Iso-Scalar States from Lattice QCD

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FOR 5269

Spectroscopy in Lattice QCD

- 2pt-Function in a Euclidean path integral

$$\langle \mathcal{O}_\alpha(x_0)\bar{\mathcal{O}}_\beta(y_0)\rangle = \frac{1}{Z}\int \mathcal{D}[U]\,\mathcal{D}[\bar\psi]\,\mathcal{D}[\psi]e^{-S[U,\bar\psi,\psi]}\mathcal{O}_\alpha(x_0)\bar{\mathcal{O}}_\beta(y_0)$$

  - U: lattice gauge field $U_\mu(x) = e^{iaA_\mu(x)}$
  - $\bar\psi, \psi$ fermion fields (Grassmann valued)
  - S: discretized Euclidean QCD action
  - $\mathcal{O}_\alpha, \bar{\mathcal{O}}_\beta$ annihilation and creation operators. $\alpha, \beta = 1, \ldots N_{\mathrm{op}}$ distinguish different operators for some symmetry channel.

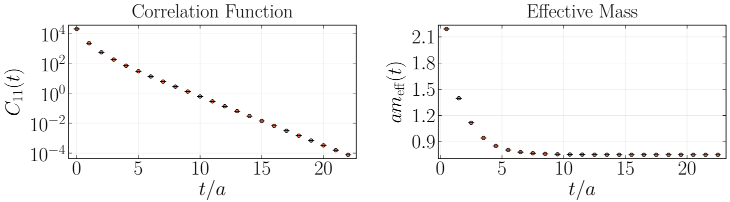
$$\text{e.g. } \mathcal{O} = \sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{p}}\bar\psi(x)\gamma_5\psi(x)$$

Ground States

Spectral decomposition

$$\langle \mathcal{O}_\alpha(t)\bar{\mathcal{O}}_\beta(0)\rangle = \sum_n c_n^{\alpha\beta}\, e^{-E_n t} \quad \propto e^{-E_0 t} + O(e^{-(E_1-E_0)t})$$

Ground states are easy to extract



- Determine correlation function $C_{11}(t) = \langle \mathcal{O}_1(t)\bar{\mathcal{O}}_1(0)\rangle$
- Fit to exponential, or compute effective mass $am_{\text{eff}}(t + a/2) = \ln\left[\frac{C_{11}(t)}{C_{11}(t+a)}\right]$
- Set the scale, here $a \approx 0.0658$ fm

  [P. Fritzsch et al, Nucl.Phys.B 865 (2012)]
- Plateau fit

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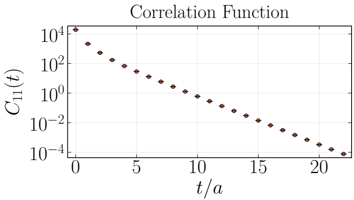

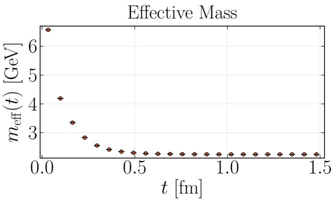

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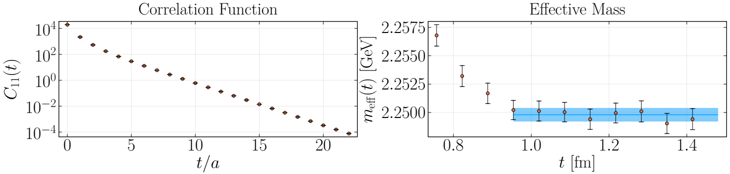

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Excited States

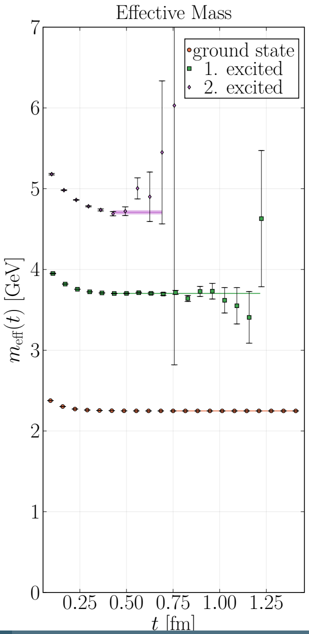
The GEVP method

[C. Michael, I. Teasdale, Nucl.Phys.B 215 (1983)]

[M. Lüscher, U. Wolff, Nucl.Phys.B 339 (1990)]

