Hadronic charm decays and CP violation

Fu-Sheng Yu

Lanzhou University

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Outline

- History of charm CP violation
- Dynamics of hadronic charm decays
- Final-state interactions and charmed baryon decays
- Topological diagrams = Irreducible representations = FSI+QCD
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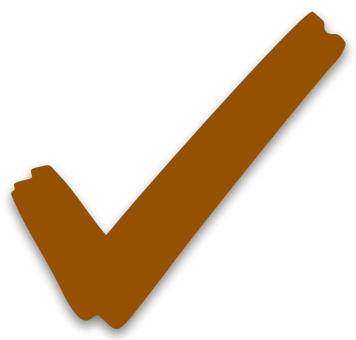
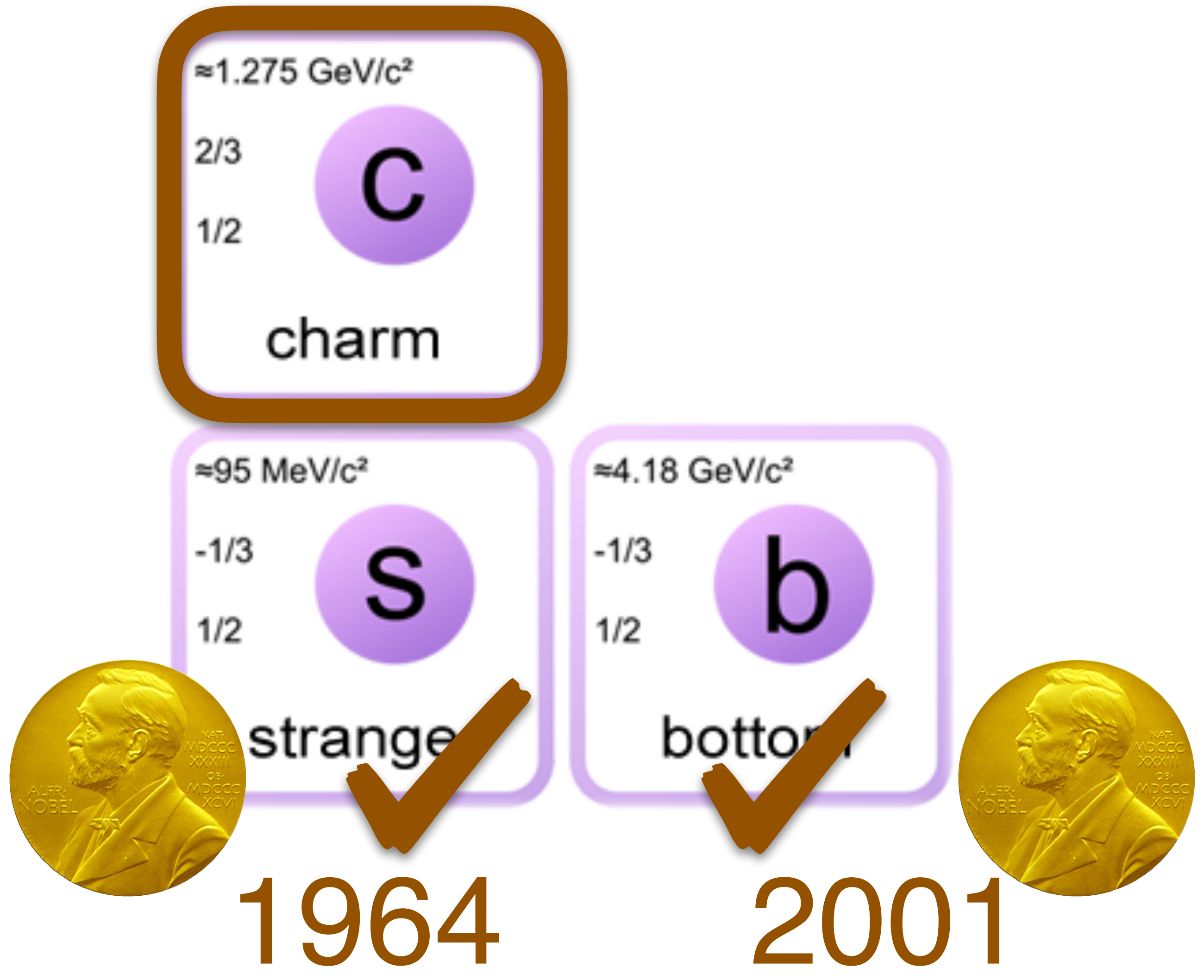
CP violation in charm?

- **CP violation** is required for the matter-antimatter asymmetry in the Universe [Sakharov, 1967]
- CPV in the SM is not large enough, thus a window to New Physics
- CPV in strange and bottom mesons have been well established.
- But how about **charm CPV?**
  - **Before 2019, Yes or No?**



  - **After 2019, SM or NP?**



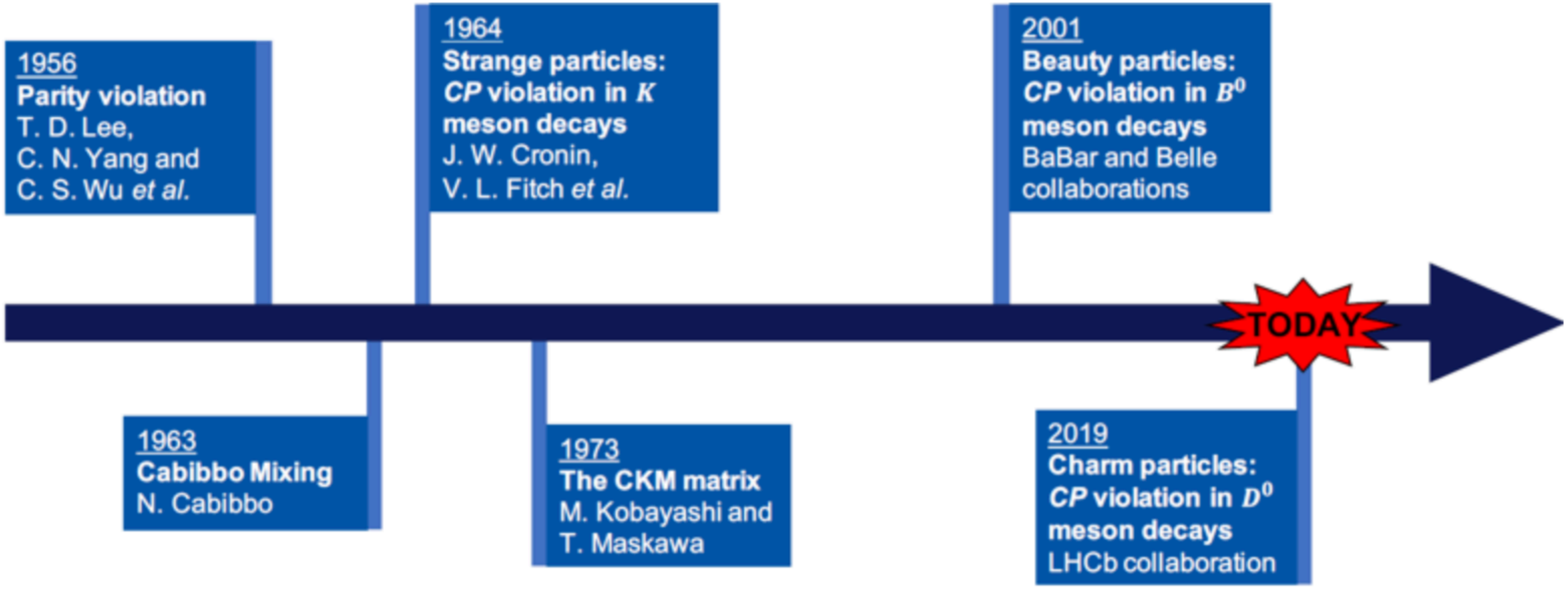
Observation of charm CPV

21 March 2019: Discovery of CP violation in charm particle decays.

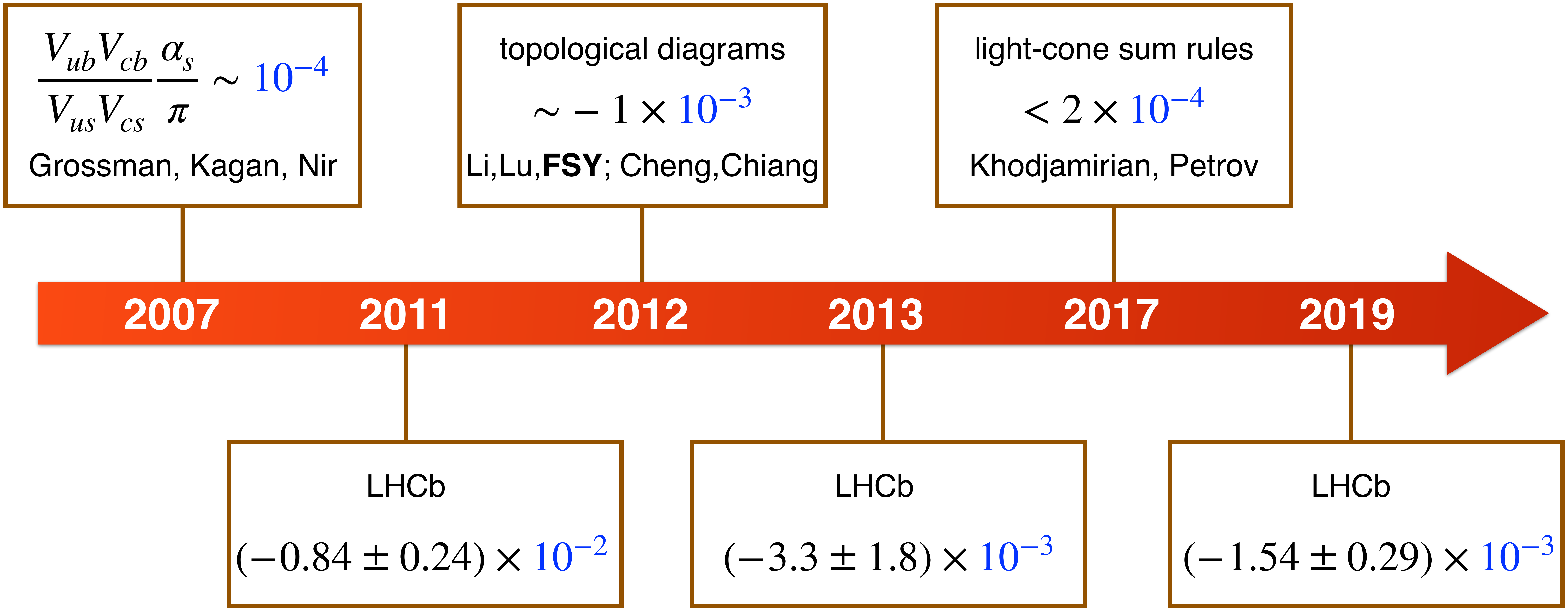
An important milestone in the history of particle physics.

[ΔA_{CP} = (-0.154±0.029)%]

LHCb, PRL122, 211803 (2019)



History of charm CPV



$$\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$$

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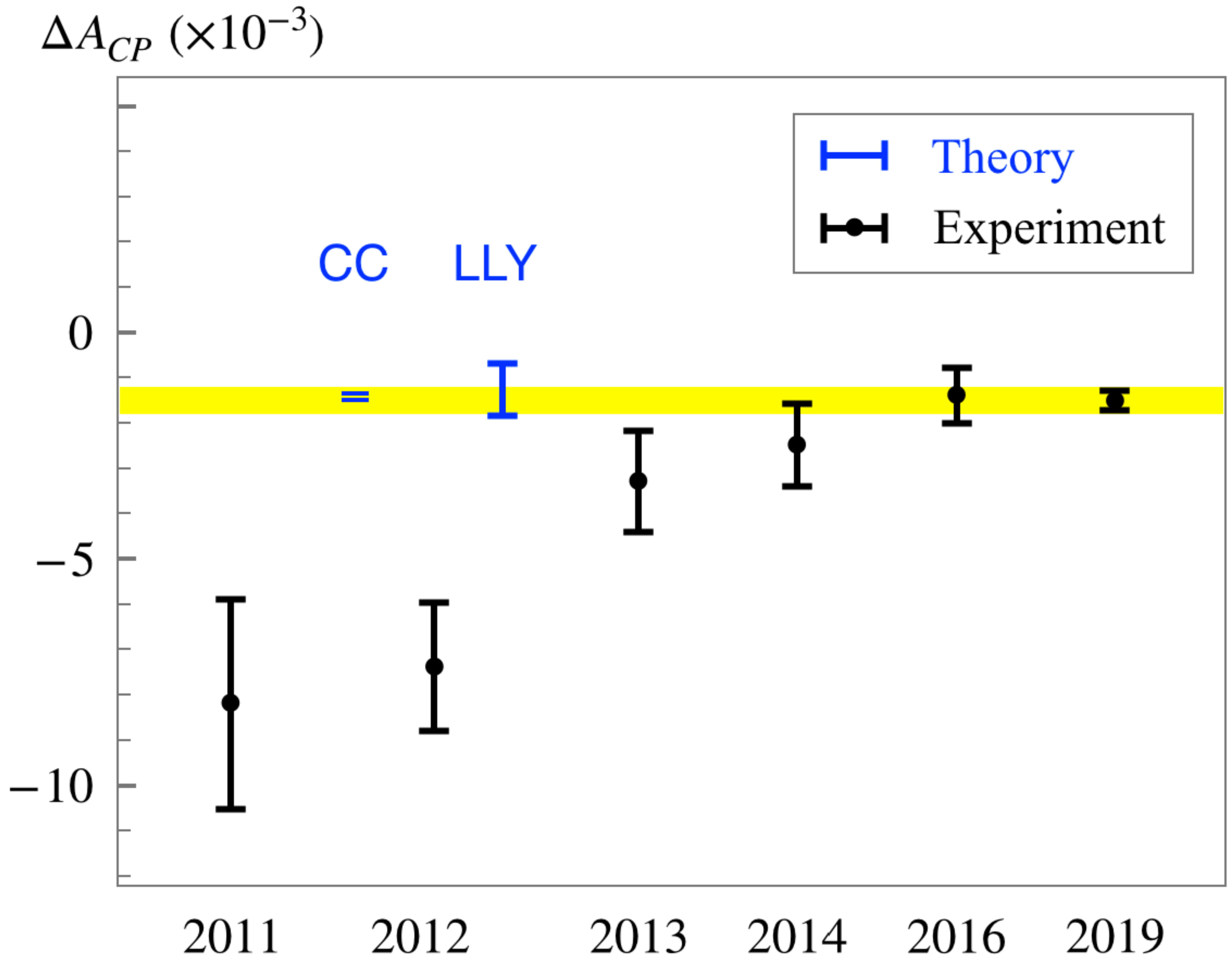

Saur, **FSY**, Sci.Bull.2020

Th: the only predictions of O(10^{-3})

CC: topological approach + QCDF

Cheng, Chiang, 2012

LLY: factorization-assisted topology (FAT)

Li, Lu, **FSY**, 2012

Exp: LHCb, PRL122, 211803 (2019)

Topological diagrammatic approach successfully predicted the charm CPV !!!

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Implications of charm CPV

$$\mathcal{A}(D^0 \to K^+K^-) = \lambda_s \mathcal{T}^{KK} + \lambda_b \mathcal{P}^{KK}, \qquad \mathcal{A}(D^0 \to \pi^+\pi^-) = \lambda_d \mathcal{T}^{\pi\pi} + \lambda_b \mathcal{P}^{\pi\pi},$$

$$\Delta A_{CP} = -2r \sin\gamma \left(\frac{|\mathcal{P}^{KK}|}{|\mathcal{T}^{KK}|} \sin\delta^{KK} + \frac{|\mathcal{P}^{\pi\pi}|}{|\mathcal{T}^{\pi\pi}|} \sin\delta^{\pi\pi} \right) \qquad r = |\lambda_b/\lambda_{d,s}|$$

$$2r\sin\gamma = 1.5 \times 10^{-3}$$

$$\Delta A_{CP} = (-1.54 \pm 0.29) \times 10^{-3}$$

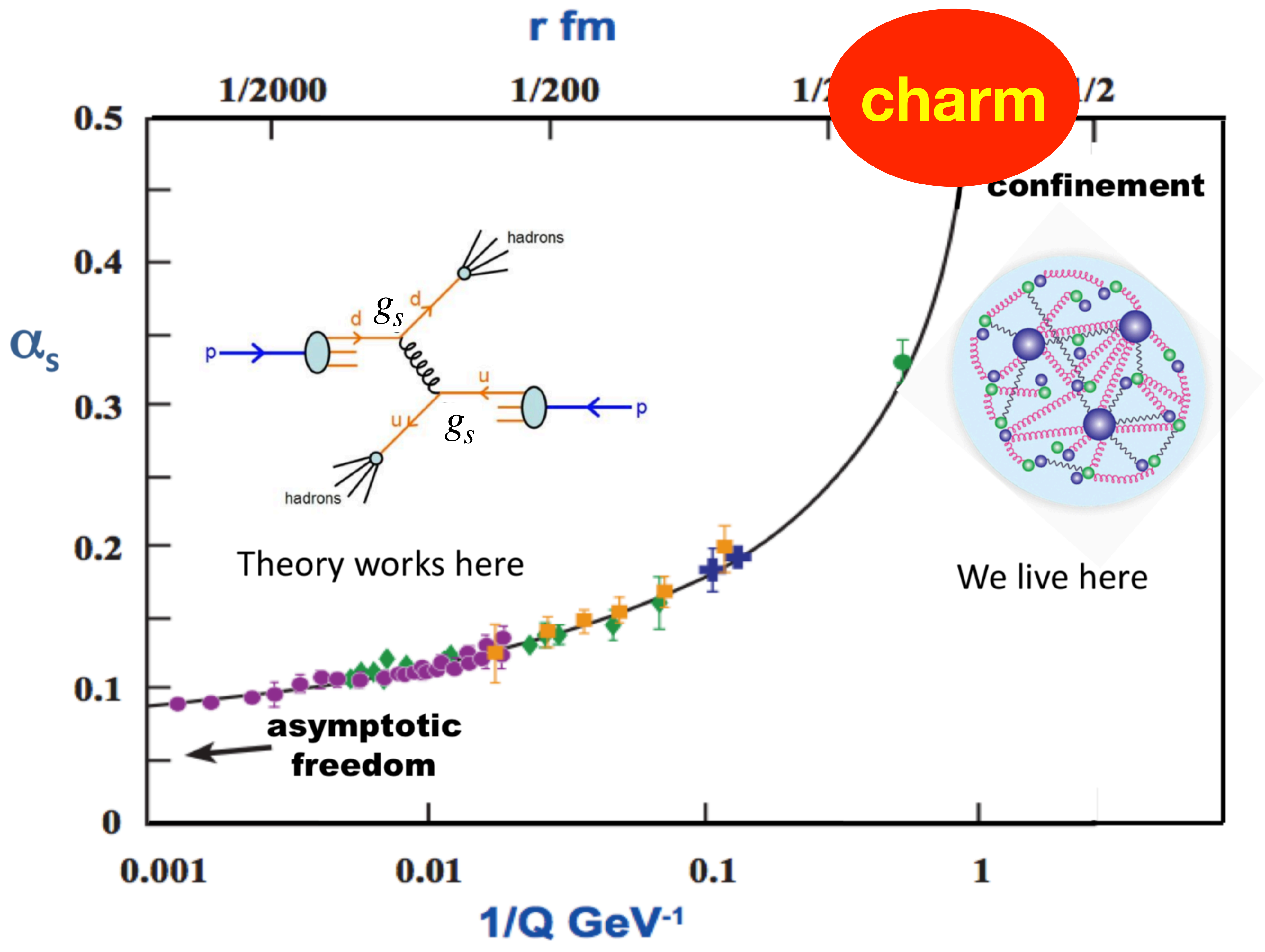
$$\Rightarrow \left(\frac{|\mathcal{P}^{KK}|}{|\mathcal{T}^{KK}|} \sin\delta^{KK} + \frac{|\mathcal{P}^{\pi\pi}|}{|\mathcal{T}^{\pi\pi}|} \sin\delta^{\pi\pi} \right) \approx 1$$

✓ **Charm is different from bottom**

$$|\mathcal{P}/\mathcal{T}|_{\text{charm}} \sim \mathcal{O}(1) \quad v.s. \quad |\mathcal{P}/\mathcal{T}|_{\text{bottom}} \sim \mathcal{O}(0.1)$$

Implications of charm CPV

$$|\mathcal{P}/\mathcal{T}|_{\text{charm}} \sim \mathcal{O}(1) \quad v.s. \quad |\mathcal{P}/\mathcal{T}|_{\text{bottom}} \sim \mathcal{O}(0.1)$$



from S.Olsen

✓ **Large non-perturbative contributions in charmed hadron decays**

$$\frac{C_{3-6}}{C_{1,2}} \sim \mathcal{O}(0.1) \quad \ll \quad \frac{\mathcal{P}}{\mathcal{T}} \sim \mathcal{O}(1)$$

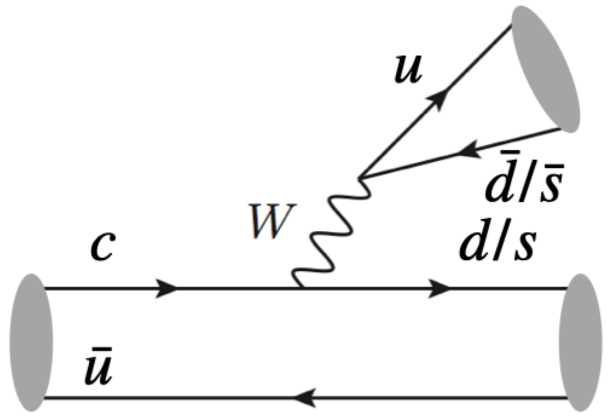
The observation of ΔA_{CP} is SM or NP?

