

Search for CP violation at Belle by measuring T -odd triple-product asymmetries in charm multi-body decays

Longke LI (李龙科)

On behalf of the Belle Collaboration

University of Cincinnati (UC)



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Outline

- ① Charm at Belle experiment
- ② $a_{CP}^{T\text{-odd}}(D_{(s)}^+ \rightarrow K_S^0 K^+ h^+ h^-)$ and $\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+ K^- \pi^+)$
- ③ $a_{CP}^{T\text{-odd}}$ and \mathcal{B} of $D_{(s)}^+ \rightarrow K^\pm h^\pm \pi^+ \pi^0$
- ④ Summary





Outline

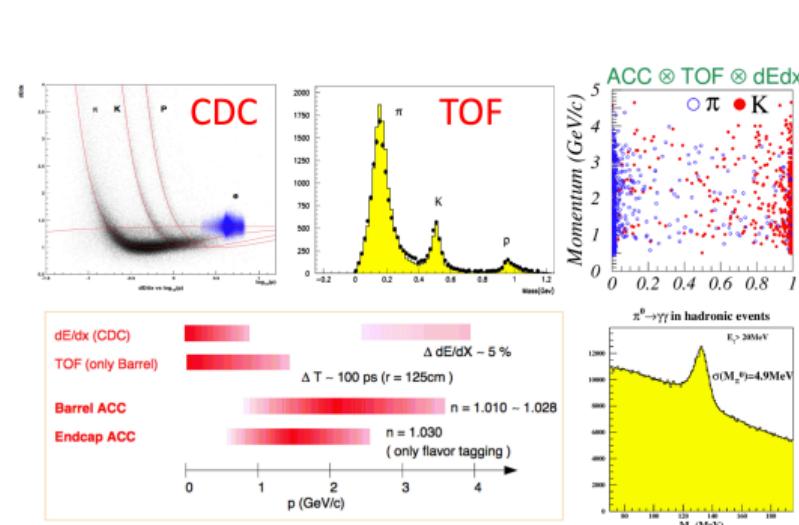
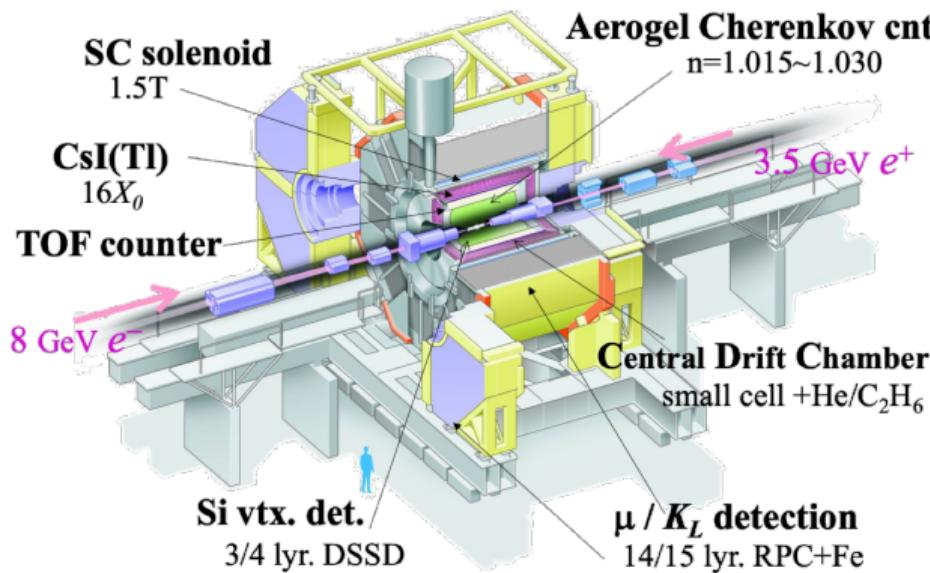
1 Charm at Belle experiment

- 2 $a_{CP}^{T\text{-odd}}(D_{(s)}^+ \rightarrow K_S^0 K^+ h^+ h^-)$ and $\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+ K^- \pi^+)$
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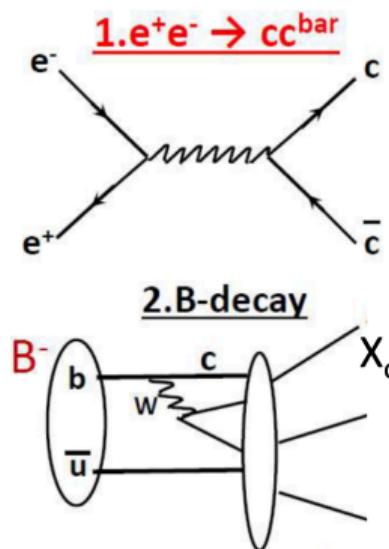
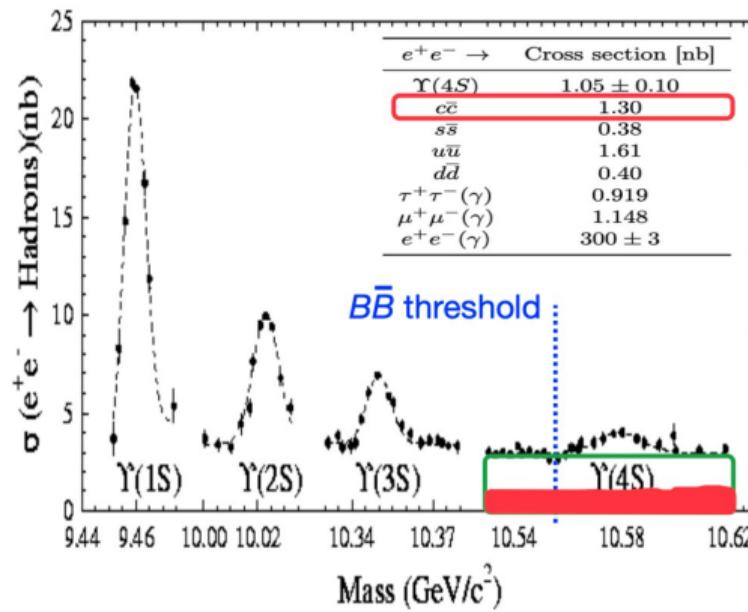
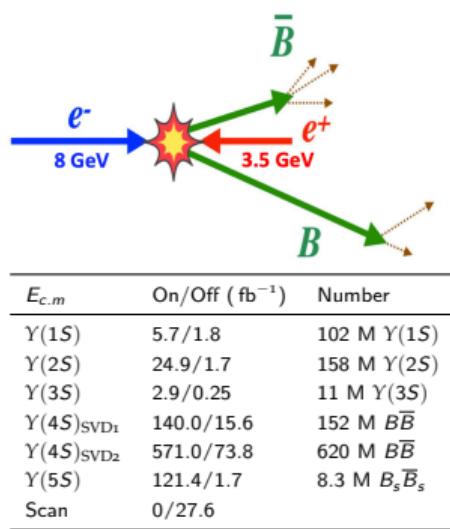
Belle experiment at KEKB

