Precise determination of the decay rates of $\eta_c \to \gamma\gamma$, $J/\psi \to \gamma\eta_c$ and $J/\psi \to \eta_c e^+ e^-$ from lattice QCD

Based on arXiv:2305.06231

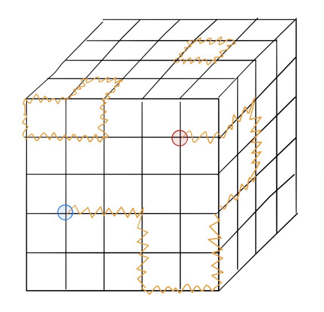
Brian Colquhoun

w/ Laurence Cooper, Christine Davies,

G. Peter Lepage

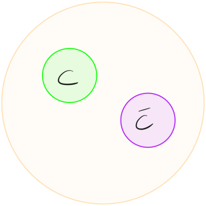
HPQCD Collaboration

18 July 2023



Charmonium decays

- ★ Decays with photons can be used as tests of our understanding of internal structure of mesons from strong interaction physics
- ★ Photons are clean objects; not as messy as QCD
- ★ $J/\Psi \to \gamma\eta_c$: Some tension between branching fractions from lattice QCD and experimental result
- ★ $\eta_c \to \gamma\gamma$ less clear
  - some lattice calculations exist (but without realistic sea quark content)
  - experimental results give no clear consensus



This work

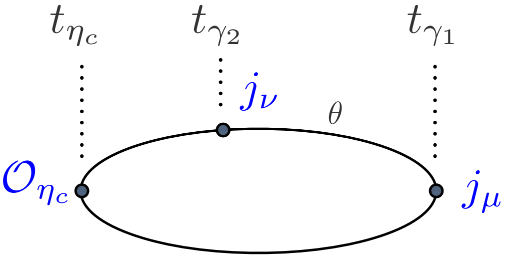
- ★ Precise calculation by using Highly Improved Staggered Quark (HISQ) action
  - Very good action for charm, c.f. previous HPQCD work
- ★ Calculate these decays with realistic sea
  - Effect of 2+1+1 quarks
- ★ 1-2% uncertainties, so more accurate now than experiment

Full details in [arXiv:2305.06231] (to appear in Phys. Rev. D. imminently!)

Lattice details

- ★ $2+1+1$ HISQ gauge ensembles provided by MILC Collaboration
- ★ Lattice spacings from ≈ 0.15 fm down to ≈ 0.06 fm
- ★ Combination of $m_s/m_l = 5$ and physical m_l
- ★ Valence charm quarks also use HISQ formalism
- ★ Charm mass accurately tuned through measurement of J/ψ meson
  - (HPQCD '20 [2005.01845])

$$\eta_c \to \gamma\gamma$$



Ji & Jung [hep-lat/0101014] & [hep-lat/0103007]:

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_c}) = a \sum_{t_{\gamma_1}} e^{-\omega_1 (t_{\gamma_1} - t_{\gamma_2})} C_{\mu\nu}(t_{\gamma_1}, t_{\gamma_2}, t_{\eta_c})$$

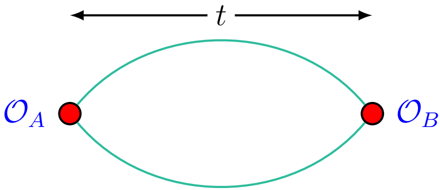
- For on-shell photons:

$$\omega_1 = |\vec{q}_1| = |\vec{q}_2| = \frac{M_{\eta_c}}{2}$$

- Impart momentum (θ in picture) to tune ω_1
- Currents require renormalisation; we use RI-SMOM scheme

Fit two sets of correlators:

$$C_{\eta_c}(t, t_{\eta_c}) = \sum_n^{N_\mathrm{n}} a_\mathrm{n}^2 \left(e^{-E_n t} + e^{-E_n (N_t - t)}\right)$$



and

$$\tilde{C}_{\mu\nu}(t_{\gamma_2}, t_{\eta_c}) = \sum_n^{N_\mathrm{n}} a_\mathrm{n} b_\mathrm{n} \left(e^{-E_n (t_{\gamma_2} - t_{\eta_c})} + e^{-E_n (N_t - t_{\gamma_2} + t_{\eta_c})}\right)$$

