

# Charmonia production in heavy ion collisions

11<sup>th</sup> International Workshop on  
Charm Physics (CHRAM 2023)

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Hörsaalzentrum Unteres Schloss



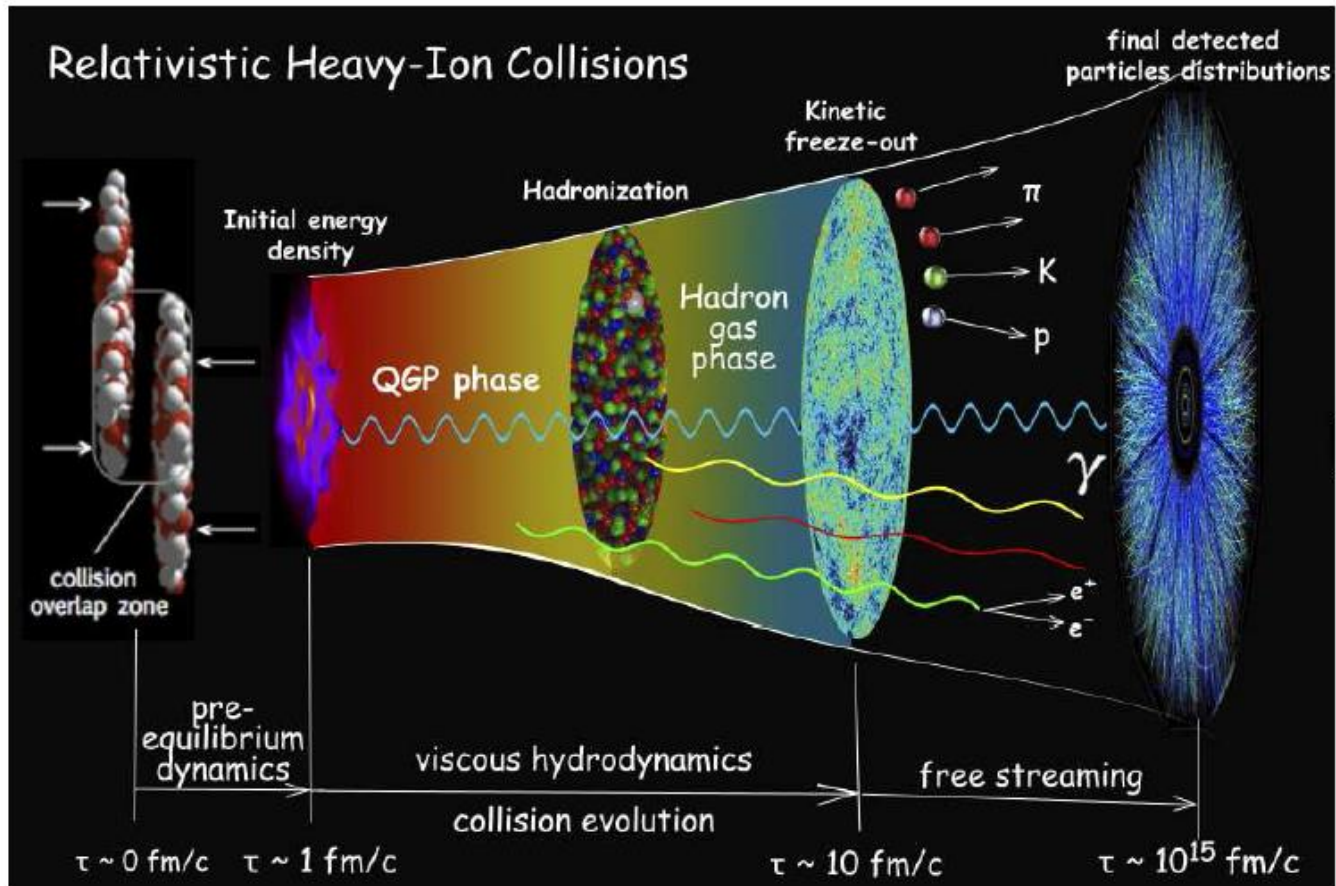
Sungtae Cho  
Kangwon National University

# Outline

- Introduction
- Charmonium states in heavy ion collisions
- Transverse momentum distributions and yields of charmonium states
- Elliptic and triangular flow of charmonium states
- Conclusion

# Introduction

## – Relativistic heavy ion collisions



U. W. Heinz, J. Phys. Conf. Ser. **455**, 012044 (2013)

11th International Workshop on Charm Physics (CHARM 2023)

# Charmonium states in heavy ion collisions

## – Quarkonia

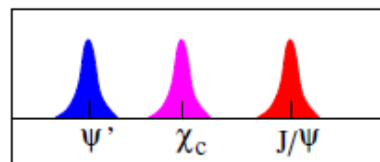
1) The bound states of a heavy quark and its anti-quark

The measured stable charmonium:

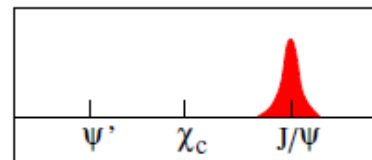
the 1S scalar  $\eta_c$  and vector  $J/\psi$ , three 1P states  $\chi_c$  (scalar, vector, and tensor), and the 2S vector state  $\psi'$

2) The different charmonium states melt sequentially as a function of their binding strength;

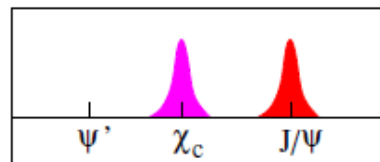
the most loosely bound state disappears first, the ground state last



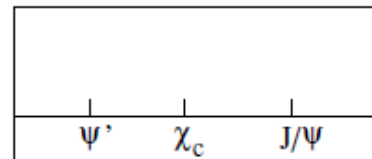
$T < T_c$



$T \sim 1.1 T_c$



$T \sim T_c$



$T \gg T_c$

H. Satz, J. Phys. G.  
**32**, R25 (2006)

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Charm Physics (CHARM 2023)

# – J/ψ suppression and Debye screening

T. Matsui and H. Satz, Phys. Lett. **B178** 416 (1986)

1) At  $T > T_C$  color charges are Debye screened in QGP

$$V = -\frac{4}{3} \frac{\alpha_s}{r} \rightarrow V = -\frac{4}{3} \frac{\alpha_s}{r} e^{-r/\lambda_D} \quad \lambda_D = \frac{1}{gT \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}}$$

Compared to the Bohr radius  $r_B$ , the Debye screening prevents the formation of the bound states when  $r_B > \lambda_D$

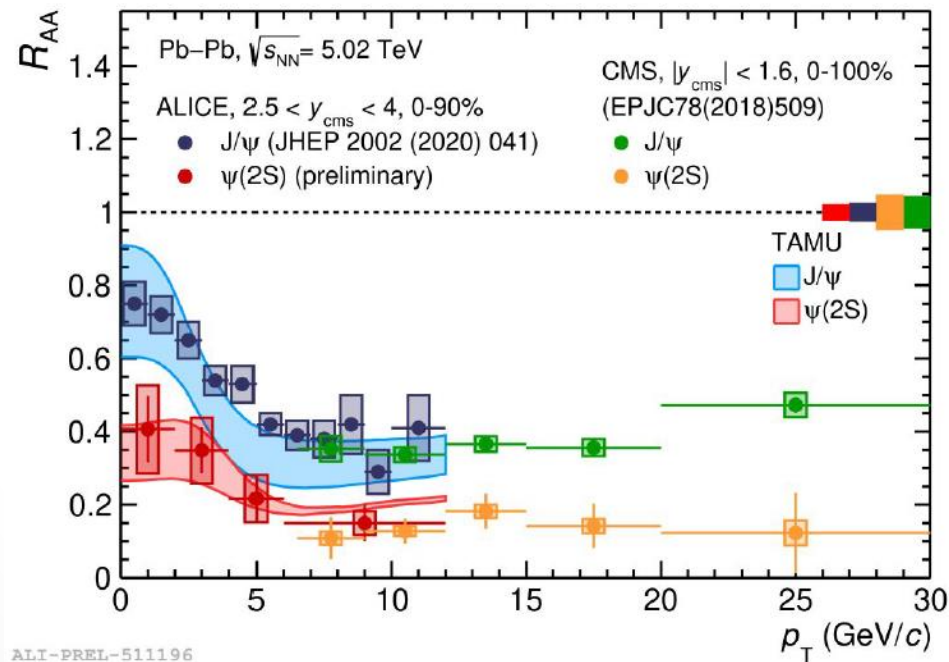
2) The measure for the effect of the quark-gluon plasma on the J/ψ suppression : the nuclear modification factor

$$R_{AA} = \frac{1}{N_{Coll}} \frac{dN_{J/\psi}^{AA}/d\vec{p}_T}{dN_{J/\psi}^{pp}/d\vec{p}_T}$$

