D-meson mixing on the lattice

Felix Erben

in collaboration with
Peter Boyle, Luigi Del Debbio, Jonathan Flynn, Andreas Jüttner,
Takashi Kaneko, Michael Marshall, Rajnandini Mukherjee,
Antonin Portelli, J Tobias Tsang, Oliver Witzel
RBC/UKQCD and JLQCD

CHARM23 Siegen

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D-MESON MIXING

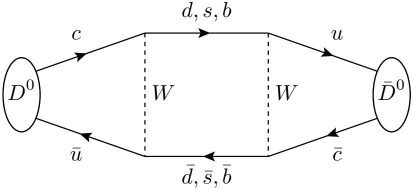
D-mesons have mass eigenstates

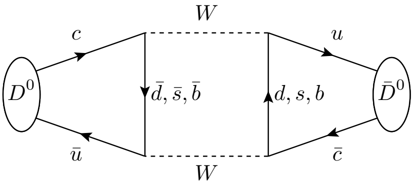
$$|D_1^0\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$
$$|D_2^0\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$

With the phase convention

$$CP|D^0\rangle = -|\bar{D}^0\rangle$$
$$CP|\bar{D}^0\rangle = -|D^0\rangle$$

and in the absence of CP violation ($p = q$), $|D_1^0\rangle$ is CP-even and $|D_2^0\rangle$ is CP-odd. Masses m_1, m_2 and decay widths Γ_1, Γ_2 of these eigenstates define mixing parameters

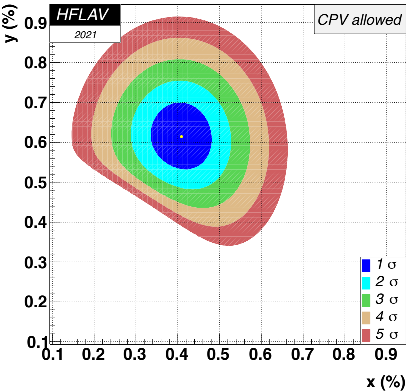


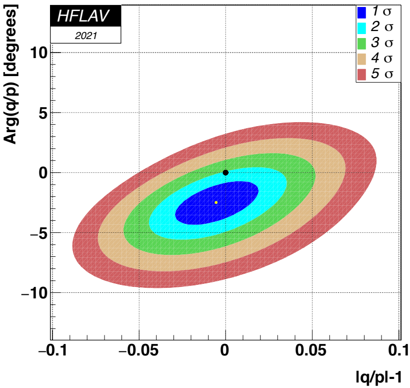


$$x = \frac{2(m_1 - m_2)}{\Gamma_1 + \Gamma_2}, \qquad y = \frac{(\Gamma_1 - \Gamma_2)}{\Gamma_1 + \Gamma_2}$$

D-MESON MIXING

Experimental results, HFLAV 2021 [Phys.Rev.D 107 (2023) 5]





- no-mixing point $x = y = 0$ excluded at $> 11.5\sigma$
- no evidence for indirect *CP* violation $|q/p| \neq 1$ or $\phi = arg(q/p) \neq 0$

D MESON MIXING

$$\begin{aligned}\langle D^0|\mathcal{H}_W^{eff}|\bar{D}^0\rangle &= \langle D^0|\mathcal{H}_W^{eff}|\bar{D}^0\rangle_{SD} + \langle D^0|\mathcal{H}_W^{eff}|\bar{D}^0\rangle_{LD} \\ &= \langle D^0|\mathcal{H}_W^{\Delta C=2}|\bar{D}^0\rangle + \sum_n \frac{\langle D^0|\mathcal{H}_W^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n}\end{aligned}$$

short-distance contribution:

- sub-leading contribution
- can validate / exclude New-Physics models modifying the $\Delta C = 2$ sector only

long-distance contribution:

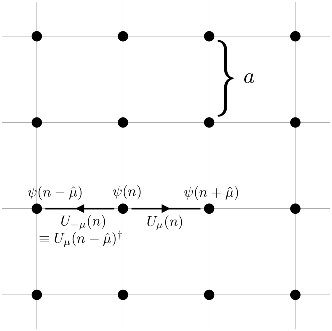
- dominant contribution
- very hard to evaluate

main subject of this talk is the short-distance contribution to *D* mixing

LATTICE QCD

Lattice QCD: method to compute correlation functions non-perturbatively and from first principles