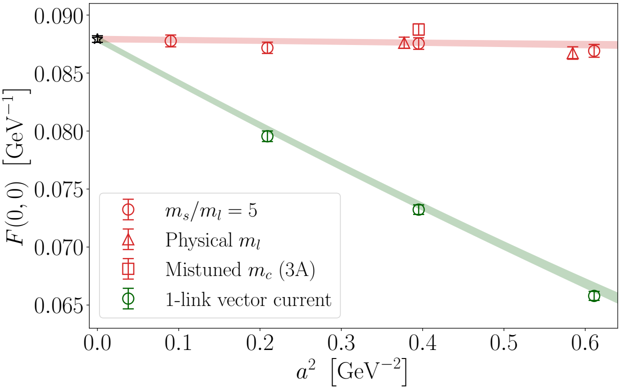
Extract form factor $F_\mathrm{latt}(0, q_2^2)$ by:

$$\frac{F_\mathrm{latt}(0, q_2^2)}{a} = b_0 Z_V^2 \frac{\sqrt{2aM_{\eta_c}^\mathrm{latt}}}{aM_{\eta_c}^\mathrm{latt} a q_1}$$

which, when $q_2^2 = 0$, relates to the width for two on-shell photons:

$$\Gamma(\eta_c \to \gamma\gamma) = \pi \alpha_\mathrm{em}^2 Q_c^4 M_{\eta_c}^3 F(0,0)^2.$$

$\eta_c \to \gamma\gamma$ in the continuum



$$\frac{F^{(t)}_{\text{latt}}(0,q_2^2)}{a} = \frac{F(0,0)}{\left(1-\frac{q_2^2}{M^2_{\text{pole}}}\right)}\left[1+\sum_{i=1}^{i_{\max}}\kappa^{(i,t)}_{a\Lambda}\left(a\Lambda^{(t)}\right)^{2i}+\kappa_{\text{val},c}\delta^{\text{val},c}+\kappa_{\text{sea},c}\delta^{\text{sea},c}\right.$$

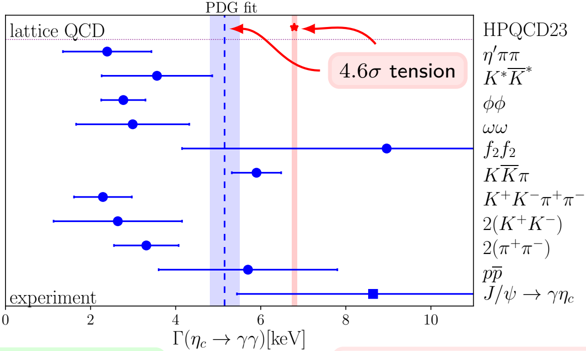
$$\left.+\kappa^{(0)}_{\text{sea},uds}\delta^{\text{sea},uds}\left\{1+\kappa^{(1,t)}_{\text{sea},uds}(a\tilde{\Lambda})^2+\kappa^{(2,t)}_{\text{sea},uds}(a\tilde{\Lambda})^4\right\}\right]$$

Continuum result gives

$$F(0,0) = 0.08793(29)_{\text{fit}}(26)_{\text{syst}}\ \text{GeV}^{-1}$$

From which we can determine the width:

$$\Gamma(\eta_c \to \gamma\gamma) = 6.788(45)_{\text{fit}}(41)_{\text{syst}}\ \text{keV}$$

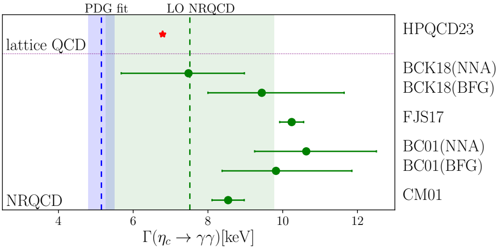


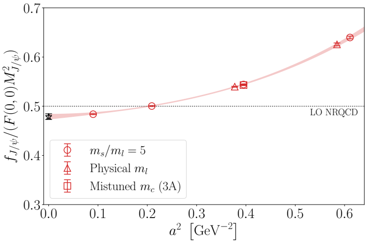
PDG fit: $5.15(35)$ keV ← $\chi^2 = 118$ for 81 d.o.f.; $p = 0.005$

Expectation in nonrelativistic limit:

$$\frac{\Gamma(J/\Psi \to e^+e^-)}{\Gamma(\eta_c \to \gamma\gamma)} \approx \frac{3}{4}; \qquad \frac{f_{J/\psi}}{F(0,0)M^2_{J/\psi}} = \frac{1}{2}\left(1 + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2/c^2)\right)$$

(Czarnecki & Melnikov '01 [hep-ph/0109054]):

