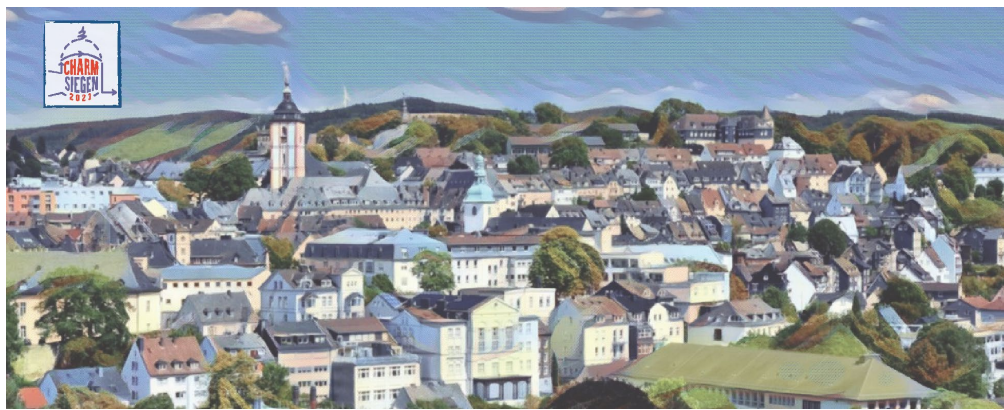


# $\tau$ Physics

Antonio Pich

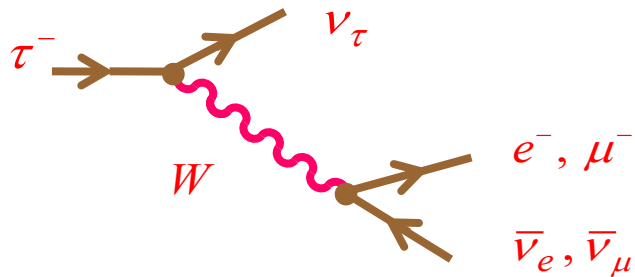
IFIC, Univ. Valencia - CSIC



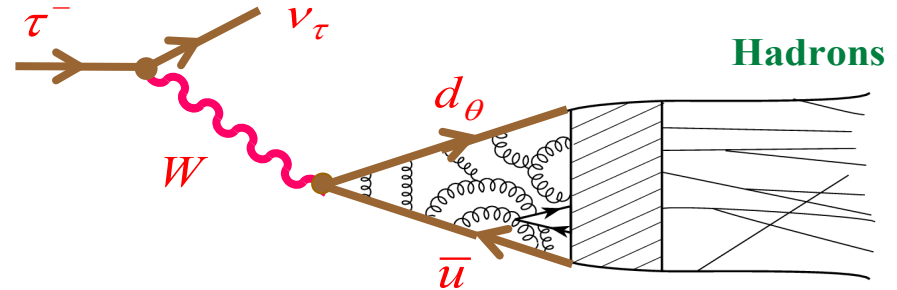
CHARM 2023  
Siegen, Germany  
17-21 July 2023

# $\tau$ Physics

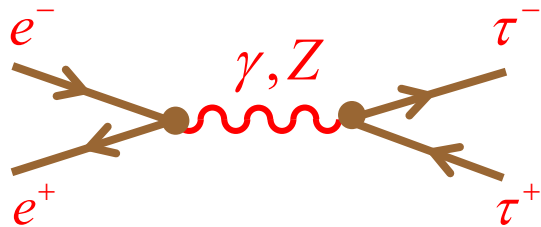
## Decay



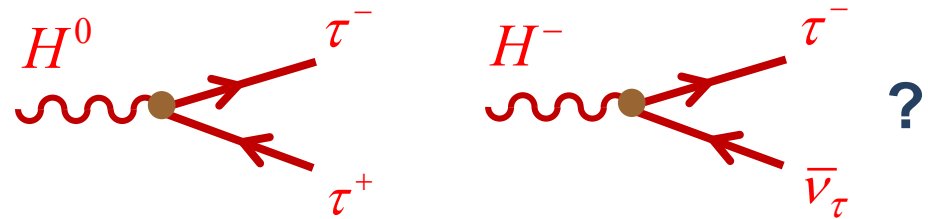
## QCD



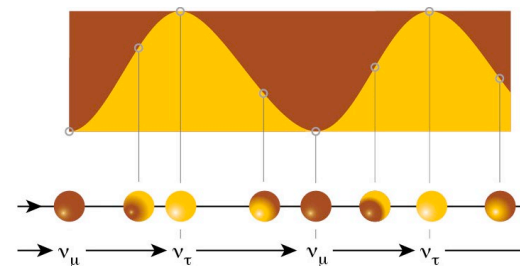
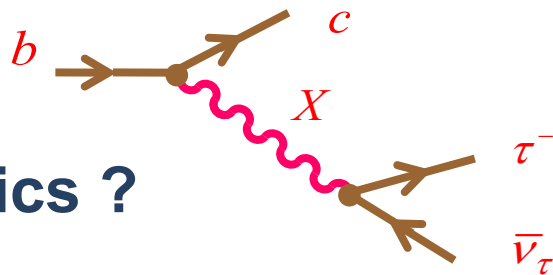
## Production



## Higgs Interactions



## New Physics ?



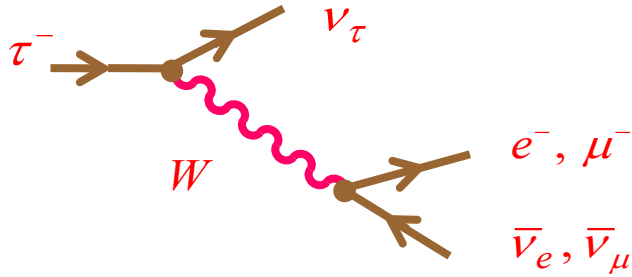
# $\tau$ Data Samples

<b>ALEPH:</b>	$3.3 \cdot 10^5$	reconstructed $\tau$ decays	
<b>BaBar / Belle:</b>	$1.4 \cdot 10^9$	$\tau^+\tau^-$ pairs	
<b>Belle-II:</b>	$4.6 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	
<b>s<math>\tau</math>cF:</b>	$2.1 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	( $10^8$ near threshold)
<b>Tera-Z:</b>	$1.7 \cdot 10^{11}$	$\tau^+\tau^-$ pairs	

**Luminosity is important. Systematics & backgrounds also!**

Different experimental conditions at different energies  
( $\tau^+\tau^-$  threshold,  $\psi$ ,  $\Upsilon$ ,  $Z$ )

# LEPTONIC DECAYS



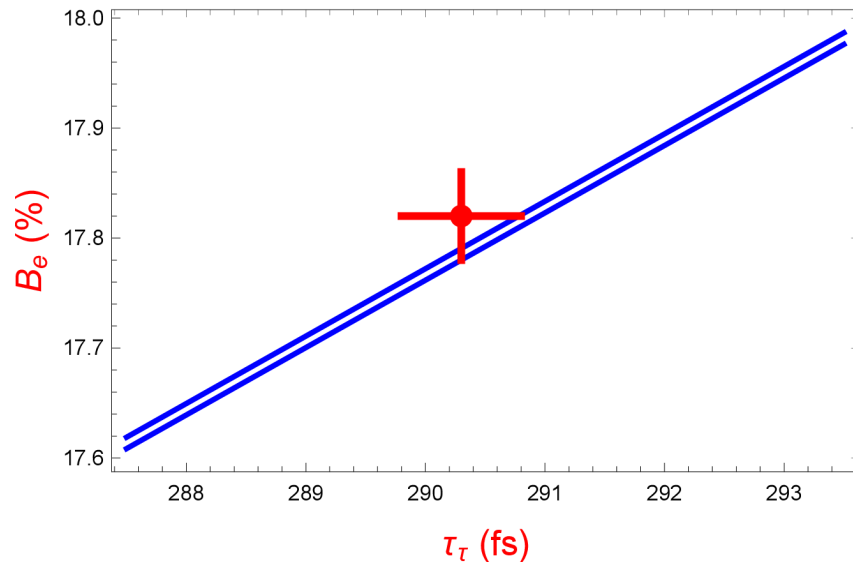
$$\Gamma(\tau \rightarrow \nu_\tau l \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192 \pi^3} f(m_l^2/m_\tau^2) (1 + \delta_{\text{RC}})$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$



$$B_e = \frac{B_\mu}{0.972564 \pm 0.000003} = \frac{\tau_\tau}{(1632.3 \pm 0.5) \times 10^{-15} \text{ s}}$$

$\tau_\tau$  (Belle),  $m_\tau$  (BesIII, BelleII)



$$\left( B_\mu / B_e \right)_{\text{exp}} = 0.9762 \pm 0.0028$$

Non-BF:  $0.9725 \pm 0.0039$

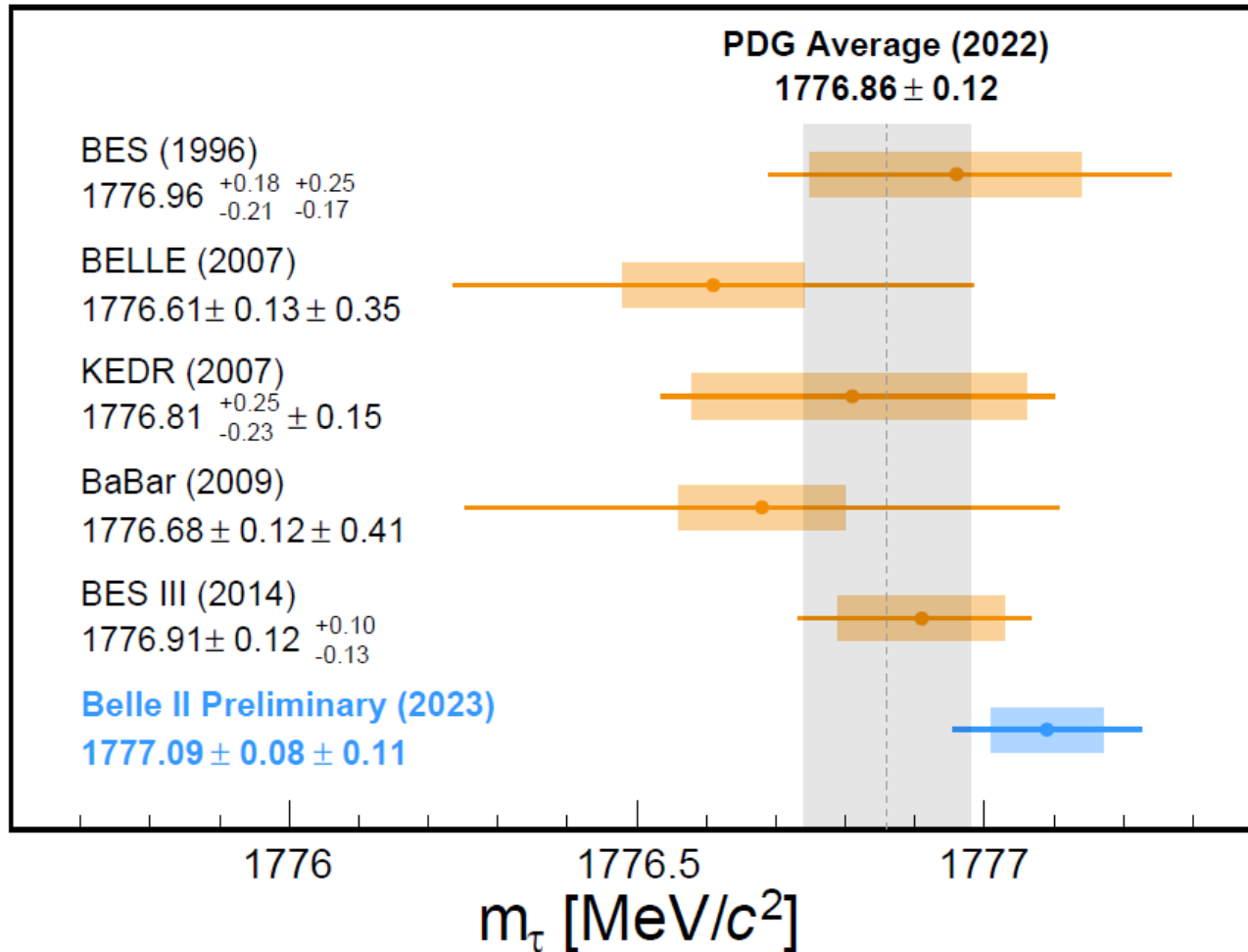
BaBar '10:  $0.9796 \pm 0.0039$



$$B_e^{\text{univ}} = (17.812 \pm 0.022)\%$$

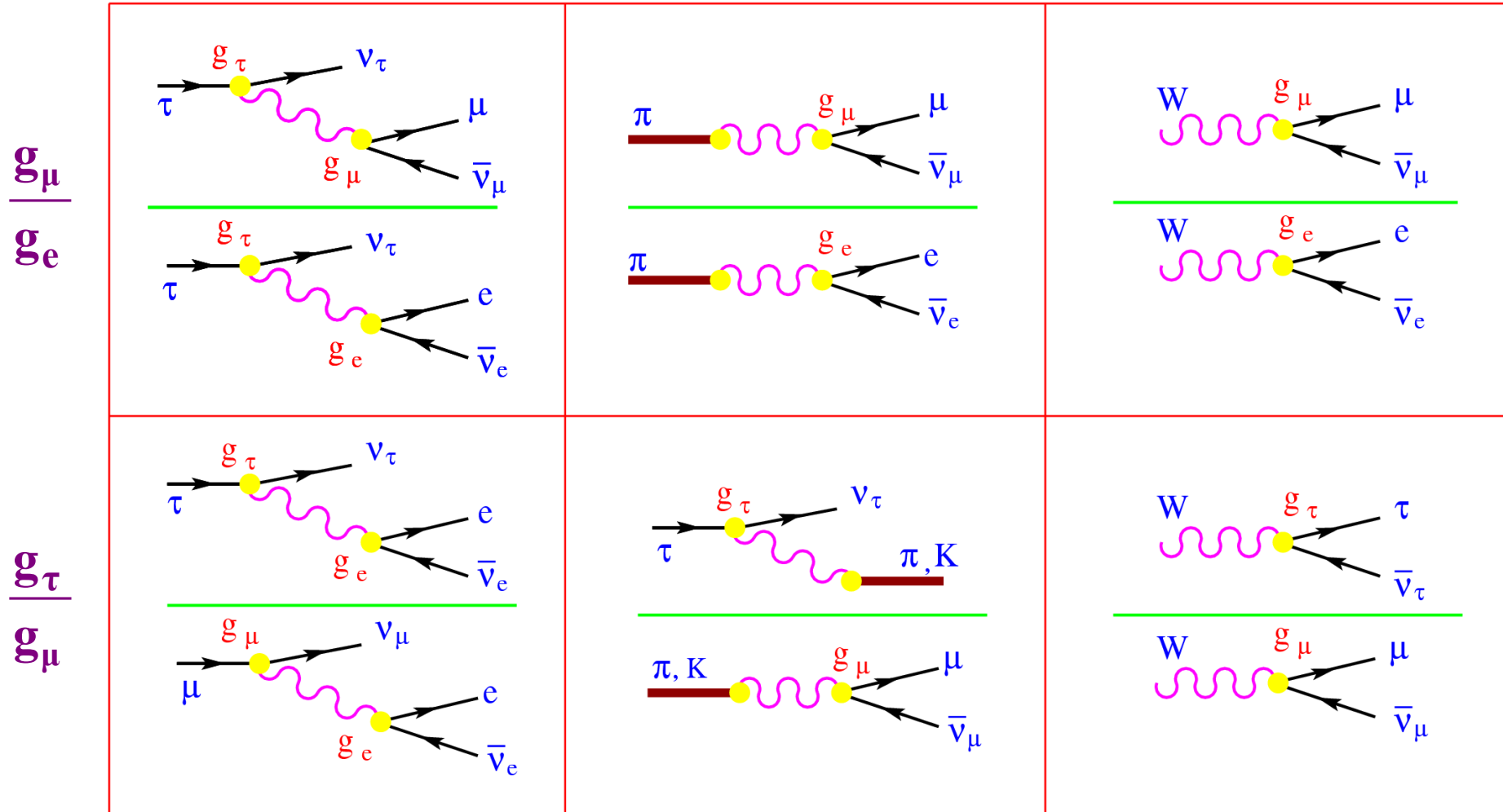


# Preliminary Belle-II measurement of $m_\tau$



$$m_\tau = (1776.96 \pm 0.09) \text{ MeV}$$

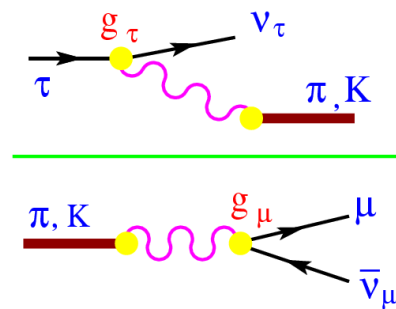
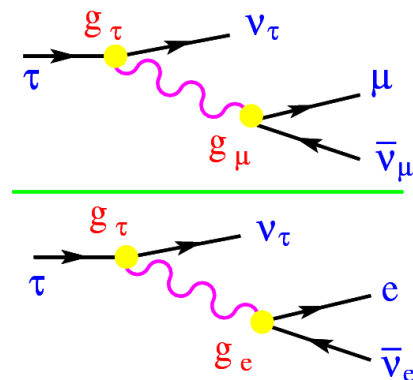
# Lepton Universality