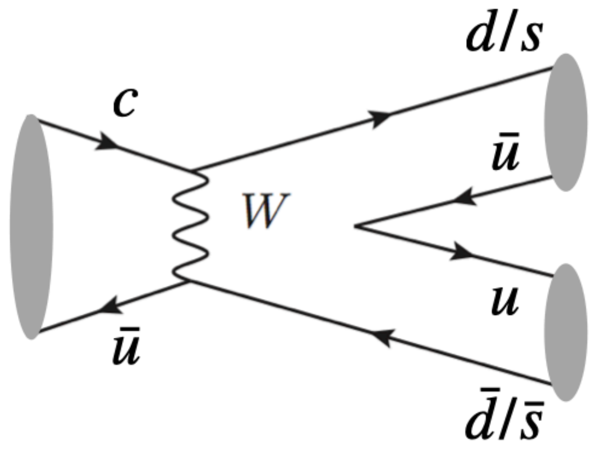
Chala, Lenz, Rusov, Scholtz, ’19

It requires dynamics !

Dynamics of hadronic charm decays

Tree

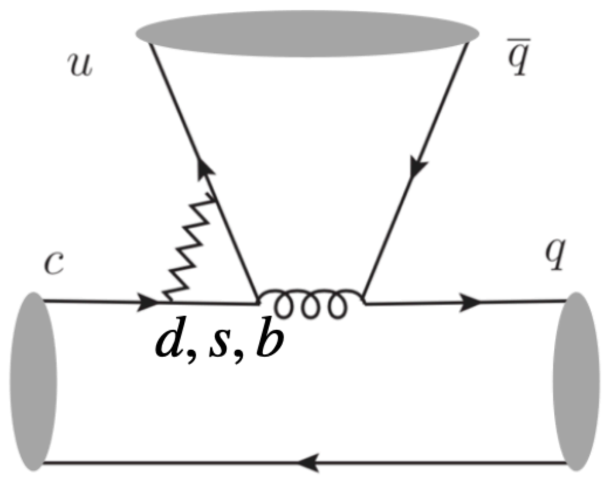


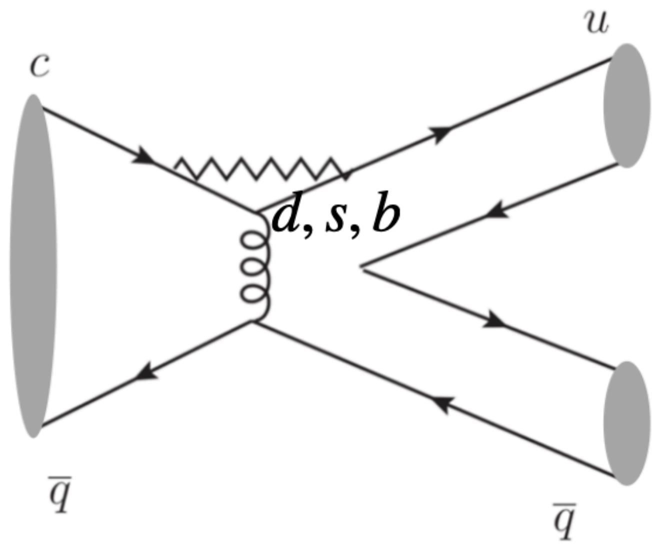


Tree diagrams are determined by data of branching fractions

Understand the dynamics at 1GeV

Penguin





Relate the penguins to the trees, with the known dynamics at 1GeV

CPV

$$\Delta A_{CP} = -2r\sin\gamma\left(\frac{|\mathcal{P}^{KK}|}{|\mathcal{T}^{KK}|}\sin\delta^{KK} + \frac{|\mathcal{P}^{\pi\pi}|}{|\mathcal{T}^{\pi\pi}|}\sin\delta^{\pi\pi}\right)$$

Then reliably predict charm CPV

Theoretical methods for hadronic weak decays

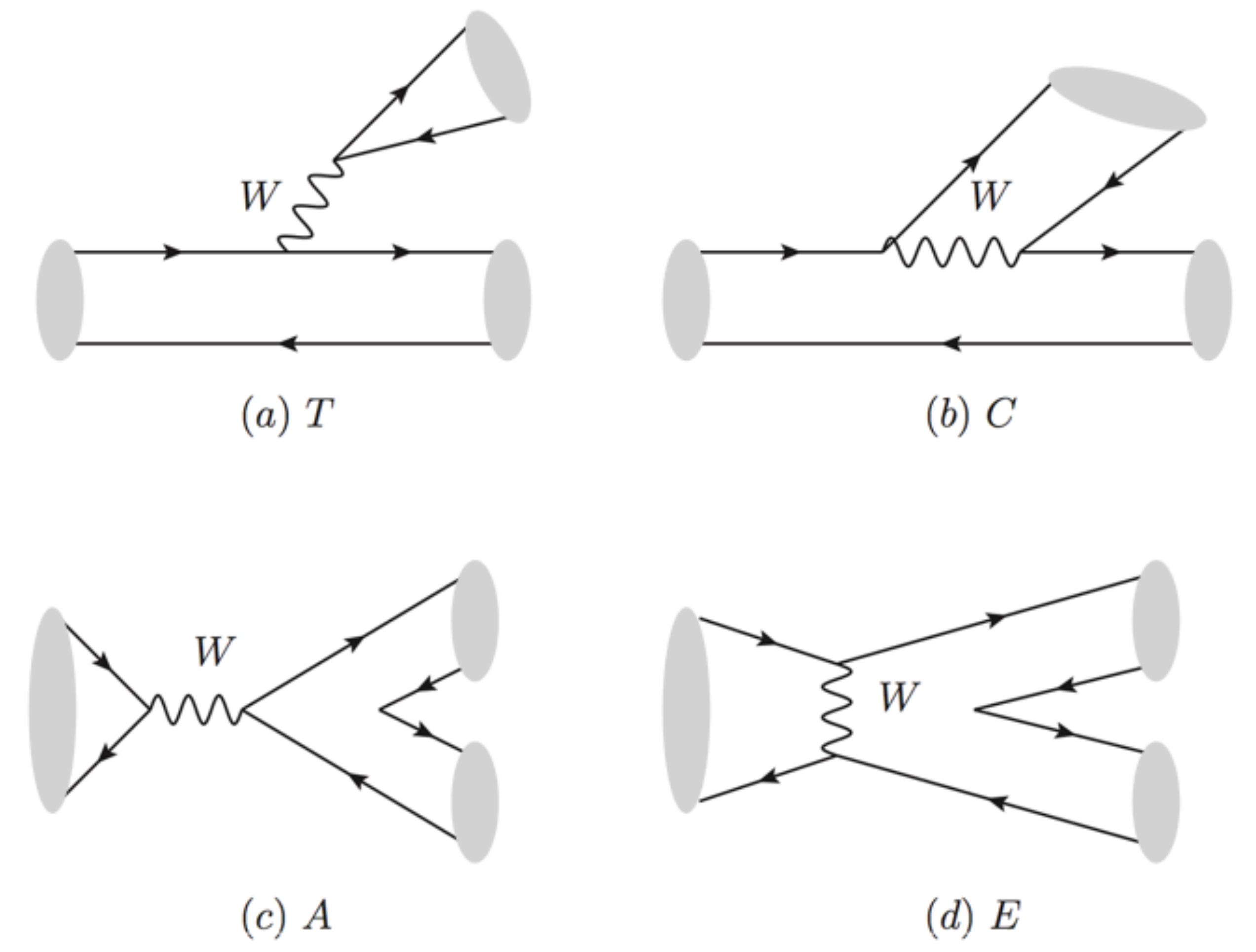
Theoretical approaches	Advantages	Disadvantages
QCD-inspired : QCDF, PQCD, SCET	(Almost) first-principle for dynamics, very predictive for B decays	Difficult for non-perturbative contributions, thus difficult for charm
Final-state interaction	Dynamics for non-perturbations	Suffer very large theoretical uncertainties
SU(3) irreducible representation	Based on approximate flavor symmetry, no additional assumptions	No link to dynamics
Topological diagrams	Include non-perturbations, successful for charm phenomenology	Mathematical foundation is not clear

Li,Lu,**FSY,** 2012; Cheng,Chiang, 2012

Topological Diagrams

- According to the weak flavour flows
- **Including all strong interaction effects** : short distance + long distance
- Amplitudes extracted from data

Chau,'86; Chau,Cheng,'87;



Meson	Mode	Representation	$\mathcal{B}_{\text{exp}}$ (%)	$\mathcal{B}_{\text{fit}}$ (%)
D^0	$K^-\pi^+$	$V_{cs}^* V_{ud}(T+E)$	3.91 ± 0.08	3.91 ± 0.17
	$\bar{K}^0\pi^0$	$\frac{1}{\sqrt{2}} V_{cs}^* V_{ud}(C-E)$	2.38 ± 0.09	2.36 ± 0.08
	$\bar{K}^0\eta$	$V_{cs}^* V_{ud}[\frac{1}{\sqrt{2}}(C+E)\cos\phi - E\sin\phi]$	0.96 ± 0.06	0.98 ± 0.05
	$\bar{K}^0\eta'$	$V_{cs}^* V_{ud}[\frac{1}{\sqrt{2}}(C+E)\sin\phi + E\cos\phi]$	1.90 ± 0.11	1.91 ± 0.09
D^+	$\bar{K}^0\pi^+$	$V_{cs}^* V_{ud}(T+C)$	3.07 ± 0.10	3.08 ± 0.36
D_s^+	$\bar{K}^0K^+$	$V_{cs}^* V_{ud}(C+A)$	2.98 ± 0.17	2.97 ± 0.32
	$\pi^+\pi^0$	0	<0.037	0
	$\pi^+\eta$	$V_{cs}^* V_{ud}(\sqrt{2}A\cos\phi - T\sin\phi)$	1.84 ± 0.15	1.82 ± 0.32
	$\pi^+\eta'$	$V_{cs}^* V_{ud}(\sqrt{2}A\sin\phi + T\cos\phi)$	3.95 ± 0.34	3.82 ± 0.36

$$T = 3.14 \pm 0.06,$$

$$C = (2.61 \pm 0.08)e^{-i(152\pm1)^\circ},$$

$$E = (1.53^{+0.07}_{-0.08})e^{i(122\pm2)^\circ},$$

$$A = (0.39^{+0.13}_{-0.09})e^{i(31^{+20}_{-33})^\circ}$$

Cheng, Chiang,'10

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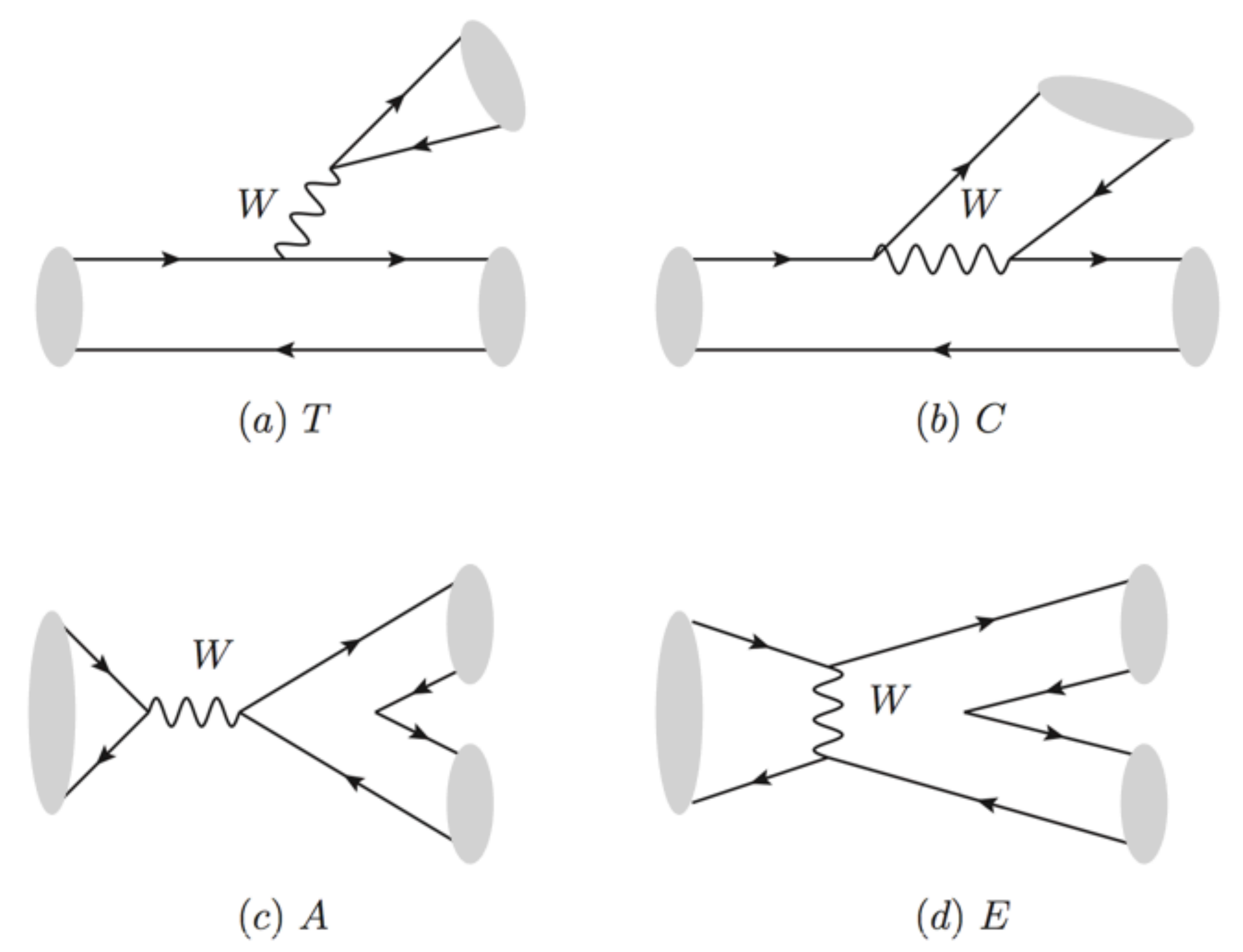

Chau,'86; Chau,Cheng,'87;

$$T = 3.14 \pm 0.06, \qquad C = (2.61 \pm 0.08)e^{-i(152\pm1)^\circ}, \qquad E = (1.53^{+0.07}_{-0.08})e^{i(122\pm2)^\circ}, \qquad A = (0.39^{+0.13}_{-0.09})e^{i(31^{+20}_{-33})^\circ}$$

$$\left|\frac{C}{T}\right| \sim 0.8 \qquad \gg \qquad \frac{a_2(\mu_c)}{a_1(\mu_c)} \sim 0.1$$

long-distance dominated in charm decays

Cheng, Chiang,'10

Li, Lu, FSY, '12

Flavor SU(3) breaking

- Flavor SU(3) symmetry breaking effects are important in the singly Cabibbo-suppressed modes

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ $(\times 10^{-3})$	$\mathcal{B}_{\rm theory}$ $(\times 10^{-3})$
D^0	$\pi^+\pi^-$	$V_{cd}^* V_{ud}(T' + E')$	1.45 ± 0.05	2.24 ± 0.10
	K^+K^-	$V_{cs}^* V_{us}(T' + E')$	4.07 ± 0.10	1.92 ± 0.08
	$K^0\bar{K}^0$	$V_{cd}^* V_{ud} E_s' + V_{cs}^* V_{us} E_d'$ [a]	0.64 ± 0.08	0

→ same in the SU(3) limit

→ vanish in the SU(3) limit

- Li, Lu, FSY, ’12: **factorization hypothesis** $\quad \frac{G_F}{\sqrt{2}} V_{\rm CKM} b_{q,s}^{E,A}(\mu) f_D m_D^2 \left(\frac{f_{P_1} f_{P_2}}{f_\pi^2}\right)$

- Cheng, Chiang, ’12, ’19: **similar to factorization** $\quad E^d = 1.10\, e^{i15.1^\circ} E\,, \qquad E^s = 0.62\, e^{-i19.7^\circ} E$

- Muller, Nierste, Schacht, ’15: **linear SU(3) breaking** $\quad H_{\cancel{\rm SU(3)}_F} = (m_s - m_d)\bar{s}s$

