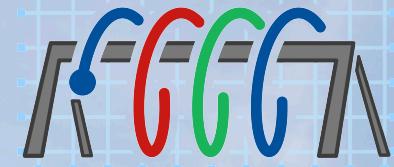


# FOR 5269 Charmonium and glueballs including light hadrons FOR 5269



Roman Höllwieser

Bergische Universität Wuppertal

J. Finkenrath, T. Korzec, F. Knechtli,  
M. Peardon, J. A. Urrea-Niño



# FOR5269: Future methods for studying confined gluons in QCD

<https://confluence.desy.de/display/for5269>

**Spokesperson: Francesco Knechtli**

## Main Goals:

- **Disconnected contributions in charmonium**
- String breaking in hybrid potentials
- **Glueballs in full dynamical QCD**
- Multilevel algorithms for glueballs
- Novel schemes for molecular dynamics
- Connection of distillation and multi-grid
- Multilevel Monte Carlo for trace estimation

## Outline:

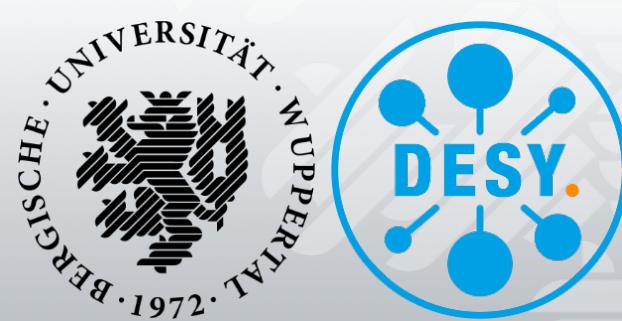
- Hadron spectroscopy and Distillation
- New ensembles for glueball measurements
- Charmonium spectrum and glueballs
- Hyperfine splitting of charmonium



Funded by

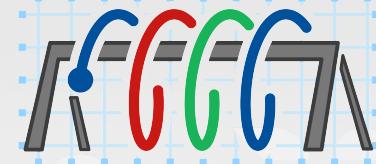


Deutsche  
Forschungsgemeinschaft  
German Research Foundation



Trinity College Dublin  
Coláiste na Tríonóide, Baile Átha Cliath  
The University of Dublin





## Hadron Spectroscopy on the lattice

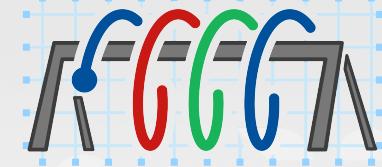
- ensemble generation using openQCD
- average over configurations, error  $\sim 1/\sqrt{N_{config}}$
- **observables:** correlation functions in terms of ‘**quark propagators**’

**Distillation:** quark field smearing  $\psi \rightarrow VV^+\psi$  using  $N_\nu$  eigenmodes of the

- 3D covariant lattice Laplace operator  $\nabla^2 v_i = \lambda_i v_i$ ,  $i = 1 \dots N_\nu = O(10^2)$
- $4 \times N_\nu \times N_T$  inversions => ‘**quark perambulators**’  $\tau_{ij}^{\alpha\beta} = v_i^+(t_0) D_{\alpha\beta}^{-1} v_j(t_1)$
- meson operators via ‘elementals’  $\Phi(t) = \rho V(t) \Gamma V(t)$ ,  $\Gamma = 1, \gamma_5, \gamma_i, \nabla_i, \dots$
- solve generalized eigenvalue problem (GEVP) to access excited states
- optimal profiles from GEVP vectors, Gaussian profiles  $\rho_k(\lambda) = e^{-\lambda^2/2\sigma_k^2}$

**Glueballs:** Laplacian eigenvalue correlator  $C^{kl}(t) = \sum_i \rho_k \lambda_i(t_0) \sum_l \rho_l \lambda_l(t_0 + t)$

- alternatively, 35 Wilson loop shapes and APE smearing



3 degenerate light quarks (up, down, strange)

1 physical charm quark =>  $\eta_c \sim 3$  GeV

light :  $m_\pi \sim 420$  MeV, heavy:  $m_\pi \sim 1$  GeV, glueball  $\sim 2$  GeV!

Coarse lattices:  $a = 0.054$  fm @  $\beta = 3.24$

- A0 - light :  $16^3 \times 72$
- A1 - light :  $32^3 \times 96$
- A2 - light :  $48^3 \times 128$

} light quark mass at physical average  
=> many decay channels for glueballs!

- A0 - heavy :  $24^3 \times 72$
- A1 - heavy :  $32^3 \times 96$
- A2 - heavy :  $48^3 \times 128$

} - 4000 configurations each (16000 MDUs)  
- eigenmodes on every 2<sup>nd</sup>/4<sup>th</sup> configuration  
=> 2000/1000 charm + light perambulators  
=> glueball can only decay into two pions!