# Lepton Universality

$$\left| g_\mu / g_e \right|$$

$B_{\tau \rightarrow \mu} / B_{\tau \rightarrow e}$	$1.0019 \pm 0.0014$
$B_{\pi \rightarrow \mu} / B_{\pi \rightarrow e}$	$1.0010 \pm 0.0009$
$B_{K \rightarrow \mu} / B_{K \rightarrow e}$	$0.9978 \pm 0.0018$
$B_{K \rightarrow \pi\mu} / B_{K \rightarrow \pi e}$	$1.0010 \pm 0.0025$
$B_{W \rightarrow \mu} / B_{W \rightarrow e}$	$1.001 \pm 0.003$



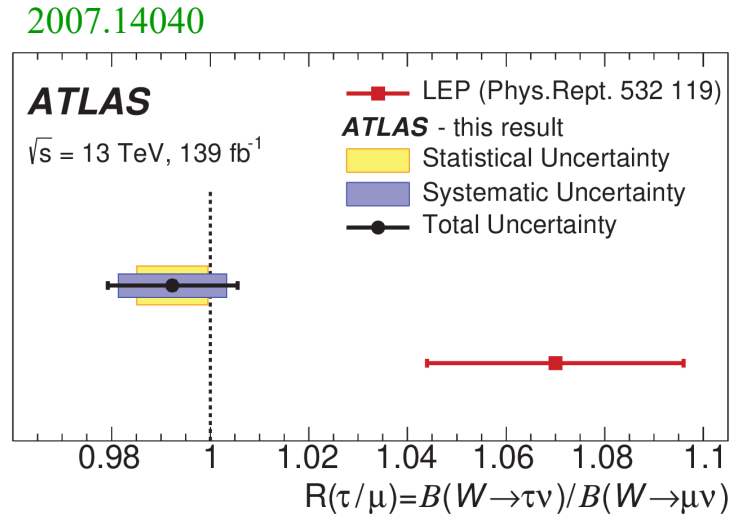
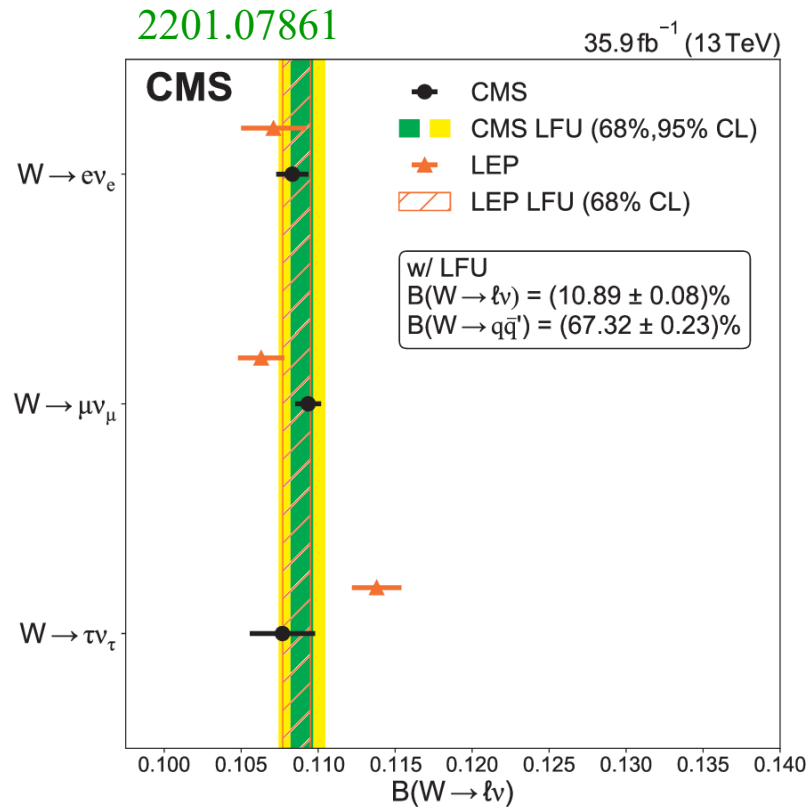
$$\left| g_\tau / g_\mu \right|$$

$$\left| g_\tau / g_e \right|$$

$B_{\tau \rightarrow \mu} \tau_\mu / \tau_\tau$	$1.0027 \pm 0.0014$
$B_{W \rightarrow \tau} / B_{W \rightarrow e}$	$1.007 \pm 0.010$

$B_{\tau \rightarrow e} \tau_\mu / \tau_\tau$	$1.0009 \pm 0.0014$
$\Gamma_{\tau \rightarrow \pi} / \Gamma_{\pi \rightarrow \mu}$	$0.9959 \pm 0.0038$
$\Gamma_{\tau \rightarrow K} / \Gamma_{K \rightarrow \mu}$	$0.986 \pm 0.008$
$B_{W \rightarrow \tau} / B_{W \rightarrow \mu}$	$1.001 \pm 0.010$

# Lepton Universality in W decays

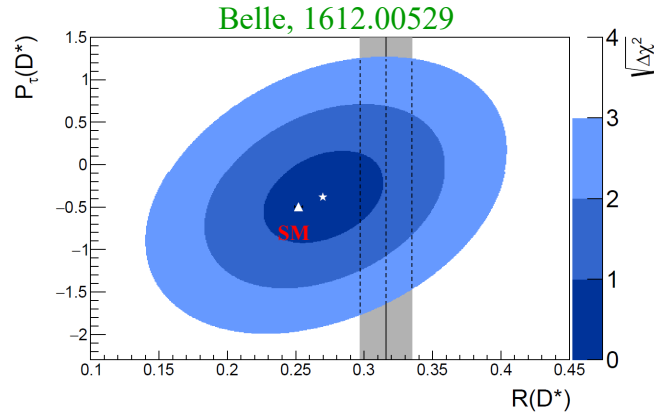
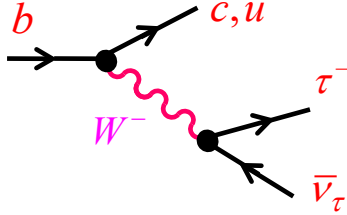


	CMS	LEP	ATLAS	LHCb	CDF	D0
$R_{\mu/e}$	$1.009 \pm 0.009$	$0.993 \pm 0.019$	$1.003 \pm 0.010$	$0.980 \pm 0.012$	$0.991 \pm 0.012$	$0.886 \pm 0.121$
$R_{\tau/e}$	$0.994 \pm 0.021$	$1.063 \pm 0.027$	—	—	—	—
$R_{\tau/\mu}$	$0.985 \pm 0.020$	$1.070 \pm 0.026$	$0.992 \pm 0.013$	—	—	—
$R_{\tau/\ell}$	$1.002 \pm 0.019$	$1.066 \pm 0.025$	—	—	—	—

# Flavour Anomaly

## 3.2 $\sigma$ discrepancy

$$R(D^{(*)}) \equiv \frac{\text{Br}(\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau)}{\text{Br}(\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell)}$$

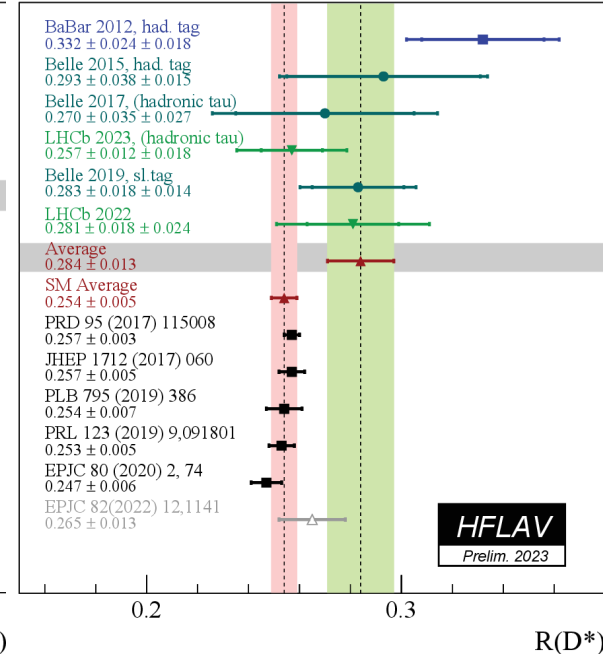
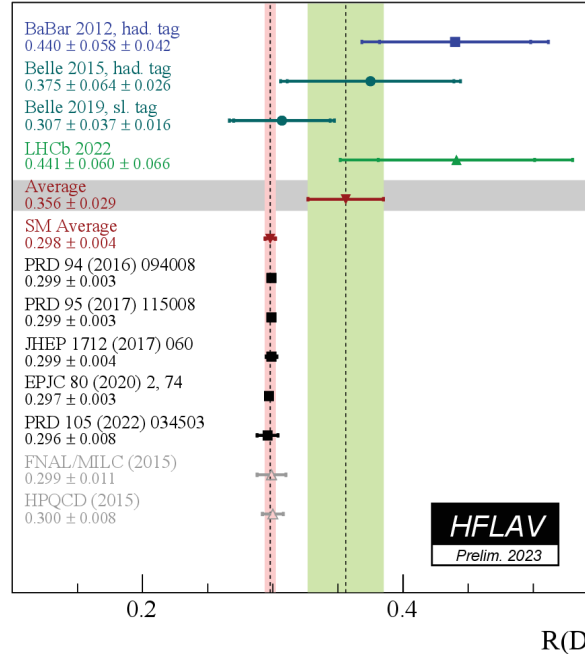
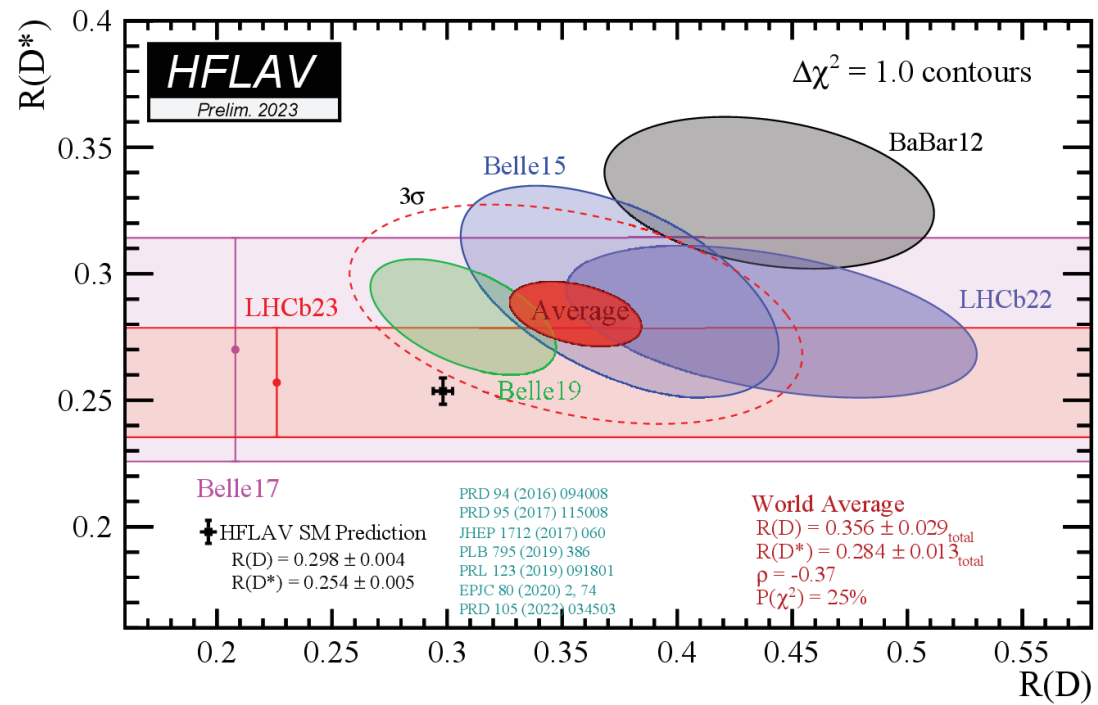


$$\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \rightarrow J/\psi \tau \bar{\nu}_\tau)}{\mathcal{B}(B_c \rightarrow J/\psi \mu \bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18 \quad (1.7 \sigma)$$

$$F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \quad (1.6 \sigma)$$

$$\mathcal{R}_{J/\psi}^{\text{SM}} \approx 0.26 - 0.28$$

$$F_{L,\text{SM}}^{D^*} = 0.455 \pm 0.003$$



# Lorentz Structure: $\ell^- \rightarrow \ell'^- \bar{\nu}_{\ell'} \nu_{\ell}$

**Effective Hamiltonian:**

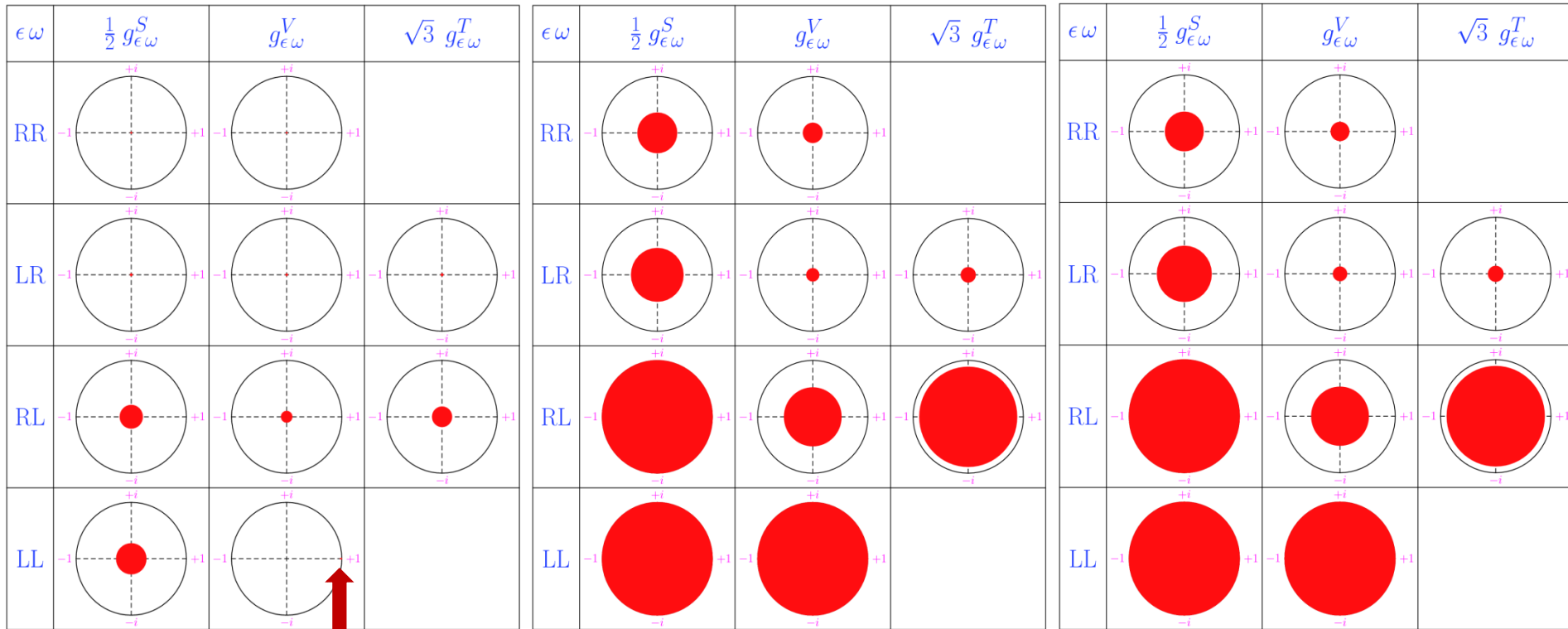
$$\mathcal{H} = 4 \frac{G_{\ell'\ell}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g_{\epsilon\omega}^n [\bar{\ell}'_{\epsilon} \Gamma^n (\nu_{\ell'})_{\sigma}] \left[ \overline{(\nu_{\ell})_{\lambda}} \Gamma_n \ell_{\omega} \right]$$