- Discrete, finite Euclidean space-time grid
  - quark fields ψ on sites n
  - gluons U_μ as gauge links
  - finite lattice spacing a (UV regulator)
  - finite volume L, T (IR regulator)
- Path integral



$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dU d\psi d\bar{\psi}\, \mathcal{O}[U,\psi,\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$$

- even relatively small grids have size $\Lambda = (L/a) \times (T/a) = 24^3 \times 48$
  - exact evaluation prohibitively expensive

  $\Rightarrow$ stochastic sampling of ensembles

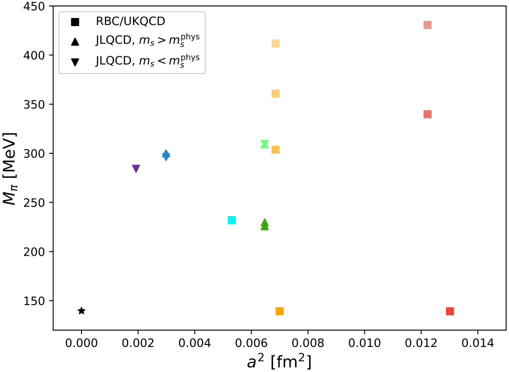
JOINT PROJECT: RBC/UKQCD AND JLQCD

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings
  $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

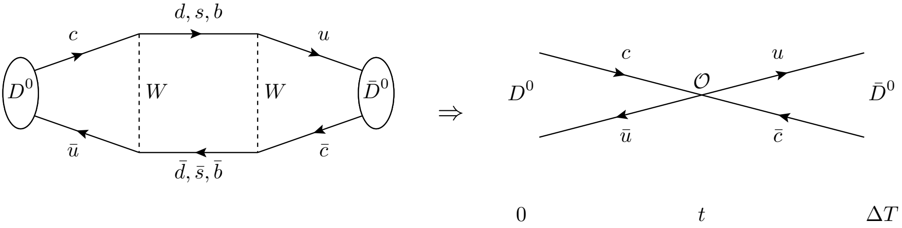
JLQCD:

- 7 ensembles
- 3 lattice spacings
  $a = 0.044 - 0.081\text{fm}$
- one pair of ensembles with $M_\pi L \sim 3$ and $M_\pi L \sim 4$



[PoS LATTICE2021 (2022) 224]

THEORY



- $\Delta C = 2$ process
- OPE shrinks box diagram to local four-quark operator

$$\langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=2} | D^0 \rangle \sim \langle \bar{D}^0(\Delta T) | \mathcal{O}_i(t) | D^0(0) \rangle$$

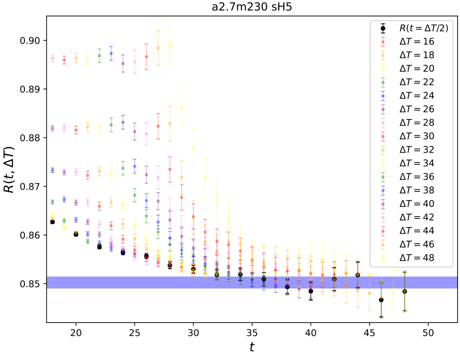
- 5 parity-even, dimension 6, $\Delta C = 2$ operators $\mathcal{O}_i$

Fits to lattice correlation functions

- this projects includes $B_{(s)}$ mixing, leading to:
- 15 ensembles
- 5 operators
- 4-6 heavy-quark masses per ensemble
- heavy-light and heavy-strange sector

$\Rightarrow$ over 700 combined fits

- multiple values for ΔT to control fits better
- two independent analyses by FE and J.T. Tsang



- Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble

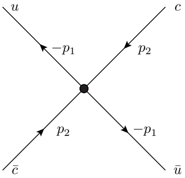
NON-PERTURBATIVE RENORMALISATION

$$\langle \mathcal{O} \rangle_i^{\mathrm{S}}(\mu) = \lim_{a^2 \to 0} \sum_{j=1}^{5} [Z_{\mathcal{O}}^{\mathrm{S}}(a, \mu)]_{ij} \langle \mathcal{O} \rangle_j^{\mathrm{bare}}(a)$$