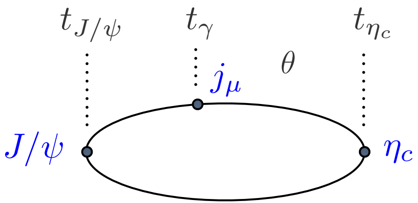




$$\text{Our result}: \frac{f_{J/\psi}}{F(0,0)M^2_{J/\psi}} = 0.4786(57)_{\text{fit}}(14)_{\text{syst}}$$

$M_{J/\psi}$ & $f_{J/\psi}$ from HPQCD '20 [2005.01845]:

$$J/\Psi \to \gamma\eta_c$$



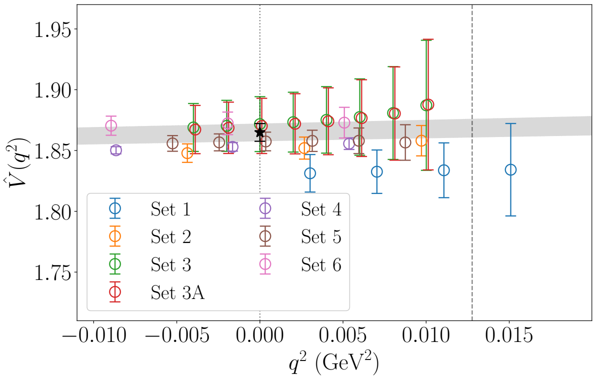
$$C_{3\text{pt}}(t,T) = \sum_{i,j}^{N_n,N_n} a_i e^{-E_i t} V_{ij} b_j e^{E_j(T-t)}$$

Form factor:

$$\hat{V}(q^2) = \frac{M_{J/\psi}^{\text{latt}} + M_{\eta_c}^{\text{latt}}}{M_{J/\psi}^{\text{latt}} q^y} Z_V \sqrt{2M_{J/\psi}^{\text{latt}}}\sqrt{2E_{\eta_c}^{\text{latt}}} V_{00}$$

From which the decay width is found:

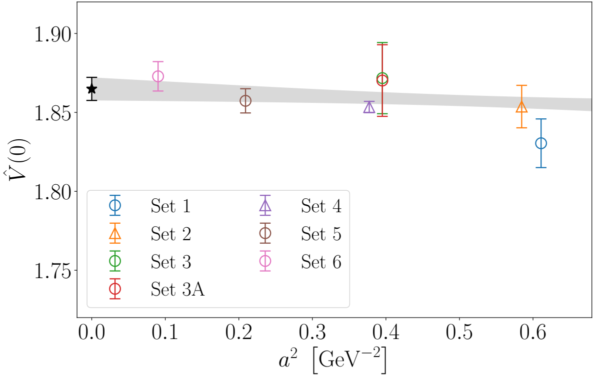
$$\Gamma\left(J/\psi \to \gamma\eta_c\right) = \alpha Q_c^2 \frac{16}{3} \frac{|\mathbf{k}|^3}{\left(M_{\eta_c} + M_{J/\psi}\right)^2} |\hat{V}(0)|^2$$



$$\hat{V}_{\mathrm{latt}}(0) = 1.8649(73)_{\mathrm{fit}}(75)_{\mathrm{syst}}$$

$$\hat{V}_{\mathrm{latt}}(q^2) = \sum_{k=0}^{2} A^{(k)} \left(\frac{q^2}{\left(M^{\mathrm{latt}}_{J/\psi}\right)^2} \right)^k \Bigg[1 + \sum_{i=1}^{i_{\max}} \kappa^{(i,k)}_{a\Lambda} (a\Lambda)^{2i} + \kappa^{(k)}_{\mathrm{val},c} \delta^{\mathrm{val},c} + \kappa^{(k)}_{\mathrm{sea},c} \delta^{\mathrm{sea},c} + \kappa^{(0,k)}_{\mathrm{sea},uds} \delta^{\mathrm{sea},uds} \left\{ 1 + \kappa^{(1,k)}_{\mathrm{sea},uds} (\tilde{\Lambda} a)^2 + \kappa^{(2,k)}_{\mathrm{sea},uds} (\tilde{\Lambda} a)^4 \right\} \Bigg]$$