**Normalization:**  $\Gamma \propto \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2) + 3 (|g_{RR}^T|^2 + |g_{RL}^T|^2) + (|g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2) \equiv 1$

$\mu \rightarrow e \bar{\nu}_e \nu_{\mu}$

$\tau \rightarrow \mu \bar{\nu}_{\mu} \nu_{\tau}$

$\tau \rightarrow e \bar{\nu}_e \nu_{\tau}$



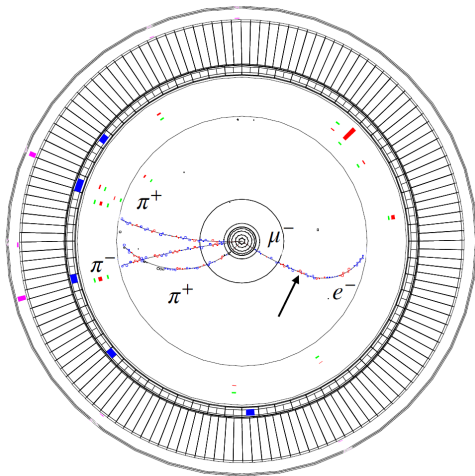
$|g_{LL}^V| > 0.960$  (90% CL)

**High-precision  $\tau$  data needed!**

# $\mu^-$ Longitudinal Polarization in $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$

Probability to decay into a right-handed muon:

$$Q_{\mu_R} = Q_{RR} + Q_{RL} = \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2) + 3|g_{RL}^T|^2 + |g_{RR}^V|^2 + |g_{RL}^V|^2 = \frac{1}{2} (1 - \xi')$$



MC  $\tau^+\tau^-$  event

Belle, 2303.10570

Tiny probability of muon decaying inside the detector compensated by huge statistics

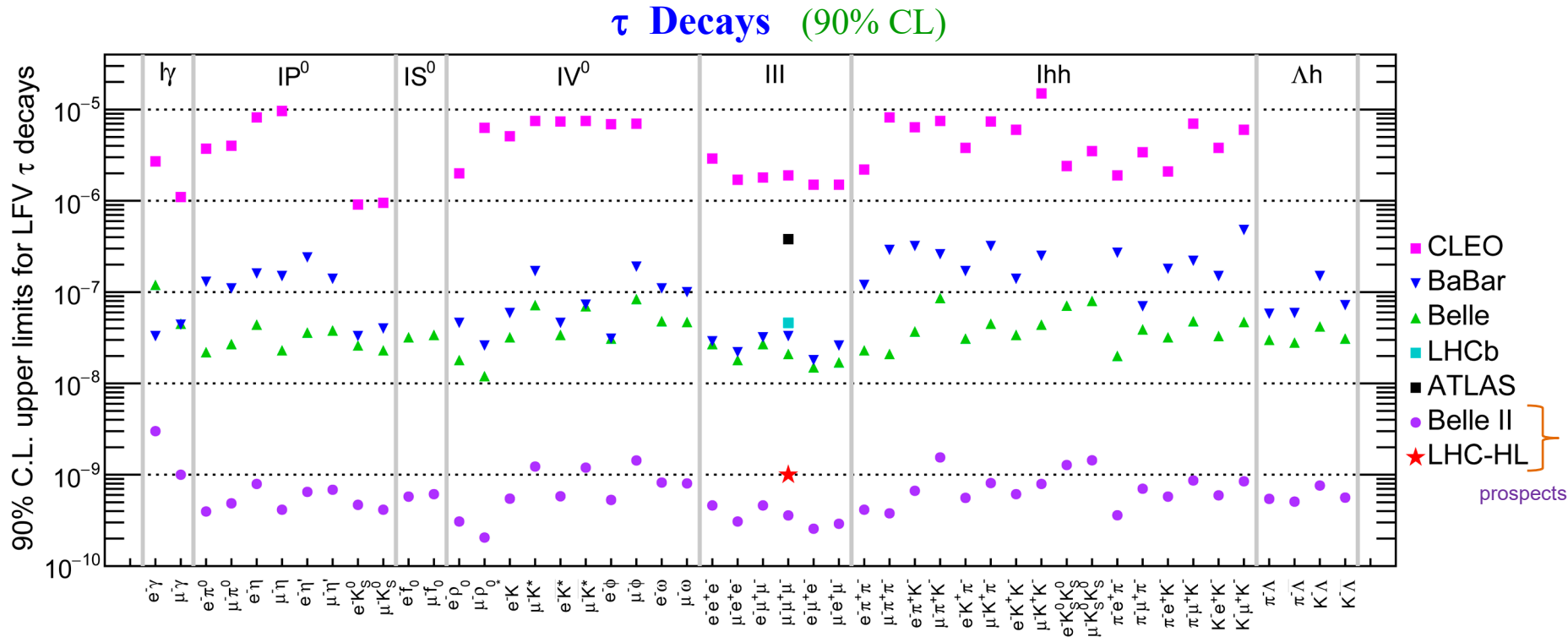
$$\xi' = 0.22 \pm 0.94 \pm 0.42$$



$$Q_{\mu_R} \leq 1.23 \quad (90\% \text{ CL})$$

**Not yet constraining. Error dominated by statistics...**

# Bounds on Lepton Flavour Violation



$$\text{Br}(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13} \quad (\text{MEG, 90\% CL})$$

,

$$\text{Br}(K_L \rightarrow \mu e) < 4.7 \times 10^{-12} \quad (\text{BNL-E871, 90\% CL})$$

,

$$\text{Br}(B^0 \rightarrow e \mu) < 1.0 \times 10^{-9} \quad (\text{LHCb, 90\% CL})$$

,

$$\text{Br}(\mu \rightarrow 3e) < 1.0 \times 10^{-12} \quad (\text{SINDRUM, 90\% CL})$$

$$\text{Br}(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11} \quad (\text{BNL-E865, 90\% CL})$$

$$\text{Br}(D^0 \rightarrow e \mu) < 1.3 \times 10^{-8} \quad (\text{LHCb, 90\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \mu) < 7.5 \times 10^{-7} \quad (\text{ATLAS, 95\% CL})$$

,

$$\text{Br}(H \rightarrow e \mu) < 6.1 \times 10^{-5} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \tau) < 5.0 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

,

$$\text{Br}(H \rightarrow e \tau) < 2.2 \times 10^{-3} \quad (\text{CMS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow \mu \tau) < 6.5 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

,

$$\text{Br}(H \rightarrow \mu \tau) < 1.5 \times 10^{-3} \quad (\text{CMS, 95\% CL})$$





# CP Asymmetry

$$A_\tau \equiv \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S \nu_\tau)} = (-3.6 \pm 2.3 \pm 1.1) \cdot 10^{-3}$$

**BaBar'11**  
( $\geq 0\pi^0$ )

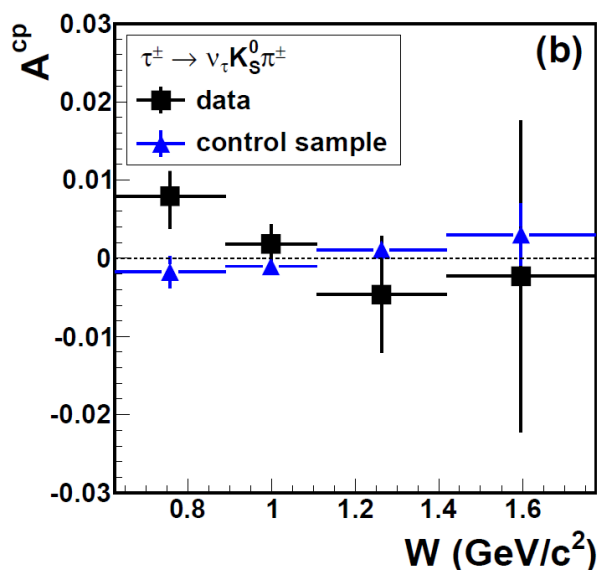
$$A_\tau^{\text{SM}}(\tau^+ \rightarrow \pi^+ K_S \bar{\nu}_\tau) = (3.6 \pm 0.1) \cdot 10^{-3}$$

Bigi-Sanda, Grossman-Nir

**2.8  $\sigma$  discrepancy**



**Belle does not see any asymmetry at the  $10^{-2}$  level**



$$A_i^{\text{CP}} \simeq \langle \cos \beta \cos \psi \rangle_i^{\tau^-} - \langle \cos \beta \cos \psi \rangle_i^{\tau^+}$$

bins ( $i$ ) of  $W = \sqrt{Q^2}$

$\beta = K_S$  direction in hadronic rest frame

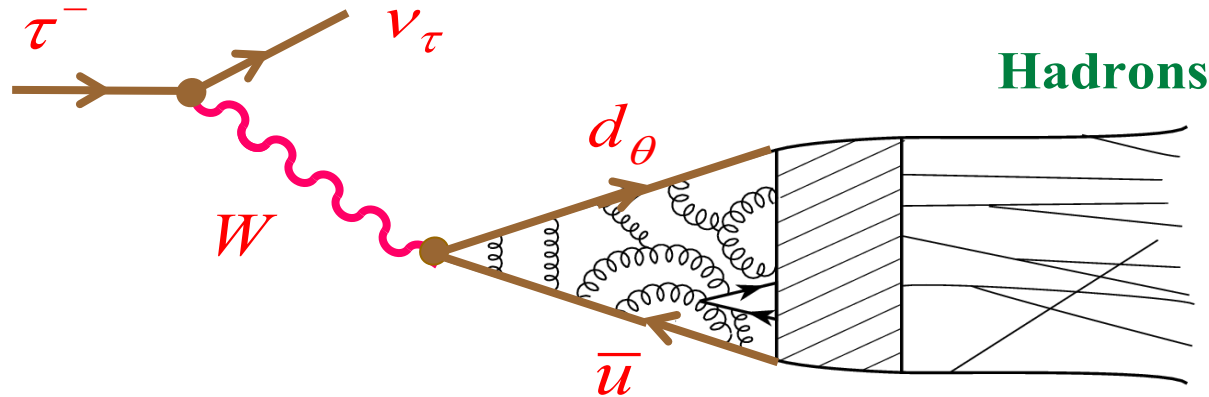
$\psi = \tau$  direction

**BaBar signal incompatible (with EFT)  
with other sets of flavour data**

Cirigliano-Crivellin-Hoferichter, 1712.06595

Rendón-Roig-Toledo, 1902.08143

# HADRONIC TAU DECAY



$$d_\theta = V_{ud} d + V_{us} s$$

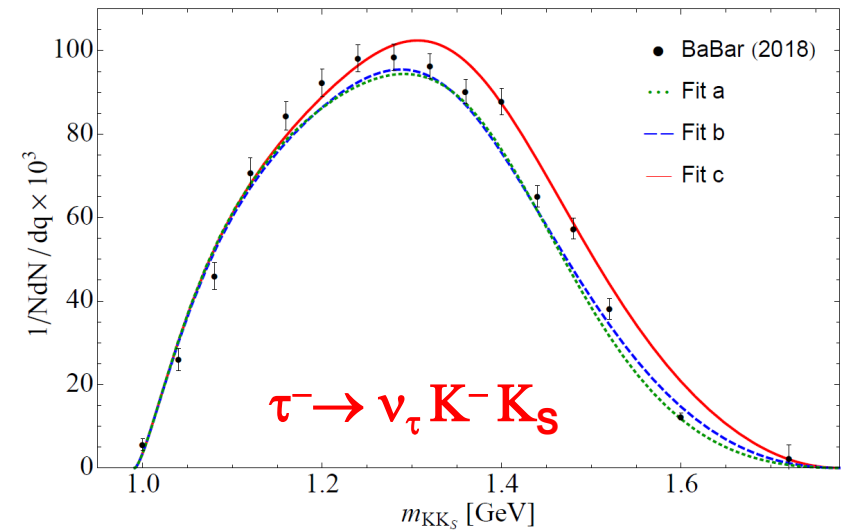
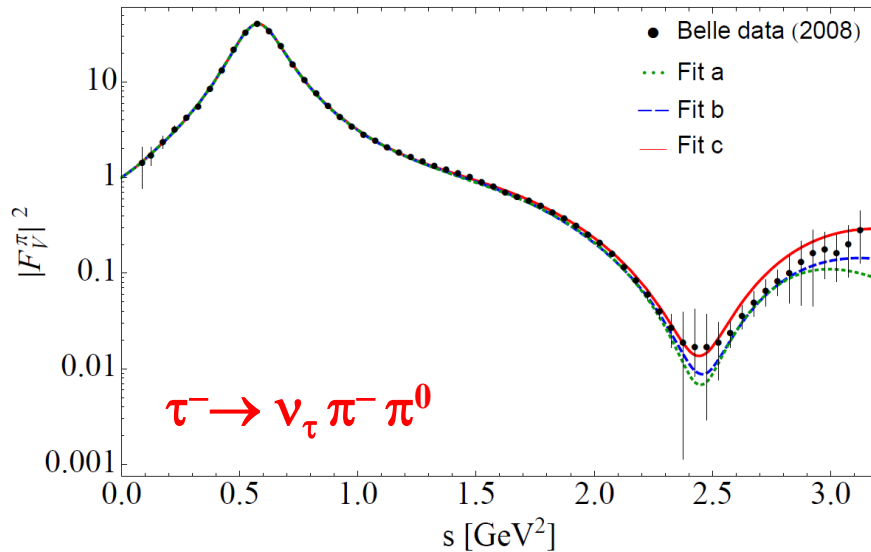
Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \quad ; \quad R_\tau = \frac{1 - B_e - B_\mu}{B_e^{\text{univ}}} = 3.6381 \pm 0.0075$$

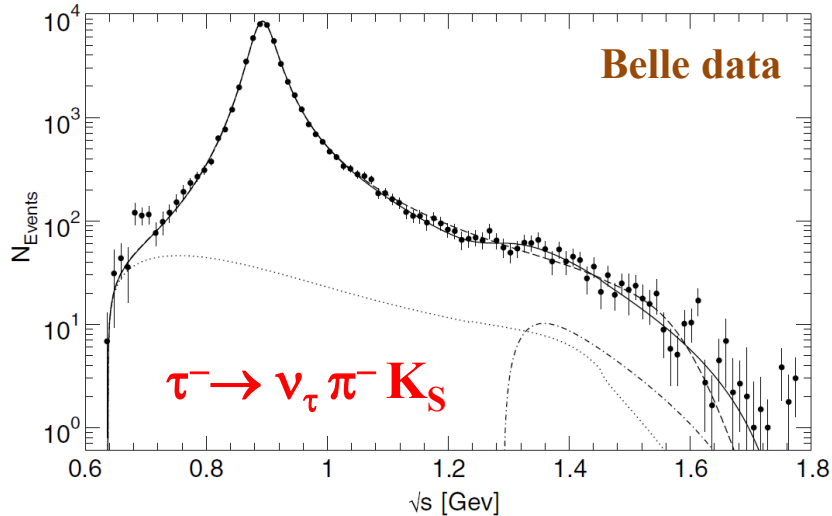
$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.972564 = 3.6417 \pm 0.0070 \quad ; \quad R_\tau = \frac{\text{Br}(\tau^- \rightarrow \nu_\tau + \text{Hadrons})}{B_e^{\text{univ}}} = 3.6343 \pm 0.0082$$

# Invariant Mass Spectra

González-Solís – Roig, 1902.02273



Jamin-Pich-Portolés, 0803.1786

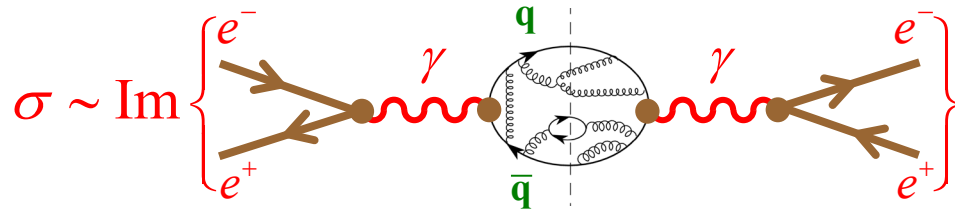


Useful tests of QCD Dynamics

Form Factors

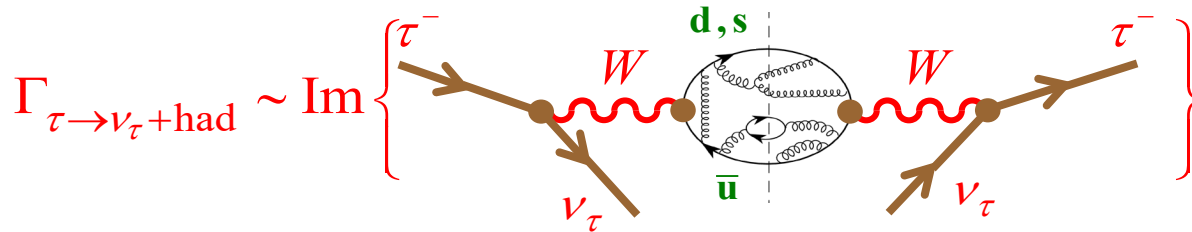
Non-perturbative parameters

Resonance Chiral Theory (R $\chi$ T)



$$\frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \operatorname{Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x \, e^{iqx} \langle 0 | T[J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$



