simulation based on Pythia

plus explicit resonances  $B \rightarrow \rho$  and  $B \rightarrow \omega$

MC-results are in good agreement with the HQE results

Wide spread between MC for higher moments

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner

## Remarks:

DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all

BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction

- should include higher moments of the shape-function model?
- include subleading shape functions?

our HQE: interesting to include  $s$  to HQE parameters,  $s^2$ ?

Contribution from ve-body charm decay  $b \rightarrow c \ell \bar{\nu}$  via

$$B(p_B) \times X_c(p_{X_c}) \times (q_{\ell}) \times (q_{\bar{\nu}}) \times (q_{\ell}) \times (q_{\bar{\nu}}) \times (q_{\ell}) \times (q_{\bar{\nu}})$$

:  
Phase space suppressed:

$$\frac{\text{tot}(b \rightarrow c \ell \bar{\nu})}{\text{tot}(b \rightarrow c \ell \bar{\nu})} = 4.0\%$$

Experimentally effects diminished by cutting on the invariant mass of the  $B$

Can be calculated exactly in the HQE

$$\frac{d^8}{dq^2 dq^2 dp_{X_c}^2 d^2 d d^2} = \frac{3G_F^2 V_{cb}^2 2^D (q^2 - m^2)(m^2 - q^2) B(\dots)}{2^{17} 5 m^8 m_b^3 q^2} W L$$

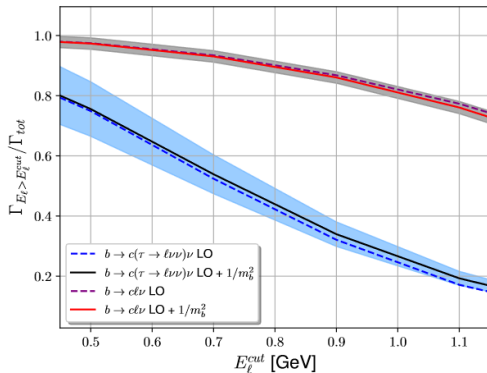
- L ve-body leptonic tensor (narrow-width limit for  $\ell$ )
- W standard hadronic tensor including HQE parameters

Interesting to search for new physics!

Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

# Five-body contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];



No MC data available to test with

## Lowest State Saturation Approximation (LSSA)

$$hBjO_1 O_2 jBi = \sum_n hBjO_1 jn i hnjO_2 jBi$$

$$\frac{3}{D} = \frac{2}{G}; \quad \frac{3}{LS} = \frac{2}{G}; \quad \frac{1}{G} = 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

LSSA estimated as priors (60% gaussian uncertainty)

$O(1=m_b^4; 1=m_b^5)$  can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$jV_{cb}j_{incl} = (42.00 \pm 0.64) \cdot 10^{-3}$$

0.25% shift due to power corrections



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Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

LSSA estimated as priors (60% gaussian uncertainty)

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$$jV_{cbj}^{\text{incl}} = (42.00 \pm 0.64) \cdot 10^{-3}$$

## Towards the Ultimate Precision in $jV_{cbj}$

Include  $\mathcal{O}(1/m_b^5)$  corrections to for  $\frac{3}{D}$  Mannel, Pivovarov [in progress]; Gambino [in progress]

Full determination up to  $1=m_b^4$  from data possible?

## Lowest State Saturation Approximation (LSSA)

$$hBjO_1O_2jBi = \sum_n hBjO_1jnihnjO_2jBi$$

$$\frac{3}{D} = \frac{2}{G}; \quad \frac{3}{LS} = \frac{2}{G}; \quad \text{'' } 0.4 \text{ GeV}$$

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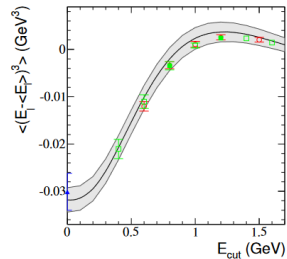
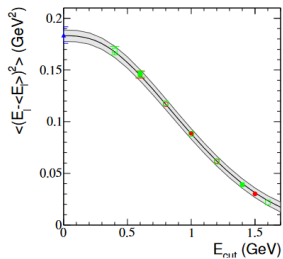
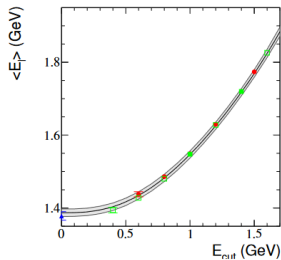
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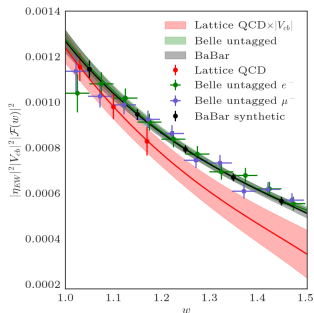
$$jV_{cb}j_{incl} = (42.00 \quad 0.64) \quad 10^{-3}$$

0.25% shift due to power corrections

# Moments of the spectrum

Gambino, Schwanda Phys. Rev. D 89, 014022 (2014)





Tension between the slope of the lattice and experimental data

Same form factors determine SM predictions for  $R_{D(\ast)}$

New experimental and lattice data needed!