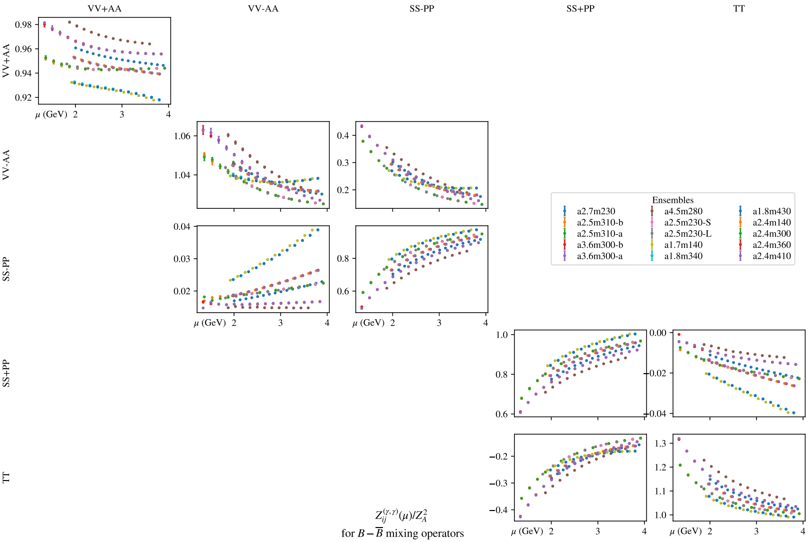
for some regularisation independent scheme S at mass scale μ.
Continuum perturbation theory can then match

$$\langle \mathcal{O} \rangle_i^{\overline{\mathrm{MS}}}(\mu) = R^{\overline{\mathrm{MS}} \leftarrow \mathrm{S}} \langle \mathcal{O} \rangle_i^{\mathrm{S}}(\mu)$$

We use the "RI-SMOM" scheme. Requires computation of four-quark vertices for $(\bar{c}u) \to (\bar{u}c)$. [Boyle et al., JHEP 10 (2017) 054]

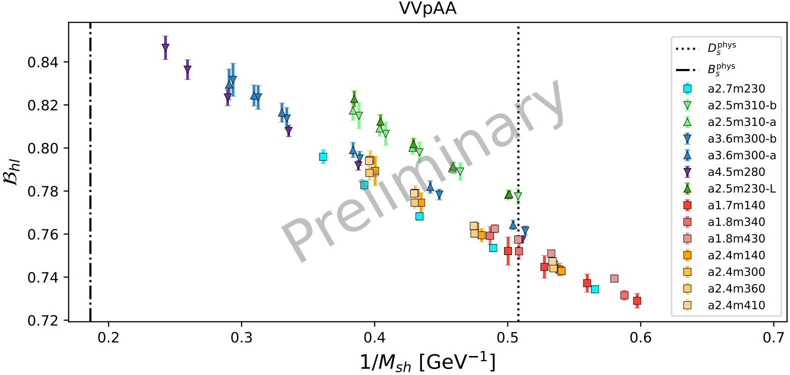


Non-perturbative renormalisation - full matrix



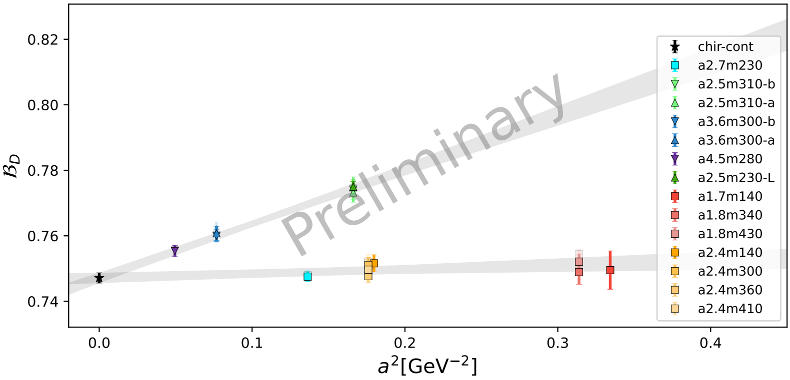
plot and work by Rajnandini Mukherjee (University of Southampton)