$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[ \left(1 + 2\frac{s}{m_\tau^2}\right) \operatorname{Im} \Pi^{(1)}(s) + \operatorname{Im} \Pi^{(0)}(s) \right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 [\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s)] + |V_{us}|^2 [\Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s)]$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x \, e^{iqx} \langle 0 | T[J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,J}^{(0)}(q^2)$$

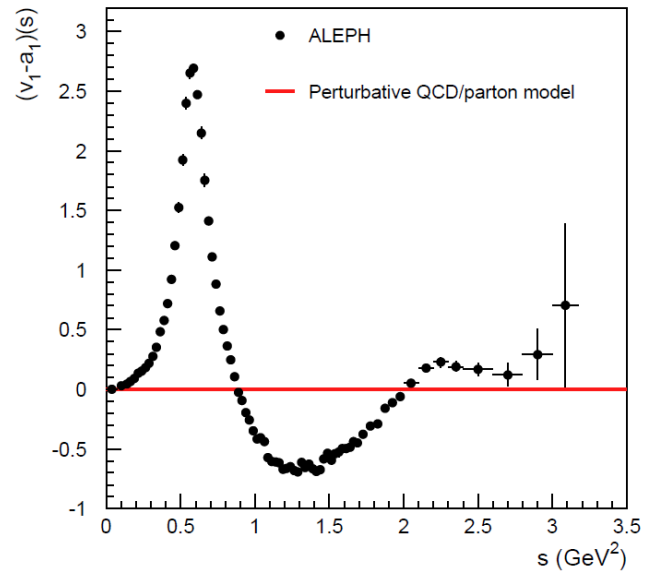
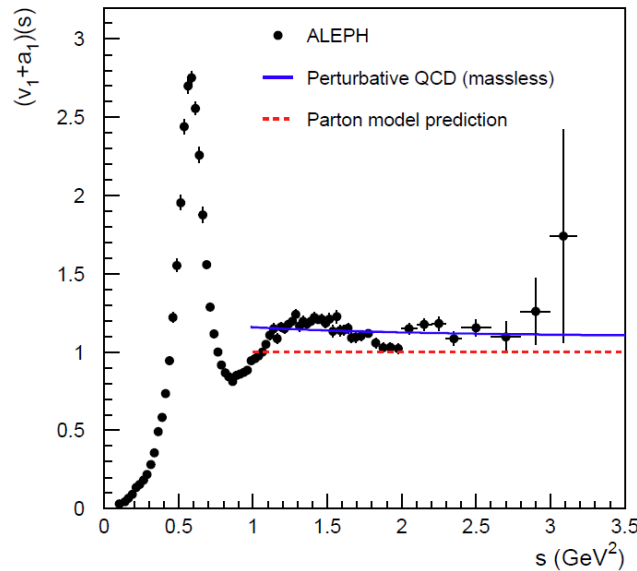
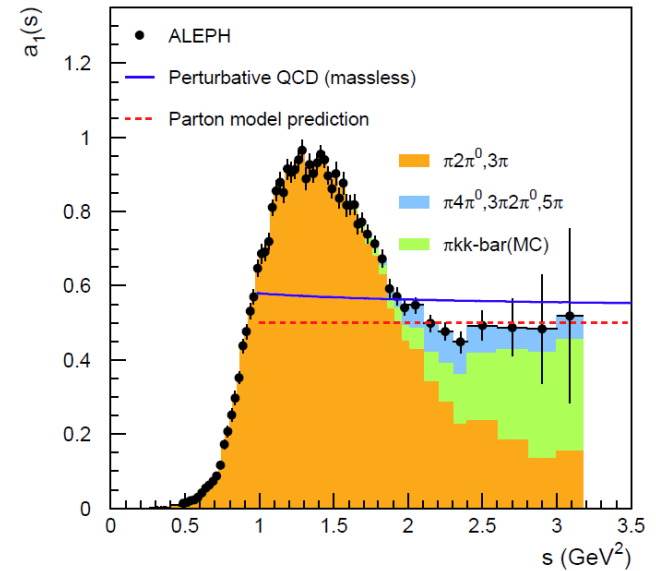
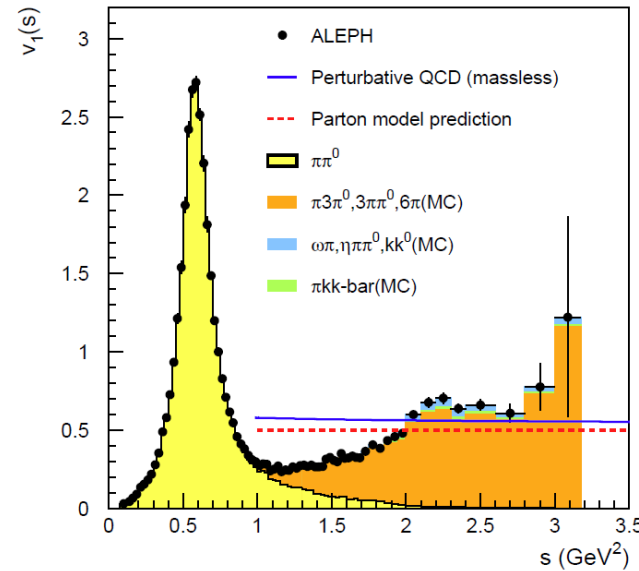
# SPECTRAL FUNCTIONS

Davier et al, 1312.1501

$$v_1(s) = 2\pi \text{Im} \Pi_{ud,V}^{(0+1)}(s)$$

$$a_1(s) = 2\pi \text{Im} \Pi_{ud,A}^{(0+1)}(s)$$

**Better  
data  
needed**

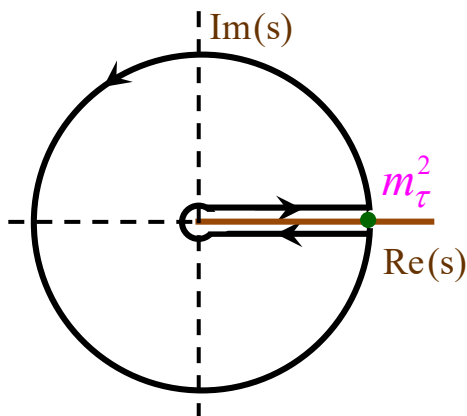


# QCD Prediction of $R_\tau$

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{(1)}(x m_\tau^2) + \text{Im} \Pi^{(0)}(x m_\tau^2) \right]$$

$$x \equiv s/m_\tau^2$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[ (1+2x) \Pi^{(0+1)}(x m_\tau^2) - 2x \Pi^{(0)}(x m_\tau^2) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s, \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C S_{\text{EW}} (1 + \delta_P + \delta_{\text{NP}}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$S_{\text{EW}} = 1.0201 (3)$$

Marciano-Sirlin, Braaten-Li, Erler

;

$$\delta_{\text{NP}} = -0.0064 \pm 0.0013$$

Fitted from data (Davier et al)

$$\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$$

;

$$a_\tau \equiv \alpha_s(m_\tau) / \pi$$

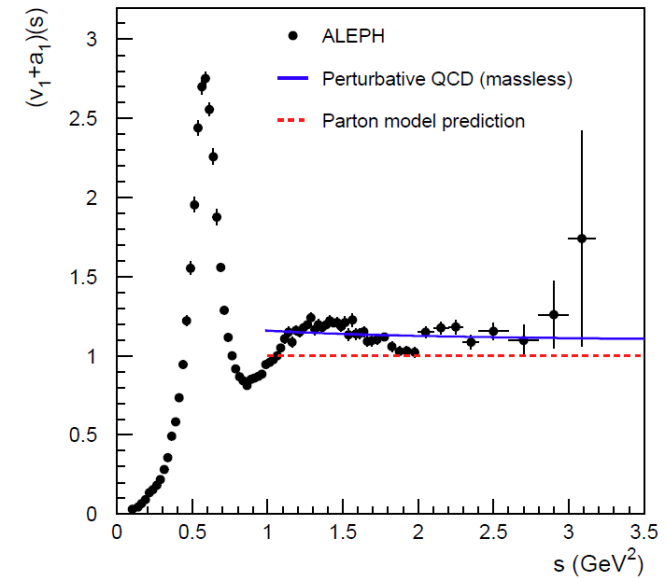
Baikov-Chetyrkin-Kühn

# Spectral Function Distribution

Moments:

$$R_{\tau}^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_{\tau}^2}\right)^l \frac{dR_{\tau}}{ds}$$

Sensitivity to power corrections (k,l)



The non-perturbative contribution to  $R_{\tau}$  can be obtained from the invariant-mass distribution of the final hadrons

Detailed analyses by ALEPH, CLEO and OPAL

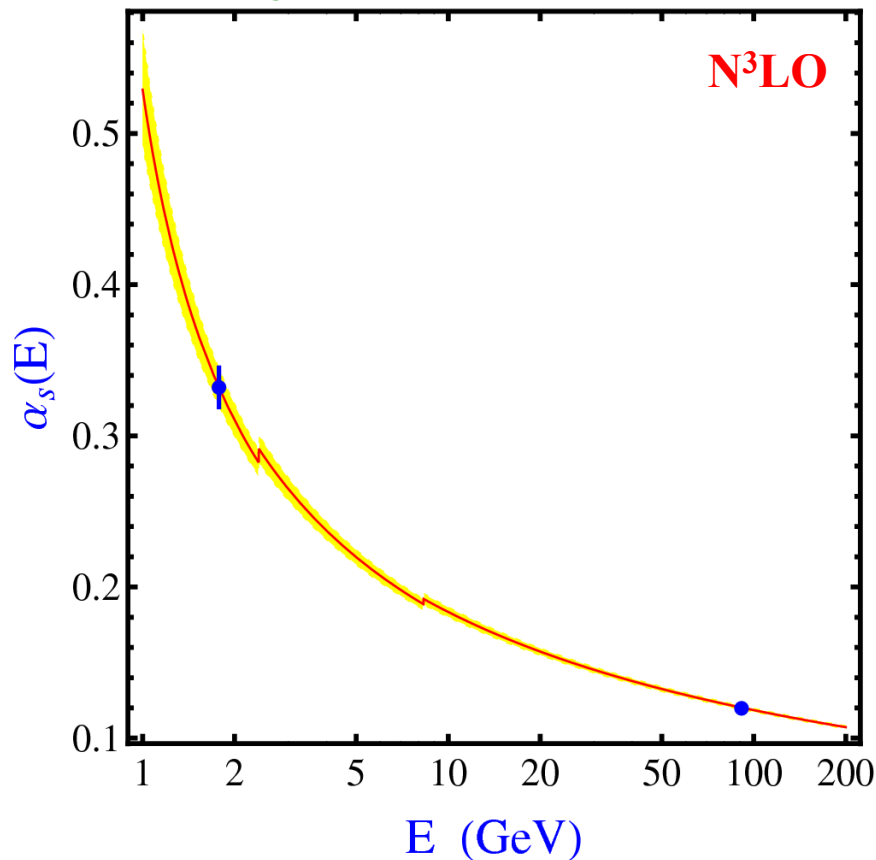
$$\delta_{\text{NP}} = -0.0064 \pm 0.0013$$

$$\alpha_s(m_{\tau}^2) = 0.332 \pm 0.005_{\text{exp}} \pm 0.011_{\text{th}}$$

Davier et al., 1312.1501  
(ALEPH data)

# $\alpha_s$ at N<sup>3</sup>LO from $\tau$ and Z

Rodríguez-Sánchez, Pich, 1605.06830



$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z_{\text{width}}} = 0.1199 \pm 0.0029$$

**Very precise test of  
Asymptotic Freedom**

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0002 \pm 0.0015_\tau \pm 0.0029_Z$$

Improved spectral function data



Better control of non-perturbative contributions

Rodríguez-Sánchez, Pich, 2205.07587

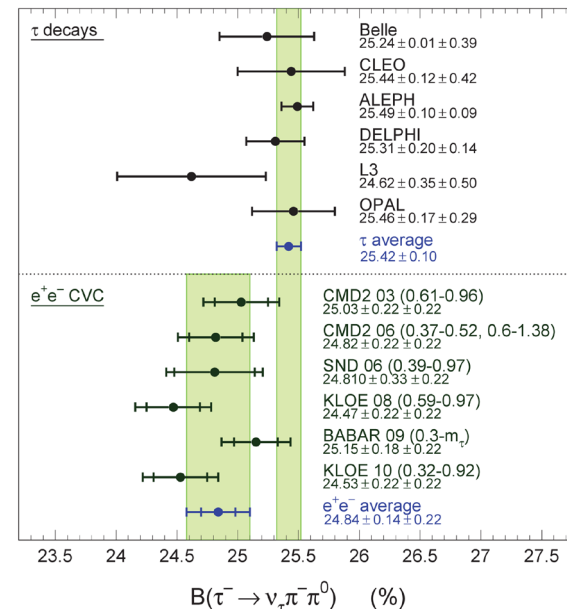
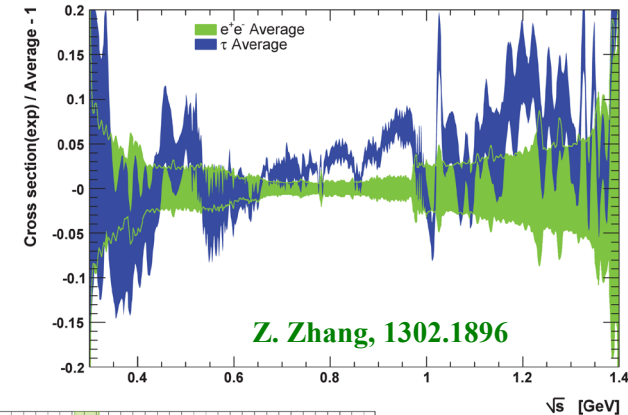
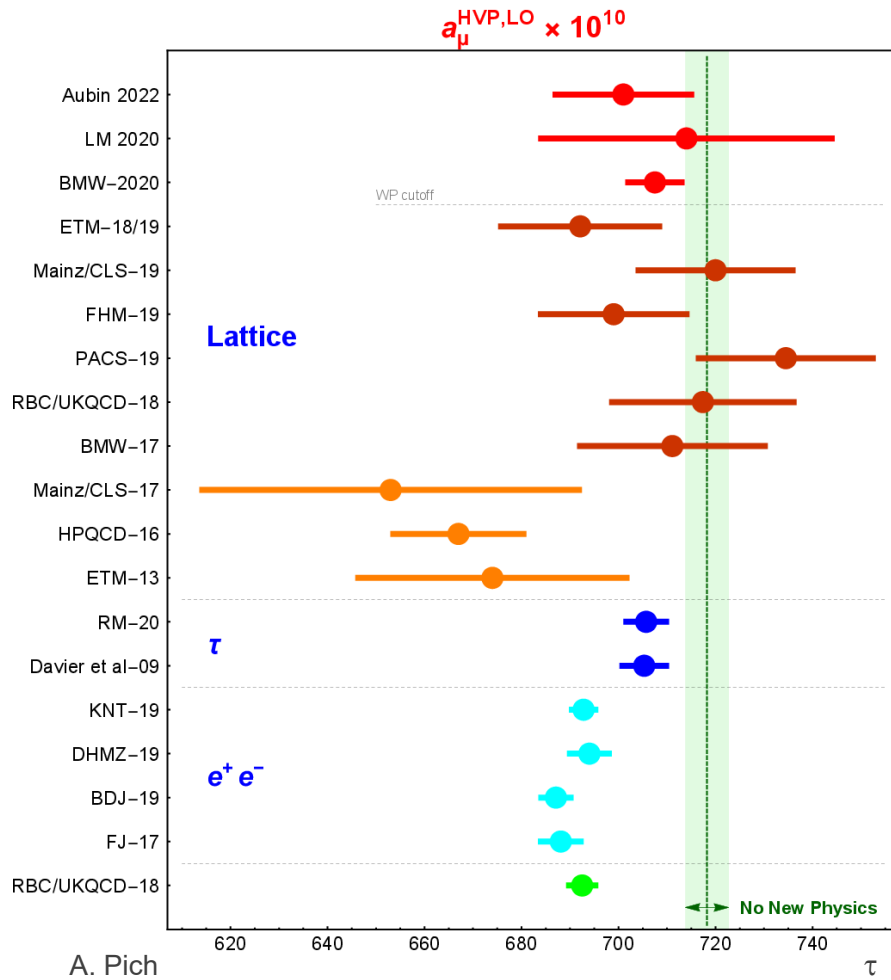
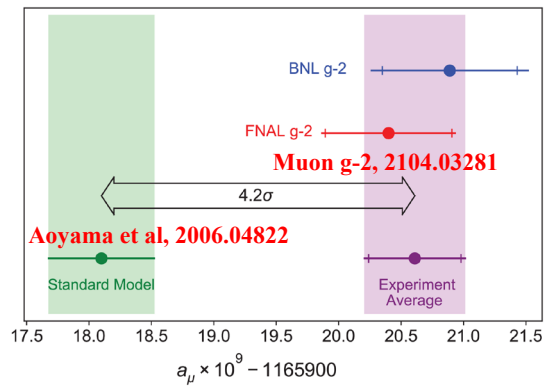
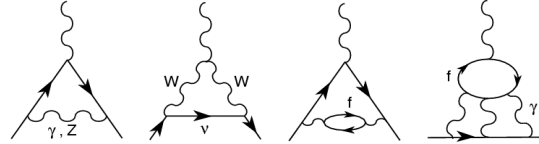
Better theoretical understanding of higher-order perturbative corrections needed  
(CIPT / FOPT, K<sub>5</sub>, renormalons...) Hoang et al, Golterman et al...