Modes	Br(exp)	Br(this work)	$A_{CP}^{SM} \times 10^{-3}$
$D^0 \to \pi^+\pi^-$	1.45 ± 0.05	1.43	0.58
$D^0 \to K^+K^-$	4.07 ± 0.10	4.19	−0.42
$D^0 \to K^0\bar{K}^0$	0.320 ± 0.038	0.36	1.38
$D^0 \to \pi^0\pi^0$	0.81 ± 0.05	0.57	0.05
$D^0 \to \pi^0\eta$	0.68 ± 0.07	0.94	−0.29
$D^0 \to \pi^0\eta'$	0.91 ± 0.13	0.65	1.53
$D^0 \to \eta\eta$	1.67 ± 0.18	1.48	0.18
$D^0 \to \eta\eta'$	1.05 ± 0.26	1.54	−0.94
$D^+ \to \pi^+\pi^0$	1.18 ± 0.07	0.89	0
$D^+ \to K^+\bar{K}^0$	6.12 ± 0.22	5.95	−0.93
$D^+ \to \pi^+\eta$	3.54 ± 0.21	3.39	−0.26
$D^+ \to \pi^+\eta'$	4.68 ± 0.29	4.58	1.18
$D_S^+ \to \pi^0K^+$	0.62 ± 0.23	0.67	0.39
$D_S^+ \to \pi^+K^0$	2.52 ± 0.27	2.21	0.84
$D_S^+ \to K^+\eta$	1.76 ± 0.36	1.00	0.70
$D_S^+ \to K^+\eta'$	1.8 ± 0.5	1.92	−1.60

$$\Delta A_{CP}^{SM} = -1 \times 10^{-3}$$

@ BESIII & CLEO

1. Understand QCD dynamics @ 1GeV by Branching Ratios

2. then predict charm CPV

H.n.Li, C.D.Lu, F.S.Yu, PRD2012

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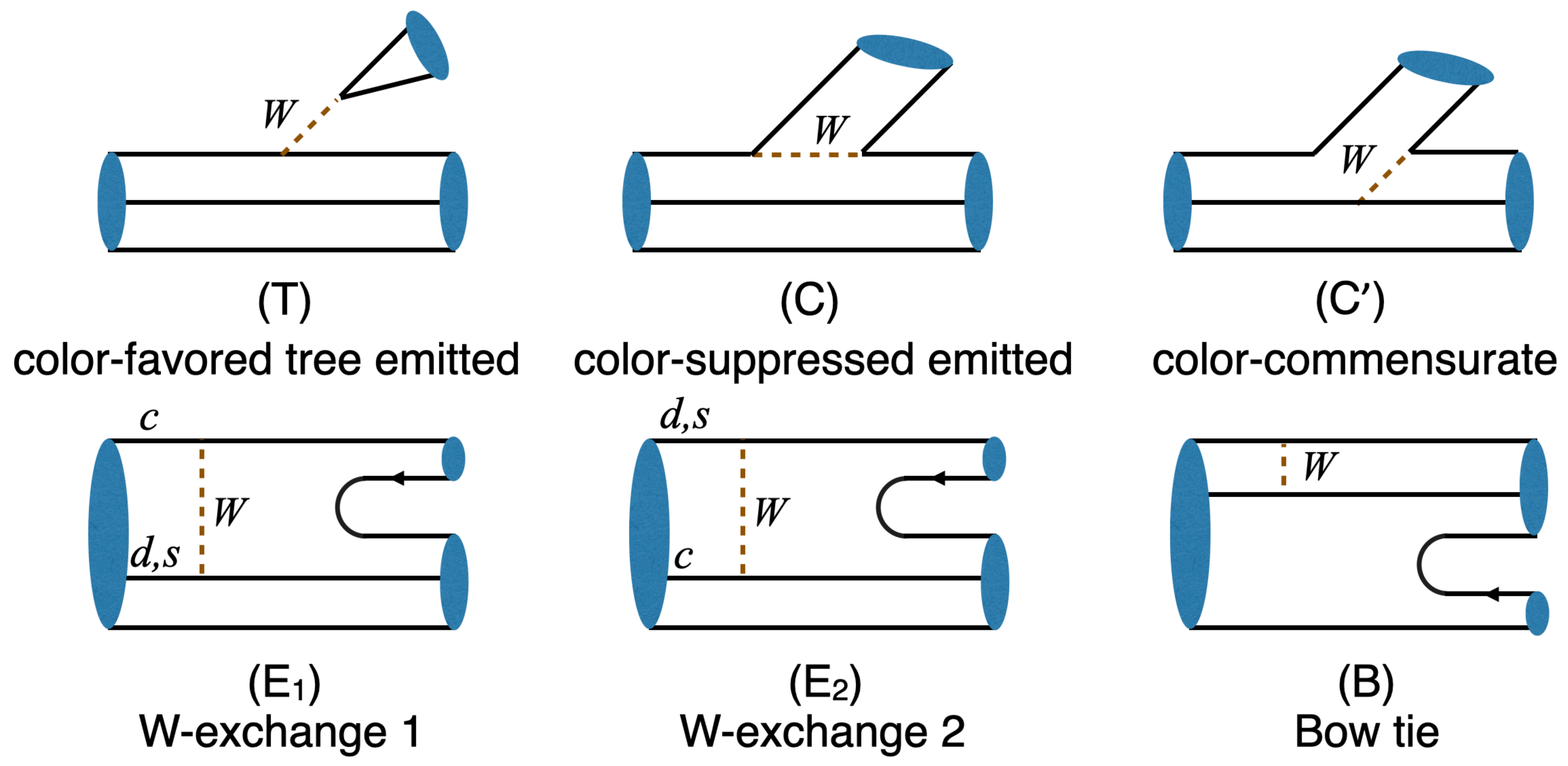
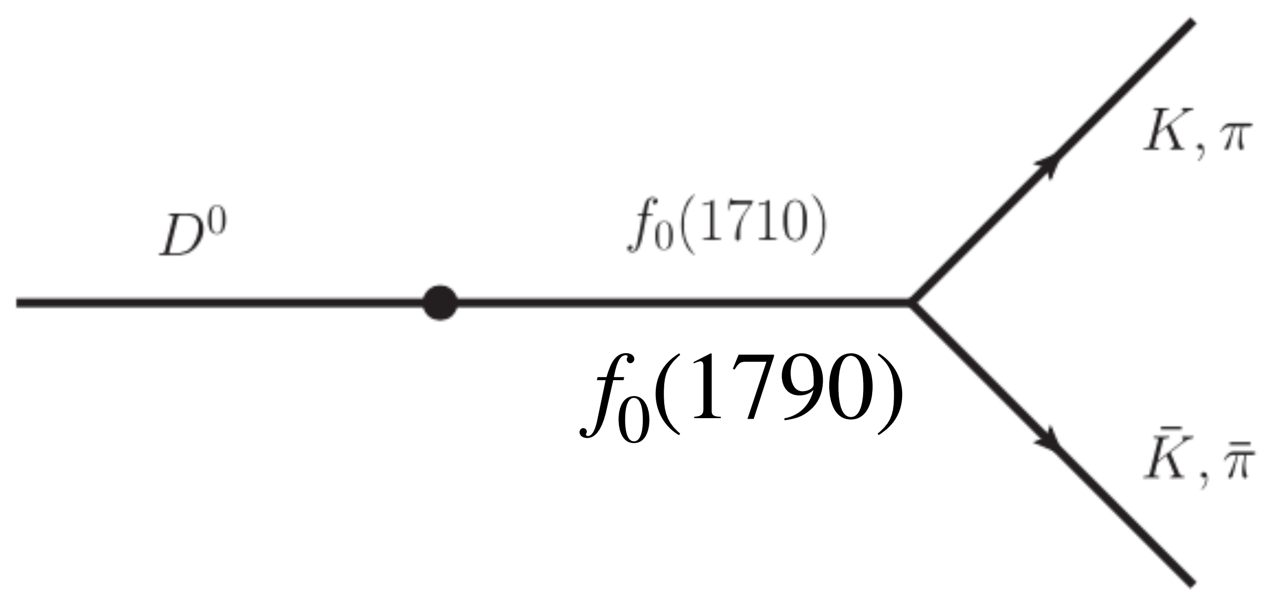
Charmed baryon decays

- Charmed baryon decays are the next opportunity and challenge of charm physics
- No any real CPV predictions
- Dynamics are more complicated
    - Many more topological diagrams + more partial waves
    - SU(3) irreducible representations cannot provide information on penguins
    - Final-state interactions (FSI) are necessary

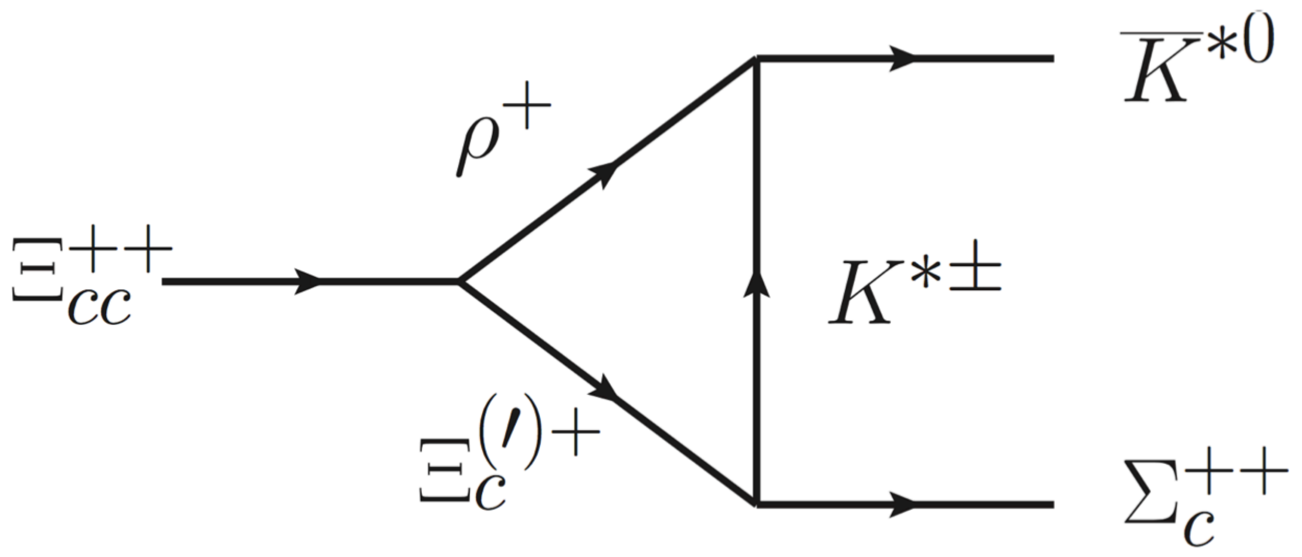


Final-state interactions

- FSIs of resonant contributions have been considered for charm CPV [Schacht, Soni, '22]
- But lack of enough information on the resonances

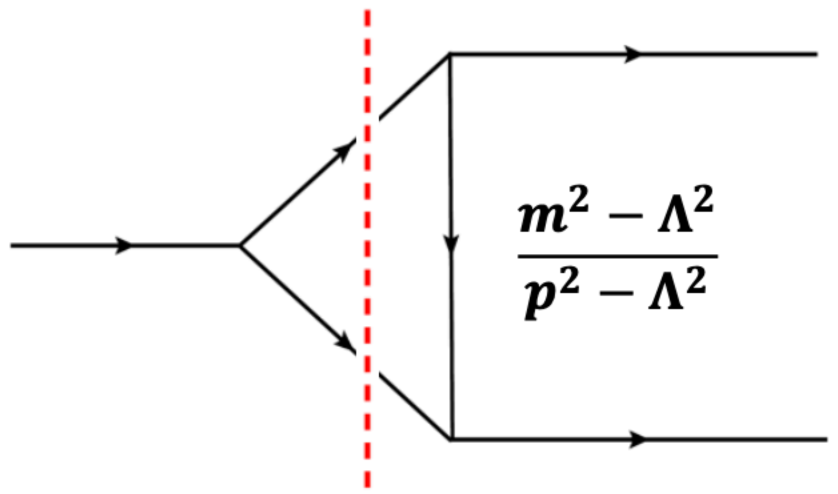


- FSIs of rescattering mechanism have been successfully used to predict the discovery channel of $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ [FSY, et al, '17]
- It deserves to develop the rescattering mechanism for CPV of charmed baryon decays



➢ **Conventional method：** optical theorem + Cutkosky cutting rule

- H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005)........



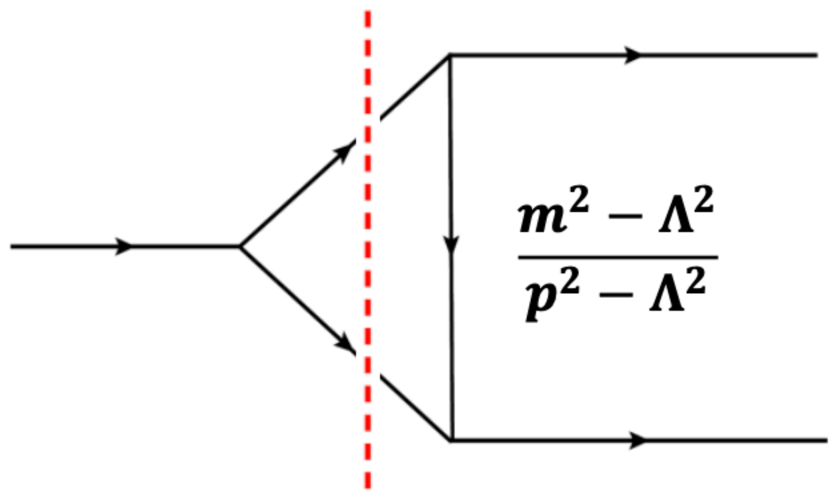
$$\Lambda = m_k + \eta \Lambda_{QCD}$$

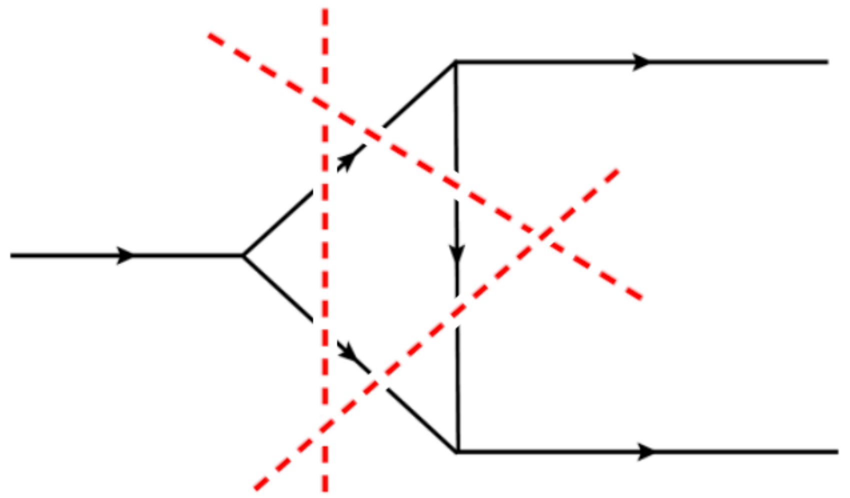
$$\mathcal{A}bs[\mathcal{M}(P_i \to P_3 P_4)] = \frac{1}{2} \sum_{\{P_1 P_2\}} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_3 + p_4 - p_1 - p_2) \cdot M(P_i \to \{P_1 P_2\}) T^*(P_3 P_4 \to \{P_1 P_2\}).$$

- **Strong model-dependent in charmed baryon decay：**

decay mode	Topology diagram	Experiment(%)	Short-distance	η
$\Lambda_c^+ \to \Sigma^+ \phi$	E_1	0.39 ± 0.06	-	6.5
$\Lambda_c^+ \to p\omega$	C, C', E_1, E_2, B	0.09 ± 0.04	2.83×10^{-6}	0.65

- **Only a part of the imaginary contribution is included……**

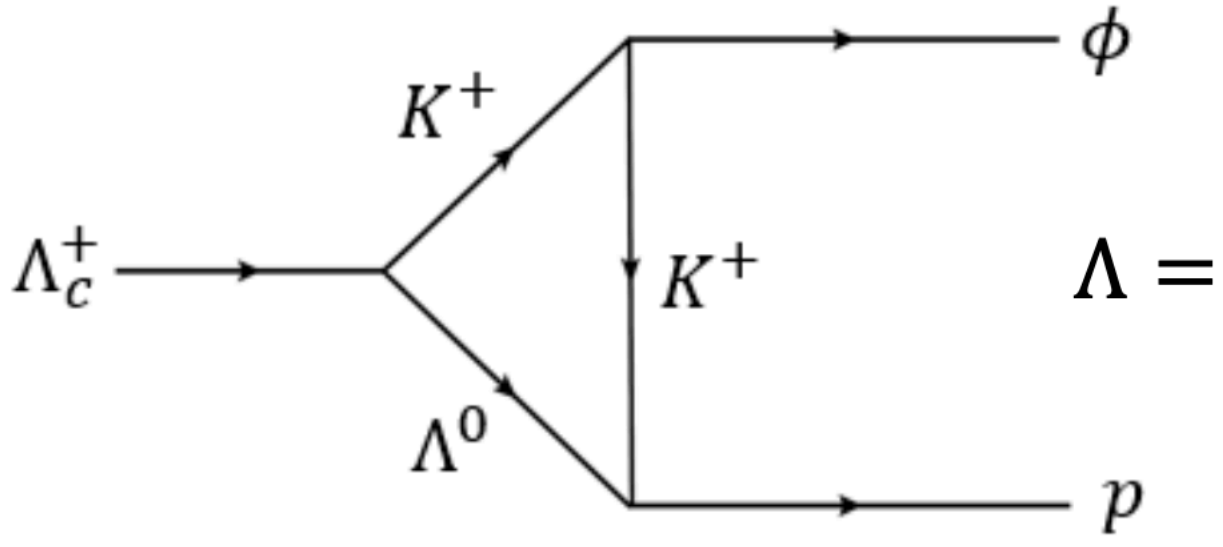




- Off-shell effects
- Lost contribution

- J.J. Han, H.Y. Jiang, W. Liu, Z.J. Xiao, and F.S. Yu, ” Chin. Phys. C 45, 053105 (2021).

➢ Improving method： Loop integral



$\Lambda = m_k + \eta \Lambda_{QCD}$

$$\mathcal{M}[P,B;V]$$
$$= -i \int \frac{d^4 p_1}{(2\pi)^4} g_{BBP} g_{VPP} \bar{u}(p_4,s_4)\gamma_5(\not{p}_2 + m_2)(A + B\gamma_5)u(p,s)\epsilon^\star_\mu(p_3,\lambda_3)(p_1+k)^\mu$$
$$\times \frac{1}{(p_1^2 - m_1^2 + i\epsilon)(p_2^2 - m_2^2 + i\epsilon)(k^2 - m_k^2 + i\epsilon)} \left(\frac{\Lambda_1^2 - m_1^2}{\Lambda_1^2 - p_1^2}\right)\left(\frac{\Lambda_2^2 - m_2^2}{\Lambda_2^2 - p_2^2}\right)\left(\frac{\Lambda_k^2 - m_k^2}{\Lambda_k^2 - k^2}\right)$$

- The complete amplitudes with real part and strong phase

$$\begin{pmatrix} \{0., 0., -1.57956\times10^{-7} + 6.40596\times10^{-8}\, i\} & \{4.65132\times10^{-7} + 1.10998\times10^{-6}\, i, 0., 0.\} \\ \{0., -1.00635\times10^{-6} + 1.46048\times10^{-7}\, i, 0.\} & \{0., 0., 4.56956\times10^{-7} - 2.83047\times10^{-7}\, i\} \end{pmatrix}$$

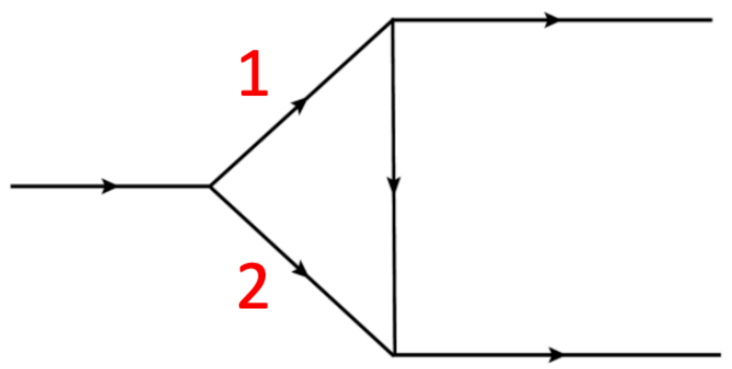
- The process dependence of the parameters is greatly reduced

The contribution of the real part is on the same order as the contribution of the imaginary part!