# – Regeneration of charmonium states

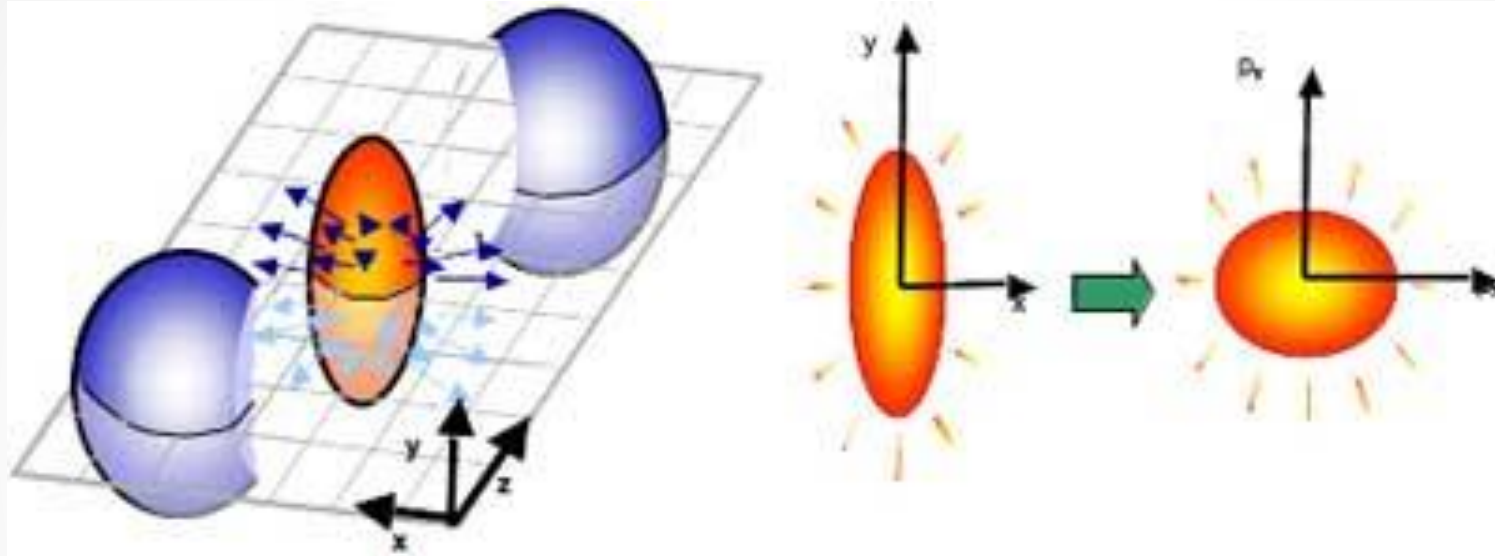
## 1) The nuclear modification factor of the $J/\psi$ and $\psi(2S)$ meson

Jon-Are Saetre (Univ. of Bergen), Quark Matter 2022, Krakow, April 4-10



ALI-PREL-511196

# - Non-central collisions, anisotropic flows

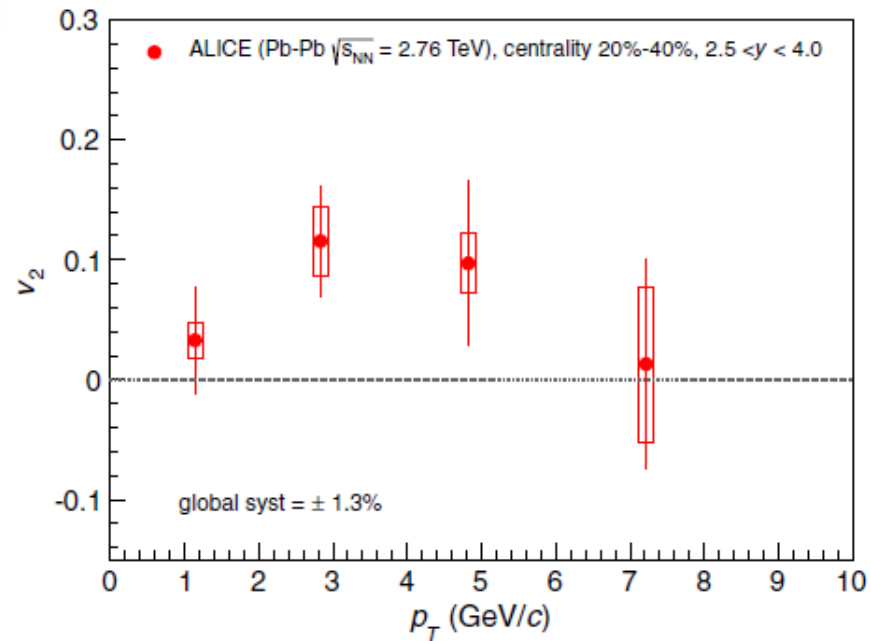


$$\begin{aligned}
 E \frac{d^3 N_q}{d^3 p} &= \frac{dN_q}{p_T dp_T d\varphi dy} = \frac{1}{2\pi} \frac{dN_q}{p_T dp_T dy} \left[ 1 + \sum_{n=1} 2v_{n,q}(p_T) \cos(n\varphi) \right] \\
 &= \frac{1}{2\pi} \frac{dN_q}{p_T dp_T dy} \left[ 1 + 2v_{1,q}(p_T) \cos(\varphi) + 2v_{2,q}(p_T) \cos(2\varphi) + \dots \right]
 \end{aligned}$$

A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998)

# 1) Elliptic flow of the J/ψ

E. Abbas et al, Phys. Rev. Lett. **111**, 162301 (2013)



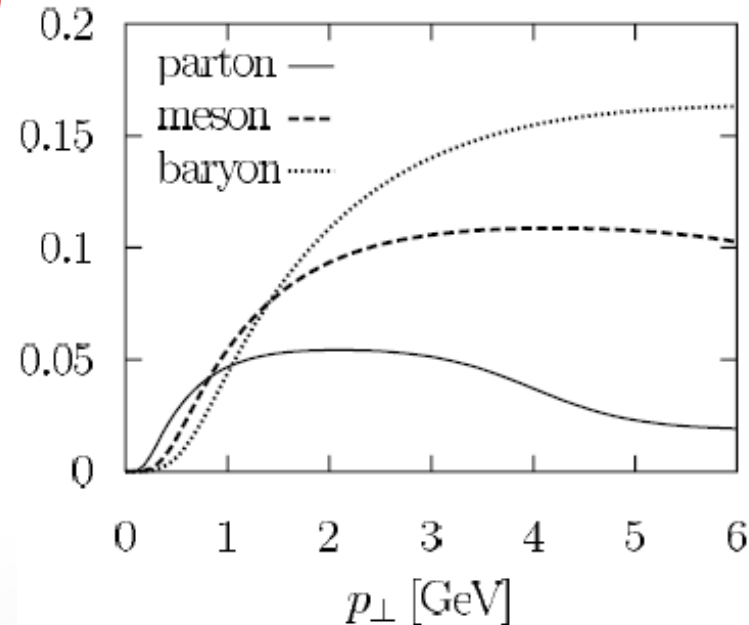
# 2) Quark number scaling of the elliptic flow