# $\mu$ Anomalous Magnetic Moment

$$a_{\mu}^{\text{HVP,LO}} = \frac{\alpha^2 m_{\mu}^2}{9\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s^2} \hat{K}(s) R(s)$$

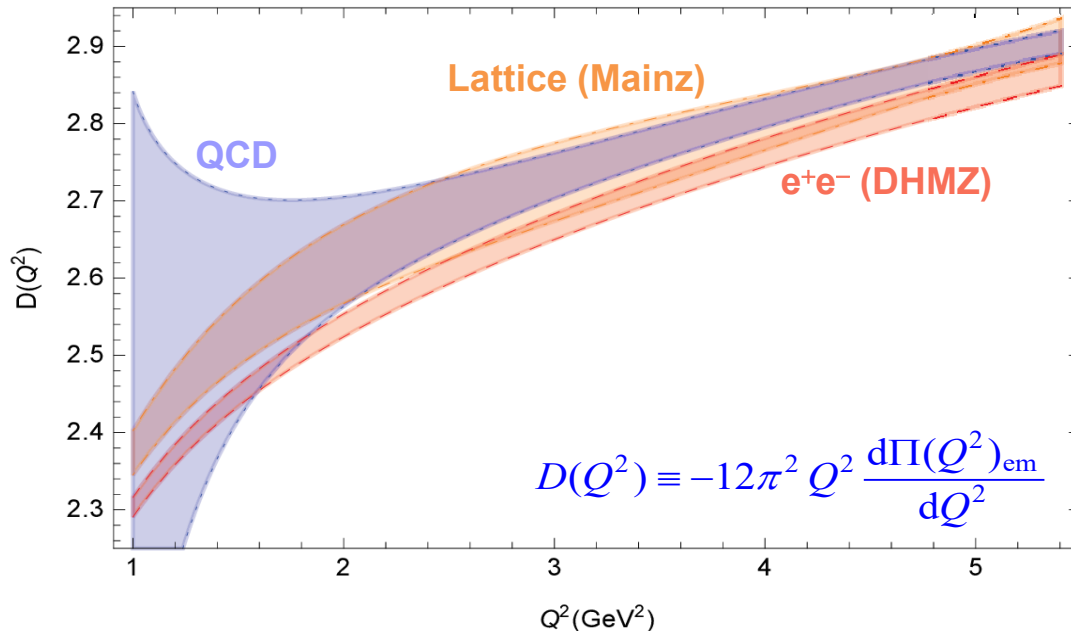
Dominated (75%) by  $2\pi$



M. Davier  
Tau 10

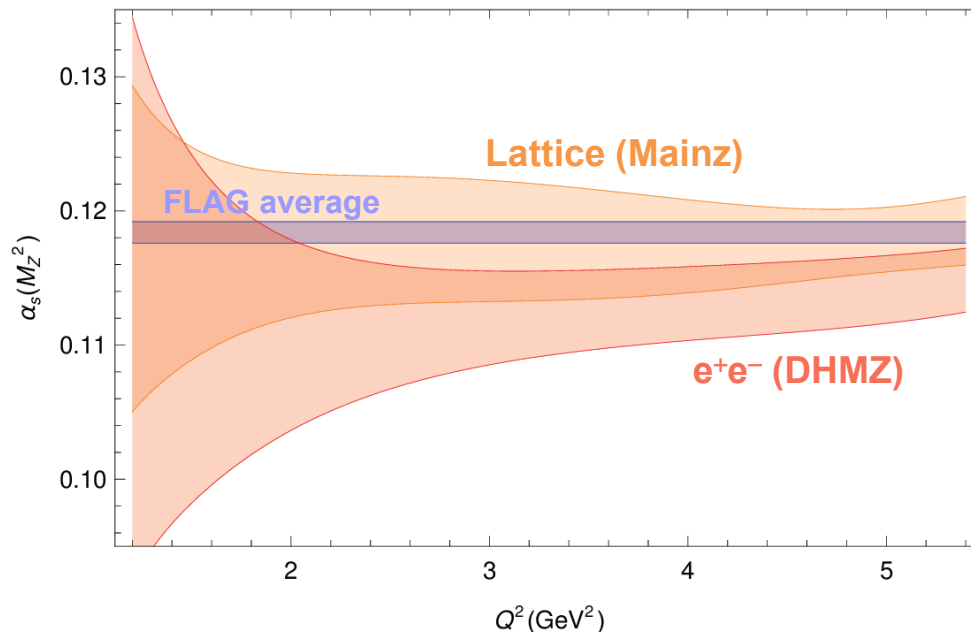
# Euclidean Adler Function

$(Q^2 = -q^2)$



M. Davier, D. Díaz-Calderón, B. Malaescu,  
A. Pich, A. Rodríguez-Sánchez,  
Z. Zhang, 2302.01359

**2 $\sigma$  discrepancy  
between  
 $e^+e^-$  data & QCD**

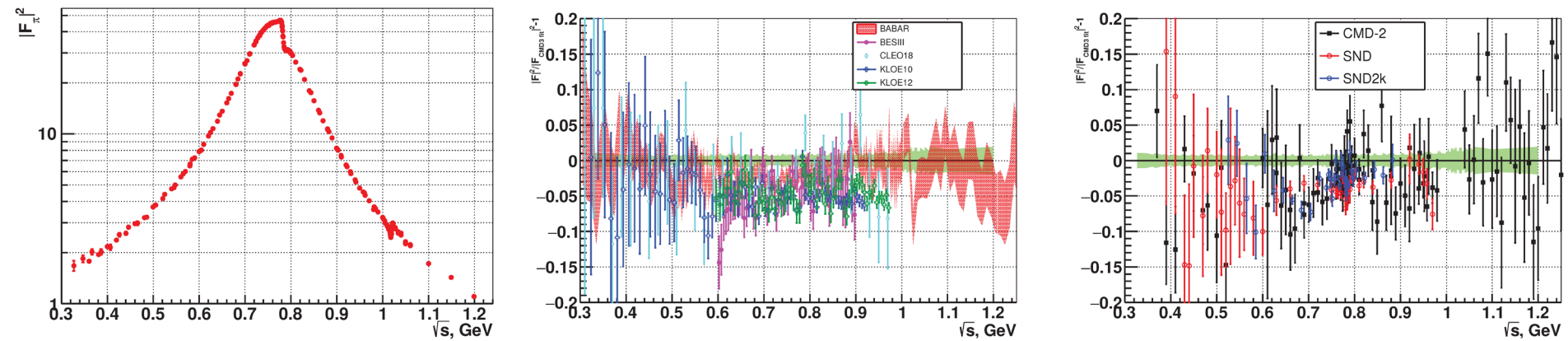


**FLAG average:**  $\alpha_s(M_Z^2) = 0.1184 \pm 0.0008$

$$\alpha_s(M_Z^2) = \begin{cases} 0.1136 \pm 0.0025 & (e^+e^- \text{ data}) \\ 0.1179 \pm 0.0025 & (\text{Lattice}) \end{cases}$$

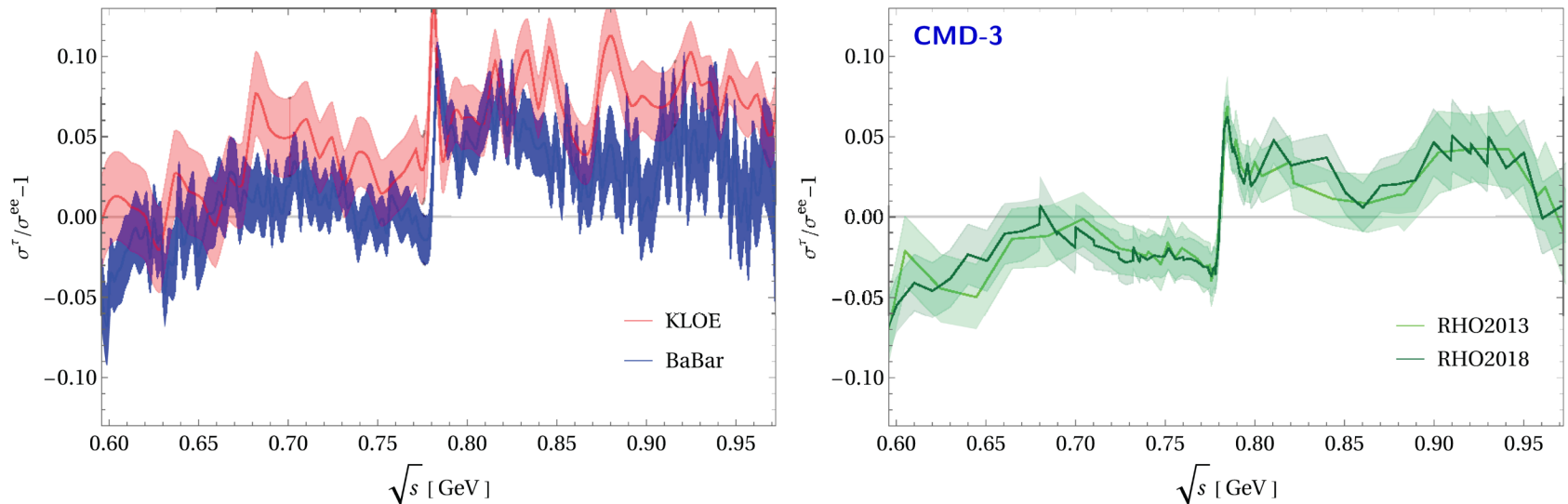
# 2023 CMD-3 $e^+e^- \rightarrow \pi^+\pi^-$ Data

2302.08834

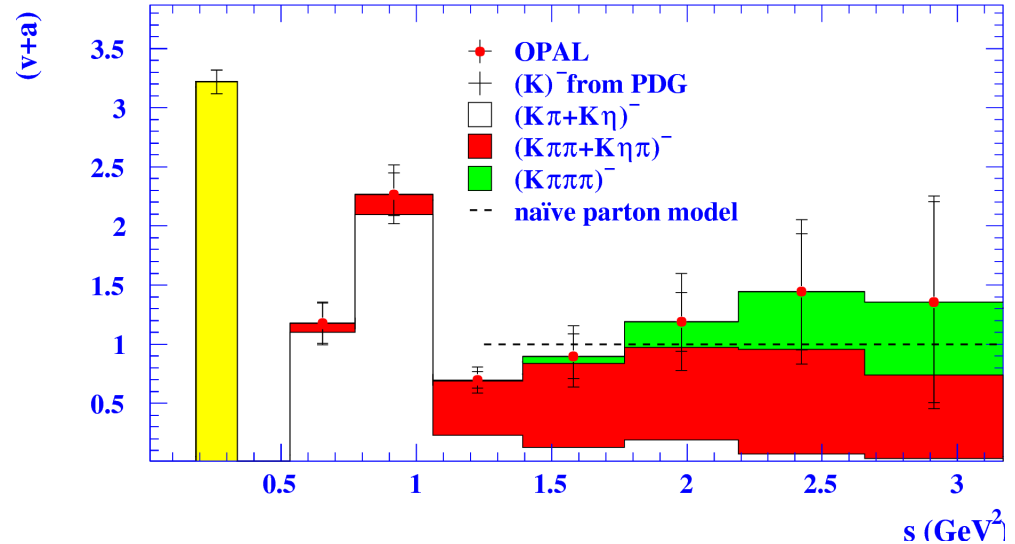
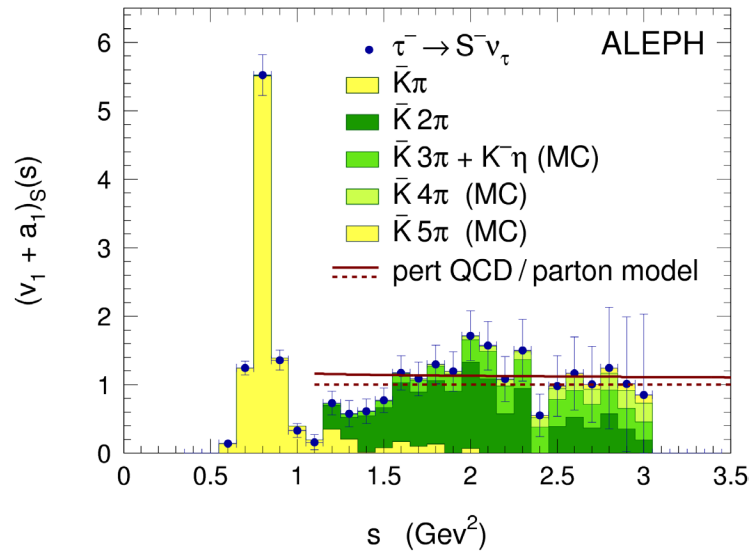


## $\tau \rightarrow 2\pi\nu_\tau$ & $e^+e^- \rightarrow 2\pi$ Spectral Functions

Masjuan-Miranda-Roig 2305.20005



# Strange Spectral Function



Very low statistics. Large experimental uncertainties

Sensitive to SU(3) breaking:  $m_s$ ,  $V_{us}$

# $V_{us}$ Determination

Gámiz-Jamin-Pich-Prades-Schwab '03

$$|V_{us}|^2 = \frac{R_{\tau,us}}{\frac{R_{\tau,ud}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

$$\delta R_{\tau,th} \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta(\alpha_s)$$

$$\delta R_{\tau,th} \equiv \underbrace{0.1544(37)}_{J=0} + \underbrace{0.084(33)}_{m_s(2 \text{ GeV}) = 93.0(8.5) \text{ MeV}} = 0.238(33)$$

$$R_{\tau,uq} = \Gamma(\tau^- \rightarrow \nu_\tau \bar{u}q) / \Gamma(\tau^- \rightarrow \nu_\tau \bar{\nu}_e e^-)$$

