Lattice Results for Semileptonic Decays of Charmed Hadrons

William I. Jay — MIT

CHARM 2023: Universität Siegen
Hörsaalzentrum am Unteren Schloss
17-21 July 2023

Outline

- Connections to program at CHARM 2023
- Experimental & theoretical motivation
- Lattice QCD
- Semi-leptonic Decays of D-mesons
- Semi-leptonic Decays of D-baryons
- Summary

Outline

- Connections to program at CHARM 2023
- Experimental & theoretical motivation
- Lattice QCD
- Semi-leptonic Decays of D-mesons
- Semi-leptonic Decays of D-baryons
- Summary

Enormous lattice literature on D-hadrons weak decays.

Impossible to be entirely comprehensive.

Talk is unavoidably selective, focusing attention on published results from the past 5-6 years.

Apologies for any omissions

Adjacent talks at CHARM 2023

Plenary

- Sara Collins, T 11:00, “Meson and baryon spectroscopy with charm quarks from lattice QCD”
- Maxwell Hansen, F 14:00, “Future Theory”
- Keri Vos, Th 9:00, “Semi-leptonic decays of decays of charmed hadrons”
- Daniel Unverzagt, Th 9:45, “Rare leptonic and semileptonic decays at LHCb”
- Shulei Zhang, Th 11:45, “Overview of leptonic and semi-leptonic decays of charmed hadrons”

Parallel

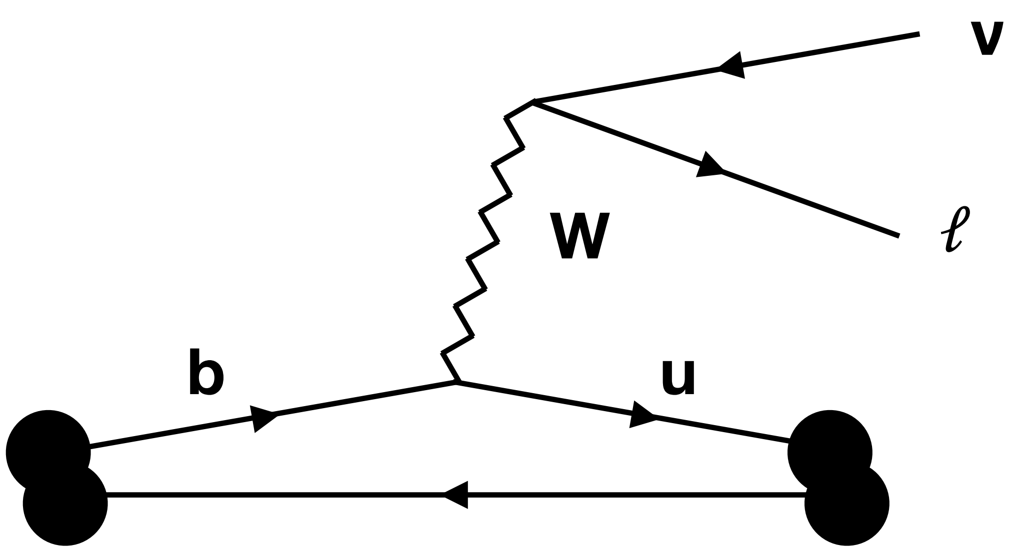
- Felix Erben, M 14:20, “D-meson mixing from lattice QCD”
- Juan Andreas Urrea Nino, T 14:00 “Toward the physical charmonium spectrum with improved distillation”
- Brian Colquhoun, T 14:20, “Precise determination of the decay rates of $\eta_c \to \gamma\gamma$, $J/\psi \to \gamma\eta_c$, $J/\psi \to \eta_c e^+ e^-$, from lattice QCD”
- Tomas Korzek, Th 14:40 “Iso-scalar states from LQCD”
- Roman Höllwieser, Th 15:20 “Charmonium and glueballs including light hadrons”

Context & Motivation

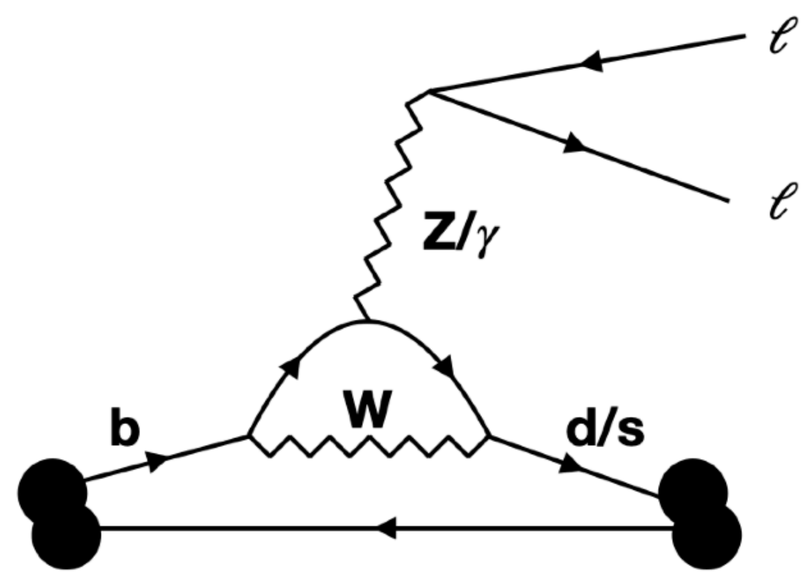
Quark Flavor and Lattice QCD

Two complementary roles

$$d\Gamma = \begin{pmatrix}\text{CKM}\\ \text{factor}\end{pmatrix}\begin{pmatrix}\text{kinematic}\\ \text{factor}\end{pmatrix}\begin{pmatrix}\text{QCD}\\ \text{factor}\end{pmatrix} + \begin{bmatrix}\text{BSM}\\ \text{term}\end{bmatrix}$$



Determine CKM matrix elements via tree-level decays



Test the CKM paradigm of the SM via rare decays

Quark Flavor and Lattice QCD

Tree level: CKM Matrix Elements

"Gold-plated processes" $\Longleftrightarrow$

Single-hadron initial state.

Zero- or one-hadron final state.

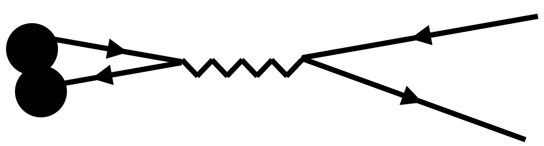
All hadrons stable under QCD.

Quark Flavor and Lattice QCD

Tree level: CKM Matrix Elements

V_{ud}	V_{us}	V_{ub}
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \ell\nu$
	$K \to \pi\ell\nu$	$B \to \pi\ell\nu$
		$\Lambda_b \to p\ell\nu$
V_{cd}	V_{cs}	V_{cb}
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^\star\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$	$\Lambda_b \to \Lambda_c\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$	
V_{td}	V_{ts}	V_{tb}
$\langle B_d \vert \bar{B}_d \rangle$	$\langle B_s \vert \bar{B}_s \rangle$	

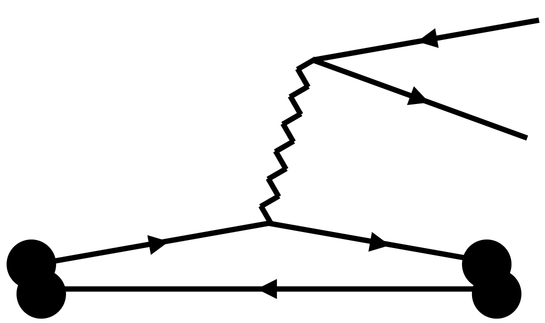
Leptonic decays



(Decay constants)

$$\langle 0 | A^\mu | H(P) \rangle = i f_H \, p^\mu$$

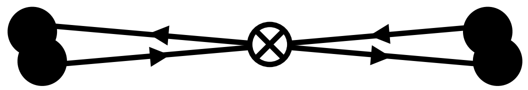
Semi-leptonic decays



(Form factors)

$$f_J(p) \propto \langle \text{final} | J(p) | \text{initial} \rangle$$

Neutral-meson mixing



(Matrix elements)

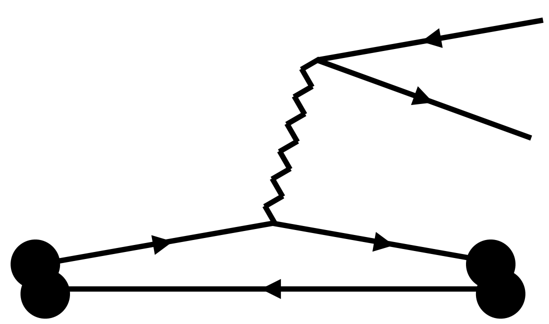
$$\langle \bar{B}^0 | \mathcal{H}_{\text{eff}} | B^0 \rangle$$

Quark Flavor and Lattice QCD

Tree level: CKM Matrix Elements

$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$
$D \to \ell\nu$	$D_s \to \ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$

Semi-leptonic decays



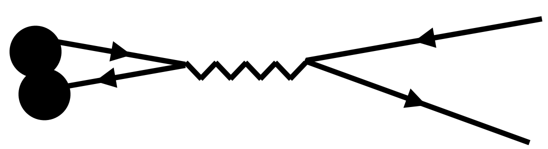
(Form factors)

$$f_J(p) \propto \langle \text{final} |\, J(p) \,| \text{initial} \rangle$$

Quark Flavor and Lattice QCD

Loop level: Flavor-Changing Neutral Currents

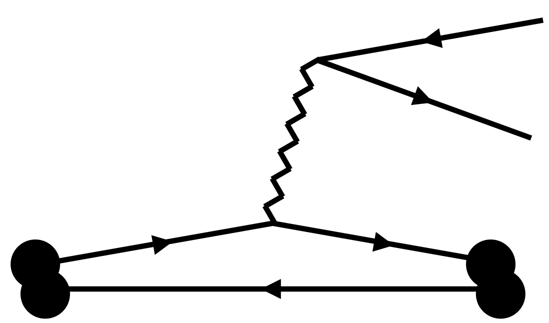
Leptonic decays



(Decay constants)

$$\langle 0|\, A^{\mu}\, |H(P)\rangle = i f_H\, p^{\mu}$$

Semi-leptonic decays



(Form factors)

$$f_J(p) \propto \langle \text{final}|\, J(p)\, |\text{initial}\rangle$$

$B_s \to \ell^+\ell^-$

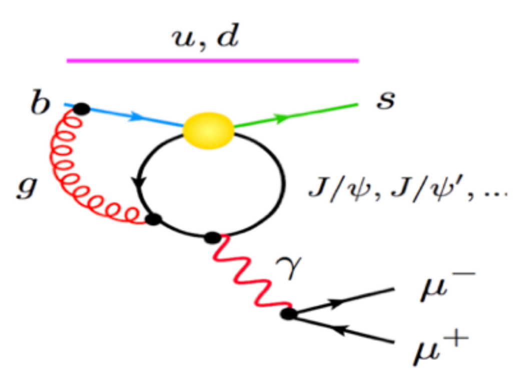
$B \to K\ell\nu$

$B \to K^{\star}\ell\nu$ — $b \to s\ell\ell$

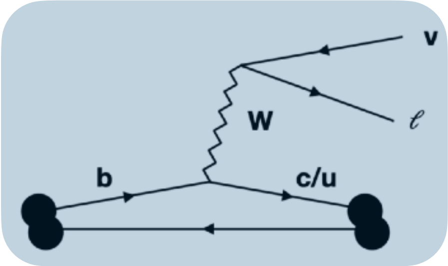
$\Lambda_b \to \Lambda\ell\nu$

$\Lambda_c \to p\mu^+\mu^-$ — $c \to u\ell\ell$

Hard-to-compute (=presently incalculable) long-distance charm loops render rare charm decays very difficult theoretically



Experimental Motivation: B-anomalies



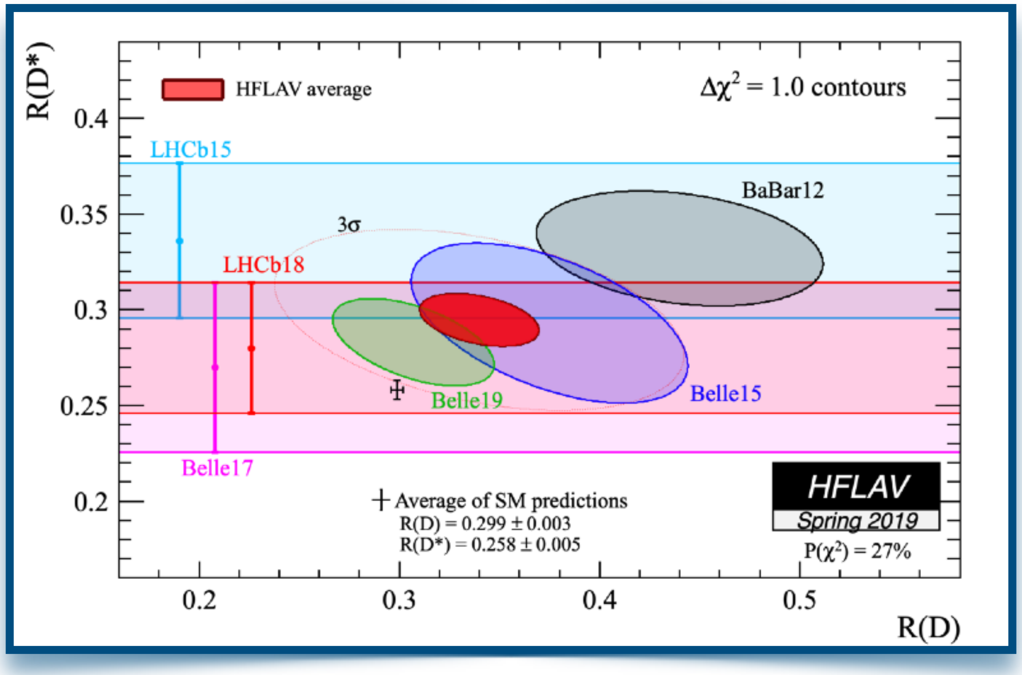
- **Tree level**: Lepton Flavor Universality: $R(D)$, $R(D^\star)$

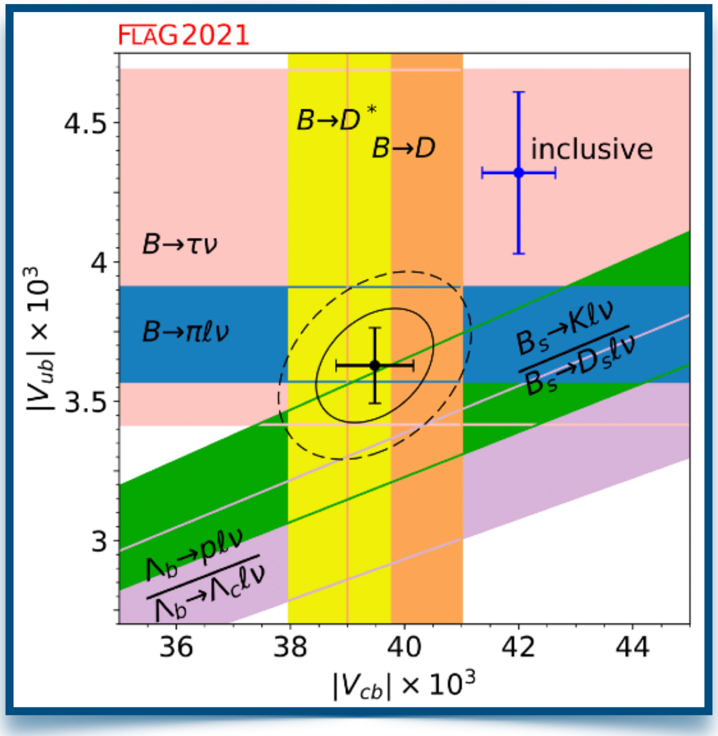
$$R(D) = \frac{\mathscr{B}(B \to D\tau\bar{\nu})}{\mathscr{B}(B \to D\mu\bar{\nu})}$$

- **Tree level**: Exclusive (LQCD) vs Inclusive (OPE+HQE) determinations of CKM matrix elements
  - $|V_{cb}|$ from $B \to D^\star \ell \nu$, $B \to D \ell \nu$
  - $|V_{ub}|$ from $B \to \pi \ell \nu$

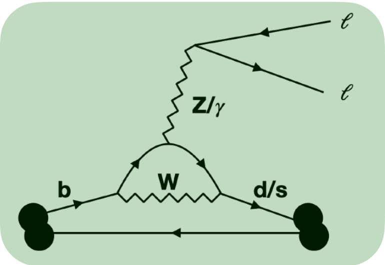
- **Loop level**: $b \to s\ell\ell$ FCNC branching fractions:
  - $B^0 \to K^{\star 0}\mu\mu$, $B_s^0 \to \varphi\mu\mu$, $\Lambda_b^0 \to \Lambda^0\mu\mu$, $B^+ \to K^+\mu\mu$, $B^0 \to K^0\mu\mu$, $B^+ \to K^{\star +}\mu\mu$

- **Loop level**: $b \to s\ell\ell$ FCNC angular observables
  - $B^0 \to K^{\star 0}\mu\mu$, $B^+ \to K^{\star +}\mu\mu$, $B_s^0 \to \varphi\mu\mu$





Experimental Motivation: B-anomalies



- **Tree level**: Lepton Flavor Universality: $R(D)$, $R(D^\star)$

$$R(D) = \frac{\mathscr{B}(B \to D\tau\bar{\nu})}{\mathscr{B}(B \to D\mu\bar{\nu})}$$

- **Tree level**: Exclusive (LQCD) vs Inclusive (OPE+HQE) determinations of CKM matrix elements
  - $|V_{cb}|$ from $B \to D^\star \ell\nu$, $B \to D\ell\nu$
  - $|V_{ub}|$ from $B \to \pi\ell\nu$
- **Loop level**: $b \to s\ell\ell$ FCNC branching fractions:
  - $B^0 \to K^{\star 0}\mu\mu$, $B_s^0 \to \varphi\mu\mu$, $\Lambda_b^0 \to \Lambda^0\mu\mu$, $B^+ \to K^+\mu\mu$, $B^0 \to K^0\mu\mu$, $B^+ \to K^{\star +}\mu\mu$
- **Loop level**: $b \to s\ell\ell$ FCNC angular observables
  - $B^0 \to K^{\star 0}\mu\mu$, $B^+ \to K^{\star +}\mu\mu$, $B_s^0 \to \varphi\mu\mu$

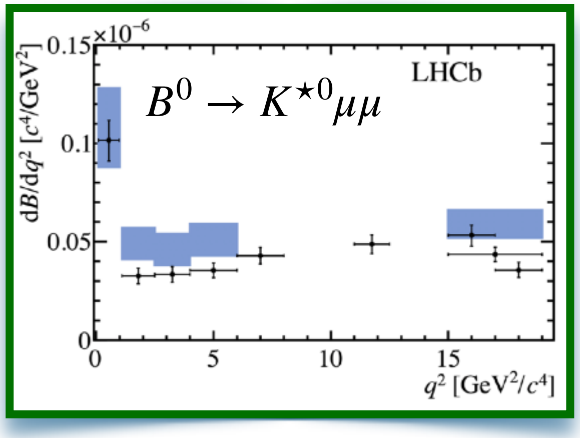
$B^0 \to K^{\star 0}\mu\mu$

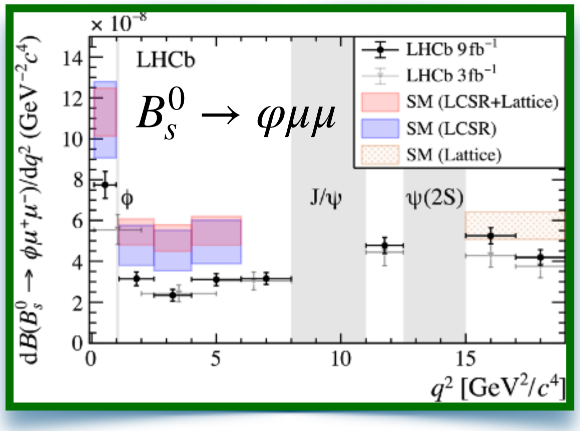
LHCb *JHEP* 11 (2016) 047
LHCb *JHEP* 04 (2017) 142
LHCb PRL 125 (2020) 011802

$B_s^0 \to \varphi\mu\mu$

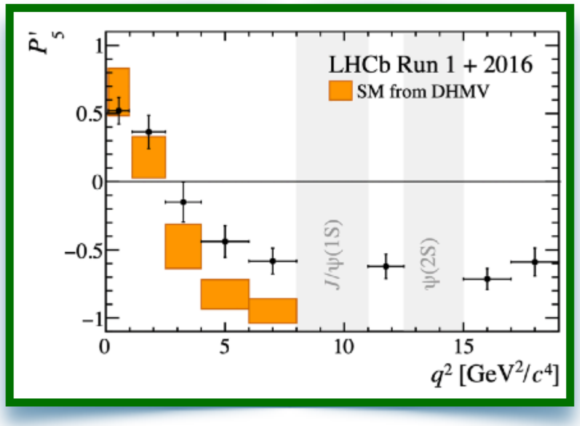
LHCb *JHEP 09 (2015) 179*
LHCb *PRL* 127 (2021) 15, 151801
LHCb JHEP 11 (2021) 043

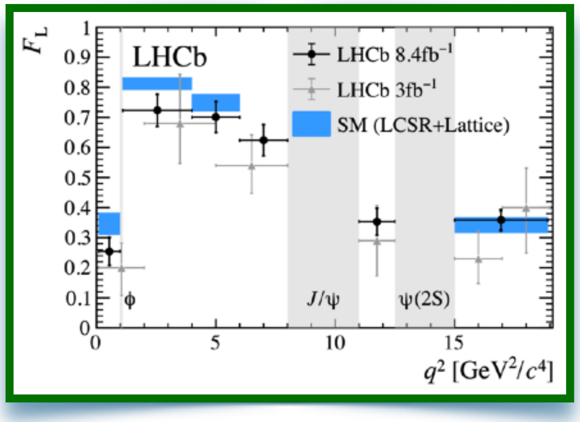
Branching fractions





Angular distributions





Experimental Motivation: CKM Unitarity

First-row unitarity?

- PDG 2022: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)^{|V_{ud}|^2}(4)^{|V_{us}|^2}$
- Quoted value has 2σ tension with unity, using as inputs
    - $|V_{ud}|$ from super-allowed $0^+ \to 0^+\ \beta$ decays
    - $|V_{us}|$ from semileptonic decay: $K_{\ell 3} \equiv K \to \pi \ell \nu$
    - Tension increases to ≈3σ if nuclear-structure uncertainties from $|V_{ud}|$ are ignored
- Similar ≈2-3σ tension if $|V_{us}|/|V_{ud}|$ taken from ratio of leptonic decays K_{l2}/π_{l2}
- Historically, similarly precise tests of second-row unitarity have been limited by experimental and theoretical precisions.
- Today's talk: recent progress in the second row via semileptonic decays

E. Blucher and W.J. Marciano
PDG 2022: 67. V_{ud}, V_{us}, the Cabibbo Angle, and CKM, Unitarity

Lattice QCD with Heavy Quarks

Lattice QCD

- Lattice QCD gives complete non-perturbative definition to the strong interactions
- This framework gives: $$\mathcal{Z} = \int \mathcal{D}[\text{fields}]e^{-S_E[\text{fields}]}$$
- **Fundamental approximations:**
  - UV cutoff: lattice spacing *a* [target: a ≪ physical scales]
  - IR cutoff: finite spacetime volume $V = L^3 \times T$ [target: $1 \ll m_\pi L$]

- **Approximations of convenience:**
  - Often: Heavier-than-physical pions: $(m_\pi)^{\text{lattice}} > (m_\pi)^{\text{PDG}}$
  - Often: Isospin limit $m_u = m_d$
  - Often: QCD interactions only, no QED
  - Often: lighter-than-physical or static heavy quarks

Lattice QCD is systematically improvable

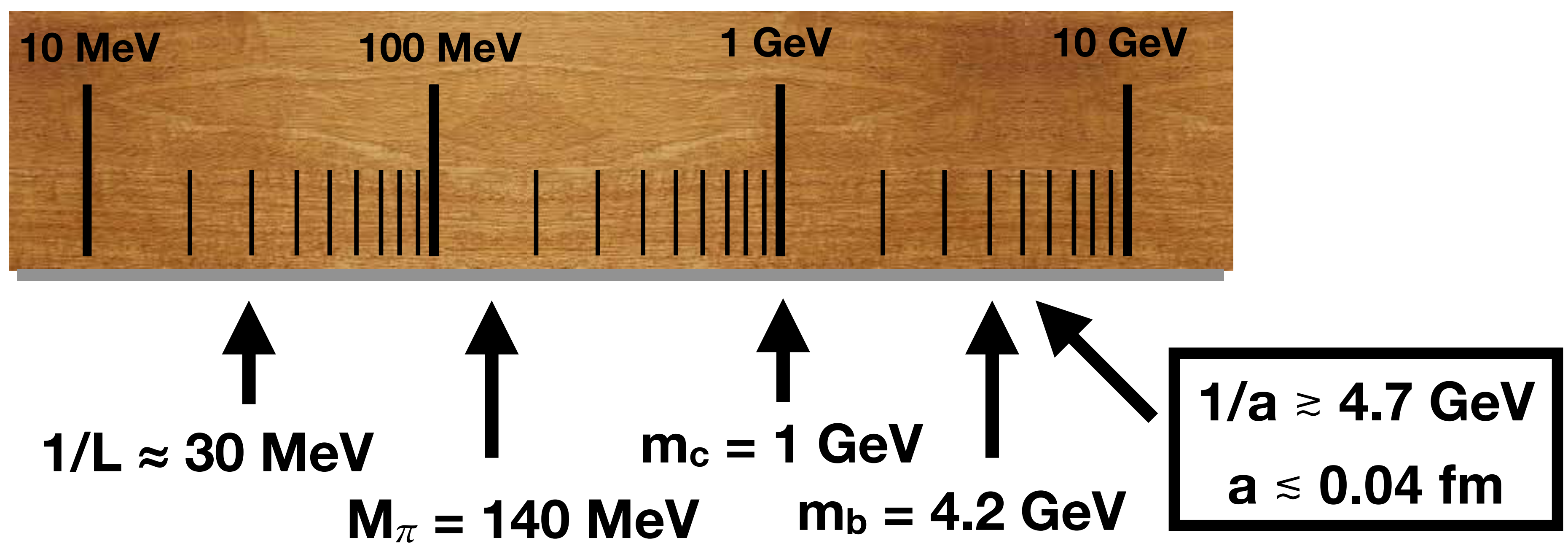
- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence $\Longleftrightarrow$ Symanzik effective theory
  - Finite-volume dependence $\Longleftrightarrow$ Finite-volume χPT
  - Chiral extrapolation / interpolation $\Longleftrightarrow$ χPT
  - Heavy quark extrapolation / interpolation $\Longleftrightarrow$ HQET, NRQCD, etc…
  - QED, isospin breaking $\Longleftrightarrow$ perturbative expansion of path integral
- Careful treatment of all systematic effects is key to modern high-precision lattice QCD
- Technical advances in controlling these systematics have been drivers of progress in lattice QCD, especially in charm physics

Lattice QCD is systematically improvable

- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence $\Longleftrightarrow$ Symanzik effective theory
  - Finite-volume dependence $\Longleftrightarrow$ Finite-volume χPT
  - Chiral extrapolation / interpolation $\Longleftrightarrow$ χPT
  - Heavy quark extrapolation / interpolation $\Longleftrightarrow$ HQET, NRQCD, etc…
  - QED, isospin breaking $\Longleftrightarrow$ perturbative expansion of path integral
- Careful treatment of all systematic effects is key to modern high-precision lattice QCD
- Technical advances in controlling these systematics have been drivers of progress in lattice QCD, especially in charm physics

Lattice QCD with Heavy Quarks

A challenging multi-scale problem



Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

$$\frac{1}{L} \ll M_\pi \ll m_h \ll \frac{1}{a}$$

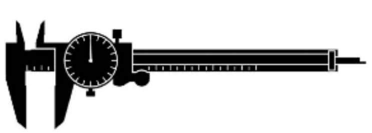
Lattice QCD with Heavy Quarks

A challenging multi-scale problem

Solutions to the cutoff challenge?

1. Use an "effective theory" for heavy quarks (b, sometimes c)
   - "FNAL interpretation," NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching and/or parameter tuning, which introduces systematic effects
   - (1-3)% total errors

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass
   - (< 1)% total errors possible



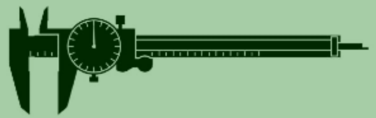
Lattice QCD with Heavy Quarks

A challenging multi-scale problem

Solutions to the cutoff challenge?