D. Molnar and S. A. Voloshin, Phys. Rev. Lett **91**, 092301 (2003)

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1 + 2v_{2,q}^2(p_T/2)}$$

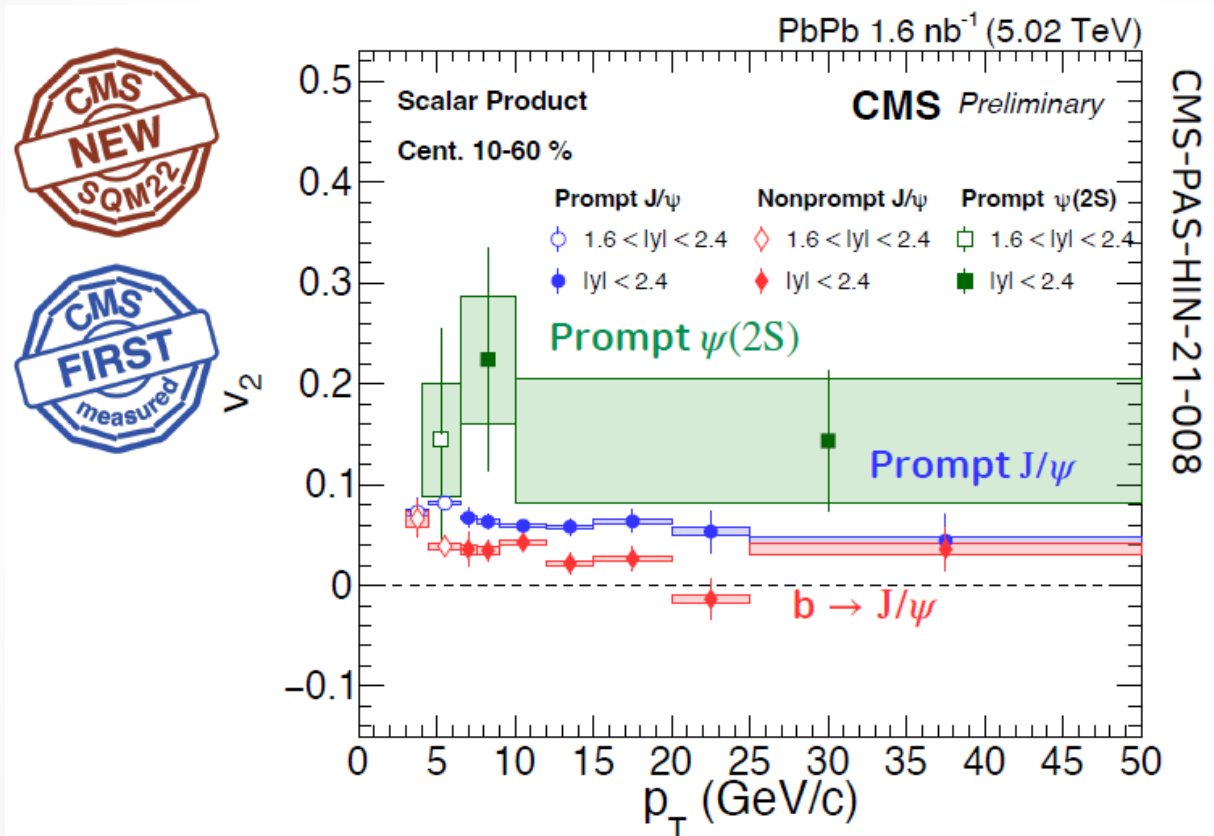
$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3) + 3v_{2,q}^3(p_T/3)}{1 + 6v_{2,q}^2(p_T/3)}$$

$$v_{2,h}(p_T) \approx nv_{2,q}\left(\frac{1}{n} p_T\right)$$





### 3) Recent measurements of elliptic flow of charmonium states, the $J/\psi$ and $\psi(2S)$ meson at LHC by CMS Collaboration



G. Bak [CMS Collaboration], Strangeness in Quark Matter 2022, Busan, June, 13-17  
 CMS-PAS-HIN-21-008

# Transverse momentum distributions and yields of charmonium states

## – Yields of hadrons in the coalescence model

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)

R. J. Freis, B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g \int \left[ \prod_{i=1}^n \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3 p_i}{E_i} f(x_i, p_i) \right] f^W(x_1, \dots, x_n : p_1, \dots, p_n)$$

### 1) The Wigner function, the coalescence probability function

$$f^W(x_1, \dots, x_n : p_1, \dots, p_n) = \int \prod_{i=1}^n dy_i e^{p_i y_i} \psi^* \left( x_1 + \frac{y_1}{2}, \dots, x_n + \frac{y_n}{2} \right) \psi \left( x_1 - \frac{y_1}{2}, \dots, x_n - \frac{y_n}{2} \right)$$

### 2) A Lorentz-invariant phase space integration of a space-like hyper-surface constraints the number of particles in the system

$$\int p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

# – Production of charmonium states by recombination

S. Cho, Phys. Rev. C **91**, 054914 (2015)

## 1) Coalescence production of charmonium states

$$N_\psi = g_\psi \int p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3 \vec{p}_c}{(2\pi)^3 E_c} \frac{d^3 \vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_\psi(r_c, r_{\bar{c}}; p_c, p_{\bar{c}}),$$

The transverse momentum distribution of the charmonium yield

$$\frac{dN_\psi}{d^2 \vec{p}_T} = \frac{g_\psi}{V} \int d^3 \vec{r} d^2 \vec{p}_{cT} d^2 \vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d^2 \vec{p}_{cT}} \frac{dN_{\bar{c}}}{d^2 \vec{p}_{\bar{c}T}} W_\psi(\vec{r}, \vec{k})$$

## 2) Gaussian Wigner functions for different charmonium states

$$\begin{aligned} W_s(\vec{r}, \vec{k}) &= 8e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2} \\ W_p(\vec{r}, \vec{k}) &= \left( \frac{16}{3} \frac{r^2}{\sigma^2} - 8 + \frac{16}{3} \sigma^2 k^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2} \\ W_{\psi_{10}}(\vec{r}, \vec{k}) &= \frac{16}{3} \left( \frac{r^4}{\sigma^4} - 2 \frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 \right. \\ &\quad \left. - 2r^2 k^2 + 4(\vec{r} \cdot \vec{k})^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}. \end{aligned}$$

## 4) Integration of the Wigner function over the spatial coordinates

$$\int d^3\vec{r} W_\psi(\vec{r}, \vec{k}) = \begin{cases} (2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} & \psi_s^G; J/\psi \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \sigma^2 k^2 & \psi_p^G; \chi_c \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \left(\sigma^2 k^2 - \frac{3}{2}\right)^2 & \psi_{10}^G; \psi(2S) \\ 64\pi \frac{a_0^3}{(a_0^2 k^2 + 1)^4} & \psi_{1S}^C; J/\psi \\ 8\pi a_0^3 \frac{(a_0^2 k^2 - 1/4)^2}{(a_0^2 k^2 + 1/4)^6} & \psi_{2S}^C; \psi(2S) \end{cases}$$

$$\int d^3\vec{r} W(\vec{r}, \vec{k}) = |\tilde{\psi}(\vec{k})|^2$$

M. Hillery, R. F. O'Connell, M. O. Scully and E. P. Wigner, Phys. Rept. **106**, 121 (1984)

$$\frac{dN_\psi}{d\vec{p}_T} = \frac{g_\psi}{V} \int d\vec{p}_{cT} d\vec{p}_{\bar{c}T} \delta(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d\vec{p}_{\bar{c}T}} |\tilde{\psi}(\vec{k})|^2$$