Fine lattice:  $a = 0.043$  fm @  $\beta = 3.43$

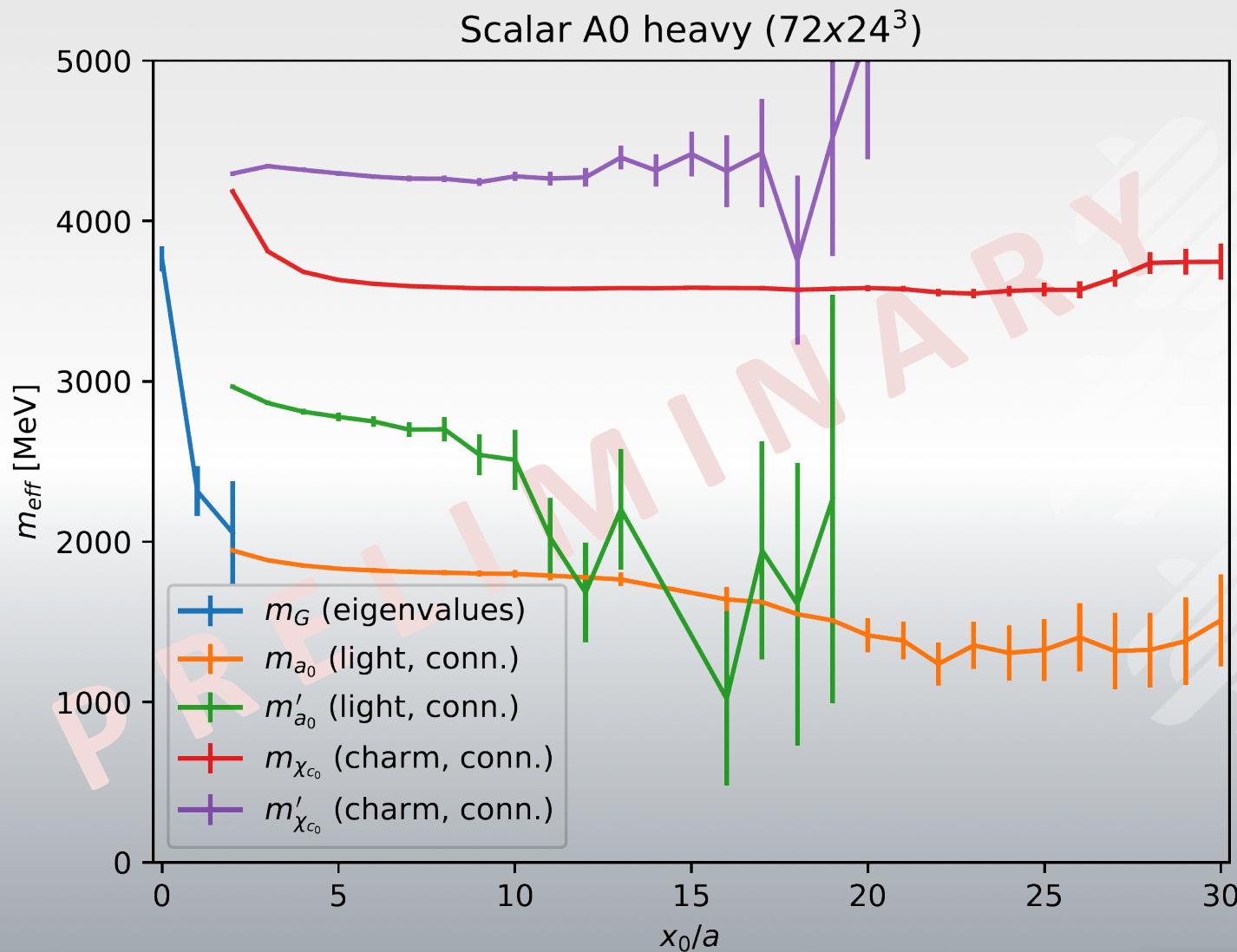
- B - light :  $48^3 \times 144$

R. Höllwieser et al., Eur. Phys. J. C **80** (2020) no.4. 349

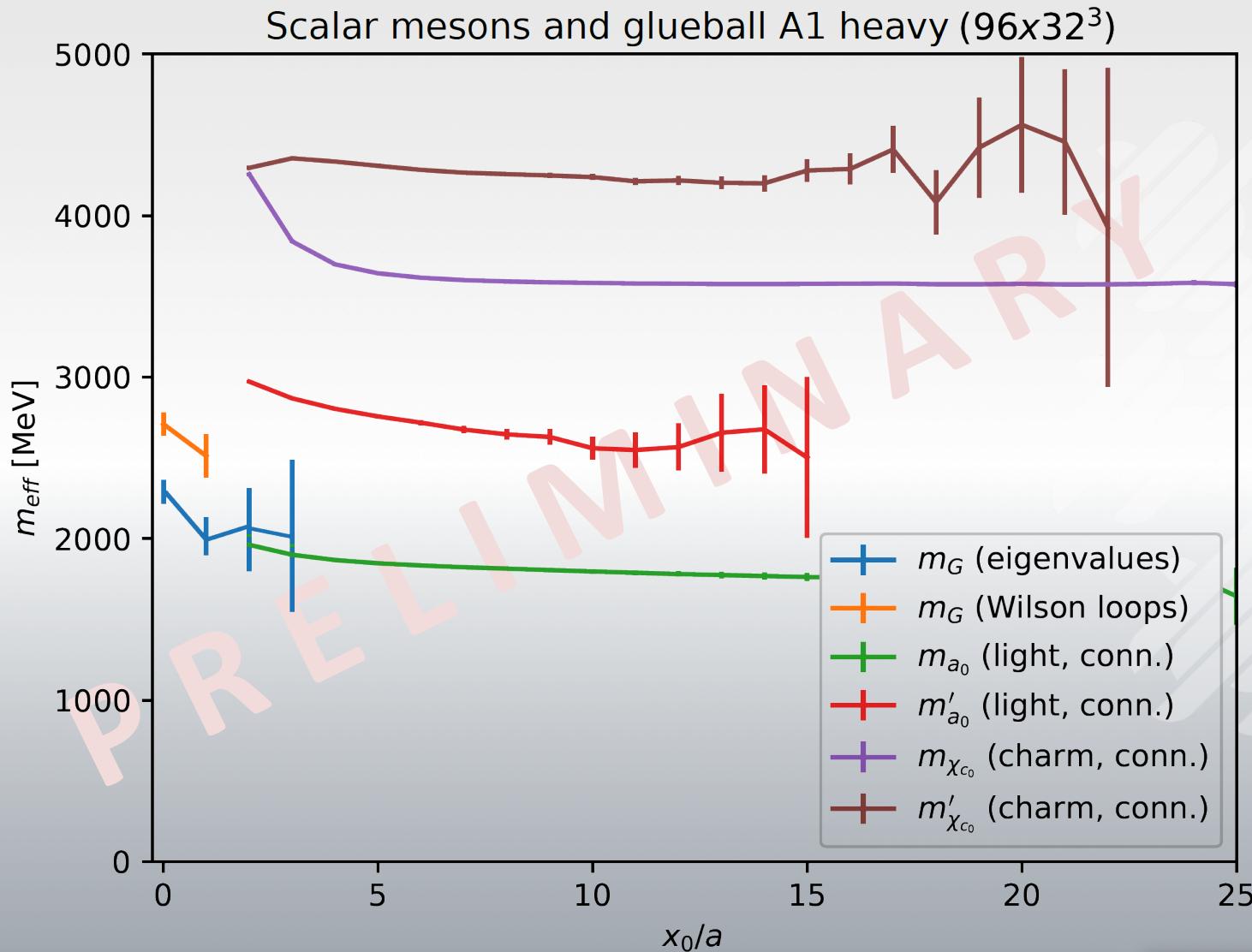
# New „heavy pion“ ensembles, preliminary (connected only) results

name	A0 heavy	A1 heavy	A2 heavy
volume	$24^3 \times 72$	$32^3 \times 96$	$48^3 \times 144$
$P_{acc}$	0.971(1)	0.947(1)	0.924(2)
configs	2000+2000	2000+2000	1000+1000
#eigenvectors	100	200	400
per. T-range	20-52	24-72	30-114
# perc perl	2000/2000	1000/1000	15/5
$t_0/a^2$	5.115(30)	5.074(16)	5.1093(79)
$\chi^{1/4}$ [MeV]	146(1)	148(1)	147(1)
$m_\pi$ [MeV]	<b>1002(4)</b>	<b>1000(2)</b>	<b>998(2)</b>
$m_{\eta_c}$ [MeV]	<b>2979(3)</b>	<b>2980(1)</b>	<b>2982(1)</b>
$m_{J/\Psi}$ [MeV]	<b>3132(2)</b>	<b>3132(1)</b>	<b>3132(1)</b>
$\frac{m_{J/\Psi} - m_{\eta_c}}{m_{\eta_c}}$	<b>0.0512(13)</b>	<b>0.0510(8)</b>	<b>0.0503(4)</b>
$m_{a_0}$ [MeV]	1786(41)	1795(25)	very preliminary
$m_{\chi_{c0}}$ [MeV]	3541.5(6.0)	3542.7(4.6)	glueball mass from
”glueball“ $0^{++}$	2090(310)	2120(140)	Laplacian eigenvalues