- KEKB (1999-2010), located in Tsukuba, is an asymmetric-energy e^+e^- collider operating at or near $\sqrt{s}=10.58$ GeV.
- It reached highest peak luminosity $2.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ (former world record, already excessed by SuperKEKB).
- Belle detector has good performances on momentum/vertex resolution; particle identification (up to 3.5 GeV/c), etc.



Charm production at Belle and Belle II

- Belle (II) has two ways to produce the charm sample: $e^+ e^- \rightarrow c\bar{c}$ ($\sigma = 1.3 \text{ nb}$) and $b \rightarrow c$ transition.
- Belle accumulated a dataset of $\sim 1 \text{ ab}^{-1}$, which provides a large $B\bar{B}$ sample (772 millions), and also a large charm sample to study charm physics, e.g. $N_{\text{prod}}^{D^+} \sim \mathcal{O}(10^9)$, $N_{\text{prod}}^{\Lambda_c^+} \sim \mathcal{O}(10^8)$, etc.



Charm publications in 2022(3) at Belle and Belle II

- Belle finished its final data accumulation > 13 years ago, but is lasting to produce fruitful physical results. The recent charm results (not covering charmonium) from Belle (II) are listed below.
- Here I talk about the recent $a_{CP}^{T\text{-odd}}$ results of several $D_{(s)}^+$ decays at Belle.

(Belle II) D_s^+ lifetime $a_{CP}^{T\text{-odd}}$ and \mathcal{B} of $D_{(s)}^+ \rightarrow K^\pm h^\mp \pi^+ \pi^0$ $a_{CP}^{T\text{-odd}}(D_{(s)}^+ \rightarrow K^+ K_S^0 h^- \pi^+)$ and $\mathcal{B}(D_s^+ \rightarrow K^+ K^- K_S^0 \pi^+)$ (Belle II) Novel method for tagging D^0 flavor M/Γ of $\Lambda_c(2625)^+$ $\Lambda\pi^\pm$ signals near $\bar{K}N(I=1)$ in $\Lambda_c^+ \rightarrow \Lambda\pi^+\pi^+\pi^-$ \mathcal{B} of $\Lambda_c^+ \rightarrow p K_S^0 K_S^0$ and $\Lambda_c^+ \rightarrow p K_S^0 \eta$ \mathcal{B} of $\Omega_c^0 \rightarrow \Xi^-\pi^+, \Xi^-\kappa^+, \Omega^-\kappa^+$ a threshold cusp at $\Lambda\eta$ in $\Lambda_c^+ \rightarrow p K^- \pi^+$ \mathcal{B}/α of $\Lambda_c^+ \rightarrow \Sigma^+(\pi^0, \eta, \eta')$ $\mathcal{B}/A_{CP}^{\text{dir}}/\alpha/A_{CP}^\alpha$ of $\Lambda_c^+ \rightarrow \Lambda h^+, \Sigma^0 h^+$ (Belle II) Ω_c^0 lifetime $\mathcal{B}/a_{CP}^{T\text{-odd}}/A_{CP}$ of $D^0 \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$ \mathcal{B} of $D^+ \rightarrow K^- K_S^0 \pi^+ \pi^+ \pi^0$ (Belle II) Λ_c^+ lifetime \mathcal{B} of $\Lambda_c^+ \rightarrow \Sigma^+\gamma$ and $\Xi_c^0 \rightarrow \Xi^0\gamma$ $\Lambda_c(2910)^+$ in $B \rightarrow \Sigma_c(2455)\pi p$ \mathcal{B} of $\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$ \mathcal{B} of $\Lambda_c^+ \rightarrow p\eta'$ \mathcal{B} of $\Xi_c^0 \rightarrow \Lambda K_S^0, \Sigma^0 K_S^0$, and $\Sigma^+ K^-$ [arXiv:2306.00365](#)[arXiv:2305.12806, PRD 107, 033003 \(2023\)](#)[arXiv:2305.11405](#)[PRD 107, 112010 \(2023\)](#)[PRD 107, 032008 \(2023\)](#)[PRL 120, 151903 \(2023\)](#)[PRD 107, 032004 \(2023\)](#)[JHEP 01, 055 \(2023\)](#)[arXiv:2209.00050 \(accepted by PRD\)](#)[PRD 107, 032003 \(2023\)](#)[Science Bulletin 68 \(2023\) 583-592](#)[PRD 107, L031103 \(2023\)](#)[PRD 107, 052001 \(2023\)](#)[arXiv:2207.06595](#)[PRL 130, 071802 \(2023\)](#)[PRD 107, 032001 \(2023\)](#)[PRL 130, 031901 \(2023\)](#)[PRD 107, 032005 \(2023\)](#)[JHEP 03 \(2022\) 090](#)[PRD 105, L011102 \(2022\)](#)