BAG PARAMETER $\mathcal{B}_{hl}$ - $VV + AA$



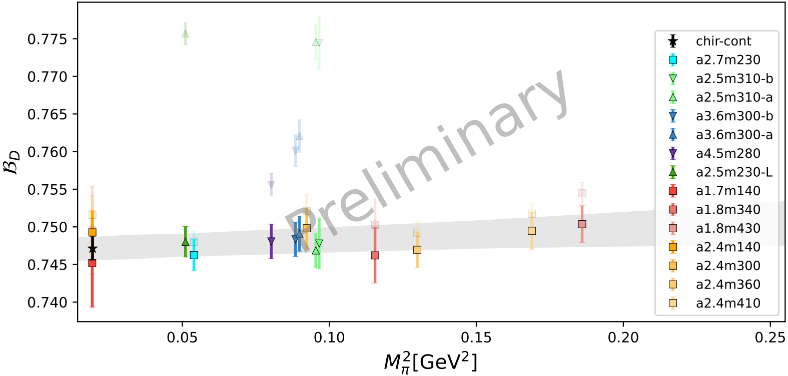
- very fine JLQCD ensembles reach far towards am_b^{phys} (a4.5m280)
- two ensembles at M_π^{phys} control chiral limit (a1.7m140, a2.4m140)
- 3 lattice spacings each for RBC/UKQCD and JLQCD sets
- for D-mixing: interpolate data to be at physical $m_c \lesssim m_h < m_b$ (JLQCD set has data points tuned to be at m_c^{phys})

Bag parameter $\mathcal{B}_D$ - $VV + AA$



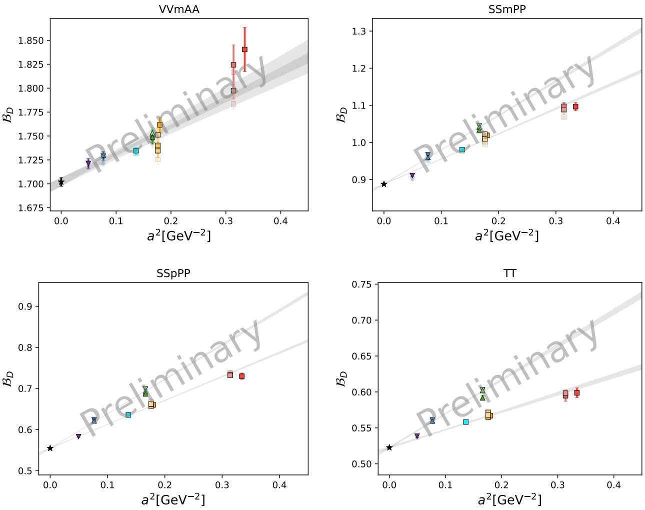
- data points at m_c^{phys}
- transparent: original data
- opaque: data corrected to physical M_π^{phys}
- two trajectories due to different actions / discretisation effects in JLQCD and RBC/UKQCD data sets

Bag parameter $\mathcal{B}_D$ - $VV + AA$



- data points at m_c^{phys}
- fit to a^2, M_π and additionally m_s dependence (JLQCD ensembles)
- transparent: original data
- opaque: data corrected to physical $a^2 = 0$

Bag parameter $\mathcal{B}_D$ - $\mathcal{O}_{2345}$



Different discretisation effects in operators O_2, O_3, O_4, O_5 - note the different y-axis scales!

LONG-DISTANCE CONTRIBUTION

$$\langle D^0|\mathcal{H}_W^{eff}|\bar{D}^0\rangle_{LD} = \sum_n \frac{\langle D^0|\mathcal{H}_W^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n}$$

- for $K - \bar{K}$ mixing, concepts have been derived in [Christ et al., Phys.Rev.D 91 (2015)]
  - the Euclidean lattice finite-volume estimator has growing exponentials for $E_n(L) < M_D$
  - careful treatment requires knowledge of the FV spectrum $E_n(L)$
- formalism can be extended to more general cases, including $D - \bar{D}$, e.g. [Briceño et al., Phys. Rev. D 101 (2020)], [FE et al., JHEP 04 (2023) 108)], [Jackura et al., PoS LATTICE2022 (2023) 062]
- problem for $D - \bar{D}$ mixing: **finite-volume effects** are not yet controllable above 4-particle thresholds like $D^0 \to K\pi\pi\pi$
- first steps towards accounting for these effects in a programme towards hadronic $D \to K\pi$ decays: [Joswig, Hansen, FE et al., PoS LATTICE2022 (2023) 063]
- see talk by Max Hansen, ”Future Theory”, Fri 14:00