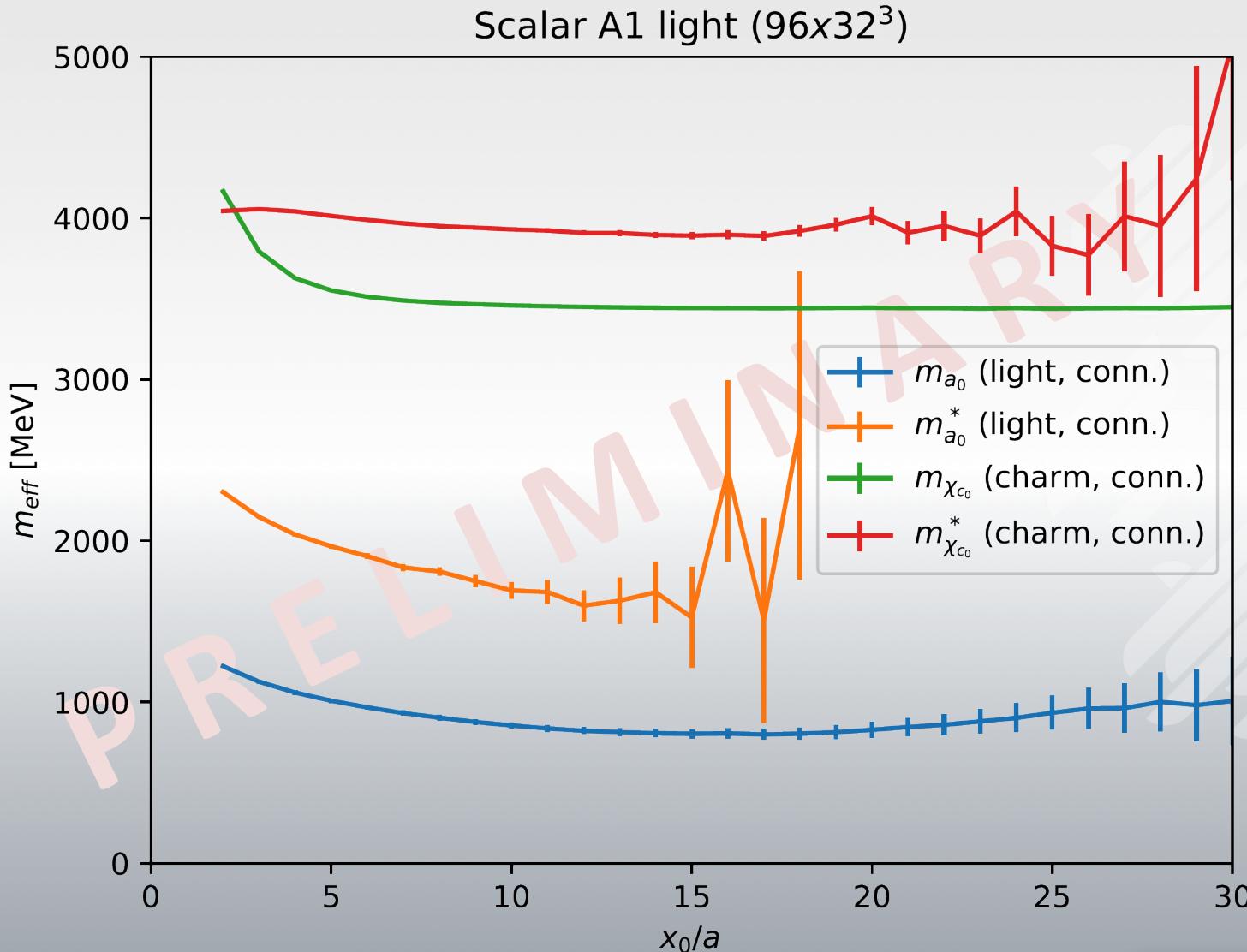
# Light and charm scalar mesons and glueball on A0 heavy ( 72 x 24<sup>3</sup> )



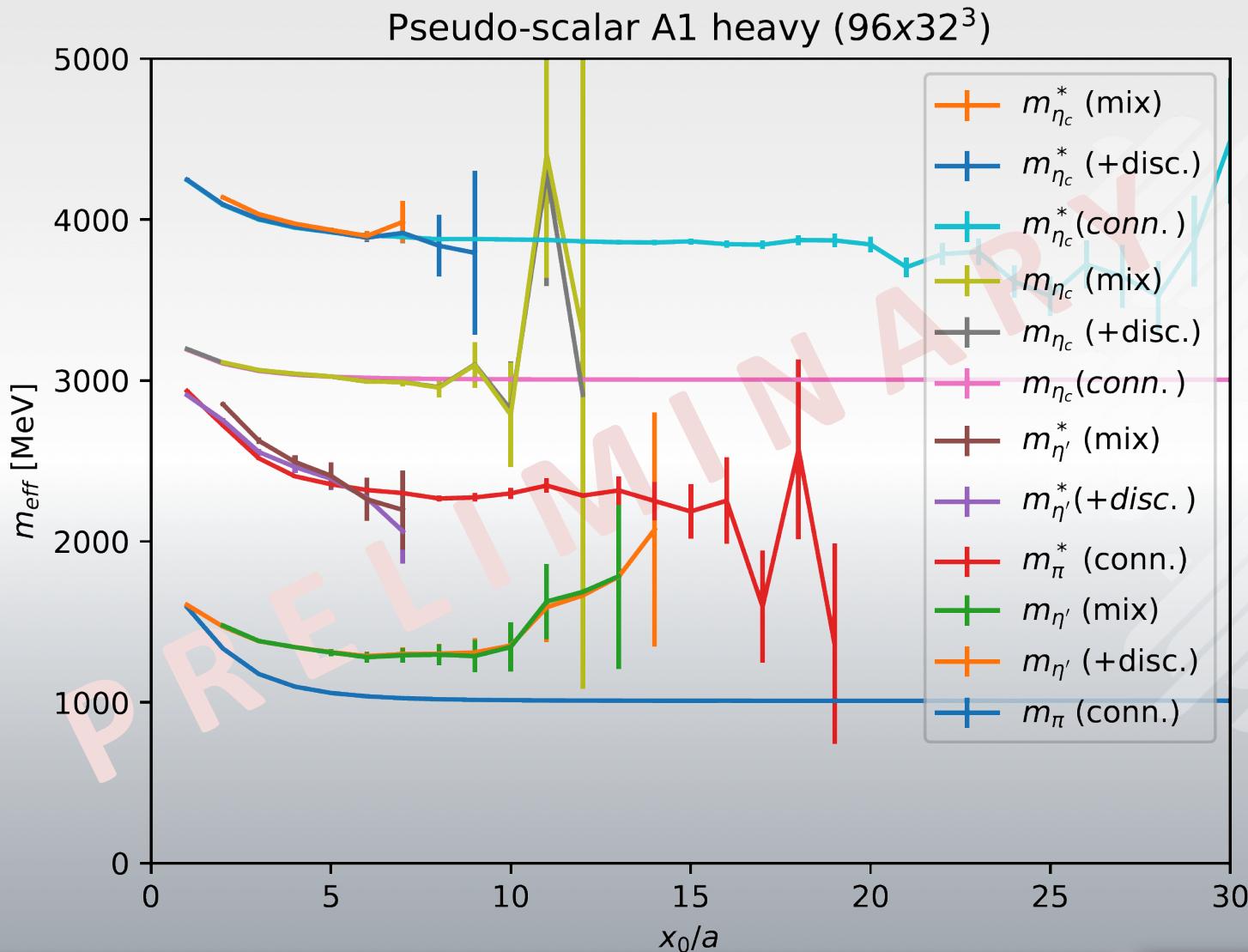
# Light and charm scalar mesons and glueball on A1 heavy ( 96 x 32<sup>3</sup> )