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$a_{CP}^{T\text{-odd}}$ with T -odd correlations for charm multi-body decays

- ▶ T -odd correlations provides a powerful tool to indirectly search for CP violation under CPT symmetry conservation:
- ▶ C_T observable defined by a triple mixed product $C_T = (\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3$, satisfying $CP(C_T) = -C(C_T) = -\bar{C}_T$. Define T -odd asymmetries for $D_{(s)}^+$ or $D_{(s)}^-$ decays:

$$A_T = \frac{\Gamma_+(C_T > 0) - \Gamma_+(C_T < 0)}{\Gamma_+(C_T > 0) + \Gamma_+(C_T < 0)} \quad \bar{A}_T = \frac{\Gamma_-(-\bar{C}_T > 0) - \Gamma_-(-\bar{C}_T < 0)}{\Gamma_-(-\bar{C}_T > 0) + \Gamma_-(-\bar{C}_T < 0)}$$

In $B \rightarrow VV$, these A_T are $\propto \sin(\phi + \delta)$ and $\propto \sin(-\phi + \delta)$, respectively^[1].

- ▶ **T-odd CP asymmetry** is defined as (to veto FSI effects):

$$a_{CP}^{T\text{-odd}} = \frac{1}{2}(A_T - \bar{A}_T) \quad \text{can be nonzero if CPV}$$

$\propto \sin \phi \cos \delta$ for $B \rightarrow VV$ ^[1]: largest value when $\delta = 0$, vs. $A_{CP}^{\text{dir}} \neq 0$ needs $\delta \neq 0$,

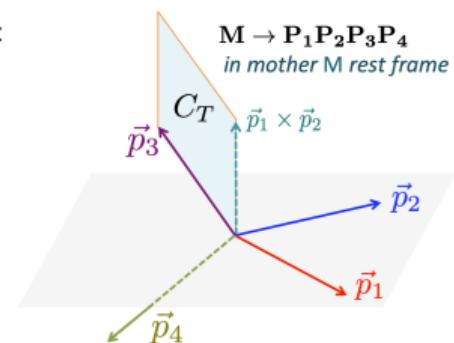
- ▶ Status of $a_{CP}^{T\text{-odd}}$ measurements in charmed mesons decay-rates:

$$D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0 \quad a_{CP}^{T\text{-odd}} = (-0.28 \pm 1.38^{+0.23}_{-0.76}) \times 10^{-3}$$

$$D^0 \rightarrow K^+ K^- \pi^+ \pi^- \quad a_{CP}^{T\text{-odd}} = (+1.7 \pm 2.7) \times 10^{-3}$$

$$D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^- \quad a_{CP}^{T\text{-odd}} = (-1.10 \pm 1.09) \times 10^{-2}$$

$$D_s^+ \rightarrow K_S^0 K^+ \pi^+ \pi^- \quad a_{CP}^{T\text{-odd}} = (-1.39 \pm 0.84) \times 10^{-2}$$



Belle and Belle II may improve some of these $a_{CP}^{T\text{-odd}}$ results and measure $a_{CP}^{T\text{-odd}}$ in more channels based on the large charm sample.

[1] A. Datta, D. London, *Int. J. Mod. Phys. A* **19** (2004) 2505

[2] K. Prasanth et al.(Belle Collab.), *Phys. Rev. D* **95**, 091101(R) (2017)

[3] R. Aaij et al.(LHCb Collab.), *JHEP* **10**, 5 (2014)

[4] P. del Amo Sanchez et al.(BaBar Collab.), *Phys. Rev. D* **81**, 111103(R) (2010)

[5] J.M. Link et al.(FOCUS Collab.), *Phys. Lett. B* **622**, 239 (2005)

[6] J.P. Lees et al.(BaBar Collab.), *Phys. Rev. D* **84**, 031103(R) (2011)



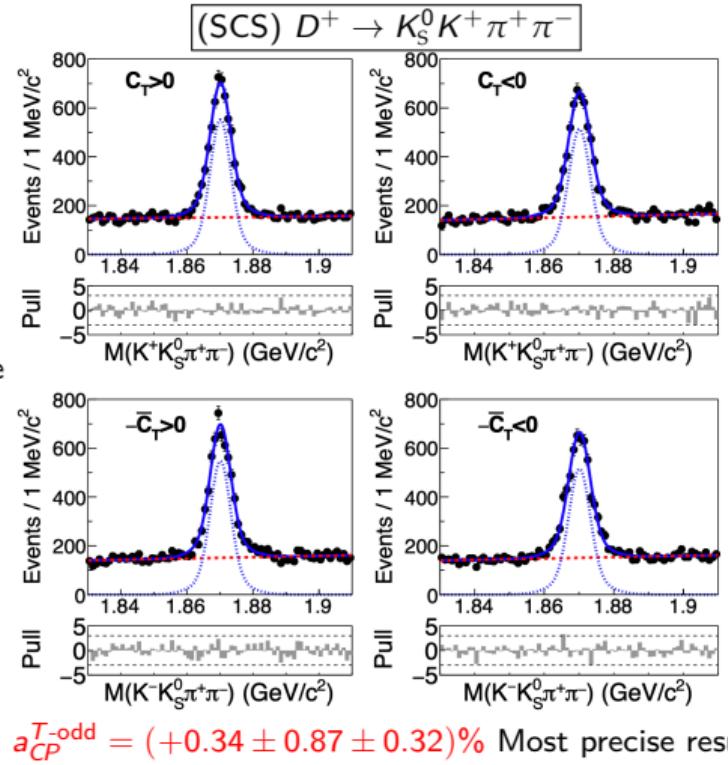
Measurement of $a_{CP}^{T\text{-odd}}$ for $D_{(s)}^+ \rightarrow K_S^0 K^+ h^+ h^-$