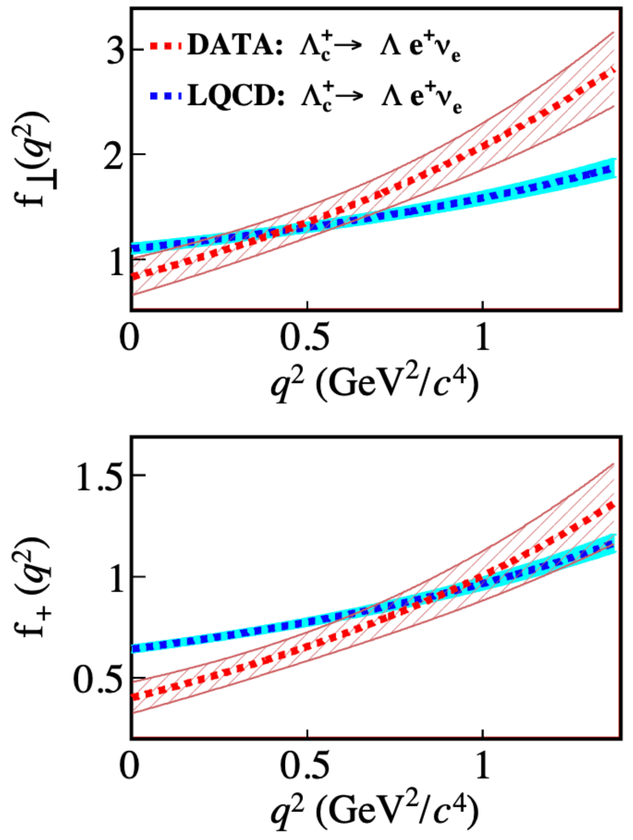
Only one parameter explain all the 8 experimental data!

- **Branching ratio:** $\eta = 0.6 \pm 0.1$

$$\Gamma(\mathcal{B}_c \to \mathcal{B}_8 V) = \frac{p_c}{8\pi m_i^2} \frac{1}{2} \sum_{\lambda\lambda'\sigma} |\mathcal{A}(\mathcal{B}_c \to \mathcal{B}_8 V)|^2$$



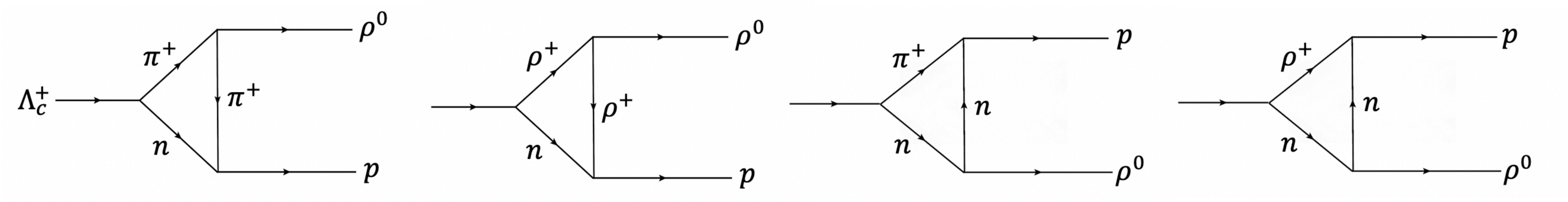
decay mode	topology	experiment(%)	Short-distance	prediction(%)
$\Lambda_c^+ \to \Lambda^0 \rho^+$	T, C', E_2, B	4.06 ± 0.52	4.91%	8 ± 0.8
$\Lambda_c^+ \to p\phi$	C	0.106 ± 0.014	1.92×10^{-6}	0.09 ± 0.03
$\Lambda_c^+ \to \Sigma^+ \phi$	E_1	0.39 ± 0.06	-	0.49 ± 0.22
$\Lambda_c^+ \to p\omega$	C, C', E_1, E_2, B	0.09 ± 0.04	2.83×10^{-6}	0.08 ± 0.04
$\Lambda_c^+ \to \Sigma^+ \rho^0$	C', E_2, B	< 1.7	-	2.0 ± 1.0
$\Lambda_c^+ \to \Sigma^0 \rho^+$	C', E_2, B	Isospin	-	Isospin
$\Lambda_c^+ \to \Sigma^+ \omega$	C', E_2, B	1.7 ± 0.21	-	1.8 ± 0.7
$\Lambda_c^+ \to p\bar{K}^{*0}$	C, E_1	1.96 ± 0.27	3.47×10^{-5}	2.9 ± 1.2
$\Lambda_c^+ \to \Sigma^+ K^{*0}$	C', E_1	0.35 ± 0.1	-	0.28 ± 0.13



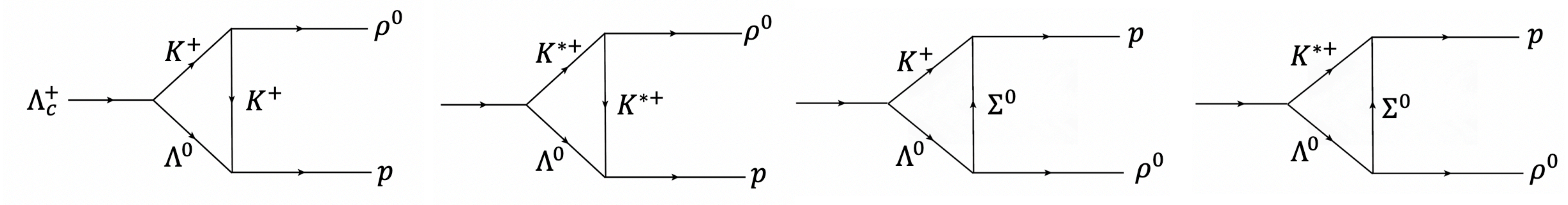
Preliminary results by C.P.Jia, H.Y.Jiang, FSY

Triangle diagrams

$V_{ud}V_{cd}^{*}$



$V_{us}V_{cs}^{*}$



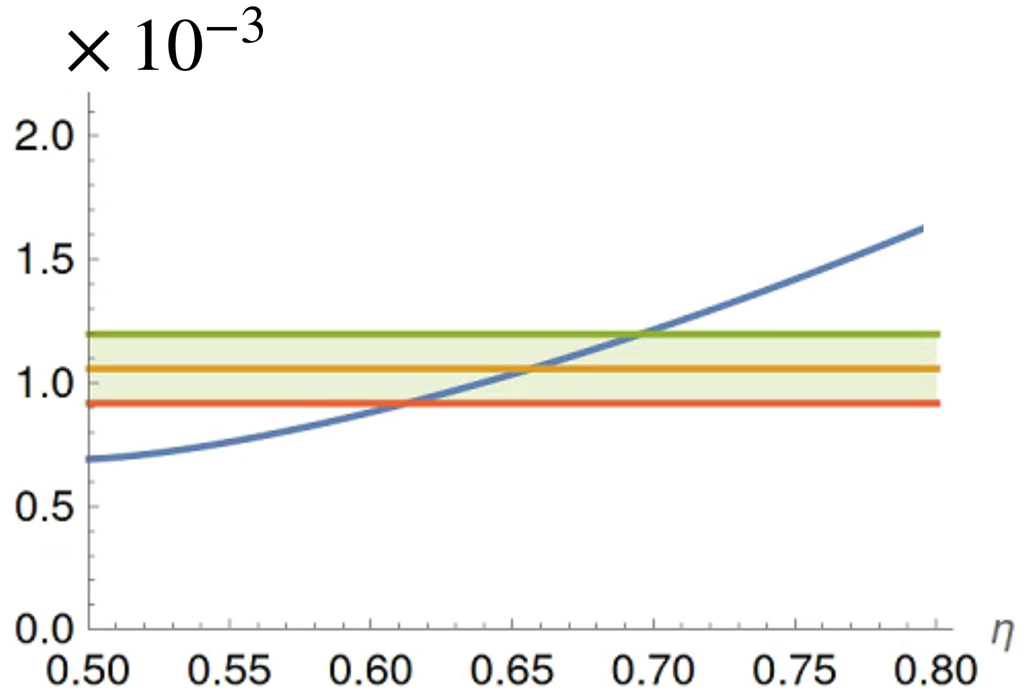
CPV can be easily obtained within the rescattering mechanism

$$\lambda_d\, A_d + \lambda_s\, A_s$$

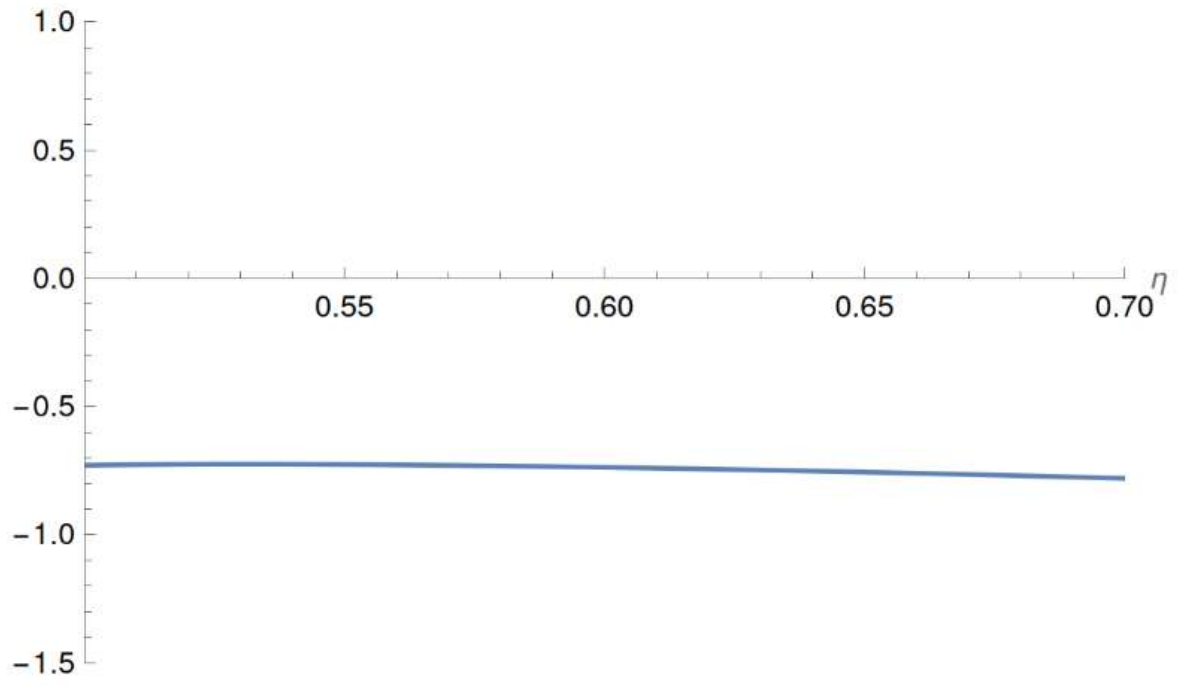
Dependence on η

$\Lambda_c^+ \to p\phi$

Branching fractions



Decay asymmetry α

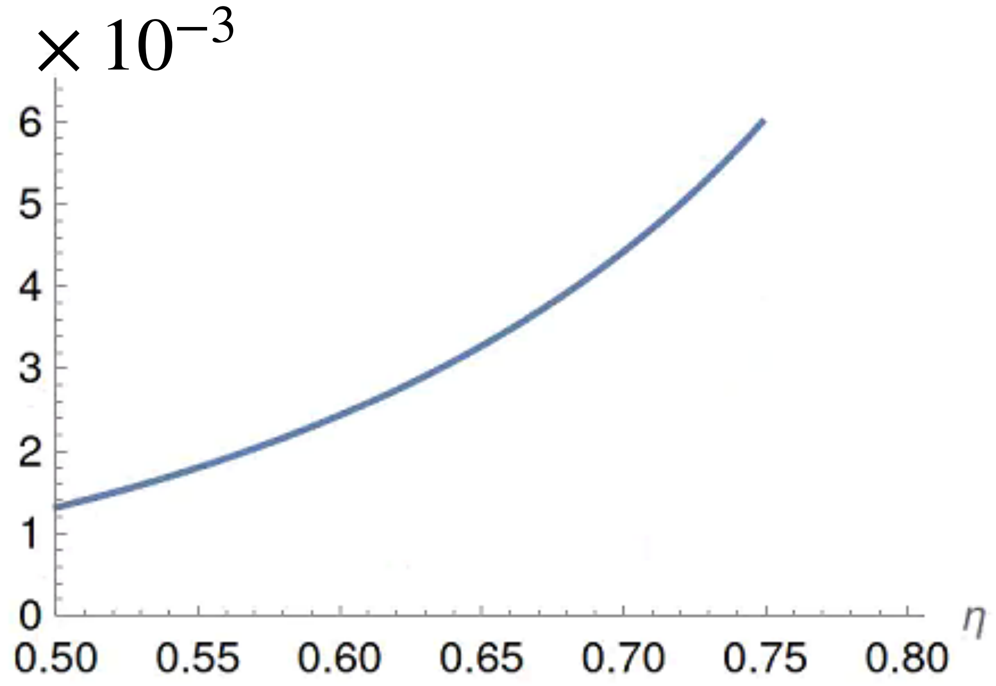


- The decay asymmetries and CPV are insensitive to η, whose dependences are mostly cancelled by the ratios

$$\alpha = \frac{\left|H_{1,\frac{1}{2}}\right|^2 - \left|H_{-1,-\frac{1}{2}}\right|^2}{\left|H_{1,\frac{1}{2}}\right|^2 + \left|H_{-1,-\frac{1}{2}}\right|^2} \qquad A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

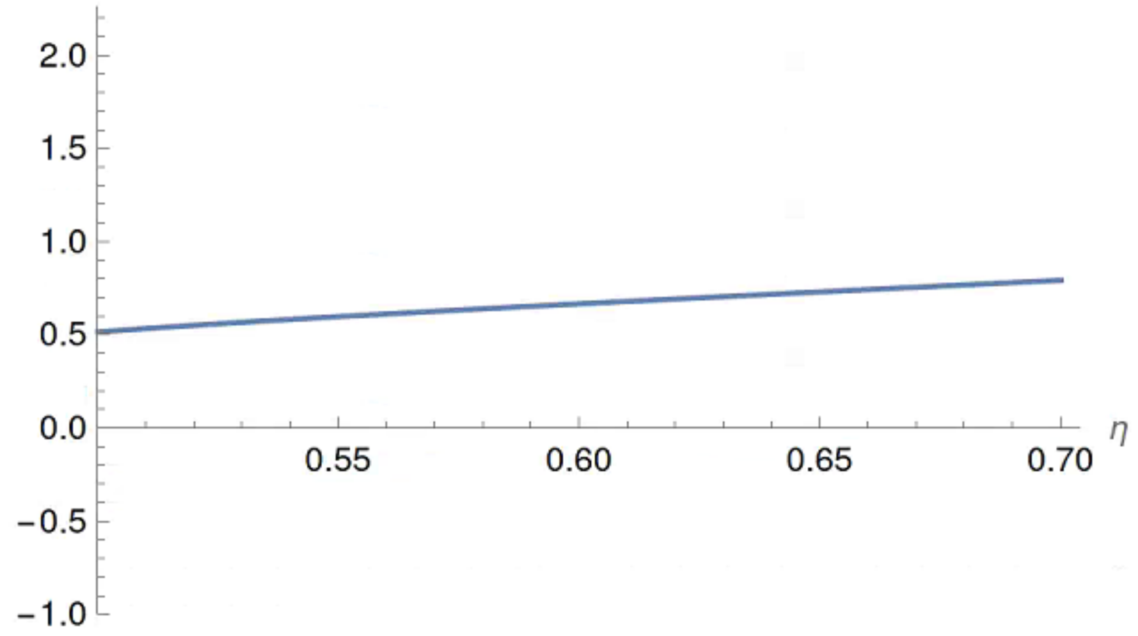
$\Lambda_c^+ \to p\rho^0$

Branching fractions

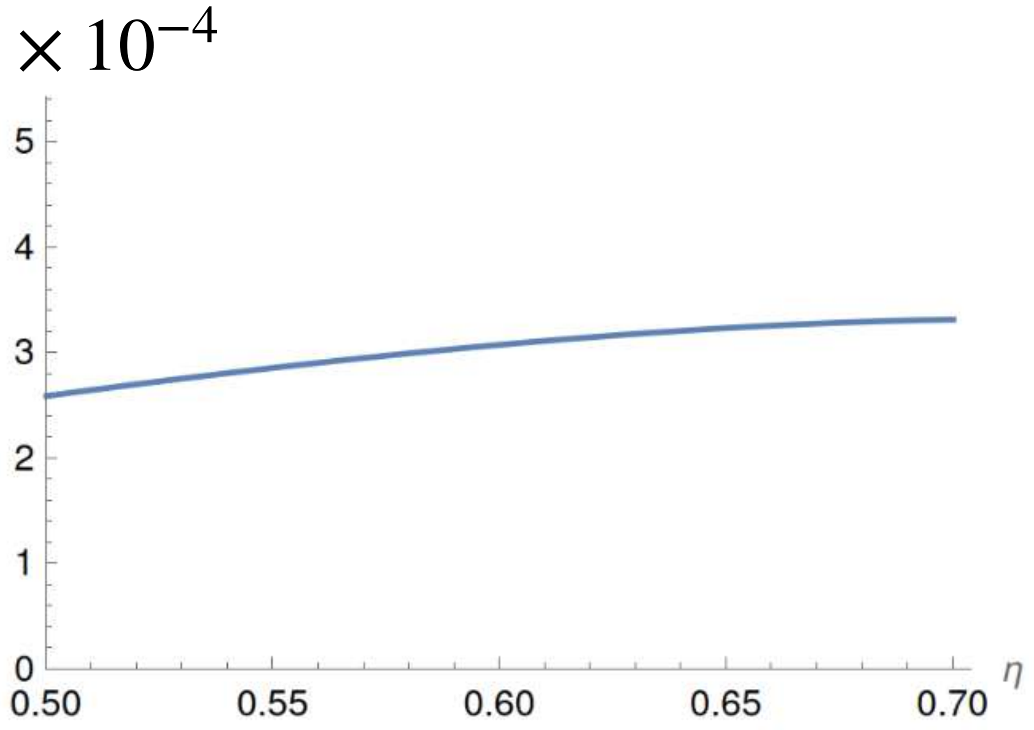


$BR(\Lambda_c^+ \to p\pi^+\pi^-) = (4.60 \pm 0.26) \times 10^{-3}$

Decay asymmetry α



Direct CPV



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- Summary

SU(3) irreducible representation approach

- Zeppendfeld, 1981
  - First SU(3) relations for B decays
  - with reduced amplitudes
- Savage and Wise, 1989
  - First **tensor contraction** formulae
  - SU(3) irreducible representation