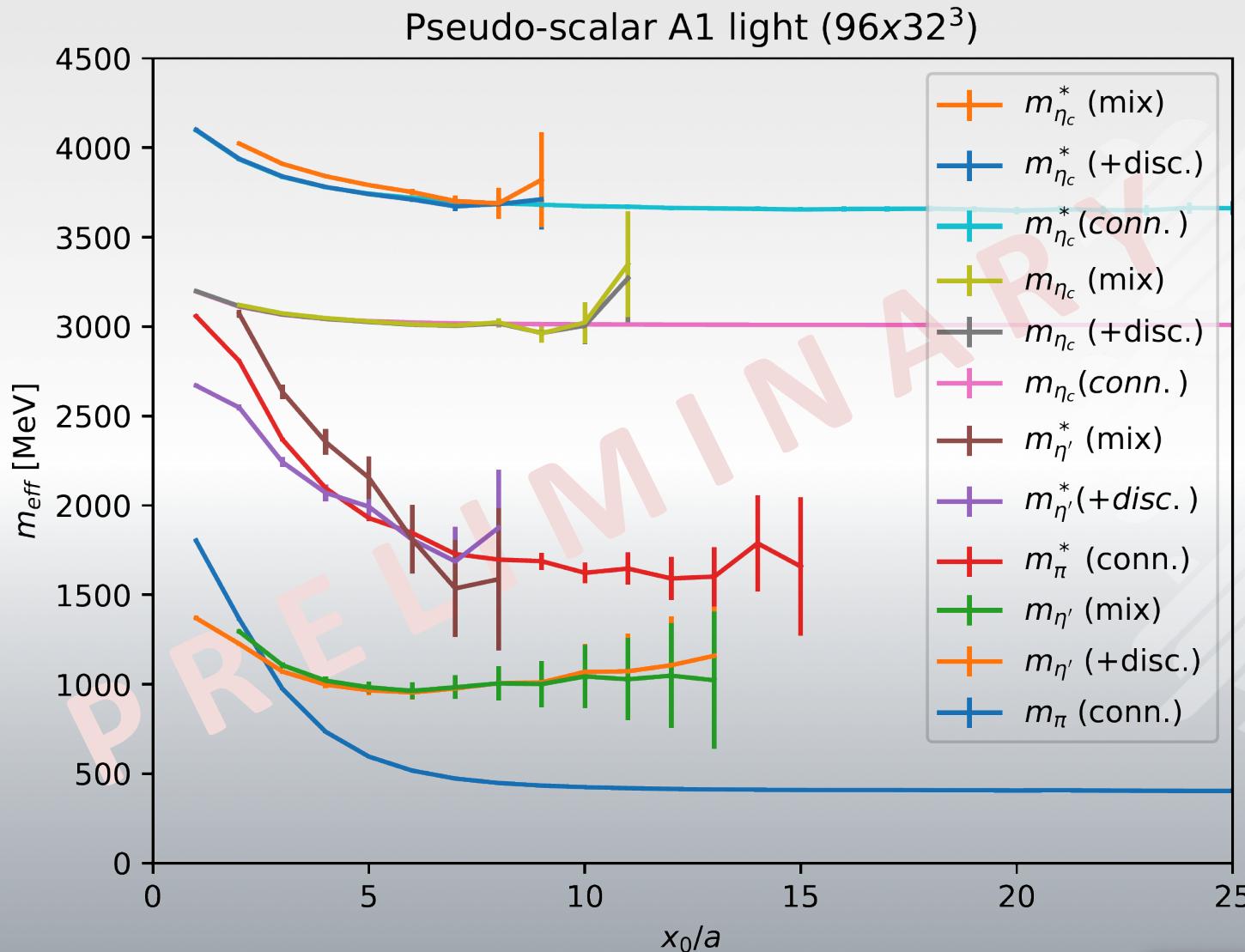
# Light and charm scalar mesons and glueball on A1 light ( 96 x 32<sup>3</sup> )



# Light and charm pseudo-scalar mesons on A1 heavy ( 96 x 32<sup>3</sup> )

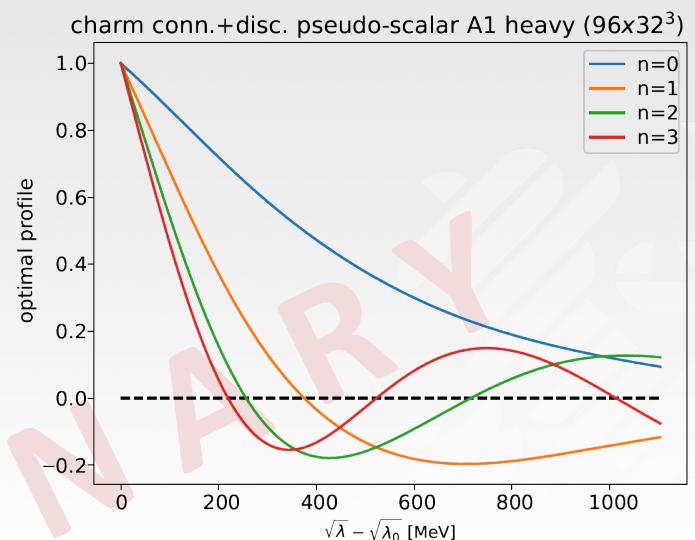
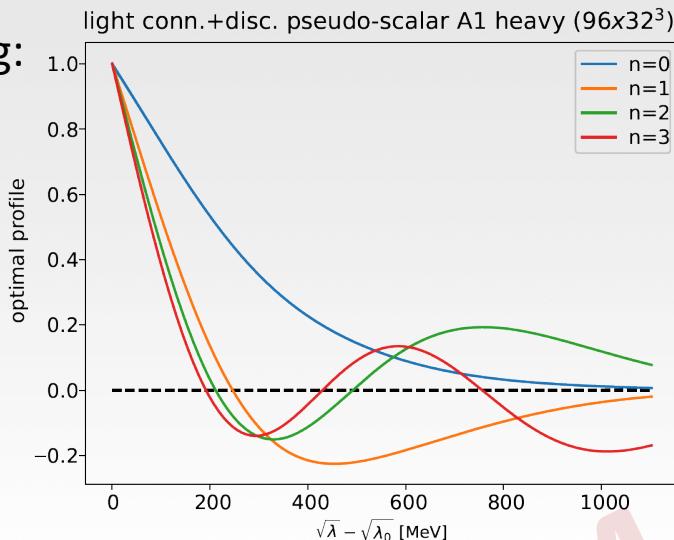


# Light and charm pseudo-scalar mesons on A1 light ( 96 x 32<sup>3</sup> )

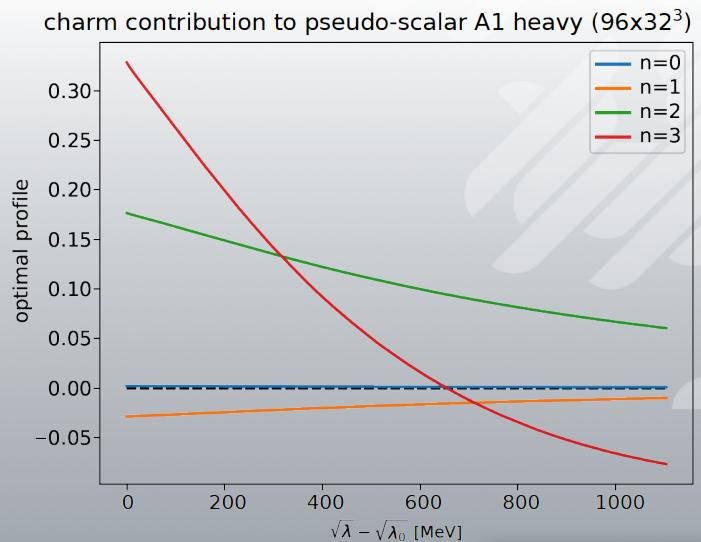
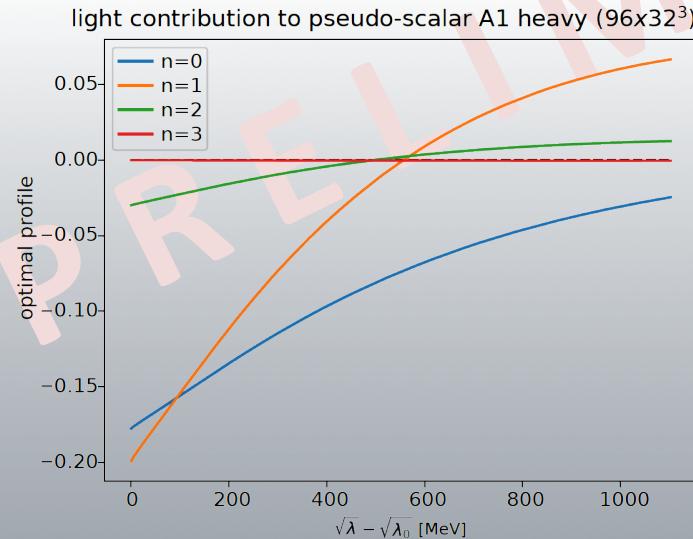


