Inclusive semileptonic D decays

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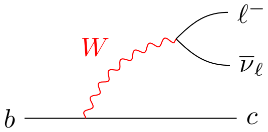
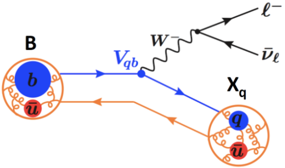
Exclusive versus Inclusive Theory



- Theory (Weak interaction): Transitions between quarks/partons

Exclusive versus Inclusive Theory

Figure from Marzia Bordone



- Theory (Weak interaction): Transitions between quarks/partons
- Observation: Transitions between hadrons

Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules
  - Measurable: from data

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established for B decays, precise framework
- Extract important CKM parameters V_{cb} and V_{ub}
- Extract power corrections from data
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B | O_i^{(k)} | B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \ldots | B \rangle$ non-perturbative matrix elements $\to$ string of iD
- operators contain chains of covariant derivatives

$$\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | \bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v | B \rangle$$

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- HQE parameters extracted from lepton energy, hadronic mass and q^2 moments

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\langle (M_X^2)^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

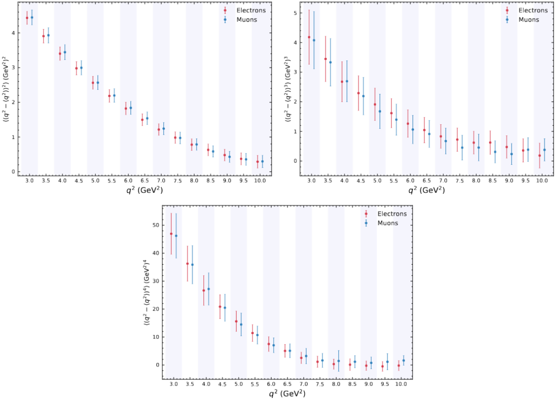
Dilepton momentum

$$\langle (q^2) \rangle_{\rm cut} = \frac{\int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use some cut on the leptons

q^2 moments

Belle Collaboration [2109.01685, 2105.08001]



Centralized moments as function of $q^2_{\rm cut}$

Determining V_{cb} and the HQE elements

$$\langle E_\ell^n \rangle, \langle (M_X^2)^n \rangle \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_d^3, r_E, r_G, s_E, s_B, s_{qB}, + \cdots$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} \right. \\ \left. + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb}$$

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

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Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\ v_\mu = \delta v_\mu \quad \text{and} \quad \delta_{RP}\ iD_\mu = -m_b \delta v_\mu$$

- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)

Heavy quark expansion for charm?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\mathrm{QCD}}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \to s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

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Challenges:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

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In short: how to handle the charm?

The HQE for charm

I: $m_Q \sim m_q \gg \Lambda_{\rm QCD}$ OPE for $b \to c\ell\bar{\nu}$

- q is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v(iD^\alpha \cdots iD^\beta)Q_v$
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \dots\right]\frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

II: $m_Q \gg m_q \gg \Lambda_{\rm QCD}$ start with q dynamical

- four-quark operators $(\bar{Q}_v\Gamma q)(q\bar{\Gamma}Q_v)$

$\rightarrow$ removed when matching onto two-quark operators

- RGE running gives $\log(m_q/m_Q)$

III: $m_Q \gg m_q \sim \Lambda_{\rm QCD}$ OPE for $c \to s\ell\bar{\nu}$

- q dynamical degree of freedom
- four-quark operators remain in OPE
- no explicit $\log(m_q/m_Q)$: hidden inside new non-perturbative HQE parameters

IV: $m_Q \gg \Lambda_{\rm QCD} \gg m_q$ for $b \to u$ and $c \to d$ transitions

HQE for Charm revisited

- The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

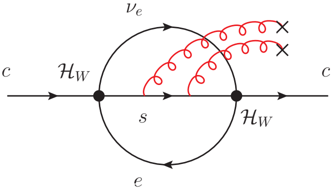
- Expansion parameters:
  - $1/m_c$
  - α_s
  - m_s/m_c