1. Use an effective theory for heavy quarks (b, sometimes c)
   - "FNAL interpretation," NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching, which introduces systematic effects
   - (1-3)% total errors

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass
   - (< 1)% total errors possible



Lattice QCD: particle masses

- 2-point correlation functions encode particle masses
- Analogy with condensed matter
    - Correlation length $\lambda \longleftrightarrow$ Particle mass 1/m

$$\langle (\bar{q}\, q)_t (\bar{q}\, q)_0 \rangle \sim \exp(-mt)$$

Lattice QCD: particle masses

- Hadronic spectrum $\longleftrightarrow$ QCD 2pt correlation functions

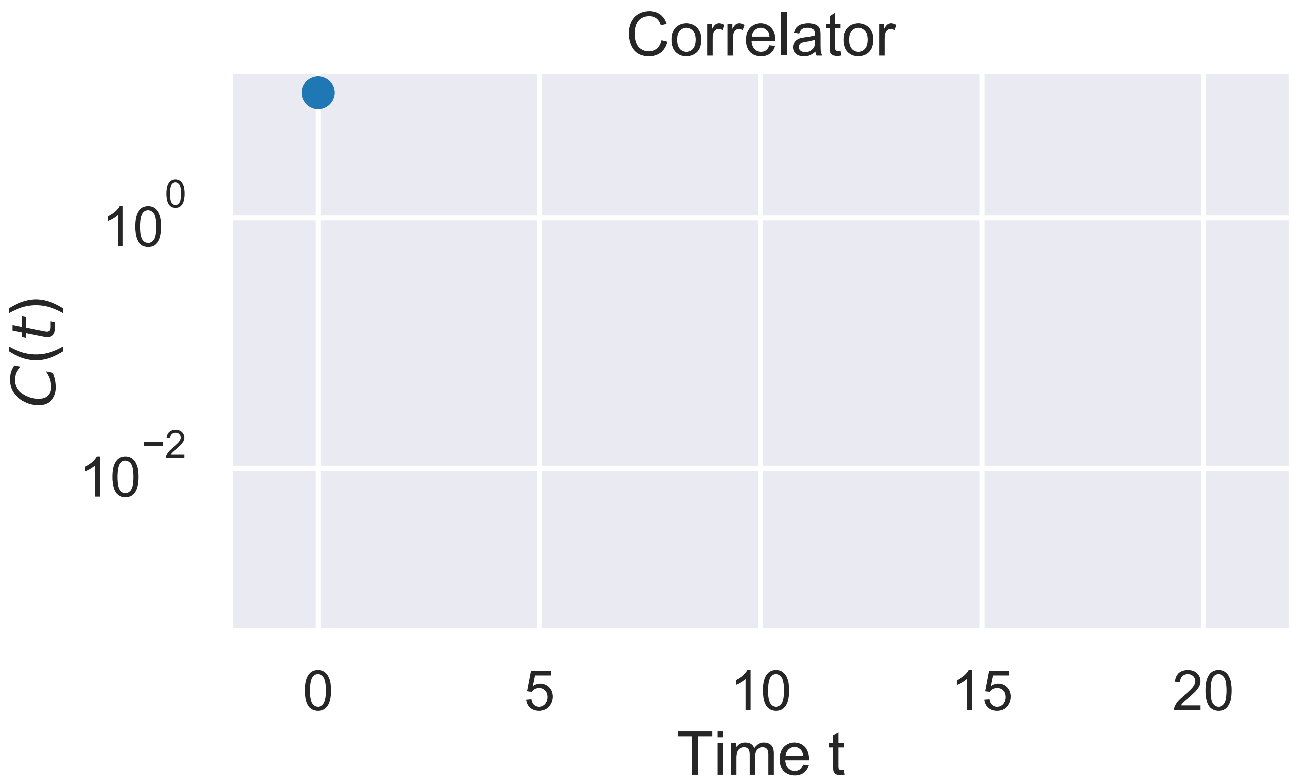
$$
\begin{aligned}
\langle O(t) O(0) \rangle &= \langle 0 | \, e^{Ht} O(0) e^{-Ht} O(0) \, | 0 \rangle \\
&= \sum_n e^{-E_n t} \langle 0 | \, O(0) \, | n \rangle \langle n | \, O(0) \, | 0 \rangle \\
&= \sum_n e^{-E_n t} \left| \langle 0 | \, O(0) \, | n \rangle \right|^2 \\
&= \sum_n |Z_n|^2 e^{-E_n t}
\end{aligned}
$$

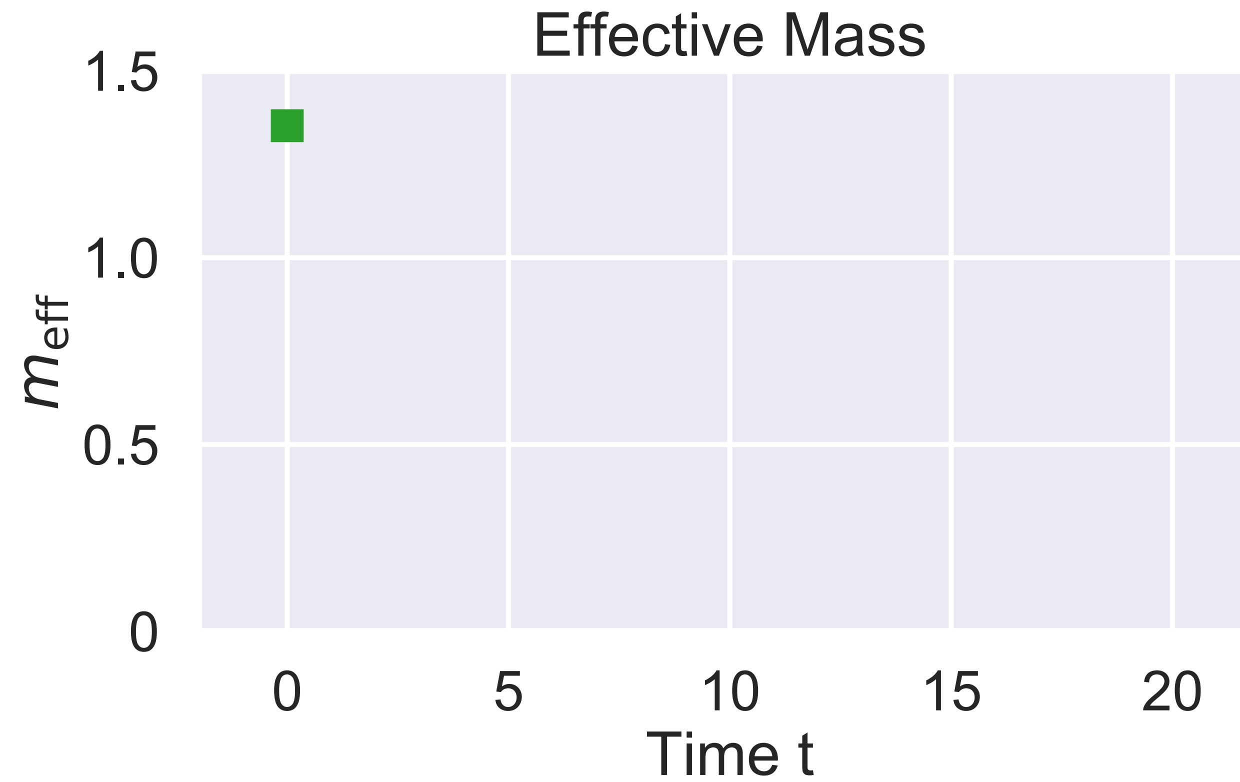
"Operators couple to an infinite tower of states."

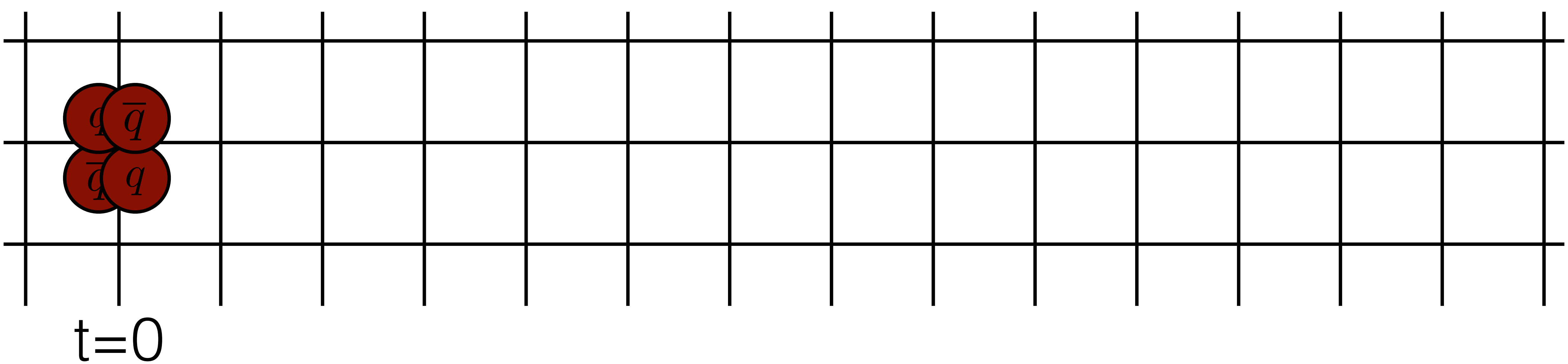
$$
m_{\text{eff}}(t) = \log C(t)/C(t+1) \overset{t \to \infty}{=} m_0
$$

"The ground state asymptotically dominates the Euclidean 2pt function."