# Optimal distillation profiles for pseudo-scalar mesons on A1 heavy

no mixing:

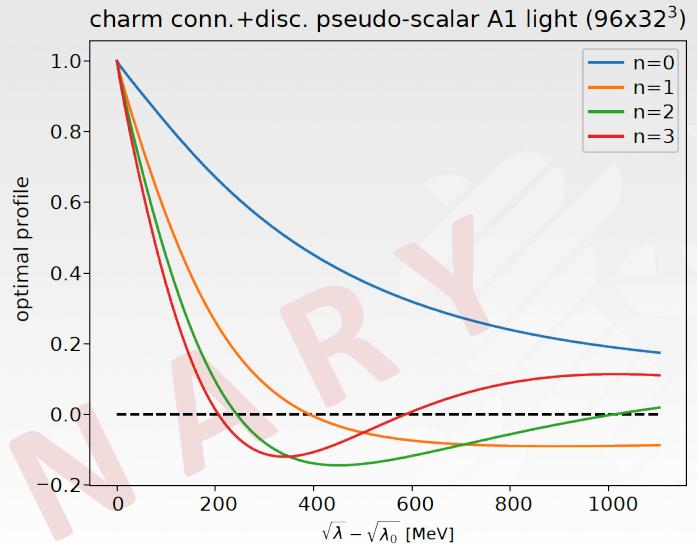
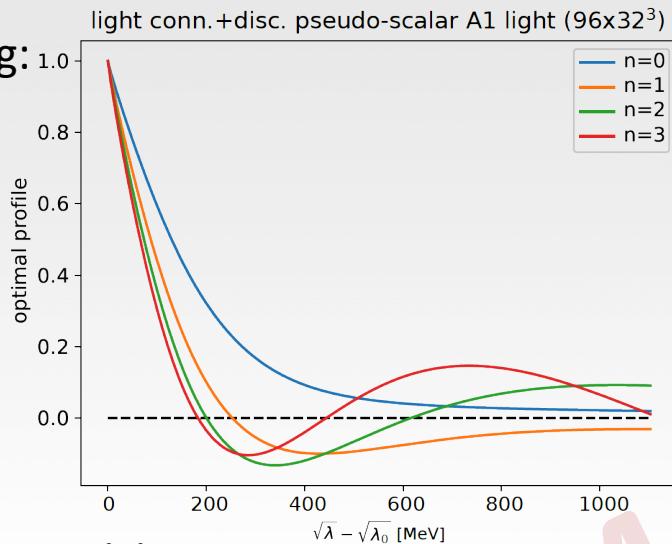


light/charm mixing:

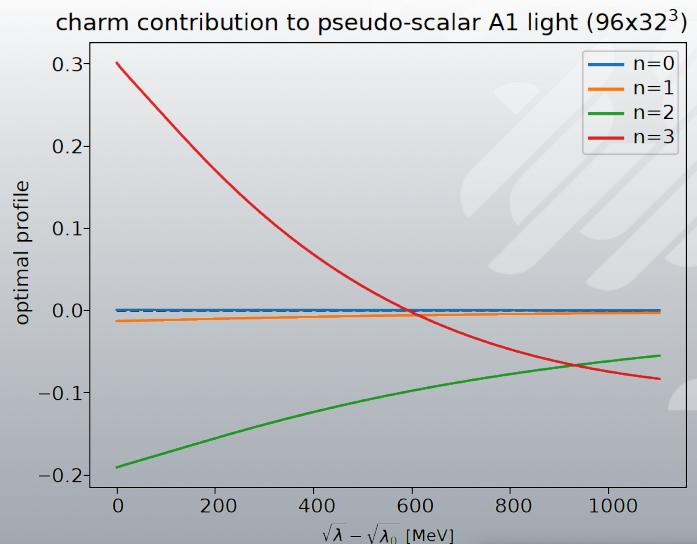
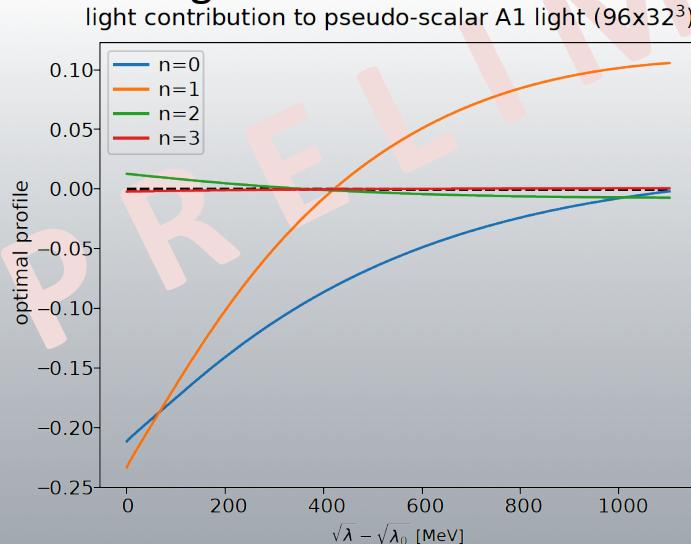


# Optimal distillation profiles for pseudo-scalar mesons on A1 light

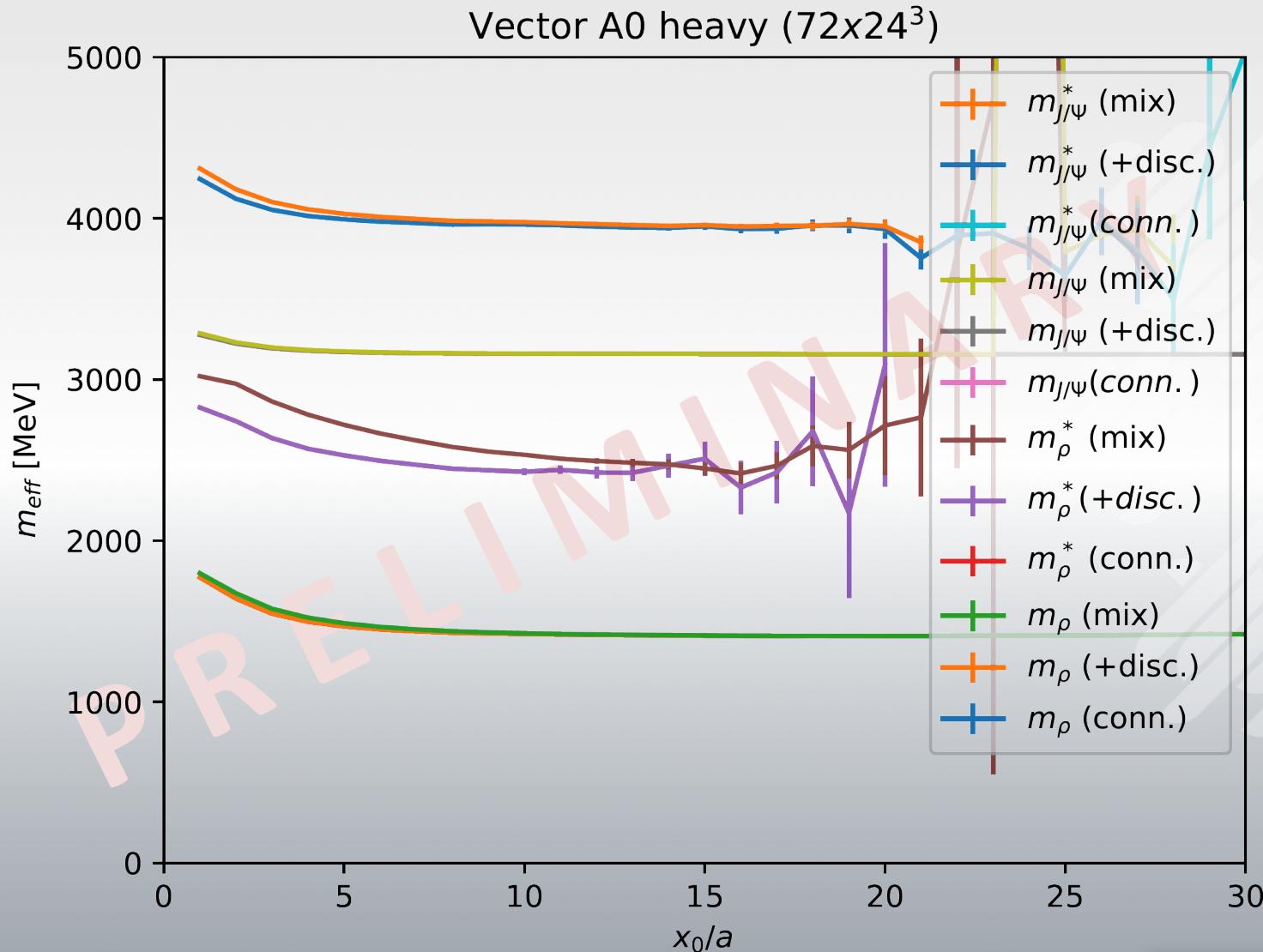
no mixing:



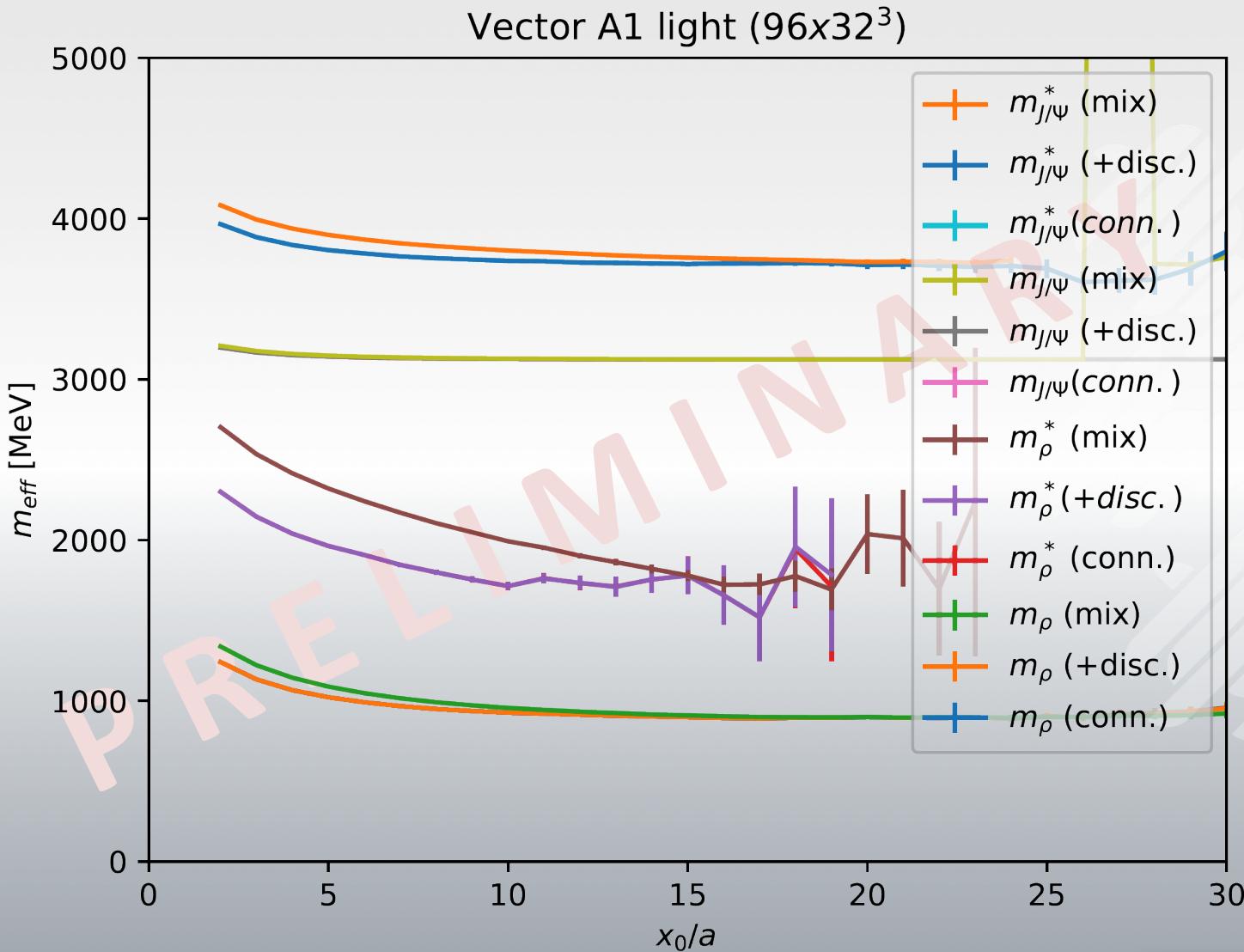
light/charm mixing:



# Light and charm vector mesons on A0 heavy ( 72 x 24<sup>3</sup> )

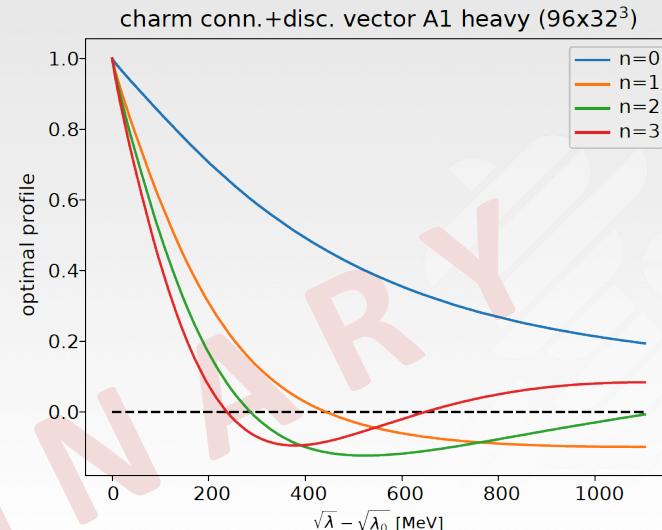
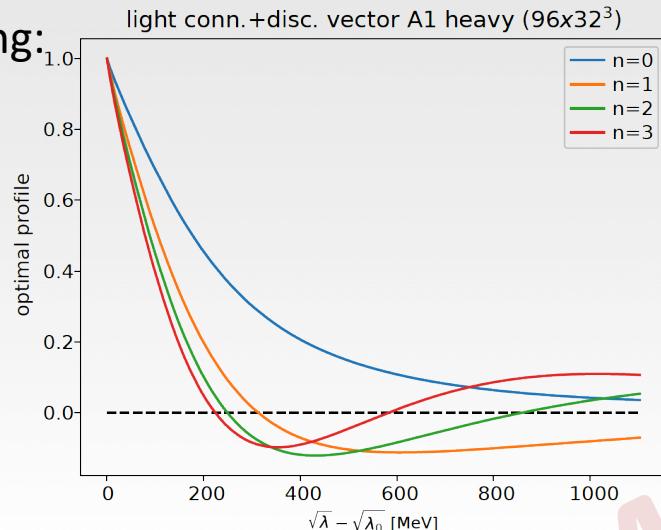