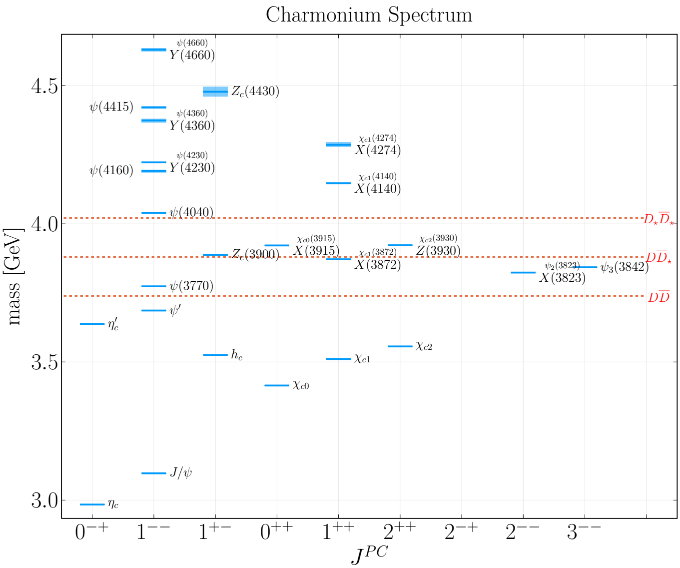
[B. Blossier et al, JHEP 04 (2009)]

- Determine correlation matrix
  $C_{\alpha\beta}(t) = \langle \mathcal{O}_\alpha(t)\bar{\mathcal{O}}_\beta(0)\rangle$
- Solve GEVP $C(t)\vec{v}_n = \lambda_n C(t_0)\vec{v}_n$
- n'th excited state:
  $\lambda_n(t,t_0) \propto e^{-E_n(t-t_0)} + O(e^{-(E_{n+1}-E_n)(t-t_0)})$
- $\vec{v}_n$: can be used to construct optimal operators for every state
- Excited states can be very noisy



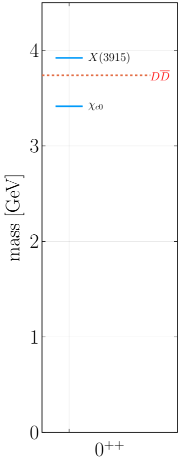
Charmonium Spectrum

[Particle Data Group, Prog. Theor. Exp. Phys. 2022]



Challenge with Iso-Scalars

- E.g. the 0^{++} channel
- There are states below χ_{c0}
  - Light iso-scalar scalars
    [Particle Data Group, Prog. Theor. Exp. Phys. 2022]
  - Glueball states?
    [Y. Chen et al, Phys.Rev.D 73 (2006)]
  - 2-meson states $\pi\pi$, KK, ...
  - 2-mesons with momentum $\pi(\vec{p})\pi(-\vec{p})$
  - ...
- Finite volume helps!
  Momenta are quantized $\vec{p} = \vec{n}\, 2\pi/L$
- Heavier pions help, e.g. SU(3) flavor symmetric point



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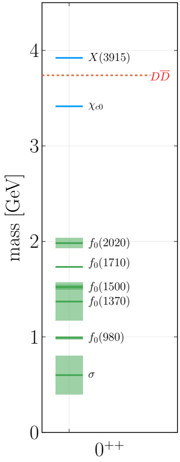

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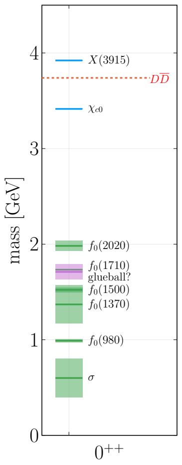

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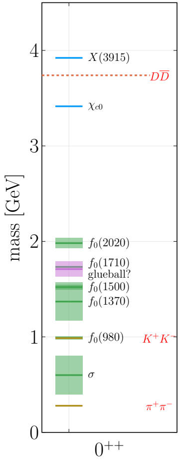

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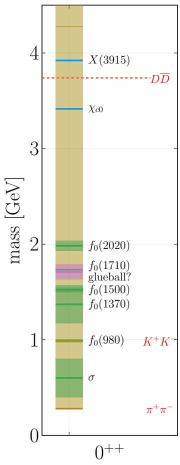

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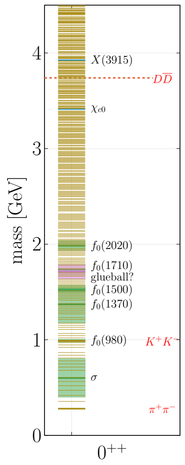

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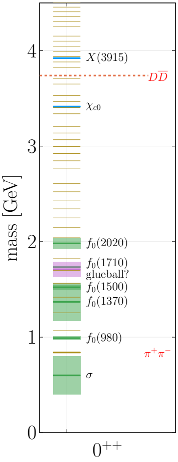


Toy Models

Requirements for realistic simulations of charmonia

- Dynamical u, d, s, c quarks with their physical masses
- Lattice size $T \times L^3$, spacing a, grid $T/a \times (L/a)^3$
  - Small finite size effects $Lm_\pi > 4$
  - Small lattice artifacts $a\overline{m}_c \ll 1$
  - For continuum extrapolation: several a