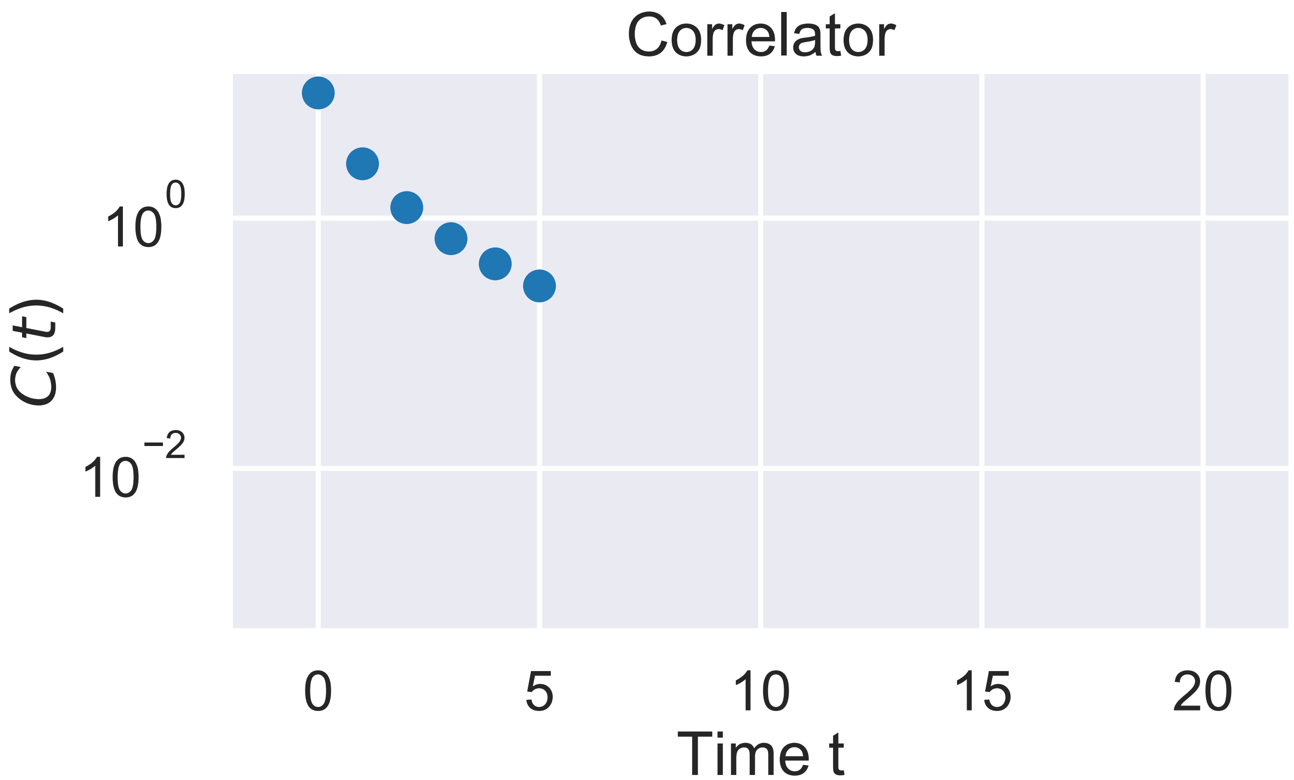
Lattice QCD: particle masses

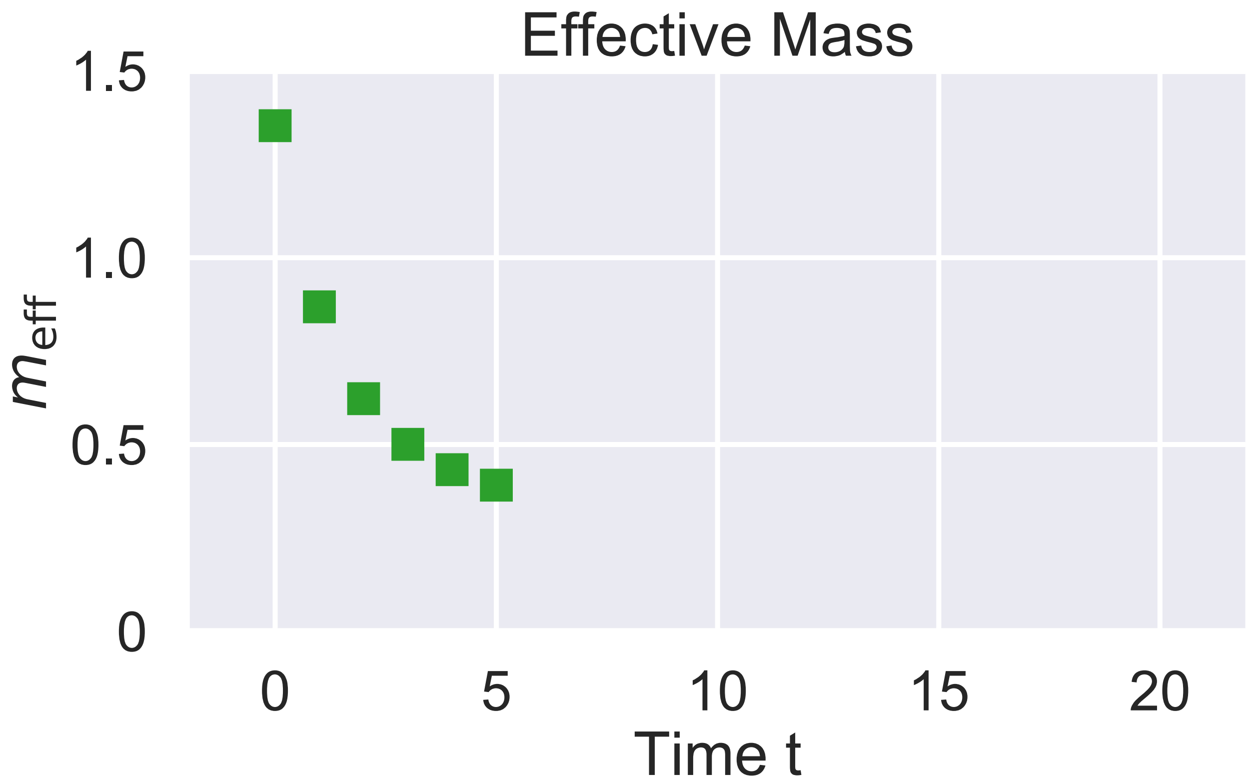


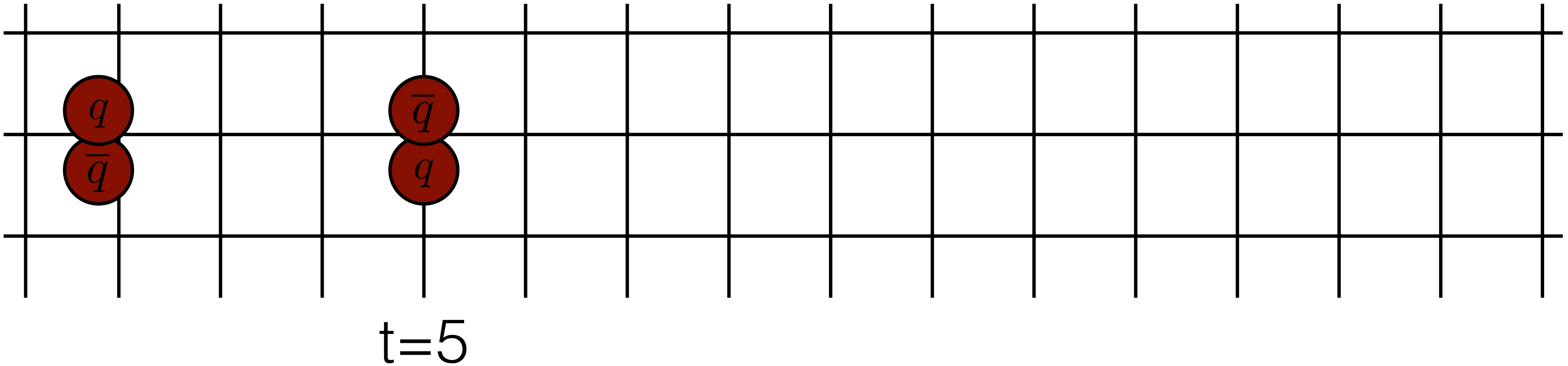




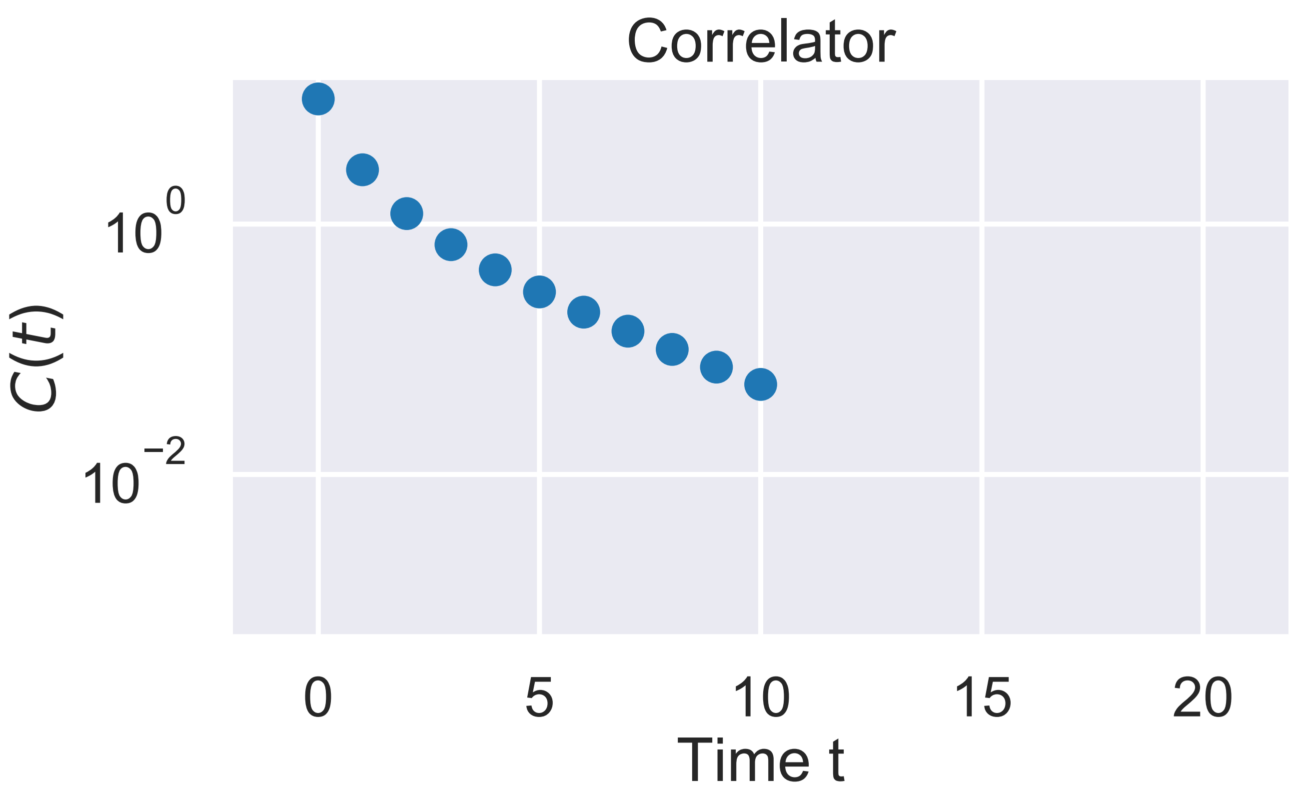
Lattice QCD: particle masses

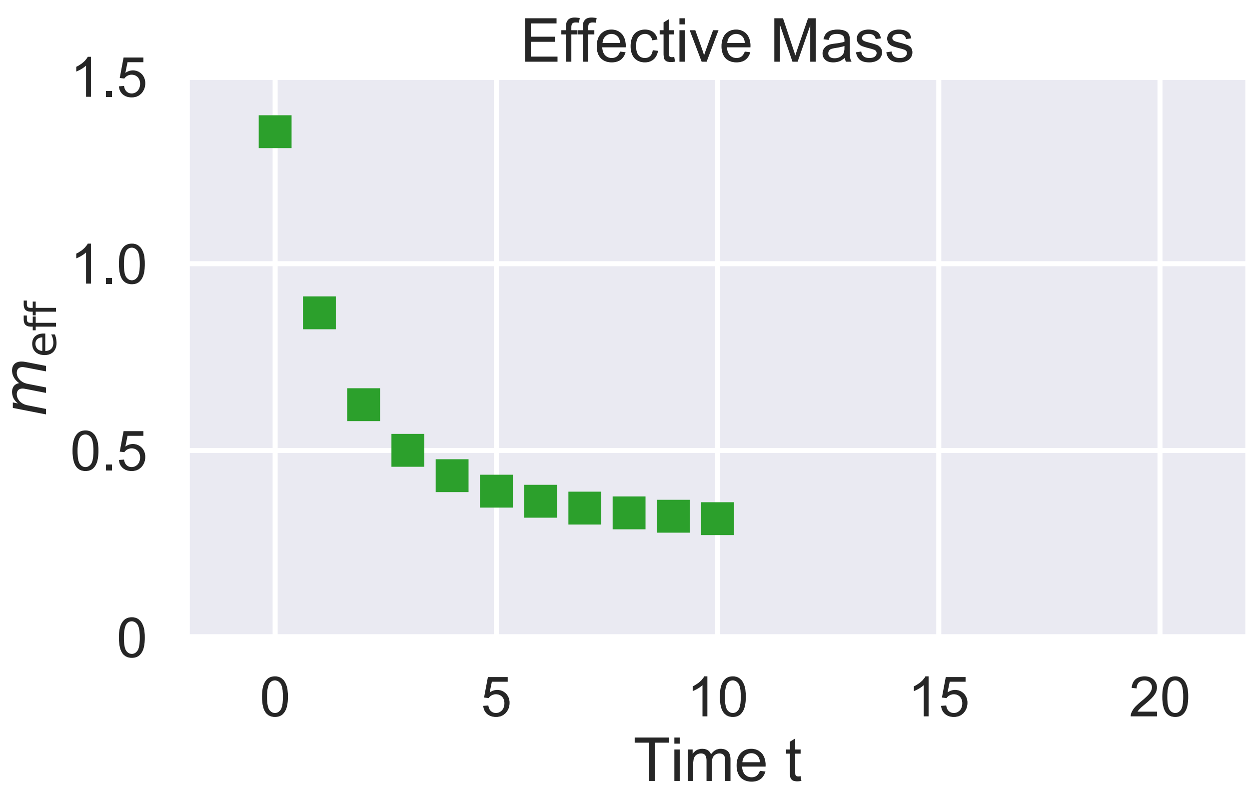


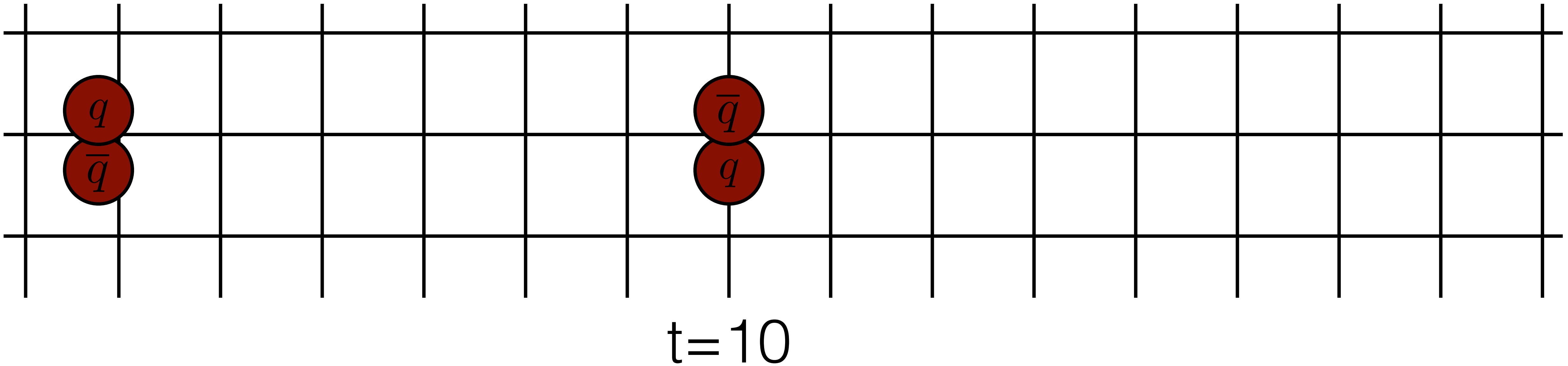




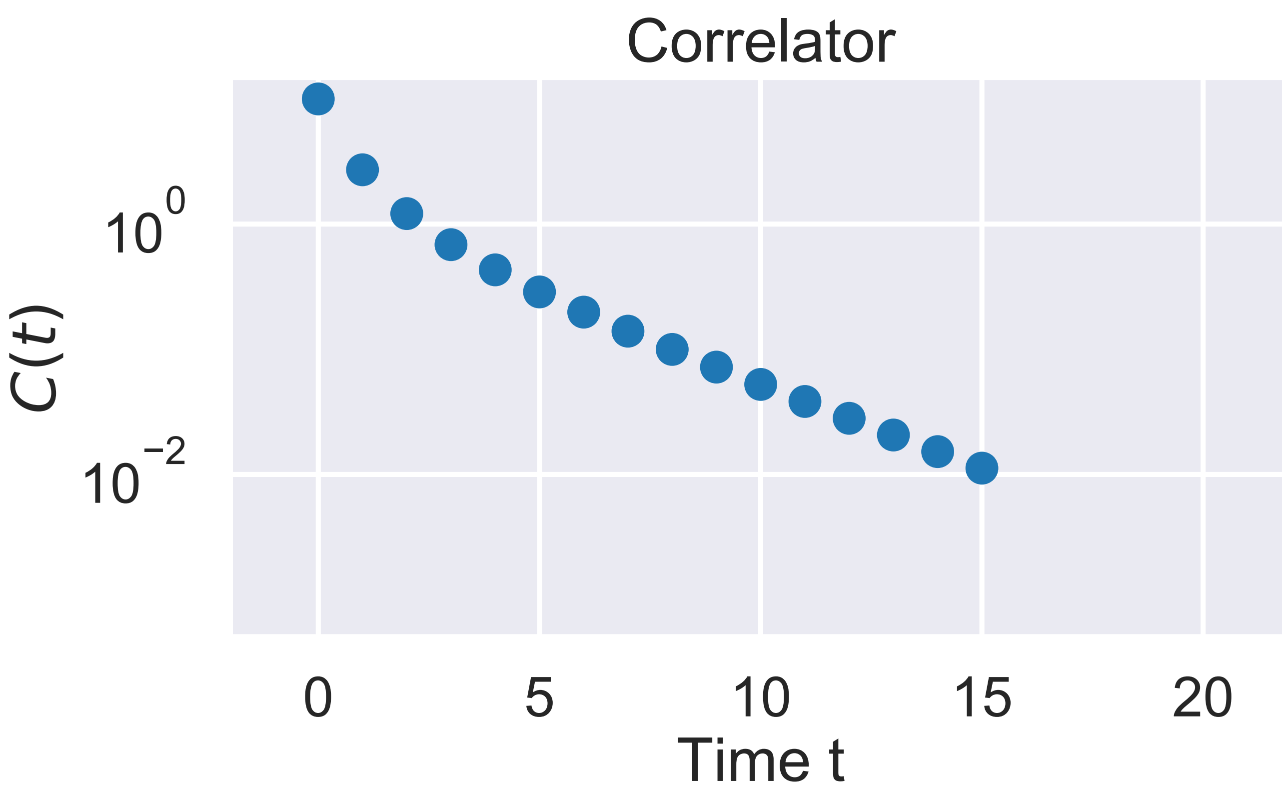
Lattice QCD: particle masses

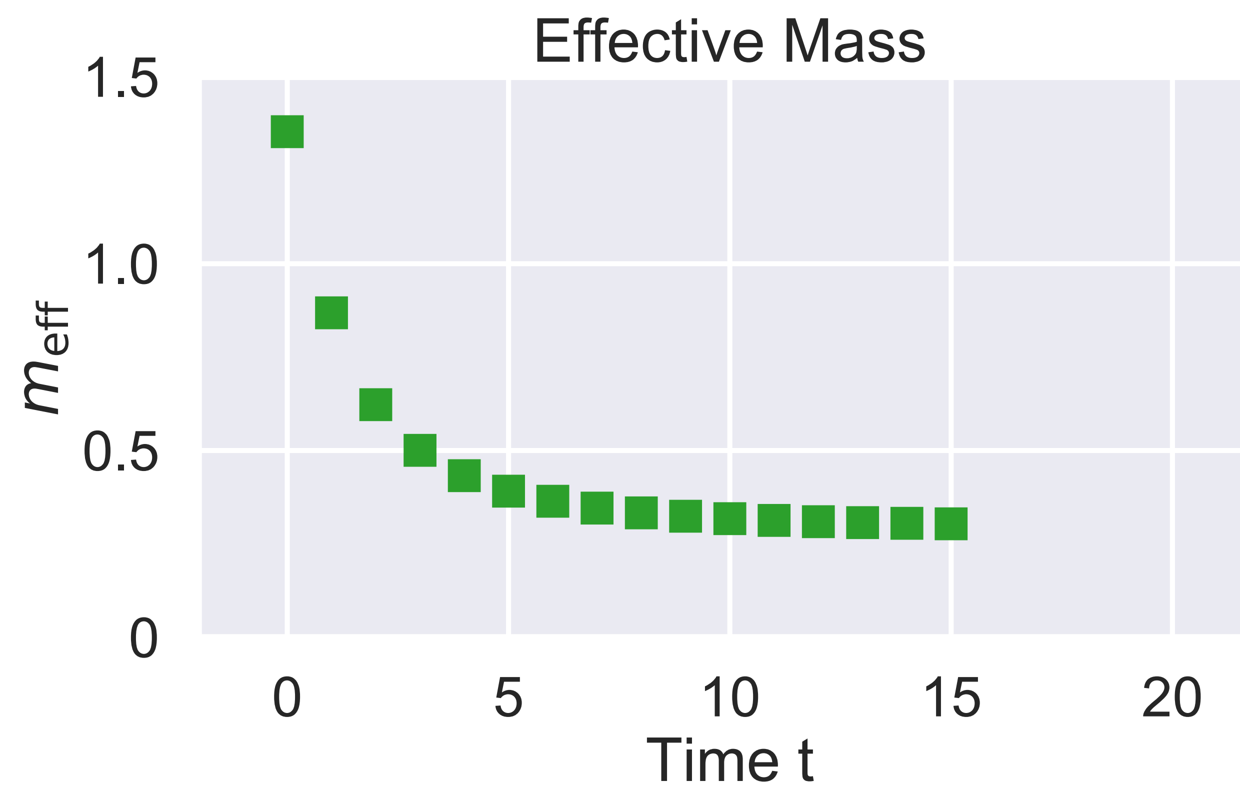


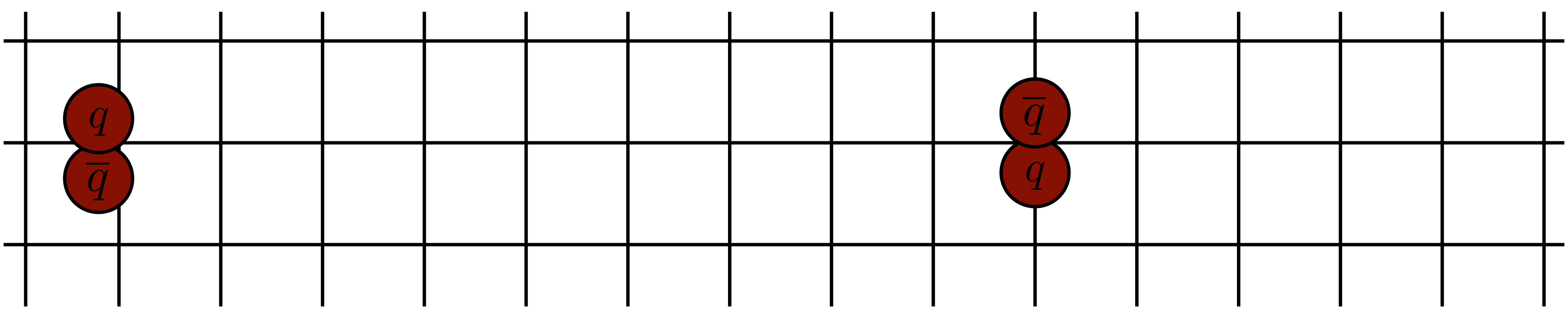




Lattice QCD: particle masses

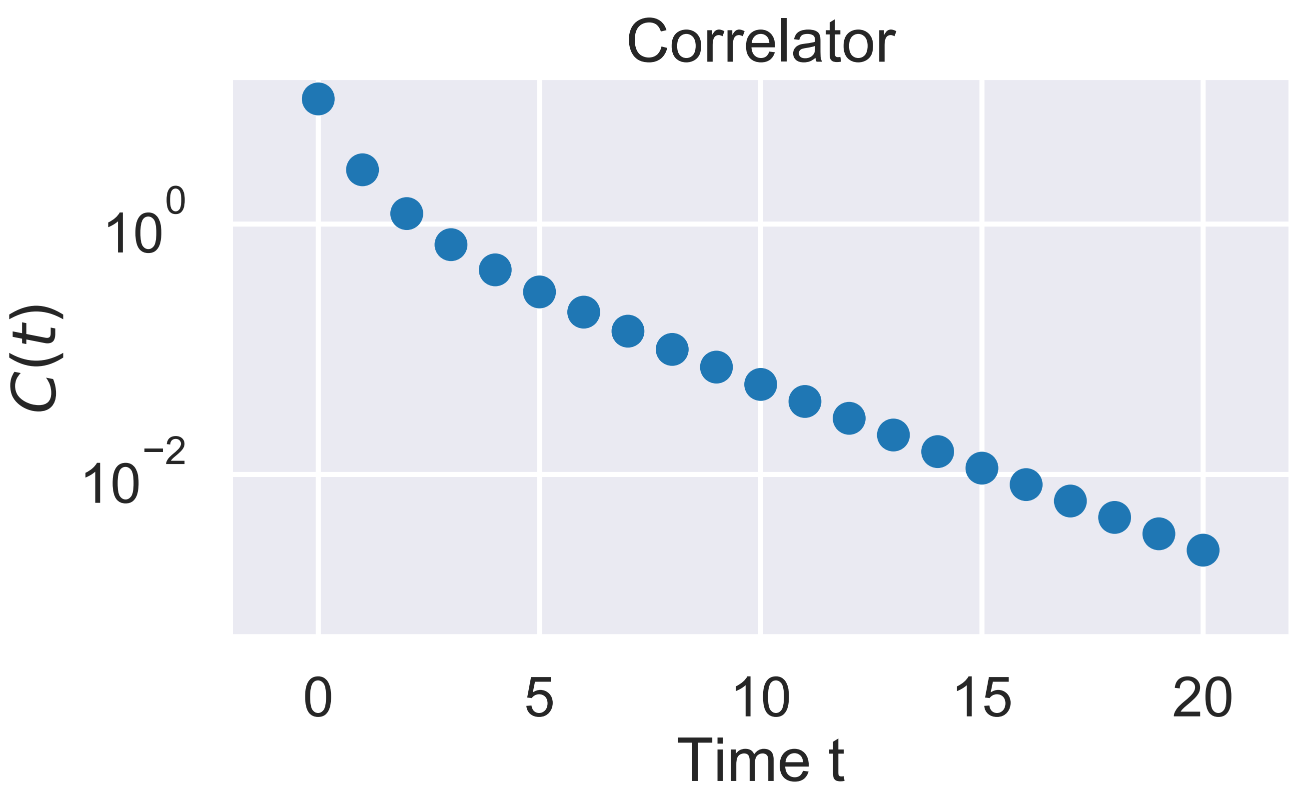


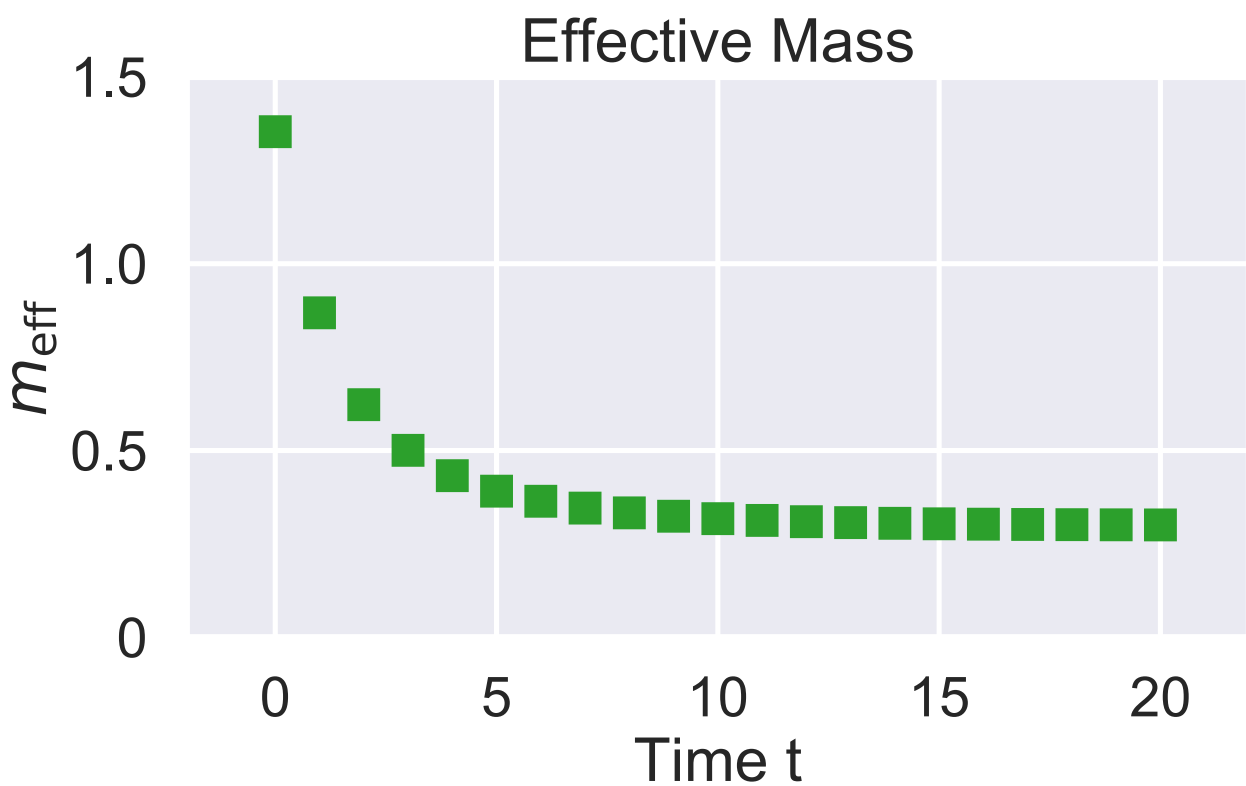


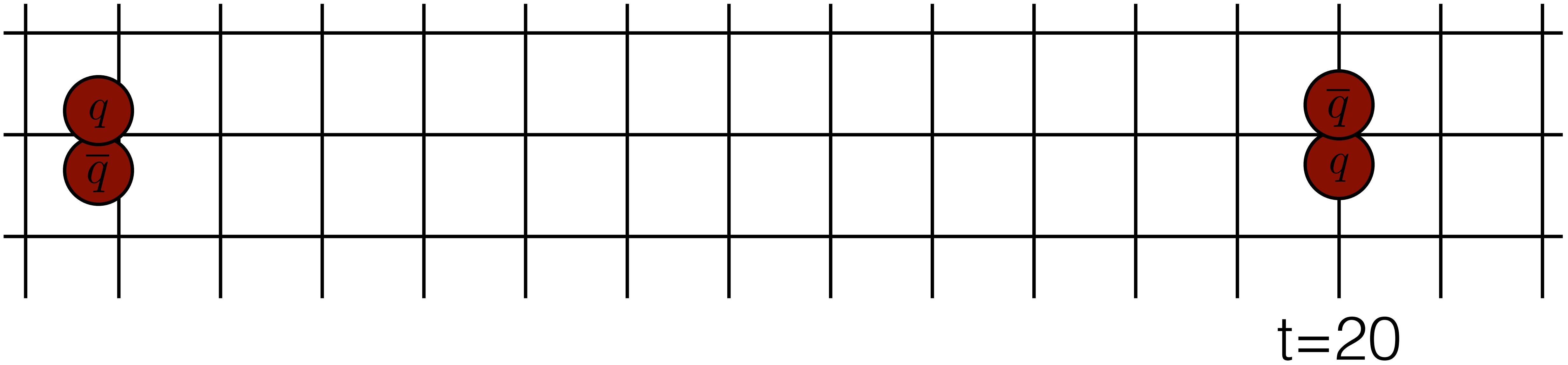


t=15

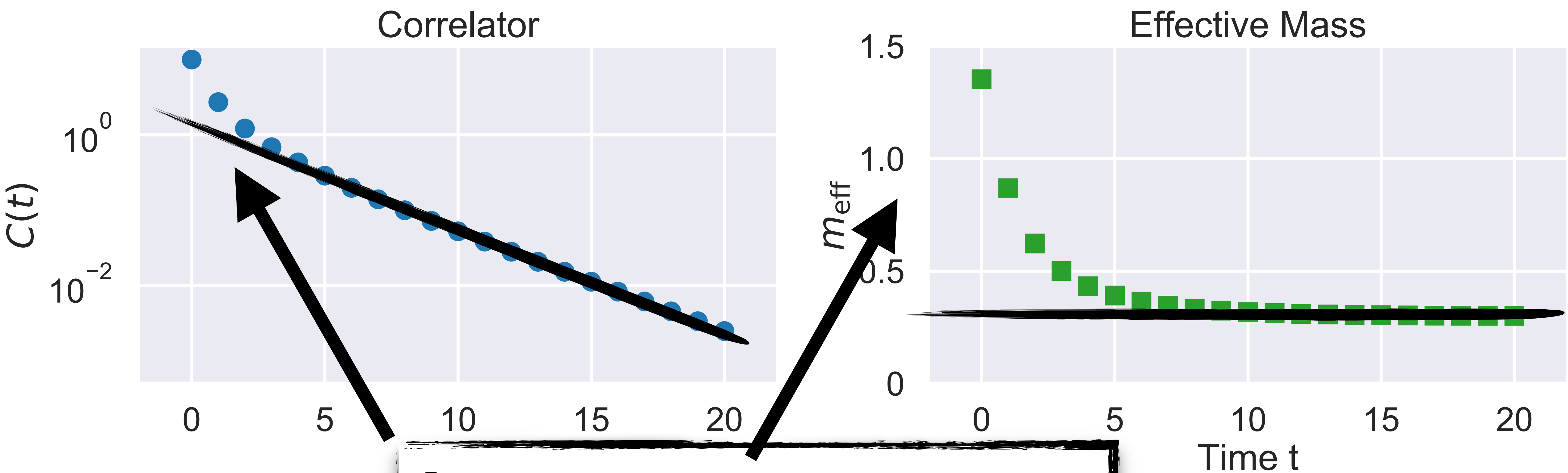
Lattice QCD: particle masses



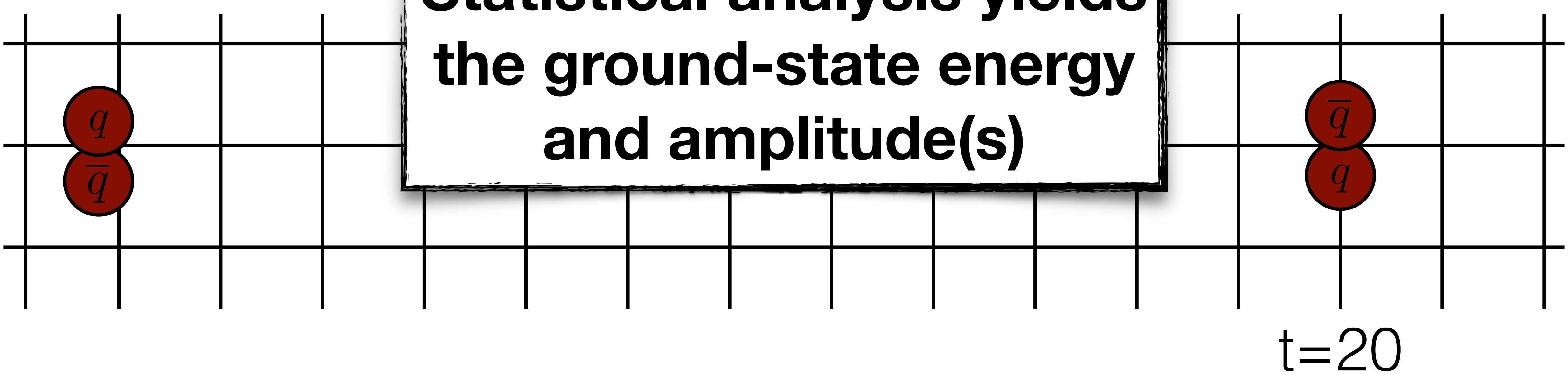




Lattice QCD: particle masses



Statistical analysis yields the ground-state energy and amplitude(s)

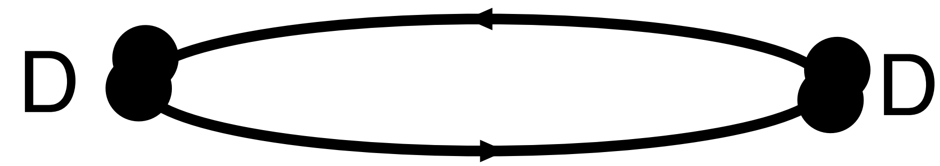


Semileptonic decays: $H \to L\ell\nu$

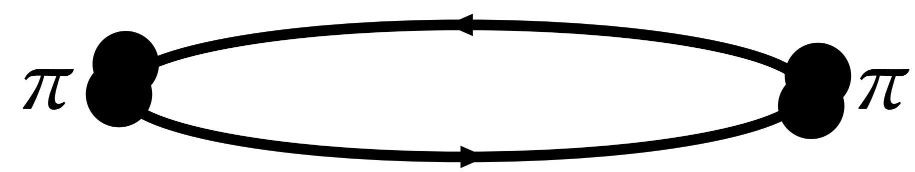
Anatomy of a calculation: correlation functions

- Hadron masses $\Longleftrightarrow$ QCD 2pt functions
- Matrix elements $\Longleftrightarrow$ QCD 3pt functions
- For concreteness: consider $D \to \pi\ell\nu$

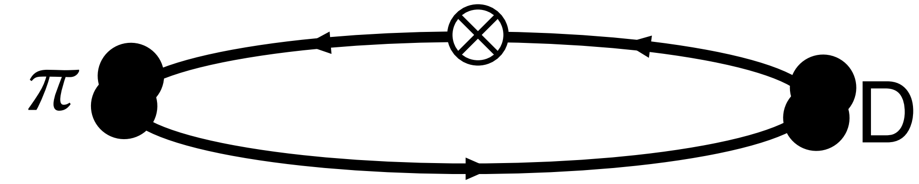
$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0})\mathcal{O}_D(t,\boldsymbol{x})\rangle$$



$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0})\mathcal{O}_\pi(t,\boldsymbol{x})\rangle$$



$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0})J(t,\boldsymbol{y})\mathcal{O}_D(T,\boldsymbol{x})\rangle$$



Semileptonic decays: $H \to L\ell\nu$

Anatomy of a calculation: correlation functions

- Hadron masses $\Longleftrightarrow$ QCD 2pt functions
- Matrix elements $\Longleftrightarrow$ QCD 3pt functions
- For concreteness: consider $D \to \pi\ell\nu$

$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0})\mathcal{O}_D(t,\boldsymbol{x})\rangle \longrightarrow \left|\langle 0|\, \mathcal{O}_D\, |D\rangle\right|^2 e^{-M_D t}$$

$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0})\mathcal{O}_\pi(t,\boldsymbol{x})\rangle \longrightarrow \left|\langle 0|\, \mathcal{O}_\pi\, |\pi\rangle\right|^2 e^{-E_\pi t}$$

$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0}) J(t,\boldsymbol{y}) \mathcal{O}_D(T,\boldsymbol{x})\rangle$$

$$\longrightarrow \langle 0|\, \mathcal{O}_\pi\, |\pi\rangle \langle \pi|\, J\, |D\rangle \langle D|\, \mathcal{O}_D\, |0\rangle\, e^{-E_\pi t} e^{M_D (T-t)}$$

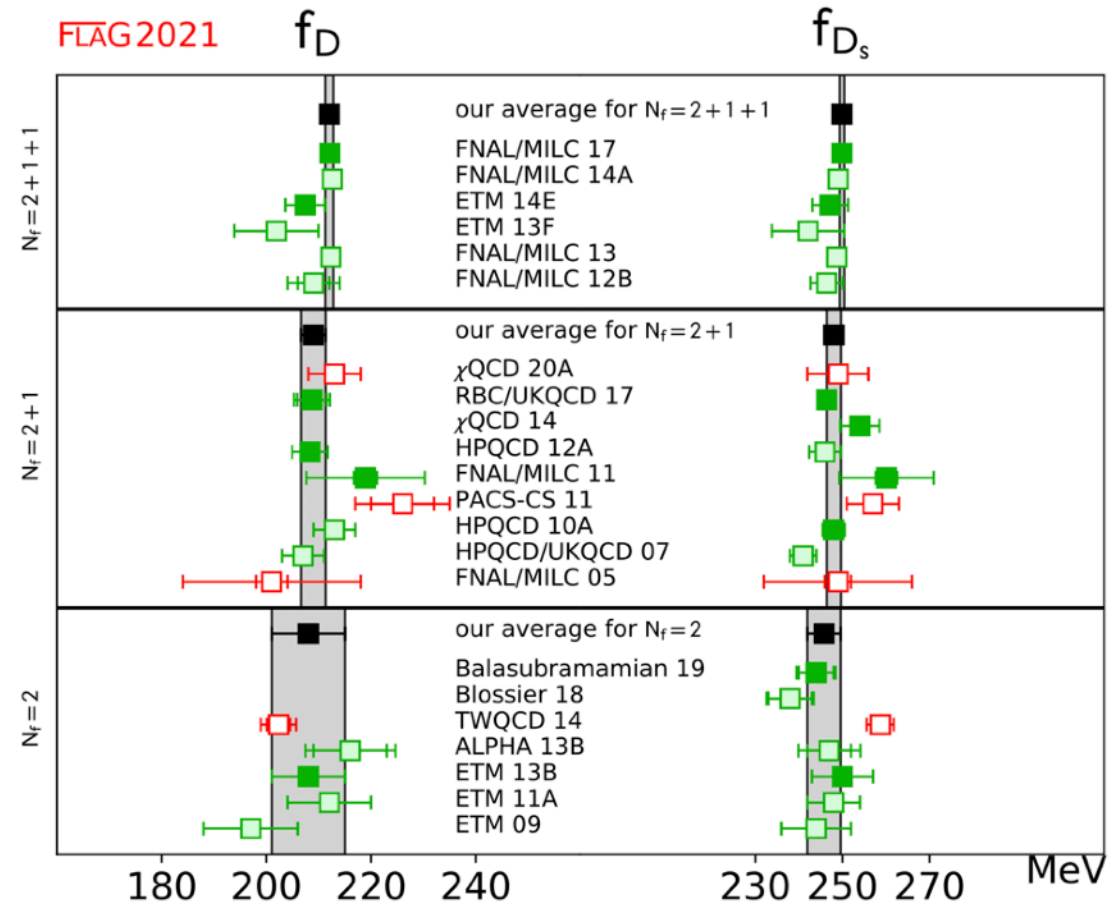
Matrix elements ⇒ Form factors

Leptonic Decays

An invitation to precision in lattice QCD

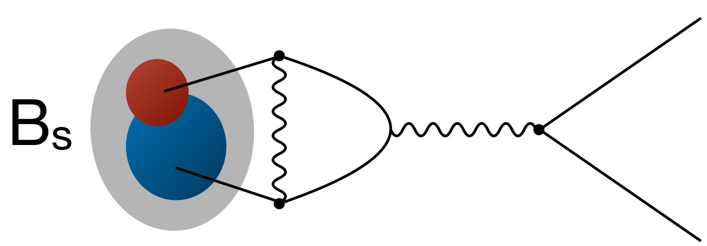
FLAG Review 21
Y. Aoki et al.
EPJC 82 (2022) 10, 869
arXiv: 2111.09849

- Sub-percent precision for $f_{D_{(s)}}, f_{B_{(s)}}$
  - Below existing/expected experimental uncertainties
  - Complementary calculations and discretizations bolster confidence in results
- "Pure QCD problem is solved"
  - Further improvement: systematic inclusion of QED, isospin breaking

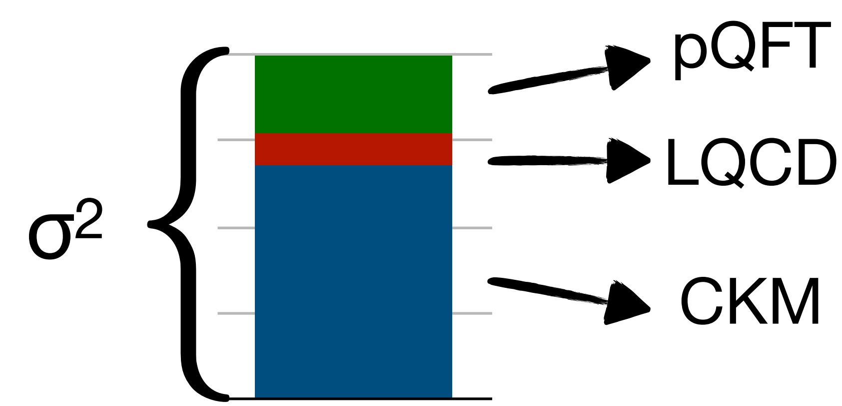


SM prediction for rare leptonic decay rate
Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$

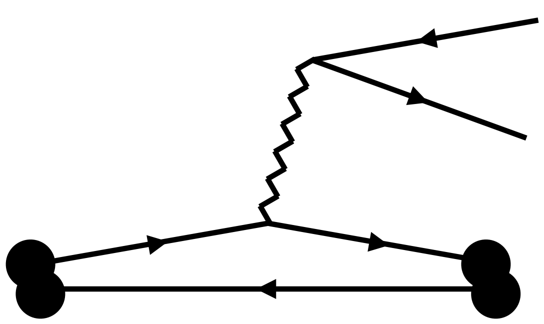


Lattice QCD value for f_{B_s} is now a sub-dominant source of uncertainty



Semileptonic Decays

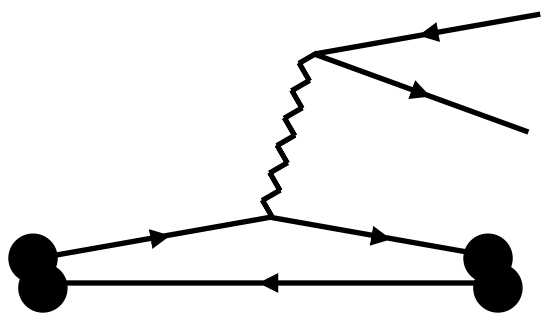
of D-mesons



$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \ell\nu$
	$K \to \pi\ell\nu$	$B \to \pi\ell\nu$
		$\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^\star\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$	$\Lambda_b \to \Lambda_c\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$	
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d \vert \bar{B}_d \rangle)$	$\langle B_s \vert \bar{B}_s \rangle)$	

Semileptonic Decays

of D-mesons

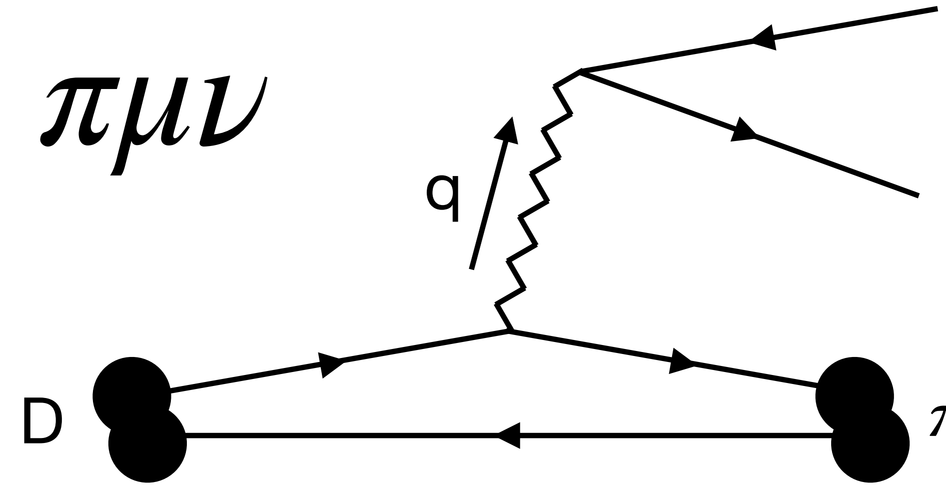


$$\langle \pi \,|\, \bar{d}\gamma^{\mu} c \,|\, D \rangle$$

Vector form factors: $f_{+,0}$

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} \left|V_{cd}\right|^2 (1-\epsilon)^2 (1+\delta_{\text{EM}}) \times$$

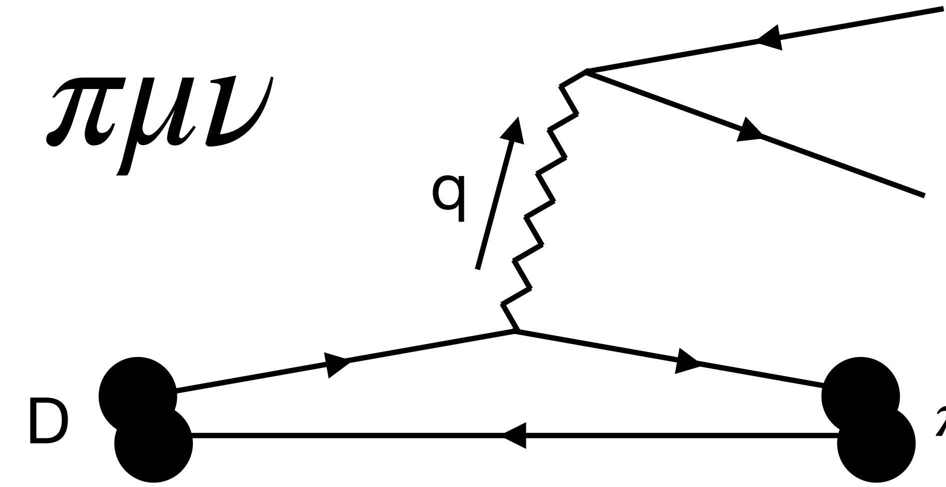
$$\left[|\mathbf{p}|^3 \left(1+\frac{\epsilon}{2}\right) \left|f_+(q^2)\right|^2 + |\mathbf{p}|\, M_D^2 \left(1-\frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} \left|f_0(q^2)\right|^2 \right]$$

: **measured decay rate**

$$\epsilon = m_\mu^2/q^2 \ll 1$$

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} \left|V_{cd}\right|^2 (1-\epsilon)^2 (1+\delta_{\mathrm{EM}}) \times$$

$$\left[|\mathbf{p}|^3 \left(1 + \frac{\epsilon}{2}\right) \left|f_+(q^2)\right|^2 + |\mathbf{p}|\, M_D^2 \left(1 - \frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} \left|f_0(q^2)\right|^2 \right]$$

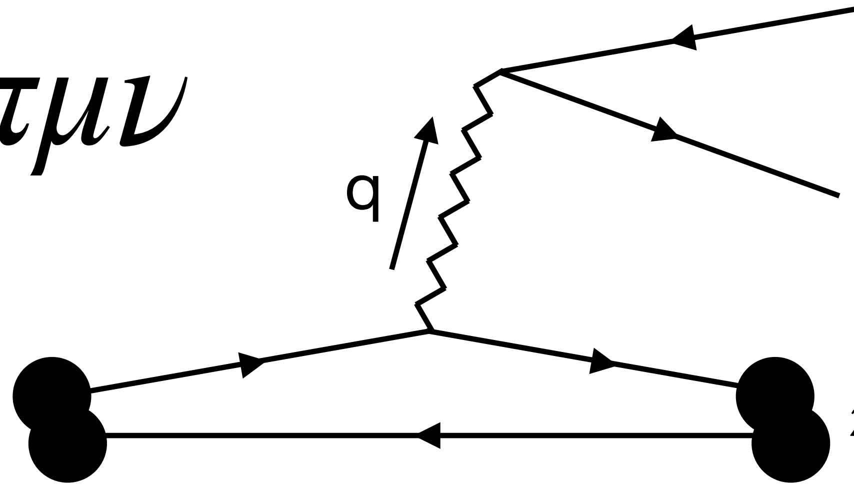
$$\epsilon = m_\mu^2/q^2 \ll 1$$

(blue box) **: measured decay rate**

(green box) **: (non-perturbative) hadronic form factors**

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} |V_{cd}|^2 (1-\epsilon)^2 (1+\delta_{\text{EM}}) \times$$

$$\left[|\mathbf{p}|^3 \left(1 + \frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\mathbf{p}| \, M_D^2 \left(1 - \frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right]$$

$$\epsilon = m_\mu^2 / q^2 \ll 1$$

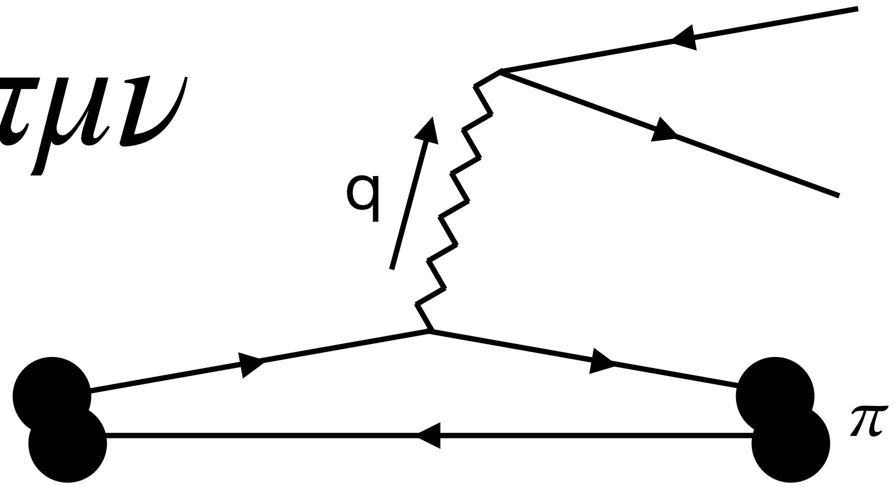
: measured decay rate (blue)

: (non-perturbative) hadronic form factors (green)

: kinematic factors (yellow)

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} |V_{cd}|^2 (1-\epsilon)^2 (1+\delta_{\mathrm{EM}}) \times \left[|\mathbf{p}|^3 \left(1 + \frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\mathbf{p}|\, M_D^2 \left(1 - \frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right]$$

$$\epsilon = m_\mu^2 / q^2 \ll 1$$

: measured decay rate (blue)

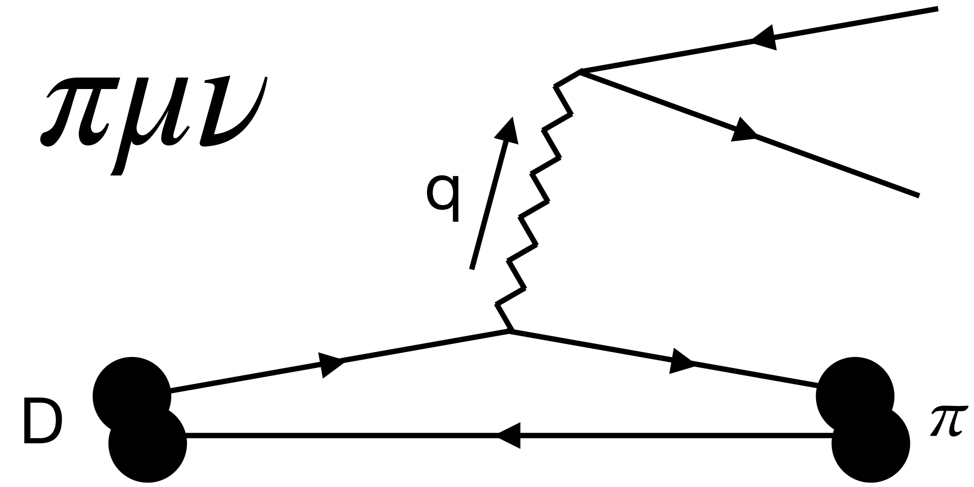
: (non-perturbative) hadronic form factors (green)

: kinematic factors (yellow)

: perturbative corrections (red)

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} |V_{cd}|^2 (1-\epsilon)^2 (1+\delta_{\text{EM}}) \times$$

$$\left[|\mathbf{p}|^3 \left(1 + \frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\mathbf{p}|\, M_D^2 \left(1 - \frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right]$$

$$\epsilon = m_\mu^2 / q^2 \ll 1$$

- **: measured decay rate** (blue)
- **: (non-perturbative) hadronic form factors** (green)
- **: kinematic factors** (yellow)
- **: perturbative corrections** (red)

> At O(1%) precision, all sectors of SM become important: QCD, QED, EW

D-meson Semileptonic Decays

Measure: Expt.