$\hat{V}(0)$

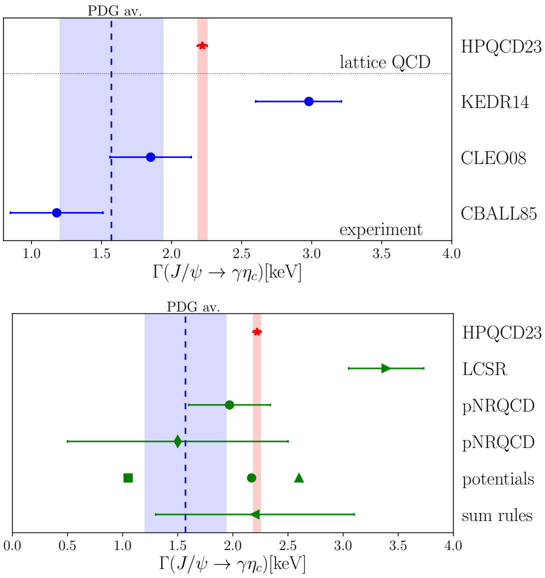


$$\hat{V}_{\mathrm{latt}}(0) = 1.8649(73)_{\mathrm{fit}}(75)_{\mathrm{syst}}$$

$$\Gamma(J/\psi \to \gamma\eta_c) = 2.219(17)_{\mathrm{fit}}(18)_{\mathrm{syst}}(24)_{\mathrm{expt}}(4)_{\mathrm{QED}}\ \mathrm{keV}$$

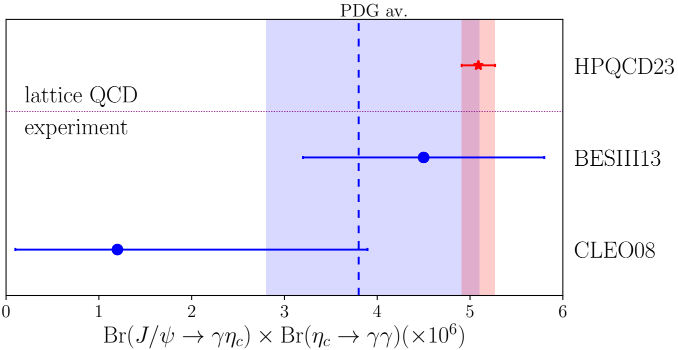
$$\mathrm{Br}(J/\psi \to \gamma\eta_c) = 2.40(3)_{\mathrm{latt}}(5)_{\mathrm{expt}}\%$$

Summary results: $J/\psi \to \gamma\eta_c$



Product of fractions

★ Product of $J/\psi \to \gamma\eta_c$ and $\eta_c \to \gamma\gamma$ measured at by CLEO and BESIII



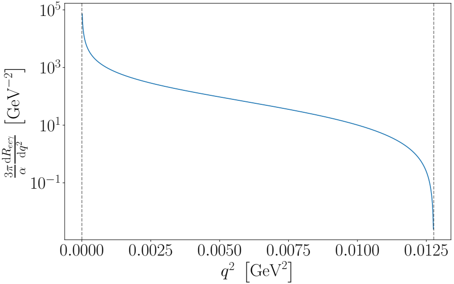
$$J/\Psi \to \eta_c e^+ e^-$$

$J/\Psi \to \eta_c e^+ e^-$

$$R_{ee\gamma} = \frac{\mathcal{B}(J/\Psi \to \eta_c e^+ e^-)}{\mathcal{B}(J/\Psi \to \gamma\eta_c)}; \quad \frac{dR_{ee\gamma}}{dq^2} = \frac{\alpha}{3\pi q^2}\left|\frac{\hat{V}(q^2)}{\hat{V}(0)}\right|^2 \left(1 - \frac{4m_e^2}{q^2}\right)^{\frac{1}{2}} \left(1 + \frac{2m_e^2}{q^2}\right)$$

Landsberg '85 Phys. Rep. 128, 301

$$\times \left(\left(1 + \frac{q^2}{M_{J/\psi}^2 - M_{\eta_c}^2}\right)^2 - \frac{4M_{J/\psi}^2 q^2}{(M_{J/\psi}^2 - M_{\eta_c}^2)^2}\right)^{\frac{3}{2}}$$



$$\Gamma(J/\Psi \to \eta_c e^+ e^-) = 0.01349(15)_{\rm latt}(15)_{\rm expt}(13)_{\rm QED}\ \text{keV}$$
$$\text{Br}(J/\Psi \to \eta_c e^+ e^-) = 1.457(16)_{\rm latt}(15)_{\rm QED}(31)_{\rm expt} \times 10^{-4}$$