$\Rightarrow$ enormously large $L/a \Rightarrow$ compromises necessary

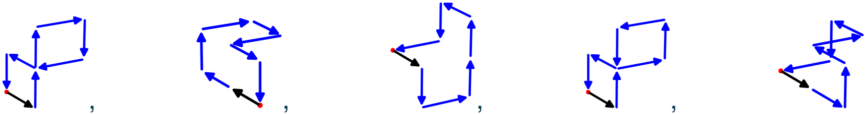
Simplified QCD for Method Development

On 48×24^3 lattices we simulate $N_f = 0$ QCD and $N_f = 2$ QCD with degenerate quarks of mass $\approx \overline{m}_c/2 \to SU(2)$ "isospin" symmetry

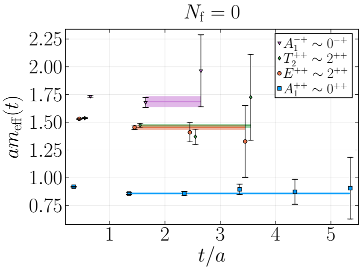
- $N_f = 2 \to$ straightforward simulation, not much tuning
- No light quarks "m_π" large $\to$ small volumes suffice, simplified spectrum
- No very heavy quarks $\to$ relatively coarse lattices suffice
- No continuum extrapolation

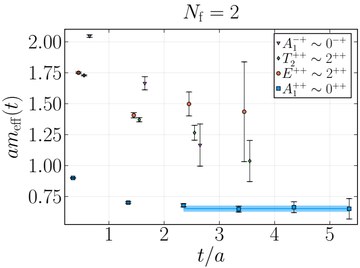
Gluonic Correlators

- Iso-scalar operators can be: gluonic, mesonic, multi-mesonic, . . .
- In $N_f = 0$ QCD only gluonic operators
  $\rightarrow$suitably chosen linear combinations of closed loops for chosen symmetry channel, e.g.



- States are glueballs
- In $N_f = 2$ QCD the same operators can be used
- But what are the states in this case?





Mesonic Correlators

The lattice QCD action is bi-linear in the quark fields

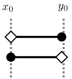
$$S = S_g[U] + \sum_{x,y} \bar{\psi}(x) D_{xy}[U] \psi(y)$$

$\rightarrow$ Perform Grassmann integration ($\rightarrow$ Wick contractions), e.g.
$\mathcal{O}^\tau(x_0) = \sum_{\vec{x}} \bar{\psi}(x)\gamma_5 \tau \psi(x)$, flavor matrix $\tau \in \{\mathbb{1}, \tau^3, \tau^\pm\}$

$$\langle \mathcal{O}^{\mathbb{1}}(x_0)\mathcal{O}^{\mathbb{1}}(y_0)\rangle = \frac{1}{Z}\int \mathcal{D}[U] e^{-S_g[U]} \det(D[U])^2 \, (2\mathfrak{c} - 4\mathfrak{d})$$

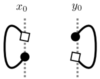
$$\langle \mathcal{O}^{\tau^{3,\pm}}(x_0)\mathcal{O}^{\tau^{3,\pm}}(y_0)\rangle = \frac{1}{Z}\int \mathcal{D}[U] e^{-S_g[U]} \det(D[U])^2 2\mathfrak{c}$$

- Connected diagram

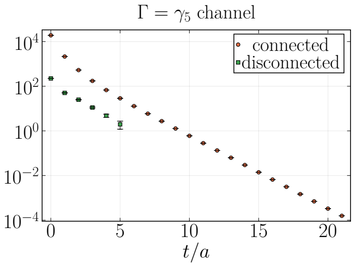


$$\mathfrak{c} = \sum_{\vec{x},\vec{y}} \mathrm{tr}[\gamma_5 D^{-1}(x,y)\gamma_5 D^{-1}(y,x)]$$

- Disconnected diagram



$$\mathfrak{d} = (\sum_{\vec{x}} \mathrm{tr}[\gamma_5 D^{-1}(x,x)])(\sum_{\vec{y}} \mathrm{tr}[\gamma_5 D^{-1}(y,y)])$$



Quark Smearing

There is a lot of freedom for choosing an operator $\mathcal{O}$ in a channel.
Smearing often increases the overlap with the ground state

$$\psi(x_0, \vec{x}) \to \sum_{\vec{y}} F(\vec{x}, \vec{y})\, \psi(x_0, \vec{y})$$

F is some covariant smearing function (diagonal in spin)

"Distillation"

Particularly convenient choice of F

[M. Peardon et al, Phys.Rev.D 80 (2009)], [F. Knechtli et al, Phys.Rev.D 106 (2022)]

$$F(\vec{x}, \vec{y}) = \sum_{i=1}^{N_v} v_i(\vec{x}) g(\lambda_i) v_i^*(\vec{y}) \,,$$