Calculate: LQCD

Pseudoscalar final state: $D_{(s)} \to \pi/K\ell\nu$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}\eta_{\mathrm{EW}}^2|V_{cx}|^2(1-\epsilon)^2(1+\delta_{\mathrm{EM}})\times$$
$$\left[|\boldsymbol{p}|^3\left(1+\frac{\epsilon}{2}\right)\left|f_+(q^2)\right|^2 + |\boldsymbol{p}|\,M_H^2\left(1-\frac{M_L^2}{M_H^2}\right)^2\frac{3\epsilon}{8}\left|f_0(q^2)\right|^2\right]$$

+tensor-current form factors for FCNC, BSM

$$|V_{cd}^{\mathrm{excl.}}| = 0.2330(0.0029)^{\mathrm{Expt}}(0.0133)^{\mathrm{QCD}}$$

- Status as of PDG 2022
- Combined precision for $D \to \pi$ ~ 6%
- Theory errors dominated
- Today: recent significant improvement

$$|V_{cs}^{\mathrm{excl.}}| = 0.972(0.007)$$

- Combined precision for $D \to K \lesssim 1\%$
- Theory errors dominant
- Percent-level total errors now possible, with QCD subdominant

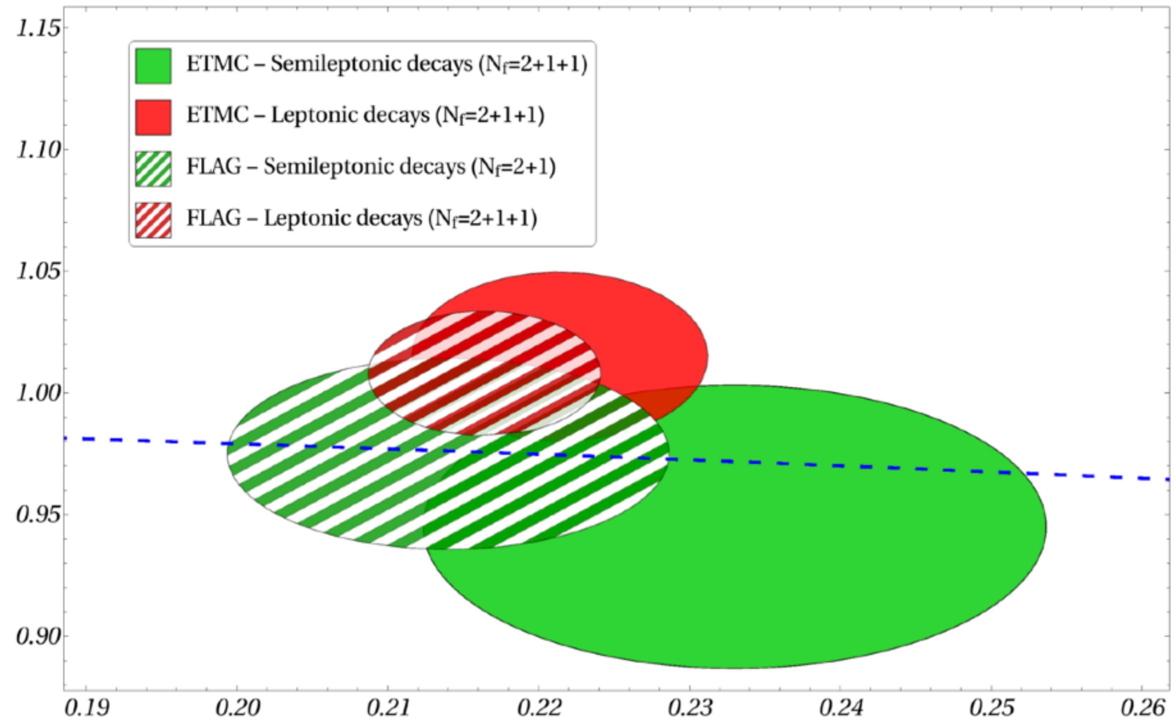
A. Ceccucci, Z. Ligeti, Y. Saka
PDG 2022: 12. CKM Quark-Mixing Matrix

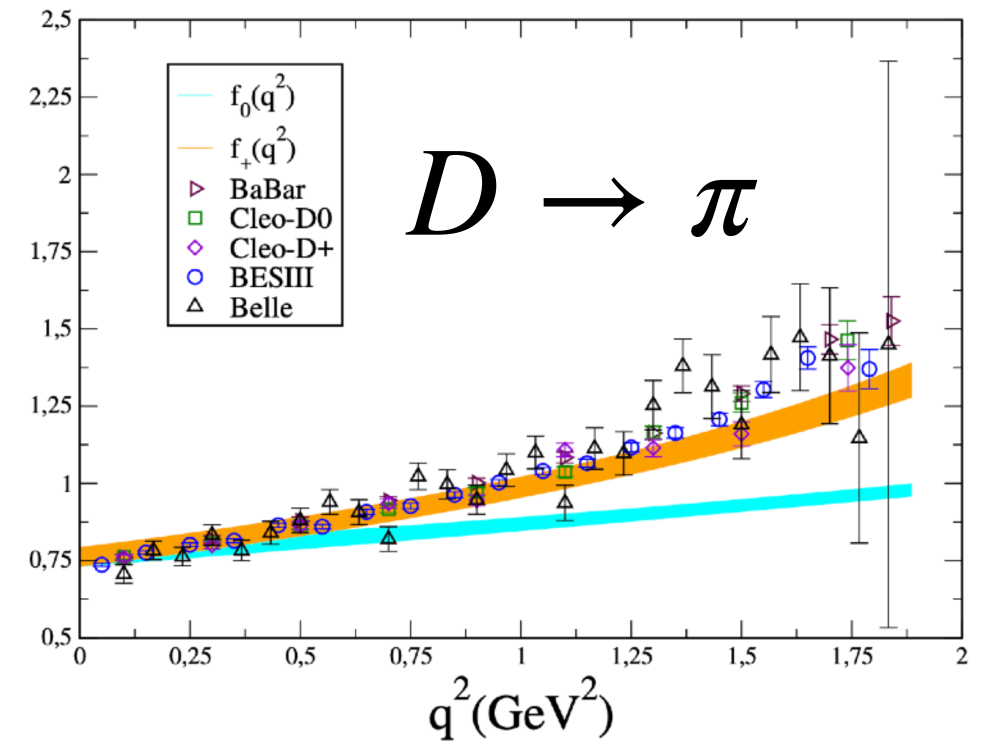
D-meson Semileptonic Decays

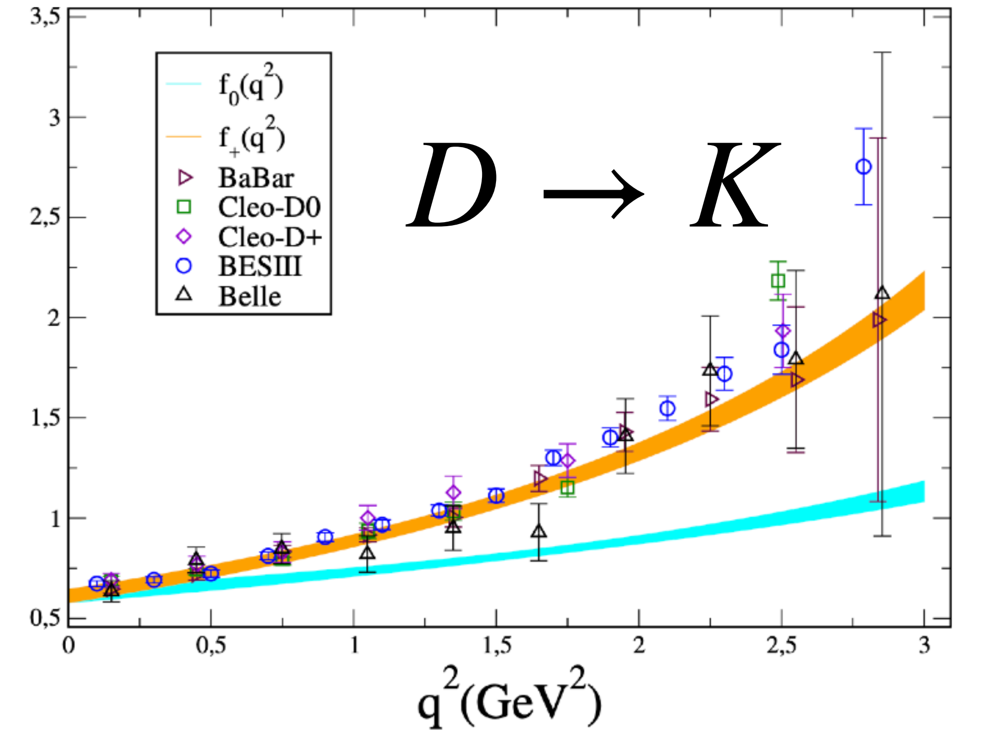
ETMC
PRD 96 (2017) 5, 054514
arXiv:1706.03017

$D \to K/\pi \ell \nu$ and |Vcd|, |Vcs|

- (N_f=2+1+1)ETMC Wilson twisted mass ensembles
- Lattice spacings: $a \in \{0.09, 0.08, 0.06\}$ fm
- $M_\pi \simeq 210 - 450$ MeV
- $\approx 4 - 6\,\%$ precision for $f_{+/0}(0)$
- $|V_{cd}| = 0.2330(133)^{\mathrm{LQCD}}(31)^{\mathrm{EXP}}$ [≈6%]
- $|V_{cs}| = 0.945(38)^{\mathrm{LQCD}}(4)^{\mathrm{EXP}}$ [≈4%]





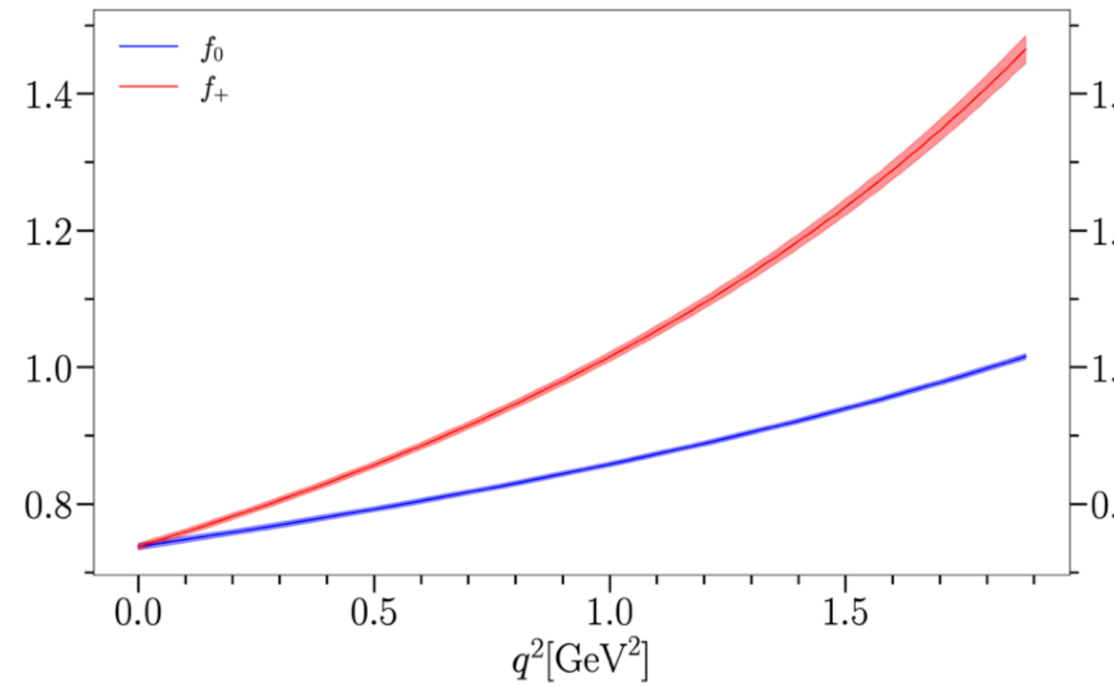


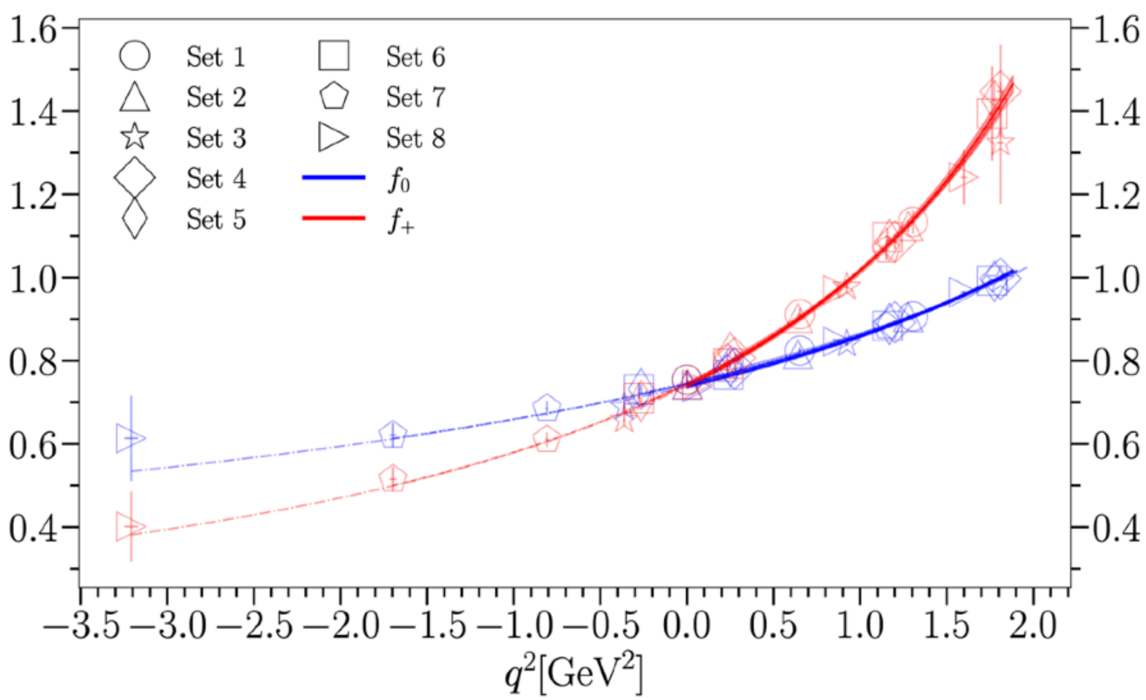
D-meson Semileptonic Decays

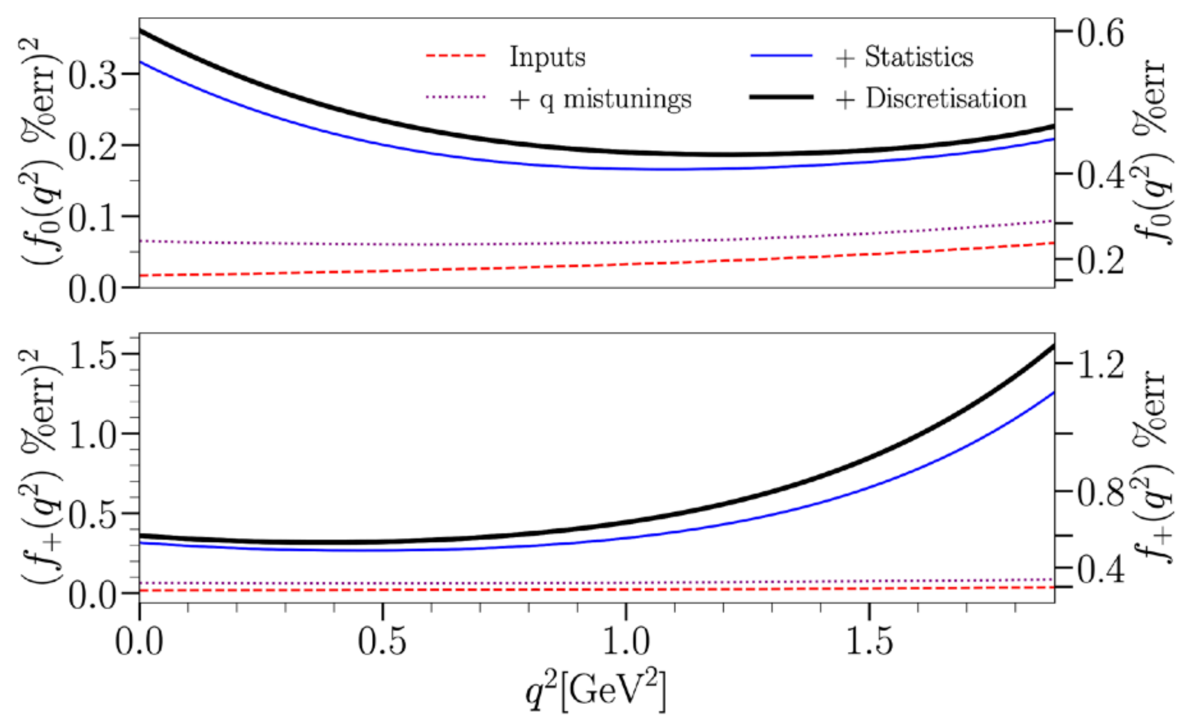
HPQCD
PRD 104 (2021) 3, 034505
arXiv:2104.09883

$D \to K\ell\nu$ and $|V_{cs}|$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: $a \in \{0.045 - 0.15\}$ fm
- $M_\pi \simeq 135 - 320$ MeV
- Valence: heavy HISQ
- Chiral-continuum analysis via "modified z-expansion"
- $\lesssim 1\,\%$ precision for $f_{+/0}(0)$
- $|V_{cs}| = 0.9663(53)^{\mathrm{LQCD}}(39)^{\mathrm{EXP}}(19)^{\mathrm{EW}}(40)^{\mathrm{EM}}$ [≈1%]





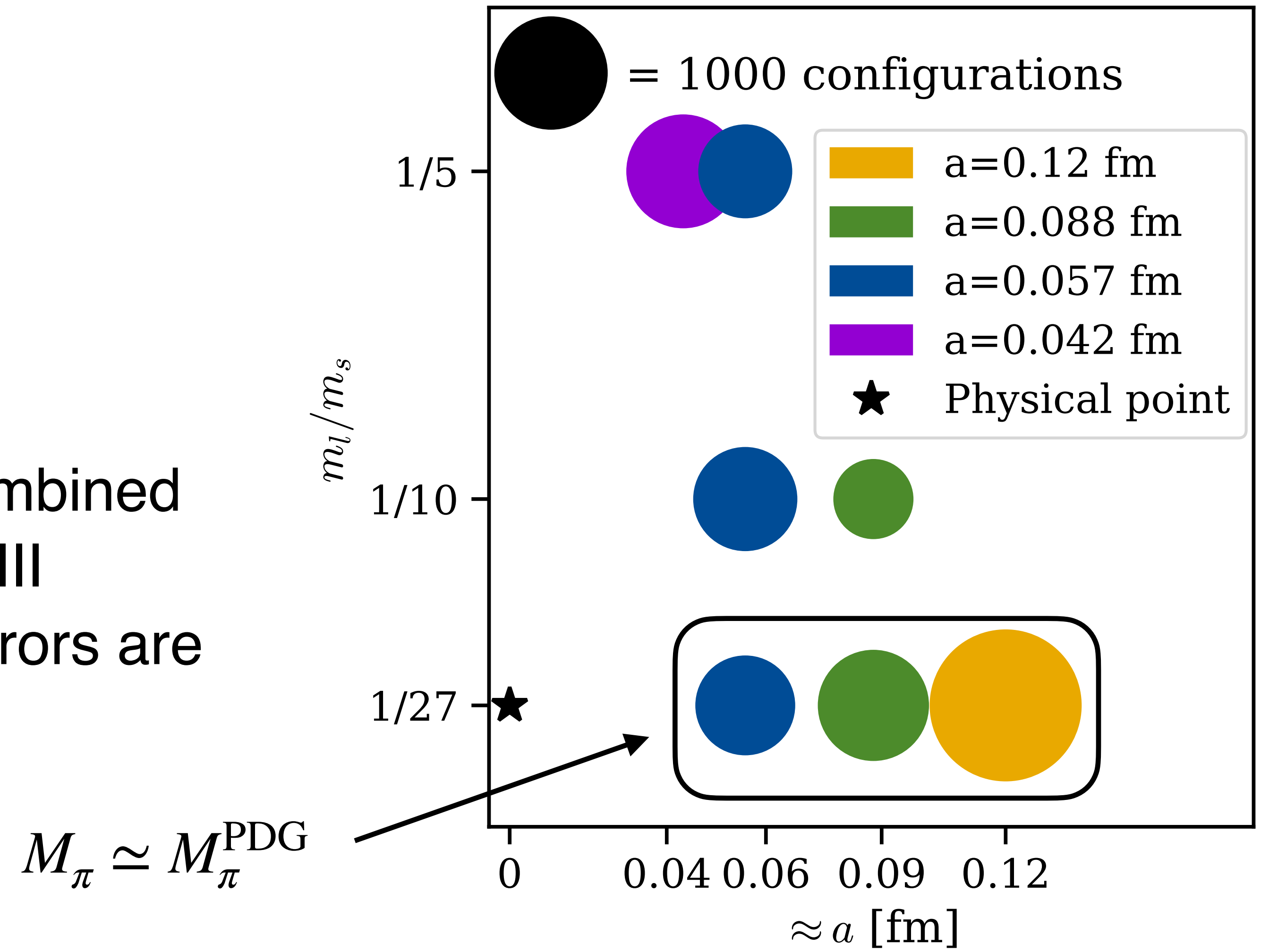


D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Valence: heavy HISQ
- Percent-level determinations of $|V_{cd}|$, $|V_{cs}|$
  - Consistent with $|V_{cs}|$ from HPQCD 2021
- First-ever $|V_{cd}|$ from $D_s \to K \ell \nu$ when combined with recent first measurements from BESIII
- First time that LQCD and experimental errors are commensurate for $D \to \pi \ell \nu$
- All results from a **<u>blinded analysis</u>**



$$|V_{cd}|^{D\to\pi} = 0.2338(11)^{\text{Expt}}(15)^{\text{LQCD}}[22]^{\text{EW/QED/SIB}}$$

$$|V_{cs}|^{D\to K} = 0.9589(23)^{\text{Expt}}(40)^{\text{LQCD}}[96]^{\text{EW/QED/SIB}}$$

Measure: Expt.

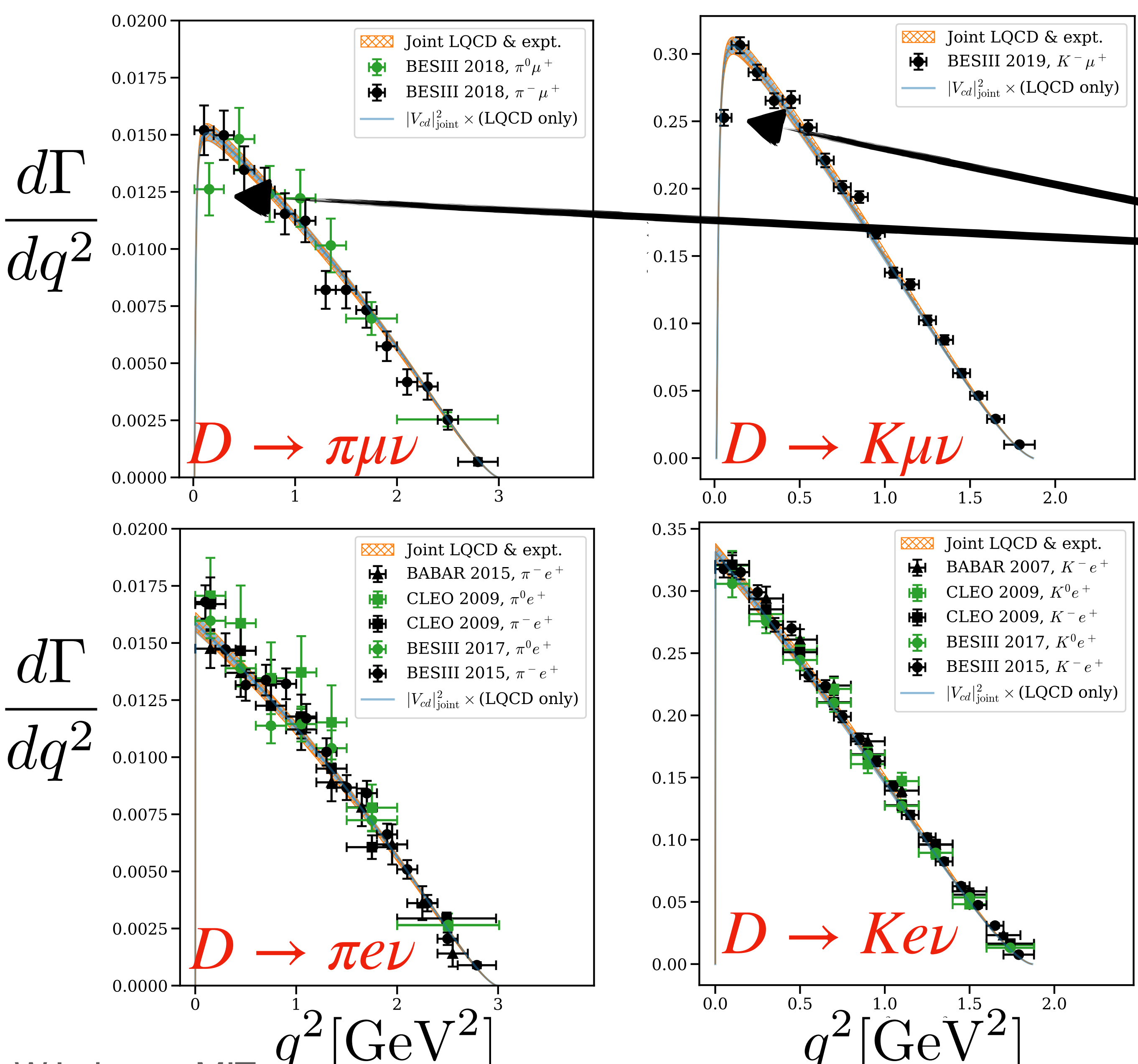
Calculate: LQCD

D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

Comparison to experimental data

$D \to \pi$ $D \to K$

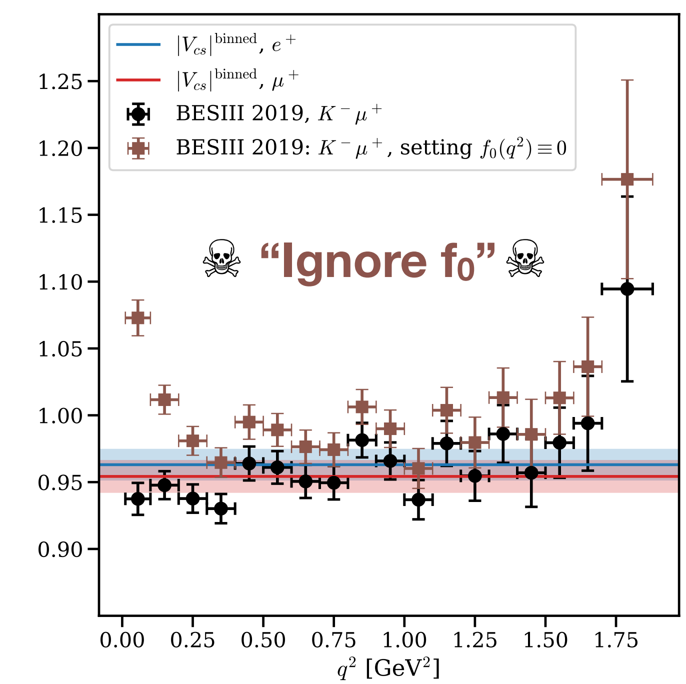


Suppressed scalar-form-factor contributions
$\propto (m_\ell^2/q^2)\,|f_0|^2$

These effects were first statistically relevant in the extraction of $|V_{cs}|$ by **HPQCD 2021**

How relevant is f_0?