Follow-on study

Follow-on study: $\eta_b \to \gamma\gamma$

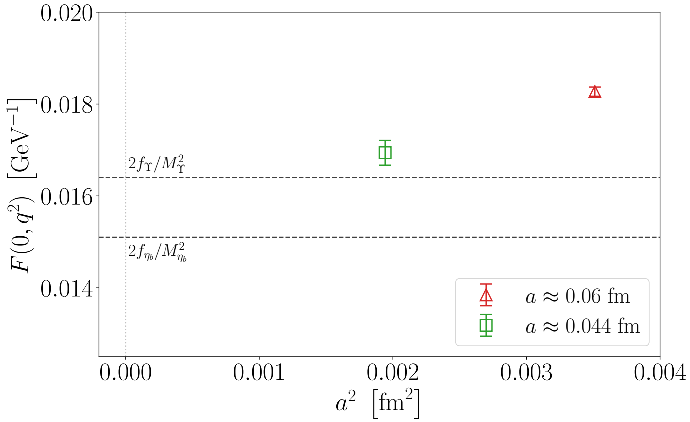
Follow-on study

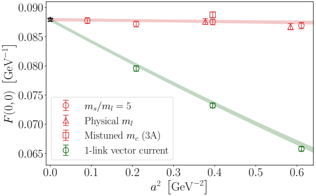
Follow-on study: $\eta_b \to \gamma\gamma$

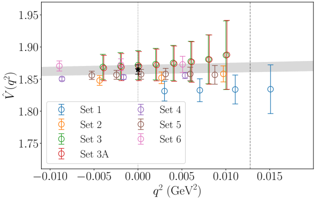
Prediction

Follow-on study: $\eta_b \to \gamma\gamma$

Prediction

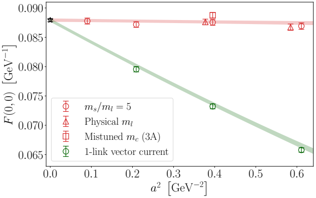


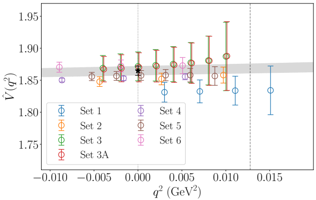




- ★ $\Gamma(\eta_c \to \gamma\gamma) = 6.788(45)_{\rm fit}(41)_{\rm syst}$ keV
  - ▶ $F(0,0) = 0.08793(29)_{\rm fit}(26)_{\rm syst}$ GeV^{-1}
- ★ $\Gamma(J/\Psi \to \gamma\eta_c) = 2.219(17)_{\rm fit}(18)_{\rm syst}(24)_{\rm expt}(4)_{\rm QED}$ keV
  - ▶ $\widehat{V}_{\rm latt}(0) = 1.8649(73)_{\rm fit}(75)_{\rm syst}$
- ★ $\Gamma(J/\Psi \to e^+e^-) = 0.01349(15)_{\rm latt}(15)_{\rm expt}(13)_{\rm QED}$ keV
- ★ New/updated information to make picture more clear from experiment side would be welcome!

Summary



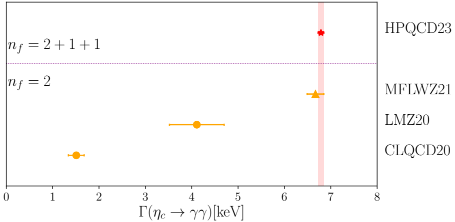


- ★ $\Gamma(\eta_c \to \gamma\gamma) = 6.788(45)_{\rm fit}(41)_{\rm syst}$ keV
  - ▶ $F(0,0) = 0.08793(29)_{\rm fit}(26)_{\rm syst}$ GeV^{-1}
- ★ $\Gamma(J/\Psi \to \gamma\eta_c) = 2.219(17)_{\rm fit}(18)_{\rm syst}(24)_{\rm expt}(4)_{\rm QED}$ keV
  - ▶ $\widehat{V}_{\rm latt}(0) = 1.8649(73)_{\rm fit}(75)_{\rm syst}$
- ★ $\Gamma(J/\Psi \to e^+e^-) = 0.01349(15)_{\rm latt}(15)_{\rm expt}(13)_{\rm QED}$ keV
- ★ New/updated information to make picture more clear from experiment side would be welcome!

Thank you!

Extra stuff

$\Gamma(\eta_c \to \gamma\gamma)$ lattice comparison



Summary plot $J/\psi \to \gamma\eta_c$: lattice results