# The $V_{cb}$ puzzle: Inclusive versus Exclusive decays

## Exclusive $B \rightarrow D^{(*)}$

Form factor required (only for  $B \rightarrow D$  available at different kinematic points)

Different parametrizations for form factors: CLN [Caprini, Lellouch, Neubert [1997]] and BGL

Boyd, Grinstein, Lebed [1995]

- BGL: model independent based on unitarity and analyticity
- CLN: Simple parametrization using HQE relations

Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone,

Gubernari [2104.02094]

## Inclusive $B \rightarrow X_c$

Determined fully data driven including  $1/m_b$  power corrections

Recently a lot of attention for the  $V_{cb}$  puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner,

Bordone, van Dyk, Gubernari

Stay tuned!

## NP in the $b \rightarrow c$ sector

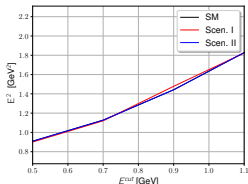
Affects also inclusive  $B \rightarrow X_c$  Rusov, Mannel, Shahriaran [2017]

Lepton and hadronic moments challenging to measure

Recently moments of the  $b \rightarrow c$  decay  $B \rightarrow X_c (\ell^+ \ell^-)$  investigated Mannel, Rahimi, KKV [2105.02163]

Would also be influenced by NP [in progress]

Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]



Preliminary!

Contribution from five-body charm decay to  $b \rightarrow c \ell \ell'$  via

$$B(p_B) \rightarrow X_c(p_{X_c})(q_{\ell'} \rightarrow (q_{\ell} \rightarrow (q_{\ell} \rightarrow (q_{\ell} \rightarrow (q_{\ell}))) \rightarrow (q_{\ell} \rightarrow (q_{\ell})))$$

:  
Phase space suppressed:

$$\frac{\Gamma_{\text{tot}}(b \rightarrow c (\ell \ell') \rightarrow (q_{\ell} \rightarrow (q_{\ell} \rightarrow (q_{\ell} \rightarrow (q_{\ell}))) \rightarrow (q_{\ell} \rightarrow (q_{\ell})))}{\Gamma_{\text{tot}}(b \rightarrow c \ell \ell')} \approx 4.0\%$$

Experimentally effects diminished by cutting on the invariant mass of the  $B$

Can be calculated exactly in the HQE

$$\frac{d^8 \Gamma}{dq^2 d^2 q^2 - dp_{X_c}^2 d^2 \Omega d\Omega d^2 \Omega} = \frac{3G_F^2 V_{cb}^2 j^2 P^-(q^2 - m^2)(m^2 - q^2) B(\ell \ell')}{2^{17} 5 m^8 m_b^3 q^2} W L$$

- $L$  five-body leptonic tensor (narrow-width limit for  $\ell \ell'$ )
- $W$  standard hadronic tensor including HQE parameters

Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Leading order shape functions

$$2m_B f(y) = \int_0^1 dx \, \langle \bar{\psi} \psi \rangle (1-x) x^2 B(x, y) + \dots$$

Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} = \int_0^1 dx \, (m_b(1-x))^2 f(x, y)$$

Moments of the shapefunction are related to HQE ( $b \rightarrow c$ ) parameters:

$$f(y) = \langle 1 \rangle + \frac{1}{6m_b^2} \langle D^2 \rangle + \frac{1}{m_b^3} \langle D^3 \rangle + \dots$$

Shape function is non-perturbative and cannot be computed



Systematic framework: Soft Collinear Effective Theory (SCET)

Separates the different scales in the problem

$$d = H \cdot J \cdot S$$

- ! H: Hard scattering kernel at  $O(m_b)$
- ! J: universal Jet function at  $O(\sqrt{m_b \Lambda_{\text{QCD}}})$
- ! S: Shape function at  $O(\Lambda_{\text{QCD}})$

Framework to include radiative corrections (+ NNLL resummation)

Introduces 3 subleading shape functions

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Other approach: OPE with hard-cuto Gambino, Giordano, Ossola, Uraltsev

- Use pert. theory above cuto and parametrize the infrared
- Different definition of the shape functions

Shape functions have to be parametrized and obtained from data

# New Physics explanation?

Too many to count: exclusive  $B \rightarrow D^{(*)}$  in combination with

$$R_{D^{(*)}} = \frac{B \rightarrow D^{(*)}}{B \rightarrow D^{(*)}}$$

For inclusive  $b \rightarrow c$  less analyses

- RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
- RH-current to moments Feger, Mannel, et. al. [2010]
- NP for moments KKV, Fael, Rahimi [in progress]