arXiv:2305.11405

- The $a_{CP}^{T\text{-odd}}$ values of singly Cabibbo-suppressed (SCS) decay $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ and Cabibbo-favored (CF) decay $D_s^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ were measured at FOCUS^[5] and BABAR^[6].
- We also measured them using the full Belle dataset (980 fb^{-1}). The data sample is divided into four subsamples (with yields):
 - $D_{(s)}^+$, $C_T > 0$ (N_1); $D_{(s)}^+$, $C_T < 0$ (N_2)
 - $D_{(s)}^-$, $-\bar{C}_T > 0$ (N_3); $D_{(s)}^-$, $-\bar{C}_T < 0$ (N_4)
- To suppress the background, we optimize the requirements of the D decay length significance, scaled momentum, vertex fit quality, etc. for each decay modes.
- We perform a binned maximum likelihood fit simultaneously to these $M(D)$ distributions of four subsamples.
- Instead of four yields, we float $N(D_{(s)}^\pm)$, A_T and $a_{CP}^{T\text{-odd}}$:

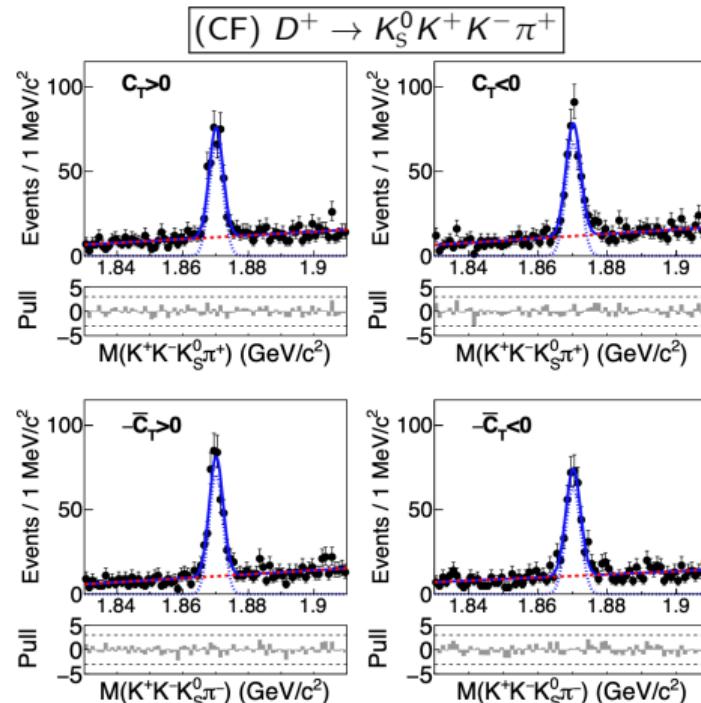
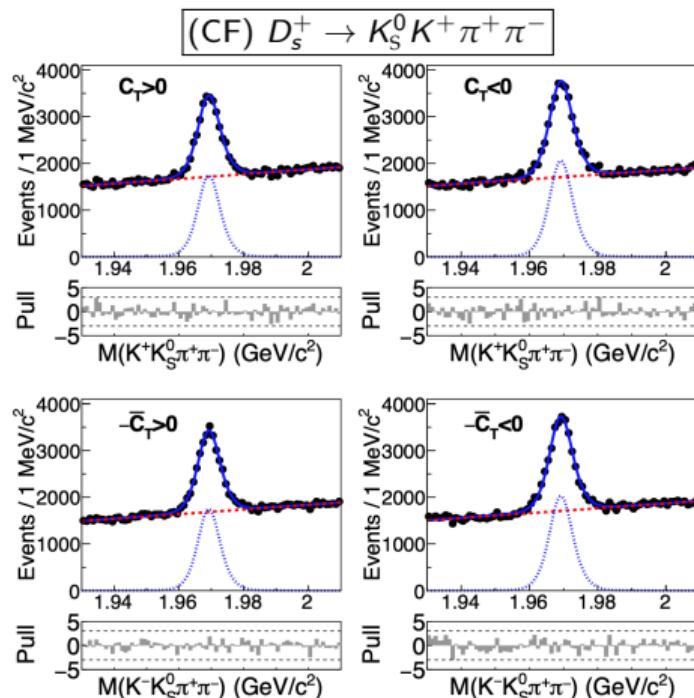
$$N_1 = N(D_{(s)}^+) \frac{1+A_T}{2}, \quad N_3 = N(D_{(s)}^-) \frac{1+A_T-2 \cdot a_{CP}^{T\text{-odd}}}{2},$$

$$N_2 = N(D_{(s)}^+) \frac{1-A_T}{2}, \quad N_4 = N(D_{(s)}^-) \frac{1-A_T+2 \cdot a_{CP}^{T\text{-odd}}}{2}.$$



Measurement of $a_{CP}^{T\text{-odd}}$ for $D_{(s)}^+ \rightarrow K_S^0 K^+ h^+ h^-$

arXiv:2305.11405



$a_{CP}^{T\text{-odd}} = (-0.46 \pm 0.63 \pm 0.38)\%$ Most precise result.

$a_{CP}^{T\text{-odd}} = (-3.34 \pm 2.66 \pm 0.35)\%$ First measurement.