# 5) Transverse momentum distributions of charm and light quarks

$$\frac{dN_c}{d^2p_T} = \begin{cases} a_0 \exp[-a_1 p_T^{a_2}] & p_T \leq p_0 \\ a_0 \exp[-a_1 p_T^{a_2}] + a_3(1 + p_T^{a_4})^{-a_5} & p_T \geq p_0 \end{cases}$$

RHIC	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$p_T \leq p_0$	0.69	1.22	1.57			
$p_T \geq p_0$	1.08	3.04	0.71	3.79	2.02	3.48
LHC	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$p_T \leq p_0$	1.97	0.35	2.47			
$p_T \geq p_0$	7.95	3.49	3.59	87335	0.5	14.31

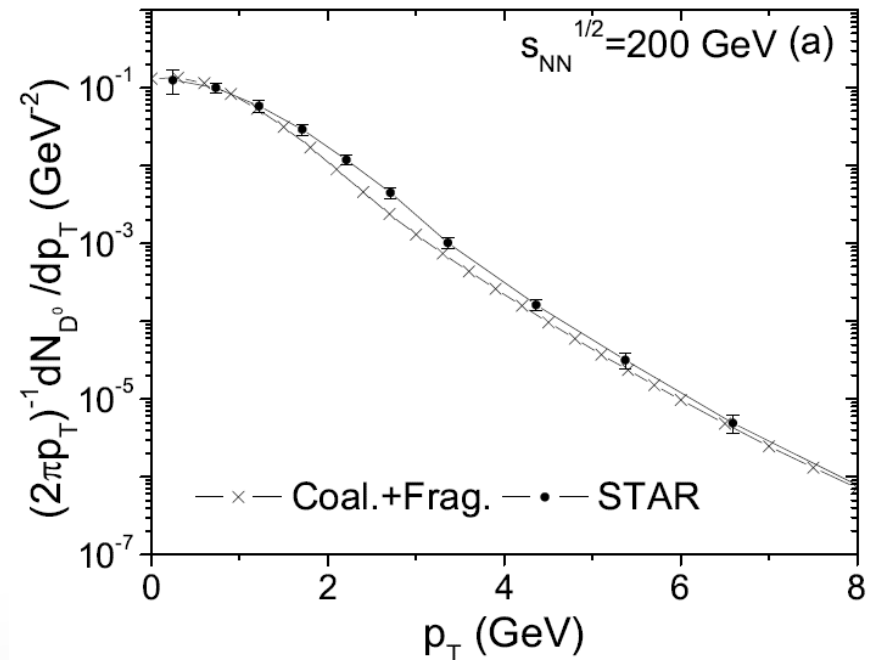
S. Plumari, V. Minissale, S. K. Das, G. Coci and V. Greco, Eur. Phys. J. C **78**:348 (2017)

Y. Oh, C. M. Ko, S.-H. Lee, and S. Yasui, Phys. Rev. C **79** 044905 (2009)

S. Cho *et al.* (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)

S. Cho and S. H. Lee, Phys. Rev. C **101**, 024902 (2020)

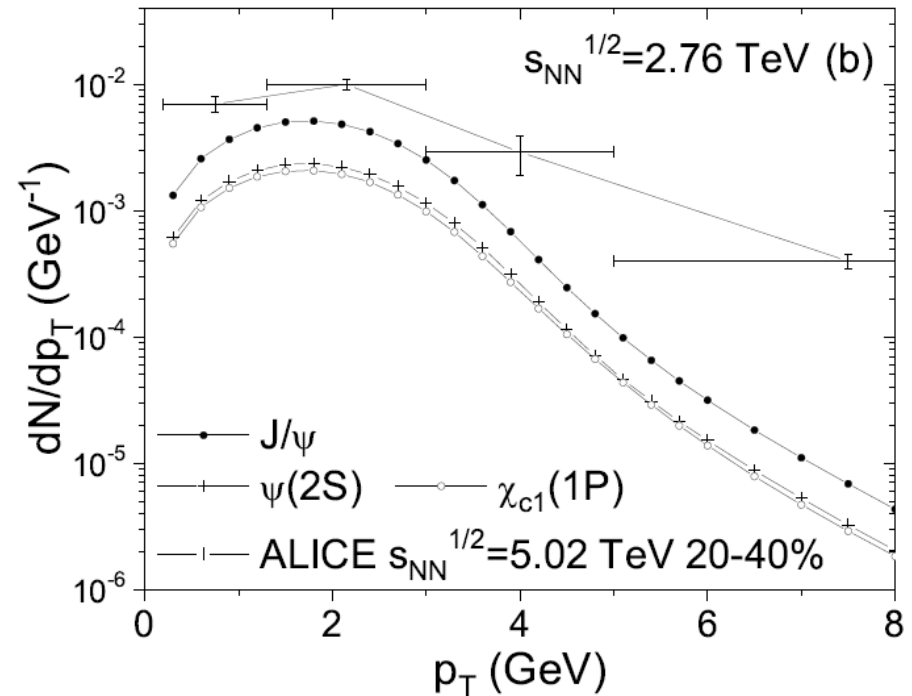
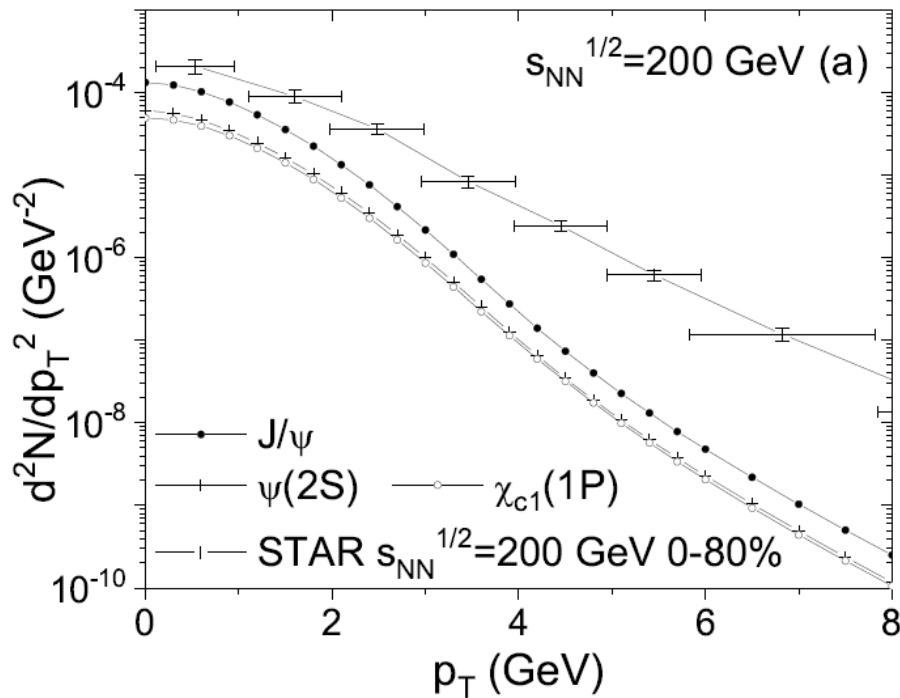
	RHIC		LHC (2.76 TeV)	
	Sc. 1	Sc. 2	Sc. 1	Sc. 2
$T_H$ (MeV)		162		156
$V_H$ (fm <sup>3</sup> )		2100		5380
$\mu_B$ (MeV)		24		0
$\mu_s$ (MeV)		10		0
$\gamma_c$				39
$\gamma_b$		$4.0 \times 10^7$		$8.6 \times 10^8$
$T_C$ (MeV)	162	166	156	166
$V_C$ (fm <sup>3</sup> )	2100	1791	5380	3533
$N_u = N_d$	320	302	700	593
$N_s = N_{\bar{s}}$	183	176	386	347
$N_c = N_{\bar{c}}$		4.1		11
$N_b = N_{\bar{b}}$		0.03		0.44