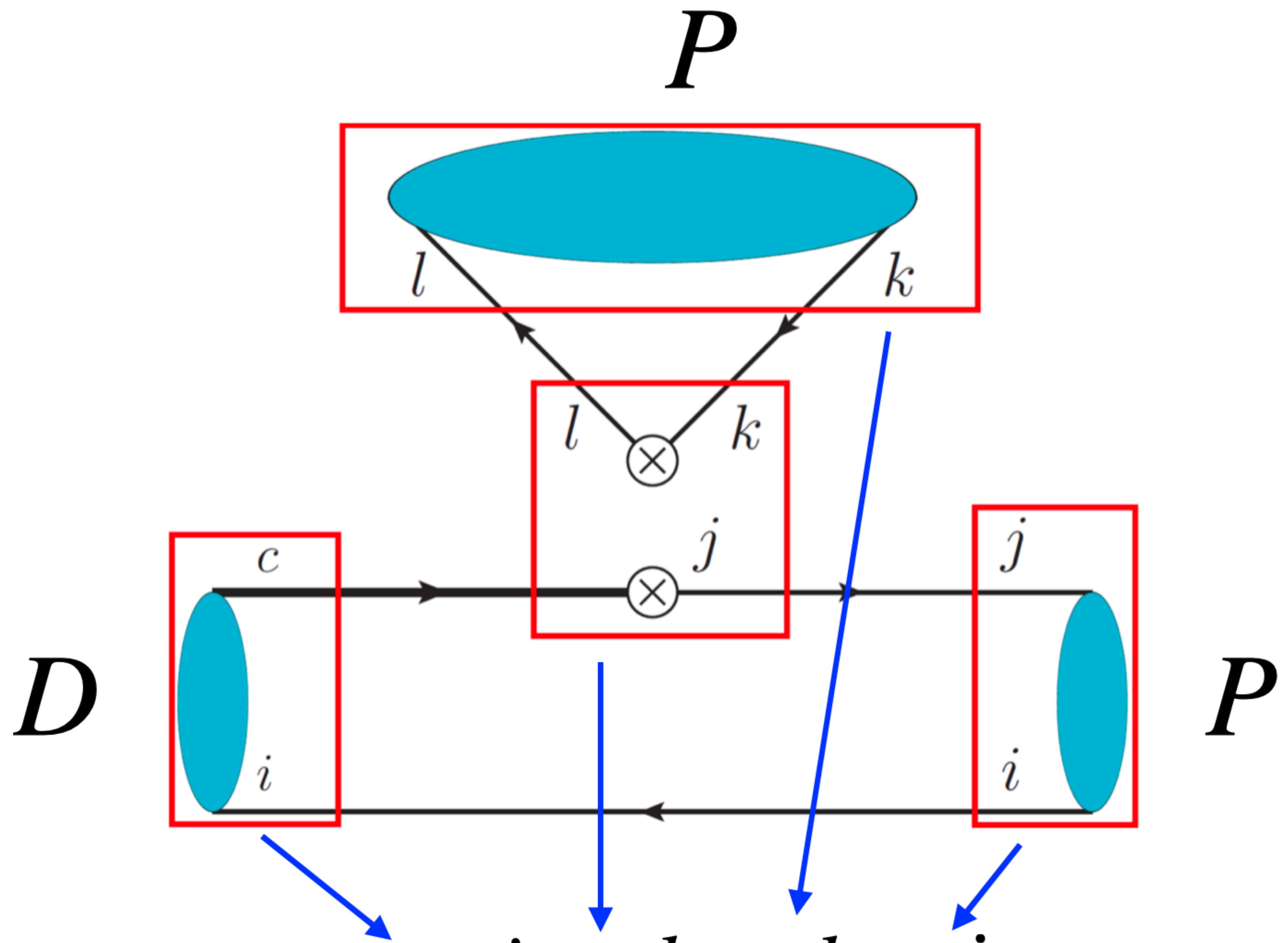
$$b(c) \to q_1\bar{q}_2q_3, \quad q_i = u, d, s$$

$$3 \otimes \bar{3} \otimes 3 = 3_p \oplus 3_t \oplus \bar{6} \oplus 15$$

$$\begin{aligned}
A = \; & a_3^p D^i \mathcal{H}(3_p)_i (P)^j_k (P)^k_j + b_3^p D^i \mathcal{H}(3_p)_i (P)^k_k (P)^j_j + c_3^p D^i \mathcal{H}(3_p)_k (P)^k_i (P)^j_j + d_3^p D^i \mathcal{H}(3_p)_k (P)^j_i (P)^k_j \\
& + a_3^t D^i \mathcal{H}(3_t)_i (P)^j_k (P)^k_j + b_3^t D^i \mathcal{H}(3_t)_i (P)^k_k (P)^j_j + c_3^t D^i \mathcal{H}(3_t)_k (P)^k_i (P)^j_j + d_3^t D^i \mathcal{H}(3_t)_k (P)^j_i (P)^k_j \\
& + a_6 D^i \mathcal{H}(\bar{6})^k_{ij} (P)^j_l (P)^l_k + b_6 D^i \mathcal{H}(\bar{6})^k_{ij} (P)^j_k (P)^l_l + c_6 D^i \mathcal{H}(\bar{6})^k_{jl} (P)^j_i (P)^l_k \\
& + a_{15} D^i \mathcal{H}(15)^k_{ij} (P)^j_l (P)^l_k + b_{15} D^i \mathcal{H}(15)^k_{ij} (P)^j_k (P)^l_l + c_{15} D^i \mathcal{H}(15)^k_{jl} (P)^j_i (P)^l_k .
\end{aligned}$$

14 SU(3) irreducible representations for $D \to PP$ modes

Topological diagrams



$$A = \quad T\, D^i\, H_{lj}^k\, P_k^l\, P_i^j \quad + \dots$$

color-favored tree-emitted amplitude, *T*

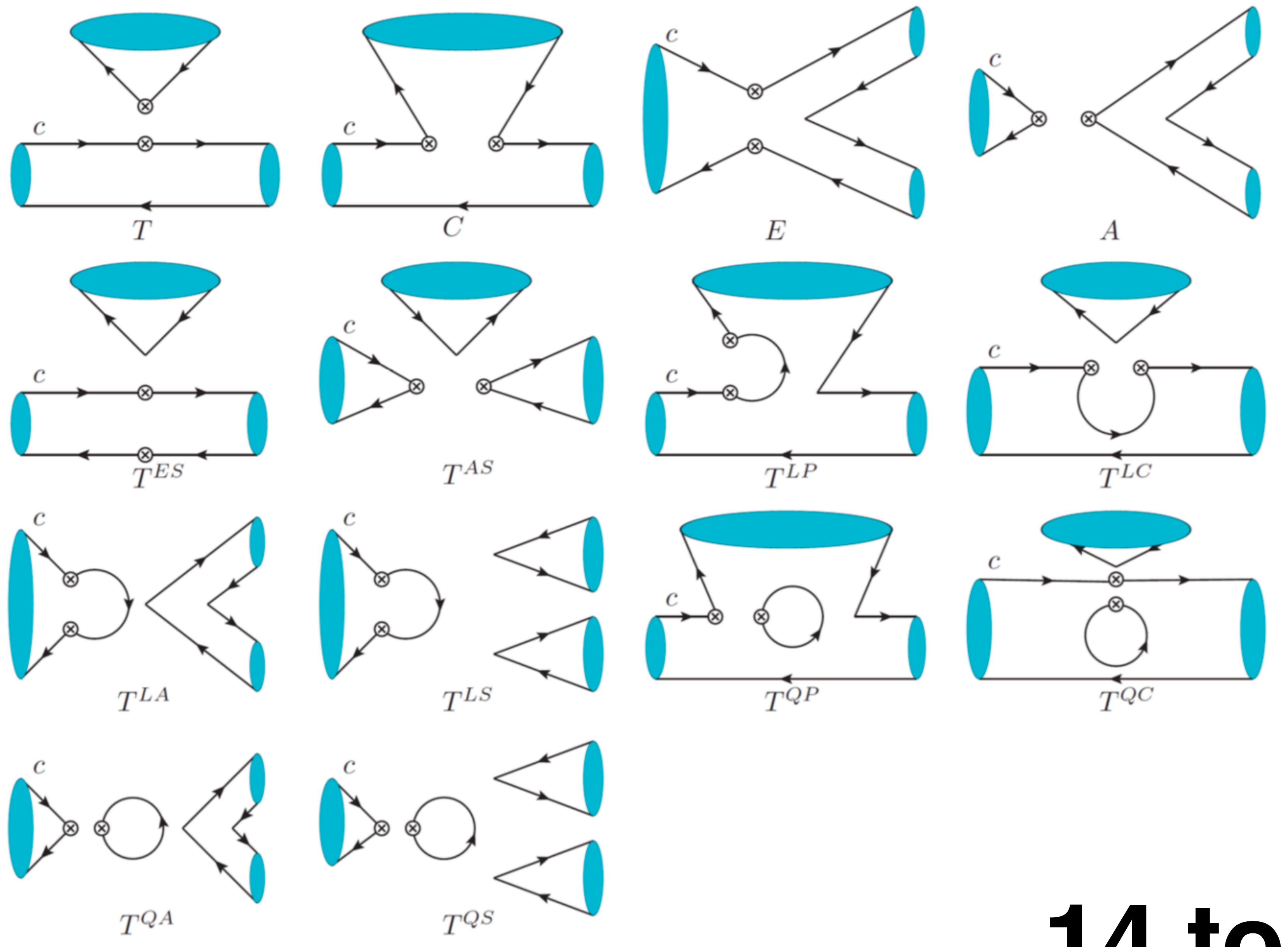
$$\mathcal{H} = \mathcal{H}_{ij}^k (\bar{q}^i q_k)(\bar{q}^j c)$$

- Under the SU(3) flavor symmetry
- Tensor indices are all contracted
- **Completeness of topological diagrams:**
  - For $D \to PP$: $A_4^4 - 2(A_3^3 - 1) = 14$ diagrams
  - For $D \to PV$: $A_4^4 = 24$ diagrams
- With the complete set of diagrams, we can then discuss the independence of diagrams

D.Wang, C.P.Jia, FSY, 2021

Topological diagrams

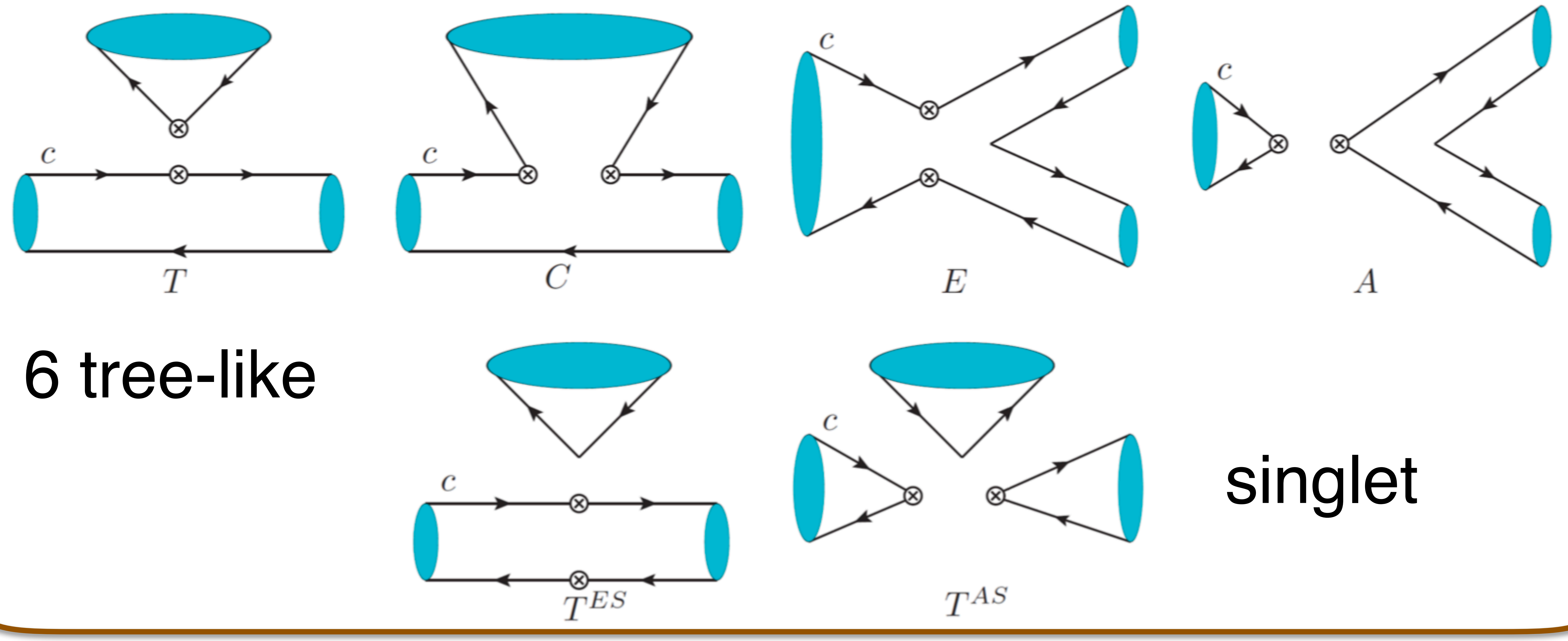
$$
\begin{aligned}
A = {} & T D^i \mathcal{H}^k_{lj} (P)^j_i (P)^l_k + C D^i \mathcal{H}^k_{jl} (P)^j_i (P)^l_k + E D^i \mathcal{H}^j_{il} (P)^k_j (P)^l_k + A D^i \mathcal{H}^j_{li} (P)^k_j (P)^l_k \\
& + T^{ES} D^i \mathcal{H}^l_{ij} (P)^j_l (P)^k_k + T^{AS} D^i \mathcal{H}^l_{ji} (P)^j_l (P)^k_k + T^{LP} D^i \mathcal{H}^l_{kl} (P)^j_i (P)^k_j + T^{LC} D^i \mathcal{H}^l_{jl} (P)^j_i (P)^k_k \\
& + T^{LA} D^i \mathcal{H}^l_{il} (P)^k_j (P)^j_k + T^{LS} D^i \mathcal{H}^l_{il} (P)^j_j (P)^k_k + T^{QP} D^i \mathcal{H}^l_{lk} (P)^j_i (P)^k_j + T^{QC} D^i \mathcal{H}^l_{lj} (P)^j_i (P)^k_k \\
& + T^{QA} D^i \mathcal{H}^l_{li} (P)^k_j (P)^j_k + T^{QS} D^i \mathcal{H}^l_{li} (P)^j_j (P)^k_k .
\end{aligned}
$$



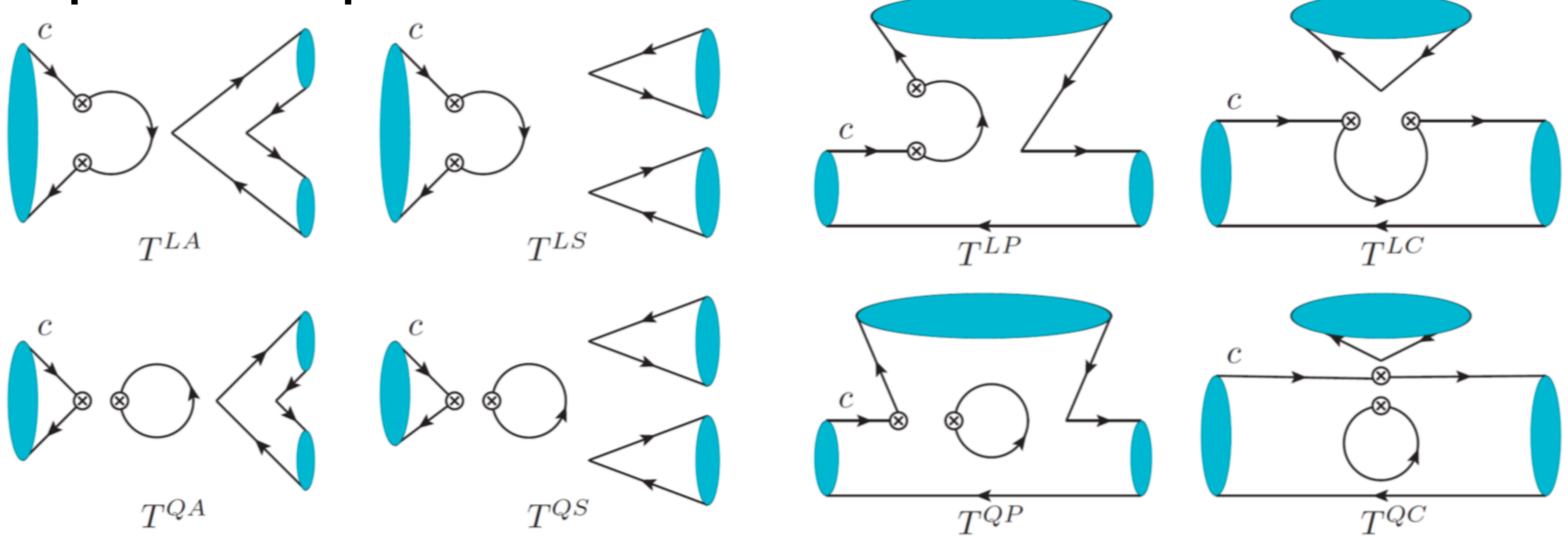
Before: directly draw all possible diagrams

Now: systematically obtain all the diagrams

14 topological diagrams



8 quark-loops



D.Wang, C.P.Jia, FSY, 2021

Topological diagrams = Irreducible representations

$$A = TD^i\mathcal{H}^k_{lj}(P)^j_i(P)^l_k + CD^i\mathcal{H}^k_{jl}(P)^j_i(P)^l_k + ED^i\mathcal{H}^j_{il}(P)^k_j(P)^l_k + AD^i\mathcal{H}^j_{li}(P)^k_j(P)^l_k$$
$$+ T^{ES}D^i\mathcal{H}^l_{ij}(P)^j_l(P)^k_k + T^{AS}D^i\mathcal{H}^l_{ji}(P)^j_l(P)^k_k + T^{LP}D^i\mathcal{H}^l_{kl}(P)^j_i(P)^k_j + T^{LC}D^i\mathcal{H}^l_{jl}(P)^j_i(P)^k_k$$
$$+ T^{LA}D^i\mathcal{H}^l_{il}(P)^k_j(P)^j_k + T^{LS}D^i\mathcal{H}^l_{il}(P)^j_j(P)^k_k + T^{QP}D^i\mathcal{H}^l_{lk}(P)^j_i(P)^k_j + T^{QC}D^i\mathcal{H}^l_{lj}(P)^j_i(P)^k_k$$
$$+ T^{QA}D^i\mathcal{H}^l_{li}(P)^k_j(P)^j_k + T^{QS}D^i\mathcal{H}^l_{li}(P)^j_j(P)^k_k.$$

topological approach

$$\mathcal{H} = \mathcal{H}^k_{ij}(\bar{q}^i q_k)(\bar{q}^j c) \qquad 3 \otimes \bar{3} \otimes 3 = 3_p \oplus 3_t \oplus \bar{6} \oplus 15$$

$$\mathcal{H}^k_{ij} = \delta^k_j\left(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\right) + \delta^k_i\left(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\right) + \epsilon_{ijl}\mathcal{H}(\bar{6})^{lk} + \mathcal{H}(15)^k_{ij}$$

$$A = a^p_3D^i\mathcal{H}(3_p)_i(P)^j_k(P)^k_j + b^p_3D^i\mathcal{H}(3_p)_i(P)^k_k(P)^j_j + c^p_3D^i\mathcal{H}(3_p)_k(P)^k_i(P)^j_j + d^p_3D^i\mathcal{H}(3_p)_k(P)^j_i(P)^k_j$$
$$+ a^t_3D^i\mathcal{H}(3_t)_i(P)^j_k(P)^k_j + b^t_3D^i\mathcal{H}(3_t)_i(P)^k_k(P)^j_j + c^t_3D^i\mathcal{H}(3_t)_k(P)^k_i(P)^j_j + d^t_3D^i\mathcal{H}(3_t)_k(P)^j_i(P)^k_j$$
$$+ a_6D^i\mathcal{H}(\bar{6})^k_{ij}(P)^j_l(P)^l_k + b_6D^i\mathcal{H}(\bar{6})^k_{ij}(P)^j_k(P)^l_l + c_6D^i\mathcal{H}(\bar{6})^k_{jl}(P)^j_i(P)^l_k$$
$$+ a_{15}D^i\mathcal{H}(15)^k_{ij}(P)^j_l(P)^l_k + b_{15}D^i\mathcal{H}(15)^k_{ij}(P)^j_k(P)^l_l + c_{15}D^i\mathcal{H}(15)^k_{jl}(P)^j_i(P)^l_k.$$