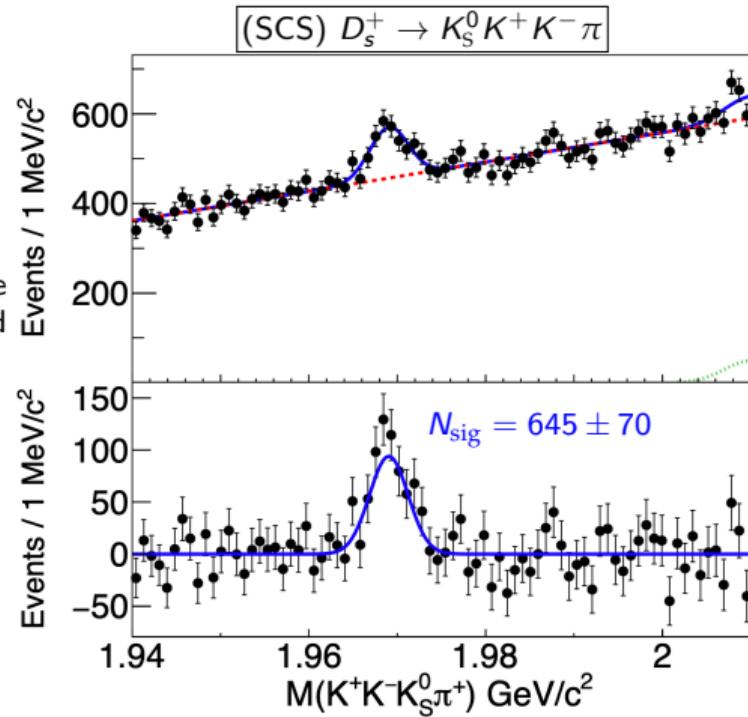


First measurement of $\mathcal{B}(D_s^+ \rightarrow K_s^0 K^+ K^- \pi^+)$ arXiv:2305.11405

- We report the first observation of (SCS) $D_s^+ \rightarrow K_S^0 K^+ K^- \pi^+$ with significance of 9.2σ .
 - Using (CF) $D_s^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$ as a normalization mode ($N_{\text{sig}} = 70080 \pm 676$), we measured the relative \mathcal{B} .
 - To take into account variation in reconstruction efficiencies due to unknown intermediate resonances, we correct the fitted yield for efficiency in bins of five-dimensional phase space.
 - Finally we have

$$\frac{\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+ K^- \pi^+)}{\mathcal{B}(D_s^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-)} = (1.36 \pm 0.15_{\text{stat.}} \pm 0.04_{\text{syst.}})\%$$

- Inserting the W.A. $\mathcal{B}(D_s^+ \rightarrow K_s^0 K^+ \pi^+ \pi^-) = (0.95 \pm 0.08)\%$, we obtain



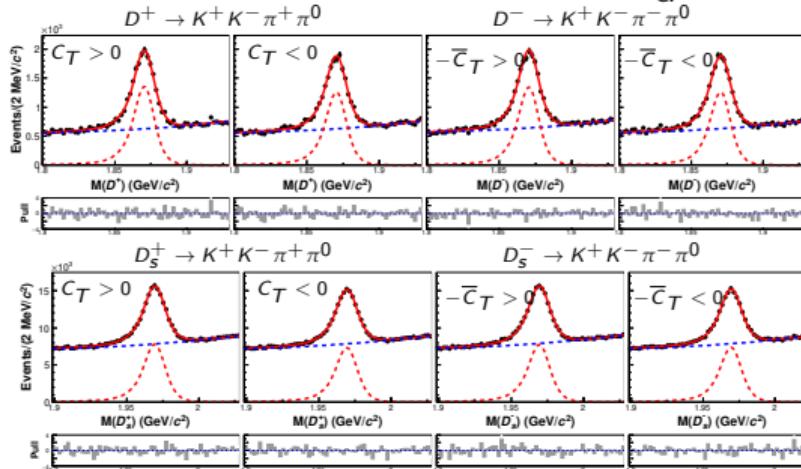
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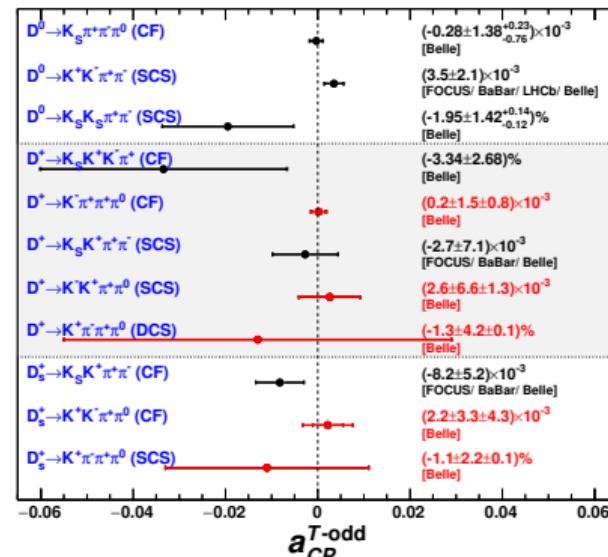
Measurement of $a_{CP}^{T\text{-odd}}$ for $D_{(s)}^+ \rightarrow K^\pm h^\pm \pi^+ \pi^0$ arXiv:2305.12806

- Simultaneous fit on four C_T subsamples to extract $a_{CP}^{T\text{-odd}}$, e.g,