See, e.g., binwise estimates of $|V_{cs}|$



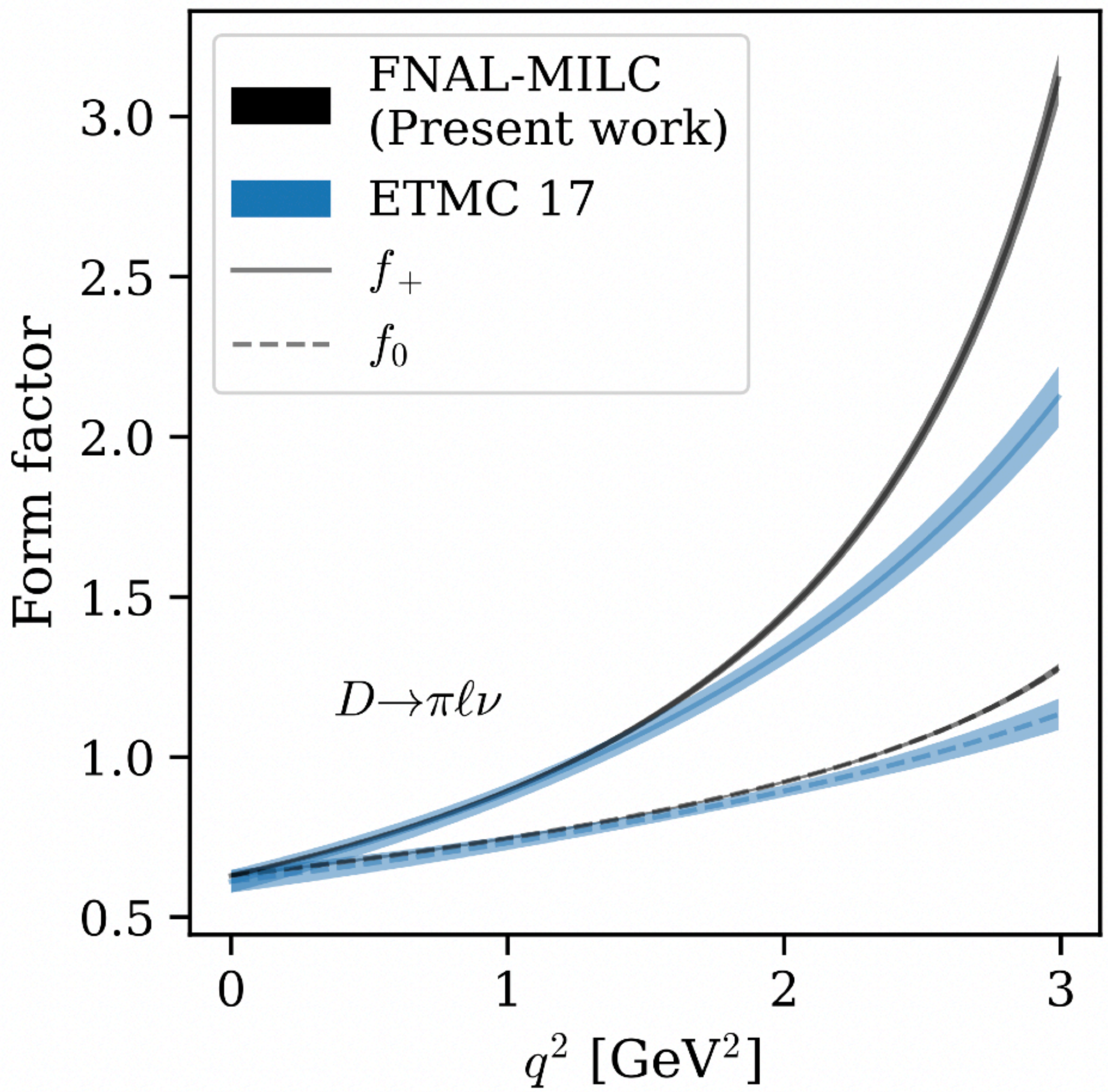
D-meson Semileptonic Decays

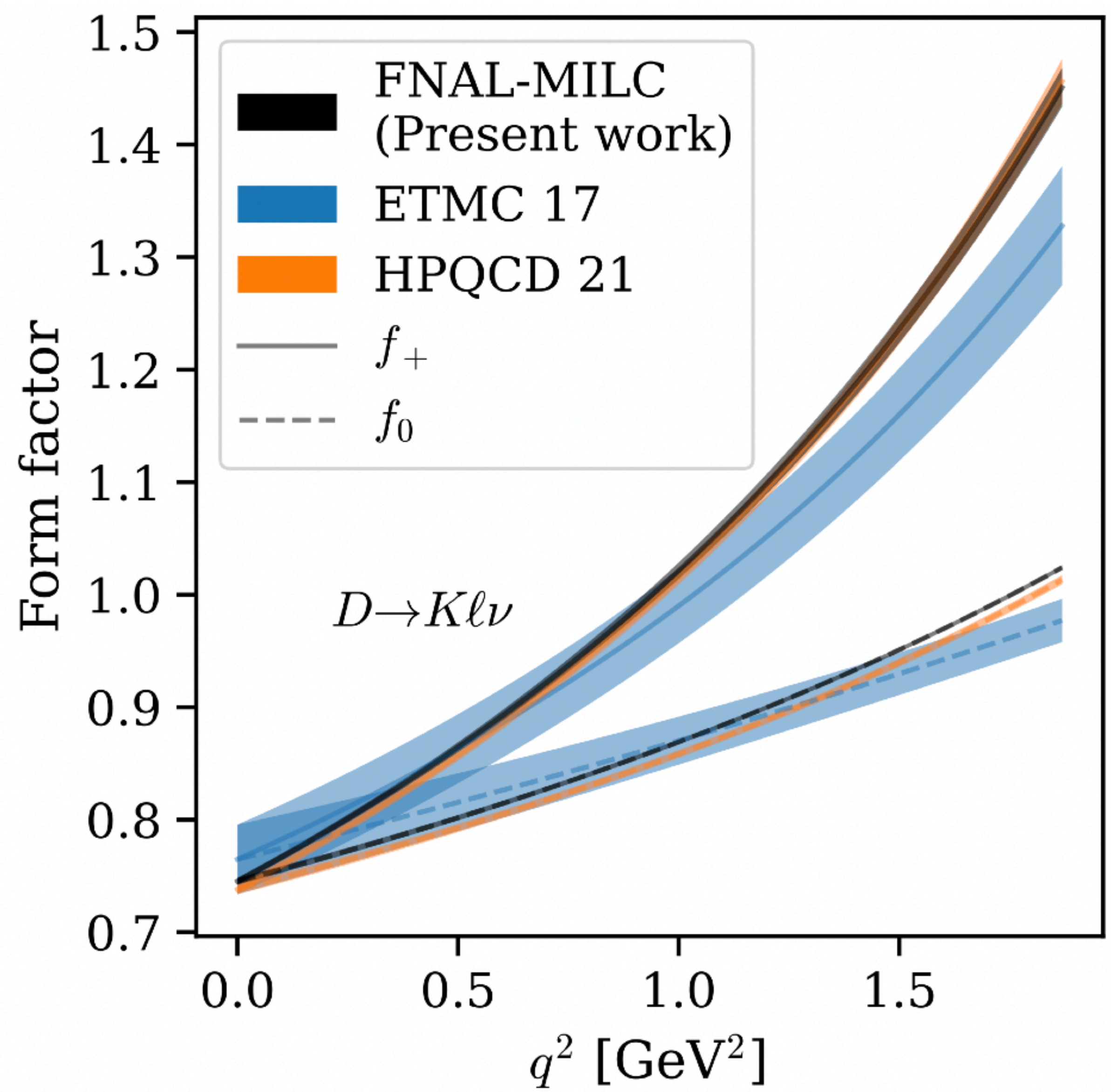
$D_{(s)} \to K/\pi\, \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

ETMC
PRD 96 (2017) 5, 054514
arXiv:1706.03017

HPQCD
PRD 104 (2021) 3, 034505
arXiv:2104.09883



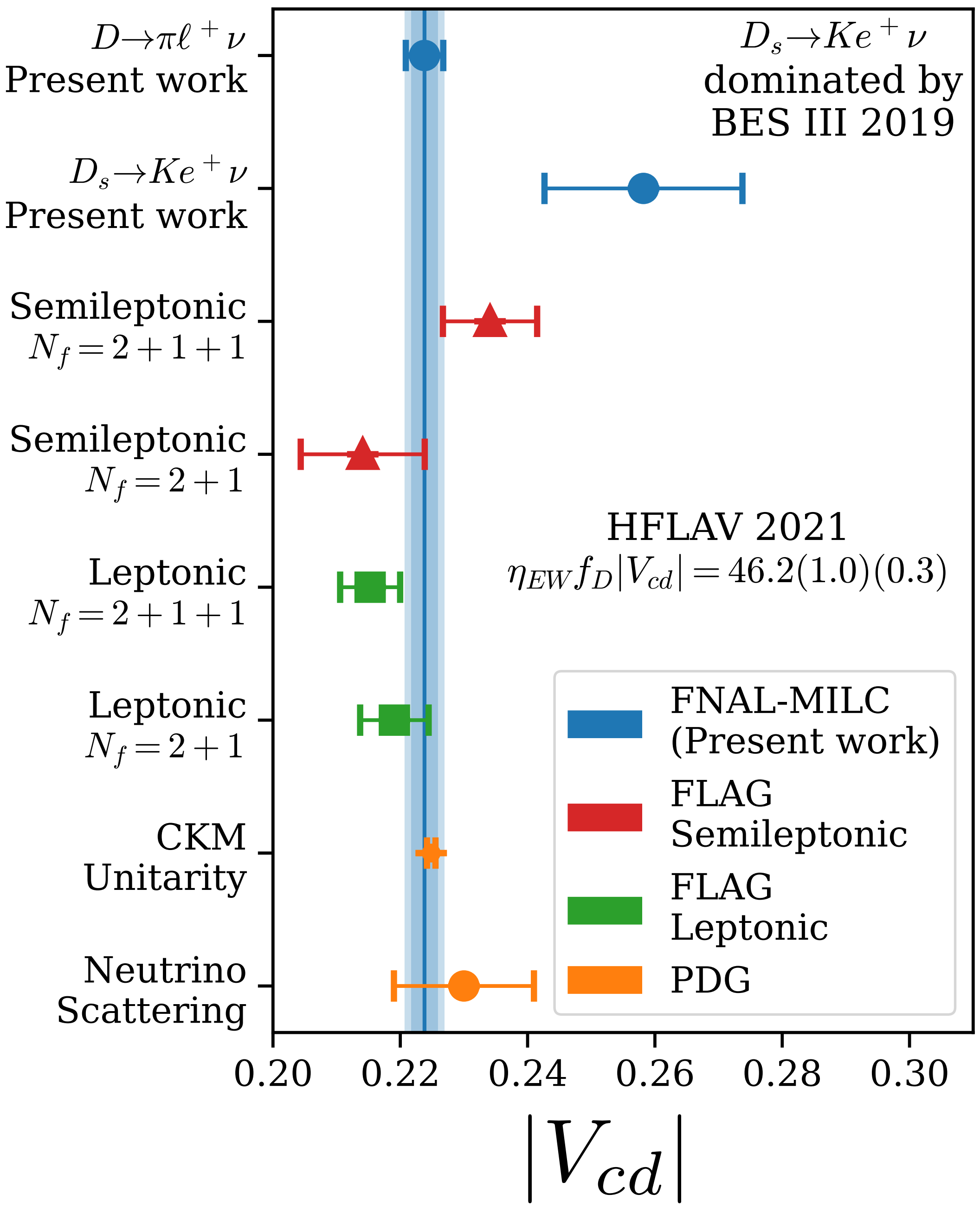


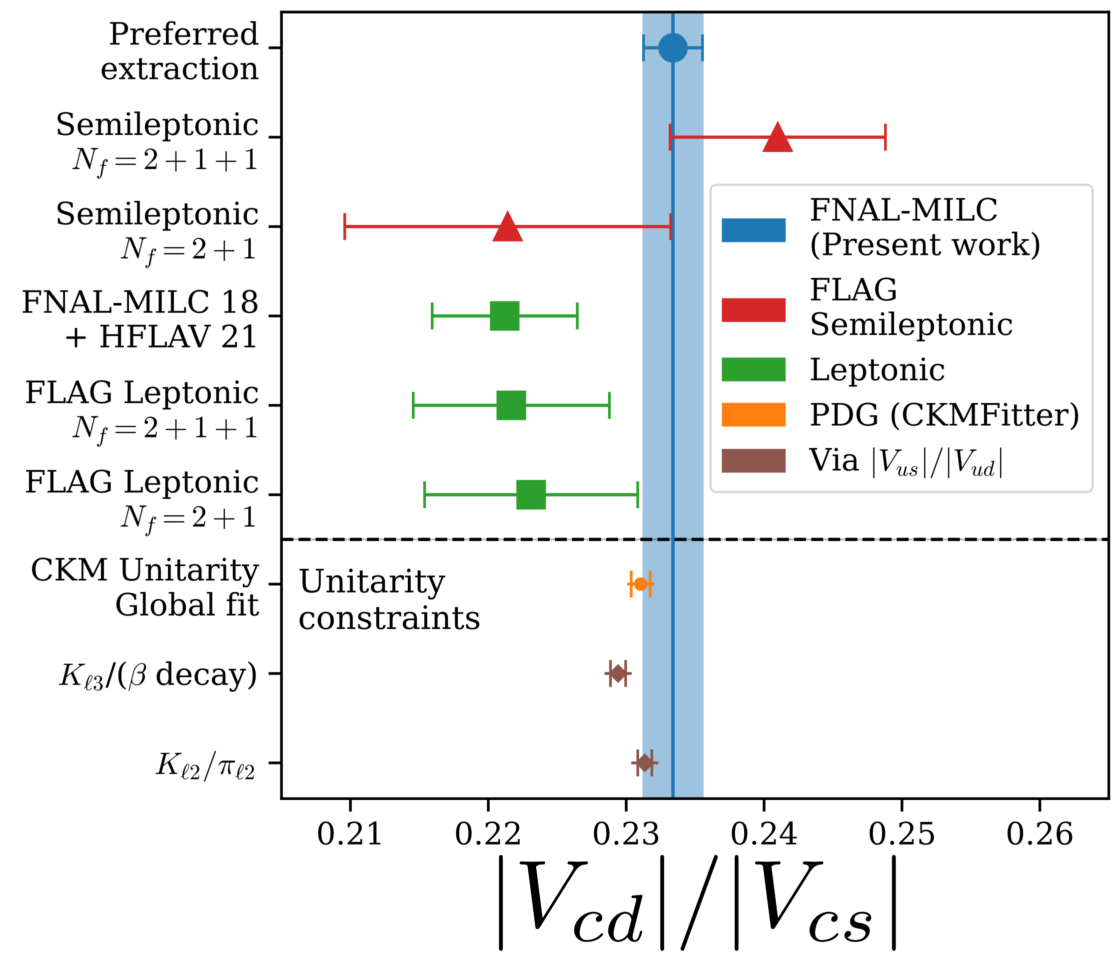
D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

PDG particle data group · HFLAV · FLAG Flavour Lattice Averaging Group





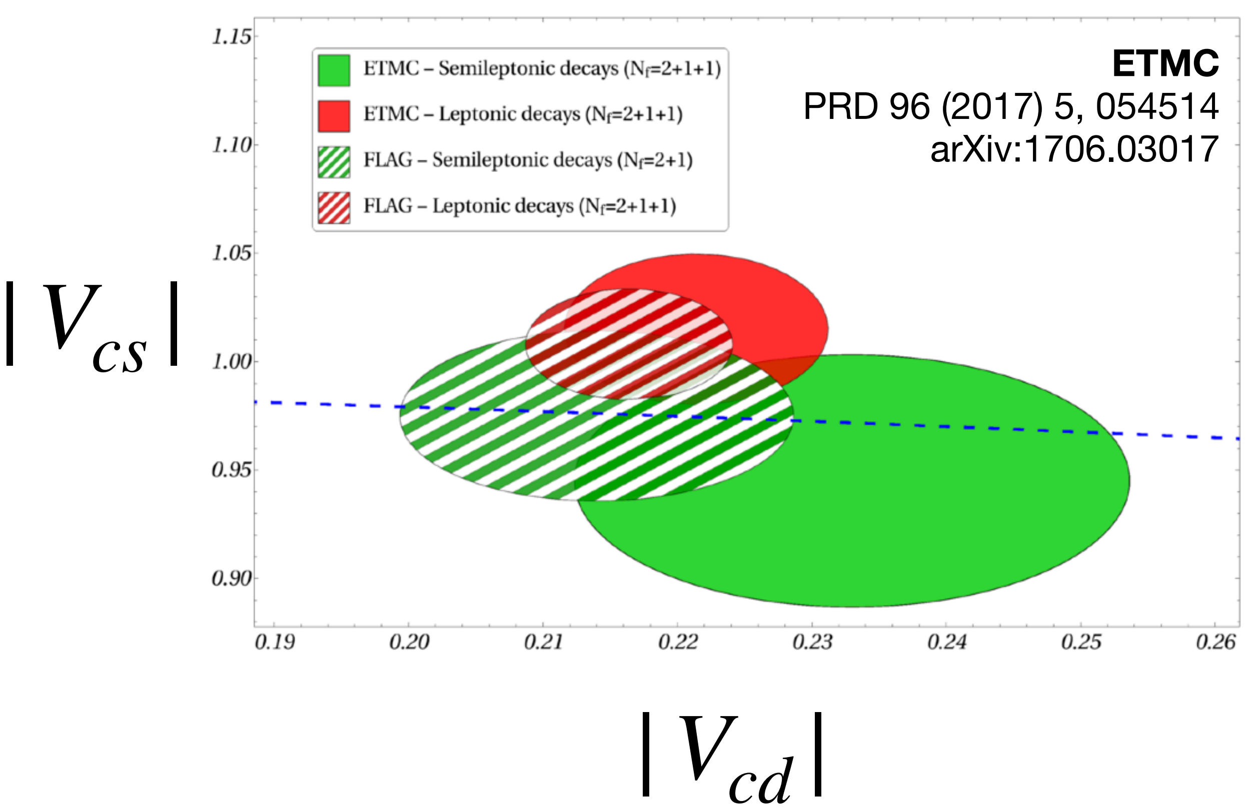
D-meson Semileptonic Decays

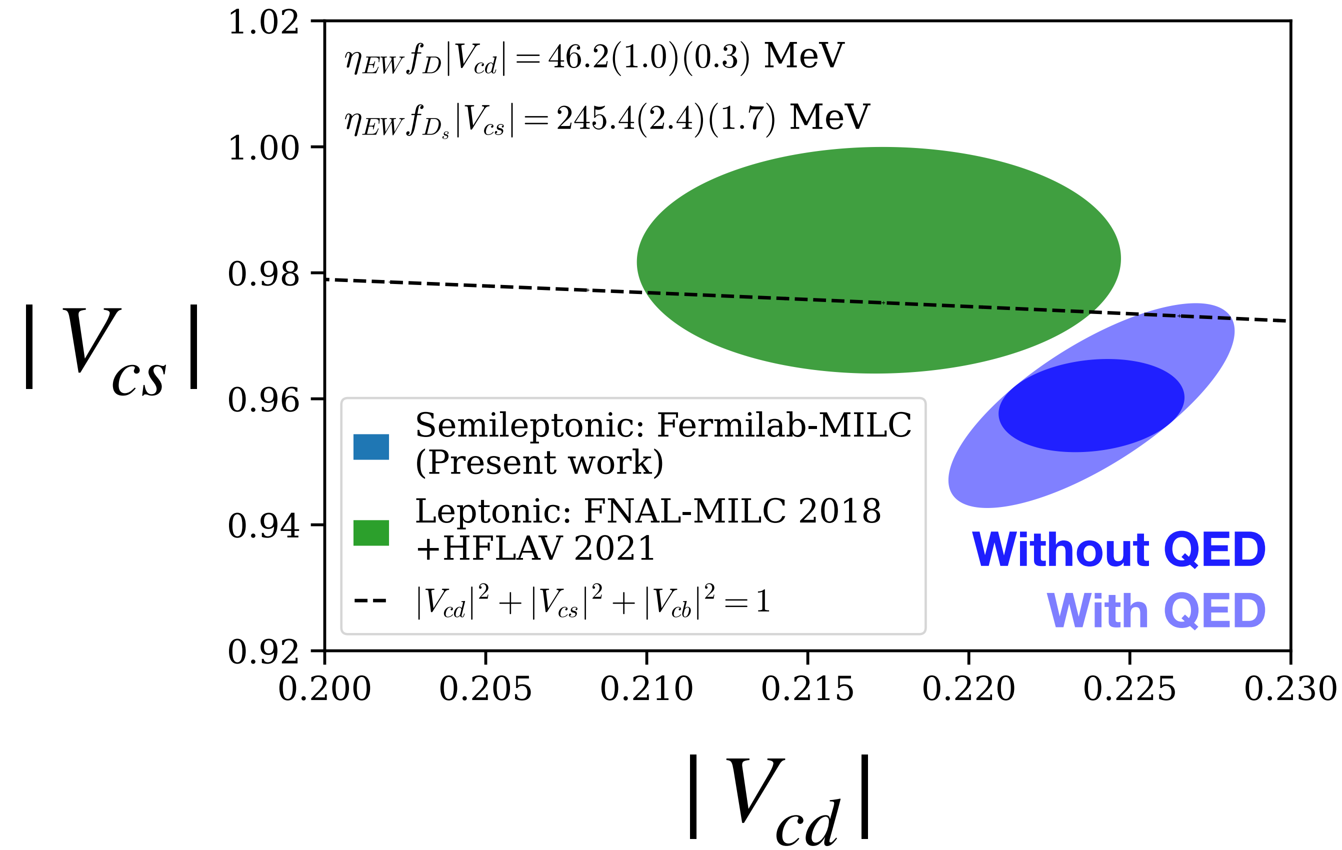
Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

Second-row unitarity tests

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.0286(44)^{\mathrm{EXP}}(78)^{\mathrm{QCD}}[194]^{\mathrm{QED}}(28)^{\mathrm{EW}}$$

- Consistent with unitarity at ≈1σ
- Uncertainty still dominated by theory
- QCD uncertainty subdominant to QED
- $|V_{cd}|/|V_{cs}|$: qualitatively similar arrangement to what was seen by ETMC 2017





D-meson Semileptonic Decays

Lepton Flavor Universality Ratios

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

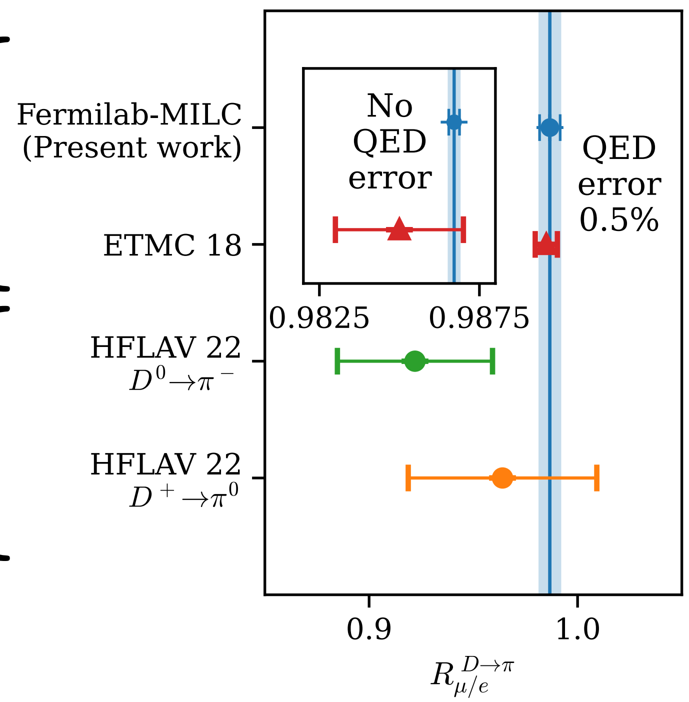
$$R_{\mu/e}^{H\to L} \equiv \frac{\mathscr{B}(H \to L\mu\nu)}{\mathscr{B}(H \to Le\nu)}$$

- CKM factors cancel in the ratio
  → pure theoretical SM predictions are available
- Theoretical uncertainties cancel in the ratio
  → lattice QCD gives very precise results

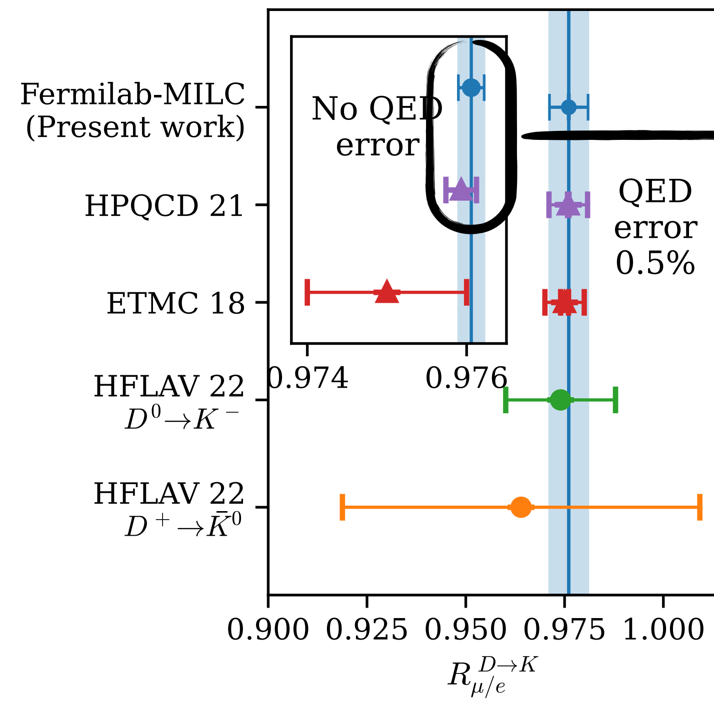
LQCD

Experiment

$D \to \pi$



$D \to K$



LQCD agreement at ≈ 0.01%

D-meson Semileptonic Decays

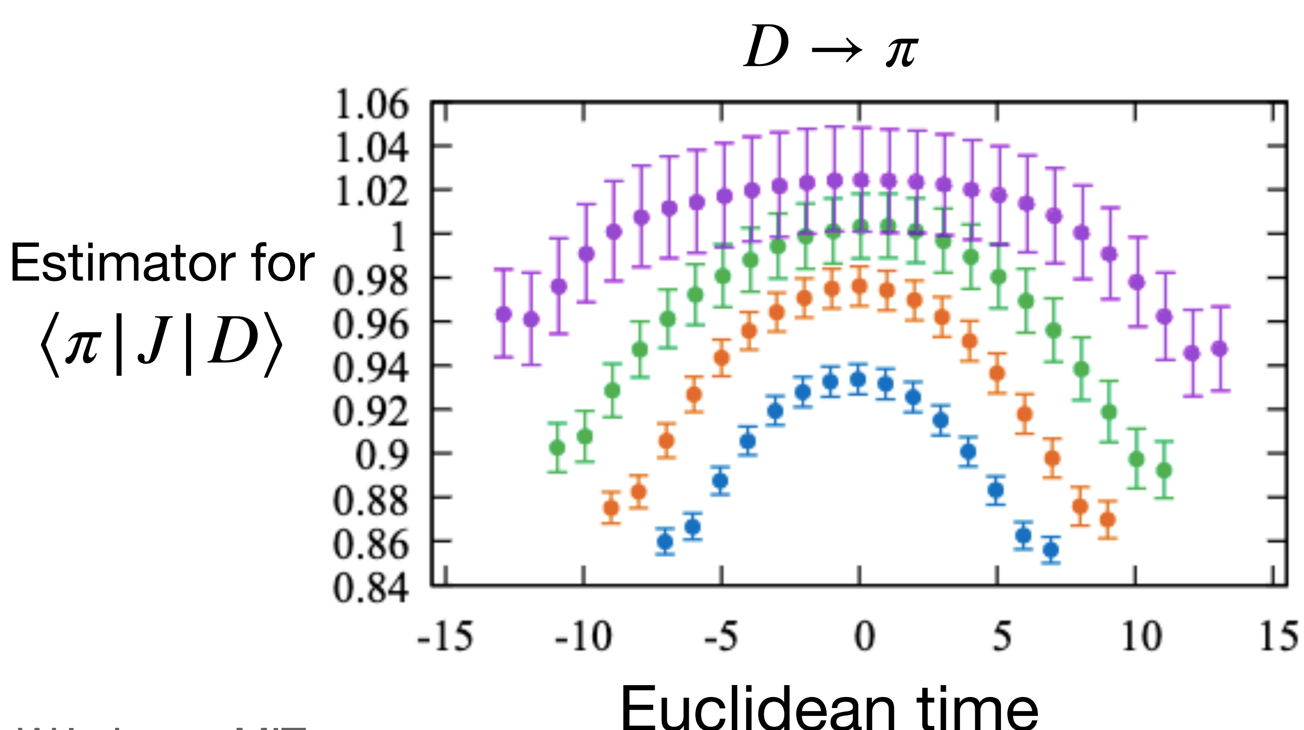
Unpublished & In-progress Calculations

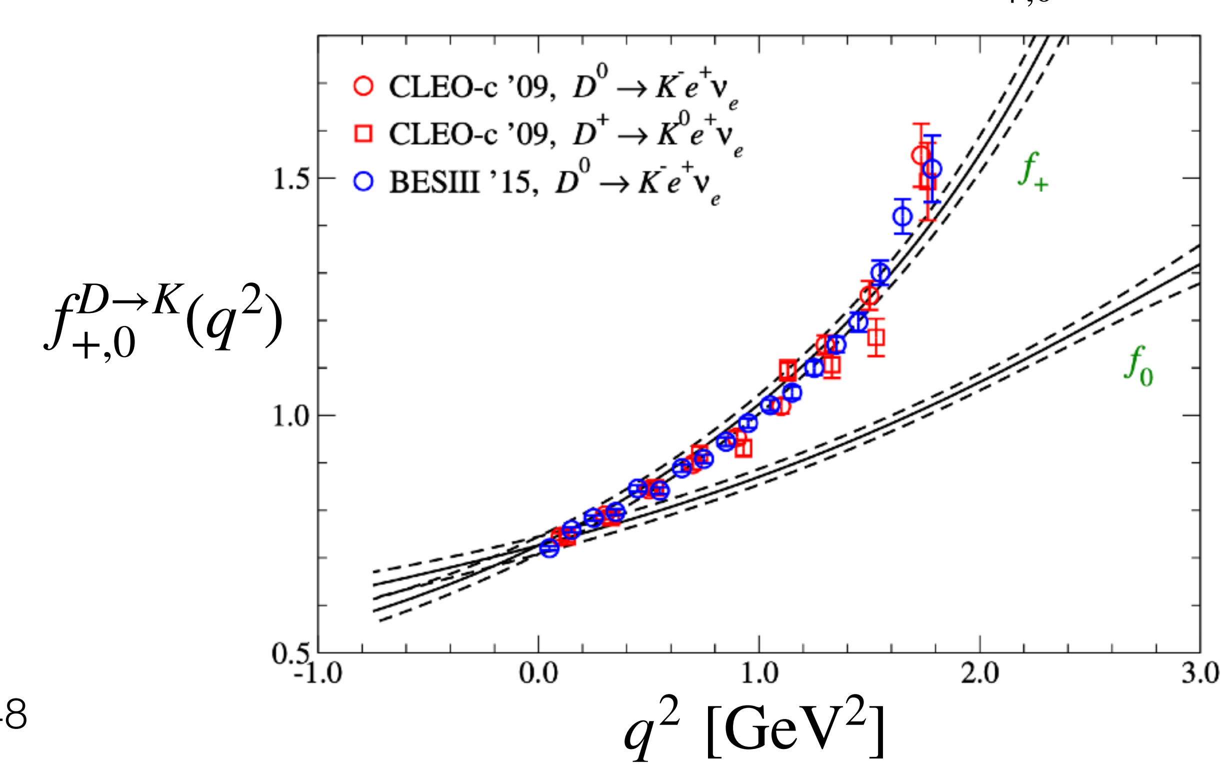
RBC/UKQCD @ Lattice 2021 [arXiv:2201.02680]