- v_i and λ_i: i'th eigenvector and eigenvalue of the 3D covariant Laplace operator on time-slice x_0
- $g(\lambda)$ profile function $\to$ tunable for best overlap with desired state
- F not only smears, but also maps to a small subspace

Distillation

Traces are now in a smaller space and be computed exactly

$$\mathrm{tr}[\gamma_5 D^{-1}(x,y)\gamma_5 D^{-1}(y,x)] \quad \rightarrow \quad \mathrm{tr}\left[\Phi[x_0]\tau[x_0,y_0]\bar{\Phi}[y_0]\tau[y_0,x_0]\right]$$

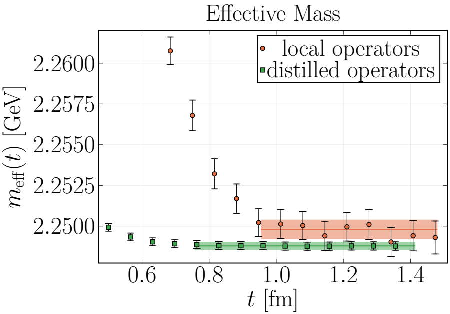
with "small" matrices ($4N_v \times 4N_v$)

- "Perambulator" $\tau[x_0,y_0]_{ij} = v_i[x_0]^\dagger D^{-1} v_j[y_0]$
  Expensive! $4\,\frac{T}{a}\,N_v$ solves of type $Dx = v$
- "Elemental" $\Phi[x_0]_{ij} = g^*(\lambda_i[x_0])g(\lambda_j[x_0])v_i[x_0]^\dagger\gamma_5 v_j[y_0]$
  Cheap, but depends on smearing profile and channel Γ

Similarly disconnected contributions

$$\mathrm{tr}[\gamma_5 D^{-1}(x,x)]\mathrm{tr}[\gamma_5 D^{-1}(y,y)] \quad \rightarrow \quad \mathrm{tr}\left[\Phi[x_0]\tau[x_0][x_0]\right]\mathrm{tr}\left[\bar{\Phi}[y_0]\tau[y_0][y_0]\right]$$

Effect of Smearing

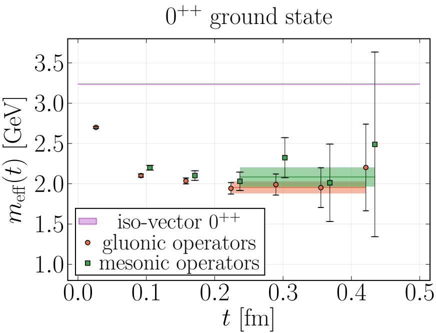


- Much earlier plateaus (crucial for disconnected signal)
- More precise disconnected signal
  Traces evaluated exactly

Example: Scalar Iso-Scalar Ground State

The best signal is in the 0^{++} channel.

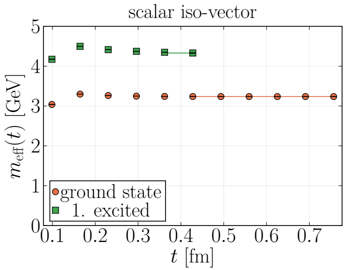
- Both gluonic and mesonic operators see the same ground state
  What is it?

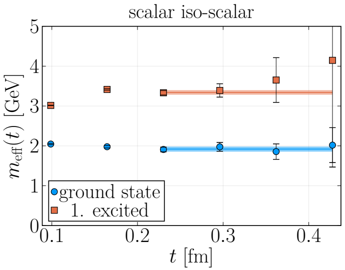


- GEVP with a large basis of gluonic and mesonic iso-scalar scalar operators (15×15)
  - 8 gluonic (5+3 with different smearing levels)
  - 7 mesonic (different smearing profiles)
- Determine ground state and 1. excited state
- Compare with iso-vector channel
  (= spectrum with neglected charm annihilation effects)

Example: Scalar Iso-Scalar States

- The "χ_{c0}" is the first excited state in the iso-scalar channel
- It agrees well with the corresponding state, when charm annihilation effects are neglected (i.e. the iso-vector ground state)
- There is one state below: glueball?





Conclusions and Outlook

Conclusions

- "Disconnected diagrams" cannot be neglected, but they are very noisy
- Lattice QCD helps clarifying the nature of a state, e.g. glueball vs meson, at least to some extent

Outlook

- These methods can be applied to a more physical setup
  light u, d, s quarks, physical c quark $\rightarrow$ talk R. Höllwieser
- Inclusion of multi-meson operators
- Larger volumes $\rightarrow$ stochastic distillation
- Development of multi-level simulation techniques
  $\rightarrow$ solution of signal/noise problem $\rightarrow$ FOR-5269