Fael, Mannel, KKV, hep-ph/1910.05234

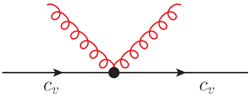
HQE for Charm revisited

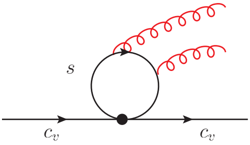
- Systematic treatment of four-quark operators order by order in $1/m_Q$
- Set up OPE directly for $\Gamma_{\rm tot}$ and $\langle M^{(n)} \rangle$

following the idea in Bauer, Falk, Luke hep-ph/9604290

$2\,\mathrm{Im}$ 

$\longrightarrow$

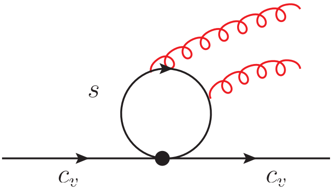
$C^{2\mathrm{q}}(\mu) \times$ 

$+\, C^{4\mathrm{q}}(\mu) \times$ 

HQE for Charm revisited

- $\log(m_c/m_b)$ in $B \to X\ell\nu$ corresponds to $\log(\mu/m_c)$ in $D \to X\ell\nu$
- caused by mixing of four-quark operators into two-quark operators:

$$C_i^{2q}(\mu) = C_i^{2q}(m_c) + \log\left(\frac{\mu}{m_c}\right) \sum_j \hat{\gamma}_{ij}^T C_j^{4q}(m_c)$$



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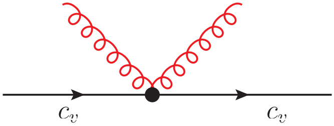

HQE for Charm revisited

Fael, Mannel, KKV, hep-ph/1910.05234

- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D} \langle D | O_i^{4\mathrm{q}} | D \rangle$
- Up to $1/m_c^3$ only <u>one</u> extra HQE param:

$$\tau_0 = 128\pi^2 \left(T_1(\mu) - T_2(\mu) - 2\frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c} \right) + \log\left(\frac{\mu^2}{m_c^2}\right) \left[8\tilde{\rho}_D^3 + \frac{1}{m_c} \left(\frac{16}{3} r_G^4 - \frac{16}{3} r_E^4 + \frac{8}{3} s_E^4 - \frac{1}{3} s_{qB}^4 - 12 m_s^4 \right) \right]$$

- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.

HQE for charm revisited

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV, hep-ph/1910.05234

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

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$$+ \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

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Key question: HQE indeed applicable to inclusive charm decays?

HQE for charm revisited

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Fael, Mannel, KKV, hep-ph/1910.05234

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Key question: How to handle the charm mass?

How to handle the charm mass?

Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- $\overline{\mathrm{MS}}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\mathrm{kin}} = m_Q^{\mathrm{Pole}} - \left[\overline{\Lambda}\right]_{\mathrm{pert}} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\mathrm{pert}} + \ldots$$

$$[\overline{\Lambda}]_{\mathrm{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\mathrm{pert}} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\mathbf{\Lambda_{QCD} < \mu < m_Q}$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??

    $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$

    $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Kinetic Mass

Putting all power corrections to zero!

- $m_c^{\mathrm{kin}}(1\text{ GeV}) = 1.16\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\mathrm{kin}} = \Gamma_0 \left[1 + 7.7\frac{\alpha_s(m_c)}{\pi} + 69\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

- $m_c^{\mathrm{kin}}(0.5\text{ GeV}) = 1.4\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\mathrm{kin}} = \Gamma_0 \left[1 + 1.2\frac{\alpha_s(m_c)}{\pi} + 17\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

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$\mu = 0.5$ GeV touches upon the non-perturbative regime?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

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- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0| T[j_\mu(x) j_\nu(0)] |0\rangle = (g_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

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- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right)$$

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$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right) = \Pi(0) + \frac{q^2}{12\pi^2}\int\frac{ds}{s}\frac{R(s)}{s-q^2}$$

- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s) \tag{1}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

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- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser [hep-ph/9705254], Penin, Pivovarov [hep-pp/9805344]