- (N_f=2+1) RBC/UKQCD domain-wall quarks
- Valence: domain wall
- Preliminary results on a single ensemble: 1/a ≈ 1.78 GeV
- Results indicate that percent-scale errors are achievable
- Plans in place to extend calculation to additional ensembles
- Precise DWF results will give a valuable check on the recent HISQ results for $D_{(s)} \to K/\pi \ell \nu$

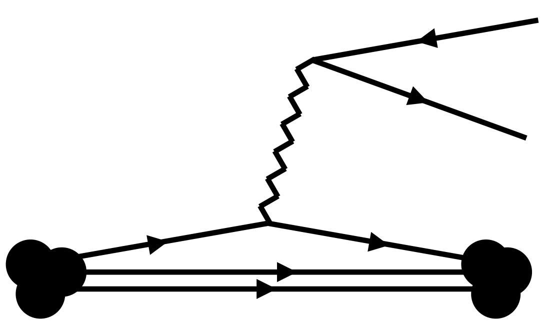
JLQCD @ Lattice 2017 [arXiv:1711.11235]

- Unpublished but quite mature/complete results
- (N_f=2+1) JLQCD ensembles with domain-wall quarks
  - 14 total ensembles
  - $1/a \in \{2.5,\ 3.6,\ 4.5\}$ GeV
  - $M_\pi \in [230,\ 500]$ MeV
  - Valence: domain wall
- Form factors in the continuum limit are reported
- Excellent control over systematic effects
- $f_{+,0}^{D\to K/\pi}(0)$ at $\approx 6\,\%$ precision
- 1σ agreement with recent HISQ results for $f_{+,0}^{D\to K/\pi}(0)$





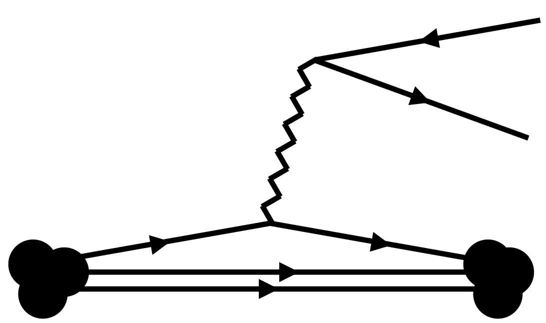
Semileptonic Decays
of D-baryons



$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \ell\nu$
	$K \to \pi\ell\nu$	$B \to \pi\ell\nu$
		$\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^{\star}\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$	$\Lambda_b \to \Lambda_c\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$	
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d \vert \bar{B}_d \rangle)$	$\langle B_s \vert \bar{B}_s \rangle)$	

Semileptonic Decays

of D-baryons



$$\langle \Lambda \,|\, \bar{s}\gamma^{\mu}(1-\gamma_5)c \,|\, \Lambda_c \rangle$$

Vector form factors: $f_{+,0,\perp}$

Axial form factors: $g_{+,0,\perp}$

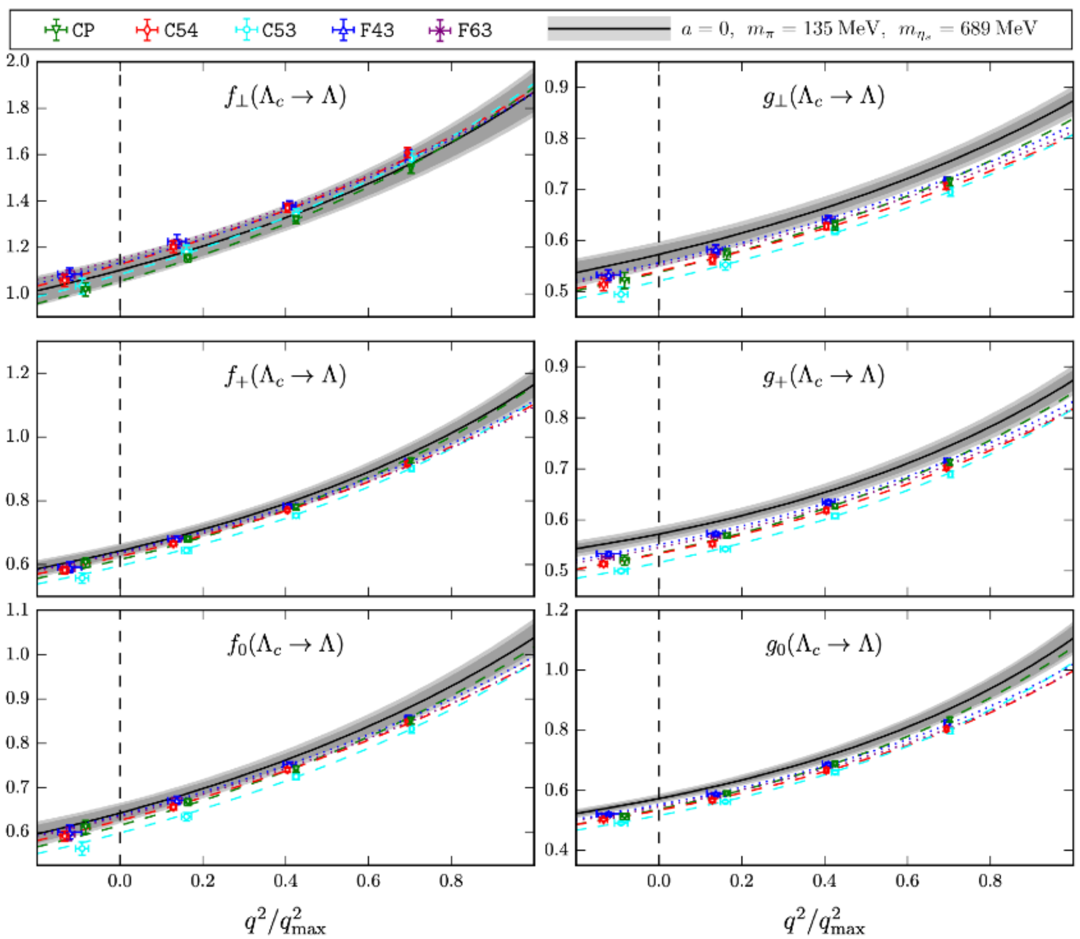
D-baryon semileptonic decays

S. Meinel

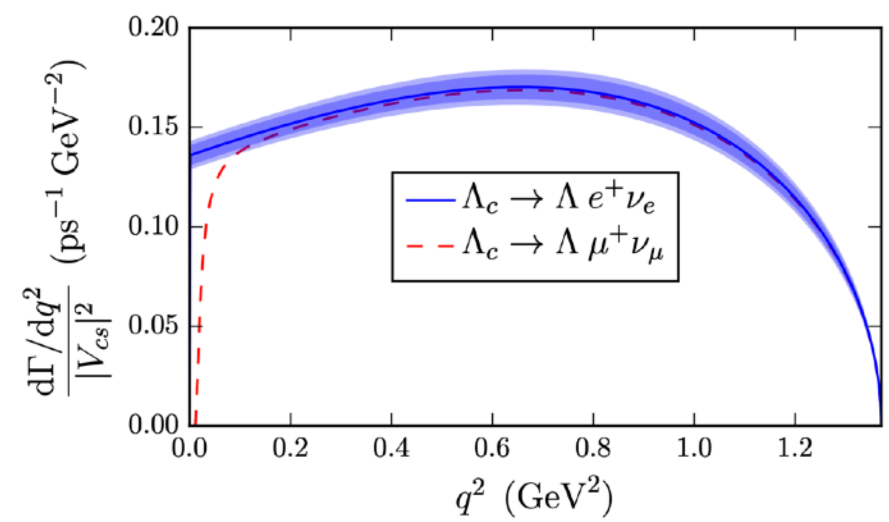
PRL 118 (2017) 8, 082001
arXiv:1611.09696

$\Lambda_c \to \Lambda \ell \nu$

- 5x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{139 - 350\}$ MeV
- Valence charm: Columbia RHQ (clover action, tuned to give J/ψ dispersion relation)
- "Mostly non-perturbative" renormalization
- First-ever determination of $|V_{cs}|$ [≈6%] from baryon decays when combined with measurements from BESIII



$$|V_{cs}| = \begin{cases} 0.951(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(56)_{\mathcal{B}}, & \ell = e, \\ 0.947(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(72)_{\mathcal{B}}, & \ell = \mu, \\ 0.949(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(49)_{\mathcal{B}}, & \ell = e, \mu, \end{cases}$$

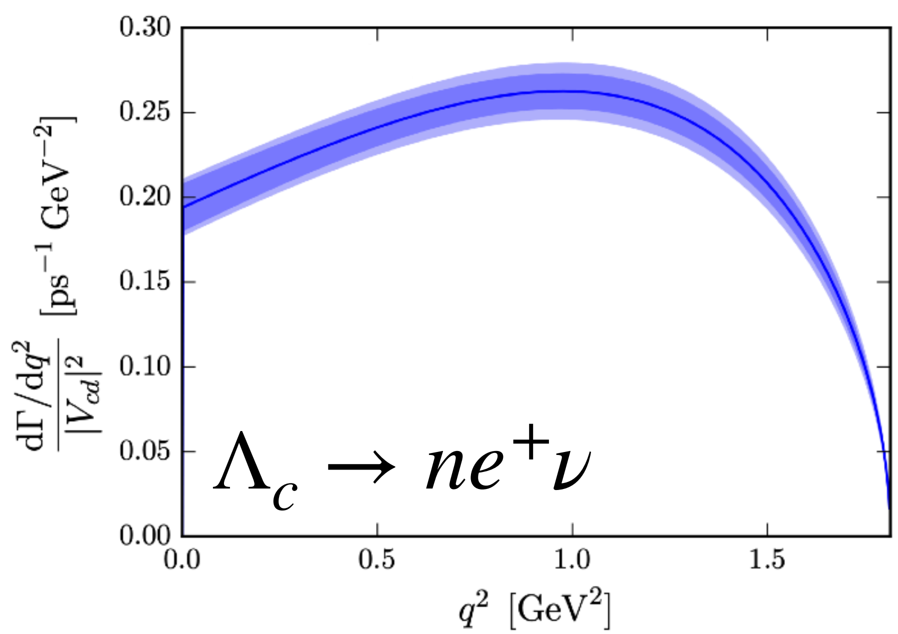


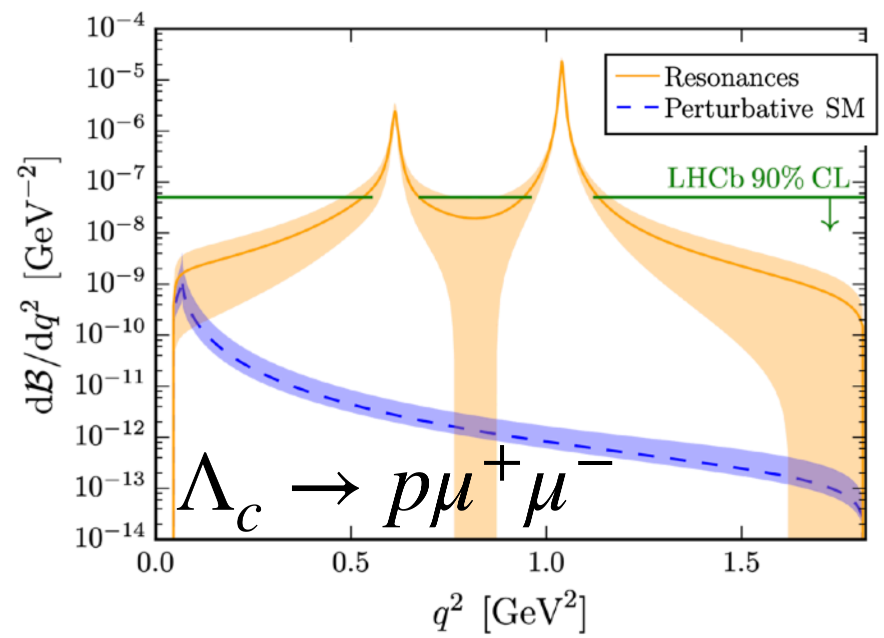
D-baryon semileptonic decays

S. Meinel
PRD 97 (2018) 3, 034511
arXiv:1712.05783

$\Lambda_c \to N$ form factors

- Isospin limit: same form factors for $\Lambda_c \to p^+$, $\Lambda_c \to n$
- 6x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{240 - 350\}$ MeV
- Valence charm: Columbia RHQ
- "Mostly non-perturbative" renormalization
- SM predictions for charged-current $\Lambda_c \to n\ell^+\nu$ rates [≈6.4%]
  - $\Gamma(\Lambda_c \to ne^+\nu)/|V_{cd}|^2 = (0.405 \pm 0.016_{\text{stat}} \pm 0.020_{\text{syst}})\ \text{ps}^{-1}$
  - Tough to measure experimentally (n and ν in final state)
  - Results larger by factor of ≈1.5–2 compared to other calculations [quark models, sum rules, SU(3)]
- Rare neutral-current decay:
  - LHCb 2018: $\mathcal{B}(\Lambda_c \to p^+\mu^+\mu^-) < 7.7 \times 10^{-8}$ [90%]
  - Comparison to LQCD with additional assumptions
    - SM Wilson coefficients at NLO
    - Breit-Wigner model for intermediate $\phi/\omega/\rho$





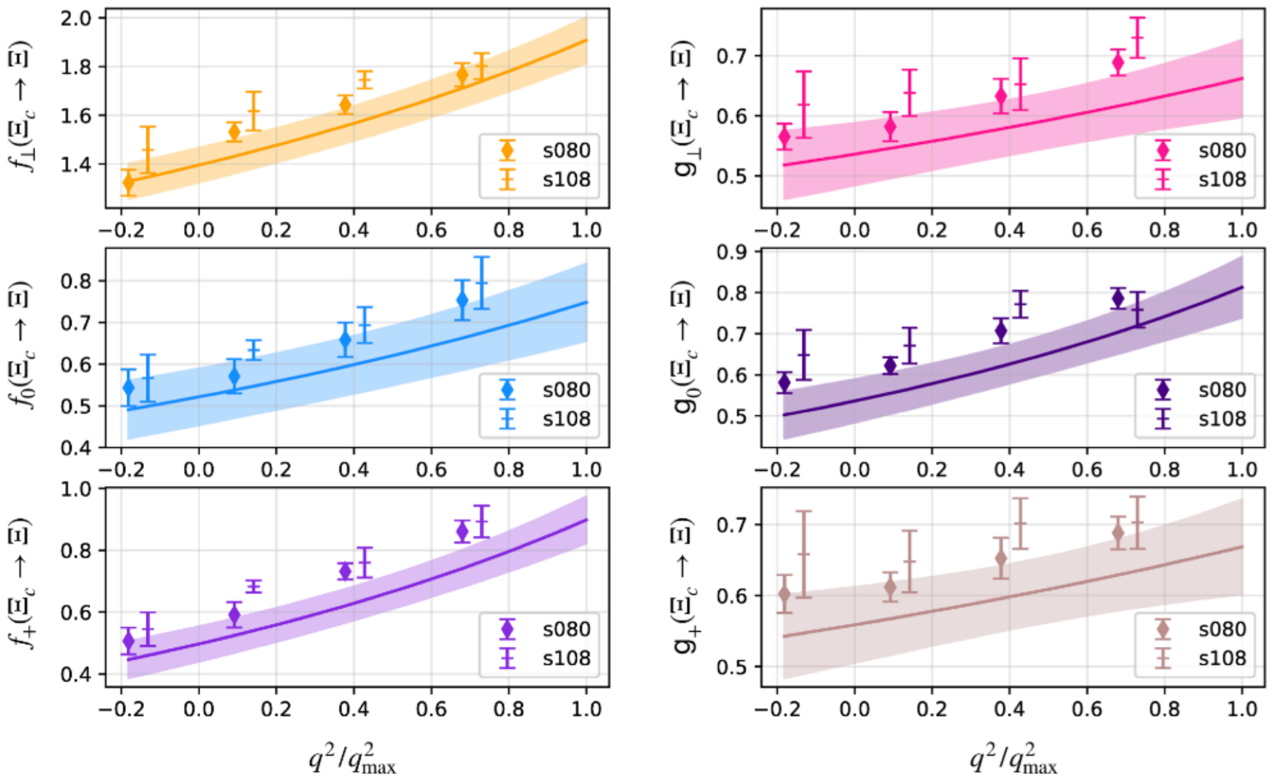
LHCb
PRD 97 (2018) 9, 091101
arXiv:1712.07938

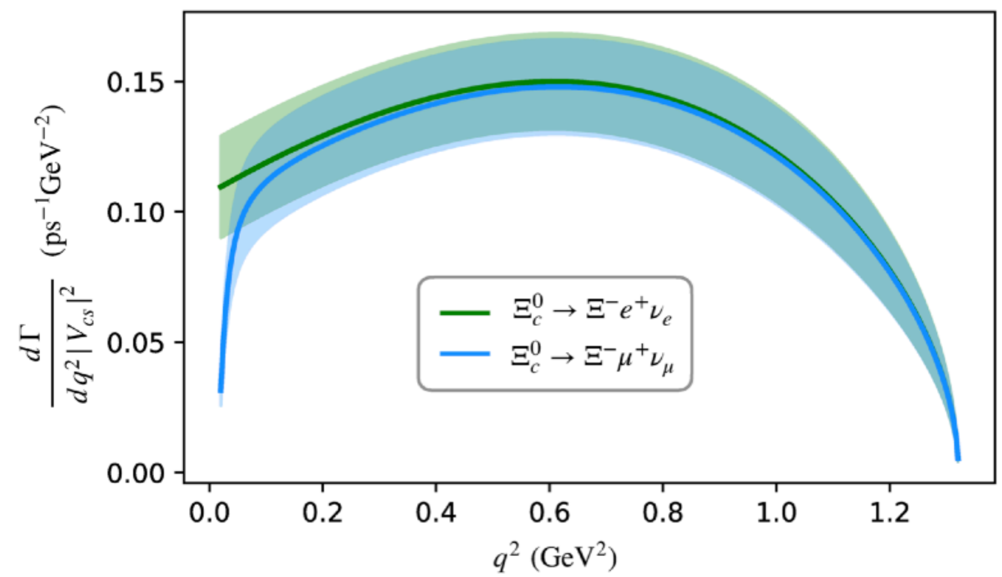
D-baryon semileptonic decays

$\Xi_c \to \Xi \ell \nu$ form factors

Q.-A. Zhang et al.
Chin.Phys.C 46 (2022) 1, 011002
arXiv:2103.07064

- 2x ensembles with N_f=2+1 Wilson clover quarks
  - $a \in \{0.11, 0.08\}$ fm
  - $M_\pi \approx 300$ MeV
- Continuum extrapolation is given
- No chiral extrapolation to physical pion mass
- Extractions of $|V_{cs}|$:
  - Using ALICE branching-fraction measurements: $|V_{cs}| = 0.983(0.060)^{\text{stat}}(0.065)^{\text{syst}}(0.167)^{\text{exp}}$ [≈19%]
  - Using Belle branching-fraction measurements $|V_{cs}| = 0.834(0.051)^{\text{stat}}(0.056)^{\text{syst}}(0.127)^{\text{exp}}$ [≈18%]





Summary & Outlook

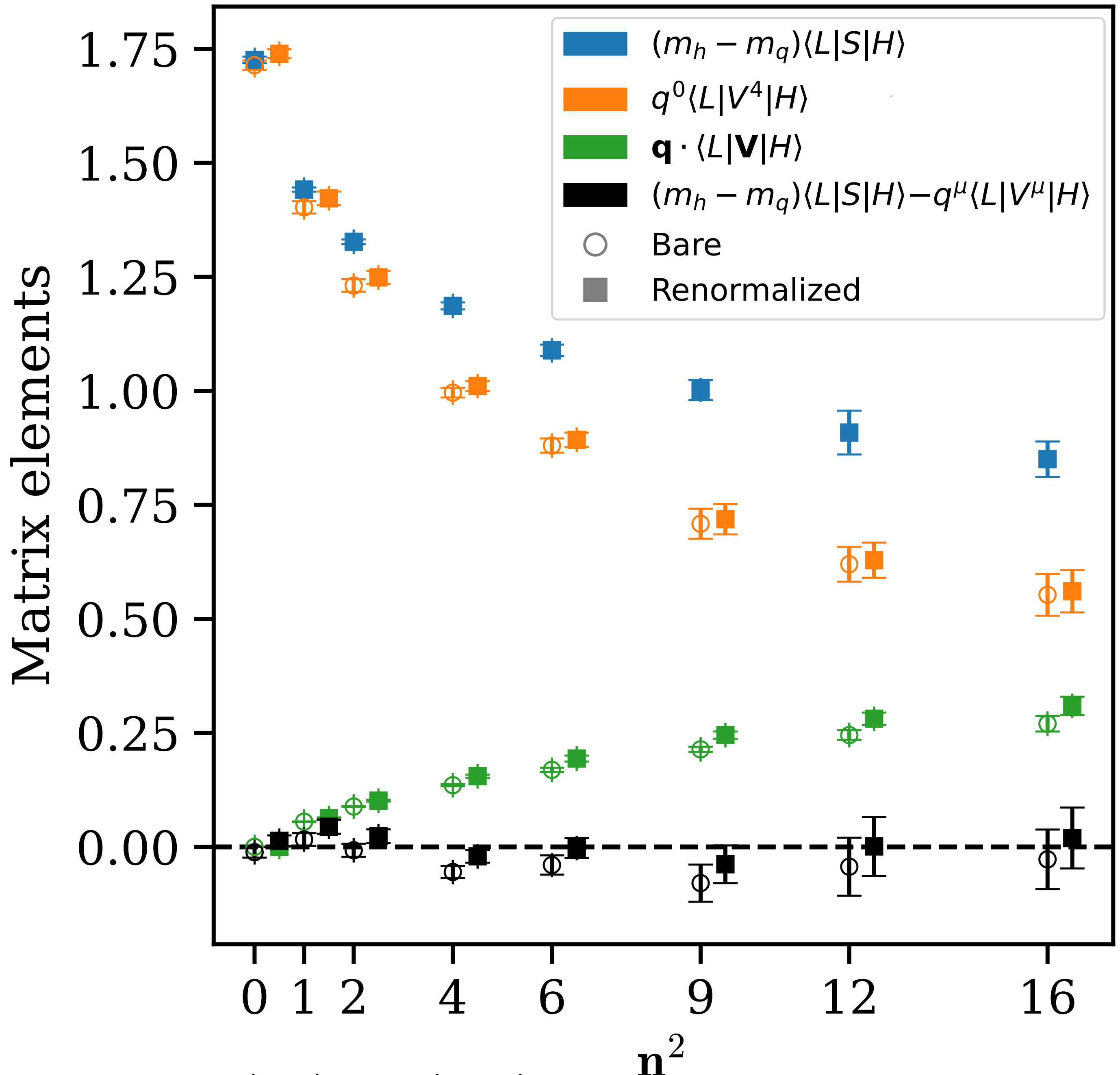
- **Lattice QCD calculations have achieved:**
  - Subpercent precision for leptonic decays
  - Percent level precision for D-meson semileptonic decays
  - 5-20% precision for D-baryon semileptonic decays
- **Enabling "technologies" for high precision include:**
  - Ensembles with physical mass pions: $M_\pi \approx 140\ \mathrm{MeV}$
  - Relativistic light-quark action(s) for charms: absolutely normalized currents
  - Highly improved actions: small discretization effects for charm
- **Precise LQCD + latest experimental results give:**
  - CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ at O(1%)
  - Improved tests of second-row unitarity
  - Precise SM predictions of LFU ratio

Backup

Renormalization semileptonic decays

Example $D \to \pi \ell \nu$

- Recall $\mathcal{J} = Z_J J$
- PVCV: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$
- For the HISQ action, the local scalar density is absolutely normalized.
- Imposing PCVC in a global fit gives values for Z_{V_0} and Z_{V_i}
- In terms of D→π matrix elements, PCVC reads:

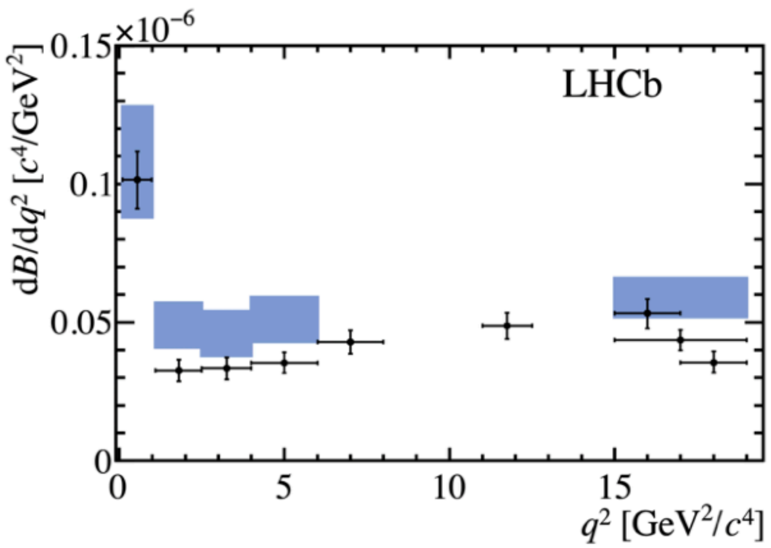


$$Z_{V^0}(M_D - E_\pi)\langle\pi|\,V^0\,|D\rangle + Z_{V^i}\,\boldsymbol{q}\cdot\langle\pi|\,\boldsymbol{V}\,|D\rangle$$
$$= (m_c - m_d)\langle\pi|\,S\,|D\rangle$$