**HFLAV 2022:**

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{u}s) = (2.908 \pm 0.048)\%$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{u}d) = (61.83 \pm 0.10)\%$$

$$\text{Br}(\tau^- \rightarrow \nu_\tau \bar{\nu}_e e^-)_{\text{univ}} = (17.812 \pm 0.022)\%$$

$$V_{ud} = 0.97373 \pm 0.00031$$

$$\Rightarrow |V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$

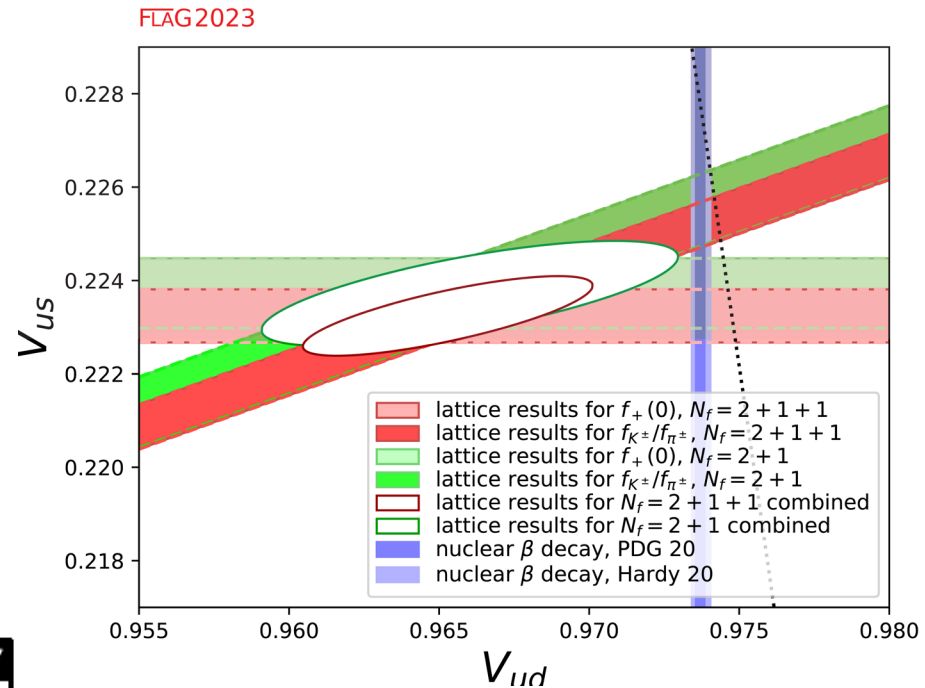
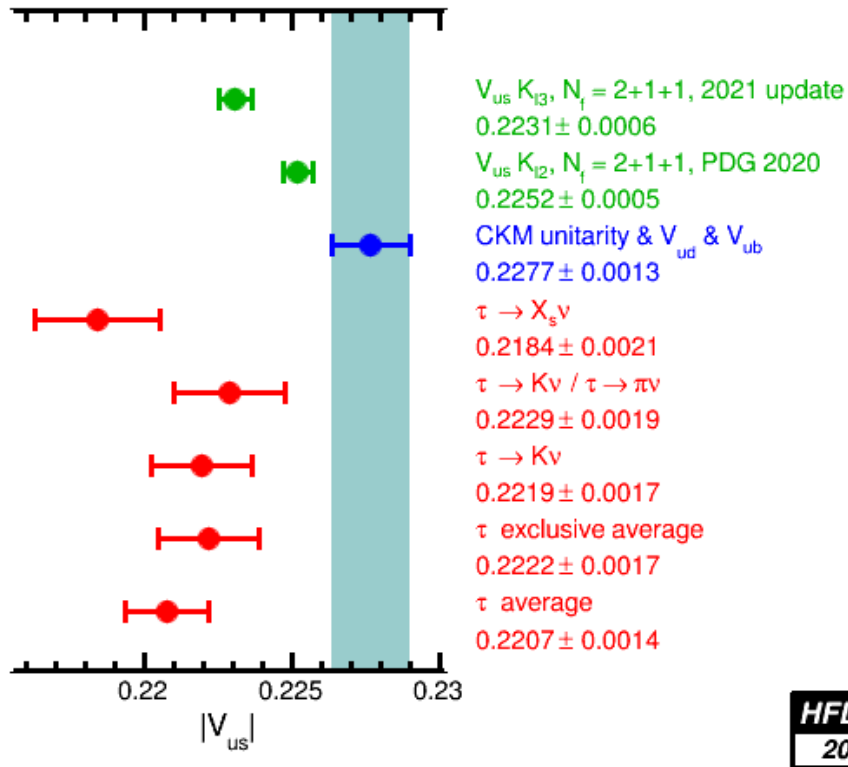
$$\mathbf{K}_{13}: |V_{us}| = 0.2232 \pm 0.0006$$

$$[f_+(0) = 0.9698 \pm 0.0017]$$

FLAG 2021

**Sizeable discrepancy. Improvements needed**

# $V_{us}$ & $V_{ud}$ Cabibbo Anomaly

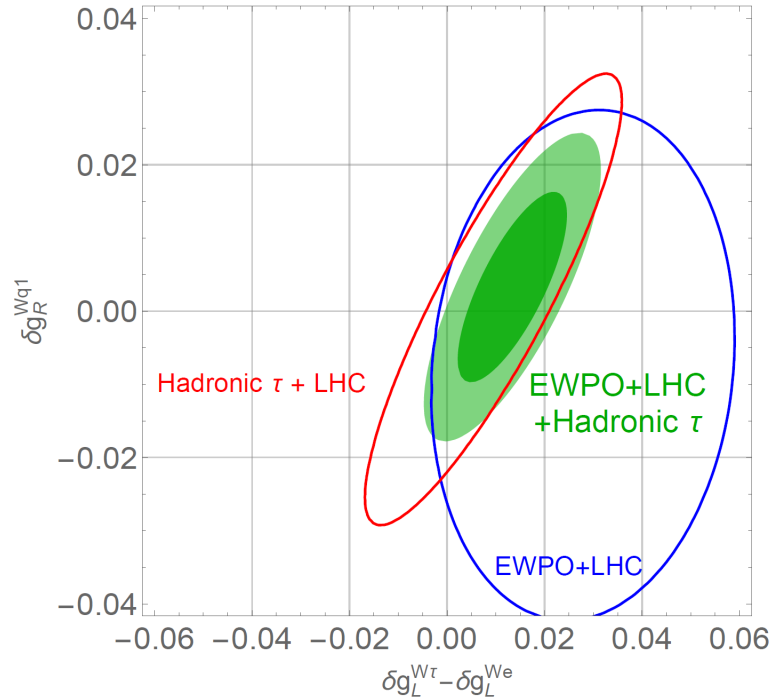


Sizeable violation of CKM unitarity

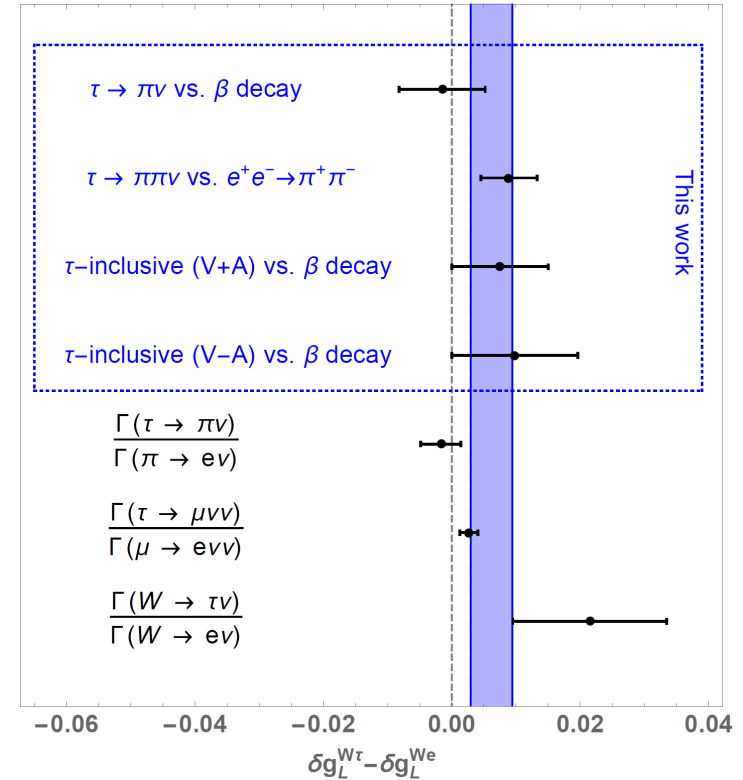
# Hadronic $\tau$ Decay & New Physics

$$\mathcal{L}_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \left[ \left(1 + \epsilon_L^\tau\right) \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma_5) d + \epsilon_R^\tau \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma_5) d \right. \\ \left. + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \left[ \epsilon_S^\tau - \epsilon_P^\tau \gamma_5 \right] d + \epsilon_T^\tau \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \right] + \text{h.c.}$$

Cirigliano, Falkowski, González-Alonso, Rodríguez-Sánchez, 1809.01161



$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee 11} \quad \epsilon_{S,P}^\tau = -\frac{1}{2} [c_{lequ} \pm c_{ledq}]_{\tau\tau 11}^* \\ \epsilon_R^\tau = \delta g_R^{Wq_1}, \quad \epsilon_T^\tau = -\frac{1}{2} [c_{lequ}^{(3)}]_{\tau\tau 11}^*,$$

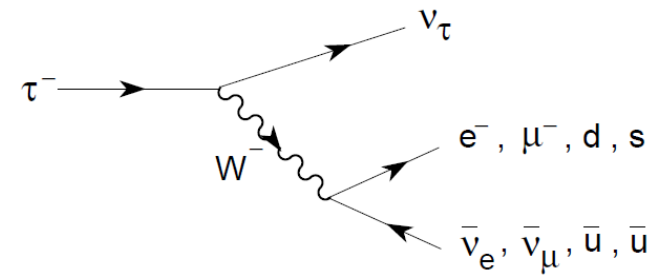


95% CL

$10^{-3}$  units

Coefficient	ATLAS $\tau\nu$	$\tau$ decays	$\tau$ and $\pi$ decays
$[c_{\ell q}^{(3)}]_{\tau\tau 11}$	[0.0, 1.6]	[-12.6, 0.2]	[-7.6, 2.1]
$[c_{lequ}]_{\tau\tau 11}$	[-5.6, 5.6]	[-8.4, 4.1]	[-5.6, 2.3]
$[c_{ledq}]_{\tau\tau 11}$	[-5.6, 5.6]	[-3.5, 9.0]	[-2.1, 5.8]
$[c_{lequ}^{(3)}]_{\tau\tau 11}$	[-3.3, 3.3]	[-10.4, -0.2]	[-8.6, 0.7]

# SUMMARY



## Many interesting $\tau$ topics

- ❑ Tests of QCD and the Electroweak Theory
- ❑ Looking for Signals of New Phenomena
- ❑ Superb Tool for New Physics Searches

**Better data samples needed**

**Lots of data will be produced @ Belle-II &  $\text{stcF}$   
& TeraZ...**

**Improving systematics brings a great reward**



# Backup



**CHARM 2023**  
**Siegen, Germany**  
**17-21 July 2023**

# LORENTZ STRUCTURE

$$\mathcal{H} = 4 \frac{G_F V_{ll'}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{\epsilon\omega}^n \left[ \bar{l}'_{\epsilon} \Gamma^n (\nu_{l'})_{\sigma} \right] \left[ \overline{(\nu_l)_{\lambda}} \Gamma_n l_{\omega} \right]$$

90% CL

Fetscher-Gerber, PDG2020

$$\mu \rightarrow e \bar{\nu}_e \nu_{\mu}$$

$ g_{RR}^S  < 0.035$	$ g_{RR}^V  < 0.017$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.050$	$ g_{LR}^V  < 0.023$	$ g_{LR}^T  < 0.015$
$ g_{RL}^S  < 0.420$	$ g_{RL}^V  < 0.105$	$ g_{RL}^T  < 0.105$
$ g_{LL}^S  < 0.550$	$ g_{LL}^V  > 0.960$	$ g_{LL}^T  \equiv 0$
$ g_{LR}^S + 6g_{LR}^T  < 0.143$	$ g_{RL}^S + 6g_{RL}^T  < 0.418$	
$ g_{LR}^S + 2g_{LR}^T  < 0.108$	$ g_{RL}^S + 2g_{RL}^T  < 0.417$	
$ g_{LR}^S - 2g_{LR}^T  < 0.070$	$ g_{RL}^S - 2g_{RL}^T  < 0.418$	
$Q_{RR} + Q_{LR} < 8.2 \times 10^{-4}$		

95% CL

Stahl, PDG2020

$$\tau \rightarrow e \bar{\nu}_e \nu_{\tau}$$

$ g_{RR}^S  < 0.70$	$ g_{RR}^V  < 0.17$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.99$	$ g_{LR}^V  < 0.13$	$ g_{LR}^T  < 0.082$
$ g_{RL}^S  < 2.01$	$ g_{RL}^V  < 0.52$	$ g_{RL}^T  < 0.51$
$ g_{LL}^S  < 2.01$	$ g_{LL}^V  < 1.005$	$ g_{LL}^T  \equiv 0$

$$\tau \rightarrow \mu \bar{\nu}_{\mu} \nu_{\tau}$$

$ g_{RR}^S  < 0.72$	$ g_{RR}^V  < 0.18$	$ g_{RR}^T  \equiv 0$
$ g_{LR}^S  < 0.95$	$ g_{LR}^V  < 0.12$	$ g_{LR}^T  < 0.079$
$ g_{RL}^S  < 2.01$	$ g_{RL}^V  < 0.52$	$ g_{RL}^T  < 0.51$
$ g_{LL}^S  < 2.01$	$ g_{LL}^V  < 1.005$	$ g_{LL}^T  \equiv 0$

$$\tau \rightarrow \pi \nu_{\tau}$$

$ g_R^V  < 0.15$	$ g_L^V  > 0.992$
------------------	-------------------

$$\tau \rightarrow \rho \nu_{\tau}$$

$ g_R^V  < 0.10$	$ g_L^V  > 0.995$
------------------	-------------------

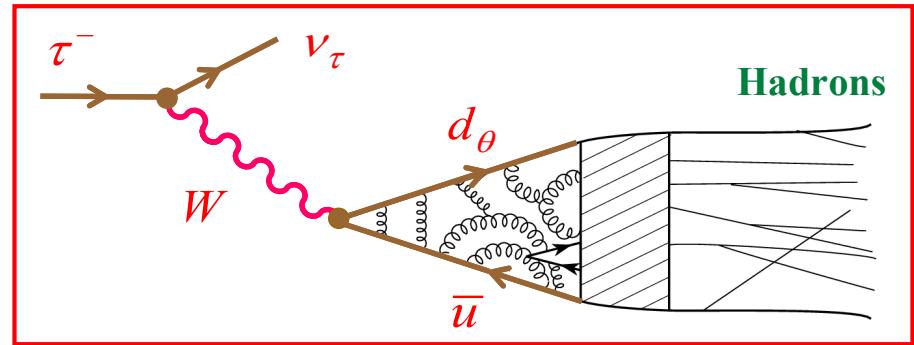
$$\tau \rightarrow a_1 \nu_{\tau}$$

$ g_R^V  < 0.16$	$ g_L^V  > 0.987$
------------------	-------------------

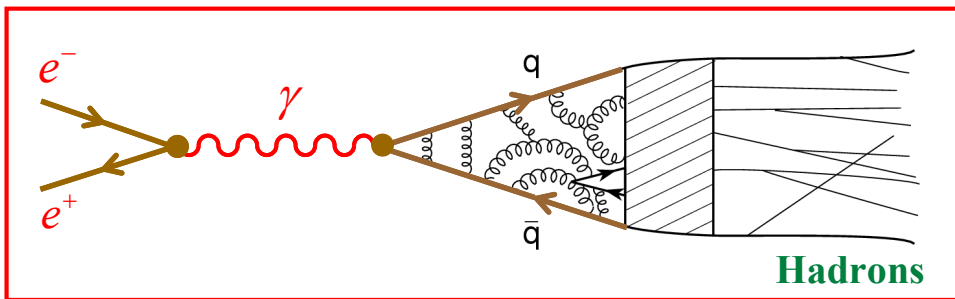
# Only Lepton Massive Enough to Decay into Hadrons

$\tau^- \rightarrow \nu_\tau H^-$  probes the  
hadronic V-A current

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1 - \gamma_5) u | 0 \rangle$$



$e^+e^- \rightarrow H^0$  probes the hadronic electromagnetic current



$$\langle H^0 | \sum_q Q_q \bar{q} \gamma^\mu q | 0 \rangle$$

**Isospin:** 
$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau V^-)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \frac{3 \cos^2 \theta_C}{2 \pi \alpha^2} S_{\text{EW}} \int_0^1 dx (1-x)^2 (1+2x) x \sigma_{e^+e^- \rightarrow V^0}^{I=1}(x m_\tau^2)$$

## Perturbative ( $m_q=0$ )

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n$$

$$K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101 \quad , \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08

$$\longrightarrow \quad \delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1-2x+2x^3-x^4) \left( \frac{\alpha_s(-s)}{\pi} \right)^n = a_\tau^n + \dots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau) / \pi$$

## Power Corrections

$$\Pi_{\text{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2} \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4 \langle O_4 \rangle \approx \frac{2\pi}{3} \langle 0 | \alpha_s G^{\mu\nu} G_{\mu\nu} | 0 \rangle$$

$$\delta_{\text{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx (1-3x^2+2x^3) \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-xm_\tau^2)^n} = -3 \frac{C_6 \langle O_6 \rangle}{m_\tau^6} - 2 \frac{C_8 \langle O_8 \rangle}{m_\tau^8}$$

Suppressed by  $m_\tau^6$  [additional chiral suppression in  $C_6 \langle O_6 \rangle^{V+A}$ ]