# 6) Transverse momentum distributions and yields of charmonium states in midrapidities at RHIC and LHC

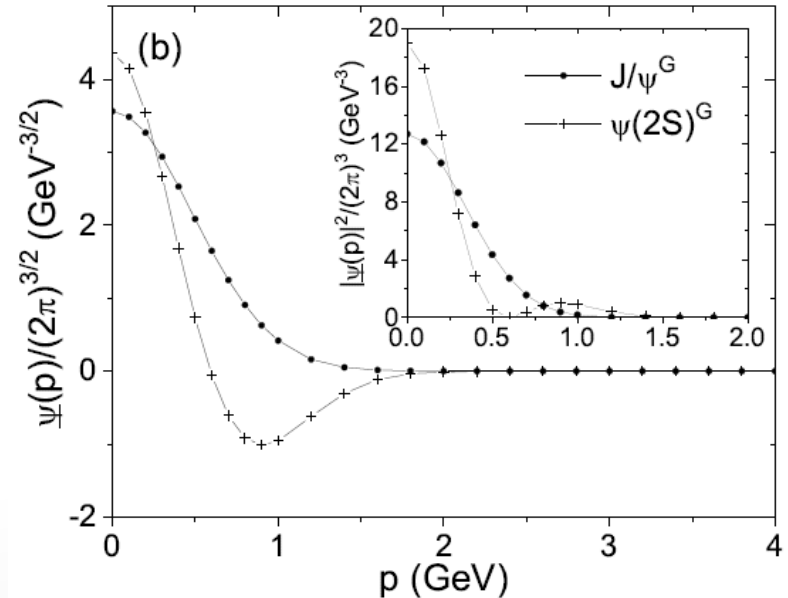
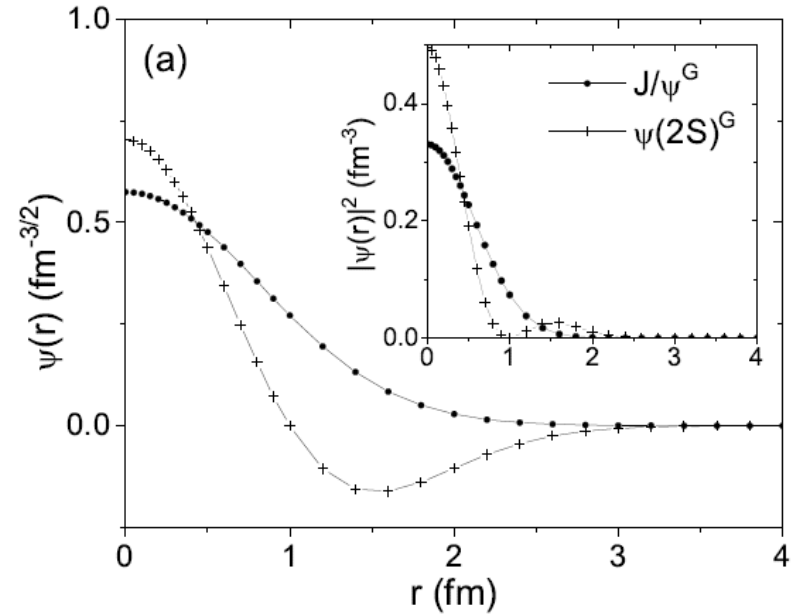
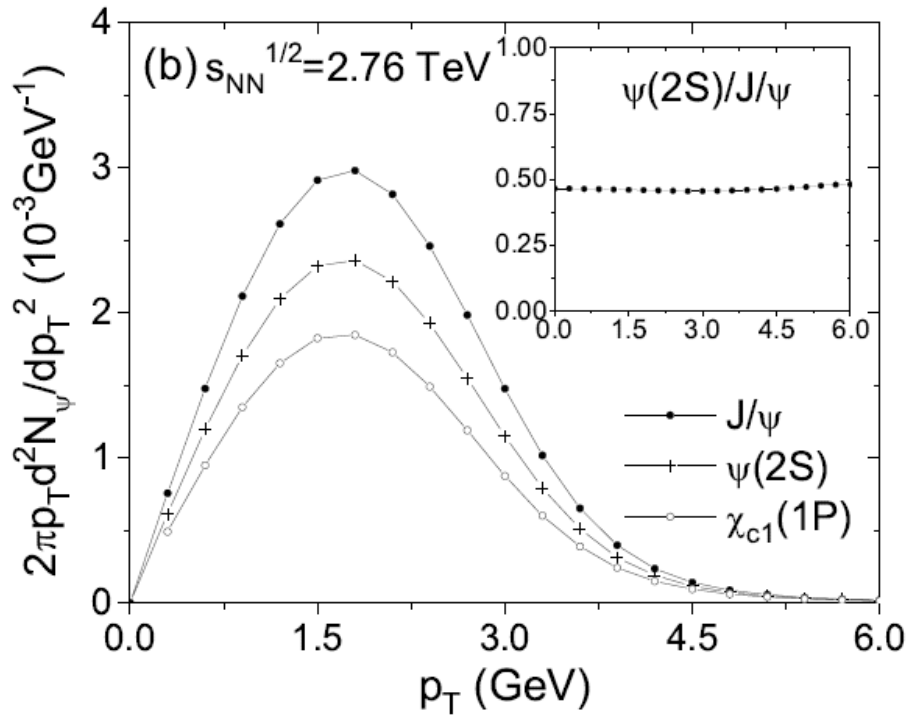
J. Adam et al. [STAR Collaboration], Phys. Lett. B **797**, 134917 (2019).

S. Acharya et al. [ALICE Collaboration], Phys. Lett. B **805**, 135434 (2020).



	RHIC	LHC
$J/\psi$	$7.6 \times 10^{-4}$	$1.3 \times 10^{-2}$
$\psi(2S)$	$3.5 \times 10^{-4}$	$5.8 \times 10^{-3}$
$\chi_{c1}(1P)$	$3.0 \times 10^{-4}$	$5.1 \times 10^{-3}$

# 7) The dependence of transverse momentum distributions and yields of charmonium states on the internal structure, or the wave function distributions



# Elliptic and triangular flow of charmonium states

– Flow harmonics of charmonium states

$$v_n(p_T) = \langle \cos(n(\psi - \Psi_n)) \rangle = \frac{\int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2 N}{dp_T^2}}{\int d\psi \frac{d^2 N}{dp_T^2}}, \quad \Psi_n = \frac{1}{n} \tan^{-1} \left( \frac{\langle p_T \sin(n\psi) \rangle}{\langle p_T \cos(n\psi) \rangle} \right),$$

1) Transverse momentum distribution of charm quarks with flow harmonics

$$\frac{d^2 N_c}{dp_{cT}^2} = \frac{1}{2\pi p_{cT}} \frac{dN_c}{dp_{cT}} \left( 1 + \sum_{n=1}^{\infty} 2v_{nc}(p_{cT}) \cos(n(\phi_c - \Psi_n)) \right),$$