Branching fraction tensions

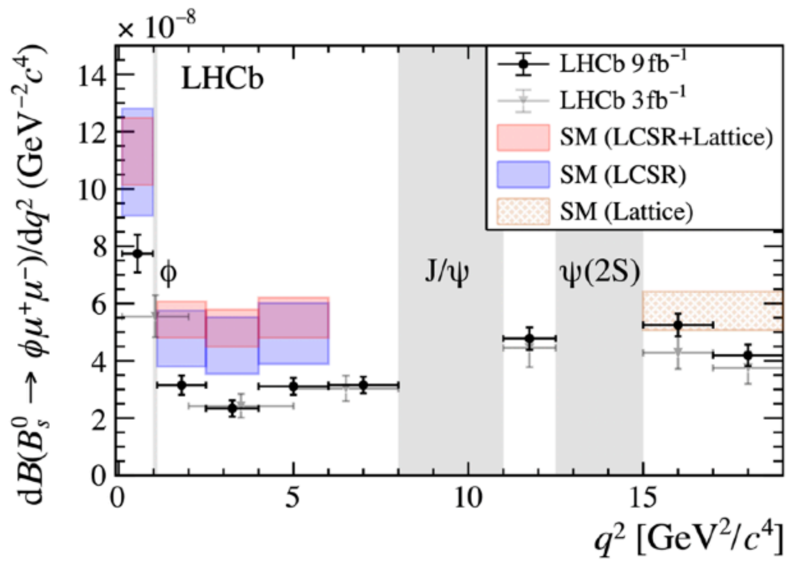
$B^0 \to K^{\star 0}\mu\mu$

LHCb *JHEP* 11 (2016) 047
LHCb *JHEP* 04 (2017) 142



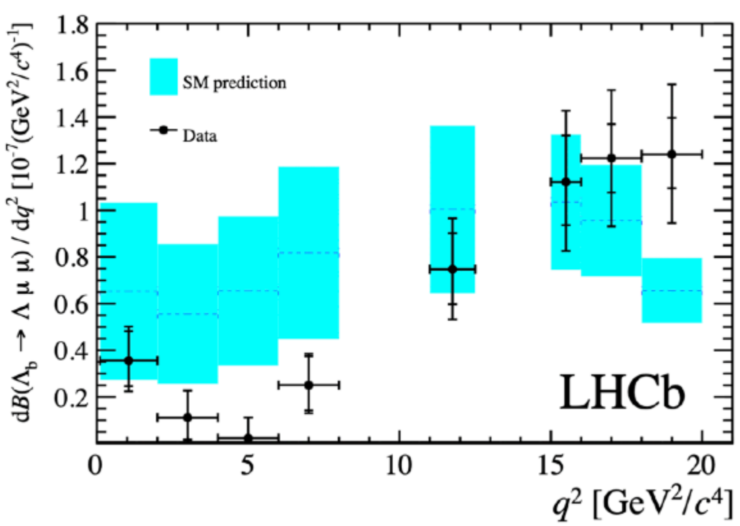
$B_s^0 \to \varphi\mu\mu$

LHCb *JHEP 09 (2015) 179*
LHCb *PRL* 127 (2021) 15, 151801

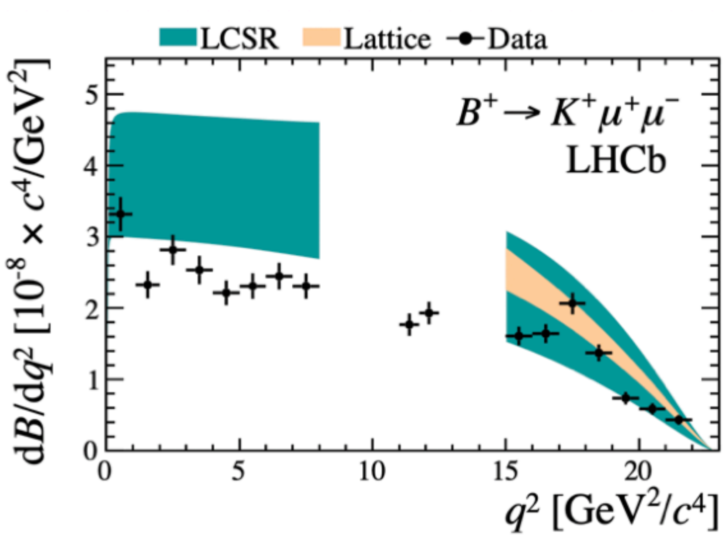


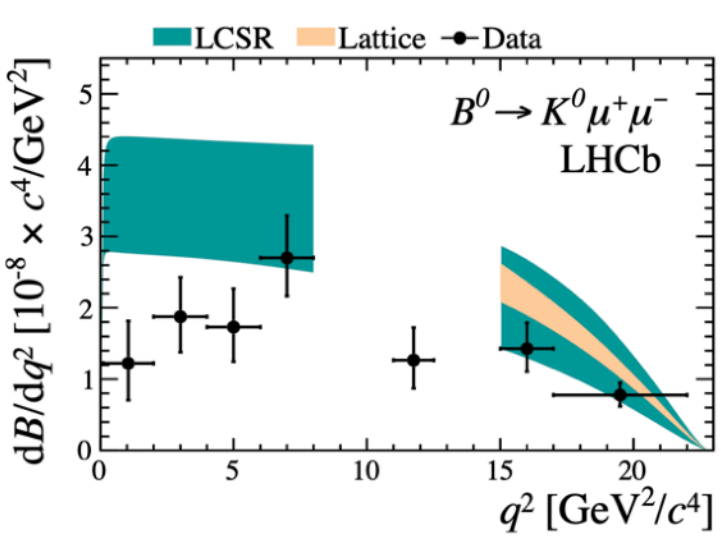
$\Lambda_b^0 \to \Lambda^0\mu\mu$

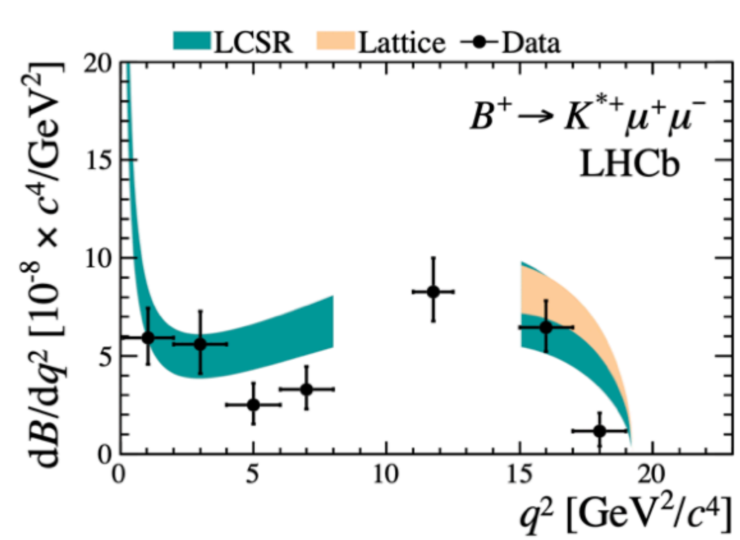
LHCb *JHEP* 06 (2015) 115



LHCb *JHEP* 06 (2014) 133



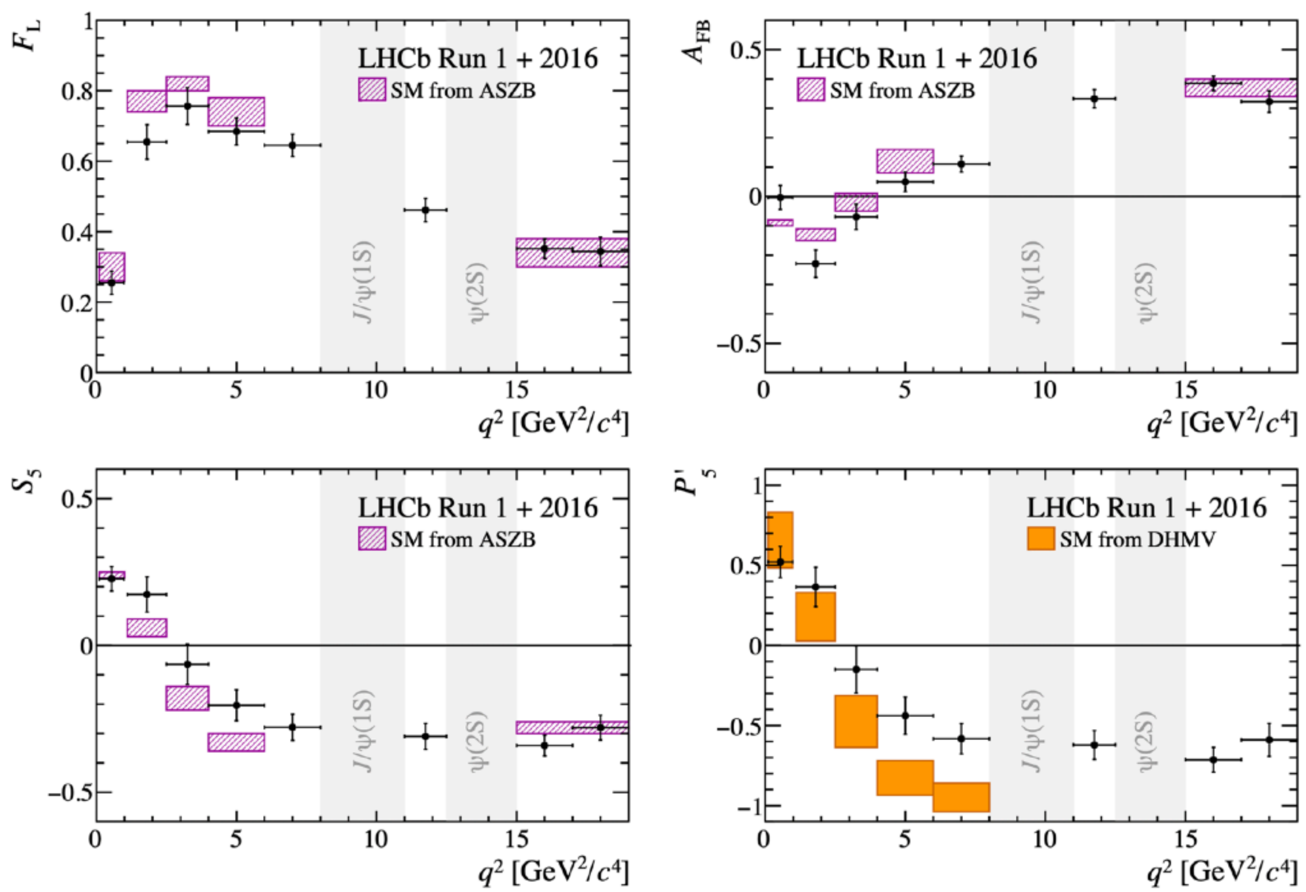




Angular Tensions

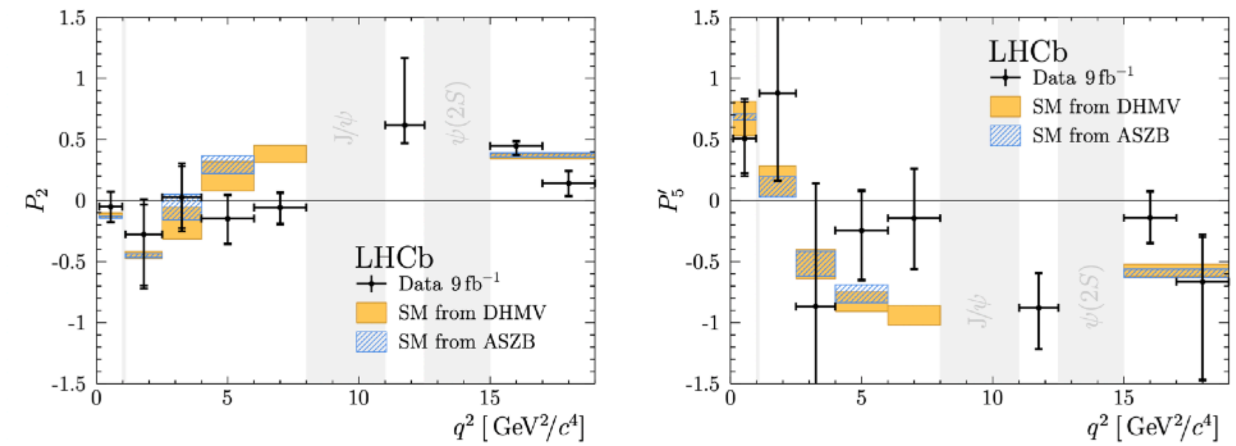
$B^0 \to K^{\star 0}\mu\mu$

LHCb *PRL* 125 (2020) 011802



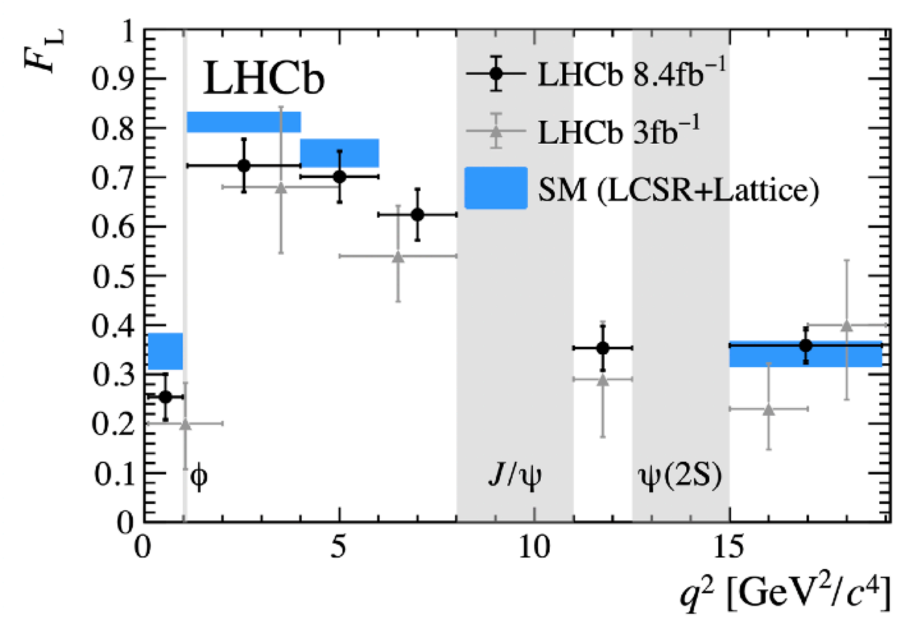
$B^+ \to K^{\star +}\mu\mu$

LHCb PRL 126 (2021) 161802



$B_s^0 \to \varphi\mu\mu$

LHCb *JHEP* 11 (2021) 043

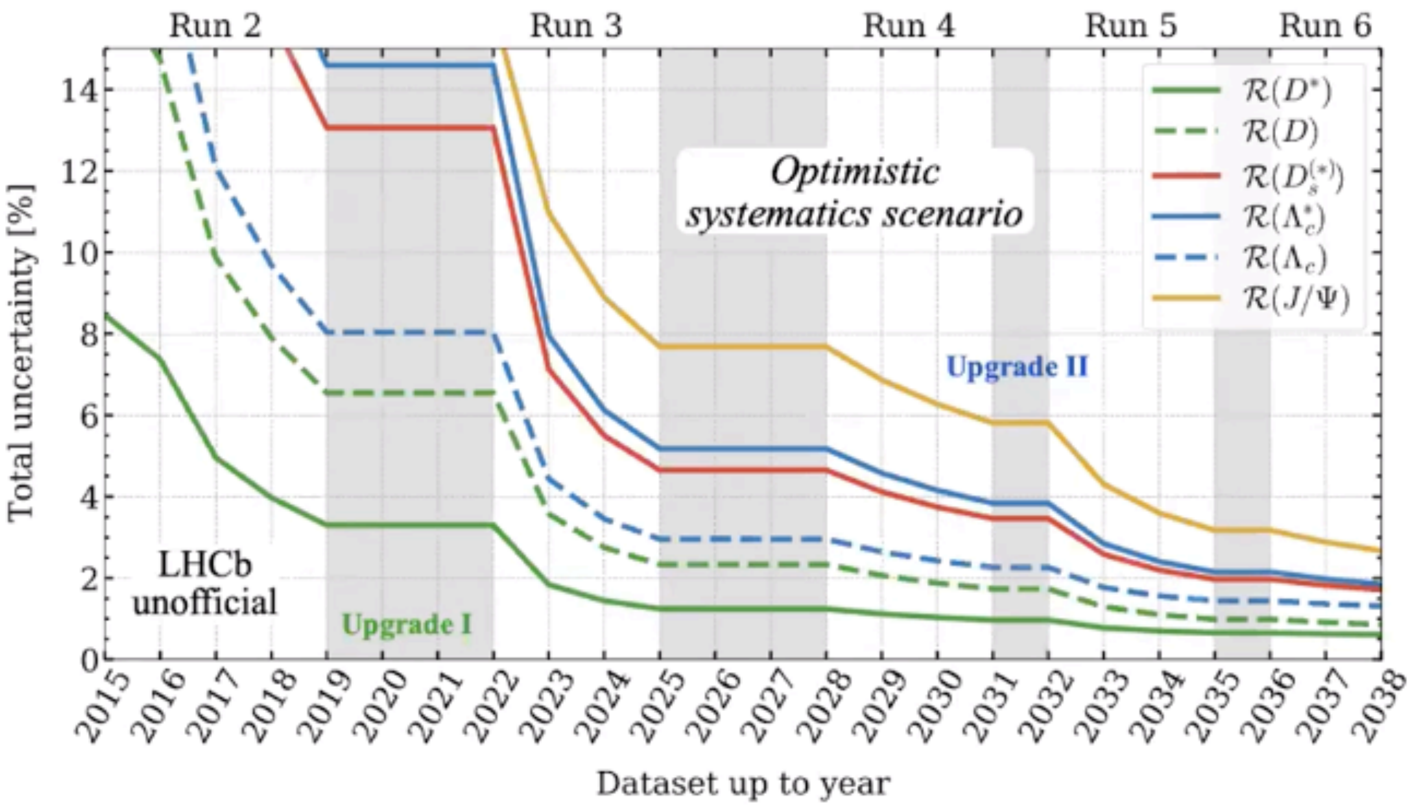


Note: Evidence for LFUV in $b \to s\ell\ell$ is gone after LHCb arXiv:2212.09153

Culprit: residual from mis-ID of hadronic backgrounds

Additional Experimental Prospects

- LHCb: pp at LHC
  - ~10^{12} b-hadrons to date (cf. ~10^{7} at LEP)
- Belle II: e^+e^- around Υ(4s) ~ 10.5 GeV
  - Goal: 50 ab^{-1} (50x Belle), roughly 215 fb^{-1} to date



Many exciting first measurements.
For example:

- BESIII: Form factors for $D_s \rightarrow K^{(*)} e \nu$
  - PRL 122, 061801 arXiv:1811.02911
- LHCb: Rare CKM suppressed $B \rightarrow \pi \mu^+ \mu^-$
  - JHEP 10 (2015) 034 arXiv:1509.00414

LQCD precision achievements over time

CSS2013: Snowmass on the Mississippi
S. Butler et al [arXiv:1311.1076]

2013 | **2013 Expected** | **Achieved**

Quantity	CKM element	~~Present~~ expt. error	2007 forecast lattice error	~~Present~~ lattice error	2018 lattice error	2021 FLAG avg
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.5%	0.15%	0.18%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.5%	0.2%	0.18%
f_D	$\|V_{cd}\|$	4.3%	5%	2%	< 1%	0.3%
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	2%	< 1%	0.2%
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%	4.4%
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%	0.6%
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	< 1%	1.7%
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%	3%
f_B	$\|V_{ub}\|$	9%	–	2.5%	< 1%	0.7%
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2-4%	4%	< 1%	1.3%
ΔM_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%	4.5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	< 1%	1.3%

Systematic inclusion of QED now becomes necessary

Recently improved!

Broad community effort to:
- keep pace with experimental needs
- achieve ~1% precision

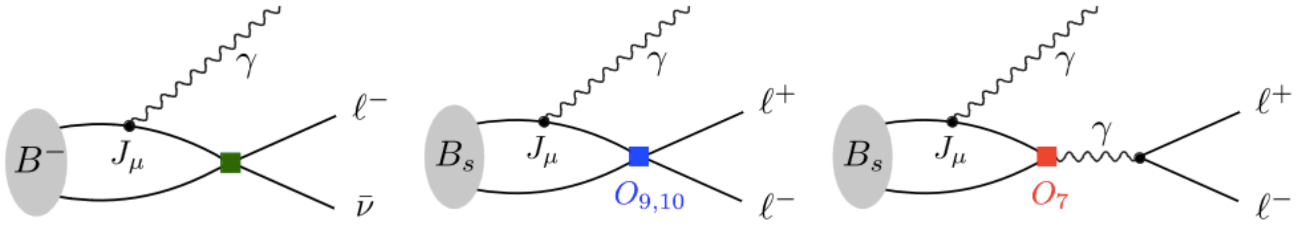
- LQCD precision: expected improvements from ~10 years ago have largely been achieved.
- In-progress calculations expect to reach ≲ 1% level for semileptonic B-decays

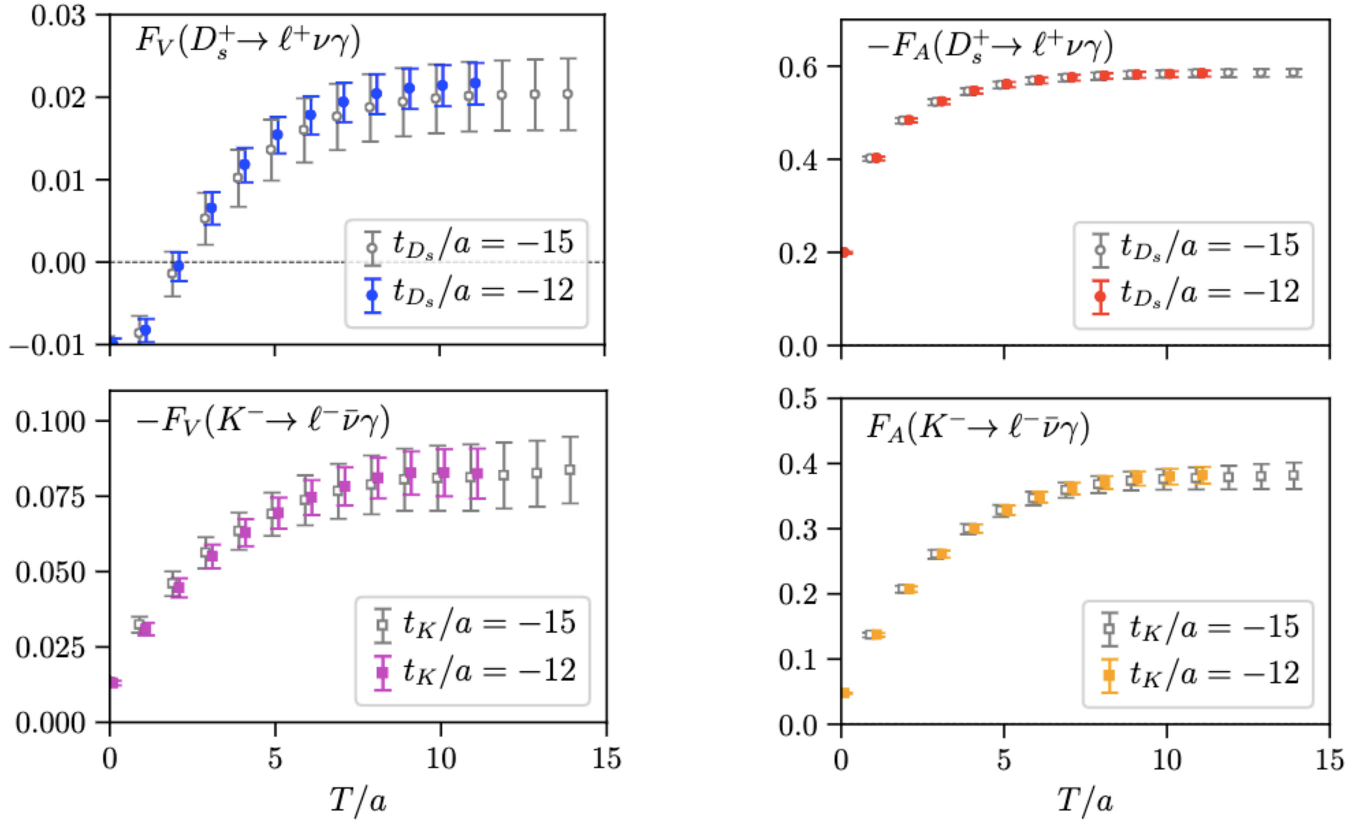
Radiative Leptonic Decays

Kane, Lehner, Meinel, Soni
Lattice 2019
arXiv:1907.00279
Kane, Giusti, Lehner, Meinel, Soni
Lattice 2021
arXiv:2110.13196

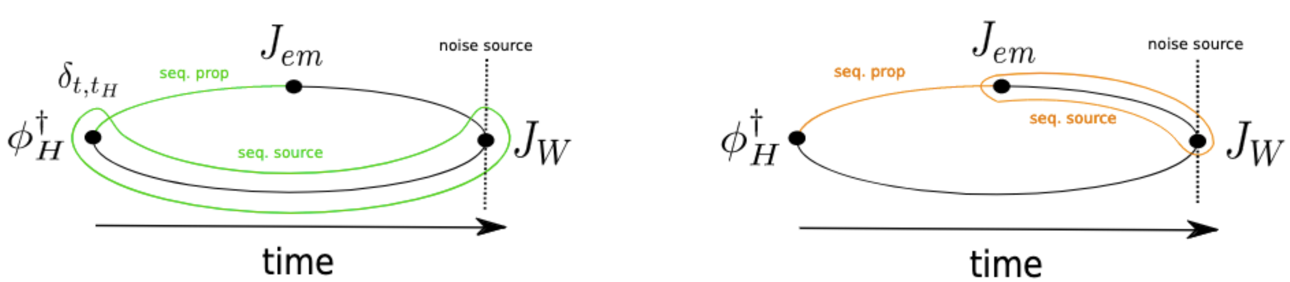
$D_s \to \ell \nu \gamma, K \to \ell \nu \gamma$

- Radiative decays probe weak interaction and hadronic structure
- Example: $B \to \ell \nu \gamma$ is sensitive to the LCDA parameter λ_B
- Radiative leptonic decays probe all Wilson coefficients in the Weak effective Hamiltonian
- Exploratory calculations developing methods



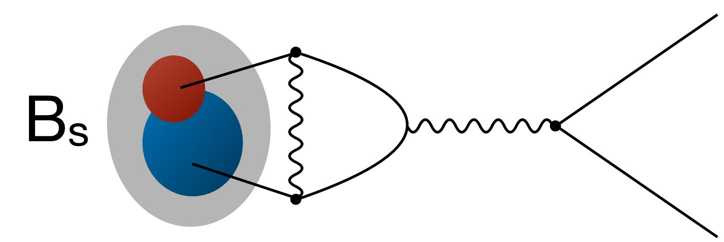


$$T_{\mu\nu} = -i \int d^4 x e^{ip_\gamma \cdot x} \langle 0 | T\{J_\mu(x) J_\nu^{\text{weak}}(0)\} | H(\mathbf{p}) \rangle$$



Rare Decay $B_s \to \mu^+\mu^-$

Uncertainty Breakdown



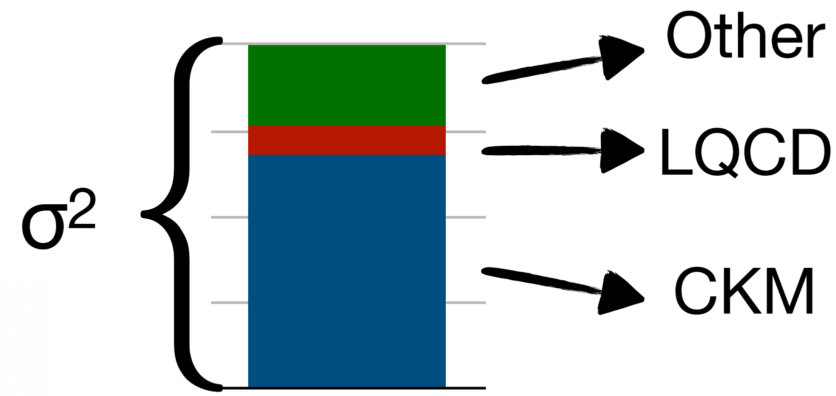
SM prediction for rare leptonic decay rate

Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$

$$\overline{\mathrm{Br}}^{(0)}_{s\mu} = \begin{pmatrix} 3.599 \\ 3.660 \end{pmatrix} \Big[1 + \begin{pmatrix} 0.032 \\ 0.011 \end{pmatrix}_{f_{B_s}} + 0.031|_{\mathrm{CKM}} + 0.011|_{m_t} + 0.006|_{\mathrm{pmr}} + 0.012|_{\mathrm{non\text{-}pmr}} {}^{+0.003}_{-0.005}|_{\mathrm{LCDA}} \Big] \cdot 10^{-9}$$

- Parametric uncertainties
  - Long distance (f_{B_s}) and short distance (CKM, m_t)
- Non-QED parametric (Γ_q, α_s)
- Non-QED non-parametric (μ_W, μ_b, and higher order)
- QED parametric: B-meson LCDA parameters (λ_B, $\sigma_{1,2}$)



Lattice QCD value for f_{Bs} is now a sub-dominant source of uncertainty

Chiral-continuum analysis

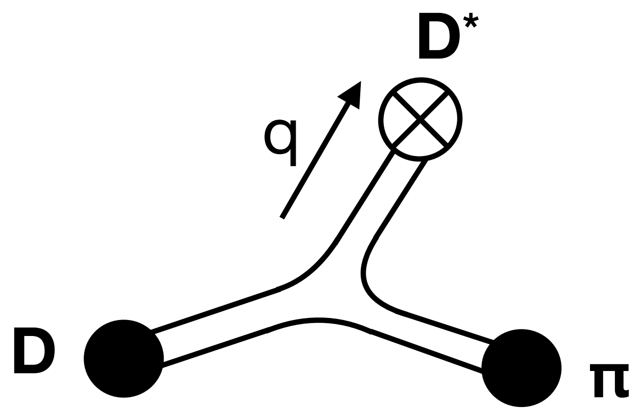
Heavy-meson rooted staggered chiral perturbation theory

- With simulations at and above the physical pion mass, the chiral fits are ***interpolations,*** not extrapolations

- The shape of the form factors can be modeled with EFT combining:
  - Chiral symmetry $\Sigma = \exp(2i\phi/f)$
  - HQET spin symmetry $H^a = \frac{1+\not{v}}{2}\left[P_\mu^{*a}(v)\gamma^\mu - P^a(v)\gamma_5\right]$
  - Light-quark discretization effects $\frac{1}{16}\sum_{\text{tastes }\xi} M_\xi^2 \log\left(\frac{M_\xi^2}{\Lambda^2}\right)$

Chiral-continuum analysis

Heavy-meson rooted staggered chiral perturbation theory



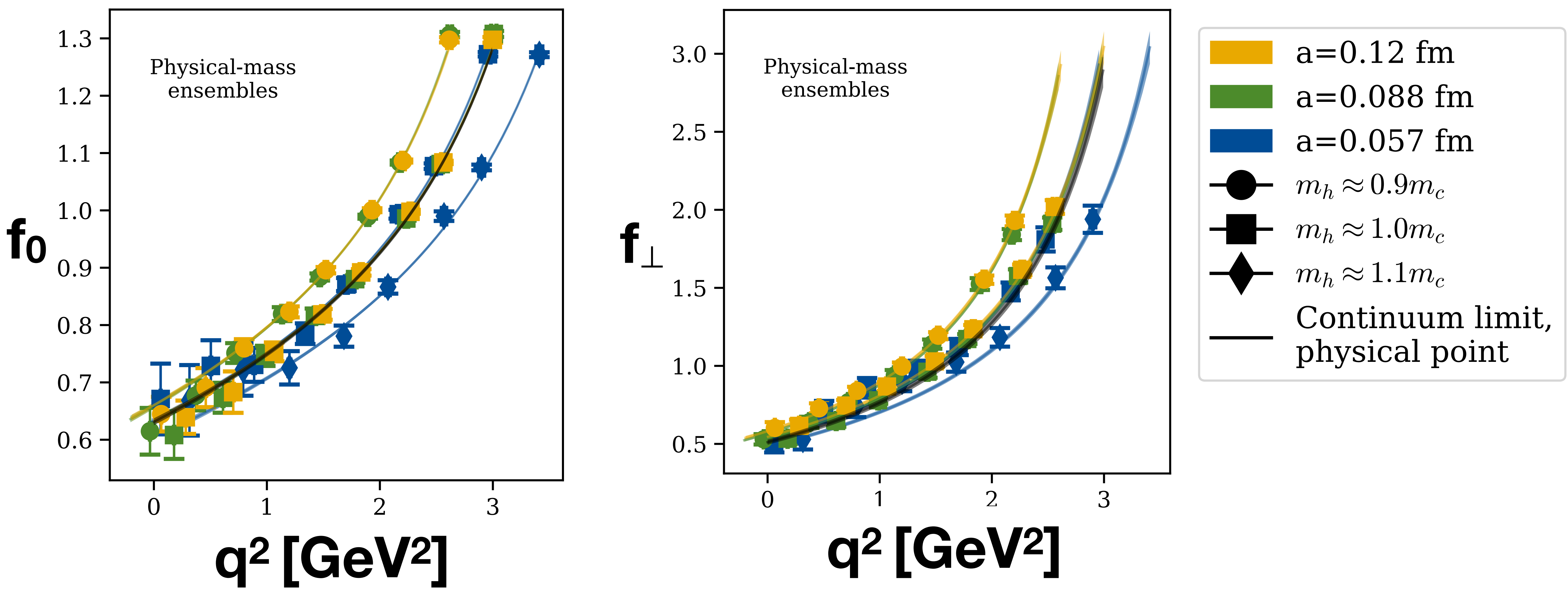
$$\sim \frac{1}{M_{D^*}^2 - q^2} \propto \frac{1}{E_\pi + \Delta}$$

$$q^2 = M_D^2 + M_\pi^2 - 2M_D E_\pi$$

- Basically: $f = \dfrac{\text{const}}{E+\Delta} \times \left(1 + \delta f_{\text{logs}} + \sum_i c_i \chi_i + \delta f_{\text{artifacts}}\right)$
- Logs computed through NLO in HMRSχPT
- Analytic terms included through N^2LO (consistent w/power counting)
- Lattice artifacts included from $O(a^2)$