Decay	$D^+ \rightarrow f$		$D_s^+ \rightarrow f$		
	$K^+ K^- \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^+ K^- \pi^+ \pi^0$
Final state (f)	$K^+ K^- \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^0$	$K^+ \pi^- \pi^+ \pi^0$	$K^+ K^- \pi^+ \pi^0$
N_D	27284 ± 254	2062 ± 127	438432 ± 947	15197 ± 484	167357 ± 786
$N_{\bar{D}}$	27177 ± 255	2044 ± 125	450667 ± 961	14945 ± 479	167064 ± 788
A_T (%)	$+3.63 \pm 0.93$	-0.4 ± 6.0	-0.76 ± 0.22	$+1.4 \pm 3.2$	$+2.96 \pm 0.47$
$a_{CP}^{T\text{-odd}}$ (%)	$+0.26 \pm 0.66$	-1.3 ± 4.2	$+0.02 \pm 0.15$	-1.1 ± 2.2	$+0.22 \pm 0.33$

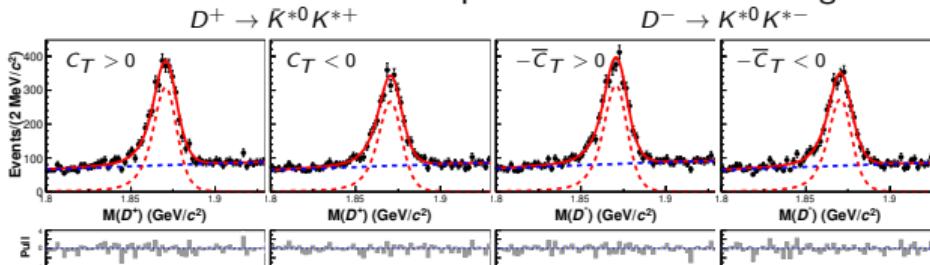
- Our $a_{CP}^{T\text{-odd}}$ results compared with other charm results from experiments. All D mesons have reached precision of $\mathcal{O}(10^{-3})$. The decays cover CF, SCS, and doubly Cabibbo-suppressed (DCS) decays.



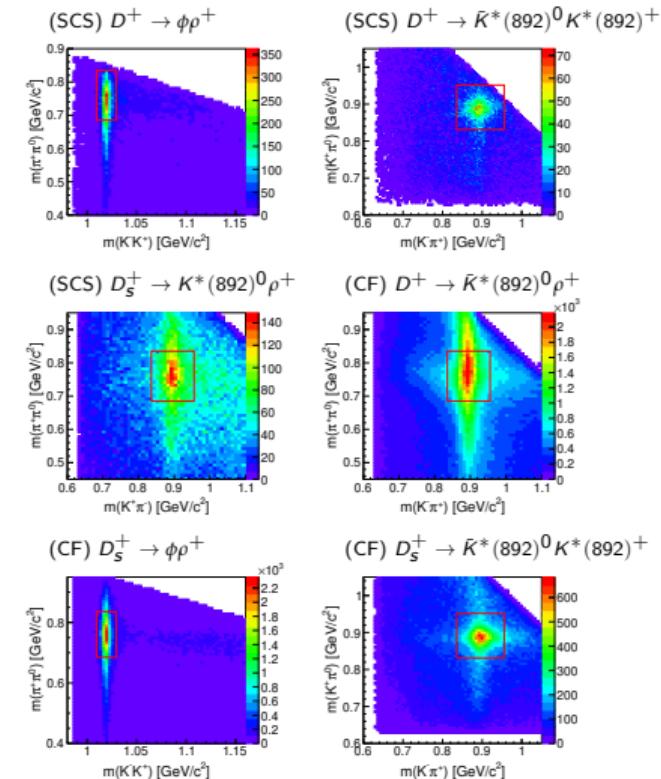
$a_{CP}^{T\text{-odd}}$ in subregions of PHSP for $D_{(s)}^+ \rightarrow K^\pm h^\pm \pi^+ \pi^0$

arXiv:2305.12806

- Similar measurement in seven subregion of phase space corresponding to dominant intermediate resonant processes of $D \rightarrow VV$. e.g.



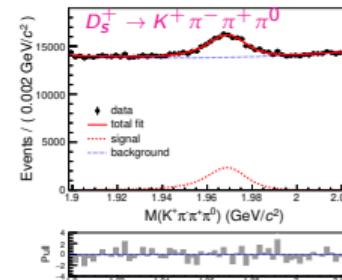
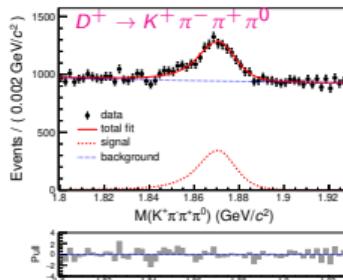
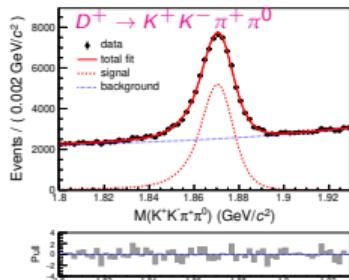
Subregion	$D_{(s)}^+ \rightarrow VV$	Signal region (SR)	$a_{CP}^{T\text{-odd}} (\times 10^{-2})$
(1) SCS	$D^+ \rightarrow \phi \rho^+$	ϕ -SR, ρ^+ -SR	$0.85 \pm 0.95 \pm 0.25$
(2) SCS	$D^+ \rightarrow \bar{K}^{*0} K^{*+}$	$K^{*(0,+)}$ -SR, veto ϕ -SR	$0.17 \pm 1.26 \pm 0.13$
(3) CF	$D^+ \rightarrow \bar{K}^{*0} \rho^+$	K^{*0} -SR, ρ^+ -SR	$0.25 \pm 0.25 \pm 0.13$
(4) SCS	$D_s^+ \rightarrow K^{*0} \rho^+$	K^{*0} -SR, ρ^+ -SR	$6.2 \pm 3.0 \pm 0.4$
(5) SCS	$D_s^+ \rightarrow K^{*+} \rho^0$	K^{*+} -SR, ρ^0 -SR	$1.7 \pm 6.1 \pm 1.5$
(6) CF	$D_s^+ \rightarrow \phi \rho^+$	ϕ -SR, ρ^+ -SR	$0.31 \pm 0.40 \pm 0.43$
(7) CF	$D_s^+ \rightarrow \bar{K}^{*0} K^{*+}$	$K^{*(0,+)}$ -SR, veto ϕ -SR	$0.26 \pm 0.76 \pm 0.37$