# Light and charm vector mesons on A1 light ( 96 x 32<sup>3</sup> )

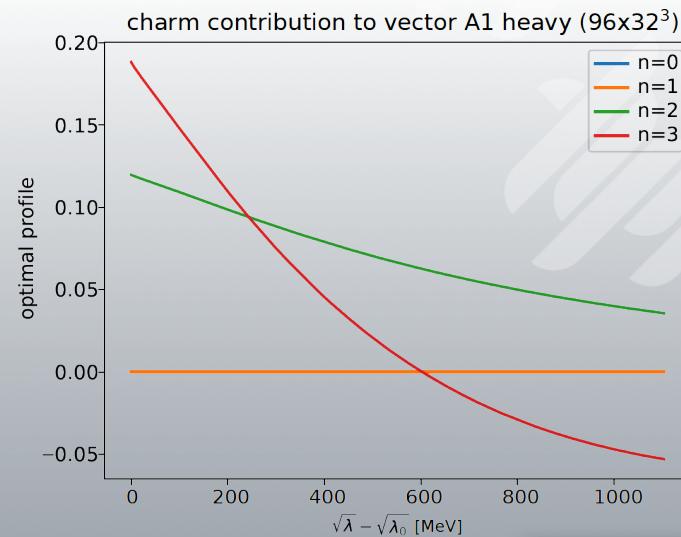
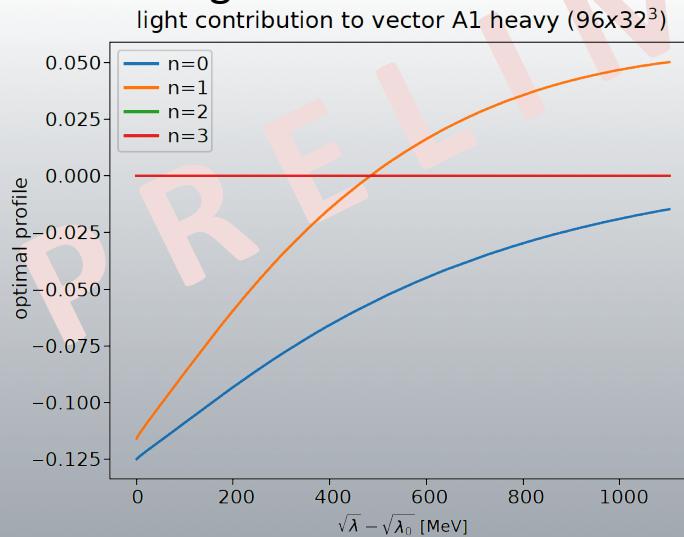


# Optimal distillation profiles for vector mesons on A1 heavy

no mixing:

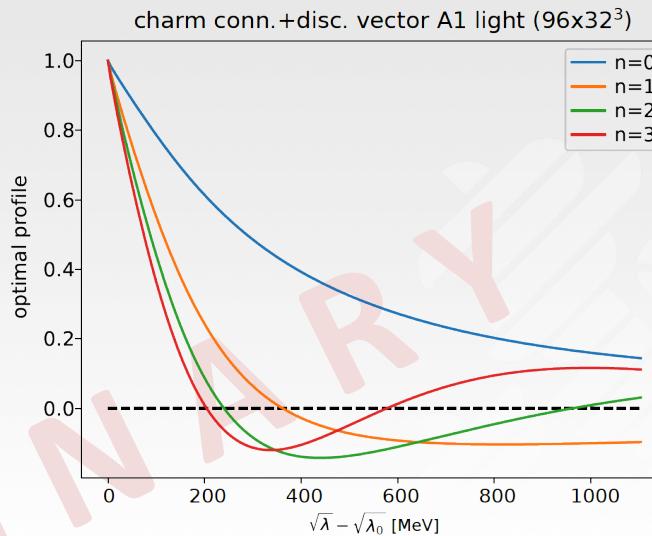
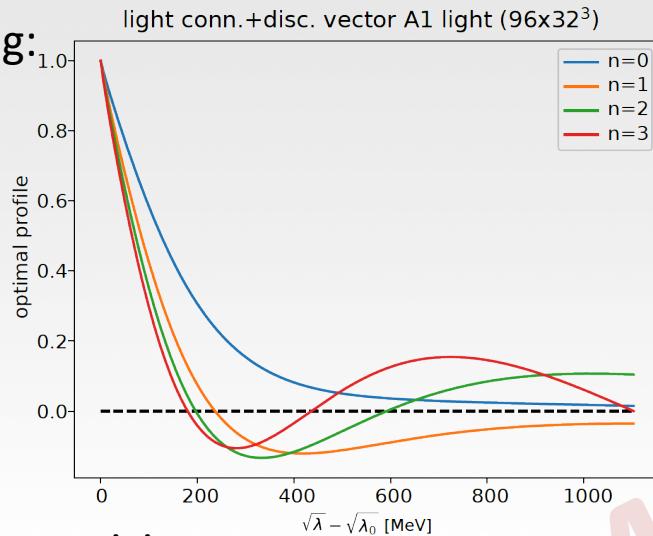


light/charm mixing:

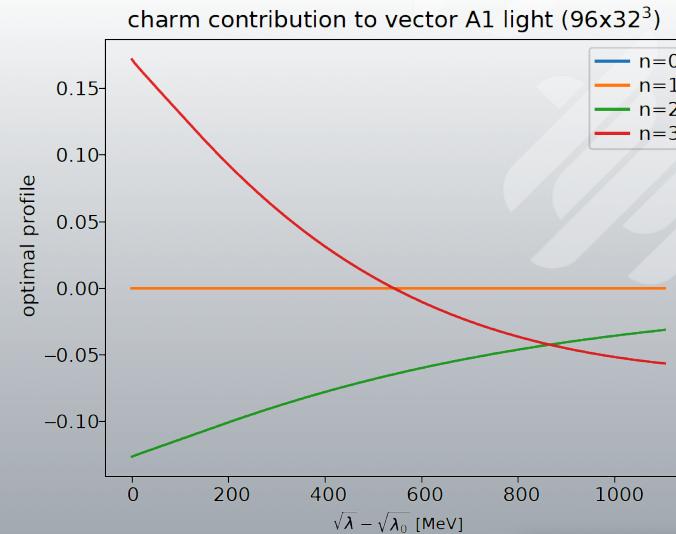
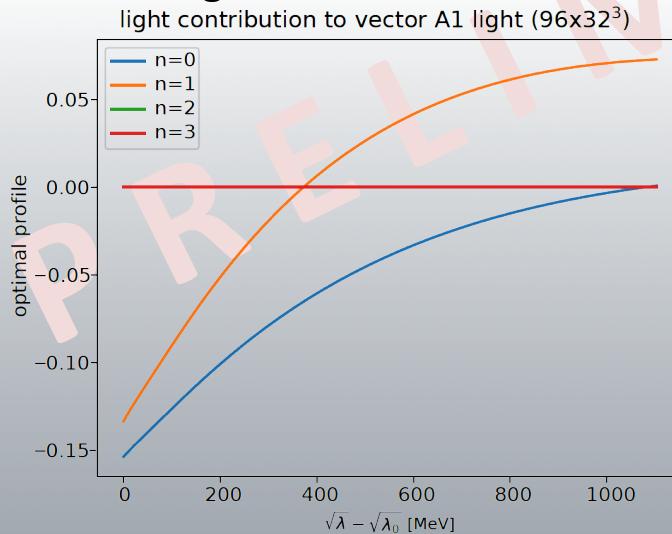