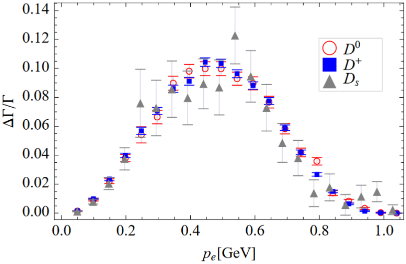
Boushmelev, Mannel, KKV [2301.05607]

$$\begin{aligned}\Gamma(c \to s\ell\nu) &= \frac{G_F^2 |V_{cs}|^2}{192\pi^3} \left(\frac{1}{2} \left(\frac{\bar{C}_n}{M_n} \right)^{1/2} \right)^5 \left(1 + \frac{\alpha_s(\mu)}{\pi} a_1 + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 a_2 + \cdots \right) \\ &= \frac{G_F^2 |V_{cs}|^2}{6144\pi^3} \left(\frac{\bar{C}_n^{(0)}}{M_n} \right)^{5/2} \left(1 + \frac{\alpha_s(\mu)}{\pi} \left[a_1 + \frac{5}{2n} \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right] \right. \\ &\quad \left. + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[a_2 + \frac{5}{2n} a_1 \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} + \frac{5}{2n} \frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}} + \frac{5}{4n} \left(\frac{5}{4n} - 1 \right) \left(\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right)^2 \right] + \cdots \right)\end{aligned}$$

- Conclusion for B: pert. series improves a bit
- Scale at which α_s^2 vanishes rather low: $0.7\ m_b$
- In progress: Similar approach for the charm + power corrections

Extracting weak annihilation from data

Gambino, Kamenik [1004.0114]



- Extrapolate data to $p_e \to 0$ and convert from lab frame to D meson rest frame
- Kinetic mass for charm at $\mu = 0.5$ GeV threshold, HQE parameters as input
- Obtain strong bounds on weak annihilation (WA) contribution
- Max 2% WA contribution to $B \to X_u \ell \nu$

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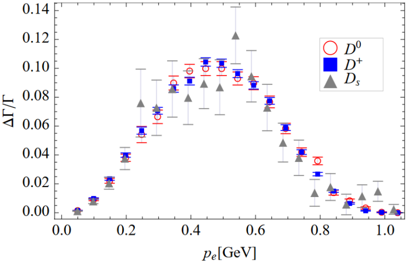

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- Max 2% WA contribution to $B \to X_u \ell \nu$
- My wish: Extract HQE and WA directly from q^2 moments at BESIII

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

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- After inclusive B decays now its time to tackle the charm!

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Close collaboration between theory and experiment necessary!

Backup

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV [arXiv: 2105.02163]

Avoid background subtraction by calculating the full inclusive width:

$$d\Gamma(B \to X\ell) = d\Gamma(B \to X_c \ell \bar{\nu}) + d\Gamma(B \to X_u \ell \bar{\nu}) + d\Gamma(B \to X_c(\tau \to \ell \bar{\nu} \nu)\bar{\nu})$$

- $b \to u\ell\nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \to c(\tau \to \mu\nu\bar{\nu})\bar{\nu}$ contribution: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \to u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \to X_c\ell\nu$ by taking $m_c \to 0$ limit
- For V_{ub} determination
  - large charm background requires experimental cuts
  - reduces the inclusivity and local OPE no longer converges
  - spectrum described by non-local OPE
  - convolution of pert. coefficients with shape function

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2 + 1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \to X_c\ell\nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

Monte Carlo:

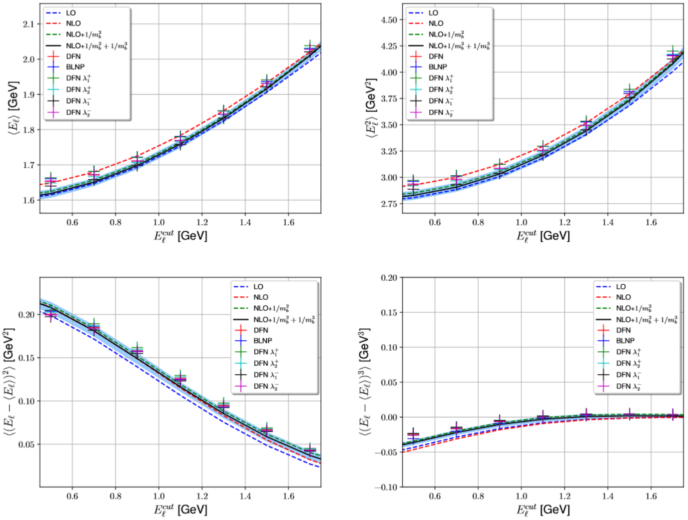
- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
  - Inputs from $B \to X_c \ell\nu$ and $B \to X_s\gamma$ data using KN-scheme Kagan, Neubert 1998
  - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi\ell\bar{\nu}$ and $\bar{B} \to \rho\ell\bar{\nu}$

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

Monte Carlo versus HQE

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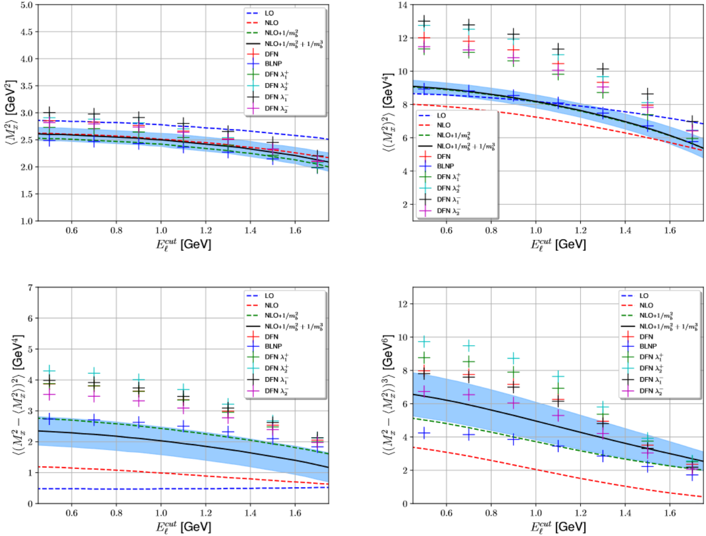
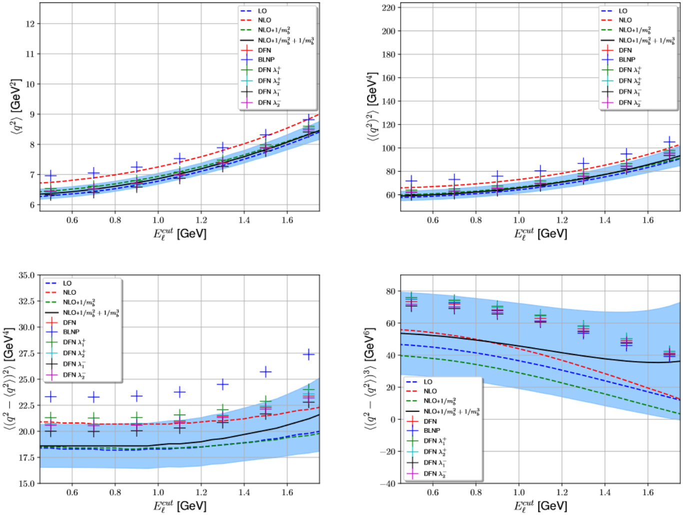


Wide spread between MC for higher moments

Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Monte Carlo versus local OPE: remarks

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
  - should include higher moments of the shape-function model?
  - include subleading shape functions?
- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Five-body τ contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\rm tot}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\rm tot}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

$$\frac{\mathrm{d}^8\Gamma}{\mathrm{d}q^2\,\mathrm{d}q^2_{\nu\bar{\nu}}\,\mathrm{d}p^2_{X_c}\,\mathrm{d}^2\Omega\,\mathrm{d}\Omega^*\,\mathrm{d}^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2-m_\tau^2)(m_\tau^2-q^2_{\nu\bar{\nu}})\mathcal{B}(\tau\to\mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu}L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Five-body τ contribution