SR: $|M_{KK} - m_\phi| < 10 \text{ MeV}/c^2$, $-90 < (M_{\pi\pi} - m_\rho) < 60 \text{ MeV}/c^2$, and $|M_{K\pi} - m_{K^*}| < 60 \text{ MeV}/c^2$.

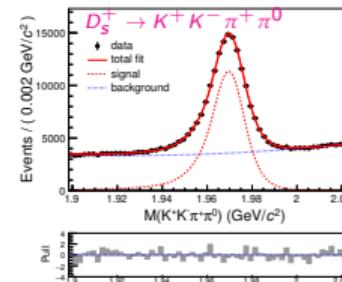
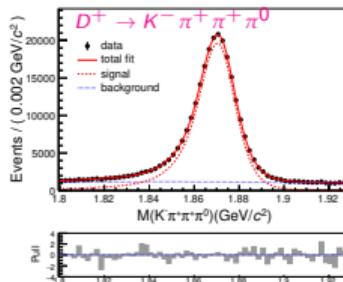
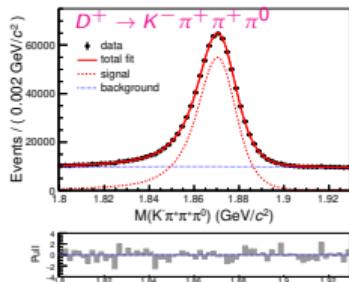
\mathcal{B} of Cabibbo-suppressed decays $D_{(s)}^+ \rightarrow K^\pm h^\pm \pi^+ \pi^0$

Phys. Rev. D 107, 033003 (2023)

- Three Cabibbo-suppressed decays



and reference modes (using same cut criteria as signal modes):



- Based on the efficiency-corrected yields, we obtain three relative $\mathcal{B}_{\text{sig}}/\mathcal{B}_{\text{ref}}$ values. e.g.

$$\frac{\mathcal{B}(D^+ \rightarrow K^+ \pi^- \pi^+ \pi^0)}{\mathcal{B}(D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0)} = (1.68 \pm 0.11 \pm 0.03)\%$$

corresponds to $(5.83 \pm 0.42) \tan^4 \theta_C$.

This ratio is significantly larger than all other known DCS/CF ratios. This confirms BESIII's discovery of this anomaly ratio PRL 125, 141802 (2020), PRD 104, 072005 (2021)

- using the W.A. \mathcal{B}_{ref} , we have three absolute \mathcal{B} results:

$$\mathcal{B}(D^+ \rightarrow K^+ K^- \pi^+ \pi^0) = (7.08 \pm 0.08 \pm 0.16 \pm 0.20) \times 10^{-3},$$

$$\mathcal{B}(D^+ \rightarrow K^+ \pi^- \pi^+ \pi^0) = (1.05 \pm 0.07 \pm 0.02 \pm 0.03) \times 10^{-3},$$

$$\mathcal{B}(D_s^+ \rightarrow K^+ \pi^- \pi^+ \pi^0) = (9.44 \pm 0.34 \pm 0.28 \pm 0.32) \times 10^{-3}.$$

Most precise to date!



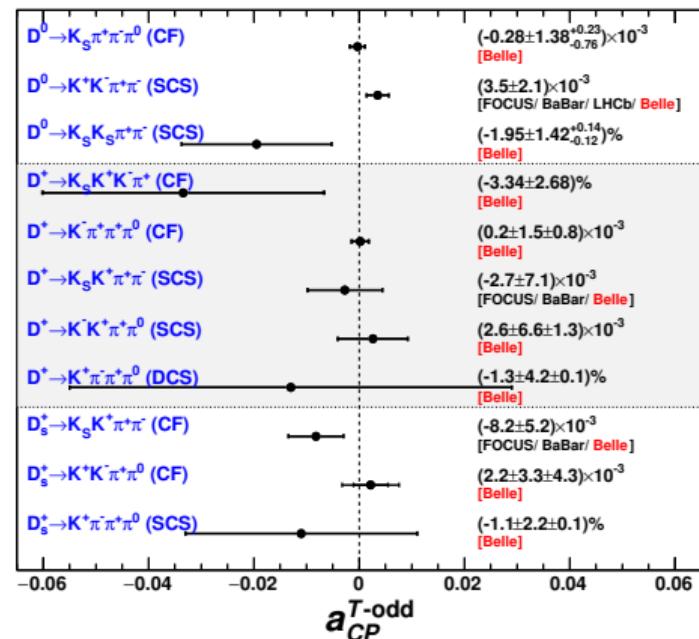
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Summary

- Belle finished her data accumulation 13 years ago, but is still lasting to produce lots of charm results to date.
- Recently searches for CP violation via T -odd correlations in eight $D_{(s)}^+$ decays (three SCS / four CF / one DCS) were reported, along with most precise measurements of branching fractions of four $D_{(s)}^+$ decays.
- Belle did make the significant contributions to $a_{CP}^{T\text{-odd}}$ results for all measured charm decays.
- The precisions of $a_{CP}^{T\text{-odd}}$ results for various charmed mesons have reached $\mathcal{O}(0.1\%)$.
- More charm CPV results are promising in the future, based on Belle and Belle II available datasets (totally 1.4 ab^{-1}), and Belle II final targeted dataset (50 ab^{-1}). Please stay tuned.