# Optimal distillation profiles for vector mesons on A1 light

no mixing:



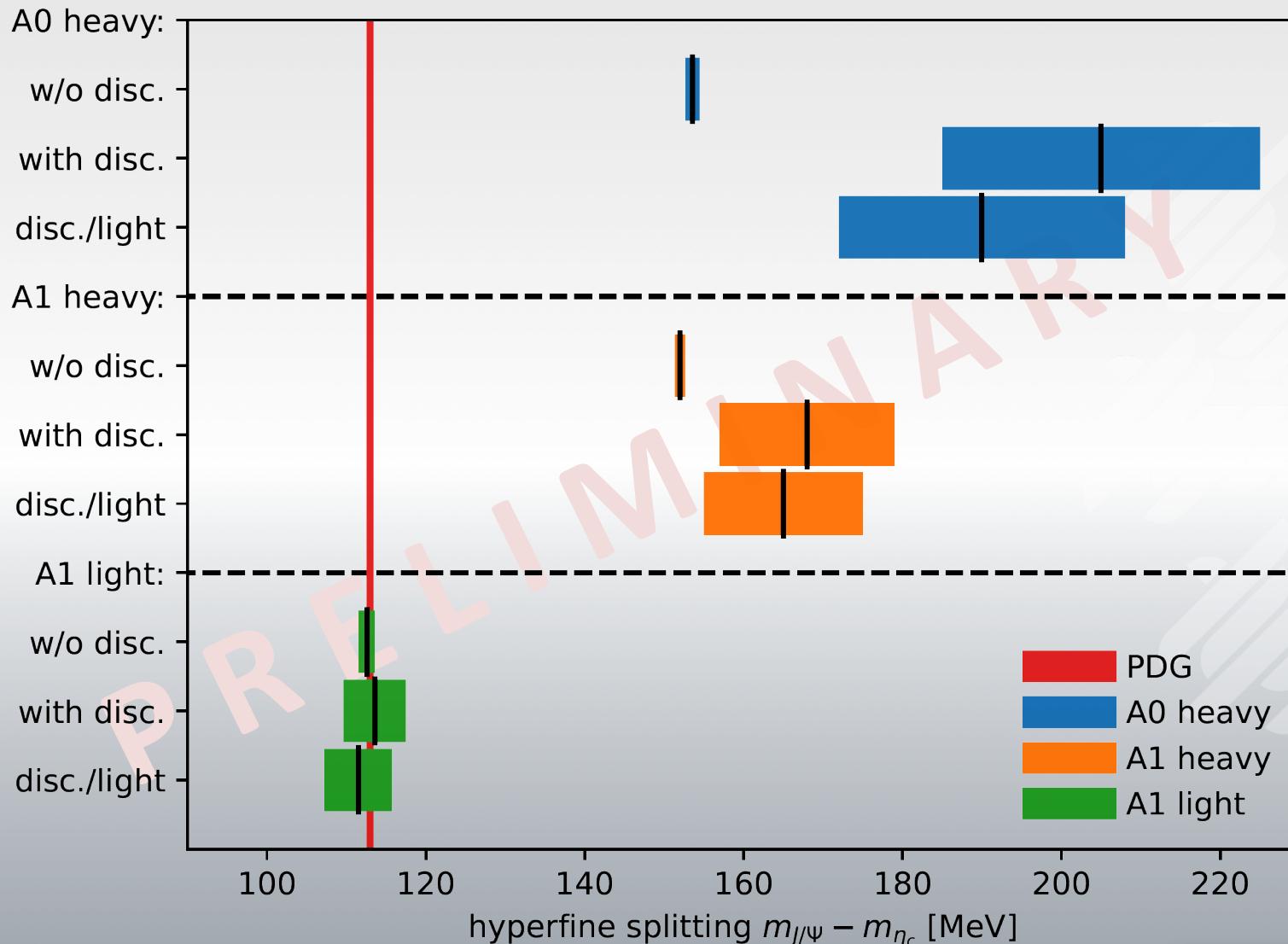
light/charm mixing:



## Overview of meson masses on light and heavy ensembles

ensemble	A0 heavy	A1 heavy	A1 light
volume	$24^3 \times 72$	$32^3 \times 96$	$32^3 \times 96$
measurements	2000	1000	2000
$m_\pi$	1008(4)	1009(3)	418(3)
$m_{\eta'}$	1317(22)	1295(35)	972(40)
	1324(21)	1287(37)	982(48)
$m_{\eta_c}$	3006(7)	3005(7)	3011(7)
	2954(21)	2989(13)	3010(8)
	2970(20)	2992(13)	3012(8)
$m_\rho$	1425(9)	1411(6)	898(11)
$m_{J/\Psi}$	3160(8)	3157(8)	3124(8)
$m_{J/\Psi} - m_{\eta_c}$	154(1)	152(1)	113(1)
	205(20)	168(11)	114(4)
	190(18)	165(10)	112(4)
$m_{a_0}$	1802(18)	1761(34)	894(12)
$m_{\chi_{c0}}$	3579(10)	3578(9)	3442(9)
"glueball" $0^{++}$	2090(310)	2120(140)	(Laplacian eigenvalues!)

# Hyperfine splitting of charmonium on light and heavy ensembles



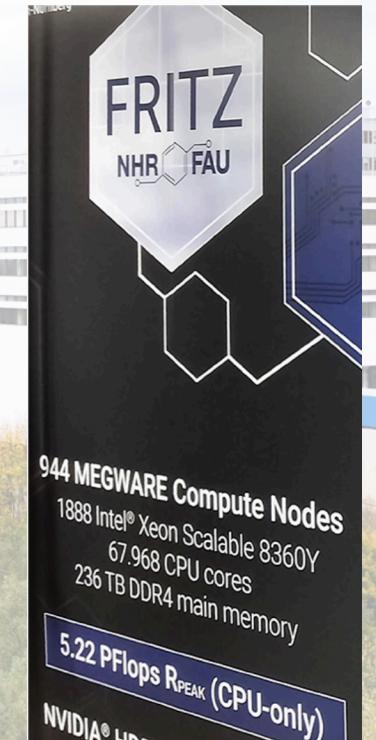
# Conclusions & Outlook

## New $N_f = 3 + 1$ lattice ensembles

- $a=0.054\text{fm}$ ,  $24^3 \times 72$ ,  $32^3 \times 96$  and  $48^3 \times 128$
- $m_\pi \sim 1 \text{ GeV}$ , physical charm  $m_{\eta c} \sim 3 \text{ GeV}$
- 4000 configurations each (16000 MDUs)
- eigenmodes on every 2<sup>nd</sup>/4<sup>th</sup> configuration
- 2000/1000 charm and light perambulators
- glueball can only decay into two pions
- mixing of charmonium with light hadrons and glueballs

## Outlook

- study glueballs / scattering via Lüscher analysis
- including two pion states in the correlation matrix
- working on multilevel glueball operators in full QCD



Thank you for your attention!

Questions?

A wide-angle night photograph of a modern university campus. The buildings are illuminated from within, showing various architectural features like glass facades and illuminated walkways. The sky is dark, suggesting it's nighttime.

**NHR**  **FAU**  
**superMUC**

Leibniz-Rechenzentrum  
der Bayerischen Akademie der Wissenschaften

**lrz**

A photograph showing a long row of server racks in a data center. The racks are black with blue liquid cooling pipes visible on the sides. The room has a high ceiling with blue lighting fixtures. The floor is made of light-colored tiles.

BERGISCHE  
UNIVERSITÄT  
WUPPERTAL