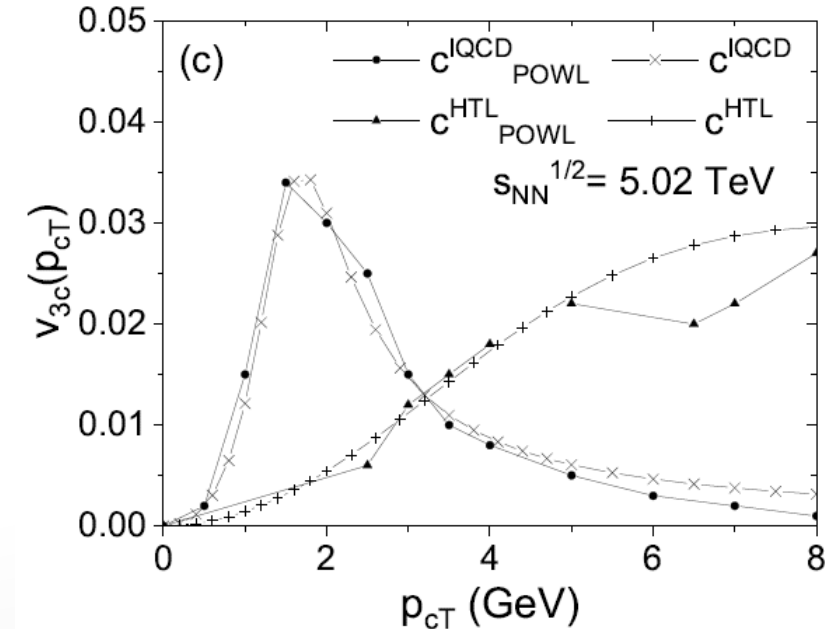
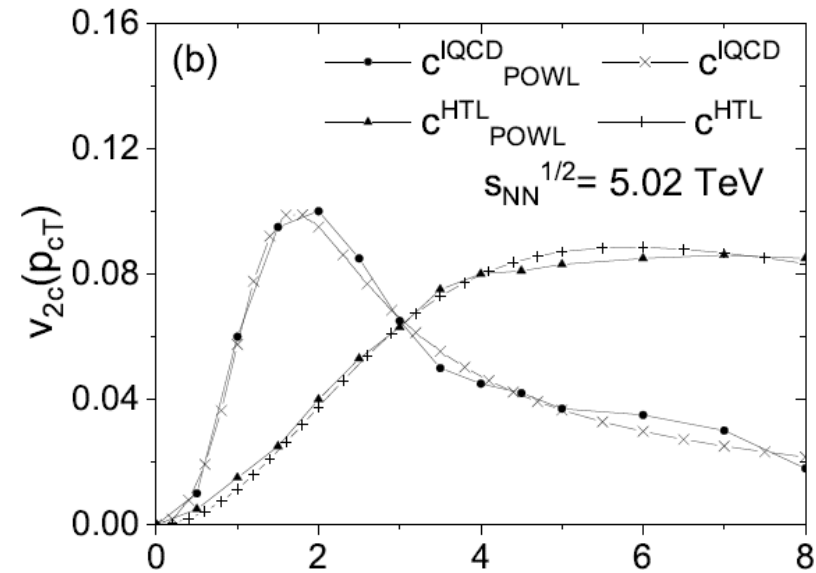
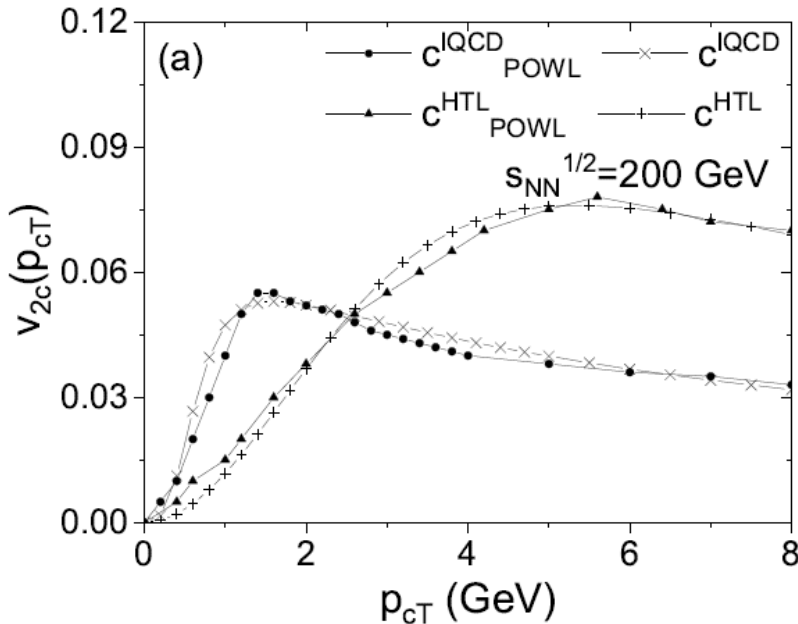
2) Event plane averaged flow harmonics

$$v_n(p_T) = \frac{\frac{n}{2\pi} \int_{-\frac{\pi}{n}}^{\frac{\pi}{n}} \int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2 N}{dp_T^2} d\Psi_n}{\frac{n}{2\pi} \int_{-\frac{\pi}{n}}^{\frac{\pi}{n}} \int d\psi \frac{d^2 N}{dp_T^2} d\Psi_n}.$$



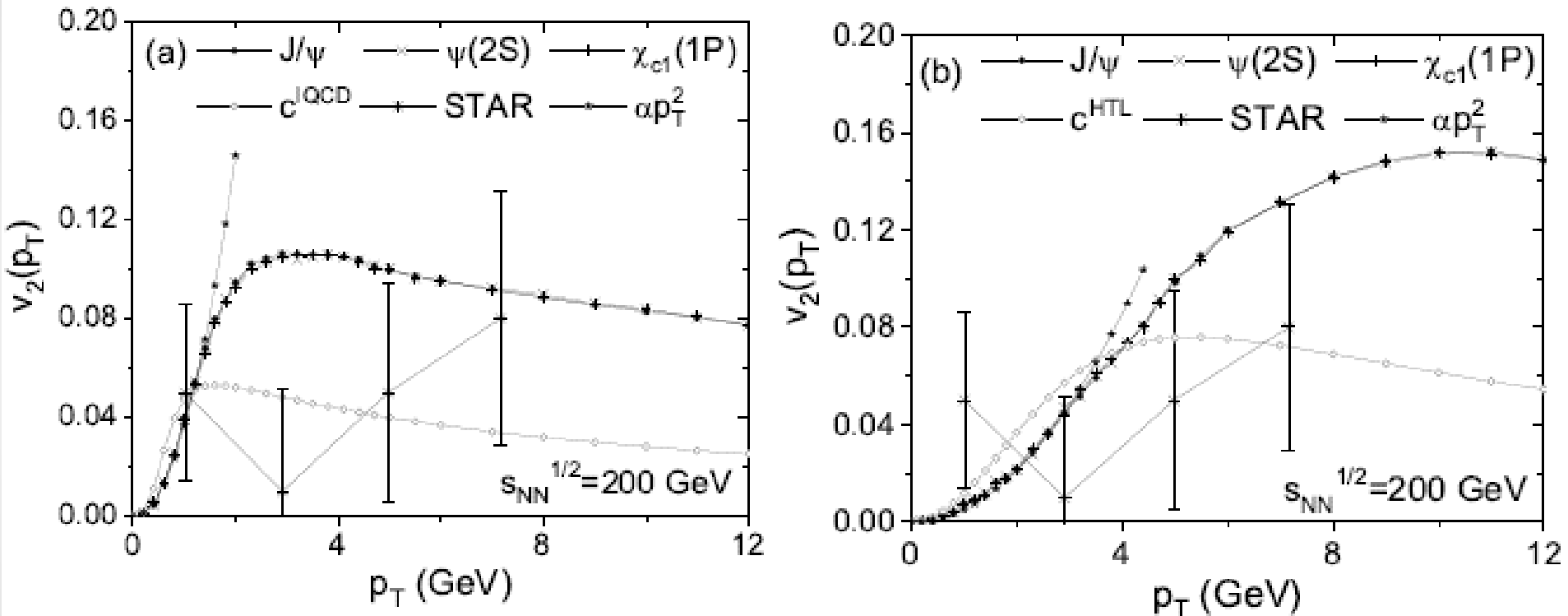
### 3) Charm quark flow harmonics from POWLANG : Pade approximation of charm quark flow harmonics

$$v_{nc}(p_{cT}) = \frac{a_3 p_{cT}^3 + a_2 p_{cT}^2 + a_1 p_{cT}}{b_4 p_{cT}^4 + b_3 p_{cT}^3 + b_2 p_{cT}^2 + b_1 p_{cT} + 1},$$