Back up

Thank you for your attentions.



谢谢!

Dr. Longke LI (李龙科)
Department of Physics,
University of Cincinnati (UC)
 [lilongke_ustc](#)
 lilk@ucmail.uc.edu



Available Charm samples from Charm factories, B-factories, hadron colliders

Experiment	Machine	Operation	C.M.	Lumin.	N_{prod}	Efficiency	Characters
	CESR (e^+e^-)	2003-2008	3.77	0.8 fb^{-1}	$D^0: 3 \times 10^6$	$\sim 10\text{-}30\%$	☺ extremely clean environment ☺ pure D-beam, almost no bkg ☺ quantum coherence ☺ no CM boost, no T-dep analyses
			4.18 GeV	0.6 fb^{-1}	$D_s^+: 6 \times 10^5$		
	BEPC-II (e^+e^-)	2010-2011(2021-)	3.77 GeV	$2.92(\rightarrow 20) \text{ fb}^{-1}$	$D^0: 10^7 (\rightarrow 10^8)$		
		2016-2019 2014+2020	4.18-4.23 GeV 4.6-4.7 GeV	7.3 fb^{-1} 4.5 fb^{-1}	$D_s^+: 5 \times 10^6$ $\Lambda_c^+: ? \text{ M}$		
						★	★★★
	SuperKEKB (e^+e^-)	2019-	10.58 GeV	$424(\rightarrow 50000) \text{ fb}^{-1}$	$D^0: 6 \times 10^8 (\rightarrow 10^{11})$ $D_s^+(\text{s}): 10^8 (\rightarrow 10^{10})$ $\Lambda_c^+: 10^7 (\rightarrow 10^9)$	$\sim 5\text{-}10\%$	☺ clear event environment ☺ high trigger efficiency ☺ high-efficiency detection of neutrals ☺ many high-statistics control samples ☺ time-dependent analysis ☺ smaller cross-section than pp colliders
	KEKB (e^+e^-)	1999-2010	10.58 GeV	1000 fb^{-1}	$D^0: 10^9$ $D_s^+(\text{s}): 10^9$ $\Lambda_c^+: 10^8$	$\sim 5\text{-}10\%$	
	PEP-II (e^+e^-)	1999-2008	10.58 GeV	500 fb^{-1}	6×10^8	★	★★
	Tevatron ($p\bar{p}$)	2002-2011	1960	9.6 fb^{-1}	10^{11}	$<0.5\%$	☺ very large production cross-section ☺ large boost ☺ excellent time resolution ☺ dedicated trigger required
	LHC (pp)	2011	7 TeV	1.0 fb^{-1}	5×10^{12} 10^{13}	$<0.5\%$	★
		2012	8 TeV	2.0 fb^{-1}			
		2015-2018	13 TeV	6 fb^{-1}			

Here uses $\sigma(D^0\bar{D}^0 @ 3.77 \text{ GeV}) = 3.61 \text{ nb}$, $\sigma(D^+D^- @ 3.77 \text{ GeV}) = 2.88 \text{ nb}$, $\sigma(D_s^*D_s @ 4.17 \text{ GeV}) = 0.967 \text{ nb}$; $\sigma(c\bar{c} @ 10.58 \text{ GeV}) = 1.3 \text{ nb}$ where each $c\bar{c}$ event averagely has 1.1 D^0 yields, 0.6 D^+ yields and 0.3 D_s^+ yields; $\sigma(D^0 @ CDF) = 13.3 \mu\text{b}$, and $\sigma(D^0 @ LHCb) = 1661 \mu\text{b}$, mainly referred to *Int. J. Mod. Phys. A* 29 (2014) 24, 14300518.

Systematic uncertainties for $a_{CP}^{T\text{-odd}}(D_{(s)}^+ \rightarrow K_S^0 K^+ h^+ h^-)$ and $a_{CP}^{T\text{-odd}}(D_{(s)}^+ \rightarrow K^\pm h^\pm \pi^+ \pi^0)$

Table: Systematic uncertainties for $a_{CP}^{T\text{-odd}}$ in % for
 (a) $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$; (b) $D_s^+ \rightarrow K_S^0 K^+ K^- \pi^+$; and
 (c) $D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$.

Source	(a)	(b)	(c)
Fit model	0.01	0.02	0.12
Detector bias	0.32	0.32	0.32
Efficiency variation with C_T, \bar{C}_T	0.03	0.20	0.06
Total syst.	0.32	0.38	0.35

Table: Systematic uncertainties for $a_{CP}^{T\text{-odd}}$ in % for five $D_{(s)}^+$ decays:
 (a) $D^+ \rightarrow K^+ K^- \pi^+ \pi^0$; (b) $D^+ \rightarrow K^+ \pi^- \pi^+ \pi^0$; (c) $D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$;
 (d) $D_s^+ \rightarrow K^+ \pi^- \pi^+ \pi^0$; and (e) $D_s^+ \rightarrow K^+ K^- \pi^+ \pi^0$.

Decay channel	(a)	(b)	(c)	(d)	(e)
C_T -dependent efficiency	0.13	0.02	0.08	0.02	0.41
C_T resolution	0.01	0.06	0.01	0.07	0.02
PDF parameters	0.01	0.07	0.01	0.07	0.04
Mass resolution	0.03	0.01	...	0.02	0.11
Fit bias	0.01	0.07	0.00	0.06	0.02
Total syst.	0.13	0.12	0.08	0.12	0.43