SU(3) decomposition

Topological diagrams = Irreducible representations

Equivalence is obvious: H^k_{ij} is decomposed or not

$$T \times D^i \mathcal{H}^k_{lj} (P)^j_i (P)^l_k = T \times D^i (P)^j_i (P)^l_k \times$$
$$\Big[\delta^k_j \big(\frac{3}{8}\mathcal{H}(3_t)_l - \frac{1}{8}\mathcal{H}(3_p)_l\big) + \delta^k_l \big(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\big) + \varepsilon_{ljm}\mathcal{H}(\overline{6})^{mk} + \mathcal{H}(15)^k_{lj}\Big]$$

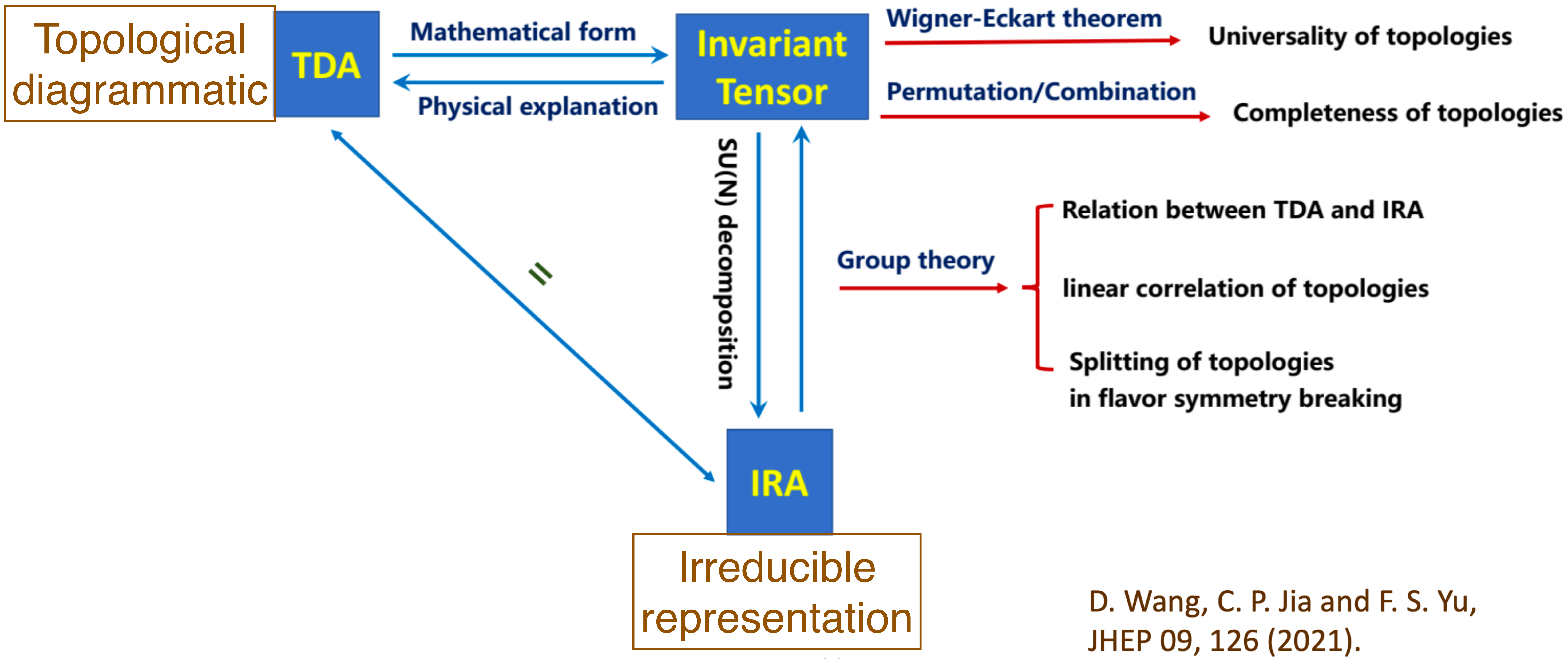
Relations of parameters

Linear combinations

$$a_6 = \frac{E-A}{4}, \quad b_6 = \frac{T^{ES}-T^{AS}}{4}, \quad c_6 = \frac{-T+C}{4},$$
$$a_{15} = \frac{E+A}{8}, \quad b_{15} = \frac{T^{ES}+T^{AS}}{8}, \quad c_{15} = \frac{T+C}{8},$$
$$a^t_3 = \frac{3}{8}E - \frac{1}{8}A + T^{LA}, \quad a^p_3 = -\frac{1}{8}E + \frac{3}{8}A + T^{QA},$$
$$b^t_3 = \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{LS}, \quad b^p_3 = -\frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{QS},$$
$$c^t_3 = -\frac{1}{8}T + \frac{3}{8}C - \frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{LC}, \quad c^p_3 = \frac{3}{8}T - \frac{1}{8}C + \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{QC},$$
$$d^t_3 = \frac{3}{8}T - \frac{1}{8}C - \frac{1}{8}E + \frac{3}{8}A + T^{LP}, \quad d^p_3 = -\frac{1}{8}T + \frac{3}{8}C + \frac{3}{8}E - \frac{1}{8}A + T^{QP}.$$

Topological diagrams = Irreducible representations

- The Equivalence was firstly pointed out by [X.G.He, W.Wang, 2018]
- The invariant tensors are the bridge between the two approaches.



D. Wang, C. P. Jia and F. S. Yu, JHEP 09, 126 (2021).

Topological diagrams = QCD + FSI

QCD = Short-distance contributions of topological diagrams

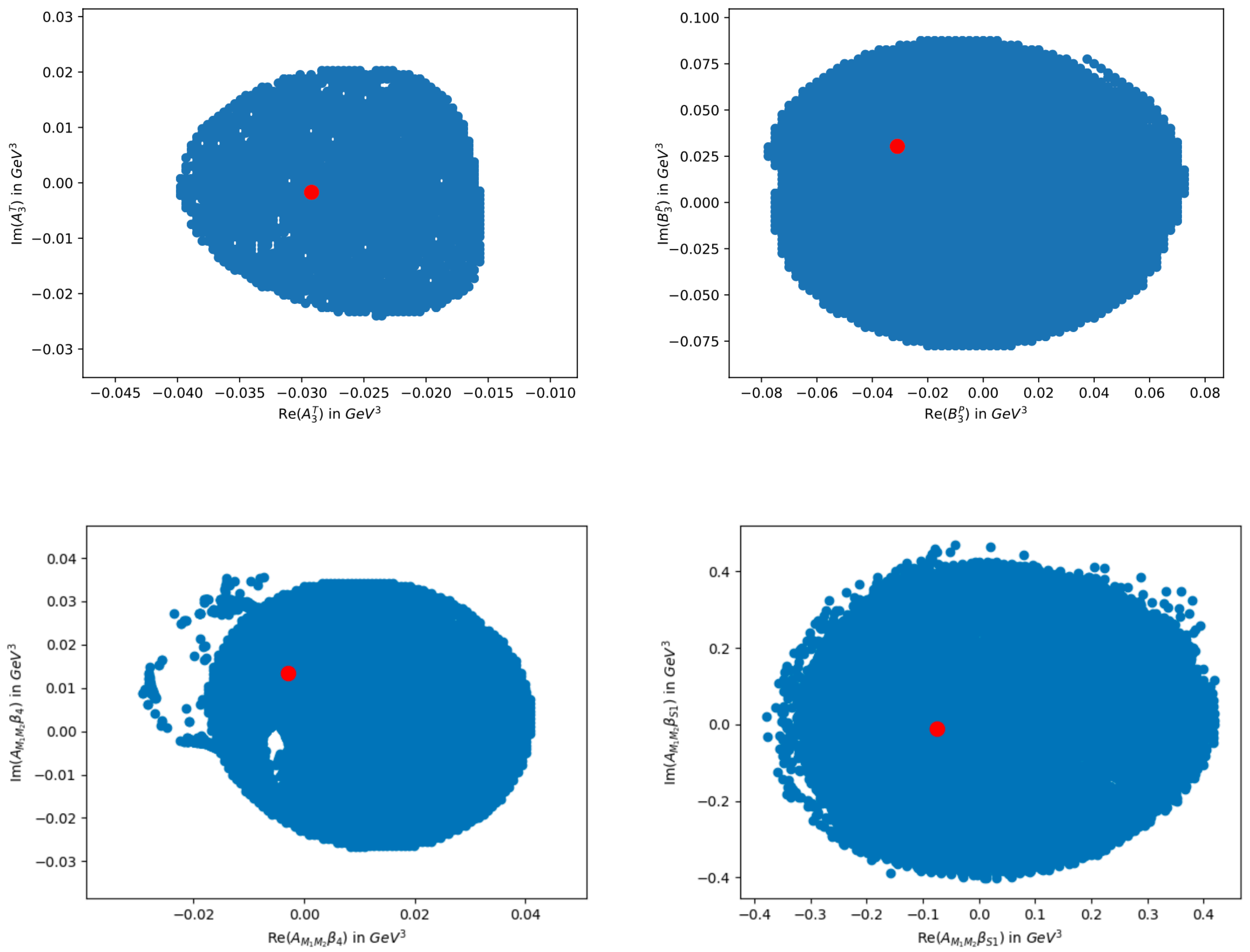
Topological diagrams = QCDF

$$T = A_{M_1M_2}\left[\alpha_1 + \frac{3}{2}\alpha^u_{4,EW} - \frac{3}{2}\alpha^c_{4,EW}\right], \qquad C = A_{M_1M_2}\left[\alpha_2 + \frac{3}{2}\alpha^u_{3,EW} - \frac{3}{2}\alpha^c_{3,EW}\right],$$

$$E = A_{M_1M_2}\left[\beta_1 + \frac{3}{2}b^u_{4,EW} - \frac{3}{2}b^c_{4,EW}\right], \qquad A = A_{M_1M_2}\left[\beta_2 + \frac{3}{2}\beta^u_{3,EW} - \frac{3}{2}\beta^c_{3,EW}\right],$$

$$T_{AS} = A_{M_1M_2}\left[\beta_{S1} + \frac{3}{2}b^u_{S4,EW} - \frac{3}{2}b^c_{S4,EW}\right], \quad T_{ES} = A_{M_1M_2}\left[\beta_{S2} + \frac{3}{2}\beta^u_{S3,EW} - \frac{3}{2}\beta^c_{S3,EW}\right]$$

$$A_{M_1M_2} = M_B^2\, F_0^{B\to M_1}(0)\, f_{M_2}$$

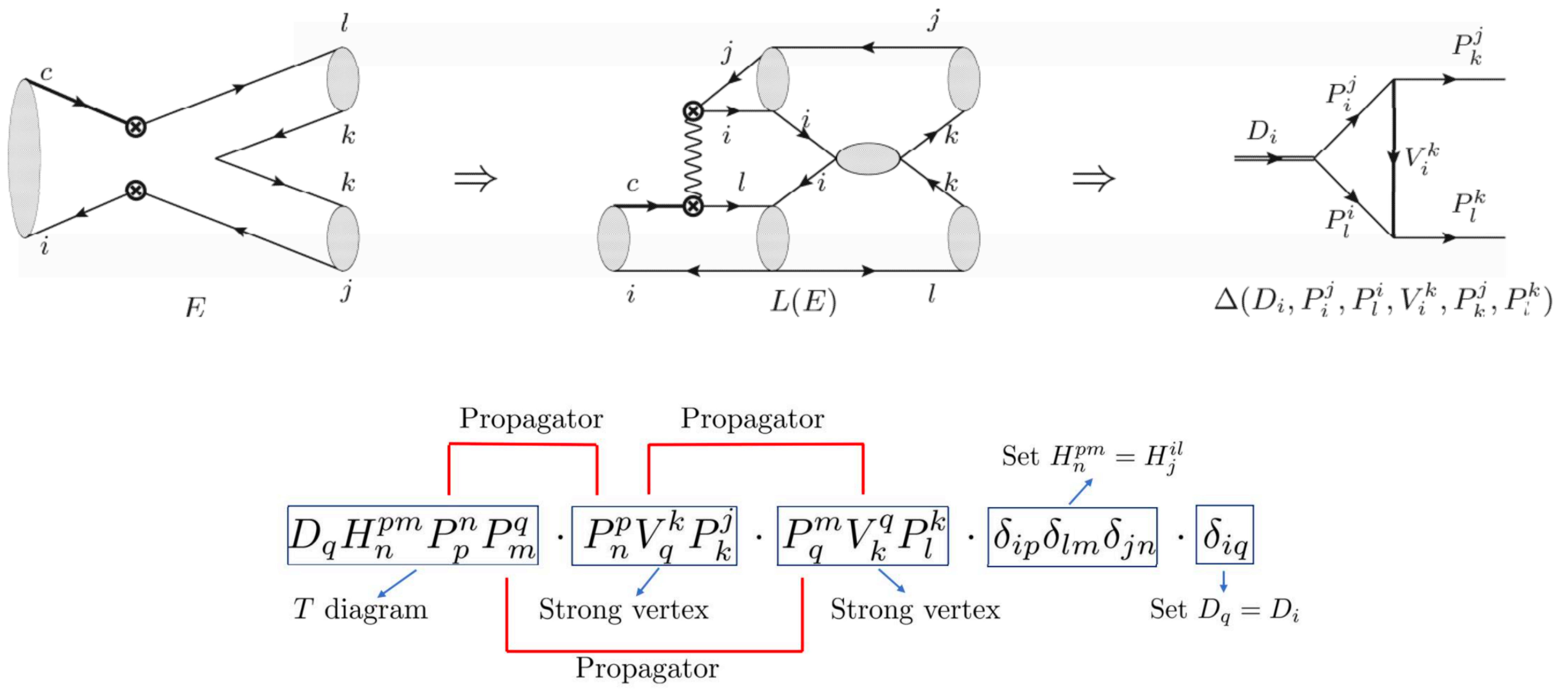


Doing a global fit

[T.Huber, Tetlalmatzi-Xolocotzi, 2021]

Topological diagrams = QCD + FSI

Final-State Interaction = Long-distance contributions of topological diagrams



➢ Topological diagram ⟹ Quark diagram ⟹ Triangle diagram

[D.Wang, 2021]

Summary

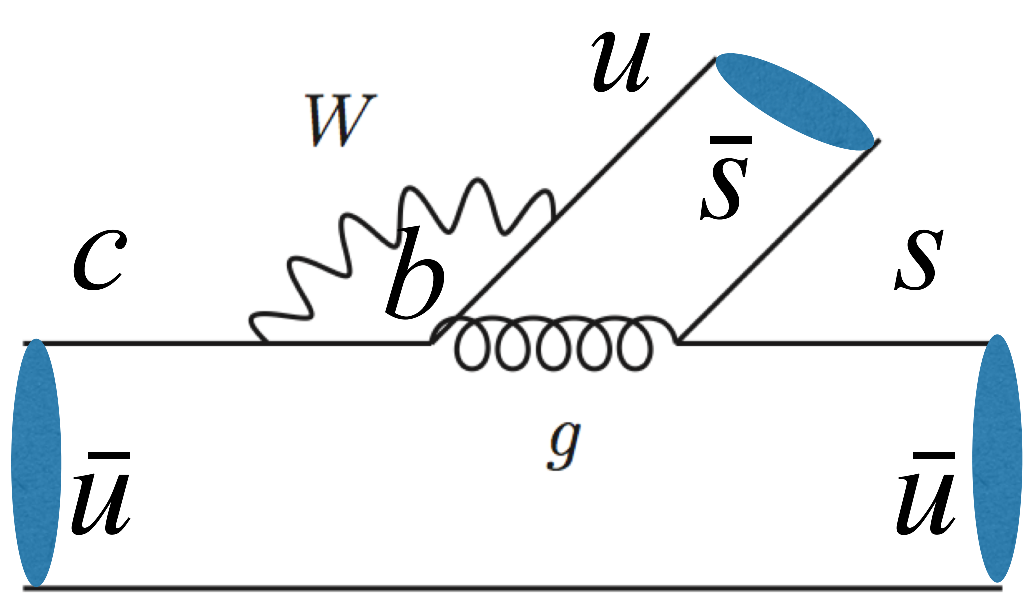
- The discovery of charm CPV is a milestone of particle physics
- To make it clear from SM or NP, it is required to know the dynamics of hadronic charm decays
- Topological diagrams approach is successful to predict the charm CPV
- Rescattering mechanism of final-state interactions is developed to predict CPV of charmed baryon decays.
- Topological approach = SU(3) irreducible representations = FSI + QCD

Thank you very much!