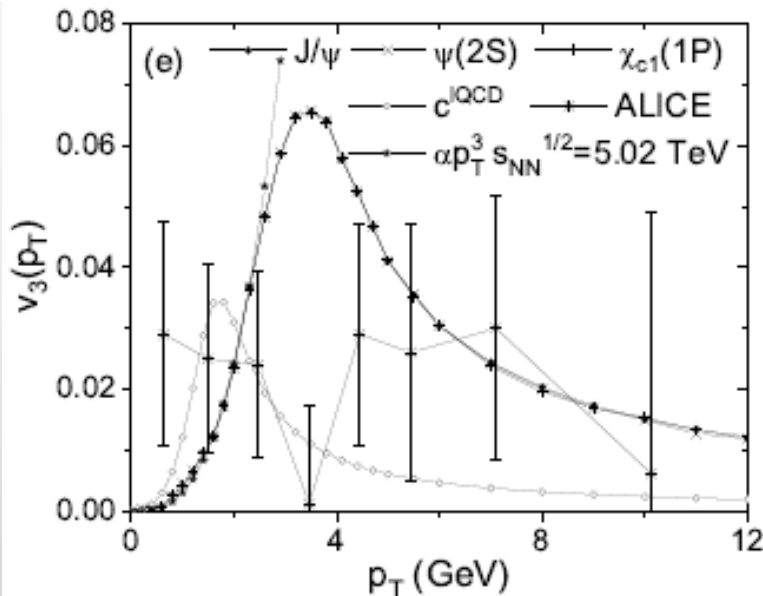
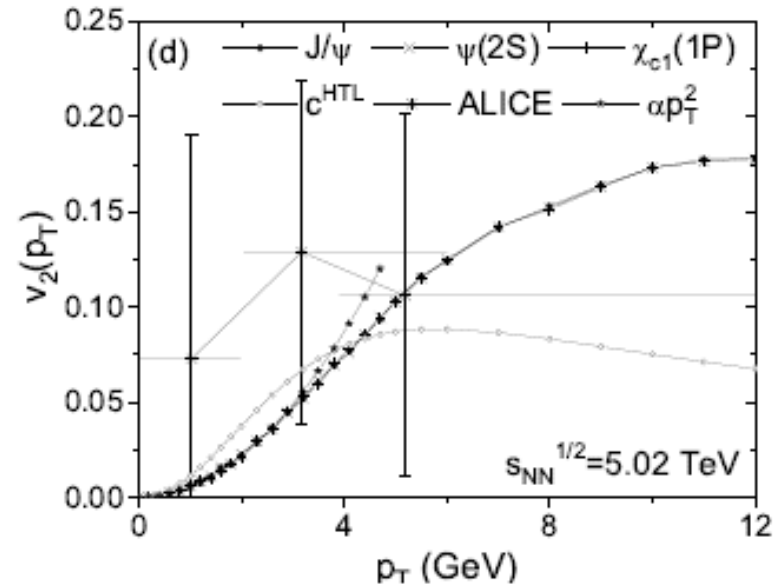
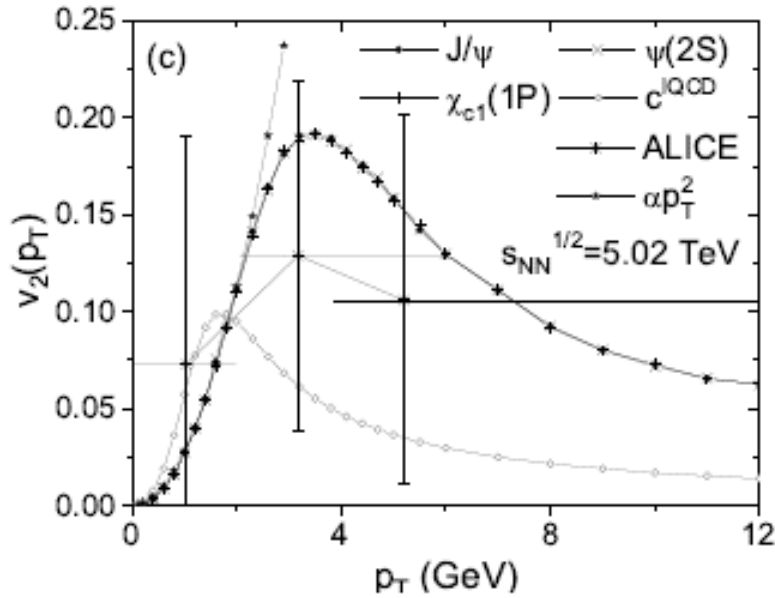
A. Beraudo, A. De Pace, M. Monteno,  
M. Nardi and F. Prino, JHEP **02**, 043 (2018).

# 4) Elliptic flow of charmonium states at RHIC



L. Adamczyk et al. [STAR Collaboration], Phys. Rev. Lett. **111**, no. 5, 052301 (2013).

# 5) Elliptic and triangular flow of charmonium states at LHC



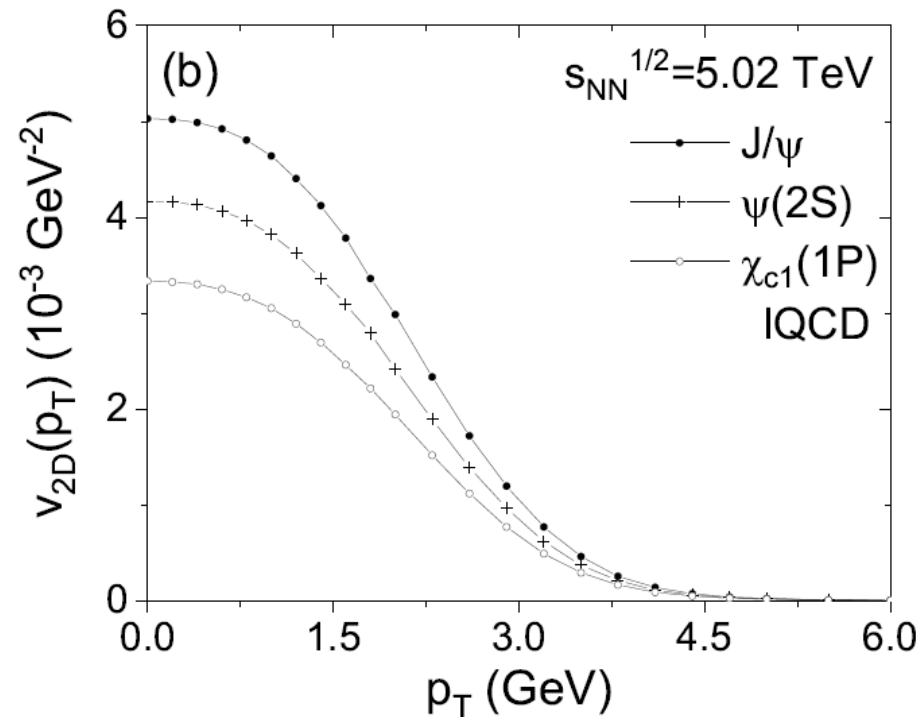
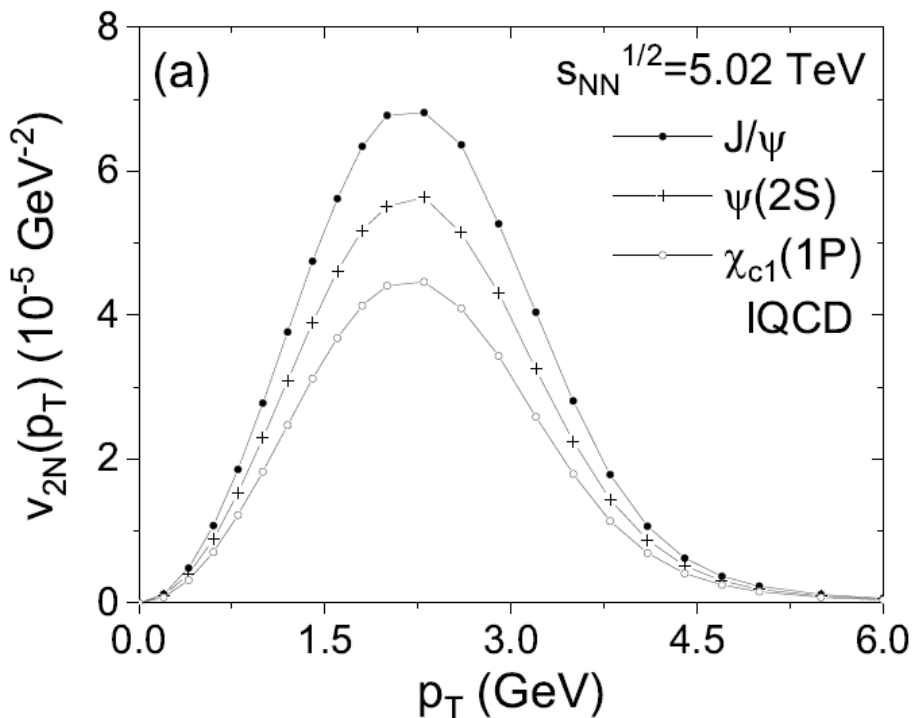
$$v_{n,c\bar{c}}(p_T) \approx 2v_{n,c}(p_T/2)$$