CONCLUSIONS

- *D*-mixing $\Delta C = 2$ bag parameters with fully relativistic *c*-quark action
- data for full 5-operator basis available
- 15 ensembles, 6 lattice spacings from 2 collaborations, including two ensembles at M_π^{phys}
- programme extends to $B_{(s)}$-mixing and K-mixing:
  - simple renormalisation for chiral Domain-Wall Fermions
  - fully relativistic treatment of heavy-quark
  - very fine lattice spacings
  - large variety of ensembles to control relevant limits
- Long-distance contribution $D - \bar{D}$ mixing very relevant
  - formalism to compute them conceptually clear but very challenging
  - *Max Hansen: Fri 21/7 14:00 Future Theory*

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BACKUP

LATTICE SETUP

- RBC-UKQCD's 2+1 flavour domain wall fermions [Blum et al. Phys.Rev.D 93 (2016) 7]
  - pion masses from $M_\pi = 139$ MeV to $M_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5m_b$, using a stout-smeared action ($\rho = 0.1$, $N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ [Boyle et al. arxiv:1812.08791]
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at M_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions [Kaneko et al. EPJ Web Conf. 175 (2018) 13007]
  - pion masses from $M_\pi = 226$ MeV to $M_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

LATTICE SETUP

	L/a	T/a	a^{-1} [GeV]	M_π [MeV]	$M_\pi L$	hits $\times N_{\rm conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48×90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32×100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32×101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64×82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32×83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32×76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32×81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24×100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16×100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16×100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16×100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48×72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24×50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24×50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32×50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last column shows names used by RBC/UKQCD ("R/U") and JLQCD ("J").

CONTINUUM LIMIT (B-MIXING)

We need to control on each ensemble

- light-quark discretisation effects $\Rightarrow M_\pi L \gtrapprox 4$
- heavy-quark discretisation effects am_h

Two approaches for heavy quark:

effective theories

- allow expansion in $1/am_b$
- truncation at some order
- "cheap"
- not easily improvable beyond certain precision

method:

- Relativistic action (HQET, RHQ, Fermilab method)
- Nonrelativistic QCD (NRQCD)

fully relativistic

- $am_h \ll 1$ needed

$\Rightarrow$ fine lattice spacing for am_b^{phys}

- "expensive"
- improvable with finer, larger boxes

method:

- extrapolation $am_h \to am_b$ for multiple $am_h < am_b$
- today impossible to reach am_l^{phys}, am_b^{phys} simultaneously

Domain-Wall Fermions

- we use "Domain-Wall Fermions"
  - automatic $O(a)$ improvement in absence of odd powers in a

  $\Rightarrow$ reduced discretisation effects
  - chirally symmetric formulation

  $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}$$

$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{2/2} & \mathcal{O}_{2/3} \\ \mathcal{O}_{3/2} & \mathcal{O}_{3/3} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{4/4} & \mathcal{O}_{4/5} \\ \mathcal{O}_{5/4} & \mathcal{O}_{5/5} \end{pmatrix} \end{pmatrix}$$

Block-structure:

- $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix
- linearly independent from each other and from $\mathcal{O}_1$
- more complicated mixing pattern for other lattice fermions

[Boyle et al., JHEP 10 (2017) 054]

Other neutral meson mixings

For other neutral mesons $M^0 \in \{K, D, B_q\}$

$$\begin{aligned}\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle &= \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} + \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD} \\ &= \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle + \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}\end{aligned}$$

short-distance contribution:

- t enhancement for K, $B_{(s)}$
- additional CKM hierachy enhancement for $B_{(s)}$
- sub-dominant for D, but ok to describe CP-violating contributions

long-distance contribution:

- relevant but smaller than short-distance for K
- dominant for D
- CKM-suppressed for $B_{(s)}$