# Exhaustive Analysis of ALEPH Data

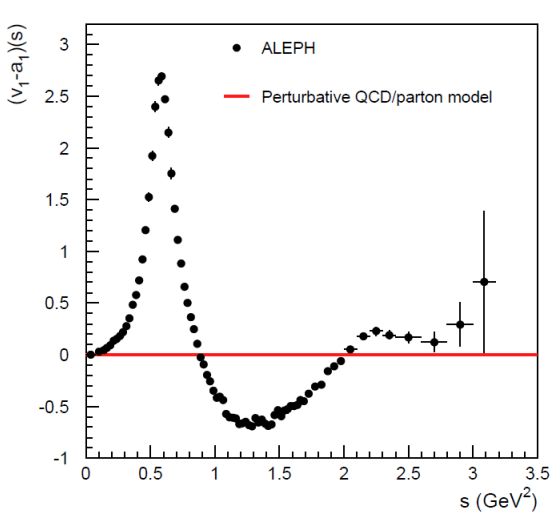
Rodríguez-Sánchez, Pich, 1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments <sup>1</sup>	$0.339^{+0.019}_{-0.017}$	$0.319^{+0.017}_{-0.015}$	$0.329^{+0.020}_{-0.018}$
Mod. ALEPH moments <sup>2</sup>	$0.338^{+0.014}_{-0.012}$	$0.319^{+0.013}_{-0.010}$	$0.329^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments <sup>3</sup>	$0.336^{+0.018}_{-0.016}$	$0.317^{+0.015}_{-0.013}$	$0.326^{+0.018}_{-0.016}$
$s_0$ dependence <sup>4</sup>	$0.335 \pm 0.014$	$0.323 \pm 0.012$	$0.329 \pm 0.013$
Borel transform <sup>5</sup>	$0.328^{+0.014}_{-0.013}$	$0.318^{+0.015}_{-0.012}$	$0.323^{+0.015}_{-0.013}$
<b>Combined value</b>	<b><math>0.335 \pm 0.013</math></b>	<b><math>0.320 \pm 0.012</math></b>	<b><math>0.328 \pm 0.013</math></b>



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

- 1)  $\omega_{kl}(x) = (1 + 2x)(1 - x)^{2+k}x^l$   $(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$
- 2)  $\tilde{\omega}_{kl}(x) = (1 - x)^{2+k}x^l$   $(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$
- 3)  $\omega^{(2,m)}(x) = (1 - x)^2 \sum_{k=0}^m (k + 1)x^k = 1 - (m + 2)x^{m+1} + (m + 1)x^{m+2}$  ,  $1 \leq m \leq 5$
- 4)  $\omega^{(2,m)}(x)$   $0 \leq m \leq 2$  , 1 single moment in each fit
- 5)  $\omega_a^{(1,m)}(x) = (1 - x^{m+1})e^{-ax}$   $0 \leq m \leq 6$



# Chiral Sum Rules

$$\Pi(s) \equiv \Pi_{VV}(s) - \Pi_{AA}(s) \quad \text{Pure non-perturbative quantity}$$

$$\lim_{s \rightarrow \infty} s^2 \Pi(s) = 0 \quad \rightarrow \quad \Pi^{\text{OPE}}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \dots$$

$$\chi\text{PT} (s \rightarrow 0): \quad \Pi(s) = \frac{2F^2}{s} - 8L_{10}^r(\mu^2) + \frac{1}{16\pi^2} \left( \frac{5}{3} - \ln \frac{-s}{\mu^2} \right) + 16C_{87}^r(\mu^2) \frac{s}{F^2} + \dots$$

$$\int_{s_{\text{th}}}^{s_0} ds \, \omega(s) \frac{1}{\pi} \text{Im} \Pi(s) + \frac{1}{2\pi i} \oint_{|s|=s_0} ds \, \omega(s) \Pi(s) = 2f_\pi^2 \omega(m_\pi^2) + \text{Res}[\omega(s)\Pi(s), s=0]$$

## Statistical analysis:

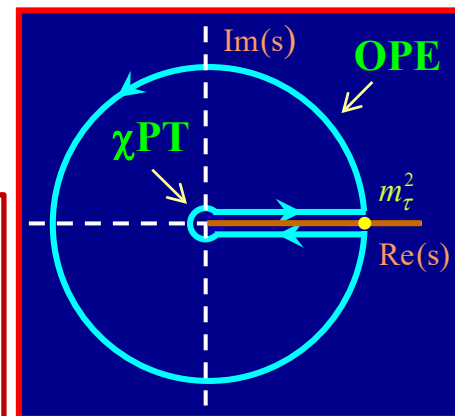
$$C_{87}^{\text{eff}} = (8.40 \pm 0.18) \cdot 10^{-3} \text{ GeV}^{-2}$$

$$L_{10}^{\text{eff}} = (-6.48 \pm 0.05) \cdot 10^{-3}$$

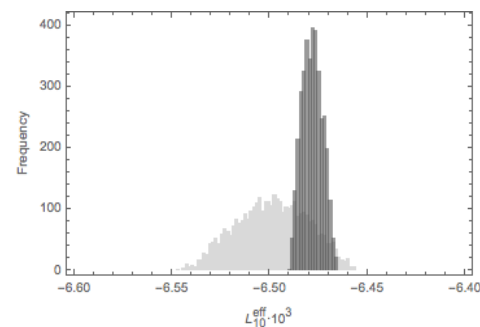
González-Pich-Rodríguez, 1602.06112

$$O_6 = (-3.6 \pm 0.7) \cdot 10^{-3} \text{ GeV}^6$$

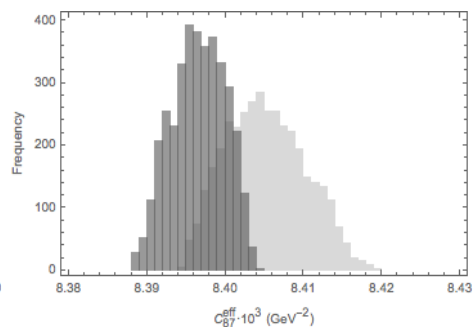
$$O_8 = (-1.0 \pm 0.4) \cdot 10^{-2} \text{ GeV}^8$$



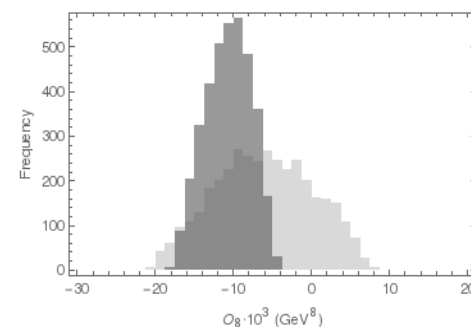
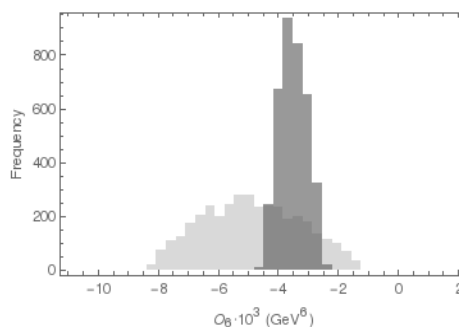
Non-pinched & pinched weights



A. Pich



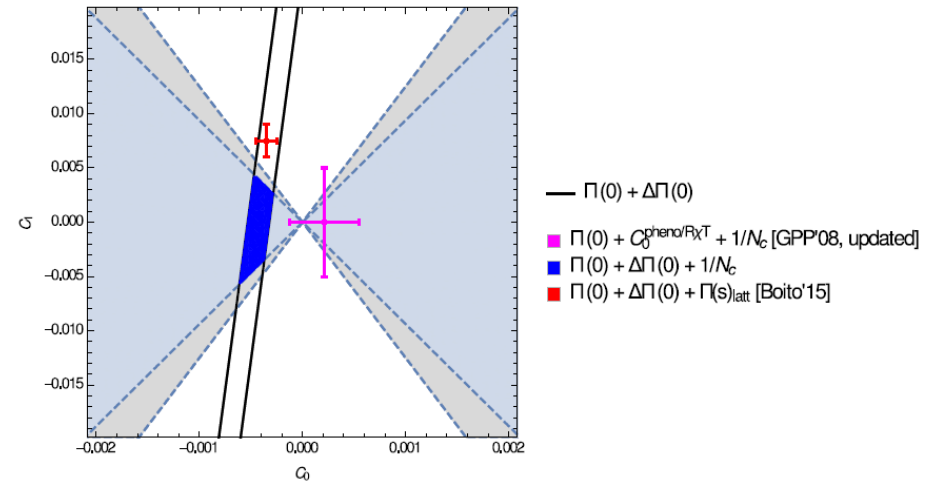
$\tau$  Physics



# • $\chi$ PT Parameters:

González-Alonso, Pich, Rodríguez-Sánchez, 1602.06112

$$\begin{aligned}
 L_{10}^{\text{eff}} &= L_{10}^r - 0.00126 + \mathcal{O}(p^6) \\
 L_{10}^{\text{eff}} &= 1.53 L_{10}^r + 0.263 L_9^r - 0.00179 \\
 &\quad - \frac{1}{8} (C_0^r + C_1^r) + \mathcal{O}(p^8) \\
 C_{87}^{\text{eff}} &= C_{87}^r + 0.296 L_9^r + 0.00155 + \mathcal{O}(p^8)
 \end{aligned}$$



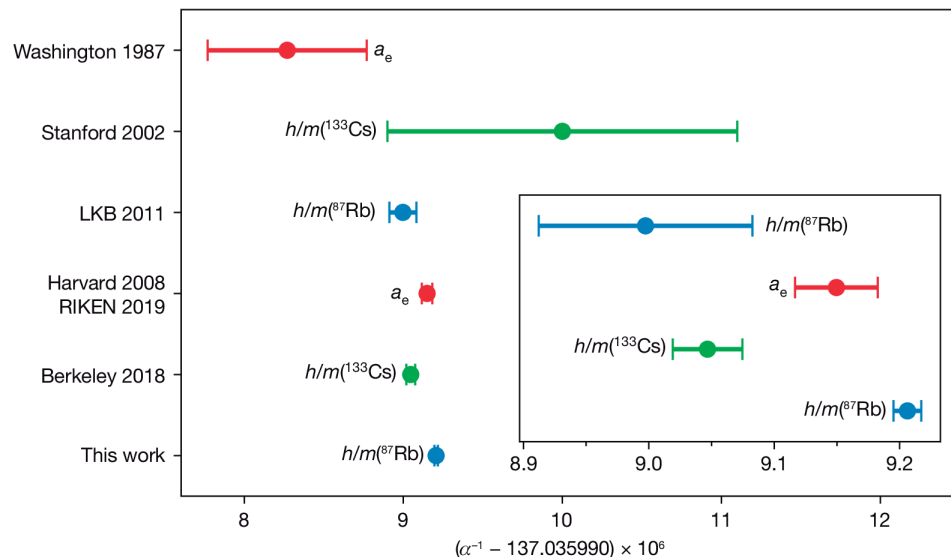
- 
- $\mathcal{O}(p^4)$  analysis:  $L_{10}^r(M_\rho) = -(5.22 \pm 0.05) \cdot 10^{-3}$
  - $\mathcal{O}(p^6)$  analysis:  $L_{10}^r(M_\rho) = -(4.1 \pm 0.4) \cdot 10^{-3}$   
 $C_{87}^r(M_\rho) = (5.10 \pm 0.22) \cdot 10^{-3} \text{ GeV}^{-2}$

- $\varepsilon'_K/\varepsilon_K$ :  $\mathcal{O}_6 \Rightarrow \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle \Rightarrow \text{e.m. penguin contribution}$

$$(\varepsilon'_K/\varepsilon_K)_{\text{EWP}}^{I=2} = (-4.5 \pm 1.8) \cdot 10^{-4}$$

Pich-Rodríguez, 2102.09308

# Electron Anomalous Magnetic Moment



Morel et al, Nature 588 (2020) 61

**New measurement of  $\alpha$**

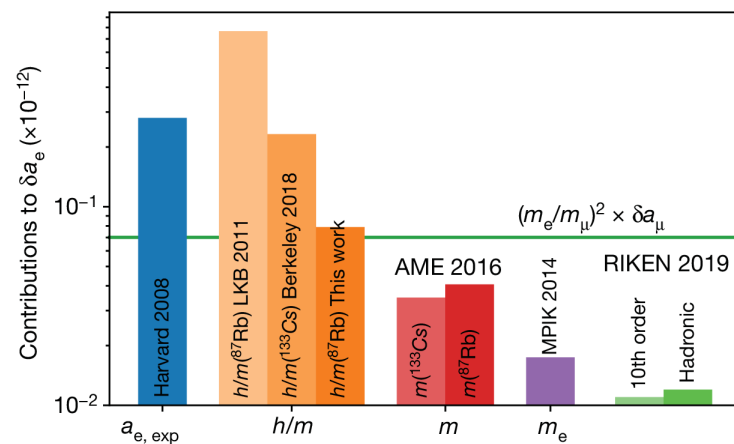
$$\alpha^{-1}(\text{Rb}) = 137.035\,999\,206\,(11)$$

$8.1 \times 10^{-11}$  accuracy

5.8  $\sigma$  discrepancy with Cs experiment

$$\Delta a_e \equiv a_e^{\text{exp}} - a_e^{\text{SM}}$$

$$= \begin{cases} (-8.8 \pm 3.6) \cdot 10^{-13} & (\text{Cs}, -2.4\sigma) \\ (+4.8 \pm 3.0) \cdot 10^{-13} & (\text{Rb}, +1.6\sigma) \end{cases}$$





# $\tau$ Anomalous Magnetic Moment

Difficult to measure!

$$a_{\tau}^{\text{exp}} = (-0.018 \pm 0.017)$$

DELPHI

$$-0.007 < a_{\tau}^{\text{New Phys}} < 0.005$$

González-Springer, Santamaria, Vidal '00 (LEP/SLD data)

Eidelman, Passera

$$\begin{aligned}
 10^8 \cdot a_{\tau}^{\text{th}} &= 117\,324 \pm 2 && \text{QED} \\
 &+ 47.4 \pm 0.5 && \text{EW} \\
 &+ 337.5 \pm 3.7 && \text{hvp} \\
 &+ 7.6 \pm 0.2 && \text{hvp NLO} \\
 &+ 5 \pm 3 && \text{light-by-light} \\
 &= \mathbf{117\,721 \pm 5}
 \end{aligned}$$

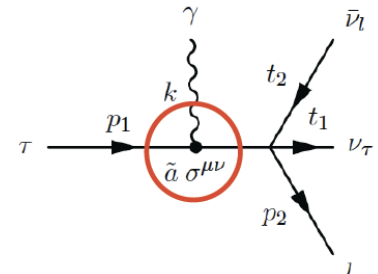
Enhanced sensitivity to

new physics:  $(m_{\tau}/m_{\mu})^2 = 283$

	Electron	Muon	Tau
$a^{\text{EW}}/a^{\text{HAD}}$	1/56	1/45	1/7
$a^{\text{EW}}/\delta a^{\text{HAD}}$	1.6	3	10

Essentially unknown

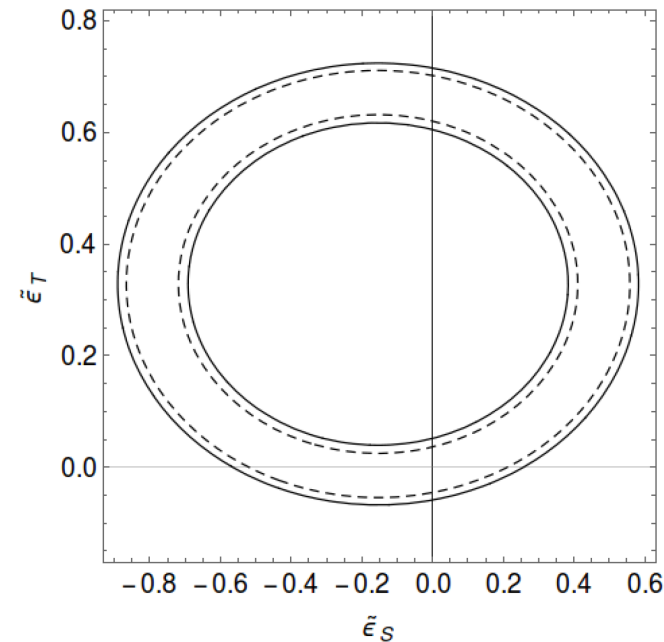
May be accessible at BF's through  
radiative leptonic decays (Fael et al) or  
with a polarized  $e^-$  beam (Crivellin et al)



# EFT analysis of $\tau \rightarrow \nu_\tau K \pi$

Rendón-Roig-Toledo, 1902.08143

$$\mathcal{L}_{cc} = -\frac{G_F V_{us}}{\sqrt{2}}(1 + \epsilon_L + \epsilon_R) \left[ \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\ell \cdot \bar{u} [\gamma^\mu - (1 - 2\hat{\epsilon}_R) \gamma^\mu \gamma_5] s \right. \\ \left. + \bar{\tau} (1 - \gamma_5) \nu_\ell \cdot \bar{u} [\hat{\epsilon}_s - \hat{\epsilon}_p \gamma_5] s + 2\hat{\epsilon}_T \bar{\tau} \sigma_{\mu\nu} (1 - \gamma_5) \nu_\ell \cdot \bar{u} \sigma^{\mu\nu} s \right] + \text{h.c.}$$