Chiral-continuum analysis: D→π

Example: $f_0(q^2)$ and $f_\perp(q^2)$

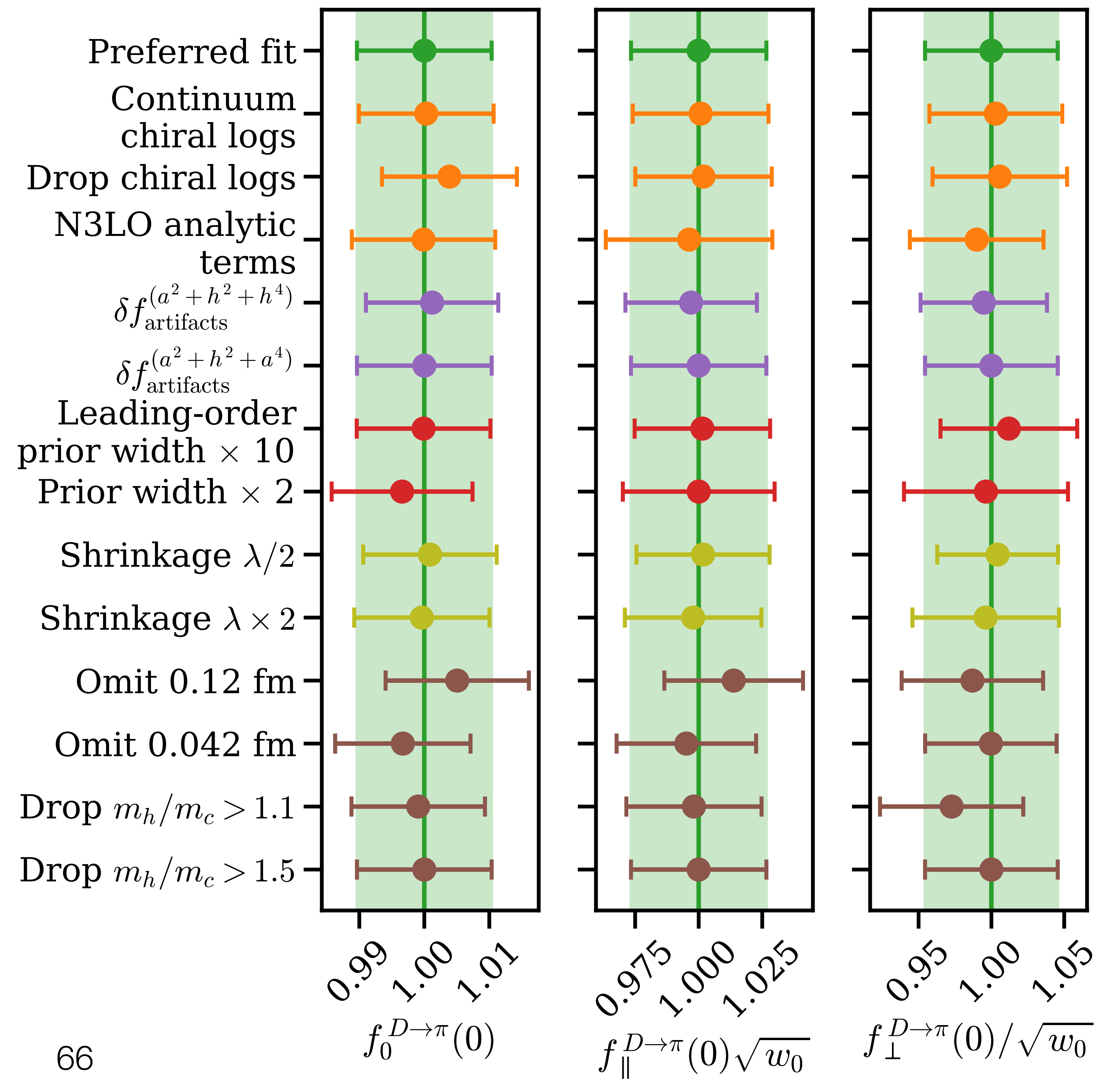


- Displayed: physical-mass ensembles only (but all ensembles included in fit)
- All fits have good quality of fit (e.g., χ^2/DOF ~ 1)
- Curve collapse at $m_h/m_c \approx 1.0$ suggests a mild approach to continuum limit

Chiral-continuum analysis: D→π

Stability of results

- Preferred analysis
- EFT variations
- Analytic discretization-term variations
- Statistical analysis variations
- Data variations



The z-expansion

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

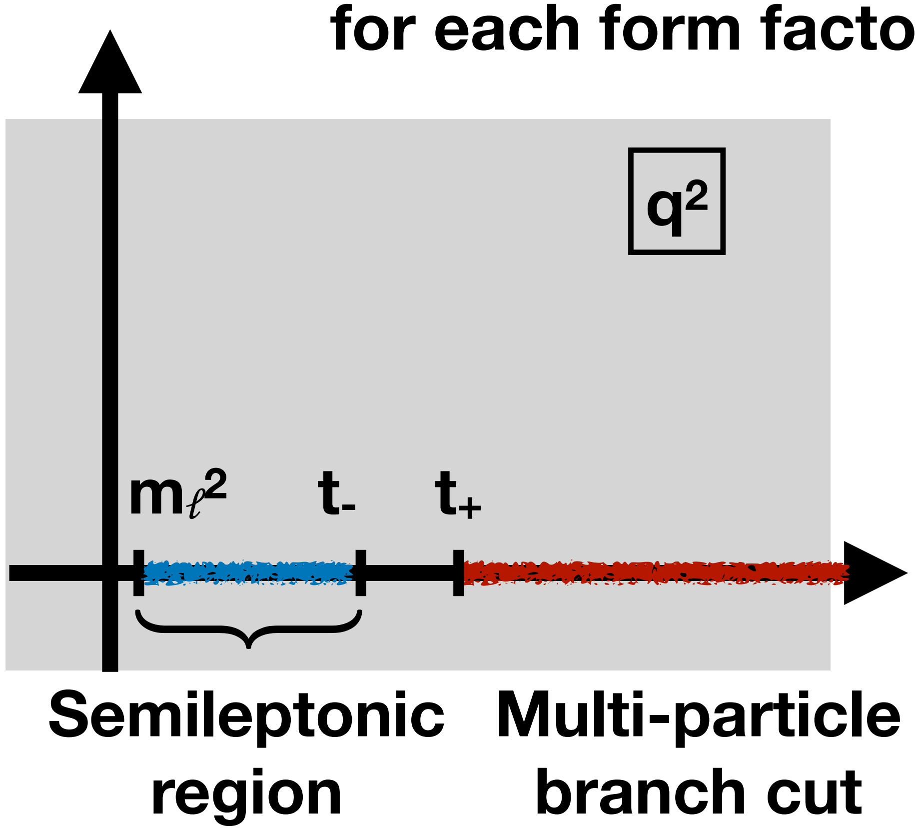
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

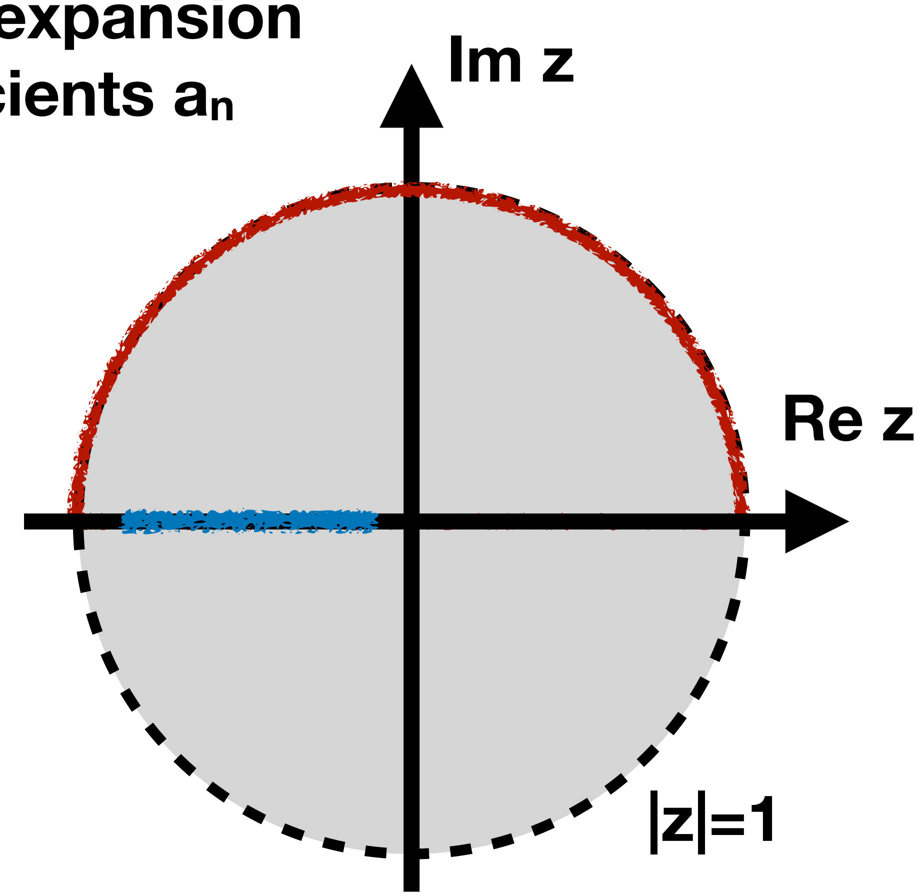
$$t_\pm = (M_D \pm M_\pi)^2$$

"Blashke factors" (contain poles)

"Outer functions" (computed analytically for each form factor)

LQCD calculations give the expansion coefficients a_n





Results at the physical point

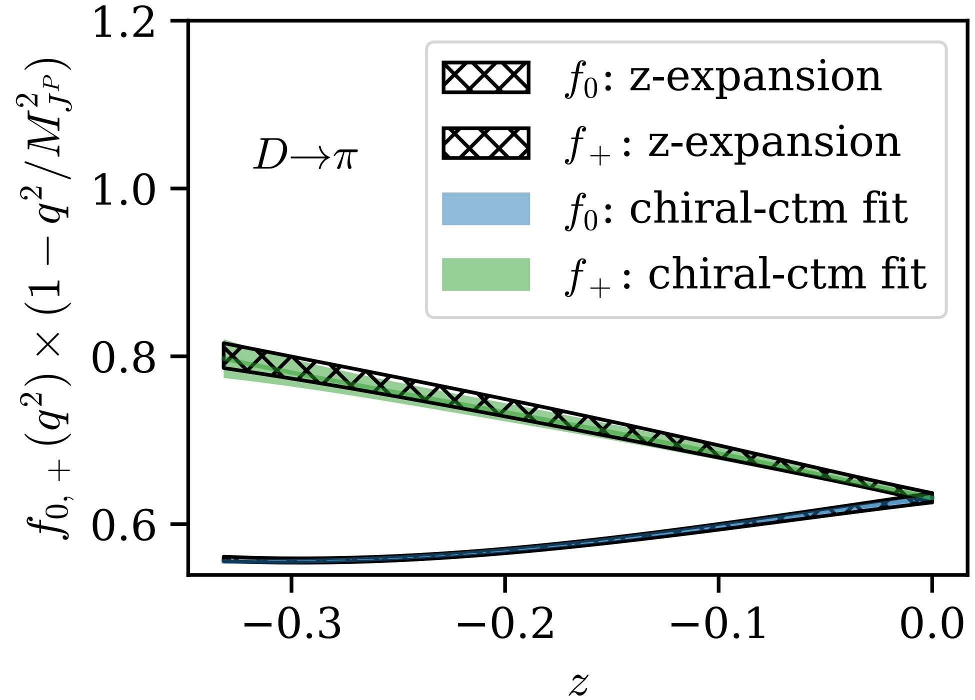
- Re-express final results using the model-independent z-expansion

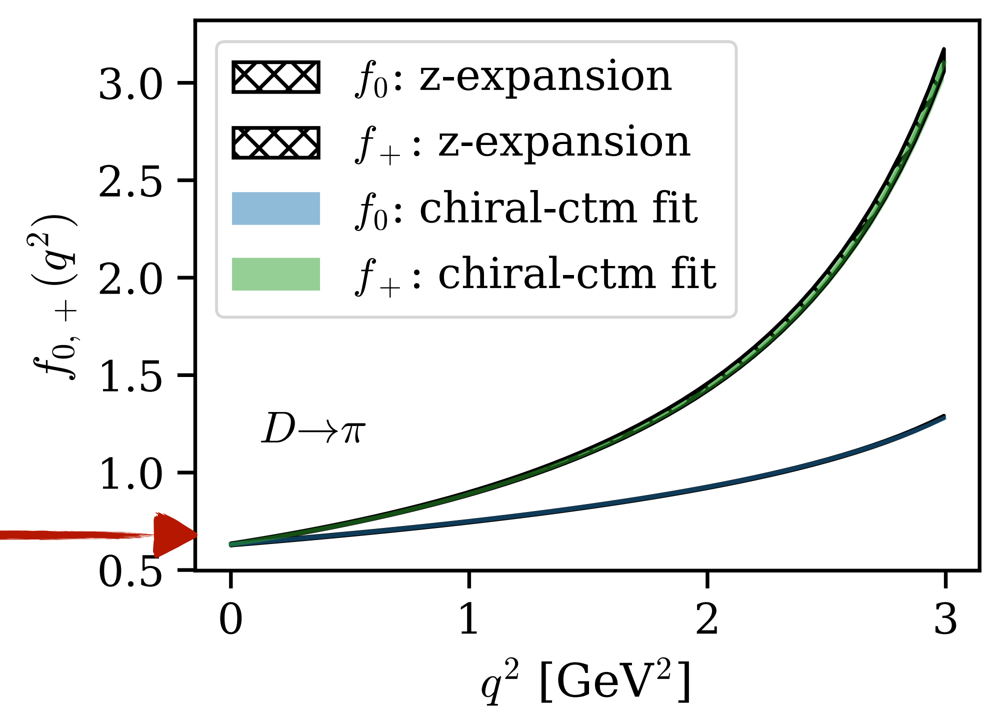
- For D-decays, the z-expansion is not an extrapolation — just a convenient change of variables

$$f_0(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{0+}^2}\right)} \sum_{n=0}^{M-1} b_m z^m,$$

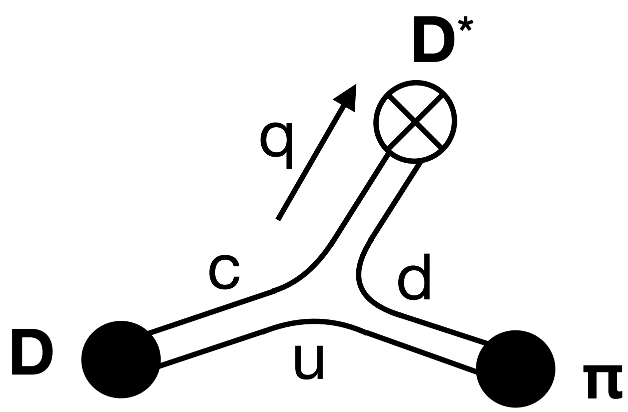
$$f_+(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{1-}^2}\right)} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right)$$

- Kinematic identity: $f_+(0) = f_0(0)$

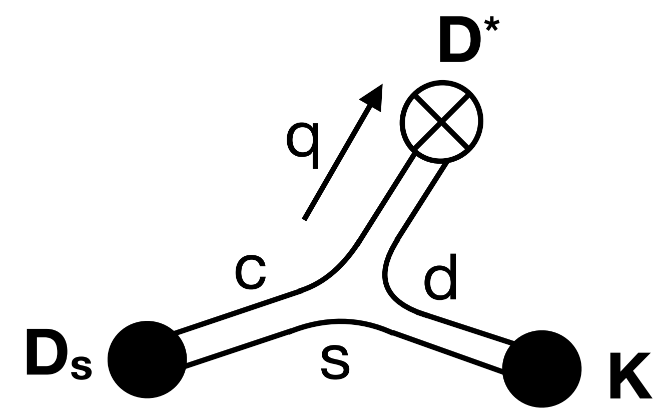




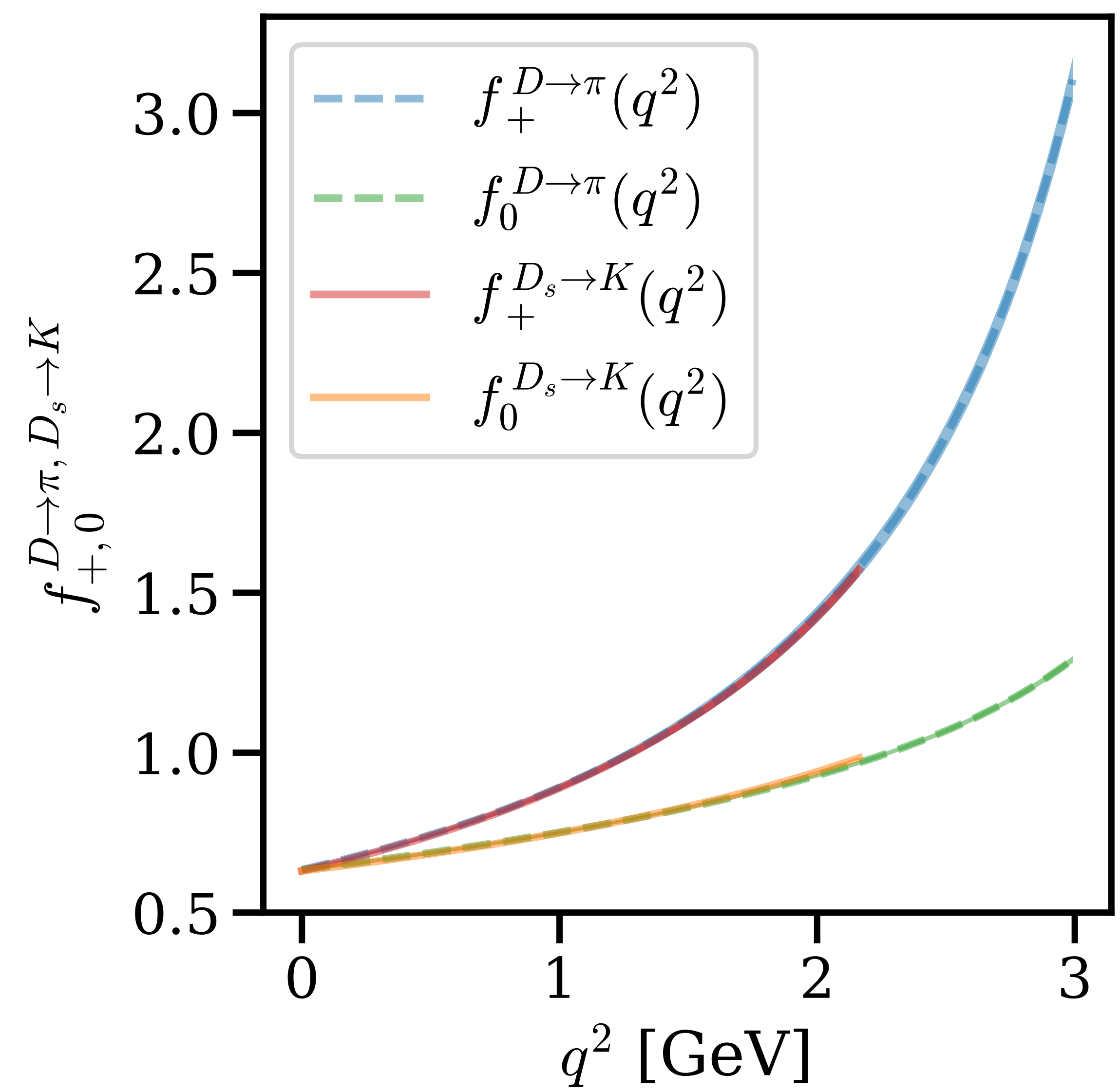
Spectator dependence: D→π vs D$_s$→K



$$\sim \frac{1}{M_{D^*}^2 - q^2} \sim$$

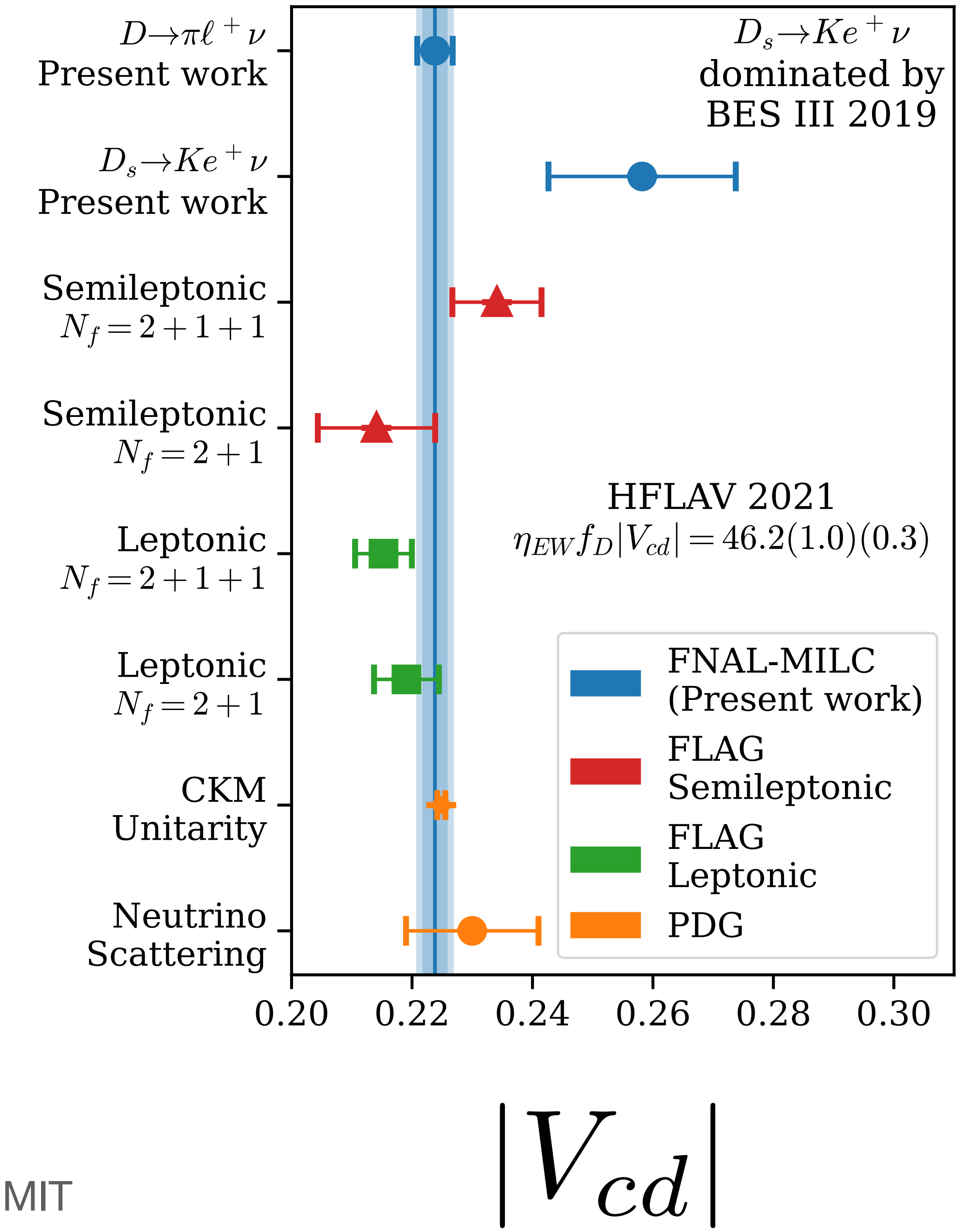


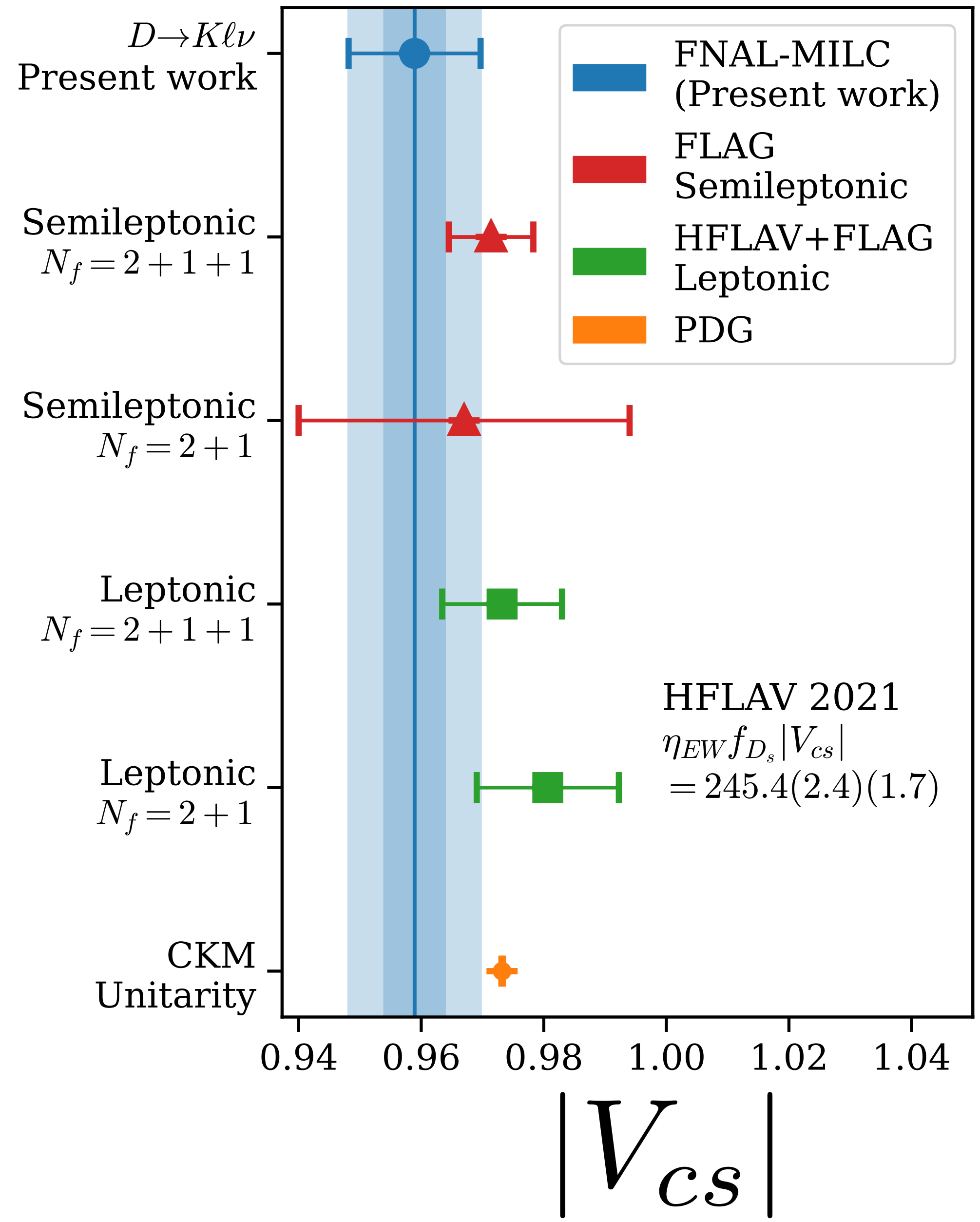
- D→π and D$_s$→K only differ by the mass of the spectator quark
- Vector and scalar form factors agree at ≲ 2% level throughout the kinematic range
- Older unpublished results by HPQCD are consistent with our findings



Comparison to Literature: $|V_{cd}|$ and $|V_{cs}|$

PDG particle data group · HFLAV · FLAG Flavour Lattice Averaging Group





Chiral-continuum fit formulae

$$f_P(E) = \frac{c_0}{E + \Delta^*_{yx,\mathrm{P}}} \times \Big[1 + \delta f_{\mathrm{P,logs}} + c_l \chi_l + c_H \chi_H + c_E \chi_E$$
$$+ c_{l^2}(\chi_l)^2 + c_{H^2}(\chi_H)^2 + c_{E^2}(\chi_E)^2$$
$$+ c_{lH}\chi_l\chi_H + c_{lE}\chi_l\chi_E + c_{HE}\chi_H\chi_E$$
$$+ \delta f^{(a^2)}_{\mathrm{artifacts}}\Big],$$

$$\chi_l = \frac{(M_\pi^{\mathrm{meas.}})^2}{8\pi^2 f^2}$$

$$\chi_E = \frac{\sqrt{2}E}{4\pi f}$$

$$\chi_H = \frac{(M_{D_{(s)}}^{\mathrm{meas.}})^2 - (M_{D_{(s)}}^{\mathrm{PDG}})^2}{8\pi^2 f^2}$$

$$\delta f^{SU(2)}_{\mathrm{P,logs}} = \left(-\frac{1}{16}\sum_\xi \mathcal{I}_1(M_{\pi,\xi}) + \frac{1}{4}\mathcal{I}_1(M_{\pi,I}) + \mathcal{I}_1(M_{\pi,V}) - \mathcal{I}_1(M_{\eta,V}) + [V \to A]\right)$$
$$\times \begin{cases} \frac{1+3g^2}{(4\pi f)^2}, D \to \pi \\ \frac{3g^2}{(4\pi f)^2}, D \to K \\ \frac{1}{(4\pi f)^2}, D_s \to K \end{cases}$$

$$x_{a^2} = \frac{a^2\bar{\Delta}}{8\pi^2 f^2}$$
$$x_h = \frac{2}{\pi} a m_h.$$

$$M^2_{\pi,\xi} = M^2_{uu,\xi} = M^2_{dd,\xi}$$
$$M^2_{ij,\xi} = \mu(m_i + m_j) + \Delta_\xi$$
$$M^2_{\eta,V(A)} = M^2_{uu,V(A)} + \frac{1}{2}\delta'_{V(A)}$$
$$\bar{\Delta} = \frac{1}{16}\sum_\xi \Delta_\xi.$$