- S. Acharya et al. [ALICE Collaboration], Phys. Rev. Lett. **119**, no. 24, 242301 (2017).
- S. Acharya et al. [ALICE Collaboration], JHEP **2010**, 141 (2020).
- G. Bak [CMS Collaboration], Strangeness in Quark Matter 2022, Busan, June, 13-17

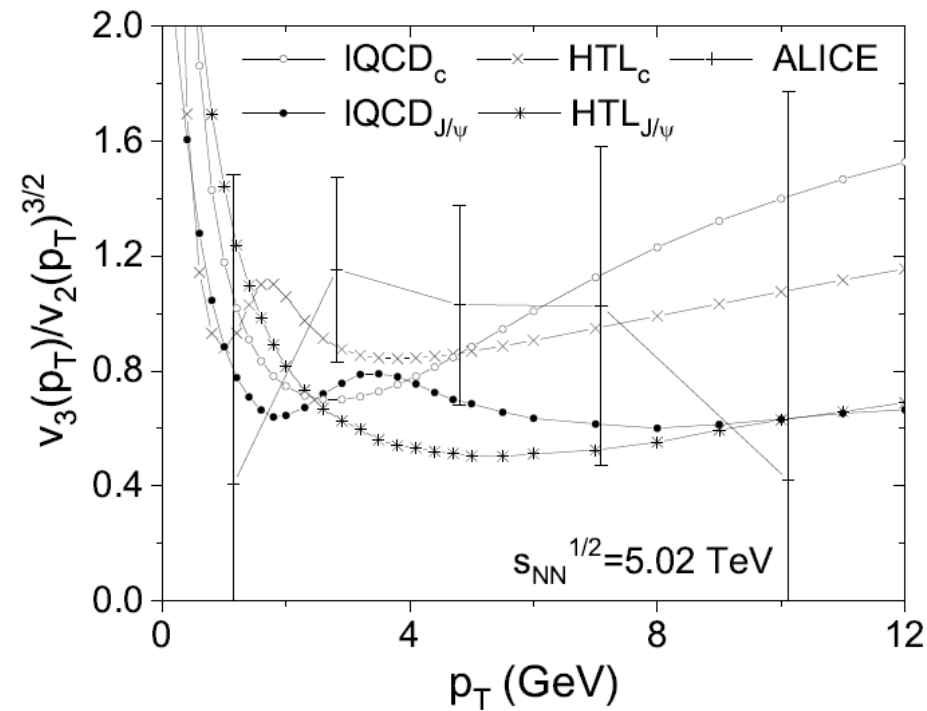
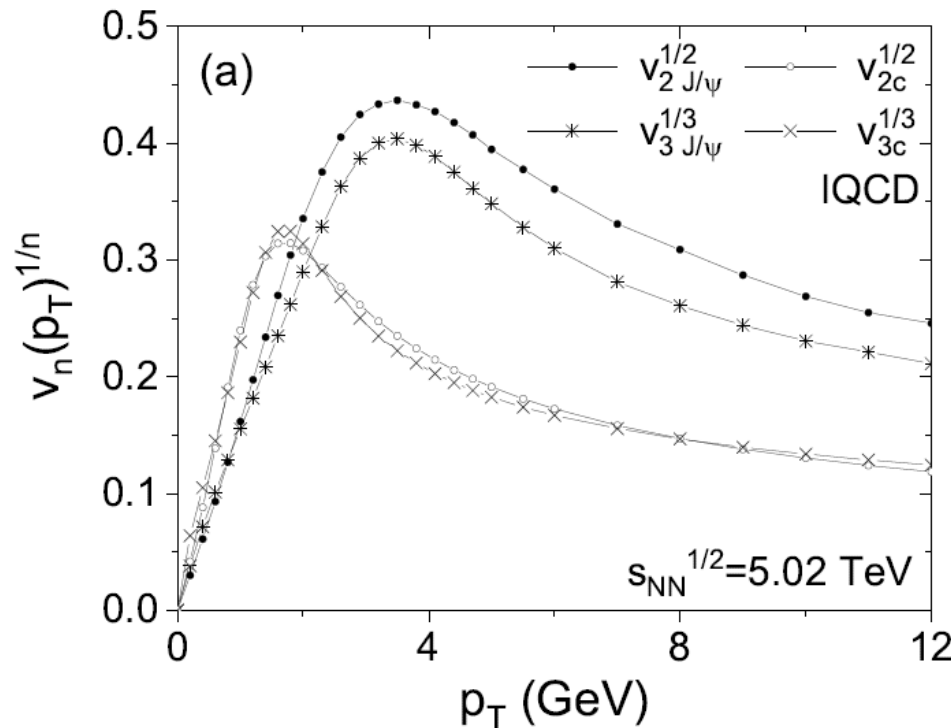
## 6) The dependence of elliptic flow of charmonium states on the internal structure, or the wave function distributions

$$v_{2N}(p_T) = \frac{2}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int d\psi \cos(2(\psi - \Psi_2)) \frac{d^2 N}{dp_T^2} d\Psi_2$$

$$v_{2D}(p_T) = \frac{2}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int d\psi \frac{d^2 N}{dp_T^2} d\Psi_2. \quad (18)$$



# 7) The $v_n(p_T)^{1/n}$ and $v_3(p_T)/v_2(p_T)^{3/2}$ of the $J/\Psi$ meson at LHC



$$v_3/v_2^{2/3} \approx 2/2^{3/2} v_{3,c}(p_T/2)/v_{2,c}^{3/2}(p_T/2),$$

$$2/2^{3/2} \approx 0.71$$

S. Acharya et al. [ALICE Collaboration],  
JHEP **2010**, 141 (2020).

# Conclusion

- Charmonia production in heavy ion collisions
  - 1) The production of heavy quarks hadrons, or charmonium states can be understood in the coalescence model.
  - 2) The transverse momentum distribution and yield are dependent on the internal structure of the hadron
  - 3) The enhanced transverse momentum distribution of  $\psi(2S)$  mesons, compared to that of  $J/\psi$  mesons, is originated from intrinsic wave function distributions between  $\psi(2S)$  and  $J/\psi$  mesons.
  - 4) The elliptic and triangular flow of charmonium states are also affected by wave function distributions of charmonium states.
  - 5) Studying charmonium states in heavy ion collisions will help us to understand many aspects in heavy ion collisions experiments.



Thank you for your attention!