Backups

CPV in SCS decays: tree *v.s.* penguin

- **Ambiguity in penguins**
  - **heavy quark expansion $1/m_c$, m_c** =1.3GeV, **converges slowly** in exclusive decays



★ **$\Delta A_{CP}(K^+K^-, \pi^+\pi^-)$ predicted from 10^{-4} to 10^{-2}**

Grossman, Kagan, Nir, '07; Bigi, Paul, '11; Isidori, Kamenik, Ligeti, Perez, '11; Brod, Grossmann, Kagan, Zupan, '11, '12; Feldmann, Nandi, Soni, '12; Bhattarcharya, Gronau, Rosner, '12; Cheng, Chiang, '12; Li, Lu, **FSY**, '12; Franco, Mishima, Silvestrini, '12; Hiller, Jung, Schacht, '12. Khodjamirian, Petrov, 17.

$$\left(\frac{|\mathcal{P}^{KK}|}{|\mathcal{T}^{KK}|}\sin\delta^{KK} + \frac{|\mathcal{P}^{\pi\pi}|}{|\mathcal{T}^{\pi\pi}|}\sin\delta^{\pi\pi}\right) \approx 1 \quad \rightarrow \quad \boxed{\frac{|\mathcal{P}|}{|\mathcal{T}|}\sin\delta \sim 1/2}$$

topological approach

Li, Lu, **FSY**, '12; Cheng, Chiang, '12

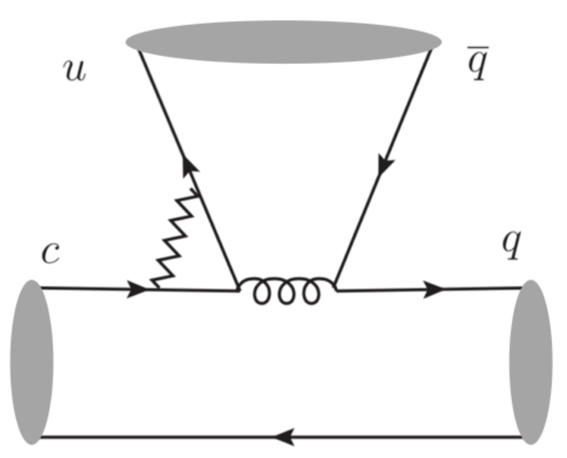
$$\frac{\mathcal{P}^{\pi\pi}}{\mathcal{T}^{\pi\pi}} = 0.66e^{i134^\circ}, \quad \text{and} \quad \frac{\mathcal{P}^{KK}}{\mathcal{T}^{KK}} = 0.45e^{i131^\circ}$$

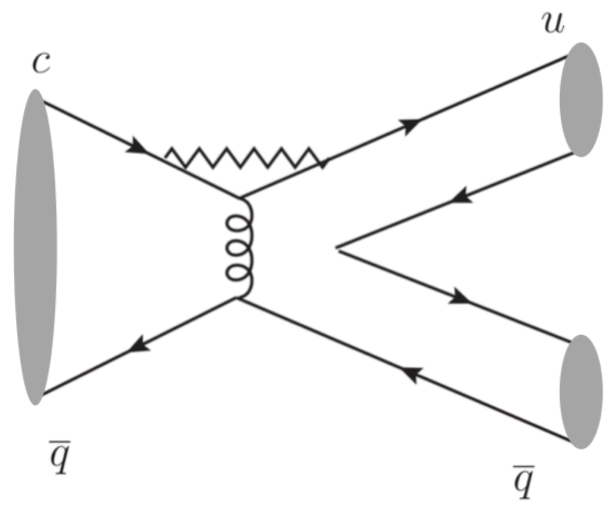
$\Delta U = 0$ over $\Delta U = 1$

Grossman, Schacht, '19

$$|\tilde{p}_0|\sin(\delta_{\text{strong}}) = 0.65 \pm 0.11$$

Key: **Long-distance non-perturbative**

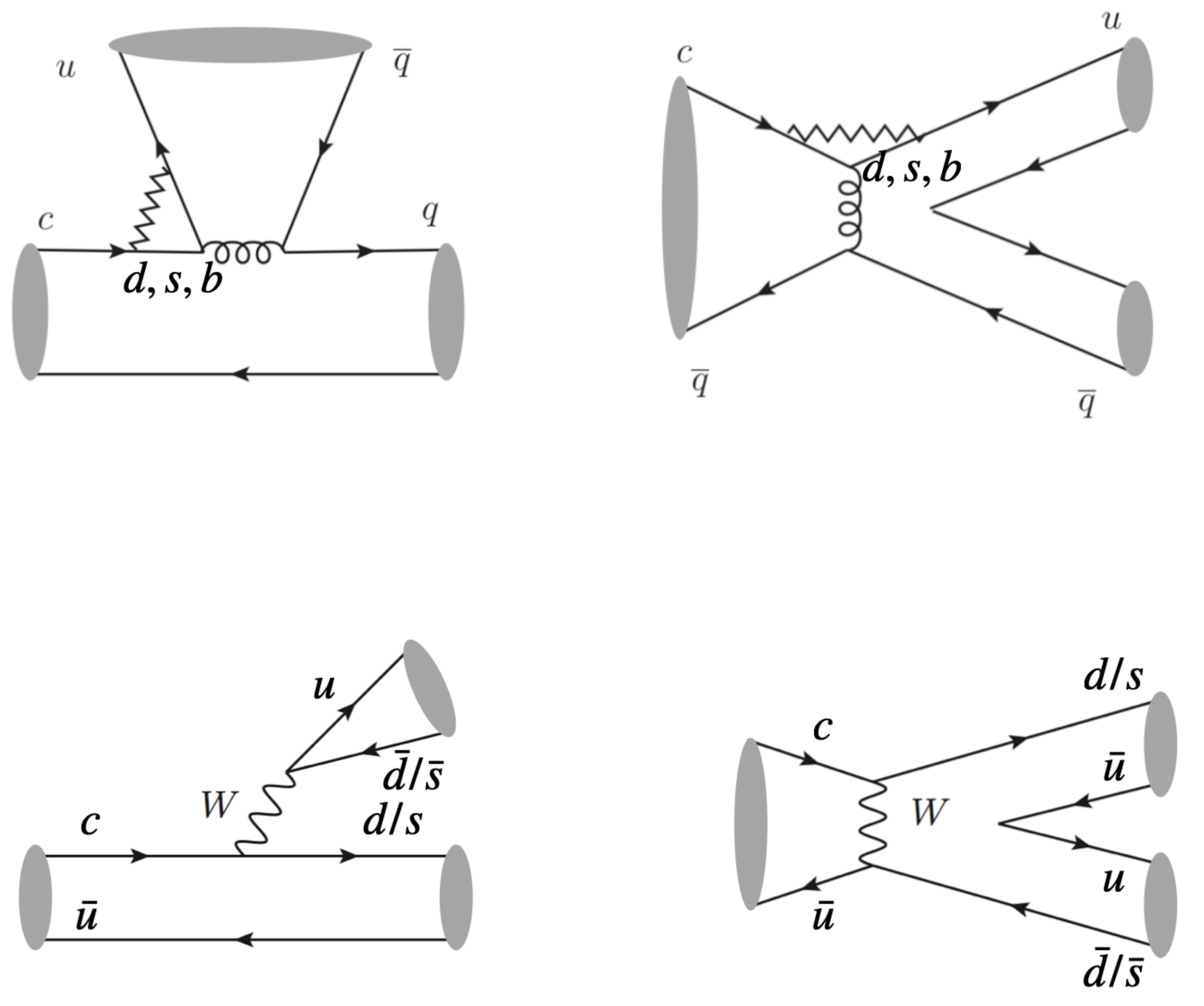




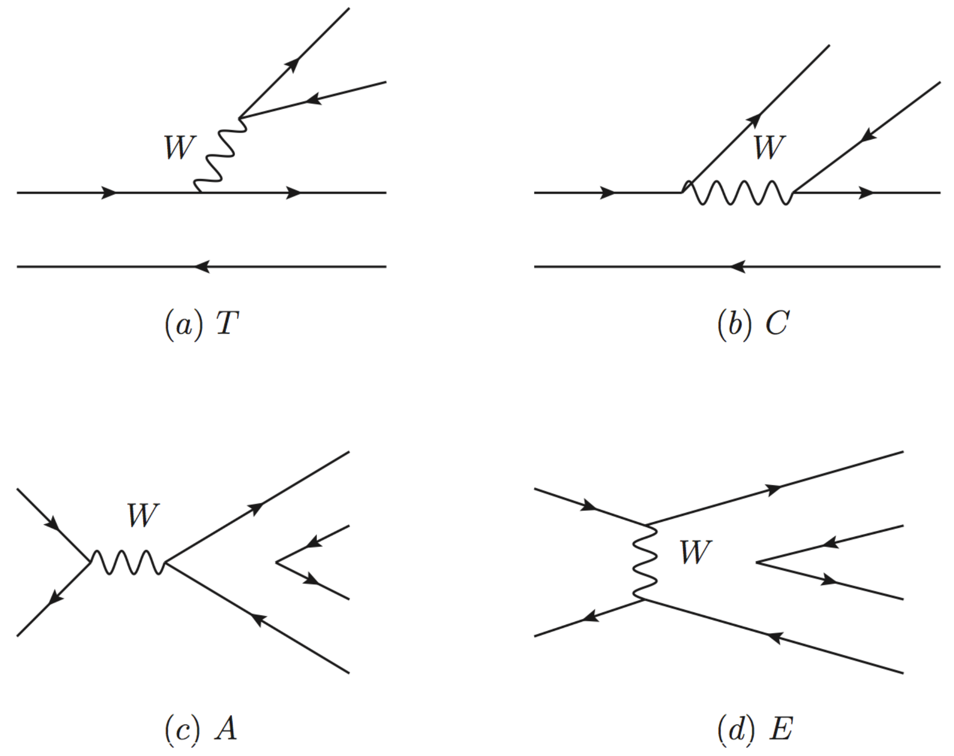
Understand: tree —> penguin; Branching ratio —> CPV

Dynamics of hadronic charm decays

- Before reliable predictions on penguin diagrams
- Firstly describe tree contributions
- Both tree and penguin are similar dynamics at 1 GeV
- Tree contributes to branching fractions, which have fruitful experimental data
- Without explanation of the data of branching fractions, no reliable prediction on the penguins and CPV



Topological Amplitudes



$$T = 3.14 \pm 0.06, \qquad C = (2.61 \pm 0.08)e^{-i(152\pm1)^\circ},$$

$$E = (1.53^{+0.07}_{-0.08})e^{i(122\pm2)^\circ}, \qquad A = (0.39^{+0.13}_{-0.09})e^{i(31^{+20}_{-33})^\circ}$$

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ (%)	$\mathcal{B}_{\rm fit}$ (%)
D^0	$K^-\pi^+$	$V^*_{cs}V_{ud}(T+E)$	3.91 ± 0.08	3.91 ± 0.17
	$\bar{K}^0\pi^0$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(C-E)$	2.38 ± 0.09	2.36 ± 0.08
	$\bar{K}^0\eta$	$V^*_{cs}V_{ud}[\frac{1}{\sqrt{2}}(C+E)\cos\phi - E\sin\phi]$	0.96 ± 0.06	0.98 ± 0.05
	$\bar{K}^0\eta'$	$V^*_{cs}V_{ud}[\frac{1}{\sqrt{2}}(C+E)\sin\phi + E\cos\phi]$	1.90 ± 0.11	1.91 ± 0.09
D^+	$\bar{K}^0\pi^+$	$V^*_{cs}V_{ud}(T+C)$	3.07 ± 0.10	3.08 ± 0.36
D_s^+	$\bar{K}^0K^+$	$V^*_{cs}V_{ud}(C+A)$	2.98 ± 0.17	2.97 ± 0.32
	$\pi^+\pi^0$	0	<0.037	0
	$\pi^+\eta$	$V^*_{cs}V_{ud}(\sqrt{2}A\cos\phi - T\sin\phi)$	1.84 ± 0.15	1.82 ± 0.32
	$\pi^+\eta'$	$V^*_{cs}V_{ud}(\sqrt{2}A\sin\phi + T\cos\phi)$	3.95 ± 0.34	3.82 ± 0.36

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ $(\times 10^{-3})$	$\mathcal{B}_{\rm theory}$ $(\times 10^{-3})$
D^0	$\pi^+\pi^-$	$V^*_{cd}V_{ud}(T'+E')$	1.45 ± 0.05	2.24 ± 0.10
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}V^*_{cd}V_{ud}(C'-E')$	0.81 ± 0.05	1.35 ± 0.05
	$\pi^0\eta$	$-V^*_{cd}V_{ud}E'\cos\phi - \frac{1}{\sqrt{2}}V^*_{cs}V_{us}C'\sin\phi$	0.68 ± 0.07	0.75 ± 0.02
	$\pi^0\eta'$	$-V^*_{cd}V_{ud}E'\sin\phi + \frac{1}{\sqrt{2}}V^*_{cs}V_{us}C'\cos\phi$	0.91 ± 0.13	0.74 ± 0.02
	$\eta\eta$	$-\frac{1}{\sqrt{2}}V^*_{cd}V_{ud}(C'+E')\cos^2\phi + V^*_{cs}V_{us}(2E'\sin^2\phi - \frac{1}{\sqrt{2}}C'\sin 2\phi)$	1.67 ± 0.18	1.44 ± 0.08
	$\eta\eta'$	$-\frac{1}{2}V^*_{cd}V_{ud}(C'+E')\sin 2\phi + V^*_{cs}V_{us}(E'\sin 2\phi - \frac{1}{\sqrt{2}}C'\cos 2\phi)$	1.05 ± 0.26	1.19 ± 0.07
	K^+K^-	$V^*_{cs}V_{us}(T'+E')$	4.07 ± 0.10	1.92 ± 0.08
	$K^0\bar{K}^0$	$V^*_{cd}V_{ud}E'_s + V^*_{cs}V_{us}E'_d$ [a]	0.64 ± 0.08	0

Cheng, Chiang,’10

Under flavor SU(3) symmetry

SU(3) breaking effects

should be considered

Flavor SU(3) breaking

- Flavor SU(3) symmetry breaking effects are important in the singly Cabibbo-suppressed modes

$$A(D^0 \to \pi^+\pi^-) = V_{cd}^* V_{ud}(T + E)$$

$$A(D^0 \to K^+K^-) = V_{cs}^* V_{us}(T + E)$$

$$A(D^0 \to K^0\overline{K}^0) = V_{cs}^* V_{us} E_d + V_{cd}^* V_{ud} E_s$$

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ $(\times 10^{-3})$	$\mathcal{B}_{\rm theory}$ $(\times 10^{-3})$
D^0	$\pi^+\pi^-$	$V_{cd}^* V_{ud}(T' + E')$	1.45 ± 0.05	2.24 ± 0.10
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}} V_{cd}^* V_{ud}(C' - E')$	0.81 ± 0.05	1.35 ± 0.05
	$\pi^0\eta$	$-V_{cd}^* V_{ud} E' \cos\phi - \frac{1}{\sqrt{2}} V_{cs}^* V_{us} C' \sin\phi$	0.68 ± 0.07	0.75 ± 0.02
	$\pi^0\eta'$	$-V_{cd}^* V_{ud} E' \sin\phi + \frac{1}{\sqrt{2}} V_{cs}^* V_{us} C' \cos\phi$	0.91 ± 0.13	0.74 ± 0.02
	$\eta\eta$	$-\frac{1}{\sqrt{2}} V_{cd}^* V_{ud}(C' + E')\cos^2\phi + V_{cs}^* V_{us}(2E'\sin^2\phi - \frac{1}{\sqrt{2}} C' \sin 2\phi)$	1.67 ± 0.18	1.44 ± 0.08
	$\eta\eta'$	$-\frac{1}{2} V_{cd}^* V_{ud}(C' + E') \sin 2\phi + V_{cs}^* V_{us}(E' \sin 2\phi - \frac{1}{\sqrt{2}} C' \cos 2\phi)$	1.05 ± 0.26	1.19 ± 0.07
	K^+K^-	$V_{cs}^* V_{us}(T' + E')$	4.07 ± 0.10	1.92 ± 0.08
	$K^0\bar{K}^0$	$V_{cd}^* V_{ud} E'_s + V_{cs}^* V_{us} E'_d$ [a]	0.64 ± 0.08	0

What's more?

- Topological diagrammatic approach is powerful at the charm scale: successfully predict the charm CPV and Xicc discovery channels. So far so good.
- Currently it is a phenomenological approach, but what is its mathematical foundation?
- Further studies: Deep understanding on the topological approach
    - What is the complete set of topological diagrams?
    - Are they all independent with each other?
    - Can the SU(3) breaking effects be systematically studied?

Topological diagrams = SU(3) irreducible representations

Everything is tensor

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V^*_{cq_1} V_{uq_2}\left(\sum_{q=1}^{2} C_i(\mu) O_i(\mu)\right) - V^*_{cb} V_{ub}\left(\sum_{i=3}^{6} C_i(\mu) O_i(\mu) + C_{8g}(\mu) O_{8g}(\mu)\right)\right]$$

$$\mathcal{H} = \mathcal{H}^k_{ij} (\bar{q}^i q_k)(\bar{q}^j c)$$

$q_{i,j,k} = u, d, s$

SU(3) symmetry

$$D^i = (D^0, D^+, D^+_s)$$

$$(P)^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{2/3}\eta_8 \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}$$

X.G.He, W.Wang, 2018; D.Wang, C.P.Jia, FSY, 2021

Implication: What next potential to observe charm CPV?

1. Charm CPV of order 10^{-3}	2. Precision of order 10^{-4}

1) Large branching fractions
2) Fully charged final particles @LHCb
3) Large production

$$Br(D^+ \to K^+K^-\pi^+) = 9.5 \times 10^{-3}$$

Compared to $Br(D^0 \to \pi^+\pi^-) = 1.4 \times 10^{-3}$

which dominates error of

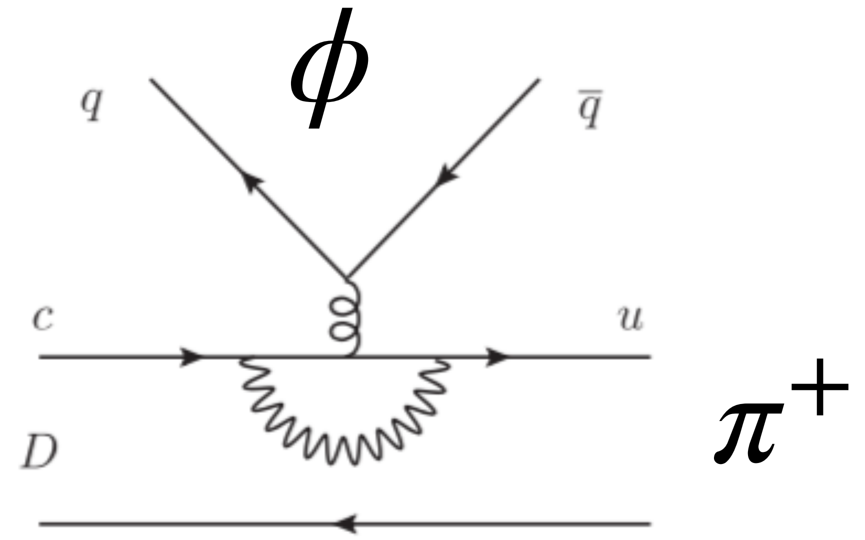
$$\Delta A_{CP} = (-1.54 \pm 0.29) \times 10^{-3}$$

Li, Lu, **FSY**, 1903.10638

What is the next potential mode to observe charm CPV?