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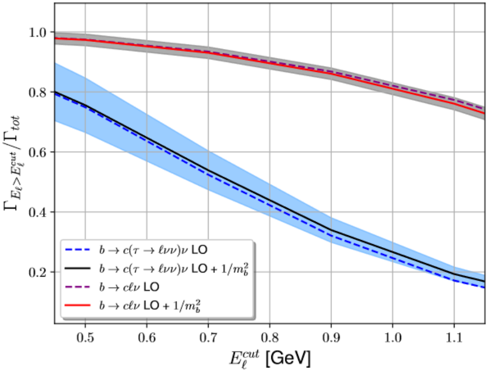

No MC data available to test with

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

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Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]; Gambino [in progress]
- Full determination up to $1/m_b^4$ from data possible?

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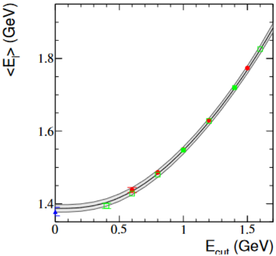
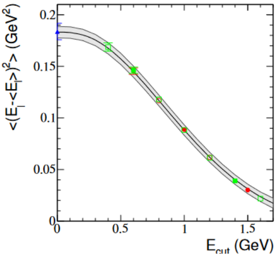
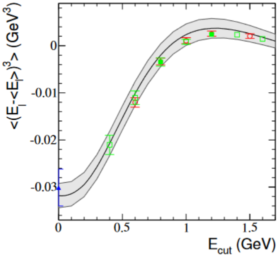
Gambino, Schwanda Phys. Rev. D 89, 014022 (2014)

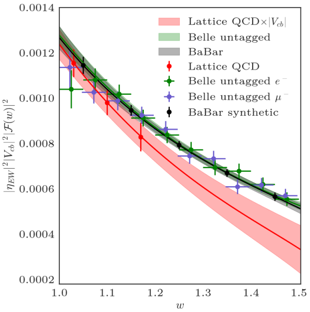






$B \to D^*$ form factors

Fermilab-MILC [2105.14019]



- Tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
- New experimental and lattice data needed!

The V_{cb} puzzle: Inclusive versus Exclusive decays

Exclusive $B \to D^{(*)}\ell\bar{\nu}$

- Form factor required (only for $B \to D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
  - BGL: model independent based on unitarity and analyticity
  - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

Inclusive $B \to X_c\ell\nu$

- Determined fully data driven including $1/m_b$ power corrections

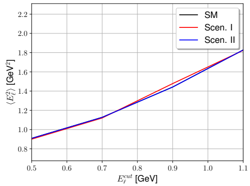
Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Stay tuned!

Mannel, Rahimi, KKV [in progress]

NP in the τ sector

- Affects also inclusive $B \to X_c \tau \nu$ Rusov, Mannel, Shahriaran [2017]
- Lepton and hadronic moments challenging to measure
- Recently moments of the five-body decay $B \to X_c \tau (\to \mu \nu \nu) \nu$ investigated Mannel, Rahimi, KKV [2105.02163]
- Would also be influenced by NP [in progress]
- Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]



Preliminary!

Five-body τ contribution

Rahimi, Mannel, KKV JHEP 09 (2021) 051 [arXiv: 2105.02163];

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- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y) - \omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
$\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

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- Other approach: OPE with hard-cutoff μ Gambino, Giordano, Ossola, Uraltsev
  - Use pert. theory above cutoff and parametrize the infrared
  - Different definition of the shape functions
- Shape functions have to be parametrized and obtained from data

New Physics explanation?

- Too many to count: exclusive $B \to D^{(*)}$ in combination with

$$R_{D^{(*)}} = \frac{B \to D^{(*)}\tau\nu}{B \to D^{(*)}\mu\nu}$$

- For inclusive $b \to c$ less analyses
  - RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
  - RH-current to moments Feger, Mannel, et. al. [2010]
  - NP for moments KKV, Fael, Rahimi [in progress]