Best fit values	$\hat{\epsilon}_S$	$\hat{\epsilon}_T$	$\chi^2$	$\chi^2$ in the SM
Excluding $i = 5, 6, 7$ bins	$(1.3 \pm 0.9) \times 10^{-2}$	$(0.7 \pm 1.0) \times 10^{-2}$	[72, 73]	[74, 77]
Including $i = 5, 6, 7$ bins	$(0.9 \pm 1.0) \times 10^{-2}$	$(1.7 \pm 1.7) \times 10^{-2}$	[83, 86]	[91, 95]



$$\Lambda_{\text{NP}} \geq 2 - 5 \text{ TeV}$$

Complementary to kaon and hyperon data analyses

# $\tau$ 's @ LHC

## □ Excellent signature to probe New Physics

Difficult to identify light objects ( $Z, W^\pm$ ) with only Jets

QCD Jets orders of magnitude larger

Must rely on leptons

## □ LHC produces high-momenta $\tau$ 's

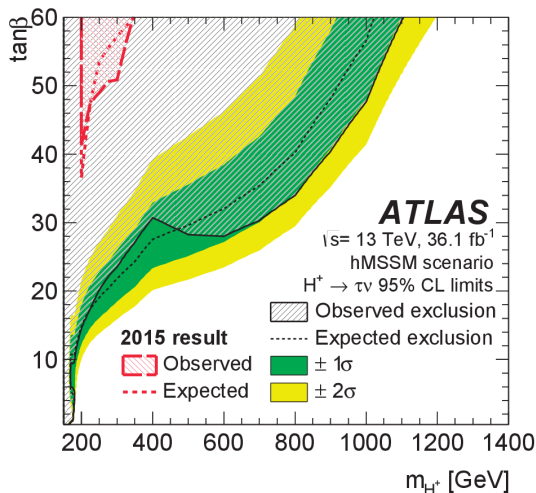
Tightly collimated decay products (mini-jet like)

Momentum reconstruction possible

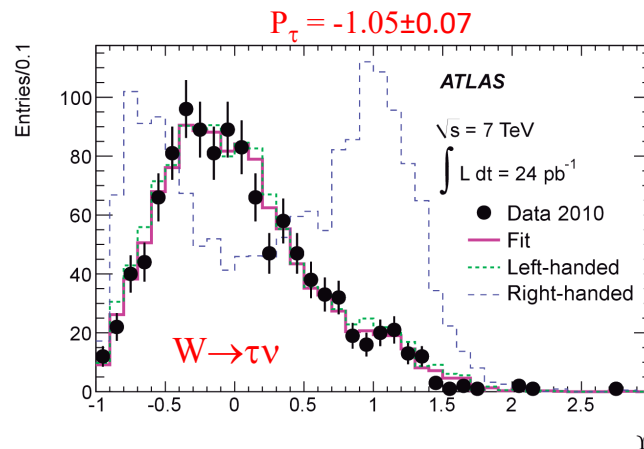
## □ Low multiplicity. Good tagging efficiency

## □ Heaviest lepton coupling to the Higgs (4<sup>th</sup> H Br)

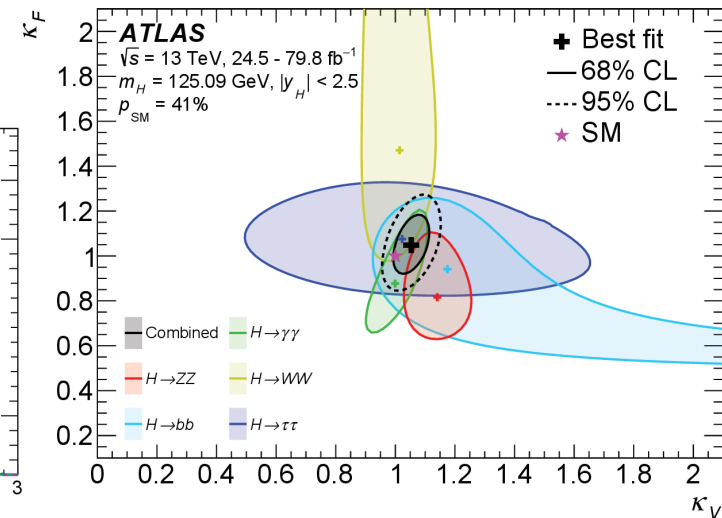
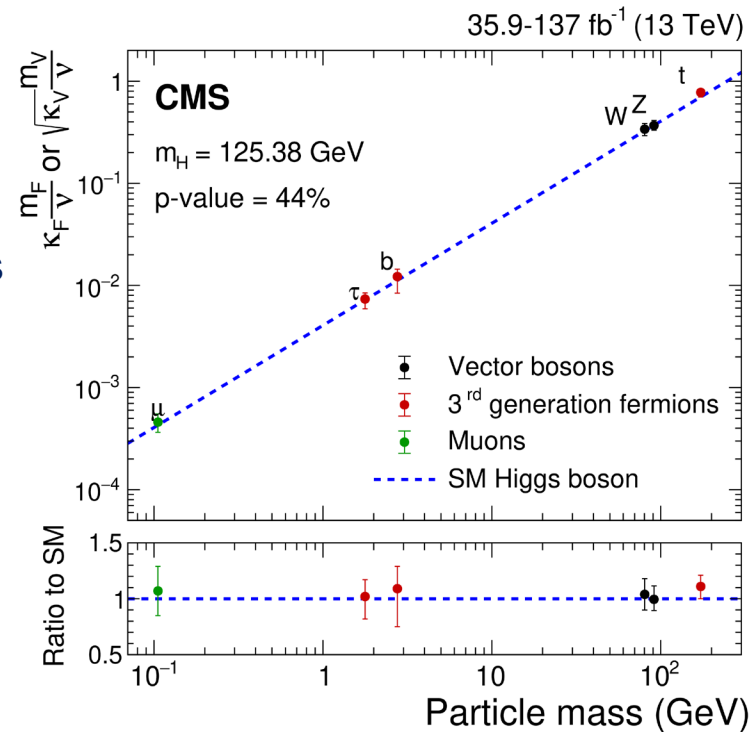
## □ Polarization information



A. Pich



$\tau$  Physics

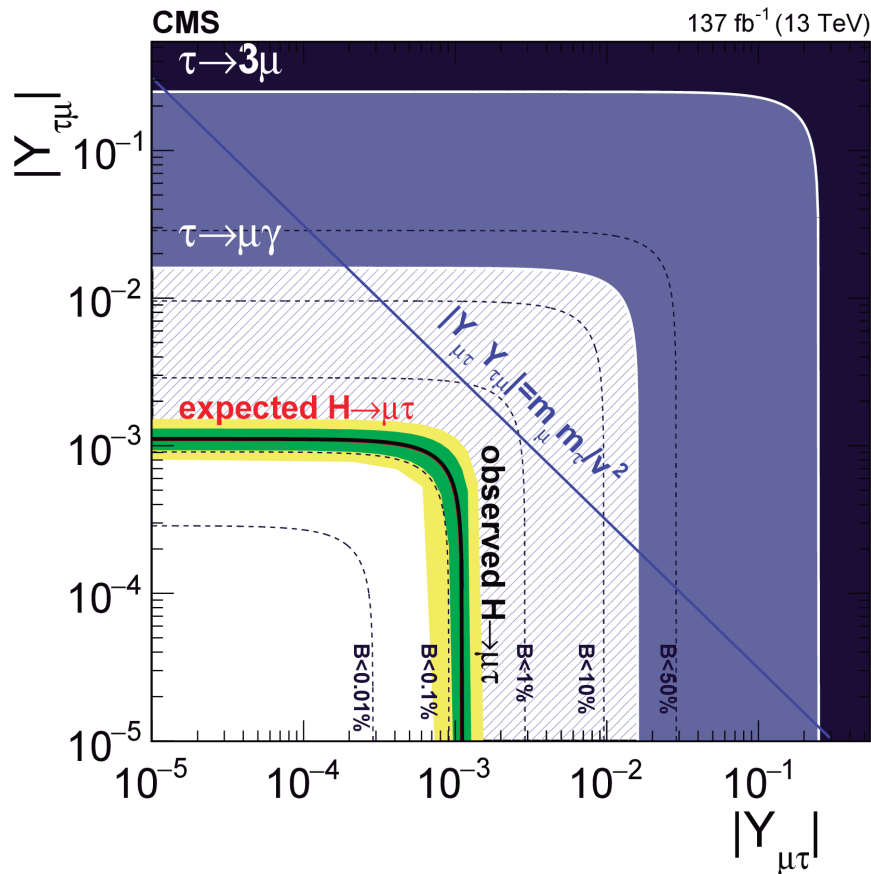


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# Flavour-Violating Higgs Couplings

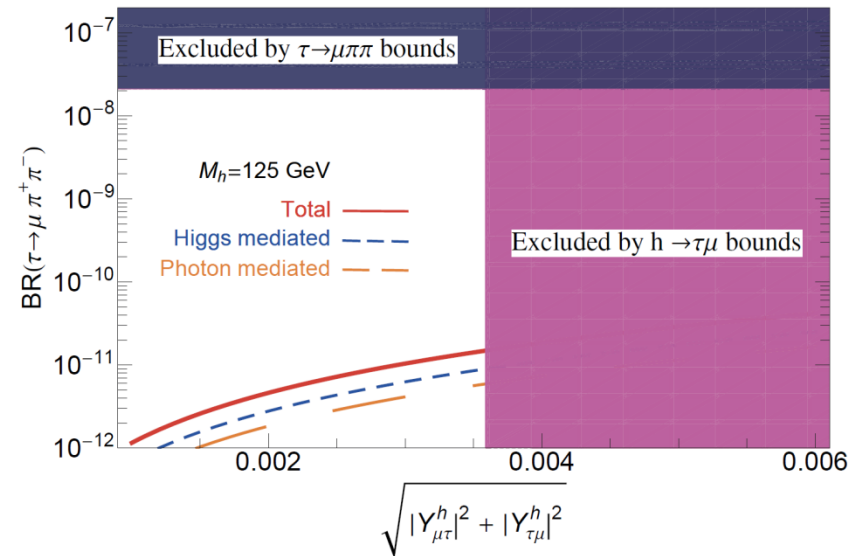
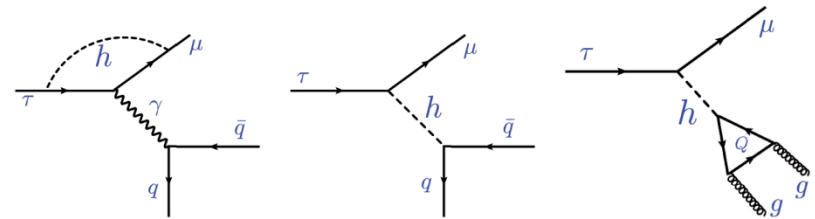
$$\mathcal{L} = -H \{ Y_{e\mu} \bar{e}_L \mu_R + Y_{e\tau} \bar{e}_L \tau_R + Y_{\mu\tau} \bar{\mu}_L \tau_R + \dots \}$$

$$\text{Br}(H \rightarrow \mu\tau) < 0.15\% \quad (95\% \text{ CL})$$



$$\tau \rightarrow \mu \pi^+ \pi^-$$

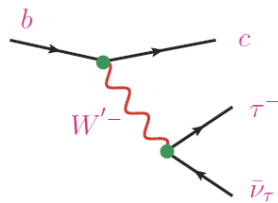
Celis et al., 1409.4439



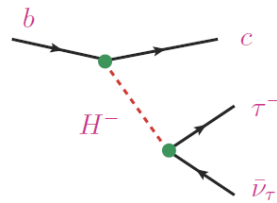
# Effective Field Theory Analysis

$$\mathcal{H}_{\text{eff}}^{b \rightarrow c \ell \nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[ (1 + C_{V_L}) \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} + C_{S_R} \mathcal{O}_{S_R} + C_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T \right] + \text{h.c.}$$

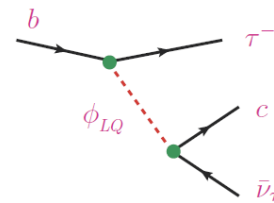
$$\mathcal{O}_{V_{L,R}} = (\bar{c} \gamma^\mu b_{L,R}) (\bar{\ell}_L \gamma_\mu \nu_{\ell L}), \quad \mathcal{O}_{S_{L,R}} = (\bar{c} b_{L,R}) (\bar{\ell}_R \nu_{\ell L}), \quad \mathcal{O}_T = (\bar{c} \sigma^{\mu\nu} b_L) (\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L})$$



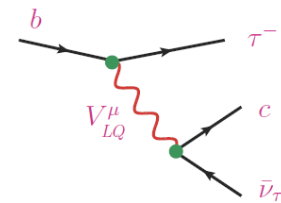
$C_{V_L}$



$C_{S_L}, C_{S_R} \quad (C_T)$



$C_{V_L}, C_{S_L}, C_T$



$C_{V_L}, C_{S_R}$

Many analyses (usually with single operator/mediator and partial data information)

Freytsis et al, Bardhan et al, Cai et al, Hu et al, Celis et al, Datta et al, Bhattacharya et al, Alonso et al, ...

**Global fit to all data**

( $q^2$  distributions included)

Murgui-Peñuelas-Jung-Pich, 1904.09311

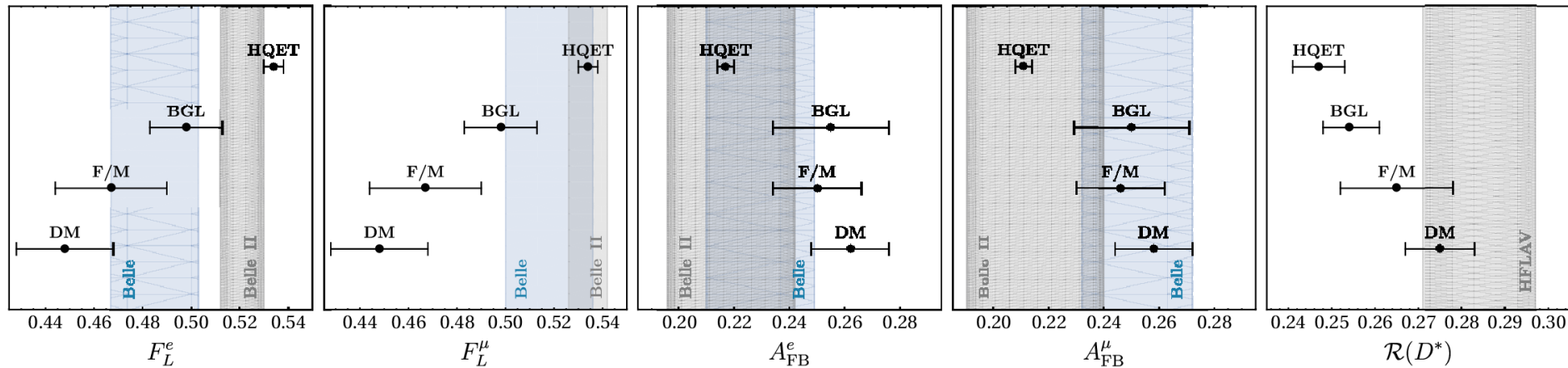
$F_L^{D^*}, \mathcal{B}_{10}$	Min 1	Min 2
$\chi^2/\text{d.o.f.}$	37.4/54	40.4/54
$C_{LL}^V$	$0.09 \pm 0.13$ $-0.12$	$0.34 \pm 0.05$ $-0.07$
$C_{RL}^S$	$0.09 \pm 0.12$ $-0.61$	$-1.10 \pm 0.48$ $-0.07$
$C_{LL}^S$	$-0.14 \pm 0.52$ $-0.07$	$-0.30 \pm 0.11$ $-0.50$
$C_{LL}^T$	$0.008 \pm 0.046$ $-0.044$	$0.093 \pm 0.029$ $-0.030$

$\mathcal{B}(B_c \rightarrow \tau \bar{\nu}) < 10\%$

$F_L^{D^*}$  included

# $B \rightarrow D^* \ell \nu$ Observables with different FFs

Fedele et al 2305.15457




Smaller [larger] discrepancy on  $R(D^*)$  [ $R(D)$ ] would bring back a possible new-physics scalar explanation

# $\tau$ Data Samples

<b>ALEPH:</b>	$3.3 \cdot 10^5$	reconstructed $\tau$ decays	
<b>BaBar / Belle:</b>	$1.4 \cdot 10^9$	$\tau^+\tau^-$ pairs	
<b>Belle-II:</b>	$4.6 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	
<b>s<b>τ</b>cF:</b>	$2.1 \cdot 10^{10}$	$\tau^+\tau^-$ pairs	( $10^8$ near threshold)

**Luminosity ( $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ ) is important. Systematics also!**

## Advantages of the threshold región:

- Ability to measure backgrounds (running below threshold)
- Free of heavy quark backgrounds
- Single-Tagging  Precise measurement of absolute branching fractions
- Monochromatic spectra for two-body decays ( $\pi$ , K)