$$Br(D^+ \to K^+K^-\pi^+) = 9.5 \times 10^{-3}$$

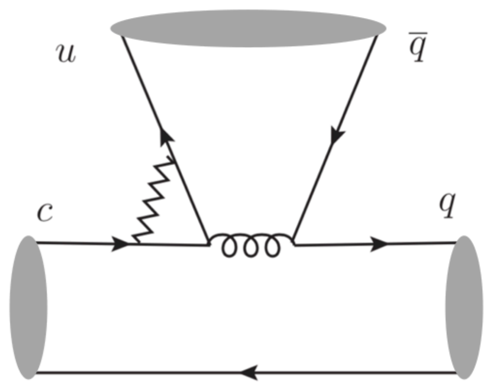
$$A_{CP}(D^+ \to \pi^+\phi) = 10^{-7}$$

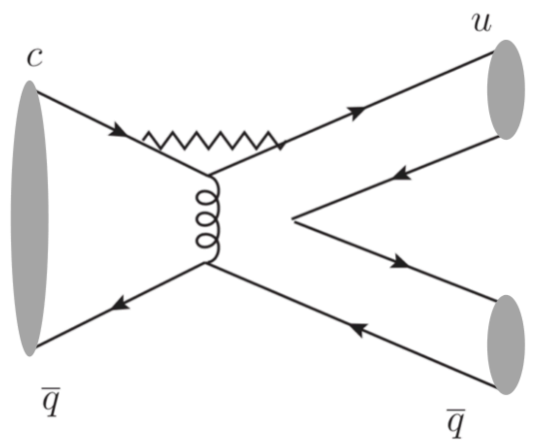


Qin, Li, Lu, **FSY**, ‘14

$$A_{CP}(D^+ \to K^+\overline{K}^{*0}) = 0.2 \times 10^{-3}$$

$$A_{CP}(D^+ \to K^+\overline{K}_0^{*0}(1430)) = -0.88 \times 10^{-3}$$

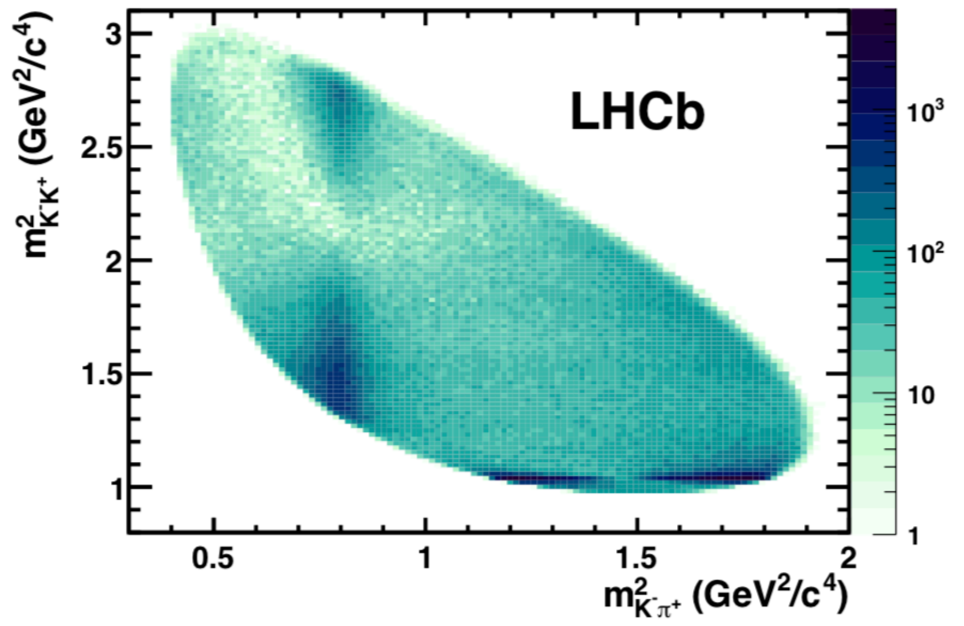




Li, Lu, **FSY**, 1903.10638

Searching Strategies

1. Binned $D^+ \to K^+K^-\pi^+$



	Branching Fractions	CP Violation
$D^+ \to \pi^+\phi$	2.6×10^{-3}	10^{-7} Benchmark
$D^+ \to K^+\overline{K}^{*0}$	2.4×10^{-3}	0.2×10^{-3}
$D^+ \to K^+\overline{K}_0^{*0}(1430)$	1.8×10^{-3}	-0.9×10^{-3}

Li, Lu, **FSY**, 1903.10638

What is the next potential mode to observe charm CPV?

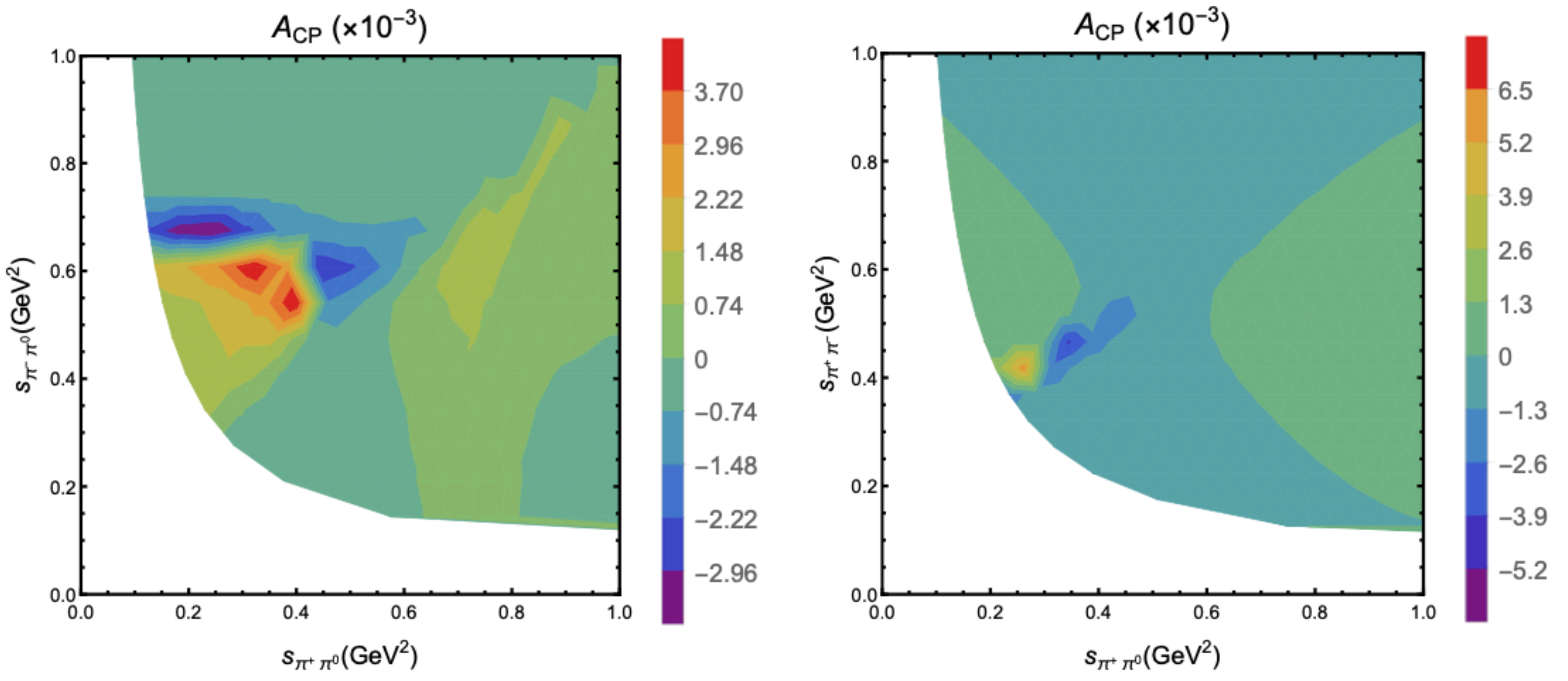
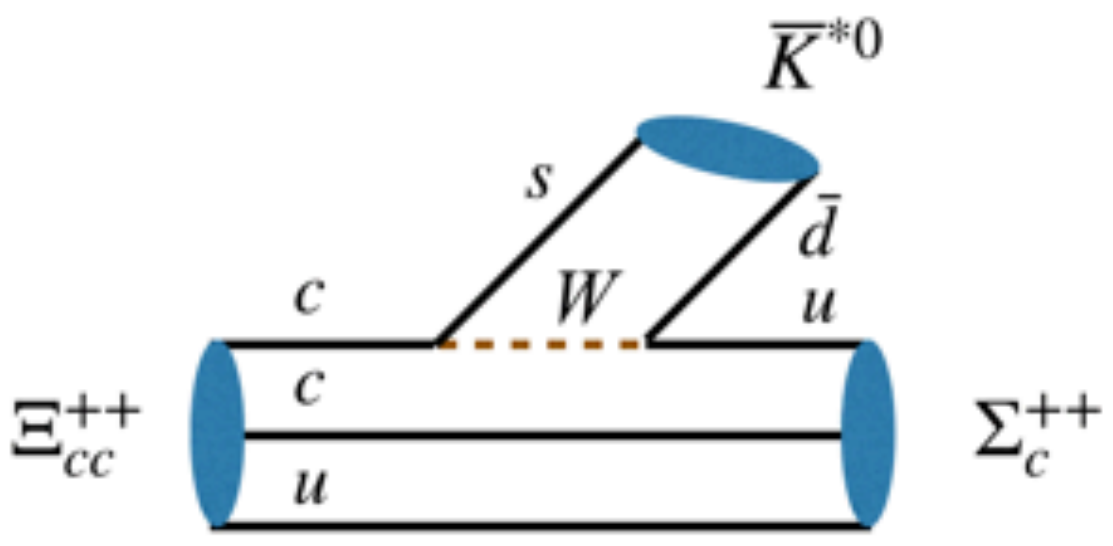


Figure 1: *CP* asymmetry distribution of $D^0 \to \pi^+\pi^-\pi^0$ in the overlapped region of $\rho(770)^\pm$ and $\rho(770)^0$ with $s_{\pi^+\pi^0}$ versus $s_{\pi^-\pi^0}$ ($s_{\pi^+\pi^-}$) in the left (right) panel.

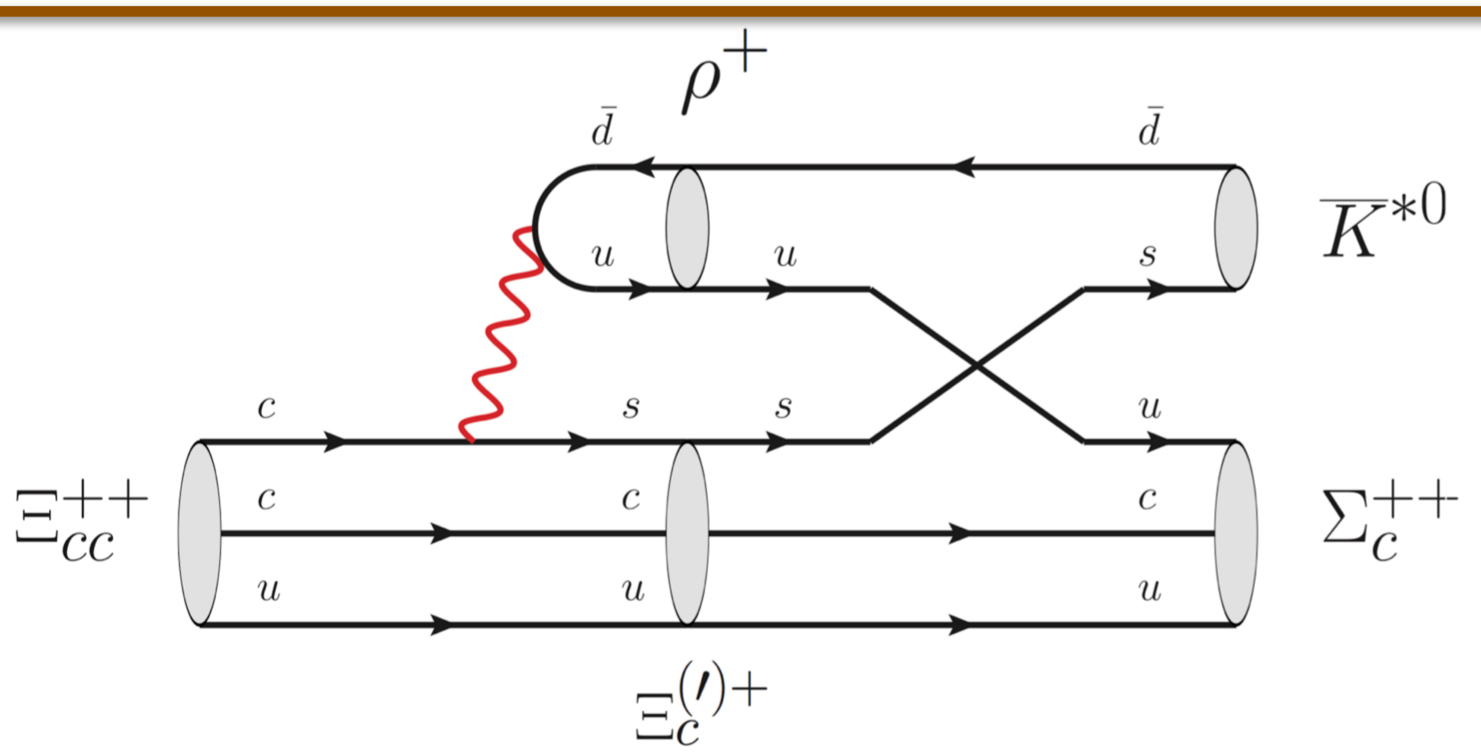
Cheng, Chiang, 2021

Decay amplitudes

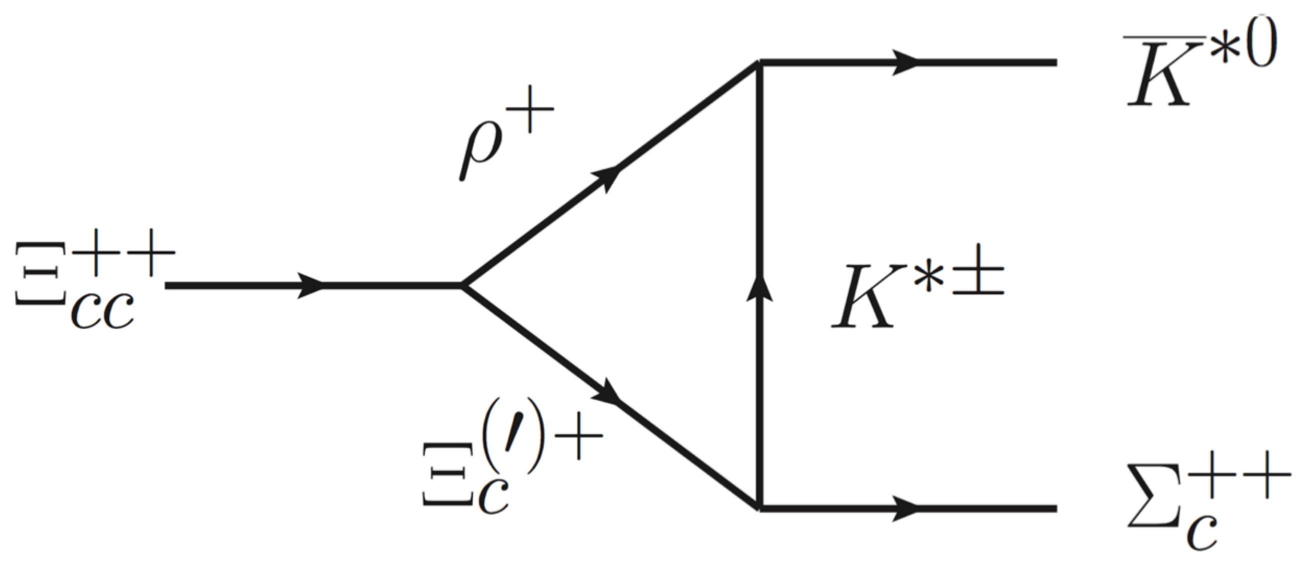
- Short-distance contributions: factorization
- Long-distance contributions: FSI rescattering



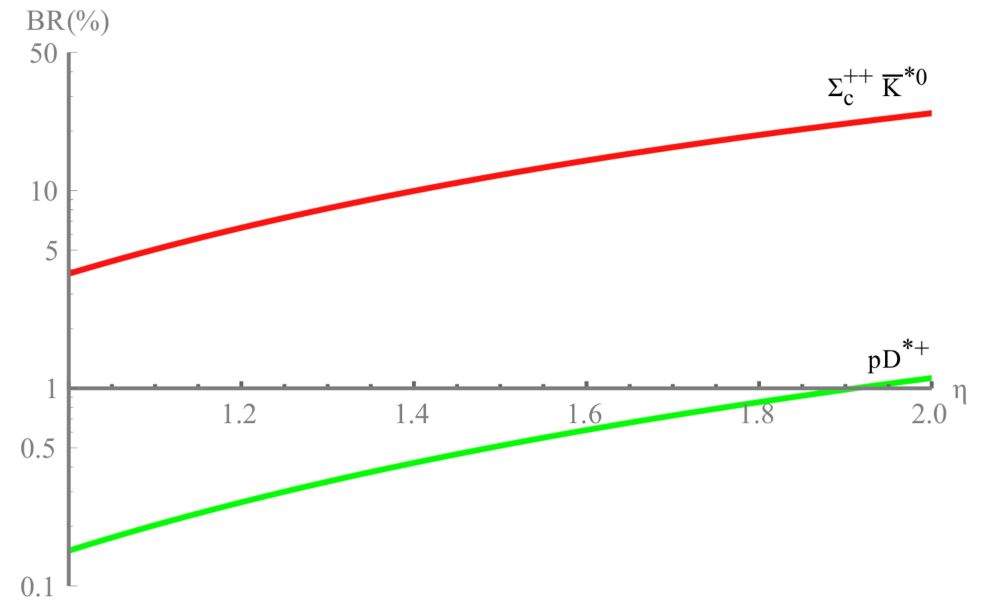
Topology: C



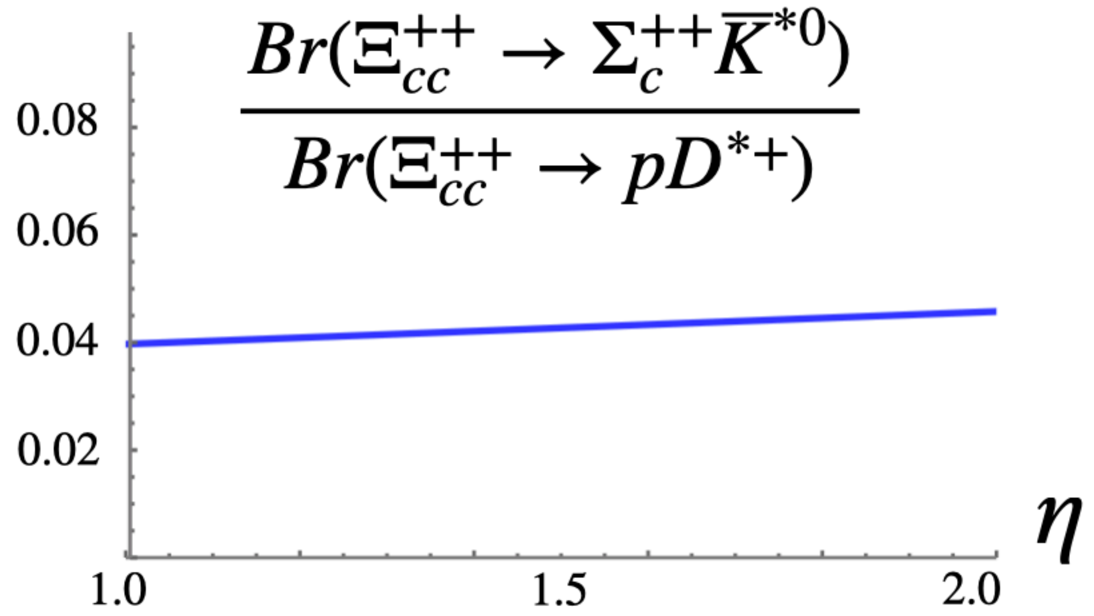
quark exchange



hadronic triangle



- Theoretical uncertainty is under control in the **ratio** of branching fractions of different processes



1703.09086
2102.00961